Social Choice and Individual Reports of Subjective Well-Being

Peter J. Hammond: p.j.hammond@warwick.ac.uk

Department of Economics, University of Warwick, Coventry CV4 7AL, UK.

Incomplete version: 2012 January 4th

Abstract

Empirical work has demonstrated that individually reported subjective well-being (SWB) may be positively correlated with objectively measured economic variables like income, and affected by non-economic personal circumstances. Assuming individuals' own SWB reports emerge from random discrete choices, one can therefore use their stochastic properties to construct a broad class of ordinal level comparable interpersonal utility measures. In turn these can be used to determine a welfarist social choice rule. Ethically acceptability, however, seems to require more than level comparability. Beyond individuals' own SWB reports, one could perhaps use data from experiments designed to elicit SWB inequality aversion. [89 words]

Acknowledgements

Generous support during 2007–10 from a Marie Curie Chair funded by the European Commission under contract number MEXC-CT-2006-041121 is gratefully acknowledged. So is funding from CAGE (The Centre for Competitive Advantage in the Global Economy) at the University of Warwick for a research project with the same title being conducted in collaboration with my colleagues Federica Liberini, Eugenio Proto, and Michela Redoano.

1 Introduction

1.1 Arrow and Sen

Six decades have passed since the first edition of Kenneth Arrow's *Social Choice and Individual Values* was published in 1951. For the first two of these there seemed to be no appealing escape from the logic of his impossibility theorem. This powerful result, however, acquires much of its force because an "Arrow social welfare function" ignores all information about any individual i except a preference ordering over the space X of social states that is assumed to represent i's "individual values".

Indeed, prompted by ideas set out in Amartya Sen's *Collective Choice and Social Welfare*, first published in 1970, more satisfactory procedures began to be found some two decades later, and received Arrow's (1977) sympathetic appraisal. The key innovation is that a "Sen social welfare functional" (or SWFL) uses more information about individuals, specifically in the form of interpersonal utility comparisons.

Sen, however, did not initially offer any kind of empirical framework on which to base interpersonal utility comparisons. Moreover, with occasional rare excursions into empirical analysis by authors such as Sen (1976), Jorgenson (1990, 1997) and Slesnick (2001), the ensuing work on social choice with interpersonally comparable measures of real income, has typically been purely theoretical. In fact, by the time Sen's attention had turned to empirical questions, he had largely turned away from "welfarist" social choice (Sen, 1979), with its reliance on the Pareto criterion that he had earlier criticised in connection with the "liberal paradox" (Sen, 1970). Instead, Sen (1985) began to use concepts like "capabilities and functionings" that, along with Dasgupta's (1993) analysis of nutrition, use data that is more clearly objective than any attempt to construct a welfarist measure of real income could ever be.

1.2 Subjective Well-Being

By contrast, largely inspired by Easterlin (1974), extensive empirical research has investigated whether there is a relationship between various self-reported measures of subjective well-being (SWB) — often called "happiness" for short — and the usual estimates of individual economic well-being based on real income. Layard, Mayraz and Nickell (2008) in particular have taken the extra step of using SWB data, along with some very specific value judgements, in order to derive a cardinal interpersonally comparable utility (ICU) function. Their work, however, presumes special functional forms both for the stochastic choice model that is assumed to determine individuals' SWB reports, as well as for the utilitarian social welfare function. Following Atkinson's (1970) approach to inequality measurement, the latter presumes constant relative inequality aversion.

The question that arises, therefore, is whether additional SWB information can and should play a role in social choice — in particular, during the often neglected preliminary stage of constructing an ICU indicator of individual welfare that can serve as the basis for an appropriate SWFL.

1.3 Outline

Section 7 of Fleurbaey and Hammond (2004) sets out a case for utilitarianism — i.e., for maximizing the sum over all existing members of society of an "ethical observer's" concept of individual welfare. Section 2 briefly reviews the argument, especially in a specific context which gives due prominence to the distinction between objectively observable features of the social consequence on the one hand, and individuals' relevant types or characteristics on the other hand.

Generally, we assume that individuals' SWB reports emerge from a multinomial stochastic choice model whose outcome is SWB measured on an $\ell + 1$-point scale (including zero). Section 3 considers the binomial case when $\ell = 1$. The probability of reporting the better outcome then leads naturally to a scalar ICU function of the kind that usually appears as an argument of a Sen SWFL.

When $\ell > 1$, however, one obtains a vector ICU function, with values in the ℓ-dimensional simplex of ℓ-point probability distributions. One important issue is whether this vector ICU function can have each of its values reduced to a single scalar. Whether such a reduction is possible or not, the key question faced in section 4 `[still very incomplete]` is how a generalized SWFL can depend on this multidimensional SWB information.

The concluding section 5 `[still to be written]` raises some other methodological issues, especially the possible use of data from experiments inspired by work such as Yaari and Bar-Hillel (1984) and by Gaertner (1994) intended to elicit subjects' views of how to distribute resources justly in a simple hypothetical allocation problem.

2 Welfarist Social Choice in an Economic Domain

2.1 Social and Personal Consequences

Though social choice theory typically considers a fixed set of individuals, here it is allowed to be variable. Individuals who never come into existence will be treated as potential rather than actual. With this convention, we can consider a society consisting of a finite set N of *potential* individuals i.

The basic task of social choice theory is to specify an appropriate decision at every decision node of every finite decision tree in which each terminal node gives rise to a determinate "social state" or *social consequence*. Rather than the usual entirely abstract set of social choice theory, we shall give the social consequence space a specific Cartesian product structure $Z^N = \prod_{i \in N} Z_i$. Here each Z_i is assumed to be itself a Cartesian product set $\mathbb{R} \times X$, whose members are *personal consequence* pairs consisting of:

1. a scalar measure $y_i \in \mathbb{R}$ of objectively measured real income or wealth;

2. a vector $x_i \in X \subset V$ in a multi-dimensional vector space V, whose different components each measure some relevant personal circumstance, also taken to be objectively measured.

We remark that some subset of the components of x may possibly be affected by policy choices. Also, some other subset may represent possible components of the public environment, which must therefore be the same for all individuals.[1] Indeed, some components of x could also be the recorded prices that are used to construct a price index deflator, which is then used to divide nominal income in order to convert it into real income.

2.2 Social Consequence Lotteries

Some contemporary social choice theorists recognize the need to consider not just social consequences $z^N \in Z^N$, but also simple lotteries $\lambda \in \Delta(Z^N)$ over such consequences. After all, quite apart from the inherent risk in any significant social decision problem, Vickrey (1945) and Harsanyi (1953, 1955) pioneered the idea of representing ethical decisions as those that would be made in a risky "initial position", behind what Rawls (1971) later called a "veil of ignorance", where an individual faces an equal chance of experiencing each individual's personal consequence.

[1] This extends the general framework of an economic domain used by Bordes, Hammond and Le Breton (2005), as well as Le Breton and Weymark (2011).

Following the ideas of Hammond (1998), one should accordingly consider an unrestricted domain of finite decision trees that include both decision and chance nodes; or rather, an "almost unrestricted domain" in which any move at a chance node has a *strictly positive* probability attached. Consider any social decision rule that specifies acceptable behaviour at every decision node in any such tree, including any continuation subtree. It can be shown to maximize a "social" preference ordering $\succsim$ over $\Delta(Z^N)$ that also satisfies the independence axiom if and only if the rule satisfies *consequentialist normal form invariance* — i.e., there is a non-empty valued "revealed" *consequence choice function* $A \mapsto C(A) \subseteq A$ which, for every finite decision tree in the almost unrestricted domain, and for each non-empty finite *feasible set* $A \subset \Delta(Z^N)$ of consequence lotteries that could result from behaviour in the tree, determines a non-empty *choice set* $C(A) \subseteq A$ of possible consequence lotteries that can result from acceptable decisions.

Suppose we add the requirement that, for any collection of finite decision trees which vary only in the positive probabilities attached to different chance moves, the graph of the correspondence mapping probabilities into chosen lotteries should be relatively closed. This additional requirement is sufficient, and also necessary, for the preference ordering $\succsim$ over $\Delta(Z^N)$ to be represented by the expected value of a "social" von Neumann–Morgenstern utility function (NMUF) $Z^N \ni z^N \mapsto w(z^N) \in \mathbb{R}$.

Definition 1. *Two NMUFs $w, \tilde{w} : Z^N \to \mathbb{R}$ are* **cardinally equivalent** *just in case there exist an additive real constant α and a positive multiplicative real constant ρ such that $\tilde{w}(z^N) \equiv \alpha + \rho w(z^N)$.*

As is well known, except in trivial cases, there is a unique cardinal equivalence class of NMUFs whose expectations represent the preference ordering $\succsim$ over $\Delta(Z^N)$.

2.3 Additive Utilitarianism

So far we have considered only the rationality implications of consequentialist normal form invariance. But the next steps of the argument exploit the social nature of the decision being considered, along with the special structure of the social consequence space. Indeed, we argue that consequences matter only because they matter to individuals. Also, by definition, only personal consequences $z_i \in Z_i$ affect individual i directly. Or, when we consider a lottery $\lambda \in \Delta(Z^N)$, all that matters to each individual $i \in N$ is the implied marginal lottery $\lambda_i \in \Delta(Z_i)$ over i's personal consequences. Moreover, suppose we consider a "one-person situation" or individualistic decision tree where, by definition, any decision that is made has no effect at all on the personal consequence lottery $\lambda_j \in \Delta(Z_j)$ of any individual $j \in N \setminus \{i\}$. Then it would seem that the appropriate decision should depend only on i's personal consequence lottery λ_i.

These considerations suggest the following two specific assumptions set out in Fleurbaey and Hammond (2004, pp. 1263–4):

IC (Individualistic Consequentialism) If the two social consequence lotteries $\lambda, \mu \in \Delta(Z^N)$ induce equal marginal personal lotteries $\lambda_i = \mu_i \in \Delta(Z_i)$ for all $i \in N$, then λ and μ are socially equivalent. In particular, they are socially indifferent, so the respective expected values of the social NMUF w must satisfy $\mathbb{E}_\lambda w(z^N) = \mathbb{E}_\mu w(z^N)$.

IW (Individual Welfarism) For each individual $i \in N$, there is a unique cardinal equivalence class of *personal* NMUFs $z_i \mapsto w_i(z_i)$ which, for each fixed profile $\bar{z}^{N\setminus\{i\}} \in \prod_{j\in N\setminus\{i\}}$ of personal consequences for individuals other than i, are all cardinally equivalent to $z_i \mapsto w(z_i, \bar{z}^{N\setminus\{i\}})$.

Definition 2. *Two families $\langle w_i \rangle_{i\in N}, \langle \tilde{w}_i \rangle_{i\in N}$ of personal NMUFs are* **co-cardinally equivalent** *just in case there exist a family $\langle \alpha_i \rangle_{i\in N}$ of additive real constants and a single positive multiplicative real constant ρ, independent of i, such that $\tilde{w}_i(z^N) \equiv \alpha_i + \rho w_i(z^N)$.*

As shown in Fleurbaey and Hammond (2004, pp. 1264–6) by adapting an argument due to Fishburn (1970, p. 176), together these two assumptions imply that, except in trivial cases, there exists a unique co-cardinal equivalence class of unit comparable personal NMUFs $z_i \mapsto w_i(z_i)$ for all $i \in N$ such that the social NMUF $z^N \mapsto w(z^N)$ can be written in the *additive utilitarian* form

$$z^N \mapsto w(z^N) = \sum_{i\in N} w_i(z_i). \tag{1}$$

2.4 Individual Welfare Characteristics

Much of what is relevant in determining an individual's well-being cannot be objectively observed. Accordingly, we assume that each individual $i \in N$ has an unobservable characteristic or *welfare type* t_i which ranges over a domain T that is independent of i. This hidden type will be treated as a parameter of i's personal NMUF.

Moreover, we assume that all individuals of the same type have the same personal NMUF. It follows that each individual i's personal NMUF can be written as $z_i \mapsto v(z_i; t_i)$ for a function v which is independent of i. Furthermore, the social NMUF in (1) takes the form

$$z^N \mapsto w(z^N; t^N) = \sum\nolimits_{i \in N} v(z_i; t_i), \tag{2}$$

whose expected values represent a preference ordering $\succsim_{t^N}$ on $\Delta(Z^N)$ which depends on the *type profile* $t^N = (t_i)_{i \in N}$ as a parameter.

Finally, since the different personal NMUFs $z_i \mapsto w_i(z_i)$ should belong to the same co-cardinal equivalence class for every possible type profile t^N, there must be a unique cardinal equivalence class of *interpersonally comparable utility functions* (ICUFs)

$$Z \times T \ni (z, t) \mapsto v(z, t) \in \mathbb{R}. \tag{3}$$

2.5 Non-Existence

A particular individual welfare type will be *non-existence*, regarded as a *null type* $t^0 \in T$. Of the set N of all potential individuals, only the set $M := \{i \in N \mid t_i \in T \setminus \{t^0\}\}$ are actual individuals who come into existence.

An obvious assumption to make is that all non-existent individuals are entirely unaffected any social decision which has no effect on their existence. Thus, there must be a *critical level of utility* $c \in \mathbb{R}$ (Blackorby, Bossert and Donaldson, 2005) such that for each $i \in N$, one has $v(z_i; t^0) = c$, independent of the (rather meaningless) personal consequence. This critical level divides the set M of actual individuals into two groups, according to whether $v(z_i; t_i) \gtrless c$. The "sub-critical" individuals $i \in N$ for whom $v(z_i; t_i) < c$ are those whose personal consequence z_i is so bad, given their welfare type t_i, that a policy which avoids their coming into existence would, *ceteris paribus*, be regarded as an improvement.

Since the ICUF $v : Z \times T \to \mathbb{R}$ is only unique up to a cardinal equivalence class, an obvious normalization is to put $c = 0$. Then the ICUF $v : Z \times T \to \mathbb{R}$ is unique up to a cardinal ratio scale — i.e., $\tilde{v}$ is equivalent to v just in case there exists a positive multiplicative constant ρ such that $\tilde{v}(z, t) \equiv v(z, t)$. Moreover, the welfare sum (2) reduces to

$$z^N \mapsto w(z^N; t^N) = \sum\nolimits_{i \in M} v(z_i; t_i) = \sum\nolimits_{i \in M} v(y_i, x_i; t_i), \tag{4}$$

where $M \subset N$ is the set of actual individuals. Also, we recall from Section 2.1 that each personal consequence z_i is a pair $(y_i, x_i) \in \mathbb{R} \times X$, where y_i is a measure of real income or wealth, and x_i indicates objectively measurable personal circumstances. Thus, we can write z^N as a pair (y^N, x^N) consisting of a real income distribution $y^N \in \mathbb{R}^N$ combined with a profile $x^N \in X^N$ of objectively measurable personal circumstances.

Of course, the expected value of (4) w.r.t. any collection $\lambda^N \in \Delta(Z^N)$ of marginal personal consequence lotteries also takes the additive form

$$\mathbb{E}_{\lambda^N} w(z^N; t^N) = \sum\nolimits_{i \in M} \mathbb{E}_{\lambda_i}[v(z_i; t_i)]. \tag{5}$$

2.6 Assessment

This section has reviewed the formal case for a social NMUF whose mathematical structure is the "utilitarian" sum of the values of an ICUF, measured on a cardinal ratio scale, over all the actual individuals in the society. This social NMUF, however, depends crucially not only on the objectively measured variables $z^N = (y^N, x^N)$, but also on the hidden welfare type profile t^N. Moreover, the variable welfare type $t \in T$ determines how the ICUF

$$\mathbb{R} \times X \ni (y, z) \mapsto v(y, z; t) \in \mathbb{R} \tag{6}$$

depends on the personal consequence (y, z).

In order to go beyond the mere mathematical structure of utilitarianism, and construct actual measures of well being that can be used for making decisions, one evidently requires two kinds of further judgement:

1. *factual judgements* of the empirical content of the unobservable welfare types;

2. *value judgements* regarding how (6) determines the rations of non-zero values of the ICUF for different combinations $(y, z, t) \in \mathbb{R} \times Z \times T$.

The question that the rest of the paper addresses is the extent to which objective survey data concerning individually reported subjective well-being (SWB) can help inform such judgements.

3 Binary Subjective Well Being

3.1 The Binomial Choice Model

Consider first the case where there is only a binary measure of SWB. This occurs when, for example, surveyed individuals can only express either general satisfaction, or general dissatisfaction, with their life circumstances. For each fixed welfare type $t \in T$, and given any personal consequence $z = (y, x)$, let $p(z,t) \in [0,1]$ denote the proportion of type t individuals facing z who are willing to express satisfaction as against dissatisfaction.

A natural hypothesis then is that, for each fixed t, the proportion $p(z,t)$ increases as the welfare level $v(z;t)$ increases. This implies that, for each welfare type $t \in T$, the function $Z \ni z \mapsto p(z,t) \in [0,1]$ can be used in principle as an ordinal utility function. Moreover, for each type $t \in T$, there must be a strictly increasing transformation $\mathbb{R} \ni \xi \mapsto \psi_t(\xi) \in \mathbb{R}$ such that $v(z;t) = \psi_t(p(z,t))$. It follows that the expectation of the additive NMUF (5) takes the form

$$\mathbb{E}_{\lambda^N}[w(z^N;t^N)] = \sum\nolimits_{i \in M} \mathbb{E}_{\lambda_i}\left[\psi_{t_i}(p(z,t_i))\right]. \tag{7}$$

From now on, let Ψ denote the class of all such transformations $\psi : T \times \mathbb{R} \to \mathbb{R}$ with the property that $\xi \mapsto \psi_t(\xi)$ is strictly increasing for each welfare type $t \in T$.

3.2 Stochastic Pareto Dominance

Of course, the information one could infer from such survey results is very limited. After all, it is formally equivalent to the situation in Arrow (1951), where each welfare type $t \in T$ has as "individual values" its own preference ordering on Z that corresponds to the ordinal utility function $z \mapsto p(z,t)$. There is no further information, however, about how to construct the ICUF $v(z;t)$, or about how to make choices in general social decision trees. There is therefore little surprise in the following results concerning the preference ordering $\succsim_{t^N}$ that the expected value of the NMUF (7) induces on $\Delta(Z^N)$.

Definition 3. *Given any welfare type $t \in T$ and personal consequence lottery $\lambda \in \Delta(Z)$, define the associated* **cumulative distribution function**

$$[0,1] \ni \xi \mapsto F(\xi;t,\lambda) := \lambda(\{z \in Z \mid p(z,t) \leq \xi\}) \in [0,1] \tag{8}$$

as the probability that the random personal consequence z induces a probability of expressing satisfaction that does not exceed ξ.

Definition 4. *Given any fixed welfare type $t \in T$ and any two personal consequence lotteries $\lambda, \mu \in \Delta(Z)$:*

1. *say that λ* **weakly stochastically dominates** *μ, and write $\lambda \succsim_t^{SD} \mu$, provided that $F(\xi;t,\lambda) \leq F(\xi;t,\mu)$ for all $\xi \in [0,1]$;*

2. *say that λ* **stochastically dominates** *μ, and write $\lambda \succ_t^{SD} \mu$, provided that $\lambda \succsim_t^{SD} \mu$ but not $\mu \succsim_t^{SD} \lambda$;*

3. *say that* λ *and* μ *are* **stochastically equivalent**, *and write* $\lambda \sim_t^{SD} \mu$, *provided that both* $\lambda \succsim_t^{SD} \mu$, *and* $\mu \succsim_t^{SD} \lambda$ *— i.e.,* $F(\xi; t, \lambda) = F(\xi; t, \mu)$ *for all* $\xi \in [0, 1]$.

Definition 5. *Given any welfare type profile* $t^N \in T^N$ *and any two social consequence lotteries* $\lambda^N, \mu^N \in \Delta(Z^N)$, *say that* λ^N **stochastically Pareto dominates** μ^N, *and write* $\lambda^N \succ_{t^N}^{SPD} \mu^N$, *just in case the respective marginal lotteries satisfy* $\lambda_i \succsim_{t_i}^{SPD} \mu_i$ *for all* $i \in N$, *with* $\lambda_i \succ_{t_i}^{SPD} \mu_i$ *for at least one* $i \in N$.

Proposition 1. *Consider any welfare type profile* $t^N \in T^N$ *and, for each transformation* $\psi \in \Psi$, *the corresponding preference ordering* $\succsim_{t^N}^{\psi}$ *on* $\Delta(Z^N)$ *represented by the expected value of the social NMUF (7). For any pair of social consequence lotteries* $\lambda^N, \mu^N \in \Delta(Z^N)$, *the following two statements are equivalent:*

ISP *(invariant strict preference)* $\lambda^N \succ_{t^N}^{\psi} \mu^N$ *for all possible transformations* $\psi \in \Psi$;

SPD *(stochastic Pareto dominance)* $\lambda^N \succ_{t^N}^{SD} \mu^N$.

Proof. Recall that the set M is finite, and the lotteries $\lambda^N, \mu^N \in \Delta(Z^N)$ both attach positive probability to only a finite set of social consequences $z^N \in Z^N$. For each $i \in M$, therefore, both λ_i and μ_i attach positive probability to only a finite set of personal consequences $z_i \in Z_i$. In particular, individual i's contribution S_i to the difference $\mathbb{E}_{\lambda^N} \tilde{w}(z^N; t^N) - \mathbb{E}_{\mu^N} \tilde{w}(z^N; t^N)$ in the values of (7) for λ^N and μ^N takes the form of a sum

$$S_i := \sum\nolimits_{z \in Z} d_i(z) \psi_{t_i}(p(z, t_i)) \tag{9}$$

where $d_i(z) := \lambda_i(z) - \mu_i(z)$ for each $z \in Z$, which is non-zero for only a finite set $H_i \subseteq Z$. The range of values $R_i := \{\psi_{t_i}(p(z, t_i)) \mid z \in H_i\}$ can therefore be written as the finite set $R_i = \{\psi_i^r \mid r \in K_i\} \subset \mathbb{R}$, where $K_i := \{1, 2, \dots, k_i\}$ and $\psi_i^1 < \psi_i^2 < \dots < \psi_i^{k_i}$. For each rank $r \in K_i$, let $Z_i^r := \{z \in Z \mid \psi_{t_i}(p(z, t_i)) = \psi_i^r\}$ and $d_i^r := \sum_{z \in Z^r} d_i(z)$. Also, define the increments $\alpha_i^r := \psi_i^r - \psi_i^{r-1} > 0$ for all $r \in K_i$, where ψ_i^0 is an arbitrary real number less than ψ_i^1. Finally, define $D_i^s := \sum_{r=s}^{k_i} d_i^r$ for all $s \in K_i$, which is the sum of the probability differences, cumulated downwards. With all these definitions, we note first that

$$D_i^s = [1 - F(\psi_i^s; t_i, \lambda)] - [1 - F(\psi_i^s; t_i, \mu)] = F(\psi_i^s; t_i, \mu) - F(\psi_i^s; t_i, \lambda), \tag{10}$$

the reverse difference in the cumulative distribution functions. Also, because $\sum_{r \in K_i} d_i^r = 0$, one has

$$S_i = \sum_{r \in K_i} d_i^r \psi_i^r = \sum_{r \in K_i} d_i^r \left(\psi_i^0 + \sum_{s=1}^r \alpha_i^s \right) = \sum_{s \in K_i} \alpha_i^s D_i^s. \quad (11)$$

Now, condition (ISP) in the proposition is evidently equivalent to the statement that $\sum_{i \in M} S_i > 0$ for every possible collection $\{\alpha_i^s \mid i \in M; s \in K_i\}$ of positive constants. But (10) implies that this is in turn equivalent to every number in the collection $\{D_i^s \mid i \in M; s \in K_i\}$ being non-negative, with at least one positive. Finally, this is equivalent to condition (SPD) in the proposition. □

3.3 Welfare Optimality without Interpersonal Comparisons

The family Ψ of possible transformations, like Arrow's original framework of individual preference profiles represented by individually ordinal utility functions, excludes interpersonal comparisons. So does the associated concept of possible welfare optimality that we now introduce.

Definition 6. *Given any welfare type profile $t^N \in T^N$ and any allowable transformation $\psi \in \Psi$, define a corresponding* **welfare optimal** *choice function $A \mapsto C(A; t^N, \psi) \subseteq A$ on the domain of non-empty feasible alternative sets $A \subset \Delta(Z^N)$ by*

$$C(A; t^N, \psi) := \arg\max_{\lambda^N} \left\{ \sum_{i \in M} \mathbb{E}_{\lambda_i} [\psi_{t_i}(p(z, t_i))] \mid \lambda^N \in A \right\}. \quad (12)$$

Any λ^N which belongs to the union $\cup_{\psi \in \Psi} C(A; t^N, \psi)$ of all the choice sets $C(A; t^N, \psi)$ is then said to be a **possible welfare optimum***; any $\lambda^N \in A$ such that $\lambda^N \notin C(A; t^N, \psi)$ for all $\psi \in \Psi$ is said to be an* **impossible welfare optimum***.*

Corollary 1. *Given any welfare type profile $t^N \in T^N$ and any non-empty finite feasible alternative set $A \subset \Delta(Z^N)$ of social consequence lotteries, the lottery $\hat{\lambda}^N \in A$ is a possible social optimum if and only if there is no $\lambda^N \in A$ that stochastically Pareto dominates $\hat{\lambda}^N$; otherwise it is an impossible social optimum.*

Proof. It is enough to note that, because of Proposition 1, the inequality

$$\sum_{i \in M} \mathbb{E}_{\lambda_i} [\psi_{t_i}(p(z, t_i))] \geq \sum_{i \in M} \mathbb{E}_{\mu_i} [\psi_{t_i}(p(z, t_i))]$$

holds simultaneously for all $\psi \in \Psi$ if and only if $\lambda^N \succsim_{t^N}^{SD} \mu^N$. Also, in case $\lambda^N \succsim_{t^N}^{SD} \mu^N$, one also has

$$\sum\nolimits_{i \in M} \mathbb{E}_{\lambda_i} \left[\psi_{t_i}(p(z, t_i)) \right] > \sum\nolimits_{i \in M} \mathbb{E}_{\mu_i} \left[\psi_{t_i}(p(z, t_i)) \right]$$

for at least one $\psi \in \Psi$ provided that $\lambda^N \succ_{t^N}^{SD} \mu^N$. □

Corollary 2. *Suppose only degenerate lotteries are feasible — i.e.,* $A \subseteq Z^N$*. Then the particular social consequence* $\hat{z} \in A$ *is a possible social optimum if and only if there is no* $z^N \in A$ *that Pareto dominates* $\hat{z}^N$*; otherwise it is an impossible social optimum.*

3.4 Stochastic Suppes Dominance

So far we have simply assumed that, for each *fixed* welfare type t, the proportion $p(z, t)$ of individuals who express satisfaction with the personal consequence $z \in Z$ increases as the welfare level $v(z; t)$ increases. A stronger value judgement, however, is that even as t also varies, higher values of $p(z, t)$ still indicate that personal utility $v(z; t)$ is higher. This implies that the function $Z \times T \ni (z, t) \mapsto p(z, t) \in [0, 1]$ can be used in principle as an *ordinal level comparable* utility function. Moreover, there must be *just one* strictly increasing transformation $\mathbb{R} \ni \xi \mapsto \phi(\xi) \in \mathbb{R}$ such that $v(z; t) = \phi(p(z, t))$. Now the expectation of the additive NMUF (5) takes the form

$$\mathbb{E}_{\lambda^N}[\tilde{w}(z^N; t^N)] = \sum\nolimits_{i \in M} \mathbb{E}_{\lambda_i} \left[\phi(p(z, t_i)) \right]. \tag{13}$$

Let Φ denote the class of all strictly increasing transformations $\phi : \mathbb{R} \to \mathbb{R}$.

Definition 7. *Given any welfare type profile* $t^N \in T^N$*, with* $m := \#M$ *as the number of actual individuals, and any social consequence lottery* $\lambda^N \in \Delta(Z^N)$*, define the associated* **interpersonal cumulative distribution function**

$$[0, 1] \ni \xi \mapsto G(\xi; t^N, \lambda^N) := \frac{1}{m} \sum_{i \in M} \lambda_i(\{z \in Z \mid p(z, t_i) \leq \xi\}) \in [0, 1] \tag{14}$$

as the expected proportion of existing individuals whose random personal consequence z *induces a probability of expressing satisfaction that does not exceed* ξ*.*

The following dominance condition builds on a concept due to Suppes (1966) that Sen (1970) introduced to social choice theory.

Definition 8. *Given any welfare type profile $t^N \in T^N$ and any fixed pair of social consequence lotteries $\lambda^N, \mu^N \in \Delta(Z^N)$, say that λ^N* **stochastically Suppes dominates** *μ^N, and write $\lambda^N \succ^{SSD}_{t^N} \mu^N$, just in case λ^N and μ^N induce respective interpersonal cumulative distribution functions that satisfy:*

1. *$G(\xi; t^N, \lambda^N) \leq G(\xi; t^N, \mu^N)$ for all $\xi \in [0, 1]$;*
2. *$G(\xi; t^N, \lambda^N) < G(\xi; t^N, \mu^N)$ for at least one $\xi \in [0, 1]$.*

Proposition 2. *Consider any welfare type profile $t^N \in T^N$ and, for any strictly increasing transformation $\phi \in \Phi$, the corresponding preference ordering $\succsim^{\phi}_{t^N}$ on $\Delta(Z^N)$ represented by the expected value of the social NMUF (13). For any pair of social consequence lotteries $\lambda^N, \mu^N \in \Delta(Z^N)$, the following two statements are equivalent:*

ISP2 *(invariant strict preference) $\lambda^N \succ^{\phi}_{t^N} \mu^N$ for all possible transformations $\phi \in \Phi$;*

SSD *(stochastic Suppes dominance) $\lambda^N \succ^{SSD}_{t^N} \mu^N$.*

Proof. This is really a special case of Prop. 1, where all existing individuals are ex ante identical, as in the Vickrey–Harsanyi original position. So all individuals share the same (dummy) type, and there is no difference between the two sets Ψ and Φ of allowable transformations. Furthermore, in this original position, each individual i's actual type t_i is regarded as emerging from the common personal consequence lottery $\tilde{\lambda}^N \in \Delta(Z \times T)$ which takes the form $\frac{1}{m} \sum_{i \in M} \tilde{\lambda}_i$, where $\tilde{\lambda}_i(z, t) = \lambda_i(z, t_i)$ if $t = t_i$, but $\tilde{\lambda}_i(z, t) = 0$ otherwise. □

3.5 Welfare Optimality with Ordinal Level Comparisons

The family Φ of possible transformations allows ordinal level interpersonal comparisons. So does the associated concept of possible welfare optimality that we move to next.

Definition 9. *Given any welfare type profile $t^N \in T^N$ and any allowable transformation $\phi \in \Phi$, define a corresponding* **welfare optimal** *choice function $A \mapsto C(A; t^N, \psi) \subseteq A$ on the domain of non-empty feasible alternative sets $A \subset \Delta(Z^N)$ by*

$$C(A; t^N, \phi) := \arg\max_{\lambda^N} \left\{ \sum\nolimits_{i \in M} \mathbb{E}_{\lambda_i} \left[\phi(p(z, t_i)) \right] \mid \lambda^N \in A \right\}. \qquad (15)$$

Any λ^N which belongs to the union $\cup_{\psi \in \Psi} C(A; t^N, \phi)$ of all the choice sets $C(A; t^N, \phi)$ is then said to be a **possible welfare optimum***; any $\lambda^N \in A$ such that $\lambda^N \notin C(A; t^N, \phi)$ for all $\phi \in \Phi$ is said to be an* **impossible welfare optimum***.*

Corollary 3. *Given any welfare type profile $t^N \in T^N$ and any non-empty finite feasible alternative set $A \subset \Delta(Z^N)$ of social consequence lotteries, the lottery $\hat{\lambda}^N \in A$ is a possible welfare optimum if and only if there is no $\lambda^N \in A$ that stochastically Suppes dominates $\hat{\lambda}^N$; otherwise it is an impossible welfare optimum.*

4 General Subjective Well Being

4.1 The Ordered Multinomial Choice Model

The basic stochastic framework we are using concerns the self-reported life satisfaction (SRLS) or *subjective well being* (SWB) level s. This typically belongs to a linearly ordered finite set S of possible values, which is usually treated as a finite set $S = \{s \in Z \mid \underline{s} \leq s \leq \bar{s}\}$ of successive non-negative integers. We introduce the harmless normalizations $\underline{s} = 0$ and $\bar{s} = \ell := \#S - 1$ whose effect is to convert S to the set $\{0, 1, 2, \dots, \ell\}$.

For each of the different levels of satisfaction $s \in S$, let $p(s|z,t)$ denote the conditional probability that an individual of welfare type $t \in T$ facing personal consequence $z \in Z$ reports that their SWB is at level s. Our analysis will focus on the *downwardly cumulated* conditional probabilities

$$P(s|z,t) := \sum\nolimits_{s'=s}^{\ell} p(s'|z,t) \tag{16}$$

that the individual reports an SWB level no lower than s. Of course, in the binomial case when $\ell = 1$, one has $P(0|z,t) = 1$ and $P(1|z,t) = p(z,t)$ in the notation of Section 3.

The "natural hypothesis" set out in Section 3.1 can now be applied. It turns into the postulate that, for each fixed welfare type $t \in T$ and SWB level s, the conditional proportion $P(s|z,t)$ increases as the welfare level $v(z;t)$ increases. This implies that, for each type $t \in T$, any of the ℓ different functions $Z \ni z \mapsto P(s|z,t) \in [0,1]$ as s varies from 1 to ℓ can be used in principle as an ordinal utility function. Moreover, for each welfare type $t \in T$ and SWB level $s \geq 1$, there must be a strictly increasing transformation $\mathbb{R} \ni \xi \mapsto \psi_t^s(\xi) \in \mathbb{R}$ such that

$$v(z;t) = \psi_t^s(P(s|z,t)) \tag{17}$$

If the different welfare types $t \in T$ could be empirically distinguished, however, this postulate could become falsifiable when $\ell > 1$. The reason is that (17) allows one to derive the logical equivalence

$$P(s|z,t) = P(s|z',t) \iff P(\tilde{s}|z,t) = P(\tilde{s}|z',t) \tag{18}$$

for all $s, \tilde{s} \in \{1, 2, \ldots, \ell\}$, all $t \in T$, and all $z, z' \in Z$. Indeed, for each $t \in T$ and $s \in \{1, 2, \ldots, \ell\}$, each set of points $z \in Z$ satisfying $P(s|z,t) = \text{constant}$ must be an indifference curve of the utility function $z \mapsto v(z;t)$.

If condition (18) is met, however, then all the results of Section 3 hold separately for each SWB level s except the lowest. In particular, the expectation of the additive NMUF (5) takes the form

$$\mathbb{E}_{\lambda^N}[w(z^N;t^N)] = \sum\nolimits_{i \in M} \mathbb{E}_{\lambda_i}\left[\psi^s_{t_i}(P(s|z,t_i))\right], \tag{19}$$

which will be independent of s.

4.2 Ordinal Random Utility

From now on we focus on a version of the "simpler form" that, as Amemiya (1985, Section 9.3.2) for instance points out, features in most applications. It includes both ordered probit and ordered logit as special cases. First, we introduce the following definition:

Definition 10. *A* **downwards cumulated distribution function** *(or DCDF) is a strictly decreasing mapping $\mathbb{R} \ni \xi \mapsto H(\xi) \in [0,1]$ for which $H(\xi) \to 0$ as $\xi \to \infty$ and $H(\xi) \to 1$ as $\xi \to -\infty$.*

Definition 11. *For each welfare type $t \in T$, the mapping $Z \ni z \mapsto u_t(z)$ is a* **random utility** *function if there exist both a DCDF $\xi \mapsto H_t(\xi)$ and a strictly increasing sequence of constants $(\xi^s_t)^\ell_{s=1}$ with the property that $P(s|z,t) = H_t(\xi^s_t - u_t(z))$ for all SWB levels $s \in S \setminus \{0\}$ and all $z \in Z$.*

Proposition 3. *If there is a random utility function $z \mapsto u_t(z)$ for the welfare type t, it is ordinally equivalent to type t's welfare NMUF $z \mapsto v(z,t)$.*

Proof. If $P(s|z,t) = H_t(\xi^s_t - u_t(z))$, then

$$\begin{aligned} P(s|z,t) = P(s|z',t) &\iff H_t(\xi^s_t - u_t(z)) = H_t(\xi^s_t - u_t(z')) \\ &\iff u_t(z) = u_t(z') \end{aligned} \tag{20}$$

where the last equivalence holds because H_t is strictly decreasing. Moreover, $P(s|z,t)$ must increase w.r.t. z as $u_t(z)$ increases. That is, $P(s|z,t)$ and $u_t(z)$ must be ordinally equivalent as functions of z. But (17) implies that $z \mapsto P(s|z,t)$ and $z \mapsto v(z,t)$ are ordinally equivalent, so the result follows. $\square$

So if a random utility function $Z \ni z \mapsto u_t(z)$ exists, it can replace the function $z \ni z \mapsto p(z,t)$ of Section 3. Indeed, all the results of that section really rely on there being a scalar objective measure of SWB.

4.3 Interpersonal Comparisons of Utility Levels

Next, we turn to the value judgement of Section 3.4. In the current context this entails that, even as t also varies, higher values of $P(s|z,t)$ indicate that personal utility $v(z;t)$ is higher. Condition (17) is replaced by the requirement that, for each $s \in S \setminus \{0\}$, there must exist a strictly increasing function $\mathbb{R} \ni \xi \mapsto \psi^s(\xi) \in \mathbb{R}$ such that

$$v(z;t) = \phi^s(P(s|z,t)) \tag{21}$$

This implies that, for any $s \in S \setminus \{0\}$, the function $Z \times T \ni (z,t) \mapsto P(s|z,t) \in [0,1]$ can be used as an ordinal level comparable utility function.

As in Section 4.1, however, the postulate that (21) holds has substantive content. Indeed, the earlier logical equivalence (18), which must hold for each fixed $t \in T$ separately, now becomes

$$P(s|z,t) = P(s|z',t') \Longleftrightarrow P(\tilde{s}|z,t) = P(\tilde{s}|z',t') \tag{22}$$

for all $s, \tilde{s} \in S \setminus \{0\}$, which must hold even as t varies along with z.

5 Conclusions

[Still to be written.]

References

Amemiya, Takeshi (1985) *Advanced Econometrics* (Cambridge, MA: Harvard University Press)

Arrow, Kenneth J. (1951; 2nd edn. 1963) *Social Choice and Individual Values* (New York: John Wiley)

Arrow, Kenneth J. (1977) "Extended Sympathy and the Possibility of Social Choice" *American Economic Review: Papers and Proceedings* 67: 219–225.

Atkinson, Anthony B. (1970) "On the Measurement of Inequality" *Journal of Economic Theory* 2: 244–263.

Blackorby, Charles, Walter Bossert, and David Donaldson (2005) *Population Issues in Social Choice Theory, Welfare Economics and Ethics* (Cambridge: Cambridge University Press).

Bordes, Georges, Peter J. Hammond and Michel Le Breton (2005) "Social Welfare Functionals on Restricted Domains and in Economic Environments" *Journal of Public Economic Theory* 7: 1–25.

Brown, Donald J. (1975) "Aggregation of Preferences" *Quarterly Journal of Economics* 89: 456–69.

Dasgupta, Partha (1993) *An Inquiry into Well-Being and Destitution* (New York: Oxford University Press).

Easterlin, Richard A. (1974) "Does Economic Growth Improve the Human Lot? Some Empirical Evidence" in Paul A. David and Melvin W. Reder (eds.) *Nations and Households in Economic Growth: Essays in Honor of Moses Abramovitz* (New York: Academic Press).

Fishburn, Peter C. (1970) *Utility Theory for Decision Making* (New York: John Wiley).

Fleurbaey, Marc, and Peter J. Hammond (2004) "Interpersonally Comparable Utility" in Salvador Barberà, Peter J. Hammond and Christian Seidl (eds.) *Handbook of Utility Theory, Vol. 2: Extensions* (Boston: Kluwer Academic Publishers), ch. 21, pp. 1181–1285.

Gaertner, Wulf (1994) "Distributive Justice: Theoretical Foundations and Empirical Findings" *European Economic Review* 38: 711–720.

Gaertner, Wulf, and Marlies Klemisch-Ahlert (1992) *Social Choice and Bargaining Perspectives on Distributive Justice* (Berlin: Springer-Verlag).

Hammond, Peter J. (1976) "Equity, Arrow's Conditions, and Rawls' Difference Principle" *Econometrica* 44: 793–804.

Hammond, Peter J. (1977a) "Dynamic Restrictions on Metastatic Choice" *Economica* 44: 337–350.

Hammond, Peter J. (1977b) "Dual Interpersonal Comparisons of Utility and the Welfare Economics of Income Distribution" *Journal of Public Economics* 7: 51–71.

Hammond, Peter J. (1979) "Equity in Two-Person Situations: Some Consequences" *Econometrica* 47: 1127–1135.

Hammond, Peter J. (1998) "Objective Expected Utility: A Consequentialist Perspective" in Salvador Barberà, Peter J. Hammond and Christian Seidl (eds.) *Handbook of Utility Theory, Vol. 1: Principles* (Boston: Kluwer Academic Publishers), ch. 5, pp. 145–211.

Hammond, Peter J., Federica Liberini and Eugenio Proto (2011) "Individual Welfare and Subjective Well-Being: Commentary Inspired by Sacks, Stevenson and Wolfers" The Warwick Economics Research Paper Series (TWERPS) 957, University of Warwick, Department of Economics.

Harsanyi, John C. (1953) "Cardinal Utility in Welfare Economics and in the Theory of Risk-Taking" *Journal of Political Economy* 61: 434–435.

Harsanyi, John C. (1955) "Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparison of Utility" *Journal of Political Economy* 63: 309–321.

Jorgenson, Dale W. (1990) "Aggregate Consumer Behavior and the Measurement of Social Welfare" *Econometrica*, **58**, 1007–1040.

Jorgenson, Dale W. (1997) *Welfare Volume 2: Measuring Social Welfare* (Cambridge, Mass.: MIT Press).

Klemisch-Ahlert, Marlies (1996) *Bargaining in Economic and Ethical Environments: An Experimental Study and Normative Solution Concepts* (Berlin: Springer-Verlag).

Layard, Richard, Guy Mayraz, and Stephen Nickell (2008) "The Marginal Utility of Income" *Journal of Public Economics* 92: 1846–1857.

Le Breton, Michel and John A. Weymark (2011) "Arrovian Social Choice Theory on Economic Domains" in K.J. Arrow, A.K. Sen and K. Suzumura (eds.) *Handbook of Social Choice and Welfare, Vol 2*, ch. 17, pp. 191–299.

Nussbaum, Martha C. and Amartya Sen, eds. (1993) *The Quality of Life* (Oxford: Clarendon Press).

Rawls, John (1971, revised 1999) *A Theory of Justice* (Cambridge, MA: Harvard University Press).

Sen, Amartya K. (1970, 1984) *Collective Choice and Social Welfare* (San Francisco: Holden Day, and Amsterdam: North-Holland).

Sen, Amartya K. (1970) "The Impossibility of a Paretian Liberal" *Journal of Political Economy* 78: 152–157.

Sen, Amartya K. (1976) "Real National Income" *Review of Economic Studies* 43: 19–39.

Sen, Amartya K. (1979) "Personal Utilities and Public Judgements: Or What's Wrong With Welfare Economics" *Economic Journal* 89: 537–558.

Sen, Amartya K. (1985) *Commodities and Capabilities* (Oxford: Oxford University Press).

Slesnick, Daniel T. (2001) *Consumption and Social Welfare: Living Standards and Their Distribution in the United States* (Cambridge: Cambridge University Press).

Suppes, Patrick C. (1966) “Some Formal Models of Grading Principles” *Synthese* 16: 284–306.

Vickrey, William S. (1945) “Measuring Marginal Utility by Reactions to Risk” *Econometrica* 13: 319–333.

Weymark, John A. (1984) “Arrow’s Theorem with Social Quasi-Orderings” *Public Choice* 42: 235–246.

Yaari, Menahem E., and Maya Bar-Hillel (1984) “On Dividing Justly” *Social Choice and Welfare* 1: 1–24.