Rhetoric in Legislative Bargaining with Asymmetric Information[1]

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Abstract

We analyze a three-player legislative bargaining game over an ideological and a distributive decision. Legislators are privately informed about their ideological intensities, i.e., the weight placed on the ideological decision relative to the weight placed on the distributive decision. Communication takes place before a proposal is offered and majority rule voting determines the outcome. We show that it is not possible for all legislators to communicate informatively. In particular, the legislator who is ideologically more distant from the proposer cannot communicate informatively, but the closer legislator may communicate whether he would "compromise" or "fight" on ideology. Surprisingly, the proposer may be worse off when bargaining with two legislators (under majority rule) than with one (who has veto power), because competition between the legislators may result in less information conveyed in equilibrium. Despite separable preferences, the proposer is always better off making proposals for the two dimensions together.

JEL classification: C78, D72, D82, D83

1 Introduction

Legislative policy-making typically involves speeches and demands by legislators that may shape the proposals made by the leadership. For example, in the 2010 health care overhaul in the U.S., one version of the Senate bill included $100 million in Medicaid funding for Nebraska and restrictions on abortion coverage in exchange for the vote of Nebraska Senator Ben Nelson. As another example, consider the threat in 2009 by seven members of the U.S. Senate Budget Committee to withhold their support for legislation to raise the debt ceiling unless a commission to recommend cuts to Medicare and Social Security is approved.[1] Would these senators indeed have let the United States default on its debt, or was their demand just a bluff? More generally, what are the patterns of demands in legislative policy-making? How much information do they convey? Do they influence the nature of the proposed bills? Who gets private benefits and what kind of policies are chosen under the ultimately accepted bills?

To answer these questions, it is necessary to have a legislative bargaining model in which legislators make demands before the proposal of the bills. One approach is to assume that the demands serve as a commitment device, that is, the legislators refuse any offer that does not meet their demands.[2] While this approach offers interesting insights into some of the questions above, it relies on the strong assumption that legislators commit to their demands.[3] In this paper, we offer a different approach that allows legislators to make speeches without commitment when casting their votes. The premise of our approach is that only individual legislators know which bills they prefer to the status quo. So even if the legislators do not necessarily carry out their threats, their demands may be meaningful rhetoric in conveying private information and dispelling some uncertainty in the bargaining process.

We model rhetoric as cheap-talk messages as in Matthews (1989). In our model, (1) three legislators bargain over an ideological and a distributive decision; (2) one of the legislators, called the chair, formulates a proposal; (3) each legislator other than the chair is privately informed about his own preferences; (4) communication takes place before a proposal is offered; (5) majority rule voting determines whether the proposal is implemented.

We assume each legislator's position on a unidimensional ideological spectrum is publicly

[1] http://thehill.com/homenews/senate/67293-sens-squeeze-speaker-over-commission

[2] This is the approach taken by Morelli (1999) in a complete information framework. He does not explicitly model the proposal-making and the voting stages. As such, the commitment assumption is implicit.

[3] Politicians often make empty threats. See, for example, http://tinyurl.com/the-hill-on-bluff.

known, but his ideological intensity, i.e., the weight he places on the ideological dimension relative to the distributive dimension is his private information. As such, the chair is unsure how much transfer she has to offer to a legislator to gain his support for a policy decision, but she can use the messages sent in the communication stage to make inferences about his ideological intensity (i.e., his type). We focus on a class of equilibria called *simple monotone equilibria* in which the types who send the same message form an interval (*monotone*), and the proposal does not depend on the message of a legislator if he receives no transfer (*simple*).

We show that in any simple monotone equilibrium: (1) At most one legislator's messages convey some information about his preferences (Proposition 4, (i)). (2) In particular, if the legislator whose position is closer to the chair's wants to move the policy in the same direction as the chair does, then it is impossible for the other legislator (i.e., the legislator whose position is further away from the chair's) to be informative (Proposition 4, (ii)). (3) Although the closer legislator may be informative, even he can convey only limited information (Proposition 5). Specifically, he sends a "fight" message when he places a relatively high weight on the ideological dimension, and the chair responds with a proposal that involves minimum policy change and gives neither legislator any private benefit since the message indicates that there is no room for making a deal. When he places a relatively low weight on the ideological dimension, he sends a "compromise" message and the chair responds by offering some private benefit in exchange for moving the policy closer to her own ideal. In contrast to the classic Crawford and Sobel (1982) model of cheap talk in which the sender conveys increasingly more precise information when the players' interests become closer, here, it is impossible for even the closer legislator to convey information more than whether he will "compromise" or "fight," no matter how close his position is to the chair's.

Surprisingly, bargaining with two legislators under majority rule may make the chair worse off than if she bargains with only one legislator (who can veto a bill). Under complete information, the chair is clearly better off when bargaining with two legislators instead of one because her bargaining position is improved. Under asymmetric information, however, the number of legislators also affects the amount of information transmission. In particular, increased competition may undermine a legislator's incentive to send the "fight" message, resulting in less information transmitted in equilibrium, and this hurts the chair.

Since the players bargain over both an ideological dimension and a distributive dimension, a natural question is whether it is better to bundle the two issues in one bill or negotiate over them separately. In our model, bundling always benefits the chair because she can exploit the differences in the other legislators' trade-offs between the two dimensions, and use private benefit as an instrument to make deals on policy changes that she wants to implement. This result, however, depends on the nature of uncertainty regarding preferences. In a related working paper (Chen and Eraslan, 2011), we show that bundling may result in informational loss when ideological positions are private information, and in that case, bundling might hurt the chair.

Before turning to the description of our model, we briefly discuss the related literature. Starting with the seminal work of Baron and Ferejohn (1989), legislative bargaining models have become a staple of political economy and have been used in numerous applications. Like our paper, some papers in the literature include an ideological dimension and a distributive dimension (e.g., Austen-Smith and Banks (1988), Banks and Duggan (2000), Jackson and Moselle (2002), and Diermeier and Merlo (2004)), but all these papers take the form of sequential offers and do not incorporate demands. A smaller strand of literature, notably Morelli (1999), instead models the legislative process as a sequential demand game where the legislators commit to their demands.[4] With the exceptions of Tsai (2009), and Tsai and Yang (2010 a, b), who do not model demands, all of these papers assume complete information.

The cheap-talk literature has largely progressed in parallel to the bargaining literature. Exceptions are Farrell and Gibbons (1989), Matthews (1989), and Matthews and Postlewaite (1989). Of these Matthews (1989) is the most closely related. Our model differs from his by having multiple legislators (rather than one) who are privately informed about their ideological intensities (rather than ideological positions); moreover, in our model the players bargain over an ideological and a distributive decision whereas in Matthews (1989), they bargain over only an ideological decision. Our paper is also related to cheap talk games with multiple senders (e.g., Gilligan and Krehbiel (1989), Austen-Smith (1993), Krishna and Morgan (2001a, b), Battaglini (2002), and Ambrus and Takahashi (2008)). Our framework differs from these papers because it has voting over the receiver's proposal and also incorporates a distributive dimension.

In the next section we describe our model. We first consider the complete information model as a benchmark in Section 3. We then study the bargaining game in which the legislators' ideological intensities are private information. In Sections 4, we analyze the simpler game with only one legislator (other than the chair) and then move on to analyze the game with two legislators in Section 5. We discuss extensions and generalizations in Section 6.

[4] See also Vidal-Puga (2004), Montero and Vidal-Puga (2007), and Breitmoser (2009).

2 Model

Three legislators play a three-stage game to collectively decide on an outcome that consists of an ideological component and a distributive component. For example, the legislators decide on the level of environmental regulation and the distribution of government spending across districts. Legislator 0 makes the proposal.[5] From now on we simply refer to legislator 0 as the chair, and use the term legislator to refer to the other two players. Let $z = (y; x)$ where y is an ideological decision and $x = (x_0, x_1, x_2)$ is a distributive decision. The set of feasible ideological decisions is $Y = \mathbb{R}$, and the set of feasible distributions is $X = \{x \in \mathbb{R}^3 : \sum_{i=0}^{2} x_i \leq c, x_1 \geq 0, x_2 \geq 0\}$ where x_i denotes the private benefit of player i and $c \geq 0$ is the size of the surplus available for division. For $i = 1, 2$, we say that proposal $(y; x)$ *includes* legislator i if $x_i > 0$ and *excludes* legislator i if $x_i = 0$.[6] The status quo allocation is $s = (\tilde{y}; \tilde{x})$ where $\tilde{y} \in Y$ and $\tilde{x} = (0, 0, 0)$.[7]

The payoff of each player $i = 0, 1, 2$ depends on the ideological decision and his/her private benefit. We assume that the players' preferences are separable over the two dimensions. Specifically, player i has a quasi-linear von Neumann-Morgenstern utility function given by

$$u_i(z, \theta_i, \hat{y}_i) = x_i + \theta_i v(y, \hat{y}_i),$$

where $z = (y; x)$ is the outcome, $\hat{y}_i \in Y$ is player i's ideal point (ideological position), and $\theta_i > 0$ is the weight that player i places on his/her payoff from the ideological decision relative to the distributive decision. The marginal rate of substitution, $(\partial u_i/\partial y)/(\partial u_i/\partial x_i) = \theta_i(\partial v/\partial y)$, measures player i's preference for ideology relative to private benefit. With fixed $\hat{y}_i$, its absolute value is increasing in θ_i, which we call legislator i's ideological intensity parameter.

Legislator $i = 1, 2$ privately observes the realization of θ_i, called his type, a random variable with probability distribution P_i. The set of possible types of legislator i is $\Theta_i = [\underline{\theta}_i, \overline{\theta}_i] \subset \mathbb{R}_+$. Let F_i denote the distribution function of θ_i, i.e., $F_i(t) = P_i(\theta_i \leq t)$. We assume that F_i is continuous and has full support on Θ_i, and the legislators' types are independently distributed. Although θ_i is legislator i's private information, its distribution and other aspects of his payoff function, including $\hat{y}_i$, are common knowledge. In the remainder of the paper, $\hat{y}_i$ is fixed and

[5] We use "she" as the pronoun for the proposer and "he" as the pronoun for legislators 1 and 2.

[6] In the remainder of the paper, when we use i and j to index the legislators, we sometimes omit the quantifiers $i = 1, 2$ or $j = 1, 2$. When we refer to both legislator i and legislator j, we implicitly assume $j \neq i$.

[7] The assumption that $\tilde{x} = (0, 0, 0)$, together with the definition of X, implies that the total surplus for reaching an agreement is non-negative, legislator 1's and legislator 2's status quo private benefits are the same, and the chair's proposal cannot offer private benefits lower than his status quo for either legislator 1 or 2.

we use $u_i(z,\theta_i)$ to denote legislator i's payoff from outcome z when his type is θ_i.

For simplicity we assume the chair's preferences are commonly known. Without loss of generality, assume $\hat{y}_0 < \tilde{y}$, i.e., the chair would like to move the policy to the left of the status quo. To simplify notation, we write $u_0(z) = x_0 + \theta_0 v(y, \hat{y}_0)$ as the chair's payoff from z.

We make the following assumptions on v: (1) v is twice differentiable; (2) for any $\hat{y}_i \in Y$, $v_{11}(y, \hat{y}_i) < 0$ for all $y \in Y$ (which implies that v is concave in y), and $v(\cdot, \hat{y}_i)$ reaches its maximum at $\hat{y}_i$; (3) v satisfies the single-crossing property in $(y, \hat{y}_i)$, i.e., for all $y, y', \hat{y}_i, \hat{y}'_i \in Y$ such that $y' > y$ and $\hat{y}'_i > \hat{y}_i$, if $v(y', \hat{y}_i) \geq v(y, \hat{y}_i)$, then $v(y', \hat{y}'_i) > v(y, \hat{y}'_i)$. This property says that if legislator i whose ideal point is $\hat{y}_i$ weakly prefers y' to y where y' is to the right of y, then any legislator whose ideal point is to the right of $\hat{y}_i$ strictly prefers y' to y. Note that the familiar quadratic-loss function, $v(y, \hat{y}_i) = -(y - \hat{y}_i)^2$, satisfies all of these assumptions.

The bargaining game has three stages. In stage one, each legislator $i = 1, 2$ observes his type θ_i and sends a private message to the chair.[8] In stage two, the chair observes the messages and makes a proposal in $Y \times X$. In stage three, the players vote on the proposal under majority rule. Without loss of generality we assume that the chair votes for the proposal. So a proposal passes if at least one of legislators 1 and 2 votes for it. Otherwise, the status quo s prevails.

The set of allowed messages for legislator i, denoted by M_i, is a finite set that has more than two elements. The messages have no literal meanings (we discuss their equilibrium meanings later); they are also "cheap talk" since they do not affect the players' payoffs directly. The assumption that M_i is finite rules out the possibility of separating equilibria, but we show that separating equilibria are not possible even if M_i's are infinite.

A strategy for legislator i consists of a message rule in the first stage and an acceptance rule in the third stage. A message rule $\mu_i : \Theta_i \to M_i$ specifies the message legislator i sends as a function of his type. An acceptance rule $\gamma_i : Y \times X \times \Theta_i \to \{0, 1\}$ specifies how legislator i votes as a function of his type: type θ_i accepts a proposal z if $\gamma_i(z, \theta_i) = 1$ and rejects it if $\gamma_i(z, \theta_i) = 0$.[9] The strategy set for legislator i consists of pairs of measurable functions (μ_i, γ_i) satisfying these properties. The chair's strategy set consists of all proposal rules $\pi : M_1 \times M_2 \to Y \times X$ where $\pi(m_1, m_2)$ is the proposal she offers when receiving (m_1, m_2). We focus on pure strategies and

[8]If the legislators send public messages simultaneously, our results still go through as long as each legislator votes for a proposal if and only if he prefers that proposal to the status quo.

[9]Technically a legislator's acceptance rule can depend on his message. However, condition (E1) in the upcoming definition of equilibrium says that legislator i accepts a proposal if and only if he prefers it to the status quo, independent of the message he sent. As such, we suppress the dependence of γ_i on m_i.

discuss conditions under which it is not restrictive to disallow mixed strategies later.

Fix a strategy profile (μ, γ, π). Say that a *message profile* $m = (m_1, m_2)$ *induces* proposal z if $\pi(m) = z$. Proposal z is *elicited* by type θ_i if it is induced by m with $m_i = \mu_i(\theta_i)$ and $\{\theta_j : \mu_j(\theta_j) = m_j\} \neq \emptyset$. If z is induced by m, then, legislator i is *pivotal with respect to* z if $\gamma_j(z, \theta_j) = 0$ for all θ_j such that $\mu_j(\theta_j) = m_j$ and *non-pivotal with respect to* z otherwise.

Equilibrium: To define an equilibrium for this game, let $\beta_i(z|m_i)$ denote the probability that legislator i votes to accept proposal z conditional on sending message m_i. Given the strategy (μ_i, γ_i) of legislator i, β_i is derived by Bayes' rule whenever possible. That is, $\beta_i(z|m_i) = \int_{\{\theta_i:\mu_i(\theta_i)=m_i\}} \gamma_i(z,\theta_i)dF_i(\theta_i) / \int_{\{\theta_i:\mu_i(\theta_i)=m_i\}} dF_i(\theta_i)$ if $\int_{\{\theta_i:\mu_i(\theta_i)=m_i\}} dF_i(\theta_i) > 0$.

An equilibrium is a strategy profile (μ, γ, π) such that the following conditions hold for all $i \neq 0$, $\theta_i \in \Theta_i$, $y \in Y, x \in X$ and $m \in M_1 \times M_2$:

(E1) $\gamma_i(z, \theta_i) = 1$ if $u_i(z, \theta_i) \geq u_i(s, \theta_i)$, and $\gamma_i(z, \theta_i) = 0$ otherwise.

(E2) $\pi(m) \in \arg\max_{z' \in Y \times X} u_0(z')\beta(z'|m) + u_0(s)(1 - \beta(z'|m))$, where

$$\beta(z'|m) = 1 - \big(1 - \beta_1(z'|m_1)\big)\big(1 - \beta_2(z'|m_2)\big)$$

is the conditional probability that z' is accepted.

(E3) If $\mu_i(\theta_i) = m_i$, then $m_i \in \arg\max_{m_i'} V_i(m_i', \theta_i)$ where

$$V_i(m_i', \theta_i) = \int_{\Theta_j} \big[\gamma_j\left(\pi\left(m_i', \mu_j(\theta_j)\right), \theta_j\right) u_i\left(\pi\left(m_i', \mu_j(\theta_j)\right), \theta_i\right)$$
$$+ \big(1 - \gamma_j\left(\pi\left(m_i', \mu_j(\theta_j)\right), \theta_j\right)\big) \max\{u_i\left(\pi\left(m_i', \mu_j(\theta_j)\right), \theta_i\right), u_i(s, \theta_i)\}\big] dF_j(\theta_j).$$

Condition (E1) requires each legislator to accept a proposal if and only if he prefers it to the status quo.[10] Condition (E2) requires that equilibrium proposals maximize the chair's payoff

[10] Condition (E1) strengthens the requirement of Perfect Bayesian Equilibrium (PBE) and is the only difference between our equilibrium solution concept and PBE. In particular, (E1) rules out the (weakly dominated) acceptance rule of accepting any proposal because a legislator expects that the other legislator accepts any proposal. Note also that (E1) assumes that a legislator accepts z whenever indifferent between z and s. If $z = s$, this is inconsequential as s would prevail whether or not legislator i accepts it. Otherwise, this is not restrictive. This is because if legislator i does not accept a proposal (not equal to s) when indifferent, then an optimal proposal does not exist for the chair. To see this, note that if the chair has an optimal proposal $(y, x) \neq s$, then at least one legislator i must strictly prefer it to s. But then there exists $\epsilon > 0$ such that either (y, x') with $x_i' = x_i - \epsilon$ or (y', x) with $y' = y - \epsilon$ is another proposal that legislator i strictly prefers to s and makes the chair better off, contradicting the optimality of (y, x).

and that her belief is consistent with Bayes' rule. Condition (E3) requires that a legislator elicits only his most preferred distribution of proposals, incorporating the acceptance rules.

For expositional simplicity, from now on we assume that in equilibrium, if $\beta(z|m) = 0$, then $\pi(m) \neq z$, i.e., if a proposal is rejected with probability 1, then the chair does not propose it.[11]

Say that a proposal z is *elicited in the equilibrium* (μ, γ, π) if there exists $(\theta_1, \theta_2) \in \Theta_1 \times \Theta_2$ such that $z = \pi(\mu_1(\theta_1), \mu_2(\theta_2))$. For any fixed strategy profile (μ, γ, π), denote by $\phi^{\mu,\gamma,\pi}(\theta_1, \theta_2)$ the *outcome* for (θ_1, θ_2); i.e., $\phi^{\mu,\gamma,\pi}(\theta_1, \theta_2) = \pi(\mu_1(\theta_1), \mu_2(\theta_2))$ if $\gamma_i(\pi(\mu_1(\theta_1), \mu_2(\theta_2)), \theta_i) = 1$ for at least one of $i = 1, 2$ and $\phi^{\mu,\gamma,\pi}(\theta_1, \theta_2) = s$ otherwise. Say that two equilibria (μ, γ, π) and (μ', γ', π') are *outcome equivalent* if $\phi^{\mu,\gamma,\pi} = \phi^{\mu',\gamma',\pi'}$.

A *babbling equilibrium* is an equilibrium (μ, γ, π) in which $\mu_i(\theta_i) = \mu_i(\theta_i')$ for all $\theta_i, \theta_i' \in \Theta_i$, $i = 1, 2$, i.e., all types of legislator i send the same message, and $\pi(m) = \pi(m')$ for all $m, m' \in M_1 \times M_2$, i.e., the chair responds to all message profiles with the same proposal. As is standard in cheap-talk models, a babbling equilibrium always exists.

3 Benchmark: complete information

We start by analyzing the benchmark game of complete information, i.e., θ_i is common knowledge. Since there is no private information, the legislators' messages are irrelevant for the chair's belief and her proposal. The modifications of the players' strategies and equilibrium conditions are straightforward and omitted. We next characterize the chair's equilibrium proposal.

If $v(\hat{y}_0, \hat{y}_i) \geq v(\tilde{y}, \hat{y}_i)$ for some legislator i, i.e., if there is a legislator who prefers the chair's ideal policy to the status quo policy, then the chair's problem is trivial: she proposes her ideal policy and keeps all the private benefit herself. From now on, we assume $v(\hat{y}_0, \hat{y}_i) < v(\tilde{y}, \hat{y}_i)$ for $i = 1, 2$. Note that since $\hat{y}_0 < \tilde{y}$, this implies that $\hat{y}_0 < \hat{y}_i$ for $i = 1, 2$.

A useful piece of notation is $e(\hat{y}_i) = \min\{y : v(y, \hat{y}_i) = v(\tilde{y}, \hat{y}_i)\}$, the left-most policy y that makes legislator i indifferent between y and $\tilde{y}$. Since $v(y, \hat{y}_i)$ is increasing in y if $y < \hat{y}_i$, given that $v(\hat{y}_0, \hat{y}_i) < v(\tilde{y}, \hat{y}_i)$, we have $\hat{y}_0 < e(\hat{y}_i) \leq \tilde{y}$, and $e(\hat{y}_i)$ is the policy y that is closest to the chair's ideal that leaves legislator i indifferent between y and $\tilde{y}$. Note that $e(\hat{y}_i)$ is nondecreasing in $\hat{y}_i$, and in particular, $e(\hat{y}_i) = \tilde{y}$ if $\hat{y}_i \geq \tilde{y}$ and $e(\hat{y}_i) < \hat{y}_i < \tilde{y}$ if $\hat{y}_i < \tilde{y}$.

[11]This is not a restrictive assumption if $c > 0$ because the chair strictly prefers the proposal $(\tilde{y}; c, 0, 0)$ (which is accepted with probability 1) to the status quo, so z is not a best response. If $c = 0$, however, it is possible that z is a best response, but not a unique one (for example, s is another best response).

To start, suppose the chair faces only legislator 1 who has veto power, i.e., for any proposal to pass, he must vote for it. Given θ_1, the chair chooses $z^1(\theta_1) = (y^1(\theta_1); x^1(\theta_1))$ to solve[12]

$$\max_{z \in Y \times X} u_0(z) = c - x_1 + \theta_0 v(y, \hat{y}_0)$$

subject to $x_1 + \theta_1 v(y, \hat{y}_1) \geq \theta_1 v(\tilde{y}, \hat{y}_1)$. Since $u_0(z)$ is decreasing in x_1, for x_1^1 to be optimal, it satisfies $x_1^1 = \theta_1 \left(v(\tilde{y}, \hat{y}_1) - v(y^1, \hat{y}_1) \right)$.[13] To satisfy $x_1^1 \geq 0$, we must have $v(\tilde{y}, \hat{y}_1) \geq v(y^1, \hat{y}_1)$. Thus, substituting for x_1 in the chair's maximization problem, y^1 must be a solution to

$$\max_{y \in Y} c - \theta_1 \left(v(\tilde{y}, \hat{y}_1) - v(y, \hat{y}_1) \right) + \theta_0 v(y, \hat{y}_0)$$

subject to $v(\tilde{y}, \hat{y}_1) \geq v(y, \hat{y}_1)$. Since $v_{11} < 0$, the objective function is strictly concave and y^1 is unique. If $\theta_1 v_1(e(\hat{y}_1), \hat{y}_1) + \theta_0 v_1(e(\hat{y}_1), \hat{y}_0) \geq 0$, i.e., if θ_1 is sufficiently high, then $v(\tilde{y}, \hat{y}_1) \geq v(y, \hat{y}_1)$ is binding, and we have a corner solution $y^1 = e(\hat{y}_1)$ and $x_1^1 = 0$. Otherwise, there exists a unique $y^1 < e(\hat{y}_1)$ such that $\theta_1 v_1(y^1, \hat{y}_1) + \theta_0 v_1(y^1, \hat{y}_0) = 0$ and $x_1^1 > 0$.

When the chair faces two legislators instead of one, her bargaining position is improved since the voting rule is the majority rule. Let $z^2(\theta_2)$ denote the chair's optimal proposal when she faces only legislator 2 with ideological intensity θ_2. If $u_0\left(z^i(\theta_i)\right) \geq u_0\left(z^j(\theta_j)\right)$, then it is optimal for the chair to propose $(y; x)$ such that $y = y^i(\theta_i)$, $x_0 = x_0^i(\theta_i)$, $x_i = x_i^i(\theta_i)$ and $x_j = 0$. when she faces both legislators i and j. Notice that it is possible that the legislator whose ideal policy is further away from the chair's is included in an optimal proposal. This can happen when he puts sufficiently less weight on ideology than the other legislator does.

We now turn to the analysis of the model with incomplete information.

4 One sender

Although our focus is on the game with three players and majority rule, it is useful to first consider a simpler game with one legislator (sender) other than the chair. In addition to providing useful intuition, the analysis is interesting in its own right because it is applicable to bilateral bargaining over two issues. Let Γ^S denote the game in which the set of legislators other than the chair is S. In this section, we consider the case with $S = \{1\}$.

The modification of the players' strategies and equilibrium conditions in $\Gamma^{\{1\}}$ are straightforward and omitted. To classify equilibria, we define the *size* of an equilibrium to be the

[12] For notational convenience, even when the chair faces only legislator $i \in \{1, 2\}$, we still assume that the chair's proposal z is in $Y \times X$ and let $x_j = 0$ for $j \in \{1, 2\}$ and $j \neq i$.

[13] To simplify notation, we suppress the dependence of y^1 and x^1 on θ_1.

number of proposals elicited in that equilibrium. To characterize equilibria, we first establish the following lemma. (Proofs omitted from the text are in the Appendix except for the proofs of Proposition 2 and Lemma 3, which are in the Supplementary Appendix.)

Lemma 1. *(i) If type θ_1 weakly prefers $z' = (y'; x')$ to $z = (y; x)$ where $x'_1 > x_1$, then any type $\theta'_1 < \theta_1$ strictly prefers z' to z. (ii) If type θ_1 weakly prefers $z'' = (y''; x'')$ to $z = (y; x)$ where $x''_1 < x_1$, then any type $\theta''_1 > \theta_1$ strictly prefers z'' to z.*

A special case of Lemma 1 is worth noting: Suppose type θ_1 is indifferent between the status quo s and $z = (y; x)$ where $x_1 > 0$. If $\theta'_1 < \theta_1$, then type θ'_1 strictly prefers z to s; if $\theta'_1 > \theta_1$, then type θ'_1 strictly prefers s to z. This immediately implies that legislator 1 does not fully reveal his type in equilibrium.[14] To see this, note that in a separating equilibrium, legislator 1 receives only his status quo payoff as the chair would make a proposal that leaves him just willing to accept. But then type θ_1 would want to mimic a higher type (i.e., exaggerate his ideological intensity) in order to get a better deal from the chair. In fact, we have a much stronger result which says that for any equilibrium, at most one proposal elicited in it gives legislator 1 some positive private benefit, and an equilibrium has at most size two. But before deriving this result and characterizing size-two equilibria, we first characterize size-one equilibria.

4.1 Size-one equilibria

We focus on babbling equilibrium since any size-one equilibrium is outcome equivalent to a babbling equilibrium. Let z' be the proposal elicited in a babbling equilibrium.

To find z', note that by Lemma 1, if $u_1\left(z, \bar{\theta}_1\right) \geq u_1\left(s, \bar{\theta}_1\right)$, then $u_1\left(z, \theta_1\right) \geq u_1\left(s, \theta_1\right)$ for all $\theta_1 \in \Theta_1$ and z is always accepted; if $u_1\left(z, \underline{\theta}_1\right) < u_1\left(s, \underline{\theta}_1\right)$, then $u_1\left(z, \theta_1\right) < u_1\left(s, \theta_1\right)$ for all $\theta_1 \in \Theta_1$ and z is always rejected; if $u_1\left(z, \bar{\theta}_1\right) < u_1\left(s, \bar{\theta}_1\right)$ and $u_1\left(z, \underline{\theta}_1\right) \geq u_1\left(s, \underline{\theta}_1\right)$, then there exists $\theta_1 \in \Theta_1$ such that $u_1\left(z, \theta_1\right) = u_1\left(s, \theta_1\right)$ and z is accepted with probability $F_1\left(\theta_1\right)$.

Let $t_1\left(z\right)$ denote the highest type who is willing to accept z if z is accepted with positive probability and set $t_1\left(z\right)$ to $\underline{\theta}_1$ if z is accepted with probability 0. Formally

$$t_1\left(z\right) = \begin{cases} \max\{\theta_1 \in \Theta_1 : u_1\left(z, \theta_1\right) \geq u_1\left(s, \theta_1\right)\} & \text{if } u_1\left(z, \underline{\theta}_1\right) \geq u_1\left(s, \underline{\theta}_1\right), \\ \underline{\theta}_1 & \text{otherwise.} \end{cases}$$

[14]To be more precise, legislator 1 does not fully reveal his type in equilibrium except in the degenerate case where $z^1\left(\theta_1\right) = \left(e\left(\hat{y}_1\right); c, 0, 0\right)$ for every $\theta_1 \in \Theta_1$. In this case, even if legislator 1 fully reveals his type, the chair still makes the same proposal $\left(e\left(\hat{y}_1\right); c, 0, 0\right)$ and we have a size-one equilibrium.

For z' to be the proposal elicited in a babbling equilibrium, it must satisfy

$$z' \in \arg\max_{z \in Y \times X} u_0(z) F_1(t_1(z)) + u_0(s)[1 - F_1(t_1(z))].$$

We can also formulate the chair's problem as choosing the highest type who is willing to accept her proposal. Let $\theta_1' = t_1(z')$, and let $V(\theta_1) = u_0\left(z^1(\theta_1)\right)$ denote the chair's highest payoff when facing legislator 1 of type θ_1. Then we have

$$\theta_1' \in \arg\max_{\theta_1 \in \Theta_1} V(\theta_1) F_1(\theta_1) + u_0(s)(1 - F_1(\theta_1)). \tag{1}$$

If the solution to (1) is unique, it is without loss of generality to consider only pure strategies. If the objective function is strictly concave, then θ_1' is unique. Another sufficient condition for uniqueness is that the objective function is strictly increasing in θ_1. Lemma 8 in the Supplementary Appendix shows that in the uniform-quadratic case (i.e., θ_1 is uniformly distributed and $v(y, \hat{y}_1) = -(y - \hat{y}_1)^2$), if $\hat{y}_1 < \tilde{y}$, then the objective function is strictly increasing in θ_1 and (1) has a unique solution at $\bar{\theta}_1$; if $\hat{y}_1 \geq \tilde{y}$ and $c > 0$, then (1) may have an interior solution and a solution at $\bar{\theta}_1$, but this happens only non-generically (i.e., fix all the parameters of the game except for c, the solution to (1) is unique except for at most one c).

4.2 Size-two equilibria

The main finding in this subsection is that legislator 1 can credibly convey some information, but only in a limited way. We first show that the number of proposals elicited in an equilibrium is at most two and then characterize size-two equilibria and provide existence conditions.

The following lemma says that there is at most one proposal elicited in equilibrium that gives legislator 1 a strictly positive transfer.

Lemma 2. *Suppose proposals $z' = (y'; x')$ and $z'' = (y''; x'')$ are elicited in an equilibrium in $\Gamma^{\{1\}}$. If $x_1' > 0$ and $x_1'' > 0$, then $z' = z''$.*

To gain some intuition, suppose there are two equilibrium proposals z' and z'' that give legislator 1 positive transfers. Then there exist a type θ_1' who elicits z' and is indifferent between z' and s, and a type θ_1'' who elicits z'' and is indifferent between z'' and s. Assume $\theta_1'' > \theta_1'$. Then by Lemma 1, type θ_1' strictly prefers to elicit z'' because he receives a payoff strictly higher than his status quo payoff by doing so, a contradiction. So only one equilibrium proposal can have $x_1 > 0$. Such a proposal must have $y < e(\hat{y}_1)$. When proposing it, the chair makes some transfer to legislator 1 in exchange for moving the policy towards her own ideal.

Now consider a proposal $(y; x)$ with $x_1 = 0$. If $e(\hat{y}_1) \leq y \leq \tilde{y}$, all types accept it; if $y < e(\hat{y}_1)$, no type accepts it. Since $v(y, \hat{y}_0)$ is decreasing in y when $y \geq e(\hat{y}_1)$, we have $y = e(\hat{y}_1)$. Hence there are at most two proposals elicited in an equilibrium: one is $(e(\hat{y}_1); c, 0, 0)$ and the other is $(y; c - x_1, x_1, 0)$ with $y < e(\hat{y}_1)$ and $x_1 > 0$. Henceforth, denote $(e(\hat{y}_1); c, 0, 0)$ by z^{NT}. Let θ_1^* be the type indifferent between $(y; c - x_1, x_1, 0)$ and z^{NT}. By Lemma 1, if $\theta_1 < \theta_1^*$, then type θ_1 strictly prefers $(y; c - x_1, x_1, 0)$ to z^{NT} and hence elicits $(y; c - x_1, x_1, 0)$. If $\theta_1 > \theta_1^*$, then type θ_1 strictly prefers z^{NT} to $(y; c - x_1, x_1, 0)$. A type $\theta_1 > \theta_1^*$ may elicit z^{NT} and accept it or elicit $(y; c - x_1, x_1, 0)$ and reject it: either way he gets his status quo payoff. To summarize:

Proposition 1. *In $\Gamma^{\{1\}}$: (i) At most two proposals are elicited in any equilibrium. (ii) In a size-two equilibrium, the elicited proposals are z^{NT} and $(y; c - x_1, x_1, 0)$ with $y < e(\hat{y}_1)$ and $x_1 > 0$. There exists a type θ_1^* such that if $\theta_1 < \theta_1^*$, type θ_1 elicits $(y; c - x_1, x_1, 0)$ and accepts it; if $\theta_1 \geq \theta_1^*$, type θ_1 either elicits $(y; c - x_1, x_1, 0)$ and rejects it or elicits z^{NT} and accepts it.*

Proposition 1 says that a type above θ_1^* may either elicit $(y; c - x_1, x_1, 0)$ and reject it, or elicit z^{NT} and accept it. Note, however, that if there were any possibility of a "tremble" by legislator 1 at the voting stage, i.e., if he might not carry out a planned veto and instead vote for a proposal even though he strictly prefers s to it, then his best message rule is to safely elicit z^{NT} if $\theta_1 > \theta_1^*$. The chair benefits if all $\theta_1 > \theta_1^*$ elicit z^{NT}, since she prefers z^{NT} to s.

Suppose the types who elicit the same proposal in equilibrium send the same message,[15] and m_1^c induces $(y; c - x_1, x_1, 0)$ and m_1^f induces z^{NT}. We can interpret m_1^c as the "compromise" message and m_1^f as the "fight" message. When the chair receives m_1^f, she infers that legislator 1 is likely to have a low ideological intensity, and responds with a "compromise" proposal that moves the policy towards her own ideal. When the chair receives m_1^f, she infers that legislator 1 is intensely ideological, and responds with a proposal that involves minimum policy change and no transfer for legislator 1. This proposal in response to the "fight" message passes with probability 1.[16] Note that multiple size-two equilibria exist with different set of elicited proposals corresponding to different thresholds θ_1^*.

Recall that $z^1(\theta_1)$ is the chair's optimal proposal when θ_1 is known.

Proposition 2. *A size-two equilibrium exists in $\Gamma^{\{1\}}$ if and only if (i) $z^1(\bar{\theta}_1) = z^{NT}$, and (ii) $z^1(\underline{\theta}_1) = (y; c - x_1, x_1, 0)$ for some $y < e(\hat{y}_1)$ and $x_1 > 0$.*

[15] This loses no generality because any size-two equilibrium is outcome equivalent to such an equilibrium.

[16] Of course, the proposal induced by the "fight" message could be just maintaining the status quo. As such, the passage of such a proposal can be interpreted as inaction by the chair on policy change.

The conditions in Proposition 2 require the chair's optimal proposal to be z^{NT} when she is sure that legislator 1 is of the highest type and to be a proposal that has $y < e(\hat{y}_1)$ and $x_1 > 0$ when she is sure that legislator 1 is of the lowest type. Intuitively, under these conditions, there exists a type $\theta_1^* \in (\underline{\theta}_1, \bar{\theta}_1)$ such that z^{NT} is optimal when the chair believes that $\theta_1 \in (\theta_1^*, \bar{\theta}_1)$ and $(y; c - x_1, x_1, 0) \neq z^{NT}$ is optimal when the chair believes that $\theta_1 \in (\underline{\theta}_1, \theta_1^*)$, which in turn guarantees that a size-two equilibrium exists. From the analysis in Section 3, the existence conditions in Proposition 2 are satisfied if $\bar{\theta}_1 > -\theta_0 v_1(e(\hat{y}_1), \hat{y}_0)/v_1(e(\hat{y}_1), \hat{y}_1) > \underline{\theta}_1$.

4.3 Comparative statics: equilibria of different sizes

A natural question is whether the players are better off in an equilibrium of a higher size. The chair clearly (weakly) prefers a size-two equilibrium to a size-one equilibrium because her decisions are based on better information in a size-two equilibrium. As to legislator 1, consider the following two cases. (i) Suppose z^{NT} is elicited in a size-one equilibrium. Then legislator 1's payoff is the same as his status quo payoff. Since in any size-two equilibrium, the payoff of any type $\theta_1 \geq \theta_1^*$ is the same as his status quo payoff and the payoff of any type $\theta_1 < \theta_1^*$ is strictly higher than his status quo payoff, legislator 1 is better off in a size-two equilibrium. (ii) Suppose $z' \neq z^{NT}$ is elicited in a size-one equilibrium. Whether legislator 1 is better off in a size-two equilibrium depends on the size-two equilibrium under consideration. But note that for any size-one equilibrium in which z' is rejected with positive probability, a size-two equilibrium exists in which every type of legislator 1 has the same payoff as in the size-one equilibrium.[17] In this sense, legislator 1 is again (weakly) better off in a size-two equilibrium.

5 Two senders

We now analyze $\Gamma^{\{1,2\}}$, the game with two legislators. Without loss of generality, assume that $\hat{y}_1 \leq \hat{y}_2$, which implies that $e(\hat{y}_1) \leq e(\hat{y}_2)$. Since legislator 1's ideal point is closer to the chair's, we call legislator 1 the *closer* legislator and legislator 2 the *more distant* legislator. As is common in the voting literature and cheap-talk literature, we restrict attention to a class

[17] To construct it, let $\theta_1' < \bar{\theta}_1$ be the type just willing to accept z'. Let $\mu_1(\theta_1) = m_1^c$ if $\theta_1 \leq \theta_1'$, $\mu_1(\theta_1) = m_1^f$ if $\theta_1 > \theta_1'$, $\pi(m_1^c) = z'$, $\pi(m_1^f) = z^{NT}$ and $\pi(m) \in \{\pi(m_1^c), \pi(m_1^f))\}$ for any other $m_1 \in M_1$. In this size-two equilibrium, the payoff for any $\theta_1 < \theta_1'$ is $u_1(z', \theta_1)$ and the payoff for any $\theta_1 \geq \theta_1'$ is $u_1(s, \theta_1)$, the same as in the size-one equilibrium.

of equilibria called **monotone equilibria**.[18] An equilibrium (μ, γ, π) is monotone if for any $\theta_i' \leq \theta_i''$ and $i = 1, 2$, $\mu_i(\theta_i') = \mu_i(\theta_i'')$ implies that $\mu_i(\theta_i) = \mu_i(\theta_i')$ for any $\theta_i \in [\theta_i', \theta_i'']$. In a monotone equilibrium, the set of types that send the same message is an interval, possibly a singleton.[19] We discuss a class of non-monotone equilibria and why they are not robust to "trembles" at the voting stage towards the end of Section 5.2.

5.1 Proposals elicited in monotone equilibria

Say that a proposal $(y; x)$ is a *one-transfer proposal* if either $x_1 > 0$ or $x_2 > 0$ but not both, a *two-transfer proposal* if both $x_1 > 0$ and $x_2 > 0$, and a *no-transfer proposal* if $x_1 = 0$ and $x_2 = 0$. The following lemma provides sufficient conditions under which no proposal elicited in a monotone equilibrium is a two-transfer proposal. Suppose F_i has a differentiable density function f_i for $i = 1, 2$. Recall that $f_i(\theta_i)/(1 - F_i(\theta_i))$ is the hazard rate and F_i satisfies the strict increasing hazard rate property (IHRP) if $f_i(\theta_i)/(1 - F_i(\theta_i))$ is strictly increasing in θ_i.

Lemma 3. *If the prior F_i $(i = 1, 2)$ satisfies the IHRP in $\Gamma^{\{1,2\}}$, then any proposal elicited in a monotone equilibrium has $x_i > 0$ for at most one $i \neq 0$.*

To see why Lemma 3 holds, consider the support of the chair's posterior. If it is a singleton for at least one of the legislators, say legislator 1, then given any proposal, the chair knows whether legislator 1 will accept or reject it. A two-transfer proposal is not optimal because if legislator 1 accepts it, then the chair is strictly better off reducing x_2 and if legislator 1 rejects it, then the chair is strictly better off reducing x_1. If the support of the posterior on θ_i is not a singleton for both $i = 1, 2$, then a two-transfer proposal results in a positive probability that both legislators vote for the proposal, which is "wasteful" for the chair because she needs only one other vote to pass her proposal. Under the IHPR, the waste is sufficiently high that it is not optimal for the chair to give transfers to both legislators. Many commonly used distribution

[18] For example, in a model of committee decision making, Perciso (2003) restricts attention to equilibria in which each juror's strategy is monotone, i.e., the probability of voting "convict" is higher if the signal is guilty than if it is innocent; in a model of cheap talk with two senders, Krishna and Morgan (2001) restrict attention to equilibria in which the receiver's action is monotone in the sender's type; and in a model of cheap talk with lying costs, Kartik (2009) restricts attention to equilibria in which the message strategy is monotone, i.e., the sender makes a higher claim when his type is higher.

[19] We call these equilibria monotone because the set of types who send the same message is an interval (possibly a singleton) if and only if there exists a partial order $\succsim$ on M_i such that the message rule μ_i is monotone (i.e. $\mu_i(\theta_i') \succsim \mu_i(\theta_i)$ if $\theta_i' > \theta_i$) for $i = 1, 2$.

functions, including uniform, normal, log-normal and beta distributions, satisfy the IHRP. This property is also frequently assumed in the economics and political science applications.[20]

The next two lemmas establish some properties of no-transfer proposals and one-transfer proposals, which are useful in equilibrium characterization.

Lemma 4. *Suppose $z = (y; x)$ is elicited in an equilibrium in $\Gamma^{\{1,2\}}$ with $x_1 = x_2 = 0$. Then (i) $y = e(\hat{y}_1)$; (ii) $u_1(z, \theta_1) = u_1(s, \theta_1)$ for any $\theta_1 \in \Theta_1$; (iii) If $e(\hat{y}_1) = e(\hat{y}_2)$, then $u_2(z, \theta_2) = u_2(s, \theta_2)$ for any $\theta_2 \in \Theta_2$; and (iv) If $e(\hat{y}_1) < e(\hat{y}_2)$, then $u_2(z, \theta_2) < u_2(s, \theta_2)$ for any $\theta_2 \in \Theta_2$ and legislator 1 is pivotal with respect to z.*

To see why Lemma 4 holds, suppose z is elicited in an equilibrium with $x_1 = x_2 = 0$. Since $e(\hat{y}_1) \leq \hat{y}_1 \leq \hat{y}_2$, if $y < e(\hat{y}_1)$, neither legislator accepts z, and if $y \geq e(\hat{y}_1)$, at least legislator 1 accepts z. Since $v(y, \hat{y}_0)$ is decreasing in y when $y \geq e(\hat{y}_1) > \hat{y}_0$, it is optimal to propose $y = e(\hat{y}_1)$. So the optimal no-transfer proposal z is equal to $(e(\hat{y}_1); c, 0, 0)$, denoted by z^{NT}. Since $x_1 = 0$ and $y = e(\hat{y}_1)$, we have $u_1(z, \theta_1) = u_1(s, \theta_1)$ for any θ_1. Similarly, if $e(\hat{y}_1) = e(\hat{y}_2)$, we have $u_2(z, \theta_2) = u_2(s, \theta_2)$ for any θ_2. Since $v(y, \hat{y}_2)$ is increasing in y when $y < \hat{y}_2$, if $e(\hat{y}_1) < e(\hat{y}_2) \leq \hat{y}_2$, we have $u_2(z, \theta_2) < u_2(s, \theta_2)$ for any θ_2, and legislator 1 is pivotal. The next lemma says that the legislator who is excluded in a one-transfer proposal rejects it, making the legislator who is included pivotal.

Lemma 5. *Suppose $z = (y; x)$ is elicited in an equilibrium in $\Gamma^{\{1,2\}}$ and $x_i > 0$, $x_j = 0$. Then $u_j(s, \theta_j) > u_i(z, \theta_j)$ for all $\theta_j \in \Theta_j$ and $u_i(z, \theta_i) \geq u_i(s, \theta_i)$ for some $\theta_i \in \Theta_i$. Hence legislator j rejects z and legislator i is pivotal with respect to z.*

If F_1 and F_2 satisfy the IHRP, then by Lemma 3, any proposal elicited in a monotone equilibrium is either a no-transfer or a one-transfer proposal. This simplifies the problem of characterizing elicited proposals in a monotone equilibrium. Specifically, recall that $t_i(z)$ is the highest type willing to accept z if some θ_i prefers z to s and $t_i(z) = \underline{\theta}_i$ otherwise. Suppose the chair's posterior is $G = (G_1, G_2)$. Let $\beta(z) = 1 - [1 - G_1(t_1(z))][1 - G_2(t_2(z))]$ and

$$z(G) \in \arg\max_{z \in Y \times X} u_0(z)\, \beta(z) + u_0(s)(1 - \beta(z)).$$

That is, $z(G)$ is an optimal proposal for the chair under belief G. Let $U_0(G)$ be the associated value function, i.e., $U_0(G)$ is the highest expected payoff for the chair under belief G.

[20] See Bagnoli and Bergstrom (2005) for a list of distribution functions that satisfy the increasing hazard rate property and references to some of the seminal papers that assume it.

Similarly, let $z^{-j}(G_i)$ be a proposal that gives the chair the highest expected payoff among all the proposals that exclude legislator j, under belief G_i $(i \neq j)$, and let $U_0^{-j}(G_i)$ be the associated value function. Note that $z^{-j}(G_i)$ does not depend on G_j because for if a proposal excludes j, either every type of legislator j accepts it or no type of legislator j accepts it.

Suppose F_1 and F_2 satisfy the IHRP. Fix a monotone equilibrium (μ, γ, π). Let $H(m) = (H_1(m_1), H_2(m_2))$ be the chair's posterior when receiving m. By Lemma 3, for any m sent in this equilibrium, $z(H(m))$ is not a two-transfer proposal and therefore $U_0(H(m)) = \max_{i=1,2} U_0^{-j}(H_i(m_i))$. Note that $U_0^{-j}(H_i(m_i)) \geq u_0(z^{NT})$ for $i = 1, 2$. Thus, if $U_0^{-j}(H_i(m_i)) > U_0^{-i}(H_j(m_j))$, then it is optimal for the chair to exclude j and include i. If $U_0^{-j}(H_i(m_i)) = u_0(z^{NT})$ for $i = 1, 2$, then z^{NT} is an optimal proposal for the chair.

Since a babbling equilibrium is a monotone equilibrium, all the results established for monotone equilibria apply to babbling equilibria. Specifically, suppose F_1 and F_2 satisfy the increasing hazard rate property. If $U_0^{-j}(F_i) > U_0^{-i}(F_j) \geq u_0(z^{NT})$, then the proposal elicited in any babbling equilibrium includes i and excludes j; if $U_0^{-2}(F_1) = U_0^{-1}(F_2) > u_0(z^{NT})$, then the proposal elicited in any babbling equilibrium is a one-transfer proposal that includes either 1 or 2; if $U_0^{-2}(F_1) = U_0^{-1}(F_2) = u_0(z^{NT})$, then there exists a babbling equilibrium in which the no-transfer proposal z^{NT} is elicited.

5.2 Informative equilibria

In this section, we characterize equilibria in $\Gamma^{\{1,2\}}$ in which some information is transmitted. Throughout this section, we assume that F_1 and F_2 satisfy the IHRP.

Fix a monotone equilibrium (μ, γ, π) and consider the proposals $\pi(m)$ and $\pi(m')$ where $m_i = m'_i$ for some $i \in \{1, 2\}$. Suppose both $\pi(m)$ and $\pi(m')$ exclude legislator $j \neq i$. That is, $z^{-j}(H_i(m_i))$ is an optimal proposal when the chair receives m and $z^{-j}(H_i(m'_i))$ is an optimal proposal when she receives m'. If $z^{-j}(H_i(m_i))$ is unique, then, since $m_i = m'_i$, and both $\pi(m)$ and $\pi(m')$ exclude j, we must have $\pi(m) = \pi(m') = z^{-j}(H_i(m_i))$. If $z^{-j}(H_i(m_i))$ is not unique, then conceivably $\pi(m) \neq \pi(m')$, but this requires that the chair chooses different proposals – none of which include legislator j – for different messages sent by legislator j, although she has the same belief about legislator i.

We now consider a refinement which rules out the preceding scenario. Call a monotone equilibrium (μ, γ, π) a **simple monotone equilibrium** (SME) if the following condition is satisfied: for any m and m' such that $m_i = m'_i$ for some $i \in \{1, 2\}$, if both $\pi(m)$ and $\pi(m')$

exclude legislator $j \neq i$, then $\pi(m) = \pi(m')$. We find this to be a reasonable refinement because when the chair optimally excludes legislator j, her proposal depends only on her belief about legislator i's type, which has nothing to do with what legislator j says. This refinement is also automatically satisfied if $z^{-j}(H_i(m_i))$ is unique. (Uniqueness of $z^{-j}(H_i(m_i))$ holds under some familiar functional forms: Lemma 8 in the Supplementary Appendix shows that if $H_i(m_i)$ is a uniform distribution and $v(y, \hat{y}_i) = -(y - \hat{y}_i)^2$, then $z^{-j}(H_i(m_i))$ is unique.)

Say that μ_i is a *size-one message rule* if $\mu_i(\theta_i) = \mu_i(\theta_i')$ for all $\theta_i, \theta_i' \in \Theta_i$, and μ_i is a *size-two message rule* if there exists a set $A_i \subset \Theta_i$ with $P_i(\theta_i \in A_i) \in (0, 1)$ such that (i) $\mu_i(\theta_i) = \mu_i(\theta_i')$ if either $\theta_i, \theta_i' \in A_i$ or $\theta_i, \theta_i' \in \Theta_i \backslash A_i$, and (ii) $\mu_i(\theta_i) \neq \mu_i(\theta_i')$ if $\theta_i \in A_i$ and $\theta_i' \in \Theta_i \backslash A_i$.

Recall that $\phi^{\mu,\gamma,\pi}(\theta_1, \theta_2)$ is the outcome for type profile (θ_1, θ_2) under (μ, γ, π). Fix an equilibrium (μ, γ, π). Say that μ_i is *equivalent to* μ_i' if for almost all $(\theta_1, \theta_2) \in \Theta_1 \times \Theta_2$ and $\mu_j' = \mu_j$, we have $\phi^{\mu,\gamma,\pi}(\theta_1, \theta_2) = \phi^{\mu',\gamma,\pi}(\theta_1, \theta_2)$. The message rule μ_i is equivalent to μ_i' in the sense that the joint distributions on type profiles and outcomes are the same under μ_i and μ_i', holding the other strategies in (μ, γ, π) fixed.

We say legislator i is *uninformative* in equilibrium (μ, γ, π) if there exists a size-one message rule μ_i^I such that μ_i is equivalent to μ_i^I, and legislator i is *informative* in (μ, γ, π) otherwise.[21] Say that (μ, γ, π) is an *informative equilibrium* if at least one legislator is informative in (μ, γ, π).

For any $z \in Y \times X$, let $I_i(z) = 1$ if z includes legislator i and $I_i(z) = 0$ if z excludes legislator i. Let $q_i(m_i) = \int_{\Theta_j} I_i(\pi(m_i, \mu_j(\theta_j))) \, dF_j$ be the probability that legislator i is included when sending m_i in (μ, γ, π).[22]

Proposition 3. *Suppose F_1 and F_2 satisfy the IHRP. Fix a simple monotone equilibrium (μ, γ, π) in $\Gamma^{\{1,2\}}$. Suppose legislator i is informative in this equilibrium. Then there exist m_i^c, $m_i^f \in M_i$ such that $q_i(m_i^c) > 0$, $q_i(m_i^f) = 0$. Moreover, μ_i is equivalent to a size-two message rule μ_i^{II} with the property that there exists $\theta_i^* \in (\underline{\theta}_i, \bar{\theta}_i)$ such that $\mu_i^{II}(\theta_i) = m_i^c$ for $\theta_i < \theta_i^*$ and $\mu_i^{II}(\theta_i) = m_i^f$ for $\theta_i > \theta_i^*$.*

Proposition 3 says that in any SME, legislator i can convey only a limited amount of

[21] It is possible that legislator i partially reveals his type in (μ, γ, π), but he is still uninformative by our definition. For example, suppose the chair optimally excludes i for θ_i sufficiently close to $\bar{\theta}_i$. Suppose also that $\mu_i(\theta_i)$ reveals θ_i if $\theta_i \in (\bar{\theta}_i - \varepsilon, \bar{\theta}_i]$ for ε sufficiently small. Although the chair updates her belief for messages sent by $\theta_i \in (\bar{\theta}_i - \varepsilon, \bar{\theta}_i]$, her proposal does not depend on $\mu_i(\theta_i)$. Since the information about θ_i is useless for the chair's decision and irrelevant for the outcome, we consider legislator i to be uninformative in this case.

[22] To simplify notation, we use $(m_i, \mu_j(\theta_j))$ to denote a message profile in which legislator i sends m_i and legislator j sends $\mu_j(\theta_j)$. We use analogous notation for other vectors of variables involving legislators i and j.

information in that even when informative, his message rule is equivalent to a size-two message rule. To give a sketch of the proof, we first show that in (μ, γ, π), there exists at most one m_i sent by a positive measure of θ_i such that $q_i(m_i) > 0$; when such a message exists, the types who send this message forms an interval at the lower end of Θ_i. We also show that there exists at most one message m_i sent by a single type such that $q_i(m_i) > 0$. So at most two m_i's have the property that $q_i(m_i) > 0$ and one of them is sent by only a single type. Consider the following two possibilities: (a) Suppose there exists no m_i sent with positive probability such that $q_i(m_i) > 0$. Then $q_i(\mu_i(\theta_i)) = 0$ for almost all $\theta_i \in \Theta_i$. Since the proposal and the resulting outcome does not depend on m_i if legislator i is excluded in a SME, it follows that μ_i is equivalent to a size-one message rule such that every θ_i sends the same message that results in zero probability that legislator i is included. (b) Suppose m_i^c is sent with positive probability and $q_i(m_i^c) > 0$. If legislator i is informative, then there exists m_i^f sent by some type such that $q_i(m_i^f) = 0$. Since in (μ, γ, π), the types who send m_i^c form an interval at the lower end of Θ_i, there exists a threshold θ_i^* such that any type below θ_i^* send m_i^c and almost every type above θ_i^* sends a message that results in zero probability that i is included. Hence μ_i is equivalent to μ_i^{II} such that $\mu_i^{II}(\theta_i) = m_i^c$ for $\theta_i < \theta_i^*$ and $\mu_i^{II}(\theta_i) = m_i^f$ for $\theta_i > \theta_i^*$.

The next proposition says that at most one legislator is informative in an SME.

Proposition 4. *Suppose F_1 and F_2 satisfy the IHRP. Fix a simple monotone equilibrium (μ, γ, π) in $\Gamma^{\{1,2\}}$. (i) At most one legislator is informative in (μ, γ, π). (ii) If $e(\hat{y}_1) < e(\hat{y}_2)$, then legislator 2 is uninformative in (μ, γ, π).*

To gain some intuition for Proposition 4, imagine that both legislators are informative in (μ, γ, π). Then, by Proposition 3, both legislators are included with positive probability. By Lemma 5, a legislator's payoff is weakly higher than his status quo payoff when included, but strictly lower than his status quo payoff when the other legislator is included. So, independent of his type, each legislator has an incentive to send the message that generates the highest probability of inclusion. But as shown in Proposition 3, if a legislator is informative, then with positive probability, he sends a message that results in zero probability of inclusion, a contradiction. As to why legislator 2 is uninformative when $e(\hat{y}_1) < e(\hat{y}_2)$, note that in this case, under the no-transfer proposal z^{NT}, legislator 2's payoff is *strictly* lower than his status quo payoff. Therefore, between m_2^c and m_2^f as described in Proposition 3, every type of legislator 2 strictly prefers to send m_2^c (with $q_2(m_2^c) > 0$) than m_2^f (with $q_2(m_2^f) = 0$), again a contradiction.

What are the proposals elicited in an informative equilibrium? Consider an SME (μ, γ, π) in which legislator i is informative. For simplicity, assume $\mu_j(\theta_j) = m_j^*$ for all $\theta_j \in \Theta_j$, and $\mu_i(\theta_i) = m_i^c$ for $\theta_i < \theta_i^*$ and $\mu_i(\theta_i) = m_i^f$ for $\theta_i > \theta_i^*$ where $q_i(m_i^c) > 0$ and $q_i(m_i^f) = 0$. Since $q_i(m_i^f) = 0$, $\pi(m_i^f, m_j^*)$ excludes legislator i. Suppose $\pi(m_i^f, m_j^*)$ includes legislator j. Then, by Lemma 5, legislator j is pivotal with respect to $\pi(m_i^f, m_j^*)$ and accepts it with positive probability, implying that by sending m_i^f, type θ_i's payoff is strictly lower than $u_i(s, \theta_i)$. Since $q_i(m_i^c) > 0$, $\pi(m_i^c, m_j^*)$ includes legislator i, and by Lemma 5, type θ_i's payoff by sending m_i^c is weakly higher than $u_i(s, \theta_i)$. Therefore any type $\theta_i > \theta_i^*$ has an incentive to deviate and send m_i^c, a contradiction. It follows that $\pi(m_i^f, m_j^*)$ excludes j as well as i and $\pi(m_i^f, m_j^*) = z^{NT}$. As to $\pi(m_i^c, m_j^*)$, it includes i and has $y < e(\hat{y}_1), x_i > 0$ and $x_j = 0$. To summarize:

Proposition 5. *Suppose F_1 and F_2 satisfy the IHRP. Fix an informative simple monotone equilibrium (μ, γ, π) in $\Gamma^{\{1,2\}}$ in which legislator i uses a size-two message rule and legislator j uses a size-one message rule. Then there exists $\theta_i^* \in \left(\underline{\theta}_i, \bar{\theta}_i\right)$ such that for any $\theta_j \in \Theta_j$, if $\theta_i > \theta_i^*$, then $\pi(\mu_i(\theta_i), \mu_j(\theta_j)) = z^{NT}$, and if $\theta_i < \theta_i^*$, then $\pi(\mu_i(\theta_i), \mu_j(\theta_j)) = (y; x)$ with $y < e(\hat{y}_1)$, $x_i > 0$, and $x_j = 0$.*

Similar to the one-sender case, we can interpret the message sent by types below θ_i^* as the "compromise" message, and the message sent by types above θ_i^* as the "fight" message. The chair responds to the "compromise" message with a proposal that gives legislator i some private benefit and moves the policy towards her own ideal and responds to the "fight" message with a proposal that involves minimum policy change and gives no private benefit to either legislator.

To illustrate what an informative equilibrium looks like, we provide the following example.

Example 1. *Suppose $\tilde{y} = 0$, $\hat{y}_0 = -1$, $\hat{y}_1 = -0.2$, $\hat{y}_2 = 0.5$, $c = 1$. Assume $\theta_0 = 1$, θ_1 and θ_2 are both uniformly distributed on $[1/4, 4]$, and player i's utility function is $x_i - \theta_i (y - \hat{y}_i)^2$.*

Suppose $\mu_1(\theta_1) = m_1^c$ if $\theta_1 \in [1/4, 1]$ and $\mu_1(\theta_1) = m_1^f$ if $\theta_1 \in (1, 4]$;[23] $\mu_2(\theta_2) = m_2^*$ for all θ_2. Given the message rules, when the chair receives m_1^c, she infers that $\theta_1 \in [1/4, 1]$. Calculation shows that $\pi(m_1^c, m_2^*) = (-0.6; 0.88, 0.12, 0)$, a proposal that gives legislator 1 a positive transfer and moves the policy towards the chair's ideal.[24] When the chair receives m_1^f,

[23] Here we let $\theta_1^* = 1$, but there are many other equilibria given by different thresholds.

[24] In this example, the proposal that the chair makes in response to (m_1^c, m_2^*) is accepted with probability 1 by legislator 1. There are examples in which a proposal made in response to a compromise message may fail to pass with positive probability. For example, suppose the distribution of θ_1 is a truncated exponential distribution

she infers that $\theta_1 \in (1,4]$. Calculation shows that it is optimal to propose $z^{NT} = (-0.4; 1, 0, 0)$. Intuitively, it is too costly for the chair to move the policy closer to her ideal because legislator 1 is too intensely ideological and legislator 2 holds an ideological position that is too far away.

Our analysis has focused on monotone equilibria. Similar to $\Gamma^{\{1\}}$, non-monotone equilibria may exist in $\Gamma^{\{1,2\}}$ in which types $\theta_i < \theta_i^*$ of legislator i elicit $(y; x)$ with $y < e(\hat{y}_1)$, $x_i > 0$, and $x_j = 0$, some types $\theta_i > \theta_i^*$ elicit z^{NT} and accept it, and other types $\theta_i > \theta_i^*$ elicit the same proposal as that elicited by types below θ_i^* and reject it, and legislator j babbles. Note that similar to $\Gamma^{\{1\}}$, these non-monotone equilibria are not robust to "trembles" by either legislator at the voting stage, i.e., if either legislator might not carry out a planned rejection, then legislator i's best message rule is to safely elicit z^{NT} when $\theta_i > \theta_i^*$.[25]

We next provide conditions for the existence of SME in which legislator 1 is informative. (The conditions are similar for SME in which legislator 2 is informative, with the additional requirement that $e(\hat{y}_1) = e(\hat{y}_2)$.)

The existence conditions for informative equilibria in $\Gamma^{\{1,2\}}$ are analogous to those in $\Gamma^{\{1\}}$, but with the additional condition that it is optimal for the chair to exclude legislator 2. This is guaranteed if $U_0^{-1}(F_2) = u_0\left(z^{NT}\right)$. To see this, recall that $U_0^{-1}(F_2)$ is the highest payoff the chair gets by excluding 1. If $U_0^{-1}(F_2) = u_0\left(z^{NT}\right)$, then no proposal that includes 2 gives the chair a higher payoff than z^{NT} and therefore it is optimal for the chair to exclude 2. Recall that $z^1(\theta_1)$ is the chair's optimal proposal when facing only legislator 1 with known θ_1. We have the following result (the proof is omitted since it is similar to that of Proposition 2):

Proposition 6. *Suppose F_1 and F_2 satisfy the IHRP. A simple monotone equilibrium in which legislator 1 is informative exists if and only if (i) $z^1\left(\bar{\theta}_1\right) = (e(\hat{y}_1); c, 0, 0)$, (ii) $z^1\left(\underline{\theta}_1\right) = (y; c - x_1, x_1, 0)$ for some $y < e(\hat{y}_1)$ and $x_1 > 0$, and (iii) $U_0^{-1}(F_2) = u_0(z^{NT})$.*

on $[1/4, 4]$ with the parameter $\lambda = 4$, i.e., $F_1(\theta_1) = \left(e^{-1} - e^{-4x}\right) / \left(e^{-1} - e^{-16}\right)$. Keep all the other parametric assumptions unchanged and assume $\mu_1(\theta_1) = m_1^c$ if $\theta_1 \in [1/4, 2]$ and $\mu_1(\theta_1) = m_1^f$ if $\theta_1 \in (2, 4]$ and $\mu_2(\theta_2) = m_2^*$ for all θ_2. Then $\pi(m_1^c, m_2^*) = (-0.585; 0.883, 0.117, 0)$ and it is rejected by all types of legislator 2 and accepted by legislator 1 if and only if $\theta_1 \leq 1.076$. Hence it fails to pass with strictly positive probability.

[25] These "trembles" at the voting stage imply that a legislator should elicit the proposal he prefers most. In addition to non-monotone equilibria, some monotone equilibria may not be robust to such trembles. For instance, in the example in footnote 24, some types below θ_1^* rejects the proposal they elicit, and they are better off by safely eliciting z^{NT}, if there is any possibility of trembles at the voting stage. If the threshold θ_1^* is sufficiently low, however, then we have a monotone equilibrium in which the proposal made in response to the "compromise" message is accepted with probability 1. Such a monotone equilibrium is robust to trembles at the voting stage.

Similar to $\Gamma^{\{1\}}$, condition (i) is satisfied if $\bar{\theta}_1$ is sufficiently high and condition (ii) is satisfied if $\underline{\theta}_1$ is sufficiently low. Condition (iii) is satisfied, roughly, if θ_2 is sufficiently likely to be high.

5.3 Comparative statics

Two comparisons seem especially interesting. The first is the comparison between informative and uninformative equilibria in $\Gamma^{\{1,2\}}$. The second is the comparison of equilibria in $\Gamma^{\{1,2\}}$ and those in $\Gamma^{\{1\}}$, which allows us to answer: is the chair always better off bargaining with more legislators? Surprisingly, we show below that although the chair needs only one legislator's support to pass a proposal, she may be worse off when facing two legislators than just one.

Comparing informative and uninformative equilibria: Let E^u be an uninformative equilibrium and E^I be an SME in which legislator i is informative in $\Gamma^{\{1,2\}}$. Since the chair benefits from information transmission, she is better off in E^I than in E^u. The welfare comparison for the informative legislator is similar to that in the one-sender case (page 12); in particular, he benefits from information transmission as well. The uninformative legislator j, however, may be worse off when legislator i is informative. To illustrate, suppose the proposal elicited in E^u is z^{NT}. Since in an informative SME the elicited proposals are z^{NT} and $(y;x)$ with $y < e(\hat{y}_1)$ and $x_j = 0$, and legislator j prefers $e(\hat{y}_1)$ to any $y < e(\hat{y}_1)$, he is better off in E^u.

Does it benefit the chair to face more legislators? Under complete information, the chair is clearly better off bargaining with two legislators than only one because she gains flexibility as to who to make a deal with, as shown at the end of section 3. Under asymmetric information, however, the answer is less clear. As illustrated in the following example, having two legislators may result in less information transmitted in equilibrium and this hurts the chair.

Example 2. *Suppose* $c = 1, \tilde{y} = 0,$ $u_0(z) = x_0 - \theta_0 (y - \hat{y}_0)^2$ *where* $\theta_0 = 1,$ $\hat{y}_0 = -1$ *and* $u_1(z, \theta_1) = x_1 - \theta_1 (y - \hat{y}_1)^2$ *where* $\hat{y}_1 = -0.2$ *and* θ_1 *is uniformly distributed on* $[1/4, 4]$.

Consider $\Gamma^{\{1\}}$, in which the chair faces only legislator 1. Size-two equilibria exists in $\Gamma^{\{1\}}$. For instance, analogous to Example 1, a size-two equilibrium exists in which $\mu_1(\theta_1) = m_1^c$ if $\theta_1 \in [1/4, 1]$ and $\mu_1(\theta_1) = m_1^f$ if $\theta_1 \in (1, 4]$. The chair's payoff in this equilibrium is 0.656. Now consider $\Gamma^{\{1,2\}}$ in which the chair faces both legislators 1 and 2.[26] Suppose $u_2(z, \theta_2) = x_2 - \theta_2 (y - \hat{y}_2)^2$ where $\hat{y}_2 = -0.201$ and θ_2 is uniformly distributed on $[5, 10]$. Since $e(\hat{y}_2) < e(\hat{y}_1)$,

[26] Although earlier we assumed that $\hat{y}_1 \leq \hat{y}_2$ for expositional convenience, in this example, in order to discuss all possibilities, we allow $\hat{y}_1 > \hat{y}_2$.

by Proposition 4 (ii) (adapted to the case with $e(\hat{y}_2) < e(\hat{y}_1)$), legislator 1 is not informative in any SME in $\Gamma^{\{1,2\}}$. Calculation shows that $z^2(\underline{\theta}_2) = (y; x)$ where $y = e(\hat{y}_2)$ and $x_2 = 0$. Since a necessary condition for the existence of SME in which legislator 2 is informative is $z^2(\underline{\theta}_2) = (y; x)$ where $y < e(\hat{y}_2)$ and $x_2 > 0$, legislator 2 is not informative in any SME either. In any uninformative equilibrium in $\Gamma^{\{1,2\}}$, the proposal $(-0.402; 1, 0, 0)$ is elicited with probability 1 and the chair's payoff is 0.642, lower than 0.656, her payoff in the size-two equilibrium that we identified in $\Gamma^{\{1\}}$.

In the preceding example, the chair is worse off when we add legislator 2 whose position is closer to the chair's (making it impossible for legislator 1 to be informative) but who is intensely ideological (making it impossible for himself to be informative). What happens if we add a legislator whose position is further away from the chair's? Can it still result in informational loss? The next example shows that the answer is yes. Suppose $\hat{y}_2 = -0.1$ and θ_2 is uniformly distributed on $[1/4, 4/5]$. Since $e(\hat{y}_1) < e(\hat{y}_2)$, by Proposition 4, legislator 2 is uninformative in any SME in $\Gamma^{\{1,2\}}$. Calculation shows that $z^{-1}(F_2) = (y; x)$ where $y = -0.6$ and $x_2 = 0.192$, i.e., conditional on excluding 1, the chair's proposal includes 2. So condition (iii) in Proposition 6 fails and it is not possible for legislator 1 to be informative in any SME in $\Gamma^{\{1,2\}}$ either.[27] In any uninformative equilibrium in $\Gamma^{\{1,2\}}$, the proposal $z^{-1}(F_2)$ is elicited with probability 1, resulting in a payoff of 0.648 for the chair, still lower than 0.656. So, the chair is again worse off when she faces two legislators rather than one.

To summarize, the chair may be better off bargaining with only one legislator when the informational loss resulting from having two legislators is sufficiently high. This contrasts with Krishna and Morgan (2001b), in which a decision maker is never worse off when facing two senders rather than one. In their model, the senders have the same information, and for any equilibrium in the one-sender case, there exists an equilibrium when another sender is added which gives the decision maker a payoff at least as high as his original equilibrium payoff.

5.4 Benefits of bundled bargaining

In the model considered so far, the chair makes a proposal on an ideological dimension and a distributive dimension, and the two dimensions are accepted or rejected together. (Call this the "bundled bargaining" game.) A natural question is whether the chair is better off

[27] Intuitively, if legislator 1 is informative in some SME, then the chair responds to his "fight" message by including 2, making 1 strictly worse off than the status quo and giving him an incentive to deviate.

bundling the two dimensions or negotiating them separately. Specifically, consider a "separate bargaining" game in which the chair, after receiving the messages, makes a proposal on only the ideological dimension and another on only the distributive dimension. The legislators vote on each proposal separately. In this game, it is possible that a proposal on one dimension passes while the proposal on the other dimension fails to pass.

The chair is better off in the bundled bargaining game. To see why, note that in the separate bargaining game, the legislators' private information is irrelevant since it is about how they trade off one dimension for the other, not about their preferences on either dimension. The resulting unique equilibrium outcome is z^{NT}. In the bundled bargaining game, z^{NT} is still feasible and will pass if proposed, and this immediately implies that the chair cannot be worse off. Indeed, bundling gives the chair two advantages: (1) Useful information may be revealed in equilibrium, as seen in Proposition 5. (2) Given the information she has, the chair can use private benefit as an instrument to make better proposals that exploit the difference in how the players trade off the two dimensions. Because of these advantages, if the chair could choose between bundled bargaining and separate bargaining, she would choose the former. Legislator 1 gets his status quo payoff and legislator 2 is worse off than the status quo in the separate bargaining game, but in the bundled bargaining game, the informative legislator is better off than the status quo whereas the other (uninformative) legislator, is worse off than the status quo. This result is reminiscent of the finding in Jackson and Moselle (2002), who also show that legislators may prefer to make proposals for the two dimensions together despite separable preferences, but their model does not have asymmetric information or communication.

6 Concluding remarks

In this paper, we develop a new model of legislative bargaining that incorporates private information about preferences and allows speech making before a bill is proposed. Although the model is simple, our analysis generates interesting predictions about what speeches can be credible even without commitment and how they influence proposals and legislative outcomes.

We believe that both private information and communication are essential elements of the legislative decision making process. Our paper has taken a first step in understanding their roles in legislative bargaining. There are many more issues to explore and many ways to generalize and extend our model and what follows is a brief discussion of some of them.

We have considered a multilateral bargaining game with only 3 players. In the Supplementary Appendix, we take a first step in generalizing our analysis to more than 3 players. Specifically, we analyze an extension in which there are $n \geq 3$ legislators other than the chair, but only two legislators have private information.[28] The main results derived in the 3-players case are robust. In particular, in the extension with n legislators, a legislator can still convey limited information, i.e., whether he will "fight" or "compromise." Under majority rule, any legislator who holds a position sufficiently distant from the chair's (specifically, a position further away from the chair's than the median position is) cannot be informative. There are equilibria in which more than one legislator is informative, but this happens only when they are either the median or hold positions closer to the chair's than the median position is.

Our motivation for incorporating private information into legislative bargaining is that individual legislators know their preferences better than others. Another possible source of private information is expertise (perhaps acquired through specialized committee work or from staff advisors) regarding the consequences of policies. Although the role of this kind of "common value" private information in legislative decision making has been studied (e.g. Austen-Smith (1990)), it is only in the context of one-dimensional spatial policy making. It would be interesting to explore it further when there is tradeoff between ideology and distribution.

In our model the chair does not have private information about her preference, consistent with the observation that bill proposers are typically established members with known positions. Sometimes, however, legislators can be uncertain about the leaders' goals, and in particular, how much compromise the leaders are willing to make in exchange for their votes. In this case, the proposal put on the table may also reveal some of the proposer's private information. This kind of signaling effect becomes especially relevant when the legislators have interdependent preferences or when the proposal is not an ultimatum but can be modified if agreement fails.

We have considered a specific extensive form in which the legislators send messages simul-

[28]Suppose at least κ votes other than the chair's are needed for a proposal to pass, and only two legislators have private information. We show that if the prior satisfies the IHRP, then any proposal elicited in a monotone equilibrium gives transfers to at most κ legislators; if a privately informed legislator is included in a proposal, then he has veto power with respect to that proposal. We then show that Propositions 3 and 4 (ii) generalize to this extension. Although a full analysis of the more general case in which more than two legislators have private information is beyond the scope of this paper, we still find the extension in the Supplementary Appendix useful because it applies to a legislature with a low turnover so that the preferences of most legislators are known through past experience and only a few new members may have private information on their preferences.

taneously. It would be interesting to explore whether and how some of our results change if the legislators send messages sequentially. In that case, the design of the optimal order of speeches (from the perspective of the proposer as well as the legislature) itself is an interesting question. Another design question with respect to communication protocol is whether the messages should be public or private. Although this distinction does not matter for the model in this paper because we assume simultaneous speeches and one round of bargaining, it would matter if either there were multiple rounds of bargaining or the preferences were interdependent.

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Appendix

Proof of Lemma 1. (i) Since type θ_1 weakly prefers z' to z, we have $x_1' + \theta_1 v(y', \hat{y}_1) \geq x_1 + \theta_1 v(y, \hat{y}_1)$, which implies that $x_1' - x_1 \geq \theta_1 (v(y, \hat{y}_1) - v(y', \hat{y}_1))$.

Suppose $v(y, \hat{y}_1) - v(y', \hat{y}_1) \leq 0$. Since $x_1' - x_1 > 0$ and $\theta_1' > 0$, it follows that $x_1' - x_1 > 0 \geq \theta_1' (v(y, \hat{y}_1) - v(y', \hat{y}_1))$, i.e., $x_1' + \theta_1' v(y', \hat{y}_1) > x_1 + \theta_1' v(y, \hat{y}_1)$.

Suppose $v(y, \hat{y}_1) - v(y', \hat{y}_1) > 0$. Then $\theta_1 (v(y, \hat{y}_1) - v(y', \hat{y}_1)) > \theta_1' (v(y, \hat{y}_1) - v(y', \hat{y}_1))$ for $\theta_1 > \theta_1'$ and hence $x_1' - x_1 > \theta_1' (v(y, \hat{y}_1) - v(y', \hat{y}_1))$, i.e., $x_1' + \theta_1' v(y', \hat{y}_1) > x_1 + \theta_1' v(y, \hat{y}_1)$.

(ii) Since type θ_1 weakly prefers z'' to z, we have $x_1'' + \theta_1 v(y'', \hat{y}_1) \geq x_1 + \theta_1 v(y, \hat{y}_1)$, which implies that $\theta_1 (v(y'', \hat{y}_1) - v(y, \hat{y}_1)) \geq x_1 - x_1''$. Since $x_1 - x_1'' > 0$, we have $v(y'', \hat{y}_1) - v(y, \hat{y}_1) > 0$. So, for $\theta_1'' > \theta_1$, we have $\theta_1'' (v(y'', \hat{y}_1) - v(y, \hat{y}_1)) > \theta_1 (v(y'', \hat{y}_1) - v(y, \hat{y}_1)) \geq x_1 - x_1''$, i.e., $x_1'' + \theta_1'' v(y'', \hat{y}_1) > x_1 + \theta_1'' v(y, \hat{y}_1)$. ■

Proof of Lemma 2. Suppose z' and z'' are elicited in equilibrium (μ, γ, π) and $x_1' > 0, x_1'' > 0$. For any z, let $\alpha(z) = \{\theta_1 : \pi(\mu_1(\theta_1)) = z \text{ and } \gamma_1(z, \theta_1) = 1\}$. Since any proposal elicited in an equilibrium is accepted by some type who elicits it (page 7), $\alpha(z') \neq \varnothing$ and $\alpha(z'') \neq \varnothing$. Let $\theta_1' = \sup \alpha(z')$ and $\theta_1'' = \sup \alpha(z'')$. Also, let $o(\theta_1) = \pi(\mu_1(\theta_1))$ if $\gamma_1(\mu_1(\theta_1), \theta_1) = 1$ and $o(\theta_1) = s$ otherwise, and let $u_1^e(\theta_1) = u_1(o(\theta_1), \theta_1)$, i.e., $u_1^e(\theta_1)$ is type θ_1's payoff in (μ, γ, π).

Claim 1. *$u_1^e(\theta_1') = u_1(z', \theta_1') = u_1(s, \theta_1')$ and $u_1^e(\theta_1'') = u_1(z'', \theta_1'') = u_1(s, \theta_1'')$.*

Proof. We show that $u_1^e(\theta_1') = u_1(z', \theta_1') = u_1(s, \theta_1')$. Similar arguments apply to θ_1''.

To show that $u_1^e(\theta_1') = u_1(z', \theta_1')$, first note that $u_1^e(\theta_1') \geq u_1(z', \theta_1')$ since type θ_1' can elicit z' and accept it. Suppose $u_1^e(\theta_1') > u_1(z', \theta_1')$. Since $u_1(o(\theta_1'), \theta_1) - u_1(z', \theta_1)$ is continuous in θ_1, there exists $\theta_1 \in \alpha(z')$ sufficiently close to θ_1' such that $u_1(o(\theta_1'), \theta_1) > u_1(z', \theta_1)$. Since for any $\theta_1 \in \alpha(z')$, $u_1^e(\theta_1) = u_1(z', \theta_1)$, this is a contradiction.

To show that $u_1^e(\theta_1') = u_1(s, \theta_1')$, first note that for any θ_1, $u_1^e(\theta_1) \geq u_1(s, \theta_1)$ since type θ_1 can reject the proposal it elicits. Suppose $u_1^e(\theta_1') > u_1(s, \theta_1')$. Since $u_1^e(\theta_1') = u_1(z', \theta_1')$, we have $u_1(z', \theta_1') > u_1(s, \theta_1')$. Since $x_1' > 0$, there exists $\hat{z} = (\hat{y}; \hat{x})$ with $\hat{y} = y'$ and $\hat{x}_1 \in (0, x_1')$ such that $u_1(\hat{z}, \theta_1') > u_1(s, \theta_1')$. Lemma 1 implies that for any $\theta_1 \in \alpha(z')$, $u_1(\hat{z}, \theta_1) > u_1(s, \theta_1)$ and $\gamma_1(\hat{z}, \theta_1) = 1$. Since $u_0(\hat{z}) > u_0(z')$, this contradicts the optimality of z'. ■

We next show that $z' = z''$. Suppose not. Consider the following two possibilities. (a) Suppose $x_1' = x_1''$ and without loss of generality, $y' < y''$. Note that $y' < y'' \leq e(\hat{y}_1) \leq \hat{y}_1$. We have $u_1(z', \theta_1) < u_1(z'', \theta_1)$ for all $\theta_1 \in \Theta_1$ since $x_1' = x_1''$ and $u_1(z, \theta_1)$ is increasing in y

for $y < \hat{y}_1$, contradicting that $\alpha(z') \neq \varnothing$. (b) Suppose $x_1' \neq x_1''$ and without loss of generality, $x_1' > x_1''$. By definition of $\alpha(z)$, for all $\theta_1 \in \alpha(z')$, $u_1^e(\theta_1) = u_1(z', \theta_1) \geq u_1(z'', \theta_1)$ and for all $\theta_1 \in \alpha(z'')$, $u_1^e(\theta_1) = u_1(z'', \theta_1) \geq u_1(z', \theta_1)$. By Lemma 1, any type in $\alpha(z')$ is strictly lower than any type in $\alpha(z'')$ and therefore $\theta_1' < \theta_1''$. Since $u_1(z'', \theta_1'') = u_1(s, \theta_1'')$ and $x_1'' > 0$, Lemma 1 implies that $u_1(z'', \theta_1') > u_1(s, \theta_1')$, contradicting $u_1^e(\theta_1') = u_1(s, \theta_1')$. Hence $z' = z''$. ■

Proof of Lemma 5. Suppose to the contrary that there exists a type θ_j' such that $u_j(z, \theta_j') \geq u_j(s, \theta_j')$. Since $x_j = 0$, this implies that $v(y, \hat{y}_j) \geq v(\tilde{y}, \hat{y}_j)$ and therefore $u_j(z, \theta_j) \geq u_j(s, \theta_j)$ for all $\theta_j \in \Theta_j$. Consider $z' = (y'; x')$ with $y' = y$ and $x_i' = x_j' = 0$. We have $u_j(z', \theta_j) \geq u_j(s, \theta_j)$ and $\gamma_j(z', \theta_j) = 1$ for all $\theta_j \in \Theta_j$. Since $x_i' < x_i$, we have $u_0(z') > u_0(z)$, contradicting the optimality of z. Hence, every type of legislator j rejects z and legislator i is pivotal. Since $x_i > 0$, if $u_i(z, \theta_i) < u_i(s, \theta_i)$ for all $\theta_i \in \Theta_i$, then proposing s is strictly better than z. Therefore $u_i(z, \theta_i) \geq u_i(s, \theta_i)$ for some $\theta_i \in \Theta_i$. ■

Proof of Proposition 3. Fix a simple monotone equilibrium (μ, γ, π) in $\Gamma^{\{1,2\}}$. Recall that $V_i(m_i, \theta_i)$ is type θ_i's expected payoff from sending m_i in (μ, γ, π) (page 6). For any $\theta_i \in \Theta_i$, $m_i \in M_i$ and interval $T \subset \Theta_j$, let $V_i(m_i, \theta_i | T)$ be type θ_i's expected payoff if he sends m_i, conditional on $\theta_j \in T$. We first state and prove the following two lemmas.

Lemma 6. *Suppose F_1 and F_2 satisfy the IHRP. Fix a simple monotone equilibrium (μ, γ, π) in $\Gamma^{\{1,2\}}$. Suppose $m = (\mu_1(\theta_1), \mu_2(\theta_2))$ for some (θ_1, θ_2), and $\pi(m) = z = (y; x)$ with $x_i > 0$. Let $\theta_i^s = \sup\{\theta_i : \mu_i(\theta_i) = m_i\}$. Then, (i) $u_i(s, \theta_i^s) \geq u_i(z, \theta_i^s)$, and (ii) $V_i(m_i, \theta_i^s | \mu_j^{-1}(m_j)) = u_i(s, \theta_i^s)$, i.e., if type θ_i^s elicits z followed by his optimal acceptance rule, he receives a payoff equal to $u_i(s, \theta_i^s)$.*

Proof. Suppose to the contrary that $u_i(z, \theta_i^s) > u_i(s, \theta_i^s)$. Since $x_i > 0$, there exists $\varepsilon > 0$ and $\hat{z} = (y; x_0 + \varepsilon, x_i - \varepsilon, x_j)$ such that $u_i(\hat{z}, \theta_i^s) > u_i(s, \theta_i^s)$. Since $\theta_i \leq \theta_i^s$ for any $\theta_i \in \mu_i^{-1}(m_i)$, by Lemma 1, $\hat{z}$ is accepted by any $\theta_i \in \mu_i^{-1}(m_i)$. Since $u_0(\hat{z}) > u_0(z)$, this contradicts the optimality of z. So $u_i(s, \theta_i^s) \geq u_i(z, \theta_i^s)$. If $u_i(z, \theta_i^s) = u_i(s, \theta_i^s)$, then $\gamma_i(z, \theta_i^s) = 1$. If $u_i(z, \theta_i^s) < u_i(s, \theta_i^s)$, then $\gamma_i(z, \theta_i^s) = 0$. Since $x_i > 0$, by Lemmas 3 and 5, legislator i is pivotal with respect to z. Therefore $V_i(m_i, \theta_i^s | \mu_j^{-1}(m_j)) = \max\{u_i(z, \theta_i^s), u_i(s, \theta_i^s)\} = u_i(s, \theta_i^s)$. ■

Lemma 7. *Suppose F_1 and F_2 satisfy the IHRP. Fix a simple monotone equilibrium (μ, γ, π) in $\Gamma^{\{1,2\}}$. Let $\theta_i' < \theta_i''$, $m_i' = \mu_i(\theta_i')$, and $m_i'' = \mu_i(\theta_i'')$. Suppose $q_i(m_i'') > 0$. (i) If $\mu_i^{-1}(m_i'')$ is not a singleton, then $m_i' = m_i''$. (ii) If $\mu_i^{-1}(m_i'')$ is a singleton, then $\mu_i^{-1}(m_i')$ is not a singleton.*

Proof. Part (i): Suppose to the contrary that $m_i' \neq m_i''$. We first prove the following claim: for any m_j sent by some $\theta_j \in \Theta_j$, if $\pi(m_i'', m_j)$ includes i then $\pi(m_i', m_j)$ also includes i. This claim implies that $q_i(m_i') \geq q_i(m_i'')$. We then use this inequality to establish a contradiction.

Proof of the claim: Suppose $\pi(m_i'', m_j) = (y''; x'')$ includes i. By Lemmas 3 and 5, legislator i is pivotal with respect to $\pi(m_i'', m_j)$. Since $\pi(m_i'', m_j)$ is accepted with positive probability and $\mu_i^{-1}(m_i'')$ is a non-degenerate interval, we have $P_i\left(\theta_i \in \mu_i^{-1}(m_i'') | u_i(\pi(m_i'', m_j), \theta_i) \geq u_i(s, \theta_i)\right) > 0$. By Lemma 1, if $u_i(\pi(m_i'', m_j), \theta_i) \geq u_i(s, \theta_i)$, then $u_i(\pi(m_i'', m_j), \tilde{\theta}_i) > u_i(s, \tilde{\theta}_i)$ for all $\tilde{\theta}_i < \theta_i$. Hence $P_i\left(\theta_i \in \mu_i^{-1}(m_i'') | u_i(\pi(m_i'', m_j), \theta_i) > u_i(s, \theta_i)\right) > 0$.

Given any $\varepsilon \in (0, x_i'')$, let $z_\varepsilon = (y''; x_0'' + \varepsilon, x_i'' - \varepsilon, x_j'')$. Since $x_i'' > 0$, and u_i is continuous in x_i, it follows that for ε sufficiently small, $P_i\left(\theta_i \in \mu_i^{-1}(m_i'') | u_i(z_\varepsilon, \theta_i) > u_i(s, \theta_i)\right) > 0$ and therefore $u_i(z_\varepsilon, \theta_i) > u_i(s, \theta_i)$ if $\theta_i = \inf\{\mu_i^{-1}(m_i'')\}$.

Since $\theta_i' < \theta_i''$, $m_i' \neq m_i''$ and (μ, γ, π) is a monotone equilibrium, $\sup\{\mu_i^{-1}(m_i')\} \leq \inf\{\mu_i^{-1}(m_i'')\}$. Let $\varepsilon \in (0, x_i'')$ be such that $u_i(z_\varepsilon, \theta_i) > u_i(s, \theta_i)$ for $\theta_i = \inf\{\mu_i^{-1}(m_i'')\}$. By Lemma 1, $u_i(z_\varepsilon, \theta_i) > u_i(s, \theta_i)$ for all $\theta_i \in \mu_i^{-1}(m_i')$. Since $u_0(z_\varepsilon) > u_0(\pi(m_i'', m_j))$ and z_ε is accepted by all $\theta_i \in \mu_i^{-1}(m_i')$, we have $U_0(H(m_i', m_j)) > U_0(H(m_i'', m_j))$. Since $\pi(m_i'', m_j)$ includes i, it follows that $U_0(H(m_i'', m_j)) \geq U_0^{-i}(H_j(m_j))$. So $U_0(H(m_i', m_j)) > U_0(H(m_i'', m_j)) \geq U_0^{-i}(H_j(m_j))$, implying that $\pi(m_i', m_j)$ includes i. This completes the proof of the claim.

The claim implies that $q_i(m_i') \geq q_i(m_i'')$, and $\Theta_j = \Theta_j^a \cup \Theta_j^b \cup \Theta_j^c$ where $\Theta_j^a = \{\theta_j \in \Theta_j |$ both $\pi(m_i', \mu_j(\theta_j))$ and $\pi(m_i'', \mu_j(\theta_j))$ exclude $i\}$, $\Theta_j^b = \{\theta_j \in \Theta_j |\ \pi(m_i', \mu_j(\theta_j))$ includes i and $\pi(m_i'', \mu_j(\theta_j))$ excludes $i\}$, and $\Theta_j^c = \{\theta_j \in \Theta_j |$ both $\pi(m_i', \mu_j(\theta_j))$ and $\pi(m_i'', \mu_j(\theta_j))$ include $i\}$. This claim also implies that $\Theta_j^c = \{\theta_j \in \Theta_j |\ \pi(m_i'', \mu_j(\theta_j))$ include $i\}$, and $P_j(\Theta_j^c) = q_i(m_i'')$. Since $q_i(m_i'') > 0$, we have $P_j(\Theta_j^c) > 0$.

Let $\theta_i^l = \sup \mu_i^{-1}(m_i')$ and $\theta_i^h = \sup \mu_i^{-1}(m_i'')$. Since $\mu_i^{-1}(m_i'')$ is not a singleton and $m_i' \neq m_i''$, we have $\theta_i^h > \theta_i^l$. Note that if $\pi(m_i', \mu_j(\theta_j))$ includes i, then by Lemma 6, $u_i(s, \theta_i^l) \geq u_i(\pi(m_i', \mu_j(\theta_j)), \theta_i^l)$. Since $\theta_i^h > \theta_i^l$, by Lemma 1, $u_i(s, \theta_i^h) > u_i(\pi(m_i', \mu_j(\theta_j)), \theta_i^h)$. Similarly, if $\pi(m_i'', \mu_j(\theta_j))$ includes legislator i, then there exists a type $\theta_i > \theta_i^l$ such that $u_i(\pi(m_i'', \mu_j(\theta_j)), \theta_i) \geq u_i(s, \theta_i)$, and by Lemma 1, $u_i(\pi(m_i'', \mu_j(\theta_j)), \theta_i^l) > u_i(s, \theta_i^l)$. In what follows, we show that if $V_i(m_i', \theta_i^l) \geq V_i(m_i'', \theta_i^l)$, then $V_i(m_i', \theta_i^h) > V_i(m_i'', \theta_i^h)$, which implies that either type θ_i^l or type θ_i^h has a strictly profitable deviation.

If $\theta_j \in \Theta_j^a$, then both $\pi(m_i', \mu_j(\theta_j))$ and $\pi(m_i'', \mu_j(\theta_j))$ exclude i, and therefore $\pi(m_i', \mu_j(\theta_j)) = \pi(m_i'', \mu_j(\theta_j))$. So $V_i(m_i', \theta_i | \Theta_j^a) = V_i(m_i'', \theta_i | \Theta_j^a)$ for all θ_i. If $\theta_j \in \Theta_j^c$, then both $\pi(m_i', \mu_j(\theta_j))$ and $\pi(m_i'', \mu_j(\theta_j))$ include i. By Lemma 6, $V_i(m_i', \theta_i^l | \Theta_j^c) = u_i(s, \theta_i^l)$ and $V_i(m_i'', \theta_i^h | \Theta_j^c) =$

$u_i\left(s,\theta_i^h\right)$. As shown in the previous paragraph, $V_i(m_i'',\theta_i^l|\Theta_j^c) > u_i(s,\theta_i^l)$ and $u_i(s,\theta_i^h) > u_i(\pi\left(m_i',\mu_j\left(\theta_j\right)\right),\theta_i^h)$ if $\theta_j \in \Theta_j^c$. By Lemmas 3 and 5, if $\theta_j \in \Theta_j^c$, legislator i is pivotal with respect to $\pi\left(m_i',\mu_j\left(\theta_j\right)\right)$ and therefore $V_i(m_i',\theta_i^h|\Theta_j^c) = u_i\left(s,\theta_i^h\right)$. So $V_i(m_i',\theta_i^l|\Theta_j^a\cup\Theta_j^c) < V_i(m_i'',\theta_i^l|\Theta_j^a\cup\Theta_j^c)$ and $V_i(m_i',\theta_i^h|\Theta_j^a\cup\Theta_j^c) = V_i(m_i'',\theta_i^h|\Theta_j^a\cup\Theta_j^c)$. Since $P_j(\Theta_j^c) > 0$, it follows that if $V_i(m_i',\theta_i^l) \geq V_i(m_i'',\theta_i^l)$, then $P_j(\Theta_j^b) > 0$ and $V_i(m_i',\theta_i^l|\Theta_j^b) > V_i(m_i'',\theta_i^l|\Theta_j^b)$.

If $\theta_j \in \Theta_j^b$, then $\pi\left(m_i',\mu_j\left(\theta_j\right)\right)$ includes i. Similar arguments as those for Θ_j^c show that $V_i(m_i',\theta_i|\Theta_j^b) = u_i\left(s,\theta_i\right)$ for $\theta_i = \theta_i^l,\theta_i^h$. Note that for any z and z' such that both exclude i, if $u_i\left(z,\theta_i\right) \geq u_i\left(z',\theta_i\right)$ for some $\theta_i \in \Theta_i$, then $u_i\left(z,\theta_i\right) \geq u_i\left(z',\theta_i\right)$ for all $\theta_i \in \Theta_i$. Since $\pi\left(m_i'',\mu_j\left(\theta_j\right)\right)$ excludes i if $\theta_j \in \Theta_j^b$, if $V_i(m_i',\theta_i^l|\Theta_j^b) = u_i\left(s,\theta_i^l\right) > V_i(m_i'',\theta_i^l|\Theta_j^b)$, then $V_i(m_i',\theta_i^h|\Theta_j^b) = u_i\left(s,\theta_i^h\right) > V_i(m_i'',\theta_i^h|\Theta_j^b)$. Suppose $V_i(m_i',\theta_i^l) \geq V_i(m_i'',\theta_i^l)$. Recall that this implies $P_j(\Theta_j^b) > 0$ and $V_i(m_i',\theta_i^l|\Theta_j^b) > V_i(m_i'',\theta_i^l|\Theta_j^b)$. It follows that $V_i(m_i',\theta_i^h|\Theta_j^b) > V_i(m_i'',\theta_i^h|\Theta_j^b)$. Since $V_i(m_i',\theta_i^h|\Theta_j^a\cup\Theta_j^c) = V_i(m_i'',\theta_i^h|\Theta_j^a\cup\Theta_j^c)$, we have $V_i(m_i',\theta_i^h) > V_i(m_i'',\theta_i^h)$, a contradiction. Hence $m_i'' = m_i'$.

Part (ii): Proof is similar to that of part (i). Suppose to the contrary that $\mu_i^{-1}\left(m_i'\right)$ is a singleton. As in part (i), we first prove the claim that for any m_j sent by some $\theta_j \in \Theta_j$, if $\pi(m_i'',m_j)$ includes i, then $\pi(m_i',m_j)$ also includes i. To show this, suppose $\pi(m_i'',m_j)$ includes i. Since $\mu_i^{-1}\left(m_i''\right)$ is a singleton, $\pi(m_i'',m_j)$ is accepted by θ_i'', i.e., $u_i(\pi(m_i'',m_j),\theta_i'') \geq u_i(s,\theta_i'')$. Since $\theta_i' < \theta_i''$, by Lemma 1, $u_i(\pi(m_i'',m_j),\theta_i') > u_i(s,\theta_i')$. Given any $\varepsilon \in (0,x_i'')$, let $z_\varepsilon = (y'';x_0''+\varepsilon,x_i''-\varepsilon,x_j'')$. Since $x_i'' > 0$ and u_i is continuous in x_i, we have $u_i\left(z_\varepsilon,\theta_i'\right) > u_i\left(s,\theta_i'\right)$ for ε sufficiently low. The rest of the proof is the same as that of part (i). ■

Let $\hat{M}_i = \{m_i|m_i = \mu_i\left(\theta_i\right)$ for some θ_i, and $q_i\left(m_i\right) > 0\}$. Lemma 7 implies that $|\hat{M}_i| \leq 2$, and if $|\hat{M}_i| = 2$, then there exists an $\hat{m}_i \in \hat{M}_i$ such that $\{\theta_i|\mu_i\left(\theta_i\right) = \hat{m}_i\}$ is a singleton.

We next show that if $P_i\left(q_i\left(\mu_i(\theta_i)\right) > 0\right) = 1$ or $P_i\left(q_i\left(\mu_i(\theta_i)\right) = 0\right) = 1$, then legislator i is uninformative in (μ,γ,π). Suppose $P_i\left(q_i\left(\mu_i(\theta_i)\right) > 0\right) = 1$. Lemma 7 implies that there exists a message m_i^c such that $q_i(m_i^c) > 0$ and $P_i(\mu_i(\theta_i) = m_i^c) = 1$. Hence μ_i is equivalent to the size-one message rule $\mu_i^I(\theta_i) = m_i^c$ for all θ_i. Next, suppose $P_i\left(q_i\left(\mu_i(\theta_i)\right) = 0\right) = 1$. Consider a type $\hat{\theta}_i$ such that $q_i(\mu_i(\hat{\theta}_i)) = 0$ and a size-one message rule $\mu_i^I\left(\theta_i\right)$ such that $\mu_i^I\left(\theta_i\right) = \mu_i(\hat{\theta}_i)$ for all θ_i. To see that μ_i is equivalent to μ_i^I, consider any θ_i such that $q_i\left(\mu_i\left(\theta_i\right)\right) = 0$. Note that in an SME, for any m_j, $\pi(\mu_i(\hat{\theta}_i),m_j) = \pi\left(\mu_i\left(\theta_i\right),m_j\right)$ if they both exclude legislator i. Since $q_i(\mu_i(\hat{\theta}_i)) = q_i\left(\mu_i\left(\theta_i\right)\right) = 0$, it follows that $\pi(\mu_i(\hat{\theta}_i),\mu_j\left(\theta_j\right)) = \pi\left(\mu_i\left(\theta_i\right),\mu_j\left(\theta_j\right)\right)$ for almost all $\theta_j \in \Theta_j$. Since $P_i\left(q_i\left(\mu_i(\theta_i)\right) = 0\right) = 1$, it follows that μ_i is equivalent to μ_i^I.

Hence, if legislator i is informative in (μ,γ,π), then we have $P_i\left(q_i\left(\mu_i(\theta_i)\right) = 0\right) \in (0,1)$ and

$P_i\left(q_i\left(\mu_i(\theta_i)\right) > 0\right) \in (0,1)$. By Lemma 7, there exists a message m_i^c and a type $\theta_i^* \in \left(\underline{\theta}_i, \bar{\theta}_i\right)$ such that $q_i(m_i^c) > 0$ and $\mu_i(\theta_i) = m_i^c$ for all $\theta_i < \theta_i^*$ and $q_i(\mu_i(\theta_i)) = 0$ for almost all $\theta_i \geq \theta_i^*$. Pick any $\hat{\theta}_i$ such that $q_i(\mu_i(\hat{\theta}_i)) = 0$, and let $m_i^f = \mu_i(\hat{\theta}_i)$. Then μ_i is equivalent to μ_i^{II} such that $\mu_i^{II}(\theta_i) = m_i^c$ for $\theta_i < \theta_i^*$ and $\mu_i^{II}(\theta_i) = m_i^f$ for $\theta_i > \theta_i^*$. ■

Proof of Proposition 4. Let $\Theta_i^0 = \{\theta_i \in \Theta_i | q_i(\mu_i(\theta_i)) = 0\}$. Proposition 3 and Lemma 7 imply that if legislator i is informative, then there exists $m_i^c \in M_i$ such that $q_i\left(m_i^c\right) > 0$, $P_i(\theta_i | \mu_i(\theta_i) = m_i^c) \in (0,1)$ and $P_i(\theta_i | \mu_i(\theta_i) = m_i^c) + P_i(\Theta_i^0) = 1$. Let $\Theta_i^c = \{\theta_i \in \Theta_i | \mu_i(\theta_i) = m_i^c\}$.

Part (i): Suppose to the contrary that both legislators 1 and 2 are informative in (μ, γ, π). Consider the following two cases.

(a) Suppose $\pi\left(m_1^c, m_2^c\right)$ excludes 1. Consider any $\tilde{m}_1 \in M_1$ such that $q_1(\tilde{m}_1) = 0$. Since $P_2(\Theta_2^c) > 0$, $\pi(\tilde{m}_1, m_2^c)$ excludes 1. Thus, $\pi(m_1^c, m_2^c) = \pi(\tilde{m}_1, m_2^c)$. Note that this holds for any $\tilde{m}_1$ with $q_1(\tilde{m}_1) = 0$. Since $q_2(m_2^c) > 0$ and $P_1(\Theta_1^c) + P_1(\Theta_1^0) = 1$, we have $q_2(m_2^c) = 1$.

Since $P_1(\Theta_1^c) > 0$, the proposal $\pi(m_1^c, \mu_2(\theta_2))$ excludes 2 for all $\theta_2 \in \Theta_2^0$, and $\pi(m_1^c, \mu_2(\theta_2))$ is the same for all $\theta_2 \in \Theta_2^0$. Since $\pi(m_1^c, m_2^c)$ excludes 1 and $q_1(m_1^c) > 0$, we have $P_2(\theta_2 | \theta_2 \in \Theta_2^0$ and $\pi(m_1^c, \mu_2(\theta_2))$ includes $1) > 0$. Hence $\pi(m_1^c, \mu_2(\theta_2))$ includes 1 for all $\theta_2 \in \Theta_2^0$. Recall that $V_i\left(m_i, \theta_i\right)$ is type θ_i's expected payoff from sending m_i in (μ, γ, π) (page 6). Consider any type $\theta_2 \in \Theta_2^0$. Since $q_2\left(m_2^c\right) = 1$, Lemmas 3 and 5 imply that $V_2\left(m_2^c, \theta_2\right) \geq u_2\left(s, \theta_2\right)$. Since $\pi\left(m_1^c, \mu_2(\theta_2)\right)$ includes 1 and $P_1(\Theta_1^c) > 0$, Lemmas 3 and 5 imply that $V_2\left(\mu_2\left(\theta_2\right), \theta_2\right) < u_2\left(s, \theta_2\right)$. Hence any type $\theta_2 \in \Theta_2^0$ is strictly better off by sending m_2^c, a contradiction.

(b) Suppose $\pi\left(m_1^c, m_2^c\right)$ includes 1. Then the same arguments as in case (a) show that $q_1\left(m_1^c\right) = 1$ and any $\theta_1 \in \Theta_1^0$ is strictly better off by sending m_1^c than $\mu_1(\theta_1)$, a contradiction.

Part (ii): Suppose to the contrary that legislator 2 is informative. Fix any type $\theta_2 \in \Theta_2^0$ and let $\Theta_1' = \{\theta_1 |$both $\pi\left(\mu_1\left(\theta_1\right), m_2^c\right)$ and $\pi\left(\mu_1\left(\theta_1\right), \mu_2\left(\theta_2\right)\right)$ exclude $2\}$ and $\Theta_1'' = \{\theta_1 | \pi\left(\mu_1\left(\theta_1\right), m_2^c\right)$ includes 2 and $\pi\left(\mu_1\left(\theta_1\right), \mu_2(\theta_2)\right)$ excludes $2\}$. If $\theta_1 \in \Theta_1'$, then $\pi\left(\mu_1\left(\theta_1\right), \mu_2\left(\theta_2\right)\right) = \pi\left(\mu_1\left(\theta_1\right), m_2^c\right)$. If $\theta_1 \in \Theta_1''$, then $\pi\left(\mu_1\left(\theta_1\right), m_2^c\right)$ includes 2 (and excludes 1 by Lemma 3). By Lemma 5, conditional on $\theta_1 \in \Theta_1''$, type θ_2's payoff is weakly higher than $u_2\left(s, \theta_2\right)$ if he sends m_2^c. Since $e(\hat{y}_1) < e(\hat{y}_2)$, by Lemmas 4 and 5, conditional on $\theta_1 \in \Theta_1''$, type θ_2's payoff is strictly lower than $u_2\left(s, \theta_2\right)$ if he sends $\mu_2\left(\theta_2\right)$. Since $\theta_2 \in \Theta_2^0$, we have $q_2(\mu_2(\theta_2)) = 0$, which implies $P_1\left(\Theta_1'\right) + P_1\left(\Theta_1''\right) = 1$. Note that $P_1\left(\Theta_1''\right) = q_2\left(m_2^c\right) > 0$. So $V_2\left(\mu_2\left(\theta_2\right), \theta_2\right) < V_2\left(m_2^c, \theta_2\right)$, i.e., any type $\theta_2 \in \Theta_2^0$ is strictly better off by sending m_2^c, a contradiction. ■

Supplementary Appendix

Proof of Proposition 2

For any $z \in Y \times X$ and $\theta_1', \theta_1'' \in \Theta_1$ with $\theta_1' < \theta_1''$, let $\tau(z, \theta_1', \theta_1'') = \max\{\theta_1 \in [\theta_1', \theta_1''] : u_1(z, \theta_1) \geq u_1(s, \theta_1)\}$ if $u_1(z, \theta_1') \geq u_1(s, \theta_1')$ and $\tau(z, \theta_1', \theta_1'') = \theta_1'$ if $u_1(z, \theta_1') < u_1(s, \theta_1')$. Let $k(\theta_1', \theta_1'')$ be the set of optimal proposals for the chair if she knows that $\theta_1 \in [\theta_1', \theta_1'']$, i.e.,

$$k\left(\theta_1', \theta_1''\right) = \underset{z \in Y \times X}{\arg\max}\, u_0(z)\left[F_1\left(\tau\left(z, \theta_1', \theta_1''\right)\right) - F_1\left(\theta_1'\right)\right] + u_0(s)\left[F_1\left(\theta_1''\right) - F_1\left(\tau\left(z, \theta_1', \theta_1''\right)\right)\right].$$

Let $k(\theta_1, \theta_1) = \{z^1(\theta_1)\}$. We first establish the following claim.

Claim 2. *Let $\theta_l', \theta_l'', \theta_h', \theta_h'' \in \Theta_1$ be such that $\theta_l' < \theta_l''$, $\theta_h' < \theta_h''$, $\theta_l' \leq \theta_h'$ and $\theta_l'' \leq \theta_h''$. (i) If $z^{NT} \in k(\theta_l', \theta_l'')$, then $z^{NT} \in k(\theta_h', \theta_h'')$. (ii) If $k(\theta_h', \theta_h'') \neq \{z^{NT}\}$, then $k(\theta_l', \theta_l'') \neq \{z^{NT}\}$.*

Proof. Fix $\theta_l', \theta_l'', \theta_h', \theta_h''$. For any $z \in Y \times X$, let $p(z) = (F_1(\tau(z, \theta_h', \theta_h'')) - F_1(\theta_h')) / (F_1(\theta_h'') - F_1(\theta_h'))$ and $r(z) = (F_1(\tau(z, \theta_l', \theta_l'')) - F_1(\theta_l')) / (F_1(\theta_l'') - F_1(\theta_l'))$. Note that $r(z^{NT}) = p(z^{NT}) = 1$. We first show that $r(z) \geq p(z)$ for any z. If $\tau(z, \theta_l', \theta_l'') = \theta_l''$, then $r(z) = 1 \geq p(z)$. If $\tau(z, \theta_h', \theta_h'') = \theta_h'$ then $p(z) = 0 \leq r(z)$. If $\tau(z, \theta_l', \theta_l'') < \theta_l''$ and $\tau(z, \theta_h', \theta_h'') > \theta_h'$, then by Lemma 1, we have $\tau(z, \theta_l', \theta_l'') = \tau(z, \theta_h', \theta_h'')$ and $r(z) \geq (F_1(\tau(z, \theta_l', \theta_l'')) - F_1(\theta_l')) / (F_1(\theta_h'') - F_1(\theta_l')) \geq p(z)$.

Part (i): Since $z^{NT} \in k(\theta_l', \theta_l'')$, we have $u_0(z^{NT}) \geq u_0(z) r(z) + u_0(s)(1 - r(z))$ for any $z \in Y \times X$. Since $r(z) \geq p(z)$, for any z such that $u_0(z) \geq u_0(s)$, we have $u_0(z) r(z) + u_0(s)(1 - r(z)) \geq u_0(z) p(z) + u_0(s)(1 - p(z))$. Hence $u_0(z^{NT}) \geq u_0(z) p(z) + u_0(s)(1 - p(z))$, which implies that $z^{NT} \in k(\theta_h', \theta_h'')$.

Part (ii): If $k(\theta_h', \theta_h'') \neq \{z^{NT}\}$, then there exists $z \neq z^{NT}$ such that $u_0(z) \geq u_0(s)$ and $u_0(z) p(z) + u_0(s)(1 - p(z)) \geq u_0(z^{NT})$. Since $r(z) \geq p(z)$, we have $u_0(z) r(z) + u_0(s)(1 - r(z)) \geq u_0(z^{NT})$ and therefore $k(\theta_l', \theta_l'') \neq \{z^{NT}\}$. ■

"if" part:

Let $t_1' = \sup\{\theta_1 \in [\underline{\theta}_1, \bar{\theta}_1]$ such that $k(\underline{\theta}_1, \theta_1) \neq \{z^{NT}\}\}$ and $t_1'' = \inf\{\theta_1 \in [\underline{\theta}_1, \bar{\theta}_1]$ such that $z^{NT} \in k(\theta_1, \bar{\theta}_1)\}$. Under conditions (i) and (ii) in Proposition 2, t_1' and t_1'' are well defined. We next show that $t_1' \geq t_1''$. Suppose to the contrary $t_1' < t_1''$. Then there exists $\theta_1 \in (t_1', t_1'')$ such that $k(\underline{\theta}_1, \theta_1) = \{z^{NT}\}$ and $z^{NT} \notin k(\theta_1, \bar{\theta}_1)$, contradicting Claim 2. Hence $t_1' \geq t_1''$. Fix $\tilde{\theta}_1 \in [t_1'', t_1']$, $m_1^c, m_1^f \in M_1$ and let $\mu_1(\theta_1) = m_1^c$ if $\theta_1 < \tilde{\theta}_1$ and $\mu_1(\theta_1) = m_1^f$ if $\theta_1 > \tilde{\theta}_1$, $\pi(m_1^c) \in k(\underline{\theta}_1, \tilde{\theta}_1)$

such that $\pi(m_1^c) \neq z^{NT}$, $\pi(m_1^f) = z^{NT}$, $\pi(m_1) \in \{\pi(m_1^c), \pi(m_1^f)\}$ for any other $m_1 \in M_1$. Also, let γ_1 satisfy (E1). Since this is an equilibrium profile, a size-two equilibrium exists.

"only if" part:

Suppose a size-two equilibrium (μ, γ, π) exists. By Proposition 1, two proposals z^{NT} and $z^* = (y^*; x^*)$ are elicited in this equilibrium where $y^* < e(y_1)$ and $x_1^* > 0$.

Since z^* is elicited in this equilibrium, there exists a type $\theta_1 \in \Theta_1$ such that $u_1(z^*, \theta_1) \geq u_1(s, \theta_1)$. Since $x_1^* > 0$, by Lemma 1, we have $u_1(z^*, \underline{\theta}_1) \geq u_1(s, \underline{\theta}_1)$. Note also that since z^* is optimal for the chair under some belief, and z^{NT} is accepted by all types θ_1, we have $u_0(z^*) \geq u_0(z^{NT})$. It follows that if the chair is sure that legislator 1's type is $\underline{\theta}_1$, then it is better to propose z^* than z^{NT} and therefore $z^1(\underline{\theta}_1) \neq z^{NT}$.

Since z^{NT} is elicited in this equilibrium, there exists a type $\theta_1 \in \Theta_1$ such that $z^1(\theta_1) = z^{NT}$. As shown in section 3, if $z^1(\theta_1) = z^{NT}$, then $z^1(\theta_1') = z^{NT}$ for all $\theta_1' > \theta_1$. Hence $z^1(\bar{\theta}_1) = z^{NT}$. ∎

Proof of Lemma 3

Fix a monotone equilibrium (μ, γ, π). Consider any message profile m sent in this equilibrium. We show below that $\pi(m_1, m_2) = z^* = (y^*; x^*)$ is not a two-transfer proposal.

Case (i): Suppose $\mu_i^{-1}(m_i)$ is a singleton for some $i \in \{1, 2\}$. Without loss of generality, suppose $\mu_1^{-1}(m_1)$ is a singleton and let $\theta_1 = \mu_1^{-1}(m_1)$. If $u_1(z^*, \theta_1) \geq u_1(s, \theta_1)$, then type θ_1 accepts z^* and we must have $x_2^* = 0$. If $u_1(z^*, \theta_1) < u_1(s, \theta_1)$, then type θ_1 rejects z^*, and we must have $x_1^* = 0$.

Case (ii): Suppose $\mu_i^{-1}(m_i)$ is a non-degenerate interval for $i = 1, 2$. Let G_i be the posterior distribution function of the chair when receiving m_i and let g_i be the associated density. Recall that for any z, $t_1(z)$ denotes the highest type of legislator 1 willing to accept z. Define $t_2(z)$ analogously. Let $\beta(z) = 1 - (1 - G_1(t_1(z)))(1 - G_2(t_2(z)))$. Let $d = x_1^* + x_2^*$ and consider the following problem

$$\max_{x \in X} (c - d + \theta_0 v(y^*, \hat{y}_0))\beta(y^*; x) + \theta_0 v(\tilde{y}, \hat{y}_0)(1 - \beta(y^*; x)) \tag{2}$$

subject to $x_1 + x_2 = d$. Since z^* is an optimal proposal when the chair receives m, x^* is a solution to equation (2).

Suppose, towards a contradiction, that $x_1^* > 0$ and $x_2^* > 0$. Let $v_i^* = v(\tilde{y}, \hat{y}_i) - v(y^*, \hat{y}_i)$. Since $x_i^* > 0$, we must have $v_i^* > 0$. Note also that $G_i(t_i(z^*)) \in (0, 1)$ for $i = 1, 2$. This is because if $G_i(t_i(z^*)) = 0$, then $x_i^* = 0$; and if $G_i(t_i(z^*)) = 1$, then $x_j^* = 0$. In the rest of the proof, abusing notation, we let $G_i = G_i(t_i(z^*))$ and $g_i = g_i(t_i(z^*))$.

Substituting x_2 with $(d - x_1)$ in (2), we have the following first order necessary condition:

$$(\theta_0 v(y^*, \hat{y}_0) - \theta_0 v(\tilde{y}, \hat{y}_0)) \frac{\partial \beta(z^*)}{\partial x_1} = 0. \tag{3}$$

Since $d > 0$, we have $v(y^*, \hat{y}_0) - v(\tilde{y}, \hat{y}_0) > 0$, and therefore $\partial\beta(z^*)/\partial x_1 = 0$. Since $\partial\beta(z^*)/\partial x_1 = g_1(1 - G_2)/v_1^* - g_2(1 - G_1)/v_2^*$ and $G_i \in (0, 1)$, we can rearrange (3) to obtain $v_1^* = g_1(1 - G_2)v_2^*/(g_2(1 - G_1))$. The second order necessary condition for an interior maximum requires that $\partial^2\beta(z^*)/\partial x_1^2 \leq 0$. Substituting for v_1^*, we simplify the second order condition to have

$$(\frac{g_2}{g_1})^2 \left(g_1'(1 - G_1) + (g_1)^2\right) + \left(g_2'(1 - G_2) + (g_2)^2\right) \leq 0. \tag{4}$$

By Corollary 5 in Bagnoli and Bergstrom (2005), a truncation of a distribution preserves the IHRP. Since F_i satisfies the IHRP, G_i satisfies the IHRP, implying that $g_i'(t_i(z^*))(1 - G_i(t_i(z^d)) + g_i(t_i(z^*))^2 > 0$ for $i = 1, 2$. But this violates inequality (4), a contradiction. ∎

Uniqueness in the uniform-quadratic setup

Recall that $z^1(\theta_1) = \left(y^1(\theta_1); x^1(\theta_1)\right)$ denotes the chair's optimal proposal under complete information when she faces only legislator 1, and $V(\theta_1) = u_0\left(z^1(\theta_1)\right)$.

Lemma 8. *Suppose $v(y, \hat{y}_i) = -(y - \hat{y}_i)^2$, and θ_1 is uniformly distributed on $[\underline{t}_1, \bar{t}_1] \subseteq \Theta_1$, where $\bar{t}_1 > \underline{t}_1$. Let G_1 be the cumulative distribution function of θ_1 and let $W(\theta_1) = V(\theta_1) G_1(\theta_1) + u_0(s)(1 - G_1(\theta_1))$.*

(i) If $\hat{y}_1 < \tilde{y}$, then $\bar{t}_1$ is the unique solution to $max_{\theta_1 \in [\underline{t}_1, \bar{t}_1]} W(\theta_1)$.

(ii) If $\hat{y}_1 \geq \tilde{y}$ and $c > 0$, then the solution to $\max_{\theta_1 \in [\underline{t}_1, \bar{t}_1]} W(\theta_1)$ is generically unique in the following sense: fix all the parameters except for c, the solution to $\max_{\theta_1 \in [\underline{t}_1, \bar{t}_1]} W(\theta_1)$ is unique except for at most one c.

Proof. Without loss of generality, let $\tilde{y} = 0$. When $v(y, \hat{y}_i) = -(y - \hat{y}_i)^2$, straightforward calculation shows that $y^1(\theta_1) = \min\{(\theta_0\hat{y}_0 + \theta_1\hat{y}_1)/(\theta_0 + \theta_1), e(\hat{y}_1)\}$ and $x_1^1(\theta_1) = \theta_1\left(v(\tilde{y}, \hat{y}_1) - v(y^1(\theta_1), \hat{y}_1)\right)$. Hence, if $(\theta_0\hat{y}_0 + \theta_1\hat{y}_1)/(\theta_0 + \theta_1) \geq e(\hat{y}_1)$, then $V(\theta_1) = c - \theta_0(e(\hat{y}_1) - \hat{y}_0)^2$, and $V'(\theta_1) = 0$; and if $(\theta_0\hat{y}_0 + \theta_1\hat{y}_1)/(\theta_0 + \theta_1) < e(\hat{y}_1)$, then

$$V(\theta_1) = c - \theta_1(-\frac{\theta_0\hat{y}_0 + \theta_1\hat{y}_1}{\theta_0 + \theta_1})(2\hat{y}_1 - \frac{\theta_0\hat{y}_0 + \theta_1\hat{y}_1}{\theta_0 + \theta_1}) - \theta_0(\frac{\theta_0\hat{y}_0 + \theta_1\hat{y}_1}{\theta_0 + \theta_1} - \hat{y}_0)^2.$$

In this case $V'(\theta_1) = -\left(v(\tilde{y}, \hat{y}_1) - v(y^1(\theta_1), \hat{y}_1)\right)$ by the envelope theorem. Since $v(y^1(\theta_1), \hat{y}_1) < v(e(\hat{y}_1), \hat{y}_1)$, it follows that $V'(\theta_1) < -\left(v(\tilde{y}, \hat{y}_1) - v(e(\hat{y}_1), \hat{y}_1)\right) = 0$.

Note that

$$W'(\theta_1) = \frac{V'(\theta_1)\,\theta_1 - V'(\theta_1)\,\underline{t}_1 + V(\theta_1) - u_0(s)}{\bar{t}_1 - \underline{t}_1}.$$

Part (i): Suppose $\hat{y}_1 \le \tilde{y}$. It suffices to show that $W'(\theta_1) > 0$ for all $\theta_1 \in [\underline{t}_1, \bar{t}_1]$.

Since $V'(\theta_1) \le 0$, to show that $W'(\theta_1) > 0$, we only need to show that $V'(\theta_1)\,\theta_1 + V(\theta_1) - u_0(s) > 0$. If $(\theta_0\hat{y}_0 + \theta_1\hat{y}_1)/(\theta_0 + \theta_1) > e(\hat{y}_1)$, then $V'(\theta_1)\,\theta_1 + V(\theta_1) - u_0(s) > 0$ since $V'(\theta_1) = 0$ and $V(\theta_1) - u_0(s) > 0$. If $(\theta_0\hat{y}_0 + \theta_1\hat{y}_1)/(\theta_0 + \theta_1) \le e(\hat{y}_1)$, then $y^1(\theta_1) = (\theta_0\hat{y}_0 + \theta_1\hat{y}_1)/(\theta_0 + \theta_1) \le e(\hat{y}_1)$ and

$$V'(\theta_1)\,\theta_1 + V(\theta_1) - u_0(s) = c + \theta_0(y^1(\theta_1))^2 + 2\theta_1 y^1(\theta_1)\hat{y}_1.$$

Since $c \ge 0, \theta_0 > 0$, $\theta_1 > 0$, and $y^1(\theta_1) \le e(\hat{y}_1) < \hat{y}_1 < \tilde{y} = 0$, it follows that $V'(\theta_1)\,\theta_1 + V(\theta_1) - u_0(s) > 0$ and therefore $W'(\theta_1) > 0$ and $\bar{t}_1$ is the unique solution to $\max W(\theta_1)$.

Part (ii): Suppose $\hat{y}_1 > \tilde{y}$. Note that if $V'(\hat{\theta}_1) = 0$, then $V'(\theta_1) = 0$ for any $\theta_1 > \hat{\theta}_1$. Since $c > 0$, if $V'(\theta_1) = 0$, then $W'(\theta_1) > 0$. It follows that if $V'(\theta_1) = 0$ and $\theta_1 \neq \bar{t}_1$, then $\theta_1 \neq \arg\max W(\theta_1)$. We next show that for θ_1 such that $V'(\theta_1) < 0$, the second derivative of $W(\theta_1)$ crosses 0 only once and from below. It is straightforward to verify that

$$\begin{aligned} W''(\theta_1) &= \frac{V''(\theta_1)(\theta_1 - \underline{t}_1) + 2V'(\theta_1)}{\bar{t}_1 - \underline{t}_1} \\ &= \frac{2\theta_1(\hat{y}_1)^2\left(3\theta_0\theta_1 + 3(\theta_0)^2 + (\theta_1)^2\right) + C}{(\theta_0 + \theta_1)^3(\bar{t}_1 - \underline{t}_1)} \end{aligned}$$

where C does not depend on θ_1. Hence, if $W''(\theta_1) = 0$, then $W''(\theta_1') > 0$ for any $\theta_1' > \theta_1$, i.e., $W''(\theta_1)$ crosses 0 at most once and from below. Consider the following two possibilities.

(a) Suppose $W''(\theta_1) > 0$ for all $\theta_1 \in [\underline{t}_1, \bar{t}_1]$ such that $V'(\theta_1) < 0$. Then $W(\theta_1)$ does not have an interior maximum, and therefore $\bar{t}_1 = \arg\max W(\theta_1)$.

(b) Suppose $W''(\theta_1) = 0$ for some $\theta_1 \in [\underline{t}_1, \bar{t}_1]$ such that $V'(\theta_1) < 0$. Then there is at most one interior maximum of $W(\theta_1)$ at $\tilde{\theta}_1$ where $W'(\tilde{\theta}_1) = 0$ and $W''(\tilde{\theta}_1) < 0$. If $W(\tilde{\theta}_1) = W(\bar{t}_1)$, then both $\tilde{\theta}_1$ and $\bar{t}_1$ are solutions to $\max W(\theta_1)$. In what follows, we show that generically $W(\theta_1)$ has only one maximum.

With some abuse of notation, let $V(\theta_1, c) = u_0\left(z^1(\theta_1)\right) = c - x_1^1(\theta_1) - \theta_0\left(y^1(\theta_1) - \hat{y}_0\right)^2$ and $W(\theta_1, c) = V(\theta_1, c)\,G_1(\theta_1) + u_0(s)(1 - G_1(\theta_1))$. Since the cross derivative $\partial^2 W/\partial c\partial\theta_1 = dG_1/d\theta_1 > 0$, the function W satisfies the strict increasing difference in (θ_1, c). Then, results from monotone comparative statics literature (see, e.g., Theorem $4'$ in Milgrom and Shannon

(1994)) imply that for any $\theta_1' \in \arg\max_{\theta_1 \in [\underline{t}_1, \bar{t}_1]} W(\theta_1, c')$ and $\theta_1'' \in \arg\max_{\theta_1 \in [\underline{t}_1, \bar{t}_1]} W(\theta_1, c'')$, we have $\theta_1'' \geq \theta_1'$ if $c'' > c'$. This implies that if for some c', $\arg\max_{\theta_1 \in [\underline{t}_1, \bar{t}_1]} W(\theta_1, c') = \{\tilde{\theta}_1, \bar{t}_1\}$ where $\tilde{\theta}_1 < \bar{t}_1$, then for any $c'' > c'$, $\arg\max_{\theta_1 \in [\underline{t}_1, \bar{t}_1]} W(\theta_1, c'') = \{\bar{t}_1\}$. Hence the solution to $\max_{\theta_1 \in [\underline{t}_1, \bar{t}_1]} W(\theta_1)$ is generically unique. ■

Extension to more than three players

Suppose the set of legislators other than the chair is $N = \{1, \ldots, n\}$ where $n \geq 3$ and the voting rule requires $(\kappa + 1) \geq 2$ votes for a proposal to pass. Since we can assume without loss of generality that the chair votes for any proposal she makes, a proposal needs κ out of the n legislators to vote for it to pass. Call this the κ voting rule. Assume that $1 \leq \kappa < n$. If $\kappa = \min\{x \in N : x \geq n/2\}$, then the voting rule is the majority rule, but our results apply to more general voting rules. Suppose only legislators 1 and 2 have private information on their types, that is, for legislator $i \in \{3, \ldots, n\}$, the distribution of θ_i, still denoted by F_i, is degenerate. We maintain the same assumptions on the players' preferences as in the main text. Denote this game by Γ^N. The definition of equilibrium is analogous to that in the main text and is omitted.

Assume that $\hat{y}_0 < \tilde{y}$. If at least κ legislators prefer $\hat{y}_0$ to $\tilde{y}$, the chair's optimal proposal is $(\hat{y}_0; c, 0, \ldots, 0)$. If $k < \kappa$ legislators prefer $\hat{y}_0$ to $\tilde{y}$, then the analysis of Γ^N with the κ voting rule is similar to that of the game in which the chair faces the $(n - k)$ legislators who strictly prefer $\tilde{y}$ to $\hat{y}_0$ and the voting rule is the $(\kappa - k)$ voting rule. Henceforth, we assume that every legislator $i \in N$ strictly prefers $\tilde{y}$ to $\hat{y}_0$, and as before, let $e(\hat{y}_i) = \min\{y : v(y, \hat{y}_i) = v(\tilde{y}, \hat{y}_i)\}$.

For an integer $1 \leq q \leq n$, say that a proposal $(y; x)$ is a *q-transfer proposal* if $x_i > 0$ for exactly $q \geq 1$ legislators in N. Say that a proposal $(y; x)$ is a no-transfer proposal if $x_i = 0$ for all $i \in N$. As in the main text, we focus on monotone equilibria (defined analogously). Suppose the prior F_i $(i = 1, 2)$ has a differentiable density function f_i. In the following lemma, we show that if F_1 and F_2 satisfy the IHRP, then no proposal elicited in a monotone equilibrium is a q-transfer proposal where $q > \kappa$. Denote legislators other than i by $-i$.

Lemma 9. *Suppose F_1 and F_2 satisfy the IHRP. Fix a monotone equilibrium in Γ^N and let $z^* = (y^*; x^*)$ be a proposal elicited in this equilibrium. (i) The proposal z^* is either a no-transfer proposal or a q-transfer proposal where $q \leq \kappa$. (ii) If any legislator $i \in \{1, 2\}$ is included in z^*, then at most κ legislators vote for z^* in this equilibrium.*

Proof. Part (i): Suppose z^* is a q-transfer proposal. Note that for $i \in \{3,\ldots,n\}$, if $x_i^* > 0$, then $u_i(z^*) \geq u_i(s)$ and legislator i votes for z^*. This is because if $u_i(z^*) < u_i(s)$, then there exists a proposal $z' = (y';x')$ where $y' = y^*$, $x_0' = x_0^* + x_i^*$, $x_{-i}' = x_{-i}^*$ and $x_i' = 0$ such that the same set of legislators vote for z^* and z' with the same probability. Since $x_0' > x_0^*$, we have $u_0(z') > u_0(z^*)$ and it is strictly better to propose z' than z^*, a contradiction. It follows immediately that z^* includes at most κ legislators in $\{3,\ldots,n\}$. Consider the following possibilities.

(a) Suppose z^* includes κ legislators in $\{3,\ldots,n\}$. Since every such legislator votes for z^*, the proposal z^* does not include legislators 1 and 2, i.e., $q = \kappa$.

(b) Suppose z^* includes $k \leq \kappa - 1$ legislators in $\{3,\ldots,n\}$. Let l be the number of legislators in $\{3,\ldots,n\}$ who vote for z^*. As established earlier, any legislators in $\{3,\ldots,n\}$ who is included in z^* votes for z^* and therefore $k \leq l$.

If $l \geq \kappa$, then $x_1^* = x_2^* = 0$ and $q = k < \kappa$.

If $l \leq (\kappa - 2)$, then $k \leq \kappa - 2$ and $q \leq k + 2 \leq \kappa$.

Suppose $l = \kappa - 1$. Then $k \leq (\kappa - 1)$. Suppose, towards a contradiction, that $q > \kappa$, which implies that $k = (\kappa - 1)$ and $x_1^* > 0$ and $x_2^* > 0$. Suppose z^* is induced by m, and for $i = 1, 2$, let G_i denote the chair's posterior on legislator i's type when receiving m_i and let g_i denote the associated density. Recall that for $i = 1, 2$ and any proposal z, $t_i(z)$ denotes the highest type of legislator i who is willing to accept z. Let $\beta(z) = 1 - (1 - G_1(t_1(z)))(1 - G_2(t_2(z)))$, the probability that at least one of legislators 1 and 2 vote for z. Since z^* is optimal, and $k = \kappa - 1$ legislators in $\{3,\ldots,n\}$ accept z^*, it follows that x^* is a solution to the problem

$$\max_{x \in X}(c - \sum_{i=1}^{n} x_i^* + \theta_0 v(y^*, \hat{y}_0))\beta(y^*; x) + \theta_0 v(\tilde{y}, \hat{y}_0)(1 - \beta(y^*; x))$$

subject to $x_1 + x_2 = x_1^* + x_2^*$ and $x_i = x_i^*$ for $i = 3, ..., n$. But as shown in the proof of Lemma 3, this is impossible in a monotone equilibrium if F_i satisfies IHRP for $i = 1, 2$. Hence $q \leq \kappa$.

Part (ii): Suppose $x_1^* > 0$ and $x_2^* > 0$. Then, as shown in the proof of part (i), at most $(\kappa - 2)$ legislators in $\{3,\ldots,n\}$ vote for z^* and therefore at most κ legislators vote for z^*.

Suppose $x_1^* > 0$ and $x_2^* = 0$ (the argument is similar if $x_1^* = 0$ and $x_2^* > 0$). Then, as shown in the proof of part (i), this happens only if $l \leq (\kappa - 1)$. Suppose $(\kappa - 1)$ legislators in $\{3,\ldots,n\}$ and legislator 2 vote for z^*. Then there exists $z' = (y';x')$ with $y' = y^*$, $x_0' = x_0^* + x_1^*$, $x_1' = 0$ and $x_{-1}' = x_{-1}^*$ such that κ legislators vote for z'. Since $u_0(z') > u_0(z^*)$, this contradicts the optimality of z^*. Hence, at most $(\kappa - 1)$ legislators other than legislator 1 vote for z^* and therefore at most κ legislators vote for z^*. ■

Lemma 9 implies that in a monotone equilibrium, if legislator $i \in \{1,2\}$ is included in z^*, then his payoff is at least as high as his status quo payoff when the chair proposes z^*. This is because by part (ii) of Lemma 9, if legislator i votes against z^*, it will fail to pass, that is, legislator i has veto power with respect to z^*. This is analogous to the result in $\Gamma^{\{1,2\}}$ that legislator i is pivotal with respect to a proposal z^* if z^* includes legislator i.

Define simple monotone equilibrium in Γ^N analogously as in $\Gamma^{\{1,2\}}$. The following Proposition says that in Γ^N, each legislator still can convey at most whether he will "fight" or "compromise," just like in $\Gamma^{\{1,2\}}$.

Proposition 7. *Suppose F_1 and F_2 satisfy the IHRP. Fix a simple monotone equilibrium (μ, γ, π) in Γ^N. Suppose legislator $i \in \{1,2\}$ is informative in this equilibrium. Then there exist m_i^c, $m_i^f \in M_i$ such that $q_i(m_i^c) > 0$, $q_i(m_i^f) = 0$. Moreover, μ_i is equivalent to a size-two message rule μ_i^{II} with the property that there exists $\theta_i^* \in \left(\underline{\theta}_i, \bar{\theta}_i\right)$ such that $\mu_i^{II}(\theta_i) = m_i^c$ for $\theta_i < \theta_i^*$ and $\mu_i^{II}(\theta_i) = m_i^f$ for $\theta_i > \theta_i^*$.*

The proof is similar to that of Proposition 3. Specifically, we extend Lemma 6 and Lemma 7 to Γ^N and we use Lemma 9 in place of Lemma 3 and Lemma 5 in the proofs of the extensions of Lemma 6 and Lemma 7. (We also replace the phrase "legislator i is pivotal with respect to z" with the phrase "legislator i has veto power with respect to z" in the relevant places.)

Define $\hat{y}^\kappa \in \{\hat{y}_1, \ldots, \hat{y}_n\}$ implicitly by $\#\{i \in N : \hat{y}_i \leq \hat{y}^\kappa\} \geq \kappa - 1$ and $\#\{i \in N : \hat{y}_i \geq \hat{y}^\kappa\} \geq n - (\kappa - 1)$. If the κ voting rule is the majority rule, then $\hat{y}^\kappa$ is the median position.

Proposition 8. *Suppose F_1 and F_2 satisfy the IHRP. Fix a simple monotone equilibrium (μ, γ, π) in Γ^N. If $e(\hat{y}_i) > e(\hat{y}^\kappa)$, then legislator i is uninformative in this equilibrium.*

The proof is similar to that of Proposition 4 (ii). Applied to the majority rule, Proposition 8 says that if the median legislator wants to move the policy in the same direction as the chair does, then any legislator whose position is to the right of the median position is uninformative in any SME.

Unlike part (i) of Proposition 4, which says that at most one legislator is informative in an SME in $\Gamma^{\{1,2\}}$, it is possible in Γ^N that both legislators 1 and 2 are informative in an SME. Below, we provide an example (part (iii) of Example 3) that illustrates what an SME looks like when both legislators 1 and 2 are informative. In this example, one of them is the median and the other is to the left of the median. (Note that this does not arise in $\Gamma^{\{1,2\}}$ since it is necessarily the case that one legislator is the median and the other is to the right of the median in $\Gamma^{\{1,2\}}$.) Part (i) of Example 3 illustrates that if a legislator is the median, then there exists an SME in which only he is informative. Part (ii) of Example 3 illustrates that if a legislator is to the left of the median, then under some conditions (roughly, when his position is sufficiently close to that of the median), then there exists an SME in which only he is informative.

Example 3. *Suppose* $n = 3$, $\kappa = 2$ *(majority rule),* $\tilde{y} = 0$, $\hat{y}_0 = -1$, $\hat{y}_3 = -0.3$, $c = 1$. *Assume that* $u_i(z, \theta_i, \hat{y}_i) = x_i - \theta_i (y - \hat{y}_i)^2$ *for* $i = 0, 1, 2, 3$, $\theta_0 = 1$, $\theta_3 = 1$, *and* θ_1, θ_2 *are both uniformly distributed on* $[1/4, 4]$.

(i) Suppose $\hat{y}_3 < \hat{y}_1 < \hat{y}_2$. Then legislator 1 is the median. By Proposition 8, legislator 2, whose position is to the right of the median, is uninformative in any SME in $\Gamma^{\{1,2,3\}}$. There are SME in which legislator 1 is informative. For instance, let $\hat{y}_1 = -0.2$ (so $e(\hat{y}_1) = -0.4$) and suppose $\mu_1(\theta_1) = m_1^c$ if $\theta_1 \in [1/4, 1]$, $\mu_1(\theta_1) = m_1^f$ if $\theta_1 \in (1, 4]$, and $\mu_2(\theta_2) = m_2^*$ for all θ_2. Given the message rules, when the chair receives m_1^f, she infers that $\theta_1 \in (1, 4]$ and proposes $(e(\hat{y}_1); c, 0, 0, 0) = (-0.4; 1, 0, 0, 0)$. Legislators 1 and 3 accept the proposal and legislator 2 rejects the proposal. When the chair receives m_1^c, she infers that $\theta_1 \in [1/4, 1]$ and proposes $(y; c - x_1, x_1, 0, 0)$ where $y = -0.6 < e(\hat{y}_1)$ and $x_1 = 0.12 > 0$. Again, only legislators 1 and 3 accept the proposal.

(ii) Suppose $\hat{y}_1 < \hat{y}_3 < \hat{y}_2$. Then legislator 3 is the median. Again, legislator 2, whose position is to the right of the median, is uninformative in any SME in $\Gamma^{\{1,2,3\}}$. Whether it is possible for legislator 1, whose position is to the left of the median, to be informative in some SME depends on how close $\hat{y}_1$ is to the chair's position relative to the median's position.

For example, suppose $\hat{y}_1 = -0.31$ (so that $e(\hat{y}_1) = -0.62$). Consider an SME in which $\mu_1(\theta_1) = m_1^c$ if $\theta_1 \in [1/4, 1]$, $\mu_1(\theta_1) = m_1^f$ if $\theta_1 \in (1, 4]$, and $\mu_2(\theta_2) = m_2^*$ for all θ_2. When the chair receives m_1^f, she infers that $\theta_1 \in (1, 4]$ and proposes $(e(\hat{y}_1); c - x_3, 0, 0, x_3)$ where $x_3 = 0.0061$. When the chair receives m_1^c, she infers that $\theta_1 \in [1/4, 1]$ and proposes $(y; c - x_1 - x_3, x_1, 0, x_3)$ where $y = -0.65 < e(\hat{y}_1)$, $x_1 = 0.0195$ and $x_3 = 0.0325$. In both cases, legislators 1 and 3 accept the proposal and legislator 2 rejects the proposal.

Suppose instead $\hat{y}_1 = -0.4$ (so that $e(\hat{y}_1) = -0.8$). Because $\hat{y}_1$ is sufficiently close to the chair's position relative to the median's position, there exists no SME in which legislator 1 is informative. To see this, suppose there is an SME in which legislator 1 follows a size two message rule: $\mu_1(\theta_1) = m_1^c$ if $\theta_1 < \theta_1^*$ and $\mu_1(\theta_1) = m_1^f$ if $\theta_1 > \theta_1^*$ for some θ_1^* such that legislator 1 is included when sending m_1^c and excluded when sending m_1^f. (By Proposition 7, if legislator 1 is informative in an SME, then μ_1 is equivalent to such a message rule.) Straightforward calculation shows that conditional on excluding legislator 1, the chair's optimal proposal is $(y; c - x_3, 0, 0, x_3)$ where $y = -0.65$, $x_3 = 0.0325$, and legislators 1 and 3 accept this proposal. Since $y = -0.65 > e(\hat{y}_1)$, legislator 1 gets a payoff strictly higher than his status quo payoff when sending m_1^f. Recall that legislator 1 is included when sending m_1^c. Arguments similar to Lemma 6 show that the threshold type θ_1^* gets a payoff equal to his status quo payoff by sending m_1^c (followed by his optimal acceptance rule). Hence type θ_1^* strictly prefers sending m_1^f to m_1^c, a contradiction. So there exists no SME in which legislator 1 is informative.

(iii) Suppose $\hat{y}_1 < \hat{y}_2 < \hat{y}_3$. Then legislator 2 is the median legislator. There exists an SME in which both legislators 1 and 2 are informative. For example, let $\hat{y}_1 = -0.35$, $\hat{y}_2 = -0.325$ (so that $e(\hat{y}_1) = -0.7$ and $e(\hat{y}_2) = -0.65$) and suppose $\mu_i(\theta_i) = m_i^c$ if $\theta_i \in [1/4, 1/2]$ and $\mu_i(\theta_i) = m_i^f$ if $\theta_i \in (1/2, 4]$ for $i = 1, 2$. When the chair receives m with $m_2 = m_2^f$, she infers that $\theta_2 \in (1/2, 4]$. In this case, independent of m_1, she proposes $(e(\hat{y}_2); c, 0, 0, 0) = (-0.65; 1, 0, 0, 0)$. When the chair receives (m_1^f, m_2^c), she infers that $\theta_1 \in (1/2, 4]$ and $\theta_2 \in [1/4, 1/2]$ and proposes $(e(\hat{y}_1); c - x_2, 0, x_2, 0)$ where $x_2 = 0.0175$. When the chair receives (m_1^c, m_2^c), she infers that $\theta_1 \in [1/4, 1/2]$ and $\theta_2 \in [1/4, 1/2]$ and proposes $(y; c - x_1 - x_2, x_1, x_2, 0)$ where $y = -0.775 < e(\hat{y}_1)$, $x_1 = 0.029$ and $x_2 = 0.048$. In all four cases, legislators 1 and 2 vote for the proposal and legislator 3 vote against the proposal.

Intuitively, when the median legislator sends the "fight" message, the chair makes no transfers and moves the policy so that the median legislator is just willing to accept. When the median legislator sends the "compromise" message, the chair's proposal depends on the message of the closer legislator. If the closer legislator says "fight", then the chair moves the policy so that the closer legislator is just willing to accept without getting any transfer, and compensates the median legislator accordingly. When the closer legislator also says "compromise", the chair moves the policy even closer to her ideal, and compensates both legislators.

References

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