APPENDIX A: BERTRAND AND COURNOT BENCHMARKS

A1: Prices, Quantities, and Profits for Dealers in a Static Bertrand Equilibrium with no Manufacturer Discount

Bertrand equilibrium in the absence of discounts can be characterized by solving dealer's profit maximization problem: $\max_{p_i} k_i(p_i - w)$ subject to $p_j = \overline{p}_j$, where $k_i = a - p_i + b\,p_j$ and $k_j = a - p_j + b\,p_i \quad (j \neq i \text{ and } i = 1,2)$. Hence, the first-order condition for an interior optimum for dealer *i* is: $a - p_i + b\,p_j - (p_i - w) = 0.$

The second-order condition (-2<0) always holds. Solving for p_i, we denote the best response to p_j as $r_i^b(p_j; w)$, for $j \neq i$ and $i = 1,2.$

Since the two reaction functions are linear and cross in the "stable" way, there exists a unique pure-strategy equilibrium. As can be verified, the following solution satisfies both first-order conditions simultaneously: $p_i^B = \dfrac{a+w}{2-b},\ i = 1, 2.$

The common price is strictly positive because the denominator and numerator are strictly positive. The two prices are identical because the dealers have identical costs and face symmetric demands. The resulting quantity pair in the Bertrand equilibrium can be determined by substituting the equilibrium prices into the demand system: $q_i^B = \dfrac{a-(1-b)w}{2-b},\ i = 1, 2.$ To have strictly positive equilibrium quantity, we assume a>(1–b)w.

To obtain each dealer's profit in the Bertrand equilibrium, we substitute each dealer's equilibrium price and quantity into his profit function: $\pi_i^B(d) = \dfrac{(1+d)(a-(1-b)w)^2}{(2-b)^2},\ i = 1,2.$ Note that when the manufacturer offers d=0, $\pi_i^B(d) = \dfrac{(a-(1-b)w)^2}{(2-b)^2},\ i = 1,2.$

A2: Prices, Quantities, and Profits for Dealers in a Static Cournot Equilibrium with Manufacturer Discounts

Recall that to preorder k_i units, dealer i must pay the manufacturer $C(k_i) = \int_{u=0}^{k_i} \max[0, w - 2ud]du.$ Hence, marginal cost decreases linearly until it reaches zero for larger preorders. Recall also that we interpret the marginal cost curve as the following limit $MC = \lim_{\varepsilon \to 0^+} \max[\varepsilon, w - 2ud].$

In the Cournot benchmark the equilibrium preorders are symmetric. As we verify below, as long as the marginal cost is strictly positive, benchmark preorders are an increasing function of the manufacturer's quantity discount, d. For any admissible $d \geq \hat{\delta},$ however, marginal cost is zero. For discounts in this range, equilibrium preorders are independent of the discount. To determine the threshold $(\hat{\delta})$, we solve for the equilibrium preorders when marginal cost is strictly positive and find that discount rate such that $w - 2\hat{\delta}k_i^C = 0.$ We perform this calculation below.

Cournot equilibrium can be characterized by solving dealer's profit maximization problem: $\max_{k_i} k_i[p_i - (w - d\,k_i)]$ subject to $k_j = \bar{k}_j$, where $k_i = a - p_i + b\,p_j$ and $k_j = a - p_j + b\,p_i \quad (j \neq i \text{ and } i = 1,2).$ We consider first the case where $d < \hat{\delta},$ which induces strictly positive marginal cost.

Using these two demand functions, we can express the prices in terms of quantities: $p_1 = \frac{a(1+b) - k_1 - b\,k_2}{1-b^2}$ and $p_2 = \frac{a(1+b) - k_2 - b\,k_1}{1-b^2}$. Hence, the first-order condition for an interior optimum is:

$$\frac{\partial \pi_i}{\partial k_i} = [\frac{a(1+b)}{1-b^2} - w] + k_i[2d - \frac{2}{1-b^2}] - \frac{b\bar{k}_j}{1-b^2} = 0,$$

for dealer $i, i = 1,2$. The second-order condition is: $2d - \frac{2}{1-b^2} < 0$. From the first-order condition, we obtain the quantity best reply of dealer i, i=1,2:

$$k_i = \frac{a(1+b) - (1-b^2)w - bk_j}{2 - 2(1-b^2)d}.$$

We assume (1) the exogenous parameters are restricted so that the second-order inequality holds and (2) the reaction functions cross in the "stable" way so that there is only a unique intersection. The second-order condition holds if and only if $d < \frac{1}{1-b^2}$. The reaction functions cross once if and only if $d < \frac{2-b}{2(1-b^2)} \equiv \delta_k$. Note that $\delta_k < \frac{1}{1-b^2}$. We take δ_k as the upper bound of d and by requiring that the discount is contained in the interval $d \in (0, \delta_k)$, we insure that the second-order condition is always satisfied and the Nash equilibrium is unique. As can be verified, the following solution satisfies both first-order conditions simultaneously: $k_i^C(d) = q_i^C(d)$ $= \frac{(1+b)(a-(1-b)w)}{(2+b-2(1-b^2)d)}, i = 1,2.$ The common quantity is strictly positive because the denominator and numerator are strictly positive. The denominator is strictly positive since the second-order condition holds. The numerator is strictly positive because we have assumed that a>(1–b)w. The two quantities are identical because the dealers have identical costs and face symmetric demands.

The resulting price pair in the Cournot equilibrium can be determined from the inverted demand functions: $p_i^C(d) = \frac{a(1\text{-}2(1\text{-}b^2)d) + (1-b^2)w}{(1\text{-}b)(2+b-2(1-b^2)d)}$, i = 1, 2. The dealers charge identical retail prices since the equilibrium is symmetric. Finally, the Cournot profits can be calculated by substituting the equilibrium price and quantity for each dealer into his profit function. Each dealer earns: $\pi_i^C(d)$

$$= \frac{(1+b)(1-(1-b^2)d)(a-(1-b)w)^2}{(1-b)(2+b-2d(1-b^2))^2}, \text{ i=1, 2.}$$

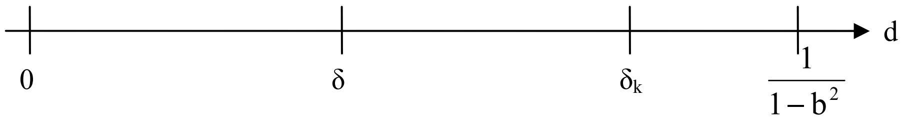
We can use the formula $w - 2\hat{\delta}k_i^C = 0$ to determine $\hat{\delta}$ as discussed above. We conclude that $\hat{\delta} = \frac{w(2+b)}{2a(1+b)}$. Thus, for any admissible $d > \hat{\delta}$, $k_i^C(d) = \frac{a(1+b)}{2+b}$. Using $p_i = \frac{a(1+b) - k_i - bk_j}{1-b^2}$, we obtain $p_i^C = \frac{a(1+b)}{2-b-b^2}$. Finally, dealer profits are $\pi_i^C(d) = \frac{a^2(1+b)}{(1-b)\,(2+b)^2} - \frac{w^2}{4d}$ for $i = 1, 2$.

A3: Critical Discount δ

Since δ is that discount which reduces the price in the Cournot equilibrium to the price in the Bertrand equilibrium in the absence of a discount (that is, $p_i^C(d) = p_i^B$), we conclude that $\delta = \frac{b^2}{2(1-b^2)}$. We refer to δ as the critical discount. It is straightforward to show that $\left.\frac{\partial p_i^c}{\partial d}\right|_{\delta} < 0$. Therefore, at d=δ, the Cournot price with discounts, $p_i^C(d)$, cuts the Bertrand price without discounts, p_i^B, from above.

Relationship among δ, $\hat{\delta}$, and $\hat{\delta}_k$

From the above results, we have $0 < \delta < \delta_k < 1/(1-b^2)$. Thus there is a non-empty Bertrand region and a non-empty Cournot region as depicted in the following figure:



It can be verified that $\hat{\delta}$ is an increasing function of w. Even for the smallest admissible value of w, however, $\hat{\delta} > \delta$. For sufficiently large w, it is possible that $\hat{\delta} > \delta_k$. In that case, preorders in the

Cournot region are always an increasing function of d because marginal cost is always strictly positive. Alternatively, w may be sufficiently small that $\hat{\delta} < \delta_k$ and thus, the upper end of the Cournot region involves zero marginal cost and preorders independent of the magnitude of d while the lower end involves preorders which increase with d. The smallest w can be obtained by the inequality $w - 2\hat{\delta}k_i^C |_{\hat{\delta}} > 0$, which gives rise to $w > \frac{ab^2}{2-b-b^2}$. The upper bound of w is obtained from the relationship that marginal revenue at its maximum, i.e. when own pre-order is zero while the rival pre-order is at its largest, is larger than w. We have $MR = \frac{a(1+b) - 2k_1 - bk_2}{1 - b^2}$

$= \frac{2a}{2-b-b^2} > w$.

APPENDIX B: DEALER 2'S RESPONSE IN THE SECOND STAGE TO DEALER 1'S FIRST STAGE DEVIATION IN THE COURNOT AND BERTRAND REGIMES

B1: In the Cournot regime, no matter how dealer 1 deviates from the benchmark in stage 1, dealer 2 always sells $k_2^C(d)$; *he neither augments nor scraps this preorder.*

To check for $d \in [\delta, \delta_k]$ dealer 2 always sells just his preorders and stays on $s^2(p_1; k_2^C) = a + bp_1 - k_2^C$, we need to show that *at* $k_2^C(d)$ his marginal revenue lies between zero and w regardless whether dealer 1 is on which one of the three parts of his best reply.

Given $s^2(p_1; k_2^C)$, we can obtain three corresponding price regions, i.e., $\underline{p}_1$, $\bar{p}_1$, and those prices lie between the two (p_1), by solving respectively each part of dealer 1's best reply and $s^2(p_1; k_2^C)$: $\underline{p}_1$ is obtained by solving $r^1(p_2) = \frac{a + bp_2}{2}$ and $s^2(p_1; k_2^C)$; $\bar{p}_1$ is obtained from $r^1(p_2; w) = \frac{a + bp_2 + w}{2}$ and $s^2(p_1; k_2^C)$; and p_1 is solved by $s^1(p_2; k_1) = a + bp_2 - k_1$ and $s^2(p_1; k_2^C)$. We divide our analysis into the following three cases which correspond to these prices $\underline{p}_1 = \frac{a(1+b) - bk_2^C}{2 - b^2}$,

$\bar{p}_1 = \frac{a(1+b) + w - bk_2^C}{2 - b^2}$, and $p_1 = \frac{a(1+b) - k_1 - bk_2^C}{1 - b^2}$.

(1). Dealer 1 is on part 1 of his best reply, i.e. $\underline{p}_1 = \frac{a(1+b) - bk_2^C}{2 - b^2}$. Dealer 2's revenue is given by $p_2 q_2 = (a + b\underline{p}_1 - q_2)q_2$ and thus his marginal revenue at k_2^C is $MR_2 = a + b\underline{p}_1 - 2k_2^C$. Substituting $\underline{p}_1$ into it, we get his marginal revenue as

$MR_2 = a + b(\frac{a(1+b) - bk_2^C}{2 - b^2}) - 2k_2^C = \frac{2a + ab - 4k_2^C + b^2 k_2^C}{2 - b^2} = \frac{(2+b)(a - (2-b)k_2^C)}{2 - b^2}$. We can further substitute $k_2^C(d) = \frac{(1+b)(a - (1-b)w)}{(2 + b - 2(1 - b^2)d)}$ to express dealer 2's marginal revenue in terms of d:

$MR_2 = \frac{(2+b)(a(b^2(1+2d)-2d))+(2-b-2b^2+b^3)w)}{(2-b^2)(2+b-2(1-b^2)d)}$. Using $\hat{\delta} = \frac{w(2+b)}{2a(1+b)}$ (see Appendix A),

we verify MR_2

$$= \frac{2(2+b)(ab^2-(2-b-b^2)w+(2-b-2b^2+b^3)w)}{(2-b^2)(2+b)(b(a\text{ - }(1\text{ - }b)w))} = \frac{ab^2(a-(1-b)w)}{(2-b^2)(a-(1-b)w)} = \frac{ab^2}{2-b^2} \geq 0 .$$

Moreover, since $\delta = \frac{b^2}{2(1-b^2)}$, it can be shown that $MR_2 - w$

$= \frac{(2+b)(2-b-b^2+b^3)w}{(2-b^2)(2+b-b^2)} - w = \frac{(2+b)(1-b)w-(2-b^2)w}{2-b^2} = \frac{-bw}{2-b^2} \leq 0$. Note that $\frac{\partial MR_2}{\partial d} =$

$-\frac{2(1+b)^2(4-b^2)(1-b)(a-(1-b)w)}{(2-b^2)(2+b-2d(1-b^2)d)^2} < 0$ at k_2^C: a>(1-b)w (see Appendix A) and all the other bracket-terms in the and numerator and denominator are strictly positive. In other words, MR_2 is decreasing in d. Furthermore, when d>$\hat{\delta}$, preorders – including k_2^C – are not sensitive to marginal changes in d. This means that $MR_2 \geq 0$ at $\hat{\delta}$ implies $MR_2 \geq 0$ when $d > \hat{\delta}$. Therefore, these two results imply $0 \leq MR_2 \leq w$ at k_2^C for all d≥δ.

(2). Dealer 1 is on part 3 of his best reply, or $\overline{p}_1 = \frac{a(1+b)+w-bk_2^C}{2-b^2}$. Dealer 2's revenue is given by $p_2q_2 = (a+b\overline{p}_1-q_2)q_2$ and thus his marginal revenue is $MR_2 = a+b\overline{p}_1-2k_2^C$ at k_2^C. Substituting $\overline{p}_1$ into it, we get his marginal revenue at k_2^C as $MR_2 = \frac{a(2+b)-(4-b^2)k_2^C+bw}{2-b^2}$.

We can further substitute $k_2^C(d) = \frac{(1+b)(a-(1-b)w)}{(2+b-2(1-b^2)d)}$ to express dealer 2's marginal revenue in terms of d:

$\mathrm{MR}_2 = \dfrac{a(2+b)(b^2-2(1-b^2)d)+(4+b(2-4b+b^3-2(1-b^2)d))w}{(2\text{-}b^2)(2+b-2(1-b^2)d)}$. Using $\hat{\delta} = \dfrac{w(2+b)}{2a(1+b)}$, we have $\mathrm{MR}_2 = \dfrac{a(ab^2-(2-b-b^2)w)+w(a(2-2b^2+b^3)-(1-b)bw)}{(2-b^2)(a-(1-b)w)}$

$= \dfrac{b(a^2b+a(1-b-b^2)w-(1-b)w^2)}{(2-b^2)(a-(1-b)w)} = \dfrac{b(ab+w)(a-(1-b)w)}{(2-b^2)(a-(1-b)w)} = \dfrac{b(ab+w)}{2-b^2} \geq 0$ when $d > \hat{\delta}$.

Using $\delta = \dfrac{b^2}{2(1-b^2)}$, we verify at d = δ, $\mathrm{MR}_2 - w = \dfrac{(2+b)(b^2-2(1-b^2)d)(a-(1-b)w)}{(2-b^2)(2+b-2(1-b^2)d} = 0$ since $b^2-2(1-b^2)d = 0$ at δ. The second result implies that when 0≤d<δ, dealer 2 will augment his preorders and thus moves to his part 3 of the best reply $r^2(p_2; w)$. Hence, the Cournot benchmarks do not sustain while the Bertrand benchmarks would be the Nash equilibrium. See below. Note that at k_2^C, $\dfrac{\partial \mathrm{MR}_2}{\partial d} = -\dfrac{8(1-b)(1+b)^2(a-(1-b)w)}{(2-b^2)(2+b-2d(1-b^2))^2} < 0$ because each bracket-term is strictly positive.

Since for d>$\hat{\delta}$, preorders – including k_2^C – are not sensitive to marginal changes in d, $\mathrm{MR}_2 \geq 0$ at $\hat{\delta}$ implies $\mathrm{MR}_2 \geq 0$ when $d > \hat{\delta}$. Therefore, these two results imply $0 \leq \mathrm{MR}_2 \leq w$ for all d≥δ at k_2^C.

(3). Dealer 1 is on the middle section of his best reply, or $p_1 = \dfrac{a(1+b)-k_1-bk_2^C}{1-b^2}$. Note that $\underline{p}_1 \leq p_1 \leq \bar{p}_1$; and $\mathrm{MR}_2 = a + bp_1 - 2k_2^C$, that is, the marginal revenue is linearly increasing in p1. Therefore, the joint results $\mathrm{MR}_2 \geq 0$ at $\underline{p}_1$ and $\mathrm{MR}_2 \leq w$ at $\bar{p}_1$ shown in (1) and (2) imply $0 \leq \mathrm{MR}_2 \leq w$ at p_1. QED.

B2: In the Bertrand regime, no matter how dealer 1 deviates from the benchmark in stage 1, dealer 2 always sells k_2^B*; he neither augments nor scraps his preorder.*

To check for $d \in [0, \delta]$ dealer 2 always sells just his preorders and stays on $s^2(p_1; k_2^B) = a + bp_1 - k_2^B$, we need to show that *at* k_2^B his marginal revenue lies between zero and w regardless whether dealer 1 is on which one of the three parts of his best reply. Given $s^2(p_1; k_2^B)$, we can obtain three corresponding price regions, i.e., $\underline{p}_1$, $\bar{p}_1$, and those prices lie between the two (p_1), by solving respectively each part of dealer 1's best reply and $s^2(p_1; k_2^B)$: $\underline{p}_1$ is obtained by solving $r^1(p_2) = \frac{a + bp_2}{2}$ and $s^2(p_1; k_2^B)$; $\bar{p}_1$ is obtained from $r^1(p_2; w) = \frac{a + bp_2 + w}{2}$ and $s^2(p_1; k_2^B)$; and p_1 is solved by $s^1(p_2; k_1) = a + bp_2 - k_1$ and $s^2(p_1; k_2^B)$. We divide our analysis into the following three cases that corresponds to these prices $\underline{p}_1 = \frac{a(1+b) - bk_2^B}{2 - b^2}$,

$\bar{p}_1 = \frac{a(1+b) + w - bk_2^B}{2 - b^2}$, and $p_1 = \frac{a(1+b) - k_1 - bk_2^B}{1 - b^2}$.

(1). Dealer 1 is on part 1 of his best reply, i.e. $\underline{p}_1 = \frac{a(1+b) - bk_2^B}{2 - b^2}$. Dealer 2's revenue is given by $p_2 q_2 = (a + b\underline{p}_1 - q_2) q_2$ and thus his marginal revenue at k_2^B is $MR_2 = a + b\underline{p}_1 - 2k_2^B$. Substituting $\underline{p}_1$ into it, we get his marginal revenue as $MR_2 = \frac{(2+b)(a - (2-b)k_2^B)}{2 - b^2}$. We can further substitute $k_2^B = \frac{a - (1-b)w}{2 - b}$ to get $MR_2 = \frac{(2+b)(a - (2-b)k_2^B)}{2 - b^2} = \frac{(2+b)(1-b)w}{2 - b^2} > 0$, since both the numerator and denominator are positive. It is also can be verified that at

$$k_2^B,\ MR_2 - w = \frac{(2+b)(1-b)w}{2 - b^2} - w = \frac{(2 - b - b^2)w - (2 - b^2)w}{2 - b^2} = \frac{-bw}{2 - b^2} \le 0.$$

Therefore, the two results imply that $0 \le MR_2 < w$ at k_2^B for all $d \le \delta$.

(2). Dealer 1 is on part 3 of his best reply, or $\bar{p}_1 = \frac{a(1+b)+w-bk_2^B}{2-b^2}$. Dealer 2's revenue is given by $p_2q_2 = (a+b\bar{p}_1 - q_2)q_2$ and thus his marginal revenue is $MR_2 = a + b\bar{p}_1 - 2k_2^B$ at k_2^B. Substituting $\bar{p}_1$ into it, we get his marginal revenue at k_2^B as $MR_2 = \frac{a(2+b)-(4-b^2)k_2^B + bw}{2-b^2}$. We can further substitute $k_2^B = \frac{a-(1-b)w}{2-b}$ to get

$$MR_2 = \frac{a(2+b)-(4-b^2)k_2^B + bw}{2-b^2} = \frac{a(2+b)-(2+b)[(a-(1-b)w]+bw}{2-b^2}$$

$= \frac{(2+b)(1-b)w+bw}{2-b^2} = w$. Since dealer 1's on $r^1(p_2;w) = \frac{a+bp_2+w}{2}$, this result implies that dealer 2 is on both his $r^2(p_1;w)$ and $s^2(p_1;k_2^B)$, or at their intersection point.

(3). Dealer 1 is on the middle section of his best reply, or $p_1 = \frac{a(1+b)-k_1-bk_2^B}{1-b^2}$. Note that $\underline{p}_1 \le p_1 \le \bar{p}_1$; and $MR_2 |_{k_2^B} = a + bp_1 - 2k_2^B$, that is, the marginal revenue is linearly increasing in p1. Therefore, the joint results $0 \le MR_2 < w$ at $\underline{p}_1$ and $MR_2 = w$ at $\bar{p}_1$ shown in (1) and (2) imply $0 \le MR_2 \le w$ at p_1. Q.E.D.

B3: At the Bertrand benchmark, if dealer 1 deviates in the first stage by lowering his preorder he would augment his preorder in the equilibrium of the second stage.

To show dealer 2 augments in the second stage when he deviates his preorder to $k_1 = k_1^B - \varepsilon$, where $\varepsilon > 0$, we need to verify that his marginal revenue is larger than w given the prevailing price of dealer 2 and his deviating preorder. At the Bertrand benchmarks (k_1^B, k_2^B), both dealers are at the intersection of part 2 and part 3 of their best replies. By preordering less than k_1^B, he falls onto part 3 of his best reply $r^1(p_2;w)$. To obtain dealer 2's price, we use Hypothesis 1 and solve $r^1(p_2;w)=\frac{a+bp_2+w}{2}$ and $s^2(p_1;k_2^B)=a+bp_1-k_2^B$ to obtain $p_2=\frac{a(2+b)+bw-2k_2^B}{2-b^2}$.

Further substituting $k_2^B=\frac{a-(1-b)w}{2-b}$, we obtain $p_2=\frac{2a-ab^2-b^2w+2w}{(2-b)(2-b^2)}$ $=\frac{a+w}{2-b}=p_2^B$. Dealer 1's revenue is given by $p_1q_1=(a+bp_2-q_1)q_1$ and thus his marginal revenue at k_1 is $MR_1=a+bp_2-2k_1$. Putting $k_1=k_1^B-\varepsilon=\frac{a-(1-b)w}{2-b}-\varepsilon$ into the expression, we have

$MR_1=a+bp_2-2k_1=a+b(\frac{a+w}{2-b})-2(\frac{a-(1-b)w}{2-b}-\varepsilon)=w+2\varepsilon>w$. Q.E.D.

B4: At the Bertrand benchmark, if dealer 1 deviates locally in the first stage by raising his preorder he would sell just his preorder in the equilibrium of the second stage

To show dealer 2 sells just his preorder in the second stage when he deviates his preorder marginally to $k_1=k_1^B+\varepsilon$, where $\varepsilon>0$, we need to verify that his marginal revenue is larger than zero but smaller than w given the prevailing price of dealer 2 and his deviating preorder. At the Bertrand benchmarks (k_1^B,k_2^B), both dealers are at the intersection of part 2 and part 3 of their best replies. By preordering more than k_1^B, he moves into part 2 of his best reply $s^1(p_2;k_1)=a+bp_2-k_1$. To obtain dealer 2's price, we use Hypothesis 1 and solve $s^1(p_2;k_1)=a+bp_2-k_1$ and $s^2(p_1;k_2^B)=a+bp_1-k_2^B$ and yield $p_2=\frac{a(1+b)-bk_1-k_2^B}{1-b^2}$.

Substituting $k_1=k_1^B+\varepsilon$ and using the fact that $k_2^B=k_1^B$, we have $p_2=\frac{a(1+b)-(1+b)k_1^B-b\varepsilon}{1-b^2}$.

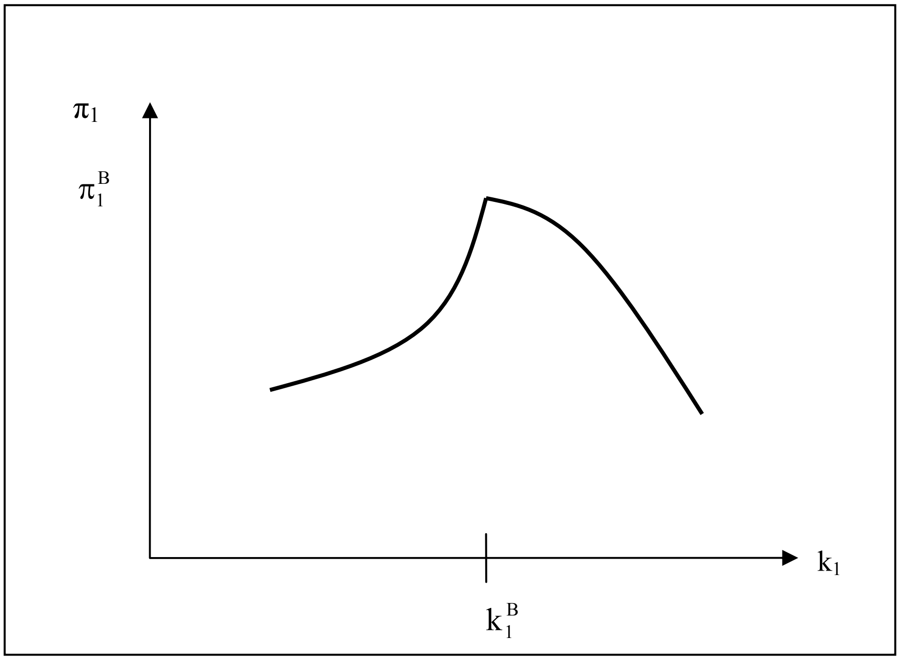
Dealer 1's revenue is given by $p_1q_1=(a+bp_2-q_1)q_1$. Thus his marginal revenue at k_1 is

$MR_1=a+bp_2-2k_1=a+bp_2-2(k_1^B+\varepsilon)=\frac{a(1+b)-(2+b-b^2)k_1^B-(2-b^2)\varepsilon}{1-b^2}$. Substituting,

$k_2^B = \frac{a-(1-b)w}{2-b}$, we obtain $MR_1 = \frac{a(1+b)-(1+b)(a-(1-b)w)-(2-b^2)\varepsilon}{1-b^2}$

$= w - \frac{2-b^2}{1-b^2}\varepsilon < w$ for small ε. Hence, $0 < MR_1 < w$ at p_2. Q.E.D.

FIGURE B1: DEALER PROFIT IN THE BERTRAND REGIME ($d \in (0,\delta)$ **)**



APPENDIX C: NO SUBGAME-PERFECT EQUILIBRIUM INVOLVES SCRAPPING OR AUGMENTING (FOR $d > 0$)

Lemma: In any subgame-perfect equilibrium, if $d > 0$, $k_i = q_i = D_i(p_i, p_j)$, $i = 1, 2$.

For, suppose that there is a subgame-perfect equilibrium where some of what is preordered is scrapped ($k_1 > q_1$). Then firm 1 can reduce its pre-order by $\varepsilon > 0$ such that $k_1 - \varepsilon > q_1$. This will strictly lower costs in the first stage since $(w - d\,k_1)k_1 - [w - d(k_1 - \varepsilon)](k_1 - \varepsilon) = \varepsilon(w - 2\,d\,k_1) + 2\,d\,\varepsilon^2 > 0$, where the inequality holds for sufficiently small $\varepsilon > 0$. We now verify that the prices in the subgame corresponding to the pre-order pair $(k_1 - \varepsilon, k_2)$ remain (p_1, p_2). Given p_1, the situation of dealer 2 is unchanged: it faces the same demand curve and has the same preorder; hence, p_2, remains the best reply. Given p_2, dealer 1 faces the same demand curve but its preorder is now $k_1 - \varepsilon > q_1$. However, p_1 is dealer 1's best response to p_2 as long as $k_1 > q_1$, which is still the case under our supposition. Hence, the unilateral deviation strictly reduces dealer 1's costs without altering its gross revenue. Since this deviation strictly increases dealer 1's profit, there can be no subgame-perfect equilibrium in which $k_1 > q_1$. Since the same argument can be made for dealer 2, there can be no subgame-perfect equilibrium in which $k_2 > q_2$.

Suppose instead there is an subgame-perfect equilibrium with $k_1 < q_1$. Then dealer 1 can increase its pre-order by $\varepsilon > 0$ such that $k_1 + \varepsilon < q_1$. This will strictly raise costs in the first stage since $[w - d(k_1 + \varepsilon)](k_1 + \varepsilon) - (w - d\,k_1)k_1 = \varepsilon(w - 2\,d\,k_1) - d\,\varepsilon^2 > 0$, where the inequality holds for sufficiently small $\varepsilon > 0$. We now verify that the prices in the subgame corresponding to $(k_1 + \varepsilon, k_2)$ remain (p_1, p_2) and since dealer 1 will not have to supplement the additional amount it preordered that its total costs fall. Given p_1, the situation of dealer 2 is unchanged: it faces the same demand curve and has the same preorder; hence, p_2 remains the best reply. Given p_2, dealer 1 faces the same demand curve but its preorder is now $k_1 + \varepsilon < q_1$. However, p_1 is dealer 1's best response to p_2 as long as its preorder requires any augmentation, which is still the case. Hence, the unilateral deviation does not alter dealer 1's gross revenue. His total costs fall, however, since what he previously augmented at the second stage at the undiscounted cost of $w\varepsilon$ he would preorder instead at the first stage at the lower cost of $\varepsilon(w - 2\,d\,k_1) - d\,\varepsilon^2 > 0$. Since this deviation strictly increases dealer 1's profit, there can be no subgame-perfect equilibrium in which $k_1 < q_1$. The identical argument could be made for dealer 2. Hence, there can be no subgame-perfect equilibrium in which $k_2 < q_2$. Q.E.D.

APPENDIX D: DEALER PRICING RULES UNDER BERTRAND AND COURNOT INFERENCES

A typical demand side model has the following form:

$$q_{it} = A_i + \beta_1 p_{it} + \lambda_{gj} (\sum_{i \neq j} p_{jt}), i = i,2,...n,\ i \neq j,$$

where p_i, p_j are own price and the retail prices of i’s rival dealers located in the same region respectively and g is region g. λ_{gj}'s are the coefficients of cross-price effects of own brand. To avoid clutter, we omit the error term and let A_i absorb non-price terms in Equation (6) in the text, viz. prices of competing manufacturers, marketing expenses, and α_i. Assuming uniform cross-price effect, the demand function can then be rewritten as

$$q_{it} = A_i + \beta_1 p_{it} + \lambda_g (\sum_{i \neq j} p_{jt}) = A_i + \beta_1 p_{it} + (n_g - 1)\lambda_g (\frac{\sum_{i \neq j} p_{jt}}{n_g - 1}).$$

Pooling for all dealers, we rewrite the national-level pooled demand equation as what we have in the text (Equation 6):

$$q_{it} = A_i + \beta_1 p_{it} + \beta_2 p_{-it}, \tag{D1}$$

where $p_{-it} \equiv \frac{\sum_{i \neq j} p_{jt}}{n_g - 1}$, the average price of i's rivals in region g, and $\beta_2 \equiv (n_g - 1)\lambda_g$, the sum of individual rivals’ cross-price effect. Note that $|\beta_1| > \beta_2 \equiv (n_g - 1)\lambda_g$ and the sign of β_1 is negative while those of β_2 and λ_g are positive.

In line with our demand specification in (D1), we work out dealer’s pricing rule by assuming the following system of demand functions for dealers located in a given region g:

$$
\begin{aligned}
q_{1t} &= A_1 + \beta_{1g} p_{1t} + \lambda_g \Big(\sum_{j \neq 1} p_{jt}\Big) \\
q_{2t} &= A_2 + \beta_{1g} p_{2t} + \lambda_g \Big(\sum_{j \neq 2} p_{jt}\Big) \\
&\ldots \\
q_{n_g t} &= A_{n_g} + \beta_{1g} p_{n_g t} + \lambda_g \Big(\sum_{j \neq n_g} p_{jt}\Big).
\end{aligned}
$$

β_{1g} is the regional-specific slope of demand obtained by interacting regional dummies with own prices (see page 25 in the text). Solving for prices, we have

$$
p_{it} = \frac{K(n_g, A_i) + [\beta_{1g} + \lambda_g (n_g - 2)] q_{it} - \lambda_g (n_g - 1) \sum_{j \neq i} q_{jt}}{(\beta_{1g} - \lambda_g)(\beta_{1g} + \lambda_g (n_g - 1))},
$$

where i=1,2, …, n_g, $i \neq j$, $i, j \in g$, and K(n_g, A_i) is a constant in terms of n_g and A_i's. Hence,

$\left.\frac{\partial q_{it}}{\partial p_{it}}\right|_{p_{jt}} = \beta_{1g}$ when p_{jt} is kept constant and $\left.\frac{\partial p_{it}}{\partial q_{it}}\right|_{q_{jt}} = \frac{\beta_{1g} + \lambda_g (n_g - 2)}{(\beta_{1g} - \lambda_g)(\beta_{1g} + \lambda_g (n_g - 1))}$

$= \frac{\beta_{1g} + \beta_2 (\frac{n_g - 2}{n_g - 1})}{(\beta_{1g} - \frac{\beta_2}{n_g - 1})(\beta_{1g} + \beta_2)}$ when q_{jt} is kept constant.

In the Cournot regime, the outcome of dealer competition is equivalent to a one-shot Cournot game and hence, dealer i maximizes its profit by choosing quantity, i.e.

$\max_{q_{it}} \pi_{it} = p_{it} \cdot q_{it} - \int_0^{q_{it}} c_{it}(u)\, du$ at given competing dealers' quantities, q_{jt}. The first-order condition

is $\frac{\partial \pi_{it}}{\partial q_{it}} = (p_{it} - c_{it}) + q_{it} \left.\frac{\partial p_{it}}{\partial q_{it}}\right|_{q_{tj}} = 0$ where c_{it} is the discounted wholesale price or marginal cost.

Substituting $\left.\frac{\partial p_{it}}{\partial q_{it}}\right|_{q_{jt}} = \frac{\beta_{1g} + \lambda_g (n_g - 2)}{(\beta_{1g} - \lambda_g)(\beta_{1g} + \lambda_g (n_g - 1))}$ and rearranging terms, we obtain the optimal pricing rule for dealer i:

(D2)
$$p_{it}^{C} = c_{it} - \frac{\beta_{1g} + \lambda_g (n_g - 2)}{(\beta_{1g} - \lambda_g)(\beta_{1g} + \lambda_g (n_g - 1))} q_{it}^{C}.$$

Due to data limitation, we approximate an individual dealer's markup with the average markup corresponding to his located region. To obtain regional-specific markups, we sum both sides of (D2) across i and t and then divide the number of dealers in a region and 12 (data periods):

$$\sum_{t=1}^{12} \sum_{i=1}^{n_g} p_{it}^{C} / (n_g \cdot 12) = \sum_{t=1}^{12} \sum_{i=1}^{n_g} c_{it} / (n_g \cdot 12) - (\sum_{t=1}^{12} \sum_{i=1}^{n_g} \frac{\beta_{1g} + \lambda_g (n_g - 2)}{(\beta_{1g} - \lambda_g)(\beta_{1g} + \lambda_g (n_g - 1))} q_{it}^{C} / (n_g \cdot 12),$$

where the second term on the right hand side is the regional-level markup. It can be rewritten as $-\frac{\beta_{1g} + \lambda_g (n_g - 2)}{(\beta_{1g} - \lambda_g)(\beta_{1g} + \lambda_g (n_g - 1))} \frac{Q_g}{12\, n_g}$. Using this and further substituting the estimates of own and cross prices obtained from Equation (6) in the text into (*), we calculate dealer i's final price by:

$$p_{it}^{C} = c_{it} - \frac{\hat{\beta}_{1g} + \hat{\beta}_2 (\frac{n_g - 2}{n_g - 1})}{(\hat{\beta}_{1g} - \frac{\hat{\beta}_2}{n_g - 1})(\hat{\beta}_{1g} + \hat{\beta}_2)} \frac{Q_g}{12\, n_g}.$$

Q.E.D.