

## **Illiquidity and Interest Rate Policy**

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The cheapest way for banks (and firms) to finance long term illiquid projects is typically by borrowing short term from households. But when household needs for funds are high, interest rates will spike, debtors will have to shut down illiquid projects, and in extremis, will face more damaging runs. Authorities may want to push down interest rates to maintain economic activity in the face of such illiquidity, but intervention may not always be feasible, and when feasible, could encourage borrowers to fund even more illiquid projects up front. Regulatory limits on leverage are no panacea either. Indeed, to restore appropriate incentives, authorities may have to commit to raising rates when low, to counter the distortions created by lowering them when high. We draw implications for interest rate policy to combat illiquidity.

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There has been substantial recent debate on the role of central banks and interest rates, not so much in controlling inflation, but in dealing with episodes of illiquidity and financial fragility. For instance, Greenspan (2002) has argued that while the Federal Reserve cannot recognize or prevent asset price booms, it can “mitigate the fallout when it occurs and, hopefully, ease the transition to the next expansion.” Others have responded that by following such an asymmetric interest rate policy – colloquially known as the Greenspan put -- a central bank can engender the kind of behavior that makes booms and busts more likely. In this paper, we ask whether a central bank can alleviate financial stress through untargeted lending at market interest rates.<sup>2</sup> We then ask whether anticipation of such intervention gives banks the incentive to undertake even more illiquid projects up front. Finally, we ask how such distortions to incentives can be rectified.

We start with a model where entrepreneurs borrow from banks to invest in long- term projects. Banks themselves borrow from risk-averse households, who receive endowments every period. Households deposit their initial endowment in banks in return for demandable deposit claims (throughout the paper, we focus on demand deposits, though any form of overnight unsecured debt could be a close substitute). There is no uncertainty about the average quality of a bank’s projects in our model, so the bank’s asset side is not the source of the problem (we relax this later). However, there is uncertainty about household endowments (or equivalently, incomes) over time. This, coupled with the mismatch between the long gestation period for the projects and the demandable nature of deposits, is the source of banking sector difficulties.

Once households have deposited their initial endowment, and projects have been started, households may have an unexpectedly high need to withdraw deposits. One obvious possibility is that they suffer an adverse shock to current endowment that causes them to want to run down financial assets to consume. But another is that they anticipate significantly higher income or endowments in the future

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<sup>2</sup> Clearly, financial fragility can be alleviated if the central bank is willing to make loans at below-market interest rates. This is tantamount to recapitalizing borrowers.

and want to smooth consumption. Thus anticipated prosperity, as well as current adversity, can increase current household demand for consumption goods substantially. We focus on the former (which has antecedents in the Austrian School of Von Mises and Hayek – see later), though the results in this paper apply for the most part to the latter also.

As households withdraw deposits to satisfy consumption needs, banks will have to call in loans to long gestation projects in order to generate the resources to pay them. The real interest rate will rise to equate the household demand for consumption goods and the supply of these goods from terminated projects.<sup>3</sup> Thus greater consumption demand will lead to higher real rates and more projects being terminated, as well as lower bank net worth. This last effect is because the bank's loans pay off only in the long run, and thus fall in value as real interest rates rise, while the bank's liabilities, that is demandable deposits, do not fall in value. Eventually, if rates rise enough, the bank may have negative net worth and experience runs. Runs can be very destructive of value because all manner of projects, including those viable at prevailing interest rates, are terminated. Anticipated future prosperity, as well as current adversity, can thus induce fragility in the banking system, and slow the real economy.

How can this tendency towards banking sector fragility be mitigated? One obvious possibility is to alter the structure of banks. If deposits were not demandable, a loss of net worth would not result in a destructive run. And if banks financed themselves with long term liabilities that fell in value as real interest rates rose, banks would be doubly stable. Not only would bank liabilities fall to offset the fall in asset values, thus protecting bank net worth, but also household wealth would fall as their holdings of long term claims on banks fell in value, offsetting their desire to consume somewhat. Even if deposits had to be demandable, banks would be more stable if they financed with lower levels of deposits, or if deposits could be made state contingent, so they fluctuated appropriately in value with the aggregate

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<sup>3</sup> While we focus on project liquidations (or equivalently, a halt in new projects) as affecting the aggregate supply of goods available for consumption, it is also plausible to think of them as affecting aggregate demand, as people lose jobs. We do not model this, though the qualitative effects would be similar.

consumption demands in each state. So can the structure of banks be altered easily while they continue to intermediate efficiently?

The answer from our previous work is no. Bank debt is demandable because it allows bankers to commit to pay out the value they collect (Diamond and Rajan (2001)). Put differently, given the skills needed to make loans to entrepreneurs and recover payments, financing with demandable debt is the cheapest form of financing for banks – financing with long term liabilities would reduce the efficiency of intermediation substantially. Given this, competitive conditions determine how much banks lever up. Banks will be willing to accept some probability of financial distress and runs in order to commit to pay depositors more, and thus attract funds. Thus the commitment value of a hard liability structure, coupled with ex ante competition for funds, can lead to highly levered banks.

But why can't the face value of deposits be made state-contingent? The problem really is that the state is hard to observe or verify in real time, and its correlation with the value of illiquid bank assets hard to determine precisely.<sup>4</sup> Making deposits contingent on the state could lead to more instability, as depositors attempt to guess the state and attempt to front run any diminution of the value of their deposits.<sup>5</sup>

If altering bank structure is not easy, what about government intervention? It is not sufficient to just keep the volume of bank lending up if consumption is not altered. In our model, given the demand for consumption, the only way the market clears is if interest rates rise, bank lending falls (because projects are no longer so attractive at the higher interest rate), projects are terminated, and goods released

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<sup>4</sup> Banks are thus unlike mutual funds holding traded assets, where there is a precise market value of assets.

<sup>5</sup> The problems caused by the need for short-term finance are not limited only to banks, although we do not separately analyze this here. When bank monitoring is not needed but the legal enforcement of debt contracts is costly, borrowers must issue short-term debt to fully exploit their debt capacity (Diamond (2004, 2008)), and leave their future fate subject to refinancing risk. Consequently, results similar to ours hold in an economy where debt is short-term, regardless of whether it is bank debt or not. Corporate debt maturity cannot be adjusted to facilitate stability without reducing access to capital.

for consumption. Put differently, there is a goods market that has to clear and any intervention has to recognize this.

The government can always violate property rights and keep the banking system intact – for instance, taxing households and gifting the proceeds to banks. This sort of bailout would reduce household consumption while limiting project termination, and therefore is politically unpopular. While recognizing that such interventions may be necessary in extremis, we want to focus on central bank interventions – lending or borrowing from the market to alter interest rates – that seemingly do less violence to property rights.

Lending takes resources. Once we recognize the fact that these resources have to ultimately come from the households, we see there are a variety of circumstances in which attempts to alter the real interest rate could be ineffective. The intuition is as follows; the authorities could tax households' current endowments to secure these resources, lend them to banks at market rates, and then repay the households in the future at the market interest rate so as to not violate their property rights. But so long as households' consumption possibility set is not affected, their consumption will not change, and such intervention will have no effect on real interest rates or bank net worth; Households will respond to any tax on their current endowment by withdrawing an equivalent amount from banks so as to keep consumption constant, so any such tax and transfer scheme does not offer net additional resources to the bank to meet withdrawals, or reduce the pressure on them to terminate projects. This is a form of Ricardian Equivalence (Barro (1974)) that limits how much governments can do.<sup>6</sup>

However, there is a way the government can have an effect on interest rates through taxing and lending. Some households may not participate in the banking system, either because they withdraw all

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<sup>6</sup> It is well understood that government borrowing to finance lump sum tax and transfer programs have effect only when this Ricardian Equivalence does not hold. Government lending at market rates of interest (liquidity lending), using its taxation authority to raise resources, with the interest augmented principal transferred back to households in a lump sum manner is almost the same transaction in reverse, and should have no effect for similar reasons.

their deposits to compensate for current taxes or because interest rates are too low, given their endowments, for them to deposit in the first place. A further tax on these households' current endowments will not be compensated for by equivalent withdrawals from the banking system, since these households already have no claims on the banking system. The consumption of these households will fall. The amount collected from this tax is then an incremental source of resources to the banking system, and by taxing and lending, the government can bring down the real interest rate, reduce project termination and maintain activity, thus bolstering the net worth of banks.

Intervention through taxing and lending, even if seemingly respectful of household property rights, has real effects only when it forcibly changes household consumption decisions. Of course, if it prevents runs, this change could be Pareto improving. But if the objective is not to head off runs, some households could be made worse off, even if they are "fully" compensated for taxes at market interest rates.

An equally important concern is the effect of anticipation of such intervention on actions. If the authorities are expected to reduce interest rates when liquidity is at a premium, banks will take on more short-term leverage or invest in more illiquid projects, thus bringing about the very states where intervention is needed, even if they would not do so in the absence of intervention.

Indeed, the current financial turmoil in the United States could be thought of as one where borrowers, anticipating a continued environment of low interest rates following the implosion of the "tech bubble" in early 2000 and the subsequent collapse of corporate investment, chose to take on more illiquid financial assets financed with short term debt. Anticipation of low interest rates may have been strengthened by the so-called Greenspan Put, whereby financial markets believed that if the markets ever came under strain because of excessive expansion, the Federal Reserve would cut interest rates. Eventually, the strains started showing as the Federal Reserve started raising interest rates, to keep pace with the growing recovery and strong consumption growth.

Returning to our model, regulatory authorities might be tempted to institute a maximum “leverage” requirement on the banks in order to combat potential financial fragility. Yet banks are unlikely to stand still in the face of such a requirement. While effective under some circumstances, the requirement could push banks into choosing more illiquid projects under others.

A partial antidote to all such aberrant behavior is to push down interest rates even below the level strictly necessary to restore stability in fragile states, while pushing up interest rates in states where the interest rate would otherwise be low. The purpose of pushing interest rates below the level strictly necessary to render banks solvent is to reward those banks that have chosen to remain relatively liquid with additional net worth. The value of pushing interest rates up in low rate states is to penalize the net worth of banks that have chosen to be illiquid. While it may be politically difficult for the authorities to raise rates in environments where there is no obvious reason for doing so, our model suggests interest rate smoothing may be useful to increase systemic stability and limit regulatory arbitrage.

In sum then, this paper attempts to derive the sources of illiquidity, and thus of economic volatility from necessary contractual rigidities in lending, and from the behavior of banks and households. In doing so, we abstract from distortions in lending resulting from banker misperceptions or incentive problems which, while no doubt important, have been explored in other work. We then focus on the consequences of intervention by the government, primarily through taxing and lending. Importantly, we focus on the real consequences of intervention, abstracting from monetary frictions. We do this deliberately, because it isolates what is going on in real terms, making for a better understanding when, in future research, money is added back to this framework. The rest of the paper is as follows. In section I, we lay out the basic model, in section II, we solve it, in section III we consider macroeconomic intervention on interest rates, and in section IV, we consider how intervention can affect the choice of assets, and what this implies for the nature of intervention. We then conclude.

## I. The Framework

### 1.1. Agents, Endowments, Technology, Preferences.

Consider an economy with risk-neutral entrepreneurs and bankers, and risk-averse households. The economy has three dates, 0, 1, and 2. Each household is initially endowed with a unit of good at date 0. Households can invest their initial endowment in banks, which will lend the resources to entrepreneurs. At date 1, households will get endowment  $e_1$ . They also learn their date-2 endowment. Some households will get a high endowment,  $e_2^H$  at date 2 while the rest will get a low endowment  $e_2^L$ . Let there be two states of the world at date 1, the *exuberant* state where  $\theta^E$  is the fraction of high endowment households and the *normal* state, where  $\theta^N$  is the fraction, with  $\theta^E > \theta^N$ . Thus E is a state with greater anticipated prosperity, where more households expect a high endowment. The probability of state E is  $p^E$ . The state of nature and household types are not verifiable and in addition, household types are private information.

Each entrepreneur has a project, which requires the investment of a unit of good at date 0. The project produces  $\tilde{Y}_2$  in goods at date 2 if it is not liquidated, and  $X_1$  at (or after) date 1 if the project is liquidated. At date 0,  $\tilde{Y}_2$  is uncertain for each entrepreneur, with outcomes becoming known at date 1 and distributed uniformly over the range  $[\underline{Y}_2, \bar{Y}_2]$ . Entrepreneurs have no goods to begin with, and the demand from entrepreneurs looking for funds at date 0 is greater than the supply with households.

Households maximize expected utility of consumption given by  $E(\log C_1 + \log C_2)$ . Risk neutral entrepreneurs and bankers maximize  $E(C_1 + C_2)$ .

### 1.2. Financing Entrepreneurs

Anyone will lend only to the extent that they can coerce borrowers into repaying, either by independently being able to generate value with the borrower's assets and thus being able to obtain some protection in case of default, or by finding ways to commit to inflict serious damage to the borrower.

Specifically, since entrepreneurs have no endowments, they need to borrow to invest. Each entrepreneur can borrow from a banker who has, or can acquire during the course of lending, knowledge about an alternative, but less effective, way to run the project. The banker's specific knowledge allows him to collect  $\gamma Y_2$  from an entrepreneur whose project just matures, with  $\gamma < 1$ . Once a banker has lent, no one else (including other bankers) has the knowledge to collect from the entrepreneur.<sup>7</sup>

Because there are more entrepreneurs than households, not all projects are funded. Banks can ask entrepreneurs to repay the maximum possible for a loan – they will lend only if the entrepreneur promises to pay  $\bar{Y}_2$  on demand. If the entrepreneur fails to pay on demand, he can make a counter-offer to the bank. If that offer is rejected, the bank takes over the project and either harvests date-2 cash flow  $\gamma Y_2$  or liquidates it.<sup>8</sup>

### **1.3. Financing Banks**

Since bankers have no resources initially, they have to raise them from households. But households have no collection skills (and bank loans are worthless in their hands), so how do banks commit to repaying households? By issuing demand deposits! In our previous work (Diamond and Rajan (2001)), we argued that the demandable nature of deposit contracts introduces a collective action problem for depositors that makes them run to demand repayment whenever they anticipate the banker cannot, or will not, pay the promised amount. Because bankers will lose all rents when their bank is run, they will repay the promised amount on deposits whenever they can.

Deposit financing introduces rigidity into the bank's required repayments. Ex ante, this enables the banker to commit to repay if he can (that is, avoid strategic defaults by passing through whatever he collects to depositors), but it exposes the bank to destructive runs if he truly cannot pay (it makes non-

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<sup>7</sup> For a relaxation of these assumptions, see Diamond and Rajan (2001).

<sup>8</sup> While we assume here that households lend via the bank, we show in Diamond and Rajan (2001) that banks and their fragile liability structures arise endogenously to facilitate the flow of credit from investors with uncertain consumption needs to entrepreneurs who have hard-to-pledge cash flows.

strategic default more costly): when depositors demand repayment before projects have matured and the bank does not have the means of payment, it will be forced to liquidate projects to get  $X_1$  immediately instead of allowing them to mature and generate  $\gamma Y_2$ . In addition to making banks fragile, short-term funding leaves households exposed to interest rate risk when reinvesting from date 1 to date 2.

Banks are competitive, and we assume that if it offers a competitive rate, each bank attracts enough entrepreneurs so that the distribution of  $\tilde{Y}_2$  among entrepreneurs it finances matches the aggregate distribution of entrepreneurs. Because the date-0 endowment is scarce relative to projects, banks will compete to offer the most attractive deposit face value  $D$  to households per unit of endowment deposited (henceforth, all values will be per unit of endowment).

*Assumption 1:* (i)  $\frac{e_2^H}{e_1} > \frac{\gamma \bar{Y}}{X_1}$  and (ii)  $\frac{e_2^L}{e_1} > \frac{\gamma \underline{Y}}{X_1}$

Assumption 1 (i) ensures that at the highest interest rate payable by firms, the H household wants to withdraw at least some of its deposits, while assumption 1 (ii) ensures that the L type wants to withdraw at the lowest interest rate payable. These assumptions can be dispensed with but limit the number of cases we have to examine.

**Timeline**

<b>Date 0</b>	<b>Date 1</b>	<b>Date 2</b>
Banks offer deposit terms and entrepreneurs offer loan repayment terms. Households get endowment and invest in banks. Banks lend to entrepreneurs.	Uncertainty over states, household date-2 endowments, and project outcomes revealed. Households decide how much to withdraw and how much to consume. Banks decide what projects to liquidate.	Projects mature, loans repaid, and deposits fully withdrawn from banks. All agents consume.

## II. Solving the basic model

In what follows, we will start by solving the bank's decision vis a vis entrepreneurs at date 1, then the households' consumption and withdrawal decisions, and finally, the bank's date 0 decision on what level of deposit repayment  $D$  to offer to maximize household willingness to deposit.

### 2.1. Bank decisions vis a vis entrepreneurs

Let us start our analysis at date 1, once uncertainty is revealed. Let the interest rate households demand in state  $s$  for re-depositing between dates 1 and 2 be  $r_{12}^s$ . The bank can get  $X_1$  at date 1 if the project is liquidated. Since the maximum it can collect from the entrepreneur is  $\gamma Y_2$  at date 2, this is the maximum the entrepreneur can commit to pay at date 1. So the bank liquidates projects with

$$Y_2 < Y_2(r_{12}^s) = \frac{r_{12}^s X_1}{\gamma} \quad (1.1)$$

and continues projects with  $Y_2 \geq Y_2(r_{12}^s)$  in return for a promised payment of  $\gamma Y_2$ . Liquidated entrepreneurs get nothing, while entrepreneurs who are continued retain  $(1 - \gamma)Y_2$ . Note that the present

value of the bank's assets at date 1 (before withdrawals) is  $\frac{1}{\bar{Y}_2 - \underline{Y}_2} \int_{\underline{Y}_2}^{Y_2(r_{12}^s)} X_1 dY_2 + \frac{1}{\bar{Y}_2 - \underline{Y}_2} \int_{Y_2(r_{12}^s)}^{\bar{Y}_2} \frac{\gamma Y_2}{r_{12}^s} dY_2$ ,

which is easily shown to fall in  $r_{12}^s$ .

### 2.2. Household decisions

Once uncertainty is revealed, and households know both the state and their endowments, they decide how much they want to withdraw and consume so as to maximize their expected utility of consumption. Of course, if they anticipate the bank will not be able to meet its obligations, they will collectively run on the bank, in which case all projects will be liquidated to pay households. We assume households can coordinate on a Pareto preferred Nash equilibrium. Thus we rule out panic based runs, which would only add to the inefficiencies we document. We start by considering situations where the bank will meet its obligations.

It is easy to show that when a household does not withdraw all its deposit, its utility is maximized when the marginal rate of substitution between consumption at dates 1 and 2 is equal to the prevailing deposit rate,  $r_{12}^S$ , that is, when  $\frac{U'(C_1)}{U'(C_2)} = \frac{C_2}{C_1} = r_{12}^S$ .<sup>9</sup> Of course, a household may have so high an endowment at date 2 (or so low an endowment at date 1) that its marginal rate of substitution, even after withdrawing all deposits, exceeds  $r_{12}^S$ .

If household H (with high date-2 endowment) withdraws amount  $w_1^{H,S}$  at date 1 (where  $w_1^{H,S} \leq D$ ), then

$$\frac{C_2^{H,S}}{C_1^{H,S}} = \frac{e_2^H + (D - w_1^{H,S})r_{12}^S}{e_1 + w_1^{H,S}} \quad (1.2)$$

Similarly, for household L with low date-2 endowment

$$\frac{C_2^{L,S}}{C_1^{L,S}} = \frac{e_2^L + (D - w_1^{L,S})r_{12}^S}{e_1 + w_1^{L,S}} \quad (1.3)$$

Inspection of the RHS of (1.2) and (1.3) suggests that for both households to have an equal marginal rate of substitution (and deposit at a common rate  $r_{12}^S$ ), it must be that H type households withdraw more. H

type households will have withdrawn everything when  $r_{12}^S$  falls below  $\frac{e_2^H}{e_1 + D}$ . After this, the deposit rate

only equals the L household's marginal rate of substitution (since the H household has withdrawn its

deposits completely), and the L household withdraws completely when  $r_{12}^S$  falls below  $\frac{e_2^L}{e_1 + D}$ .

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<sup>9</sup> The obvious intuition is that if the interest is lower, it could increase expected utility by withdrawing more at date 1 and consuming more, while if the interest rate is higher, it could increase expected utility by withdrawing less at date 1 and consuming more at date 2.

*Lemma 1:* If  $r_{12}^S \geq \frac{e_2^H}{e_1 - D}$ , both households leave all their money in the bank at date 1. If

$\frac{e_2^H}{e_1 - D} > r_{12}^S > \frac{e_2^H}{e_1 + D}$ , neither household withdraws fully from the banking system, but the H household

withdraws more than the L household. If  $\frac{e_2^H}{e_1 + D} \geq r_{12}^S > \frac{e_2^L}{e_1 + D}$ , the H household withdraws fully, while

the L household maintains some deposits. If  $\frac{e_2^L}{e_1 + D} \geq r_{12}^S$ , both households withdraw fully.

Somewhat obviously, higher the future endowment, the more will households want to withdraw from the banking system to consume today, and the higher the interest rate will have to be to keep them in.

### 2.3. Equilibrium

Assume first the bank can repay depositors without failing. In equilibrium, markets for goods at date 1 and date 2 have to clear. At date 1, goods are produced when banks liquidate projects. Because all banks have the same distribution of projects and will liquidate projects with  $Y_2 < Y_2(r_{12}^S) = \frac{r_{12}^S X_1}{\gamma}$  (see

(1.1)), date-1 liquidation proceeds are  $\frac{1}{\bar{Y}_2 - \underline{Y}_2} \int_{\underline{Y}_2}^{Y_2(r_{12}^S)} X_1 dY_2 = \left[ \frac{\frac{r_{12}^S X_1}{\gamma} - \underline{Y}_2}{\bar{Y}_2 - \underline{Y}_2} \right] X_1$ . Because this should equal

the goods consumed by withdrawing households<sup>10</sup>, it must be that (on simplifying)

$$\theta^S w_1^{H,S} + (1 - \theta^S) w_1^{L,S} = \frac{r_{12}^S X_1^2 - \gamma X_1 \underline{Y}_2}{\gamma (\bar{Y}_2 - \underline{Y}_2)} \quad (1.4)$$

where  $\theta^S$  is the fraction of H type households in state  $s$ .

<sup>10</sup> Assumption 1 rules out any consumption by either the banker or the entrepreneur at date 1 – since they are indifferent between consumption at either date, and the implied interest rate will always exceed 1, they are better off waiting.

*Lemma 2:*

(i) An equilibrium at date 1 in state  $s$  is an interest rate  $r_{12}^S$  and withdrawals by the H and L households,  $w_1^{H,S}, w_1^{L,S}$  such that the date 1 supply of goods equals the date 1 consumption, banks liquidate enough projects to meet withdrawals, and households do not want to, or cannot, withdraw more.

(ii) If it exists, the equilibrium is unique.

(iii) If there is a set  $\{r_{12}^S, w_1^{H,S}, w_1^{L,S}\}$  that solves  $\frac{e_2^H}{e_1 + 2w_1^{H,S} - D} = r_{12}^S$ ,  $\frac{e_2^L}{e_1 + 2w_1^{L,S} - D} = r_{12}^S$ ,

and (1.4), with  $r_{12}^S > 0$ ,  $w_1^{H,S} \in [0, D)$ ,  $w_1^{L,S} < D$ , then that is the equilibrium, else

if there are  $r_{12}^S$  and  $w_1^{L,S}$  that solve  $\frac{e_2^L}{e_1 + 2w_1^{L,S} - D} = r_{12}^S$ , and (1.4), with  $r_{12}^S > 0$ , and

$w_1^{L,S} \in [0, D)$ , then  $\{r_{12}^S, D, w_1^{L,S}\}$  is the equilibrium, else

$\left\{ \left[ \frac{\gamma D(\bar{Y}_2 - Y_2) + \gamma Y_2 X_1}{(X_1)^2} \right], D, D \right\}$  is the equilibrium.

Proof: See Appendix.

*Corollary 1:*

(i) H households with a higher date-2 endowment always withdraw (weakly) more than L households.

(ii) The interest rate  $r_{12}^S$ , total withdrawals,  $\theta^S w_1^{H,S} + (1 - \theta^S) w_1^{L,S}$ , and the fraction of projects liquidated all (weakly) increase in the fraction  $\theta^S$  of high endowment households and in the face value of deposits,  $D$ .

(iii) The net worth of banks decreases in the fraction  $\theta^S$  of high endowment households and in the face value of deposits,  $D$ .

Proof: See appendix.

Since H households have more date-2 endowment than L households, at any given interest rate they will consume (weakly) more at date 1, and hence will withdraw more. This means total withdrawals will (weakly) increase in the fraction  $\theta^S$  of high endowment households, which means the interest rate and liquidation will have to increase to equilibrate the market. Given the present value of the bank's assets decrease with  $r_{12}^S$ , while the value of its deposit liabilities do not, its net worth decreases with  $\theta^S$ . Turning to the effect of D, a higher face value of deposit claims increases the wealth of the depositor (provided the bank is not run), and increases his desire to consume immediately by withdrawing. The other implications follow.

Note that all these implications would also hold if households differed, not in their date-2 endowments, but in their date-1 endowments, with H households receiving a *lower* date 1 endowment and the higher marginal utility. The point is that inter-temporal marginal rates of substitution influence interest rates, and they could be high either if future prospects are good, or current conditions bad. Much of past analysis has followed Diamond-Dybvig (1983) and focused on liquidity shocks that are equivalent to poor current conditions, but it is useful to remember that exuberant views of the future could be equally problematic from the perspective of demands for liquidity. We will focus on this latter aspect through the paper, but it is useful to bear in mind that because interest rates depend on anticipated consumption growth, pressures can stem both from anticipated future prosperity or current adversity.

#### **2.4. Bank Fragility and the Ex Ante Choice of Deposit Rate**

Let us now examine each bank's choice at date 0 as to how much face value D to offer for a deposit of a unit of good. Since the market is competitive, banks will have to offer a D that maximizes household utility. Clearly, so long as the bank is not run in any state, a higher D makes households wealthier and better off. But the bank's net worth also falls by Corollary 1, and for a given D, is lower in state E because the fraction of H households is higher there than in state N (that is,  $\theta^E > \theta^N$ ). When D is high enough that the bank's net worth is completely eroded, the bank is run – which means all projects are

liquidated to generate funds to pay depositors, regardless of whether liquidation is value maximizing at the prevailing interest rate.<sup>11</sup> Each running household gets  $X_1$  immediately after a run.<sup>12</sup>

Let  $D^{E,\max}$  be the deposit level beyond which a run will be precipitated in the E state, while  $D^{N,\max}$  be the corresponding level in the N state. From our discussion above,  $D^{E,\max} < D^{N,\max}$ , so  $D^{E,\max}$  is the highest safe level of deposits, where no bank runs are experienced in either state, while with deposits set at  $D^{N,\max}$ , runs are experienced in the E state. If deposit repayments could be state contingent, then at date 0, the bank would offer to pay  $D^{E,\max}$  in state E and  $D^{N,\max}$  in state N. But state-contingent deposit contracts impose substantial demands on the contractual environment, which may not be met in practice (see the discussion earlier and Diamond and Rajan (2005)).

What would be the D competitive banks would set if they could not offer state contingent deposits? It is obvious that the only two candidates for D are  $D^{E,\max}$  and  $D^{N,\max}$ .<sup>13</sup>  $D^{E,\max}$  would mean the bank would not pay out the maximum it could in the N state. In contrast, setting the deposit level  $D^{N,\max}$  would ensure the bank would be run in the E state, which would reduce its value significantly below what would obtain if the deposit level were  $D^{E,\max}$  (since all projects are liquidated in a run even though at the prevailing interest rate, some deserve to be continued – the desire to withdraw money independent of consumption needs is what distinguishes a run from a normal withdrawal). This then is

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<sup>11</sup> This captures the idea that depositors simply want their money back before each bank runs out of resources. The results depend only on bank runs being inefficient, which could also stem from a fraction of projects being liquidated at fire-sale values, that is, at less than  $X_1$ .

<sup>12</sup> We assume that each household's deposits are evenly spread across banks and it joins enough lines to get its share of the proceeds with certainty. A run would be more problematic if it contributed to uncertainty about who gets what, with some households lucky enough to be at the front of the line getting more than households at the back. This would add to the cost of the run and complicate the algebra, but not change the results qualitatively.

<sup>13</sup> We know a higher D, provided it does not precipitate a run, makes depositors wealthier at date-1, and thus is preferred. This means one candidate for the equilibrium D offered is the maximum no-run D,  $D^{E,\max}$ . Once a run is precipitated in the E state, a higher value of D makes no difference to outcomes or payouts in that state. Which means a second candidate equilibrium D is  $D^{N,\max}$ , which maximizes payout in the N state. Of course, going beyond  $D^{N,\max}$  will reduce value because the bank will be run in all states.

the basic trade-off banks face is setting deposits at  $D^{N,\max}$  rather than  $D^{E,\max}$  – commit to paying more in the N state but have a run in the E state. The trade-off turns on the cost of a run relative to its probability.

*Lemma 3:* Ceteris paribus, the lower the probability  $p^E$  of the exuberant state, the lower the expected cost of bank runs relative to the benefit of paying out more in state N, and the more attractive is the higher face value  $D^{N,\max}$  than  $D^{E,\max}$  to households.

Proof: See appendix.

Intuitively, since  $D^{N,\max}$  and  $D^{E,\max}$  maximize payouts for their specific states, the lower the probability of state E, the more attractive does  $D^{N,\max}$  become as the ex ante choice.

## 2.5. Example & Comparative Statics

Let  $p^E=0.5$ ,  $e_1=1.2$ ,  $e_2^L=0.6$ ,  $e_2^H=3.6$ ,  $X_1=0.95$ ,  $\underline{Y}_2=0.5$ ,  $\bar{Y}_2=2.5$ ,  $\gamma=0.9$ ,  $\theta^E=0.6$ ,  $\theta^N=0.3$ . First, assume that the deposit level is set at  $D^{E,\max}$ , the maximum debt level with no run in either state, which is 1.016. In the E state, the interest rate  $r_{12}^E$  is 1.70, H households withdraw 0.97, while L households withdraw 0.08. In the N state, the interest rate  $r_{12}^N$  is 1.28, the H households withdraw their entire deposit, while L households withdraw 0.14. Now let the deposit level be set at  $D^{N,\max}=1.12$ . Now the bank is run in the E state, while in the N state, the interest rate is 1.39, the H households withdraws all their deposit while the L households withdraw only 0.18.

In Figure 1 a, b, c, we plot the difference in household expected utilities between when deposits are set at  $D^{N,\max}$  and deposits are set at  $D^{E,\max}$  varying, ceteris paribus, (a) the probability of the exuberant state,  $p^E$  (b) the fraction of H households in the exuberant state  $\theta^E$ , and (c) the fraction of H households in the normal state,  $\theta^N$ .

As Figure 1 a shows, an increase in the probability of the N state,  $(1-p^E)$ , increases the relative utility of setting  $D=1.12=D^{N,\max}$ , the debt level that maximizes depositor utility in the N state. So if the probability of the N state,  $(1-p^E)$ , increases above 0.55, the bank will set a high leverage,  $D^{N,\max}$ ,

which ensures the bank will fail in the exuberant state, else it will set low leverage  $D^{E,Max}=1.016$ , which ensures the bank is always safe.

Turn next to how changes in the fraction of high endowment households in the exuberant state,  $\theta^E$ , affect choices. As  $\theta^E$  increases, the maximum safe deposit level,  $D^{E,Max}$ , falls, while  $D^{N,max}$  remains unaffected. As  $\theta^E$  increases, therefore, relatively more is paid out to depositors in the N state if the debt level is set at  $D^{N,max}$ , with no diminution in the E state (because a run fetches the same amount regardless of the promised deposit level). Moreover, as  $\theta^E$  increases, the realized interest rate for any deposit level in the E state will be higher (because of the greater presence of impatient depositors). This implies project liquidations will be higher even if the bank stays in business, which reduces the difference between its value and the value generated by a run bank. A final effect is that impatient high endowment depositors, who want to withdraw much of their money, benefit far less from a bank staying in business than patient low endowment depositors, who would like to leave their money in. Thus the utility loss from a bank run is lower for a high endowment depositor, so as their fraction increases in the E state, the ex ante attractiveness of risky deposits,  $D^{N,max}$ , over safe deposits,  $D^{E,Max}$ , increases. For all these reasons, the risky deposit level,  $D^{N,max}$ , looks more attractive than the safe deposit level,  $D^{E,Max}$ , as  $\theta^E$  increases.

Finally turn to changes in  $\theta^N$ . Here we have two opposing effects. On the one hand, a higher  $\theta^N$  reduces the maximum no-run deposit level in the N state,  $D^{N,max}$ , and thus reduces its relative attractiveness to depositors. Put differently,  $D^{E,Max}$  does not look so bad in the N state as  $\theta^N$  approaches  $\theta^E$  from below. On the other hand, there is also a composition effect – the high endowment households who withdraw everything benefit more from higher deposit levels in the N state than do low endowment households. When  $\theta^N$  rises from low levels, the composition effect initially dominates since the deposit levels between N and E states are so different. This is why the ex ante relative attractiveness of  $D^{N,max}$  first increases in Figure 1 C, then falls, with  $\theta^N$ .

## 2.6. Discussion

The point, thus far, at one level is straightforward. In a competitive environment, the banking system can lever up to the point where it will fail with some probability when a significant fraction of households become exuberant about the future. Exuberance creates more pressure for current consumption, which the economy may be too illiquid to provide. Consequently, real interest rates rise to restore equilibrium, and projects are curbed. The more levered the banking system is to begin with, the more projects are liquidated, and higher the likelihood of bank failures.

Ex post, in the exuberant state, the banking system looks over-levered. Both in choosing leverage ex ante, and in determining which projects are discontinued ex post, banks do not internalize the non-pledgeable returns entrepreneurs generate,  $(1 - \gamma)Y_2$ . It is in this sense that there are important spillover effects in bank decisions, which might warrant intervention.

The nature of bank contracting compounds these spillover effects. If banks could write state contingent deposit contracts, they would not have to resort to levels of deposits that risked runs. Alternatively, if banks could finance substantially with long term renegotiable debt or equity, they would be able to withstand significant variations in household consumption needs without failing. We will discuss the difficulty with state contingent deposit contracts later. But long term debt or equity would not be a competitive mode of finance for banks, given how central the banker's human capital is to managing bank loans (see Diamond and Rajan (2001)). The rigidity of bank financing may therefore be an essential feature of the environment.

We are certainly not the first to place the emphasis for contraction and crises on the mismatch between the long duration before investment produces consumption goods, and the temporal pattern of consumption in an expansion. This dates back at least to Von Mises (1912) and the Austrian School. Von Mises placed the emphasis, though, on an artificially low initial rate of interest, induced by bank credit expansion, that makes the process of creating new goods excessively long compared with the tolerance of consumers to postpone consumption. While it is difficult to map the theory precisely to a rational

expectations general equilibrium model, it would appear that Von Mises (1912) places the blame for crises squarely on the heads of overly optimistic, excessively aggressive, bankers (and on central bankers who encourage aggressive credit expansion). We will examine banker reactions to likely central bank interest rate interventions shortly. Thus far, though, our emphasis has been on the frictions stemming from the lack of perfect foresight about changes in the consumption patterns of depositors, and the intrinsic illiquidity of banking.

### **III. Macroeconomic Intervention**

#### **3.1. The Purpose of Intervention**

As we have just argued, the inability of banks to offer state-contingent deposit contracts means households have to either accept too little value in one state or a destructive run in another. Moreover, the inability of banks to distinguish between H type households and L type households, at least in the contracts that are written with them, makes it difficult for banks to achieve better risk sharing between the types.

One goal of intervention, regardless of whose interests the government serves, may simply be to avoid inefficient outcomes such as runs. More controversial, is to ensure that not only are runs prevented, but also that the utility of households is maximized. Finally, if the government cares sufficiently about the welfare of entrepreneurs, it may want to minimize the date-1 interest rate so as to minimize liquidation and thus preserve the value  $(1 - \gamma)Y_2$  that entrepreneurs are unable to pledge.

Rather than take a specific stand on what the objective of intervention might be (and this may vary across countries), let us examine what intervention could accomplish, given limitations on the information governments possess and their ability to commit, and what the consequences of anticipated intervention might be.

#### **3.2. Limitations on Intervention**

Clearly, if the regulator could identify states, types, and implement any kind of tax and transfer scheme, it could achieve first-best outcomes. Yet such an omniscient, omnipotent, and error-free regulatory authority is clearly implausible. Instead, we will assume

- (i) The regulator can identify the date-1 state of the world and modulate its actions accordingly.
- (ii) The regulator cannot distinguish between households based on their future endowments.

Also, if the regulator can alter property rights, there is a lot they can do to enhance the welfare of a favored group – for instance, to rescue banks they can simply write down deposits, or tax households and transfer to banks. In extremis, this is what the authorities will do. But in more normal times, if the authorities have to return what they take from an agent, and they want to attempt a “liquidity” infusion simply through borrowing and lending at market rates (albeit forced), the scope for intervention is more limited. This is what we will focus on.

- (iii) The regulator can tax and lend endowments, or borrow and transfer them, but it cannot otherwise alter property rights – what it taxes (or transfers) today has to be transferred (or taxed) back in the future as a lump sum augmented at the market interest rate. Similarly, what it borrows or lends has to be repaid at the market interest rate. It cannot directly write-down the face value of deposits.
- (iv) The regulator cannot store goods between dates. Its net intake of goods at any date has to be zero.

The interventions we have described are typically thought of as being undertaken by different organs of the government – e.g., the central bank and the Treasury. In reality, they may be connected. For instance, we will describe an intervention which involves taxing and lending. In the real world, monetary expansion by the central bank could be thought of as a combination of a seigniorage tax and lending to

banks. At any rate, in what follows, we will refer to a single regulator performing both fiscal and monetary interventions.

### 3.3 When Interventions Do Not Work

Let us first consider what seems like an obvious intervention; Could the authorities reduce interest rates and thus keep a bank from failing? One candidate intervention could be to tax households at date 1 and lend the proceeds to the bank, transferring the repayment on the loan back to households at date 2. It turns out that a small intervention, when both households would maintain some deposits with the bank absent intervention, will have no effect on interest rates or bank solvency. To see why, let  $\Delta t$  be the small tax on all households, which is lent to the bank at the post intervention rate  $r_{12}^{S,i}$ . The bank repayment,  $\Delta t r_{12}^{S,i}$  is transferred back to households at date 2.

Post-intervention, it must be that the marginal rates of substitution for household H equals the common interest rate, so  $\frac{e_2^{H,S} + \Delta t r_{12}^{S,i} + (D - w_1^{H,S,i})r_{12}^{S,i}}{e_1 - \Delta t + w_1^{H,S,i}} = r_{12}^{S,i}$  where  $w_1^{H,S,i}$  is the post-intervention withdrawal. Simplifying, we get

$$\frac{e_2^H}{e_1 + 2(w_1^{H,S,i} - \Delta t) - D} = r_{12}^{S,i} \quad (1.5)$$

Similarly, for household L

$$\frac{e_2^L}{e_1 + 2(w_1^{L,S,i} - \Delta t) - D} = r_{12}^{S,i} \quad (1.6)$$

Finally, since the bank gets a loan of  $\Delta t$ , its date-1 resource constraint (1.4) now is

$$\theta^S w_1^{H,S,i} + (1 - \theta^S) w_1^{L,S,i} = \frac{r_{12}^{S,i} (X_1)^2 + X_1 X_2 - \gamma X_1 \underline{Y}_2}{\gamma (\bar{Y}_2 - \underline{Y}_2)} + \Delta t \quad (1.7)$$

Comparing (1.5), (1.6), and (1.7) to the equations in Lemma 2 (i), we see that if  $\{r_{12}^S, w_1^{H,S}, w_1^{L,S}\}$  is an equilibrium before intervention,  $\{r_{12}^{S,i} = r_{12}^S, w_1^{H,S,i} = w_1^{H,S} + \Delta t, w_1^{L,S,i} = w_1^{L,S} + \Delta t\}$  is an equilibrium post-intervention. Intuitively, because the wealth of households is not changed by the tax and transfer, their consumptions do not change if they can withdraw an equivalent amount to the amount they are taxed. So the interest rate does not change, nor does the amount the bank has to liquidate since the additional loan it receives from the government is completely exhausted by additional withdrawals. In short, because households have access to the capital market, government intervention has no effect on the household budget set, consumption, or the interest rate -- household choices perfectly offset government actions. Furthermore, because the interest rate does not change, the liquidity infusion has no effect on the net worth of the banking sector. To restate this, the government lending here is a form of fiscal policy, and it has effect only if Ricardian equivalence (see Barro [1974]) does not hold.

### 3.4. Interventions that work.

There is, however, a way of breaking out of this zone of neutrality. Let us now consider a larger tax (and accompanying loan to banks) so that H households withdraw their deposits fully before they can compensate for the tax, that is  $\Delta t > D - w_1^{H,S}$ . This means H households' date 1 consumption will fall by  $\Delta t - (D - w_1^{H,S})$  relative to the no-intervention case, and their marginal rate of substitution will go up. Prima facie, this seems to go in the wrong direction. But these households no longer participate in the banking system, cannot borrow against their future endowment, and do not determine interest rates. The L households do participate, and their date-1 consumption will have to go up (by the entire extent to which date-1 consumption of H households falls if the interest rate is to remain the same). In the new equilibrium, their marginal rate of substitution falls, the interest rate falls, and L households do not make up entirely for the fall in consumption of H households. Overall date-1 consumption falls, and the required liquidation to meet consumption needs falls, and bank net worth rises.

So even if households fully anticipate government policies, and even if both households would continue to deposit in the banking system after meeting their consumption needs in the absence of intervention (and in the absence of a run induced by concerns about the bank's net worth), the authorities can reduce interest rates, and enhance bank net worth by taxing households today and lending to the "money" market at market rates to the extent there is demand (from the bank).

*Lemma 4:* (i) If the prevailing no-intervention equilibrium has  $w_1^{H,S} < D$ , then government lending to the market financed by a tax of  $\Delta t \leq D - w_1^{H,S}$  has no effect on the interest rate, on consumption, or bank net worth. (ii) Government lending financed by a tax  $\Delta t > D - w_1^{H,S}$  will reduce the interest rate the bank faces, increasing bank net worth and reducing project liquidation.

Proof: See appendix.

A very small intervention can have no effect. And there are limits to how much effect the government can have. First, it cannot tax more than the endowment,  $e_1$ . Second, even before it reaches this point, it may have pushed the interest rate so low that L households withdraw entirely from the banking system. Further taxation will not help.

### **Example contd.**

Recall in the example that  $D^{N,\max} = 1.12$  which exceeds  $D^{E,\max} = 1.016$ . So if banks set  $D_1 = D^{N,\max}$ , there will be a banking run in the E state. The government could tax households in that state and lend to the banks. It turns out that the lowest interest rate it can achieve (before L households withdraw completely) is 1.00, and it has to impose a tax of 0.86 on date-1 endowments uniformly across households to achieve this. Since we know that banks become just solvent when  $R_{12} = 1.39$  (this is the interest rate at which banks are just solvent in the N state with  $D = D^{N,\max}$ ), the government has the power to restore the banking system to solvency, and it does this by setting a tax of 0.47 on households and

lending the proceeds in the open market to banks. If the government announces such a lending program at date 1, no depositor will have an incentive to run, H households will withdraw in an orderly fashion, while L households will withdraw 0.47.

Thus far, we have examined situations where the authorities want to reduce date-1 interest rates, either to reduce the extent of project liquidation, or to enhance bank net worth and thus prevent a run. What if the authorities wanted to raise rates? Here again, the authorities would have an effect only if at least the H type household withdrew its deposits entirely from the banking system in the absence of intervention. If the interest rate is so low to begin with that this is the case, authorities can reverse the above process to raise rates. They would borrow from the open market – that is from banks -- at date 1, and give the collected money back as a date-1 subsidy  $\Delta s$  to households. Date-2 taxes would be raised by  $\Delta s r_{12}^{S,i}$ .

Here is why this would raise the market interest rate. Given the marginal rate of substitution of the H household, prior to intervention, is higher than the pre-intervention market interest rate (which is why H households withdraw all deposits), they would consume a small subsidy entirely, increasing their consumption. By contrast, L households would not change consumption if the interest rate did not change. But this would mean overall consumption would be higher than pre-intervention, which would require a higher market interest rate to clear the date-1 goods market and draw forth the necessary amount of liquidation. So it must be that in the new post-intervention equilibrium, the interest rate is higher, type L households consume a little less at date 1, while type H households consume more.

*Lemma 5:* If the prevailing no-intervention equilibrium has  $w_1^{H,S} = D$ , then government borrowing from the market and paying the collected resources as a subsidy  $\Delta s$  to households (with date-2 repayment of government debt financed by date-2 taxes on households) will increase the interest rate the bank faces and depositors get, decreasing bank net worth and increasing project liquidation

Proof: See appendix.

### **3.5. Discussion**

We have seen that intervention has no effect if it does not influence the wealth of households and they have access to the capital markets. How then does government intervention have effect? Because one set of participants no longer uses the capital market at market interest rates! For example, when we tax type H households after they have withdrawn fully from the banking system, they would like to withdraw more to offset the tax at prevailing interest rates, but they cannot. Compensation at market interest rates at date-2 does not fully make up their loss (neither would compensation at their marginal rate of substitution without allowing them to borrow). More generally, a number of households do not participate in the financial system. Liquidity interventions “work” by effectively taxing them more heavily, and offering the proceeds to participants in the financial system. If the number of non-participants is small to begin with, a liquidity intervention will have an effect only once it can induce a more significant number to withdraw – so, for example, attempts to raise interest rates when all households already deposit in the system will be futile, while attempts to lower interest rates can be successful only with substantial intervention (once some households are induced to withdraw fully).

Note that the liquidity interventions that we are referring to could well be thought of as monetary policy interventions (with either seigniorage or a fiscal component attached), that are not targeted at specific banks, and are meant to bring down the real interest rate. To have effect, they must “penalize” one set of households – those who do not participate as strongly in financial markets -- in order to benefit the system. Note that this differs from direct lender-of-last-resort lending, which is targeted, and undirected lending against illiquid collateral, which is not. In the former, the central bank lends to banks the market would not lend to, while in the latter, the central bank lends against collateral the market will not touch. While the market’s aversion may be irrational, viewed at prevailing market prices the central

bank's actions in both cases have an element of subsidy. By contrast, there is no explicit subsidized lending in our framework.

### *Moral Hazard.*

What does anticipation of intervention do to the deposit levels banks choose? In Figure 1 a, we plot the expected utilities of households if banks offer  $D^{N,max}$  and households expect the authorities to intervene to prevent runs in the E state. The dashed line reflects the relative higher household utility from choosing  $D^{N,max}$  over  $D^{E,max}$  now -- essentially, by setting a high D and having the authorities reduce the interest rate in the E state to prevent runs, the bank achieves a state contingent deposit level, which clearly appeals more to households than the safe deposit level. In the absence of intervention, households prefer the risky deposit level  $D^{N,max}$  only so long as the probability of the run state,  $p^E$ , is smaller than 0.45. Now households prefer it so long as  $p^E$  is less than 0.67 – thus banks will offer a risky level of deposits, and households will accept, over a wider range of probabilities. Intervention clearly induces greater leverage, as banks know they can pay out more, relying on the authorities to rescue them in the exuberant state. Thus anticipation of intervention is a source of moral hazard.<sup>14</sup>

### *Ex-ante Intervention to Offset Moral Hazard*

If the authorities cannot commit to tie their own hands, the equilibrium will be that banks will offer a risky level of deposits in more situations than absent intervention, and authorities will be forced to intervene if the E state is realized. If intervention is costless, welfare is improved because deposit levels are effectively more state contingent, with banks paying out more in the N state and less in the E state. If intervention is costly, though, the authorities may want to intervene ex ante -- by limiting leverage, for instance, and requiring that banks not offer  $D > D^{E,max}$ .

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<sup>14</sup> Why do banks not choose the risky level of deposits all the time – after all, intervention renders them safe? The answer is that the utility of both types of households is higher in the E state with  $D^{E,max}$  than with  $D^{N,max}$  with intervention (though better than  $D^{N,max}$  with a run).

Of course, this need not be a panacea if unregulated entities (investment banks) compete with banks to take away business, or banks can place assets in unregulated entities (SIVs and conduits, for example). Leverage regulation will then only ensure that regulatory arbitrage takes place.

We have also assumed thus far that a bank's choice of assets remains constant, independent of the chosen leverage and the anticipated intervention. In this last section, let us examine bank asset choices. We will show that bank asset choices will respond to leverage and anticipated intervention. This complicates regulatory intervention because ex ante limits on leverage, or ex post reductions in the interest rate, may induce banks to choose more illiquid asset holdings. In order to induce more appropriate asset choices, the central bank may want to smooth interest rates across states.

#### **IV. Intervention and the Choice of Assets**

In this section, we examine the effects of interest rate policy (and leverage) on the incentives of banks to alter the liquidity of the assets they hold. Clearly, the more a borrower can repay at date 1 (by liquidating his project), the more liquid a loan is. One interpretation of liquid is that the bank monitors the borrower to make sure that the borrower maintains liquidity. An interpretation of illiquid is that the bank makes loans that it knows it will need to fully refinance in the future (as when it sets up a special interest vehicle (SIV) knowing that its ability to liquidate will be low). Illiquid loans are attractive assets for a bank to hold only if, despite lower short term liquidation values, they have higher long-term payoffs.

##### **A. Asset choices**

Let banks choose to lend either solely to liquid projects or to illiquid projects after issuing deposits at date 0: liquid projects have  $X_1^q = X_1$  and  $\tilde{Y}_2^q = \tilde{Y}_2$  and illiquid projects have  $X_1^i < X_1^q$  and  $\tilde{Y}_2^i = Z\tilde{Y}_2$  with  $Z > 1$ .

Let us start by assuming this choice is not observed until date 1, when the state of nature is known. In making asset choices, banks take interest rates as given and not responsive to their choice. Let us now

examine the conditions under which a bank will choose to make only loans to liquid projects (we will call such a bank and such loans “liquid”). We will later argue why such a choice may be desirable.

## B. Project Liquidation

At date 1 in state  $s$ , a solvent liquid bank chooses to liquidate projects with realization  $Y_2$  that satisfies

$$Y_2 < Y_2^q(r_{12}^s) = \frac{r_{12}^s X_1}{\gamma}. \quad \text{A solvent illiquid bank liquidates projects with } Y_2 < Y_2^i(r_{12}^s) = \frac{r_{12}^s X_1^i}{\gamma Z} < Y_2^q(r_{12}^s),$$

so at any given interest rate, illiquid banks liquidate fewer projects. The value (at date 1) of the liquid

bank's assets is  $V_1^q(r_{12}^s) = \frac{1}{\bar{Y}_2 - \underline{Y}_2} \left[ \int_{\underline{Y}_2}^{Y_2^q(r_{12}^s)} X_1 dY_2 + \frac{\gamma}{r_{12}^s} \int_{Y_2^q(r_{12}^s)}^{\bar{Y}_2} Y_2 dY_2 \right]$  while that of an illiquid bank's assets is

$$V_1^i(r_{12}^s) = \frac{1}{\bar{Y}_2 - \underline{Y}_2} \left[ \int_{\underline{Y}_2}^{Y_2^i(r_{12}^s)} X_1^i dY_2 + \frac{\gamma Z}{r_{12}^s} \int_{Y_2^i(r_{12}^s)}^{\bar{Y}_2} Y_2 dY_2 \right]. \quad \text{In what follows, we assume individual bankers do not}$$

think they will affect the date-1 rate through their actions.

## C. Relative Asset Values

Given a deposit level  $D$ , a bank will choose liquid loans at date 0 if the following incentive constraint holds:

$$\begin{aligned} & p^E [r_{12}^E (\max\{0, V_1^q(r_{12}^E) - D\} - \max\{0, V_1^i(r_{12}^E) - D\})] + \\ & (1 - p^E) [r_{12}^N p^E (\max\{0, V_1^q(r_{12}^N) - D\} - \max\{0, V_1^i(r_{12}^N) - D\})] \geq 0 \end{aligned} \quad (\text{ICq})$$

This constraint is equivalent to:

$$\frac{1}{\bar{Y}_2 - \underline{Y}_2} \left( \begin{array}{l} p^E \max \left\{ 0, \int_{\underline{Y}_2}^{Y_2^q(r_{12}^E)} (r_{12}^E X_1) dY_2 + \gamma \int_{Y_2^q(r_{12}^E)}^{\bar{Y}_2} Y_2 dY_2 - r_{12}^E D \right\} - \\ \max \left\{ 0, \int_{\underline{Y}_2}^{Y_2^i(r_{12}^E)} (r_{12}^E X_1^i) dY_2 + \gamma Z \int_{Y_2^i(r_{12}^E)}^{\bar{Y}_2} Y_2 dY_2 - r_{12}^E D \right\} + \\ \left[ (1 - p^E) \max \left\{ 0, \int_{\underline{Y}_2}^{Y_2^q(r_{12}^N)} (r_{12}^N X_1) dY_2 + \gamma \int_{Y_2^q(r_{12}^N)}^{\bar{Y}_2} Y_2 dY_2 - r_{12}^N D \right\} - \right. \\ \left. \max \left\{ 0, \int_{\underline{Y}_2}^{Y_2^i(r_{12}^N)} (r_{12}^N X_1^i) dY_2 + \gamma Z \int_{Y_2^i(r_{12}^N)}^{\bar{Y}_2} Y_2 dY - r_{12}^N D \right\} \right] \end{array} \right) \geq 0$$

The ex-ante incentive constraint (ICq) is the probability weighted average of the ex-post state contingent differences in payoffs between liquid and illiquid loans. The incentive to choose liquid loans depends on leverage, measured by  $D$ , and on interest rates. To make the intuition clear, let us examine initially the difference in the ex-post loan payoffs in the case of leverage  $D$  that is low enough that the bank will not fail in any state, for either the choice of liquid or illiquid loans (this ensures the truncation at 0 is not binding). Differentiating the incentive constraint, state by state, with respect to  $r_{12}^S$ , we get

$$\frac{X_1 - X_1^i}{\bar{Y}_2 - \underline{Y}_2} \int_{\underline{Y}_2}^{Y_2^i(r_{12}^S)} 1 dY + \frac{X_1}{\bar{Y}_2 - \underline{Y}_2} \int_{Y_2^i(r_{12}^S)}^{Y_2^q(r_{12}^S)} 1 dY + \frac{dY_2^q(r_{12}^S)}{dr_{12}^S} \{r_{12}^S X_1 - \gamma Y_2^q(r_{12}^S)\} - \frac{dY_2^i(r_{12}^S)}{dr_{12}^S} \{r_{12}^S X_1^i - \gamma Z Y_2^i(r_{12}^S)\}$$

The last two expressions are zero because,  $r_{12}^S X_1 - \gamma Y_2^q(r_{12}^S) = 0$  and  $r_{12}^S X_1^i - \gamma Z Y_2^i(r_{12}^S) = 0$ . So the expression simplifies to  $P[\tilde{Y}_2 \leq Y_2^i(r_{12}^S)](X_1 - X_1^i) + P[Y_2^i(r_{12}^S) \leq \tilde{Y} \leq Y_2^q(r_{12}^S)]X_1$ , which is positive. This implies the loan to the liquid project becomes more attractive as the interest rate prevailing at date 1,  $r_{12}^S$ ,

increases (and the left hand side of the incentive constraint is concave  $r_{12}^S$ ).<sup>15</sup> Turning to date-1 values,

we can show that  $V_1^q(r_{12}^S) - V_1^i(r_{12}^S)$  increases in  $r_{12}^S$ , and  $V_1^q(r_{12}^S), V_1^i(r_{12}^S)$  each decrease in  $r_{12}^S$ .

We know at low date-1 interest rates  $V_1^i(r_{12}^S) > V_1^q(r_{12}^S)$  and at high interest rates

$V_1^i(r_{12}^S) < V_1^q(r_{12}^S)$ . Since  $V_1^{q'} > V_1^{i'}$ , there must be an interest rate  $r_{12}^{IC}$  such that  $V_1^i(r_{12}^S) = V_1^q(r_{12}^S)$

Above  $r_{12}^{IC}$ , liquid loans are more valuable (ex post the realization of the interest rate at date 1) than

illiquid loans, and vice versa below  $r_{12}^{IC}$ .

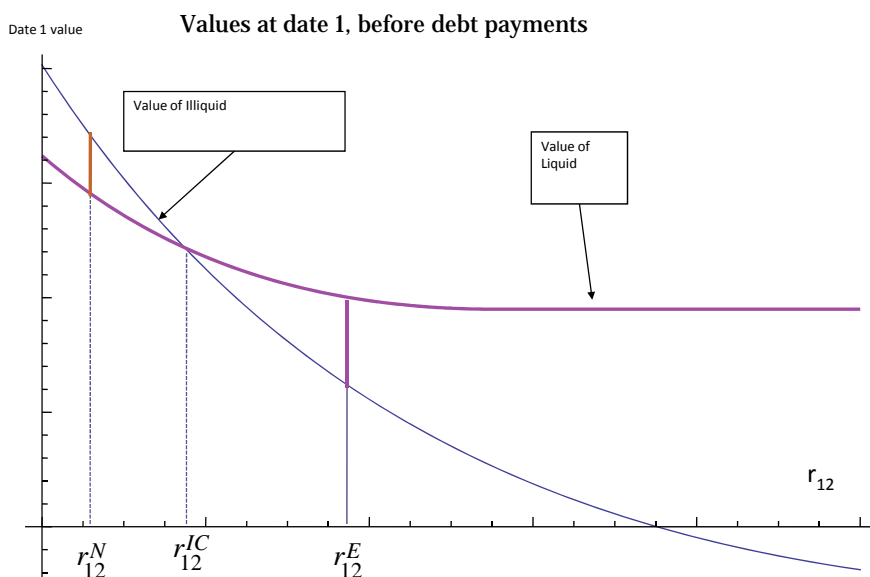
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<sup>15</sup> Differentiating again (noting that  $\frac{d^2 Y_2^u(r_{12}^S)}{dr_{12}^{S2}} = \frac{d^2 Y_2^q(r_{12}^S)}{dr_{12}^{S2}} = 0$ ), yields

$$\frac{dY_2^q(r_{12}^S)}{dr_{12}^S} (X_1 - X_1^u - X_1) + \frac{dY_2^q(r_{12}^S)}{dr_{12}^S} \left( X_1 - \gamma \frac{dY_2(r_{12}^{S_q})}{dr_{12}^S} \right) + \frac{dY_2^u(r_{12}^S)}{dr_{12}^S} X_1 - \frac{dY_2^u(r_{12}^S)}{dr_{12}^S} \left( X_1^u - \gamma Z \frac{dY_2(r_{12}^{S_u})}{dr_{12}^S} \right) =$$

$$\frac{dY_2^q(r_{12}^S)}{dr_{12}^S} \left( -X_1^u + X_1 - \gamma \frac{dY_2^q(r_{12}^S)}{dr_{12}^S} \right) + \frac{dY_2^u(r_{12}^S)}{dr_{12}^S} \left( X_1 - X_1^u + \gamma Z \frac{dY_2^u(r_{12}^S)}{dr_{12}^S} \right) =$$

$$\frac{X_1}{\gamma} \left( -X_1^u + X_1 - \gamma \frac{X_1}{\gamma} \right) + \frac{X_1^u}{\gamma Z} \left( X_1 - X_1^u + \gamma Z \frac{X_1^u}{\gamma Z} \right) = \frac{X_1}{\gamma} (-X_1^u) + \frac{X_1^u}{\gamma Z} (X_1) = \left( \frac{1}{Z} - 1 \right) \frac{X_1 X_1^u}{\gamma} < 0.$$



#### D. Bank value given the possibility of default

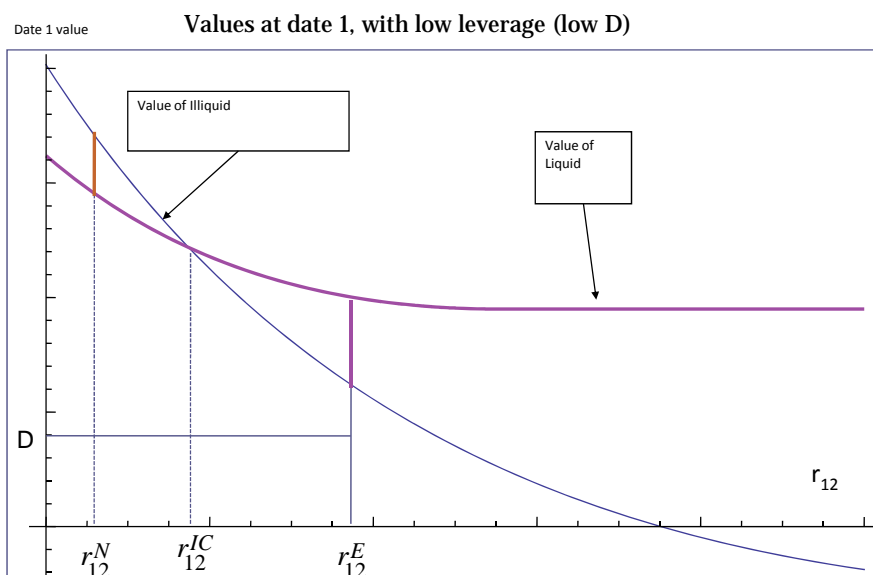
The bank has to repay deposits or fail. So long as the bank is solvent, its choice between loans depends only on their value. If, however, there is some possibility that  $D$  could exceed bank asset values for some choice of asset, then the possibility of insolvency will also affect the bank's ex ante asset choice.

Let us examine preferences at date 1, given the prevailing interest rate  $r_{12}^S$  and the deposit rate  $D$ .

We also examine the effect of moving  $r_{12}^S$  (through the interventions discussed in the last section). If

$r_{12}^S < r_{12}^{IC}$  but  $D > V_1^i(r_{12}^S) > V_1^q(r_{12}^S)$ , then small a reduction in the prevailing interest rate  $r_{12}^S$  will have no effect on relative bank values (because the bank would be in default whether it was liquid or illiquid).

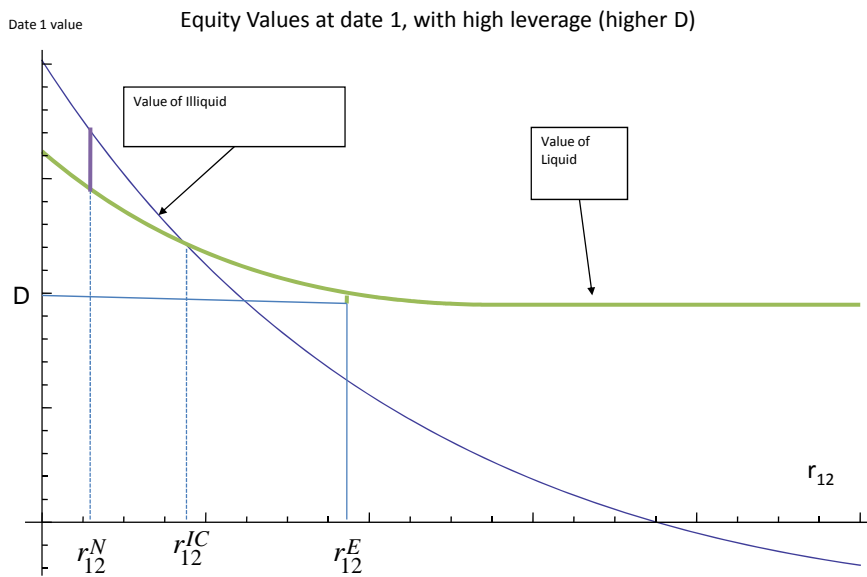
Further reductions will eventually result



a steady rise in the value of the illiquid bank relative to the liquid bank as the illiquid bank's asset value rises above  $D$ . Thus when the interest rate is below  $r_{12}^{IC}$ , lowering it further (weakly) enhances the relative value of the illiquid loan in that state, regardless of the value of  $D$ .

Suppose now the interest rate is above  $r_{12}^{IC}$ . Interestingly, if  $D > V_1^q(r_{12}^s) > V_1^i(r_{12}^s)$  and  $V_1^q(r_{12}^{IC}) > D$ , or if  $V_1^q(r_{12}^s) > D > V_1^i(r_{12}^s)$ , the liquid bank's value can be enhanced by reducing the interest rate. Intuitively, lowering the interest rate will raise both  $V_1^q$  and  $V_1^i$ . So long as  $D > V_1^q(r_{12}^s) > V_1^i(r_{12}^s)$ , neither bank will be solvent and hence changes in interest rate will not affect value. But once  $V_1^q(r_{12}^s) > D > V_1^i(r_{12}^s)$ , lowering the interest rate further will enhance the value of the liquid bank net of deposits, while it will not affect the value of the (underwater) illiquid bank. It will then make sense to lower the interest rate until  $V_1^i(r_{12}^s) = D$ , when the difference in values is maximized. Finally, if  $V_1^q(r_{12}^s) > V_1^i(r_{12}^s) \geq D$ , the liquid bank will be made relatively more attractive if the interest

rate is raised until it can be raised no more or  $V_1^i(r_{12}^s) = D$ , whichever comes first.



In sum then, if the objective of ex post intervention is to both leave a liquid bank solvent and to discourage banks from making illiquid loans, interest rates may need to be lowered or raised depending both on their level and the value of liquid/illiquid assets relative to  $D$  (effectively bank capital).

### E. Ex post intervention given ex ante incentive constraints

Having characterized the ex post possibilities, conditional on the interest rate and  $D$ , let us consider date-1 intervention considering the full date-0 incentive constraint, (ICq), and taking into account the possibility of bank default. Specifically, consider the form interest rate intervention should take if the authorities cannot allow banks to fail but can commit to any interest rate policy that satisfies the no-bank-failure condition.

If there is no date-0 leverage requirement, banks will choose  $D^{N,\max}$  over a wide range of parameter values, and will be run in the  $E$  state. In the  $N$  state, the liquid bank will be just solvent at the

interest rate,  $r_{12}^N$ . If  $r_{12}^N > r_{12}^{IC}$ , then the illiquid bank will be run in the N state also, so banks have the incentive to remain liquid at date 0 anticipating date 1 outcomes. If, however,  $r_{12}^{IC} > r_{12}^N$ , banks will prefer to be illiquid if they anticipate no intervention, or if they anticipate intervention in the E state that simply pushes interest rates down only to the point that liquid banks just survive (since being just solvent gives the banker no more rent than being run). Therefore the interest rate policy that restores date-0 incentives to choose liquid loans would have the authorities reduce interest rates in the E state below the level that would render liquid banks just solvent. The intent would be to give liquid banks a rent in the E state that offsets their disadvantage in the N state. From our earlier discussion, the maximum incentive can be given if the interest rate in the E state is pushed down to the point that the illiquid bank is just solvent.<sup>16</sup>

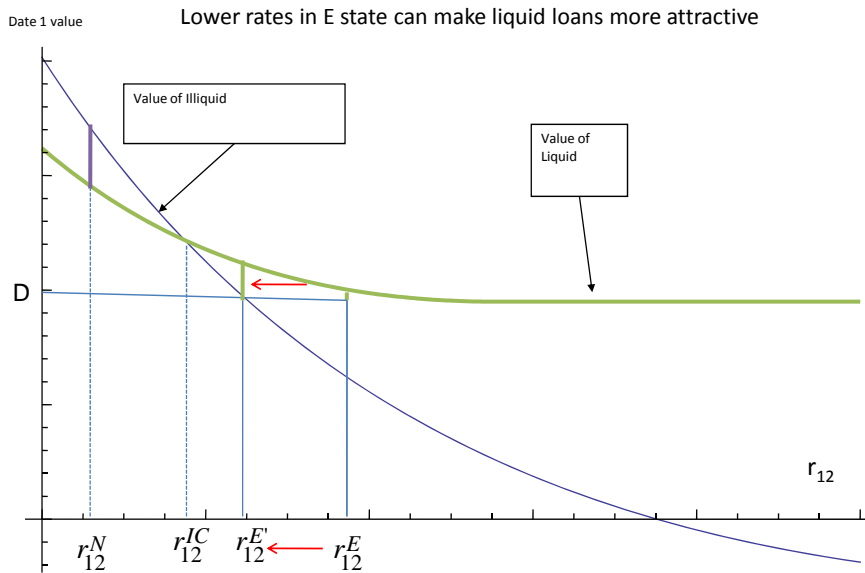
Now consider what happens if banks choose  $D^{E,max}$  (or are forced to choose it because of a maximum leverage regulation imposed by the authorities). Recall that liquid banks would be just solvent in the E state and have positive value in the N state. Because interest rates in the N state are lower than when banks choose  $D^{E,max}$ , the incentive to choose illiquid projects at date 0 can be higher when leverage is set at  $D^{E,max}$  rather than  $D^{N,max}$ . If the authorities want to restore incentive compatibility at date 0, they may have to intervene on interest rates even though liquid banks are not in danger of failing.

If  $r_{12}^{IC} > r_{12}^E > r_{12}^N$ , then no interest rate intervention can restore ex ante incentives – the authorities have to allow interest rates to go above  $r_{12}^{IC}$ , but at that interest rate all banks fail. If  $r_{12}^E > r_{12}^N \geq r_{12}^{IC}$ , ex ante incentives to choose liquid are well established. The interesting case is, again, when  $r_{12}^E > r_{12}^{IC} > r_{12}^N$ . However, now interventions in both states could help generate incentives to choose liquid projects. In the E state, the interest rate should be brought down to the point that the illiquid bank is just solvent. In the N

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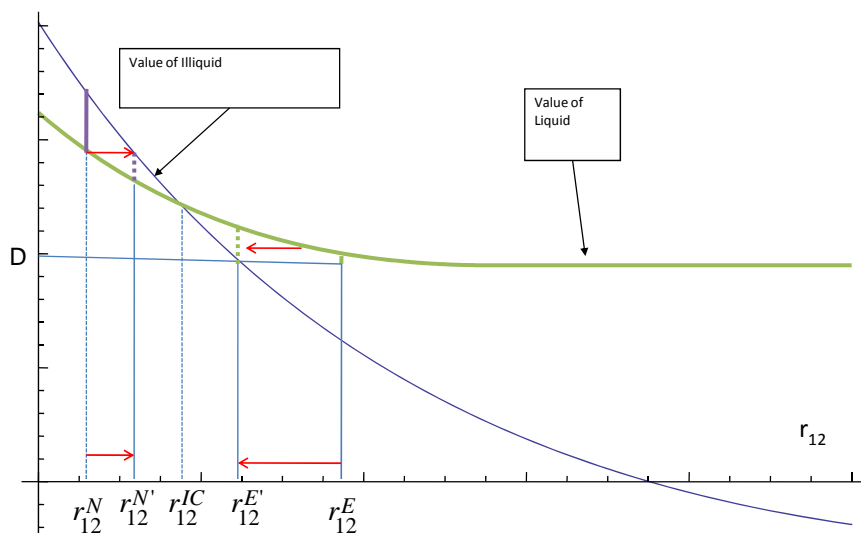
<sup>16</sup> Of course, if the interest rate required to just induce solvency in the E state is below  $r_{12}^{IC}$ , it is not possible to provide any incentives in the E state for banks to choose liquidity up front.

state, however, the interest rate should be raised – if possible to the same interest rate as in the E state, but failing that, to the highest interest rate possible.



In sum, policies on interest rates should not just take into account the possibility of bank failures, but also the potential for banks to choose excessively illiquid projects if they anticipate a low interest rate environment. Optimal interest rate policies may require committing to raising rates when relatively low and reducing them when high, in order to foster the right ex ante incentives.

Date 1 value Lower rates in E state and higher rates in N state make liquid loans attractive.



## F. Leverage

Too high a level of deposits could lead banks to become excessively fragile. This leads regulators to impose a maximum leverage ratio. But too low a leverage ratio leads to too low date-1 interest rates, which then causes banks to become excessively illiquid. The point therefore is that leverage regulations can have unintended consequences on aggregate liquidity, especially if interest rate policy does not take into account the adverse consequences of anticipated low interest rates on liquidity choice. Put differently, capital structure choice, liquidity choice, and interest rate policy are all intimately linked.

We have taken  $D$  thus far as given. Given that date-1 interest rates increase in  $D$ , banks could choose  $D$  so as to try and maintain ex ante incentive compatibility (so as to convince depositors they will not become illiquid). We have not solved for the optimal date-0 choice of deposit levels, taking into account any desire for banks to choose an incentive compatible deposit level. We could also modulate that choice taking into account interest rate policy and leverage regulations. These are left for future work.

### G. Runs and observable but verifiable liquidity.

Finally, let us consider what happens if bank choice of liquidity is observed at date 0, after the deposit but before any news about the state of nature is known. A deposit of one unit can be withdrawn for one unit immediately or for  $D$  once the state of nature on date 1 is known. Because there are always unfunded projects, if a depositor withdraws at date 1, he can reinvest at  $D$  in another bank. We seek to determine if there is an equilibrium where all banks are willing to choose liquid loans when they are interest rate takers.

If a bank is run immediately after deviation to illiquid is observed, then the threat of the run would deter deviation to illiquid loans even if the incentive constraint with unobservable liquidity is violated. If the incentive constraint (ICq) is satisfied, then a bank will make liquid loans with or without the threat of a run.

If the incentive constraint is violated and, in addition,  $r_{12}^{IC} \geq r_{12}^E \geq r_{12}^N$ , then deviation to illiquid will not make the bank worse off (than the liquid bank) in either state and the threat of runs will not commit banks to be liquid. If  $r_{12}^E > r_{12}^{IC} > r_{12}^N$ , then the threat of a run can commit a bank to be liquid if  $D$  is sufficiently *high* such that the bank will be run in the E state if it deviates to illiquid (and not so high that it would be run if it chooses liquid). Because the payoff of a solvent bank is ex-post preferable to that of a run bank, deviation to illiquid will lead to a run if other banks will choose to be liquid. This requires that the bank be solvent in state E if and only if it chooses to be liquid, or

$$\int_{\underline{Y}_2}^{Y_2^q(r_{12}^E)} (r_{12}^E X_1) dY_2 + \gamma \int_{Y_2^q(r_{12}^E)}^{\bar{Y}_2} Y_2 dY_2 \geq r_{12}^E D > \int_{\underline{Y}_2}^{Y_2^i(r_{12}^E)} (r_{12}^E X_1^i) Y_2 dY_2 + \gamma Z \int_{Y_2^i(r_{12}^E)}^{\bar{Y}_2} Y_2 dY_2 .$$

If deviation to being illiquid is observed immediately, the threat of a run can allow the bank to have maximum leverage consistent with never failing; the bank chooses  $D$  to be just solvent in state E and will

be run if it deviates to become illiquid. This only will work if the government can commit to allow these runs. If the government will intervene to reduce interest rates to stop runs, then high leverage will not serve as a commitment device and the only way to assure that banks remain liquid is to satisfy the incentive constraint, which requires that interest rates not be too low in state N.

## **H. Relation to literature**

To be written

## **V. Conclusion**

Our main effort in this paper is to paint a more careful picture of why banking systems might face stress (putting more flesh, so to speak, on the bones of early consumers and late consumers that are in models like Diamond and Dybvig (1983)), as well as how intervention might take place to reduce real interest rates. In the process, we chart a channel for transmission of policy. The focus, though, has been on how interest rate policy can affect the incentives to maintain liquidity. While our entire model is couched in terms of real good, we hope in future work to describe monetary implications.

Of course, our work has been done in very interesting times. Following a period of extremely benign financial conditions, we are in the midst of a financial panic caused by financial firms overloading on illiquid assets and taking on too much leverage. Why might we have arrived in this state and what can we do about it? While agency problems in banks and the breakdown in risk controls and compensation structures (see Kashyap, Rajan, and Stein (2008)) must be part of the explanation, our model offers another one; the anticipated benign environment, perhaps accentuated by hopes of central bank intervention if interest rates rose too high and asset prices fell substantially, must have also been an important causal explanation for both the illiquidity of assets and the excessive leverage.

Our analysis points to the dangers of one-sided prescriptions to limit such risky behavior in the future. For instance, attempts to force banks to maintain low leverage (by instituting higher capital requirements or lower leverage ratios) could move them to holding more illiquid assets. One sided interventions on interest rates (reducing them drastically when the financial sector is in trouble, but not raising them quickly as sector recovers) could also create incentives for banks to seek out more illiquidity than good for the system. In sum, both regulation as well as monetary policy should be formulated with a greater focus on the incentives of financial system participants to search for illiquidity.

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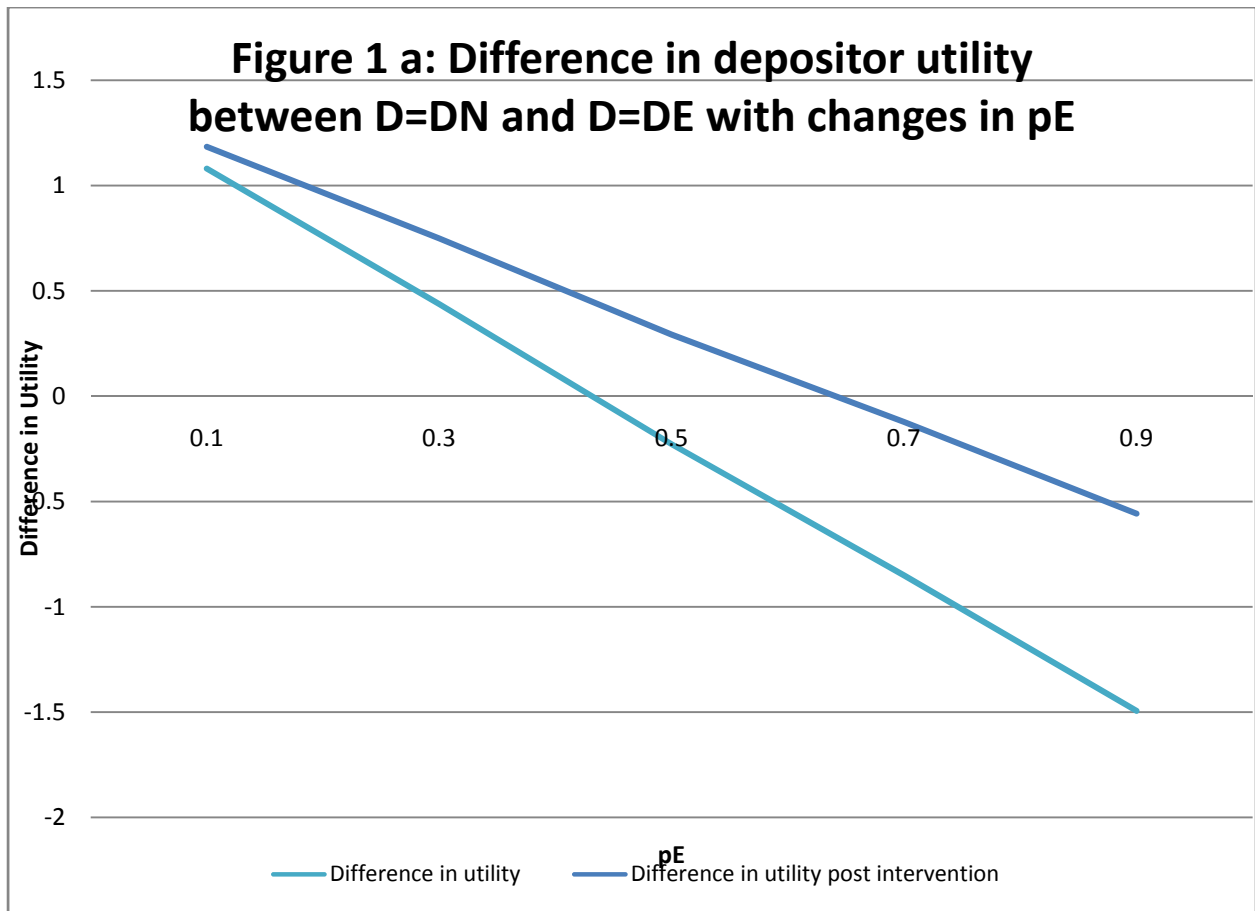
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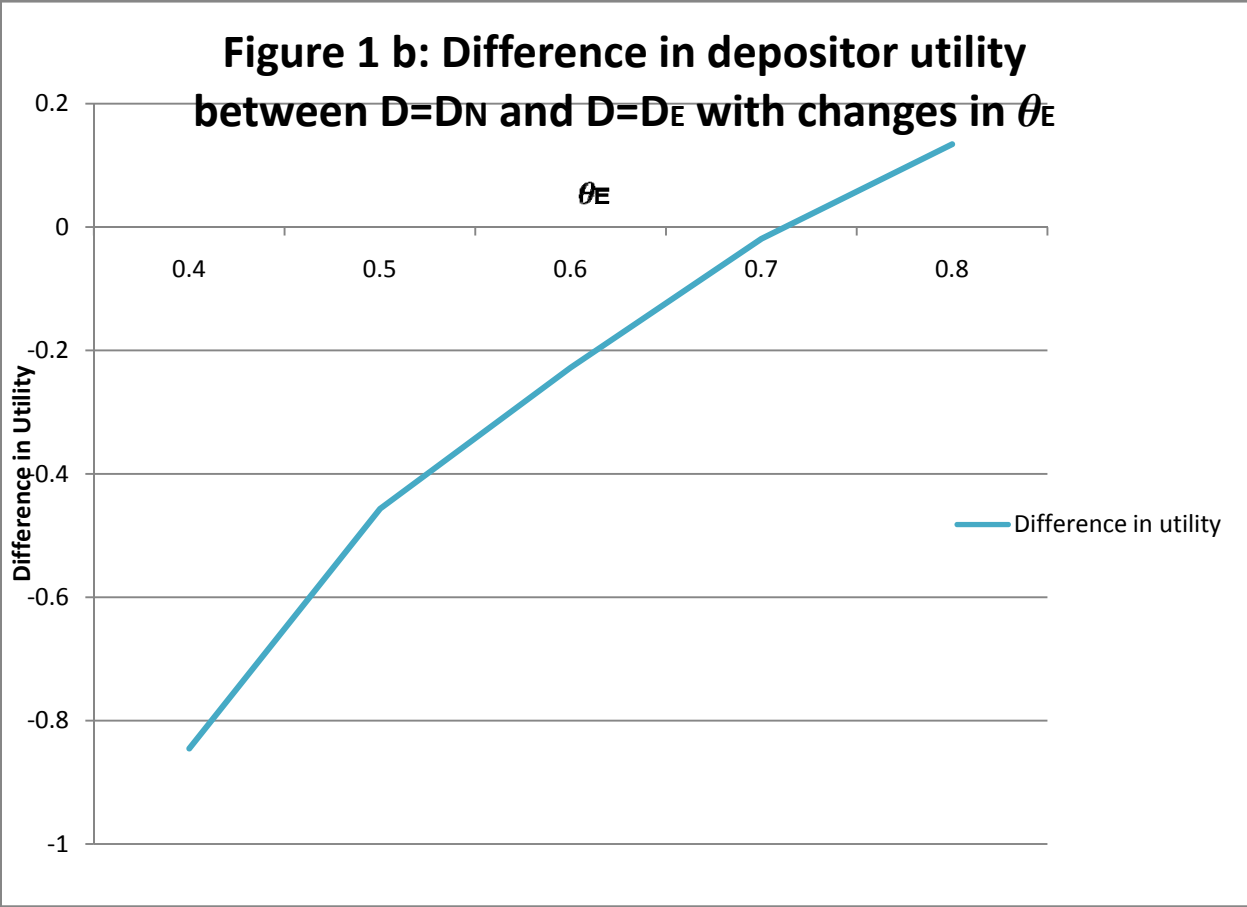
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**Figure 1 c: Difference in depositor utility between  $D=D_N$  and  $D=D_E$  with changes in  $\theta_N$**

