A Customer Management Dilemma:
When to Punish or Reward Own Customers

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November 2008

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Abstract

Firms routinely use behavior based pricing (BBP), to segment customers by past purchase behavior. This paper answers a dilemma facing such firms – When should the firm reward existing customers as opposed to new customers? Further, the paper sheds insight on when behavior based pricing will increase or decrease profits in a competitive market.

The analysis adds two features of customer behavior hitherto ignored in the literature: (1) heterogeneity in customer value and (2) mobility in preferences, i.e., customer preferences are correlated but not fixed over time. We identify conditions for when it is optimal to reward/punish own customers and when BBP can increase/decrease profits. To the best of our knowledge, we are the first to show analytically that it can be optimal to reward one's own customers under symmetric competition and that BBP can increase profits even when consumers and firms are fully strategic and forward looking.

Key Words: Targeted pricing, Behavior based pricing, 80–20 rule, Customer mobility, Forward-looking customers, Forward-looking firms, Customer relationship management, Competitive strategy, Game theory.

1. Introduction

Consider Bob's experience with WLC (We Love Our Customers) Bank. Bob obtained a home equity line of credit a few years ago from WLC at an interest rate of 'prime + 0.25'% when the prime rate was 3.5%. As the Federal Reserve raised discount rates, the prime rate grew to 8.25%, making home equity lines of credit very profitable for banks. Therefore, interest rates for new home equity lines of credit have fallen to 'prime –1'%. WLC recognized Bob as one of its most valuable customers, who maintained a high balance on his equity line of credit. To retain him against potential offers from competitors, WLC proactively reduced Bob's interest rate to 'prime – 0.5'%. Although this rate represents an improvement over the existing rate that Bob previously had been paying, it was higher than what it offers its new customers. Bob considered WLC's offer, yet switched to a rival bank as a new customer and obtained a rate of 'prime –1'%.

Was WLC right in "punishing" a valued current customer such as Bob? Should WLC have offered the same rate it provides to its new customers to retain Bob as a customer? Or given that WLC recognizes Bob as a valuable customer, should it have "rewarded" him by offering an even lower rate than for new customers? This customer management dilemma is ubiquitous across many different industries: Should a wireless carrier offer lower rates to its high volume customers who uses more minutes or to its new customers? Would it be more profitable for a magazine to offer a better price for subscription renewal as opposed to a new subscription? Should a hotel, airline, or retailer offer better rates for a high value loyalty card member or to a new customer? This paper seeks answers to two basic customer management questions: First, when should a firm offer better value to its current customers or new customers? Second, when will differential pricing to one's current versus new customers — whether one rewards own or new customers — be profitable in a competitive market?

Behavior based pricing (BBP), where firms offer different prices to customers based on past purchase behavior is now widely prevalent.[1] Over the last two decades, firms have made massive investments of organizational resources (human, technical, and financial) in information infrastructures to store and analyze customer purchase behavior, facilitating BBP. Yet surprisingly, there is little consensus on the two basic questions we address in the paper. Many

[1] The literature variously refers to it as *behavior-based price discrimination* (Fudenberg and Tirole 2000) or *pricing with customer recognition* (Villas-Boas 1999). To the extent that such targeted pricing distinguishes customers based on their prior relationships with the firm, it may be referred to as *customer relationship management pricing.*

practitioners and academic literature arrive at opposing answers. For example, O'Brien and Jones (1995, p.76) epitomize the conventional wisdom for many practitioners: "to maximize loyalty and profitability, a company must give its *best value to its best customers*. As a result, they will then become even more loyal and *profitable*" (emphasis added). If the firm can offer different prices to different customers according to their past purchases, rewarding these current customers will pay off in more efficient, mutually beneficial firm–customer relationships through a virtuous cycle. In other words, the firm rewards its current customers with better value propositions, which makes it optimal for them to deepen their relationship with the firm, which increases customer satisfaction, loyalty, and, ultimately, firm profitability (Peppers and Rogers 2004).

Yet theoretical literature remains skeptical of this conventional wisdom. In most existing models of behavior-based pricing (BBP), current customers do not receive the best value, because if the firm can price discriminate on the basis of consumers' past purchase behavior, it should charge existing consumers, who already have revealed their higher willingness to pay for the product, higher relative to competitors' customers. In this sense, customers' past purchases reveals their relative preference for each firm. If consumers are forward looking, they recognize the possibility that the firm may take advantage of this preference information to discriminate against them and may, therefore, modify their purchase behavior to prevent firms from inferring their true preferences. In one set of models (e.g., Villas-Boas 2004), when a monopolist faces strategic, forward-looking customers, customers choose not to purchase initially to prevent the firm from inferring their true preferences, which could be used to hurt them in the future. Worse, such strategic behavior can actually reduce firm profits in equilibrium, relative to the case when they commit not to use purchase information in pricing.

These issues become magnified in the presence of competition. Existing models still indicate it is never optimal for firms to offer the best value to their existing customers. Even when consumers are not strategically forward looking but are willing to end up in a relationship trap, competing firms may suffer compared with a scenario without past purchase information. Thus, in a competitive market, firms confront a prisoner's dilemma regarding the use of information about customer purchase history (Villas-Boas 1999; Fudenberg and Tirole 2000). According to Fudenberg and Villas-Boas's (2006, p. 2) succinct summary of extant literature on behavior-based price discrimination, "the seller may be better off if it can commit to ignore information about buyer's past decisions…. more information will lead to more intense competition between firms." In a recent paper, Pazgal and Soberman (2008) show that BBP can be profitable but only if one of the firms use BBP. However, they also restate the conclusion of the theoretical literature that BBP in a symmetric competitive environment "generally leads to lower profits for both firms. … form a prisoner's dilemma."

How can the discrepancy between theoretical predictions and practitioner intuition be explained? We contend that the discrepancy occurs because existing models abstract away important features of customer behavior in real world markets. By incorporating two simple but characteristic features of customer behavior in existing BBP models, we are able to reconcile the different viewpoints by identifying conditions when either or both practitioner intuition and current theoretical results hold. We describe these two key features that we add to our model below.

Customer heterogeneity

First, we incorporate the intuition that not all customers are equally valuable into our model. For example, some customers purchase more than others or contribute more to a firm's profits. We believe this customer heterogeneity is a critical feature for capturing the practitioner notion of a "best" customer. There is widespread empirical support in various categories for the 80–20 rule, which states that a small proportion of customers contribute to a large number of purchases and profit in a category (Schmittlein et al. 1993).

Incorporating customer heterogeneity in purchase quantity into the model, allows firms to learn a new dimension of information about consumers from purchase history data. Purchase history now provides two different types of information: (1) whom they bought from and (2) how much they bought. As in many previous works, "whom they bought from" provides horizontal "relative preference" information between the two firms. The new "how much they purchase" provides *vertical* information about the relative importance of consumers for the firm.

Not only does the vertical information about quantity provide additional information to firms, it generates an intrinsic customer information asymmetry among firms because a firm can identify "best customers" only among its own customers, but not among its competitor's customers. In other words, the firm knows only that there is a mix of high and low volume customers among its competitor's customers but not the exact identity of who is who. Existing models, in which all customers buy just one unit of the product and the market is fully covered, do not allow for information asymmetry. This is because if a consumer does not buy from one firm, it must buy from the other since the market is covered. These models, therefore, ignore a major reason firms invest in customer relationship management (CRM) programs in practice: to obtain an information advantage over competitors about existing customers. Hence, extending the scope of the BBP into the vertical dimension of customer type information by modeling the customer heterogeneity is critical in bridging the discrepancy between theoretical predictions and practitioner intuition.

Customer mobility

Second, we recognize that consumers' preference for a product may change across purchase occasions, independent of the marketing mix or pricing, because their needs or wants depend on the specific purchase situation, which changes over time (Wernerfelt 1994). The assumption of changing customer preferences across purchase occasions (hereafter, customer mobility) is particularly relevant in the context of store choice, because consumers' geographic locations often appear stochastic. For example, a customer may generally prefer Lowe's for home improvement products, because it is closer to home and offers superior quality offerings. However, she may still go to Home Depot on the way home from the office as it is closer to the office.

The idea of customer mobility or changing customer preferences is not restricted to store choice and geographic locations. Consider a college student who lives in New York. She generally prefers American Airlines because she likes its service and it flies a direct route to her hometown. However, when she needs to visit a friend at Houston, she may prefer Continental because there are more direct flights for that route. A similar logic may apply to consumers' choice of hotels; even if a customer prefers Marriott in general, he may find that a Sheraton satisfies his needs better on a particular trip because of its proximity to a conference venue.[2]

We note that the past research in BBP (for example, Caminal and Matutes 1990, Fudenberg and Tirole 2000) has considered customer mobility, but only for the extreme case in which customers' preferences are completely independent over time. However, when preferences are completely independent, customers' past purchase data does not have any value for firm since it does not have any predictive power of future purchase. It is obvious that for past purchase

[2]Further, consumer preferences cannot be perfectly observed or inferred by a firm, even with long purchase history (Pancras and Sudhir 2006). Lee et al. (2002) find that a consumer's preference (ideal point) varies stochastically over time.

information to be valuable to firms, preferences should be correlated across time. One also expects that in most markets preferences are likely to be correlated across time. We therefore allow preferences to be correlated across time; indeed we find that modeling this correlation is critical in obtaining insight to the questions of interest[3].

With these two features included, we identify conditions under which (1) firms should reward their own best customers or competitor's customers and (2) behavior based pricing (BBP) increases or decreases firm profits in a competitive environment when firms and consumers are strategic and forward looking. Specifically, we find that in markets with both sufficient heterogeneity in customer value *and* customer mobility, it is optimal to reward one's best customers; otherwise, firms should reward competitors' customers.[4] Our result is in contrast to extant research which claims that it is never optimal to reward one's own customers. We also show that *either* sufficient heterogeneity in customer value *or* customer mobility is sufficient for BBP to increase firm profits. This profit result is in contrast to the existing papers which claim that in a symmetric competitive setting BBP always leads to a prisoner's dilemma, reducing firm profits. Moreover, as most real markets are likely to have either customer heterogeneity in quantities or mobility, we conclude that BBP is almost always profitable in competitive environments even when firms and consumers are strategic and forward looking. It, therefore, provides a justification for the continued investments by firms in customer management technologies.

The rest of this article is organized as follows: Section 2 describes the related literature. Sections 3 and 4 describe the model and the analysis respectively. Section 5 concludes.

2. Literature Review

Our research connects with several interrelated areas of marketing and economics. The primary contribution is to the literature on behavior-based pricing (BBP), which uses customer's past purchase information to infer their preference and focuses on the issue of whether to reward own or competitor's customers (for a comprehensive review, see Fudenberg and Villas-Boas

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[3] Chen and Pearcy (2007) also consider a general relationship for consumers' preferences across periods in the context of behavior based pricing as we did. But they do not incorporate customer heterogeneity and as such their model can be regarded as a special case of our model.

[4] Another potential reason for why rewarding existing customer may be profitable could be that the selling cost to serve existing customers is lower than the cost of serving newly acquired customers (Dowling and Uncles 1997, Shin 2005). Our results here focus on a pure demand side explanation.

2006). In terms of model setup, the paper is closest to Fudenberg and Tirole (2000) who analyze a two-period duopoly model with firms at the extremes of a Hotelling line. In their model, firms can price discriminate between its own customers and competitor's customers on the basis of past purchase history that reveals the customer's relative preference between firms. Villas-Boas (1999) extends Fudenberg and Tirole (2000) to an infinite period model with overlapping generations of customers. In both papers where both firms and customers are forward looking, behavior based pricing leads to a prisoner's dilemma of lower profits than firms could credibly commit not to use past purchase information, and it is never optimal to reward one's own customers. In terms of model features, we add customer heterogeneity in purchase quantity and correlated preference mobility across time to the Fudenberg and Tirole (2000) model to get the new insights in this paper. Pazgal and Soberman (2008) also show that firms can increase profit if only one of the firms practices behavior-based price discrimination. However, it is still not profitable to reward own customers. [5]

A second related literature is on switching costs (see Farrell and Klemperer 2007 for a comprehensive literature review). Firms can price discriminate between their old locked-in customers through switching costs and customers locked-in to a rival. Chen (1997) analyses a two-period duopoly model with heterogeneous switching costs among customers. Firms charge higher prices to their own customers relative to their rival's customers, but customers with lower switching costs switch firms in equilibrium. Similar to the BBP literature, firm always charge lower price to its competitor's customers and the total profits are lower than if they could not discriminate. Taylor (2003) extends Chen's model to competition between many firms and continues to find that firms charge the lowest prices to new customers. Shaffer and Zhang (2000) consider a static game similar to the second period of the Fudenberg and Tirole (2000)'s two period model, but allow for switching costs to be asymmetric between the consumers of the two firms. With asymmetric switching costs, the firm with lower switching cost may offer lower prices to own customers, but it is never optimal for both firms to charge lower prices to their own customers. Moreover, when switching costs are symmetric, firms always charge a lower price to their rival's consumers. In their comprehensive review, Farrell and Klemperer (2007) conclude

[5] A related paper is Fudenberg and Tirole (1998) who identify conditions under which a firm selling successive generations of durable goods should punish or reward current consumers under monopoly setting.

that the literature has found it hard to explain discrimination in favor of old customers (page 1993).

In contrast to the literature in behavior based pricing and switching costs, we demonstrate (1) that profits can increase even if both firms practice behavior-based pricing and (2) it can be optimal for both firms to reward its own consumers under symmetric competition, even if consumers and firms are strategic and forward-looking.

Third, our work aligns closely with theoretical and empirical literature on targeted pricing. Unlike the behavior based price discrimination literature, competitive models on targeting are static, where firms discriminate consumers based on perfect or noisy information about their underlying preferences for products. In the behavior based pricing literature, one explicitly models how behavior in the first period provides firms information about consumer preferences, which are then used to determine discriminatory prices in the second period. In contrast, how preference information is obtained by firms is typically not modeled in targeted pricing literature. Although targeted pricing by a monopolist always leads to profits that are at least as great as those under uniform pricing, the competitive implications in oligopoly markets are subtle. Thisse and Vives (1988) and Shaffer and Zhang (1995) show that price discrimination effects get overwhelmed by price competition effects in targeted pricing, leading to a prisoner's dilemma. Liu and Zhang (2006) show that targeted pricing reduces the profits of not only competing manufacturers but the focal retailers as well. More generally, Cabral and Villas-Boas (2005) present a sufficient condition for when the strategic effect through competitors' reaction outweighs the direct effect of price discrimination. Corts (1998) shows that price discrimination by all competing firms may lead to "all-out competition," lower prices, and poor profits in all market segments. However, Corts (1998) also provides conditions in which competitive price discrimination may lead to increased profits. Chen et al. (2001) also show that targeting accuracy can moderate profitability of firms. They recognize that consumer preference information is noisy; hence targeting is imperfect. At low accuracy levels, the positive effect of price discrimination on profit is stronger, whereas at high levels, the negative effect of competition on profit is stronger. Overall, profits are greatest at moderate levels of accuracy.

Empirical literature on this topic (Besanko et al. 2003; McCulloch et al. 1996; Pancras and Sudhir 2006) indicates that firms can improve profits through targeted pricing if firms use customers' past purchase behavior to infer preferences. Interestingly, the probit and logit empirical models in these papers that support the profit improvement achieved through targeted pricing include the two key consumer features that we model, namely, customer heterogeneity in category usage (through the brand intercept terms relative to the outside good) and mobility in customer preferences over time (through the normal and extreme value distributions in consumer utility).

Fourth, our paper relates to financial models of credit markets with adverse selection (Dell'Ariccia et al. 1999; Pagano and Jappelli 1993; Sharpe 1990; Villas-Boas and Schmidt-Mohr 1999). Rather than learn about customer's relative preference (horizontal preference information), which has been the focus of existing work in behavior based pricing, here firms learn vertical information more about their own customers' type (i.e., their ability to repay loans during the lending process). Firms can use this information asymmetry to determine future loans to those customers.[6] Thus, our model combines two streams of research about information from customers' past purchase data – horizontal preference information, which has been the focus of the BBP literature, and the vertical type information, which is the main focus of adverse selection literature in economics and finance.

Finally, our work relates to the CRM literature in marketing (for a general review, see Boulding et al. 2005). A large body of research uses data about customers' purchase history to estimate their lifetime value. Thus, the firm can identify its most valuable customers and provide them with differentiated value propositions through different price and service levels (Blattberg and Deighton 1996; Gupta et al. 2004; Jain and Singh 2002; Venkatesan and Kumar 2003; Zhang 2008). Syam and Hess (2006) analyze the importance of using CRM for acquisition versus retention in a competitive setting. They find that competitive firms may differentiate with one adopting a retention strategy and the other adopting an acquisition strategy using CRM. Similarly, Musalem and Joshi (2008) analyze the problem of customer acquisition and retention in a competitive market when service is an important component. They find that development of CRM may not necessarily result in increased profits because of the more intense competition between firms.

We acknowledge other ways in which a firm may identify consumer types or preferences without using past purchase information. For example, in traditional models of third-degree price

[6] There are two related theoretical papers in marketing that have accommodated heterogeneity in purchase quantities (Kumar and Rao 2006, Kim et al. 2001). Neither paper, however, addresses behavior based pricing -– namely, offering different prices to own versus competitor's customers.

discrimination (Tirole 1988), firms offer discounts on the basis of observable and exogenous consumer characteristics (e.g., student and senior citizen discounts). Alternatively, firms can use second-degree price discrimination by offering a menu of bundles (price, quantity), such that consumers voluntarily reveal their type information. In particular, second-degree price discrimination through a price menu based on purchase quantity seems very relevant in our setting.

Given our focus on behavior-based price discrimination, we abstract away from modeling second-degree price discrimination and leave it for the future research. Nevertheless, our model is a reasonable abstraction of many real world markets (for example, airlines, departmental stores, grocery stores), where firms only use behavior based pricing with past customer purchase history data, but do not offer a menu of prices. This could be because consumers differ not in the quantity they use at any one time, but in the frequency of their use. Hence firms need to aggregate the flow of multiple purchases across a certain time period into a "past quantity" stock. In this scenario, menu pricing is not feasible and behavior based pricing is appropriate. Our analysis is in this spirit. Alternatively, menus may not be feasible if there are high menu costs. Further, our model is in the spirit of the empirical literature using purchase history based pricing (e.g., Rossi et al. 1996; Besanko et al. 2003; Pancras and Sudhir 2006), where firms differentiate customers who purchase different quantities based on past purchase behavior, without offering a menu of prices.

We also recognize that issues of fairness arise when firms charge different prices to different customers (Anderson and Simester 2007, Fehr and Schmidt 1999, Feinberg et al. 2002, Rabin 1993, 1998) and that can limit firms' ability to price discriminate. This paper abstracts away from fairness issues. Further, given our research objectives, we abstract away from the issue of long-term contracts where firms can commit to future prices, that is considered in Fudenberg and Tirole (2000).

3. Model

We follow Fudenberg and Tirole (2000) in considering a two period standard Hotelling model. There are two retailers who are geographically located on the two ends of a unit line. We denote the retailer located at point 0 as retailer *A* and the retailer at point 1 as retailer *B*. The retailers sell an identical nondurable good (e.g. ice cream, gasoline) from which the consumer receives a gross utility of v. We assume that v is large enough that in equilibrium all consumers will purchase the product from one of the retailers.[7] We assume the retailer's marginal cost for the product is constant and normalize it to zero without loss of generality. The market has two periods, and consumers make purchase decisions in both periods. We denote the prices charged by retailers A and B it period t as p_t^A and p_t^B, respectively.

We now adapt the Hotelling model to capture the two key features -- customer heterogeneity and customer mobility. First, to capture customer heterogeneity in their lifetime value parsimoniously, we adapt the Hotelling model to allow for two types of consumers in the market: a low type segment (L) that purchases only one unit of the good in each period, and a high type segment (H) that purchases q units of the good.

Both H- and L-type consumers' geographic locations or preferences (denoted θ) are uniformly distributed along the Hotelling line, $\theta \sim U[0,1]$. We normalize the size of the each market to 1 (we later relax this equal size assumption of two segments in section 4.4.), such that an H-type consumer and an L-type consumer located at θ receive the following utilities from purchasing the product:

$$U^H(p_t^A, p_t^B | \theta) = \begin{cases} q(v - p_t^A) - \theta & \text{if purchase from retailer } A, \\ q(v - p_t^B) - (1 - \theta) & \text{if purchase from retailer } B, \end{cases}$$

and

$$U^L(p_t^A, p_t^B | \theta) = \begin{cases} v - p_t^A - \theta & \text{if purchase from retailer } A, \\ v - p_t^B - (1 - \theta) & \text{if purchase from retailer } B. \end{cases}$$

In this model, we treat q as exogenous; that is some consumers exogenously need more of the product than others. For example, a household with two people will need two cones of ice cream as opposed to one cone for a single person household; or a person who lives farther from work will buy gasoline more frequently than someone who lives close to work.[8]

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[7] These assumptions of exogenous locations at the ends of the Hotelling line and full market coverage are standard in the literature (e.g., Villas-Boas 1999; Fudeberg and Tirole 2000), and they ensure that there are no discontinuities in the demand function which is a general problem in the Hotelling type models when the location choices are endogenous (d'Aspremont et al. 1979).

[8] In reality, q will arise from multiple purchases across each period. The current formulation captures the spirit of 80-20 rule parsimoniously and keeps the model analytically tractable. Further, we recognize that higher quantity of the H type does not automatically imply higher lifetime value, if the price for the high type is very low. But in equilibrium, the optimal prices will be such that per-period profits (and therefore life time value) will be higher from

Second, to capture "customer mobility," we allow for the preference for retailer to change across periods. As discussed in the introduction, the change in preference for retailer may occur due to a change in the consumer's geographic location. For example, a customer may prefer one retailer when beginning the shopping trip from home and another when starting from the office. Similar to Klemperer (1987), we allow that customer location in the second period may change with probability β and remain the same as in the first period with probability $1-\beta$. When the location changes in the second period, the new location is drawn from a uniform distribution $\varepsilon \sim U[0,1]$.

Let θ_1 be the first-period location. Then the second period location $\theta_2 = \varepsilon$ with probability β, or $\theta_2 = \theta_1$ with probability $1-\beta$. Therefore, customer locations are correlated over the two periods, and the expected location of the second period for a customer is $E(\theta_2) = \beta\varepsilon + (1-\beta)\theta_1$. When there is no customer mobility ($\beta = 0$), $\theta_2 = \theta_1$ all the time. In contrast, when $\beta = 1$, maximum mobility exists, because consumer locations are completely independent.[9]

In the first period, retailers *A* and *B* each offer a single price p_1^A and p_1^B, respectively, to all consumers. Consumers purchase from either retailer, depending on which choice is optimal for them. Note that we allow consumers to be forward looking so they can correctly anticipate how their purchase behavior will affect the prices they will have to pay in the future and strategic so they may can adjust whom they buy from in the first period to avoid being taken advantage of by the retailer in the second period.

In the second period, the retailer now can distinguish three types of customers: (1) customers who bought *q* units from it, (2) customers who bought one unit from it, and (3) customers who bought no units (and therefore must have bought from the competitor). The retailers can also infer two types of information about customers based on the purchase behavior in the first period. First, the retailers infer *horizontal information* about relative location proximity to *A* or *B*. This is

the high type (that is $q \times p_2^H > p_2^L$ for all β and $q \geq 2$) so that the high type has indeed higher lifetime value for the firm. See footnote 18.

[9] Another specification for customer mobility that we considered is as follows: the second-period location, θ_2, is the weighted average of the first-period location and the external situational shock ($\theta_2 = (1-\beta)\theta_1 + \beta\varepsilon$). However, this specification leads to too many corner solutions, which makes the analysis extremely tedious without adding insight. Nevertheless, we find a parameter space in which the "reward customer" and "increased profit" results hold, suggesting that our key results are robust to alternative specifications.

symmetric to both retailers because the market is covered. For any given set of retailers' prices, it is possible for both retailers to infer based on whether the consumer bought (or did not buy) from them, whether that consumer location was closer to *A* or *B*. Second, the retailer can infer *vertical information* about the type of the customer based on the quantity purchased. However this information is asymmetric in that a firm knows this information only about its own customers. In this way, customer heterogeneity of purchase quantity confers the retailers an endogenous information advantage about their current customers in the second period.[10] This point represents a key departure from existing models that include no heterogeneity in purchase quantity.

Though the retailer knows that consumers who purchased from it in the first period had a relative preference for it, the retailer also recognizes that it is *not* guaranteed that those consumers will continue to prefer it again in the second period because consumer preferences are not fixed. But as we show in Section 4.1, a consumer who purchased from one retailer in the first period is probabilistically more likely to prefer the same retailer in the second period as long as the customer mobility is below a certain threshold. In other words, when mobility is below a threshold, the probability that the customer remains close to the same retailer is greater than the probability of the customer moving closer to the other retailer. Thus, retailers possess useful probabilistic information about the relative preference of the customer in the second period, given the purchase information in the first period. We formally analyze relative location more precisely in Section 4.1, but for now, it suffices to say that first-period choice reveals information about relative location, even when consumer locations are stochastic.

In the second period, on the basis of its information set, each retailer offers three different prices to the three groups of customers: (1) a poaching price to the competitor's customers (p_2^{AO}, p_2^{BO}), (2) a price for its own *L*-type customers (p_2^{AL}, p_2^{BL}), and (3) a price for its own *H*-type customers (p_2^{AH}, p_2^{BH}). Consumers decide where to purchase in the second period after observing all these prices.

Figure 1 summarizes the outline of the game, and Table 1 summarizes the notation we use herein.

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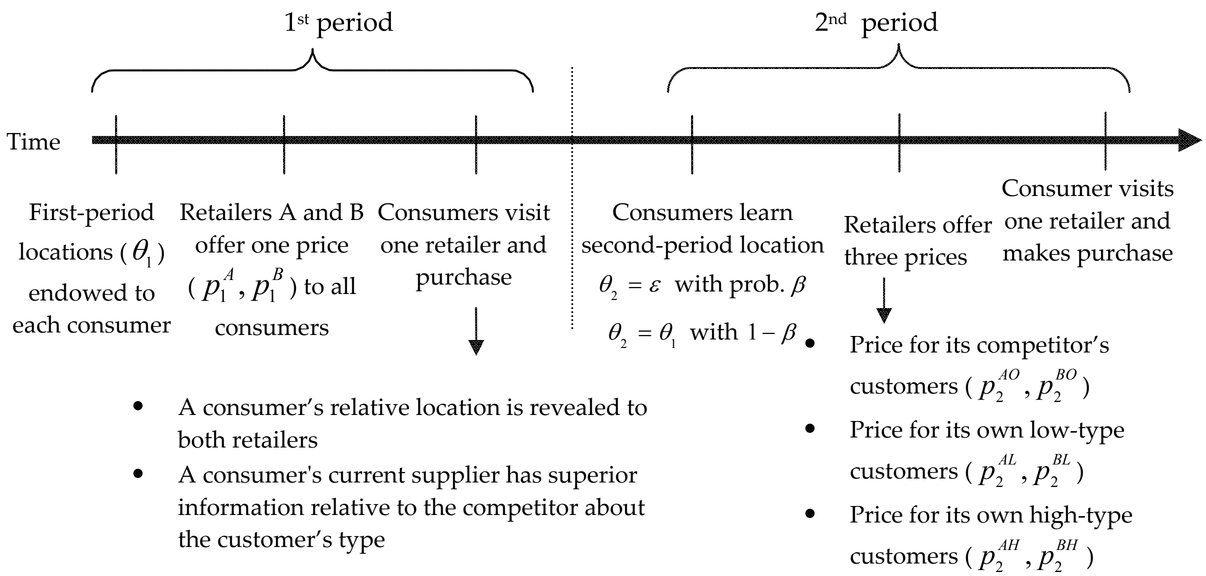
[10] Regarding the idea that firms have different amounts of information about their own customers and non-customers, Villas-Boas (1999) incorporates information asymmetry by assuming that customers can be either new customers or competitor's customers. In contrast, our information asymmetry comes from the customer type (quantity). Also, Villas-Boas and Schmidt-Mohr (1999) consider the impact of asymmetric information between firms and consumers.

Figure 1: Sequence of Events



*** Table 1 ***

4. Analysis

We solve the proposed game using backward induction, solving first for the second-period equilibrium strategies and then for the first-period strategies. Before we solve the game, we define the term "retailer turf" and formally demonstrate the relevance of customer purchase history information, even in the presence of customer mobility.

4.1. Retailer Turf and the Value of Purchase History Information

For any pair of first-period prices (all consumers purchase and both retailers have positive sales), there is a cut-off $\tilde{\theta}_1^j$ ($j \in \{L, H\}$), such that all consumers of type *j* whose $\theta \leq \tilde{\theta}_1^j$ decide to purchase from retailer *A*, and the rest purchase from retailer *B*. Following Fudenberg and Tirole (2000), we say that consumers to the left of $\tilde{\theta}_1^j$ lie in retailer *A*'s turf and those on the right lie in retailer *B*'s turf to emphasize consumers' relative locations on the Hotelling line.

For consumers who purchased from retailer *A* in the first period, retailer *A* offers prices p_2^{AH}, p_2^{AL} to *H*- and *L*-type customers, respectively, whereas retailer *B* offers price p_2^{BO} to both types at the beginning of the second period. In parallel, among consumers who purchased from retailer *B* in the first period, retailer *B* charges p_2^{BH}, p_2^{BL}, and retailer *A* charges p_2^{AO}.[11]

Given customer mobility in the second period ($E(\theta_2) = \beta\varepsilon + (1-\beta)\theta_1$, where $\beta \in [0,1]$ and $\varepsilon \sim U[0,1]$), we can compute the conditional probability that consumers will locate in a certain range of the Hotelling line, given their first-period purchase choice of *A* or *B*, as follows:

$$\Pr[\theta_2 \le x | \theta_1 \le \tilde{\theta}_1] = \begin{cases} (1-\beta) + \beta x & \text{if} \quad \tilde{\theta}_1 \le x, \\ (1-\beta)\dfrac{x}{\tilde{\theta}_1} + \beta x & \text{if} \quad \tilde{\theta}_1 > x, \end{cases} \tag{1}$$

$$\Pr[\theta_2 > x | \theta_1 > \tilde{\theta}_1] = \begin{cases} (1-\beta) + \beta(1-x) & \text{if} \quad \tilde{\theta}_1 \ge x, \\ (1-\beta)\dfrac{1-x}{1-\tilde{\theta}_1} + \beta(1-x) & \text{if} \quad \tilde{\theta}_1 < x. \end{cases} \tag{2}$$

Note that equations (1) and (2) indicate the general conditional probabilities of the second period locations, assuming a consumer purchases from retailer *A* and *B*, respectively, in the first period. Therefore, we can state the following lemma:

Lemma 1 (Value of Purchase History Information): When customer mobility is not extreme ($\beta \le \frac{1}{2(1-z)}$, where z is first-period market share), a consumer who purchases from retailer i ($i \in \{A, B\}$) in period 1 is more likely to stay in the same retailer's turf rather than move to the competitor's turf.

Proof: See Appendix.

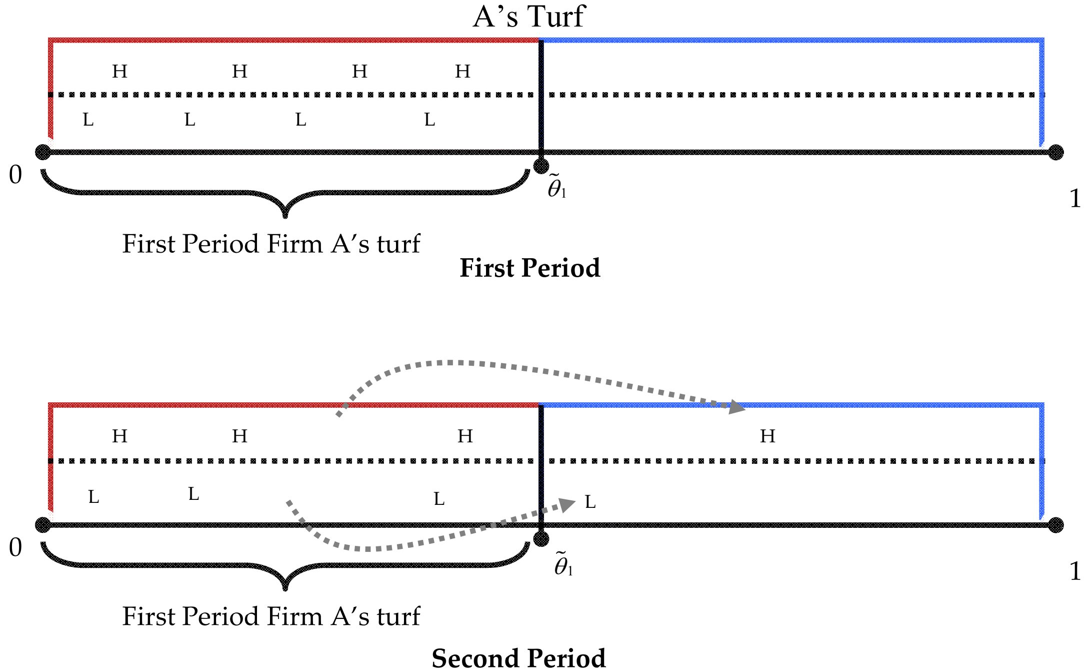
The lemma shows that the customer's past purchase is a good indication of that person's future preference since she is more likely to stay in the same turf unless customer mobility is extremely high. In this sense, the information about consumers' relative locations, revealed from first-period purchase, is valuable for retailers. In particular, when the retailers' turf is symmetric ($\tilde{\theta}_1 = \frac{1}{2}$), past purchase information is always relevant unless preferences are completely

[11] When we analyze the symmetric case, "no poaching" does not arise in equilibrium, but in the asymmetric case, when $\tilde{\theta}_1 < \frac{1}{4}$, *A*'s turf is very small and consists only of consumers with a strong preference for *A*, and retailer *A* can charge a monopoly price but not lose any sales to retailer *B*.

independent across periods, $\beta = 1$. For example, when $\tilde{\theta}_1 = \frac{1}{2}$ and $\beta = \frac{1}{2}$, a consumer in *A*'s turf relocates in the same turf with probability $\frac{3}{4}$ and relocates in *B*'s turf with only probability $\frac{1}{4}$.[12]

Figure 2 illustrates the redistribution of customer locations in the second period for four *H*-type and *L*-type first-period customers of *A* in the symmetric case when $\tilde{\theta}_1 = \frac{1}{2}$ and $\beta = \frac{1}{2}$. Each letter represents one *H*- or *L*-type consumer. Probabilistically, three *H*- and *L*-type consumers remain relatively close to retailer *A*, and one of each type changes his or her location toward retailer *B* in the second period, as indicated by the arrows in Figure 2. Remember that retailers did not know the exact locations of their customers in the first period, but they had a good sense that their customers must be somewhere in their turf (relative location). In the second period, retailers still do not know the exact locations of their first-period customers, but can assess the average probability of relative proximity (closer to *A* or *B*) of their first-period customers. In the symmetric case, for all $\beta \in [0,1)$ a customers' purchase not only indicates a current preference for the retailer, but also indicates that the same retailer will be preferred in the future.

Figure 2: Redistribution of Consumer Location for Consumers in Retailer A's Turf



[12] Lemma 1 focuses on the *horizontal* information about the customer's first-period relative location, as observed from his or her past purchases. Past purchases also reveal *vertical* information about the customer type, which is asymmetric between firms and, therefore, always relevant for retailers.

4.2. Second Period

A consumer in retailer A's turf repeat purchases from retailer A in the second period if and only if $q^j v - q^j p_2^{Aj} - \theta_2 \geq q^j v - q^j p_2^{BO} - (1-\theta_2) \Leftrightarrow \theta_2 \leq \frac{1+(q^j p_2^{BO} - q^j p_2^{Aj})}{2} \equiv \tilde{\theta}_2^{Aj}$, where $j \in \{L, H\}$ and $q^L = 1,\ \ q^H = q$. Otherwise, the consumer will switch to retailer B. We denote the repeat purchase probability of the high and low types for retailer A as $\Pr^{AH}$ and $\Pr^{AL}$, respectively. Thus, $\Pr^{AH} = \Pr[\theta_2 \leq \frac{\left[1+q(p_2^{BO}-p_2^{AH})\right]}{2} \Big| \theta_1 \leq \tilde{\theta}_1^H]$ and $\Pr^{AL} = \Pr[\theta_2 \leq \frac{\left[1+(p_2^{BO}-p_2^{AL})\right]}{2} \Big| \theta_1 \leq \tilde{\theta}_1^L]$, where $\tilde{\theta}_1^j$ is the first-period cut-off for type $j \in \{L, H\}$. The repeat purchase probabilities for retailer B similarly are $\Pr^{BH} = \Pr[\theta_2 > \frac{\left[1+q(p_2^{BH}-p_2^{AO})\right]}{2} \Big| \theta_1 > \tilde{\theta}_1^H]$ and $\Pr^{BL} = \Pr[\theta_2 > \frac{\left[1+p_2^{BL}-p_2^{AO}\right]}{2} \Big| \theta_1 > \tilde{\theta}_1^L]$.

Thus, the second-period profits of retailers A and B are

$$\begin{aligned} \Pi_2^A &= (p_2^{AH})q\tilde{\theta}_1^H \Pr^{AH} + (p_2^{AL})\tilde{\theta}_1^L \Pr^{AL} + (p_2^{AO})\left\{q(1-\tilde{\theta}_1^H)\left(1-\Pr^{BH}\right) + (1-\tilde{\theta}_1^L)\left(1-\Pr^{BL}\right)\right\} \\ \Pi_2^B &= (p_2^{BH})q(1-\tilde{\theta}_1^H)\Pr^{BH} + (p_2^{BL})(1-\tilde{\theta}_1^L)\Pr^{BL} + (p_2^{BO})\left\{q\tilde{\theta}_1^H\left(1-\Pr^{AH}\right) + \tilde{\theta}_1^L\left(1-\Pr^{AL}\right)\right\} \end{aligned} \quad (3)$$

Each retailer's second-period demand consists of three parts. For example, retailer A has demand: (1) from its own previous H-type customers ($\tilde{\theta}_1^H$), who continue to be in the retailer's turf in the second period (with probability $\Pr^{AH}$) and pay a price p_2^{AH}; (2) from its own previous L-type customers ($\tilde{\theta}_1^L$), who continue to be in the retailer's turf in the second period (with probability $\Pr^{AL}$) and pay a price p_2^{AL}; and (3) from a mix of the competitor's previous high- and low-type customers ($(1-\tilde{\theta}_1^H)+(1-\tilde{\theta}_1^L)$) who have shifted to the retailer's turf (with probability $1-\Pr^{Bj}$, where $j \in \{L, H\}$) and pay a price p_2^{AO}.

Using equations (1) and (2), we obtain the second-period prices by solving the retailers' first-order conditions. We only look for the pure strategy equilibrium in a symmetric game. In the next section of the first-period analysis, we show that the symmetric outcome, in which both firms charge the same price in the first period, is indeed the equilibrium solution.

We summarize our first key result in the following proposition:

Proposition 1: (a) Reward Competitor's Customers: When customer mobility is low ($\beta < \underline{\chi}(q) = \frac{2(3-q+2q^2)}{3+q+4q^2}$), there exists a symmetric pure strategy equilibrium in second-period prices, such that retailers charge $p_2^{AH} = p_2^{BH} = \frac{1}{2q} + \frac{(2+\beta)(1+q)}{6(q^2+1)(2-\beta)}$, $p_2^{AL} = p_2^{BL} = \frac{1}{2} + \frac{(2+\beta)(1+q)}{6(q^2+1)(2-\beta)}$, and $p_2^{AO} = p_2^{BO}$ $= \frac{(2+\beta)(1+q)}{3(q^2+1)(2-\beta)}$. The equilibrium second-period profits are $\Pi_2^A = \Pi_2^B = \frac{(q^2+1)\left(116-(52-17\beta)\beta\right)+2q(22-\beta)(2+\beta)}{144(q^2+1)(2-\beta)}$. Moreover, prices follow an ordinal relationship, $p_2^{iO} \le p_2^{iH} \le p_2^{iL}$, where $i \in \{A, B\}$; that is, a competitor's customers receive the lowest price.

(b) Reward Own Customers: When customer mobility is sufficiently high ($\beta \ge \overline{\chi}(q) = \frac{2q^2-q-6+\sqrt{4q^4-4q^3+25q^2+24q}}{4q^2+q-3}$) and consumer heterogeneity in purchase quantity exists ($q \ge 2$), there exists a symmetric pure strategy equilibrium in second- period prices such that retailers charge $p_2^{AH} = p_2^{BH} = \frac{2-\beta}{2q\beta} + \frac{(2+\beta)(1+q)}{12+6(q^2-1)\beta}$, $p_2^{AL} = p_2^{BL} = \frac{1}{2} + \frac{(2+\beta)(1+q)}{12+6(q^2-1)\beta}$, and $p_2^{AO} = p_2^{BO} =$ $\frac{(2+\beta)(1+q)}{6+3(q^2-1)\beta}$. The second-period profits are $\Pi_2^A = \Pi_2^B = \frac{1}{72\beta} + \frac{72+\beta\left(80q^2+88q+40(1+q)\beta-(1+q)^2\beta-32\right)}{144\beta\left(2+(q^2-1)\beta\right)}$.

Moreover, prices follow an ordinal relationship, $p_2^{iH} \le p_2^{iO} \le p_2^{iL}$, where $i \in \{A, B\}$; that is, the retailer's own high-type customers receive the lowest price.[13]

Proof. See Appendix.

Proposition 1 leads to several interesting implications: (1)Rewarding own high-type customers is optimal when there exist both customer heterogeneity in quantity *and* sufficient customer mobility. (2) Otherwise, it is optimal to reward competitor's customers. (3) Finally, it is not optimal to reward one's own low-type customers; they always receive the highest price (see Figure 3).

This proposition highlights the importance of the new features in our model. Specifically, when q =1, it is optimal to reward competitor's customers, because $\underline{\chi}(1) = 1$. Similarly when $\beta = 0$, it is also optimal to reward competitor's customer because $\underline{\chi}(q) > 0$ for all q. Thus consistent with existing literature, it is optimal to reward competitor's customers when there is neither heterogeneity nor customer mobility (Fudenberg and Tirole 2000). Both heterogeneity in quantity and high level of customer mobility are, therefore, necessary conditions to reward one's own customers. The intuition for this result is as follows:

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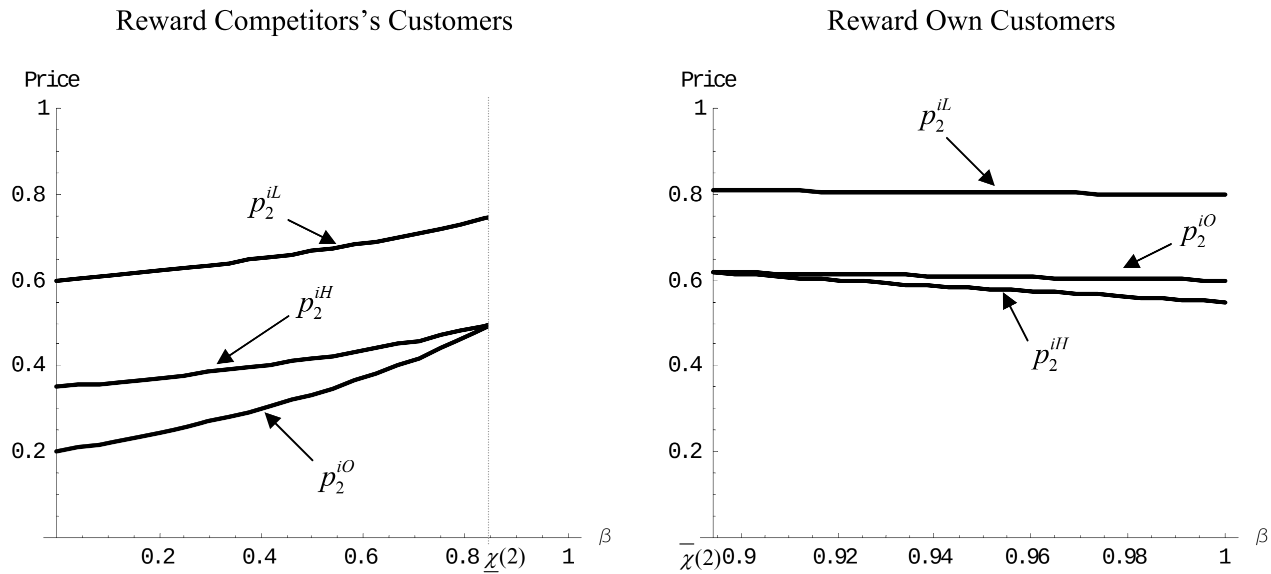
[13] In a small range ($\underline{\chi}(q) < \beta < \overline{\chi}(q)$) of β, the pure strategy equilibrium does not exist. We focus our analysis only on the pure strategy equilibrium area.

First, when customer mobility is low, customer preferences are relatively stable, and the probability of current customer retention (irrespective of high or low type) is very high. Retailers therefore spend more effort acquiring competitor customers by offering low poaching prices.

Second, even when customer mobility is high, customers are always at least as likely to stay with the retailer as they are to switch to the rival in a symmetric market (Lemma 1). Therefore, a price cut targeted toward the competitor's customers is always more effective than a price cut targeted toward own customers if the customers are all equal in value (i.e., no customer heterogeneity in quantity). Although retailers fully expect that some customers would switch from one retailer to the other in the second period, they do not find it optimal to offer better value to their current customers.[14]

Figure 3: Prices when q = 2



However, if customers are heterogeneous, retailers need to assess whether selective retention (rewarding own best customers) can be profitable. The value of own high-type customers is greater than the expected average value of the competitor's customers. Furthermore, as customer mobility increases, the monopoly power that the retailer may enjoy from horizontal preference becomes weaker. Therefore, at a certain threshold, the marginal gain in profit from cutting prices to retain high-type customers becomes greater than the marginal benefit of poaching a mix of

[14] Note that even though retailers fully expect some customers switch in the second period, they cannot identify exactly who would switch – they only know the aggregate probability of switching.

high- and low-type competitor's customers. In turn, retailers seek to retain their high-type customers when customer mobility is high. But, it is never optimal to offer a better value to own low-type customers, whose value is less than the expected average value of the competitor's customers. For this reason, both heterogeneity in quantities and high levels of customer mobility are necessary for rewarding own high-type customers to be an effective strategy.

Finally, we illustrate customer switching behavior in the second period in Figure 4. Figure 4a illustrates the case when β is low and, therefore, retailers reward competitor's customers. The second-period cut-off thresholds for both high and low types ($\tilde{\theta}_2^{AH}$ and $\tilde{\theta}_2^{AL}$) shift to the left of the first-period threshold, though the shift is greater for low-type customers who pay the highest price. Therefore, the observed level of switching is greater for both the high and the low types, relative to their customer mobility levels.

Figure 4a: Customer Switching when $\beta \leq \underline{\chi}(q)$: $\tilde{\theta}_1^L > \tilde{\theta}_2^{AL}$ and $\tilde{\theta}_1^H > \tilde{\theta}_2^{AH}$

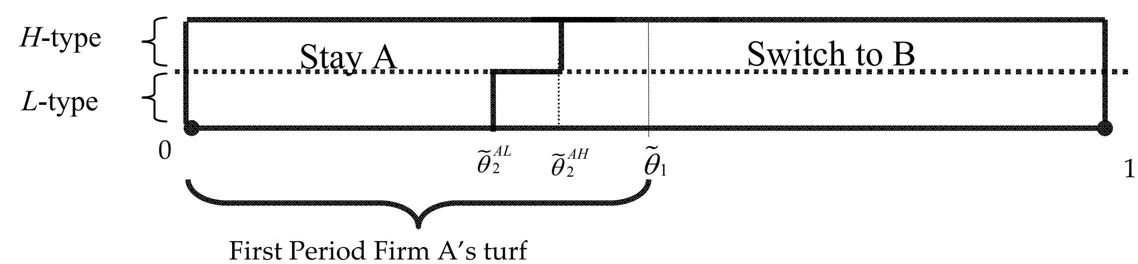
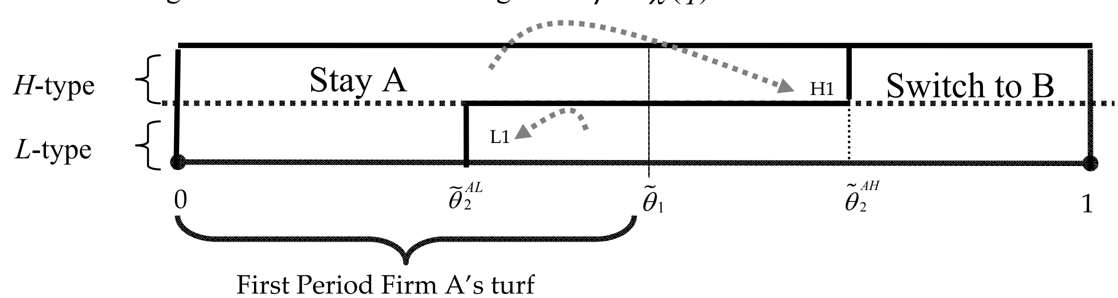


Figure 4b: Customer Switching when $\beta \geq \overline{\chi}(q)$: $\tilde{\theta}_1^L > \tilde{\theta}_2^{AL}$ and $\tilde{\theta}_1^H \leq \tilde{\theta}_2^{AH}$



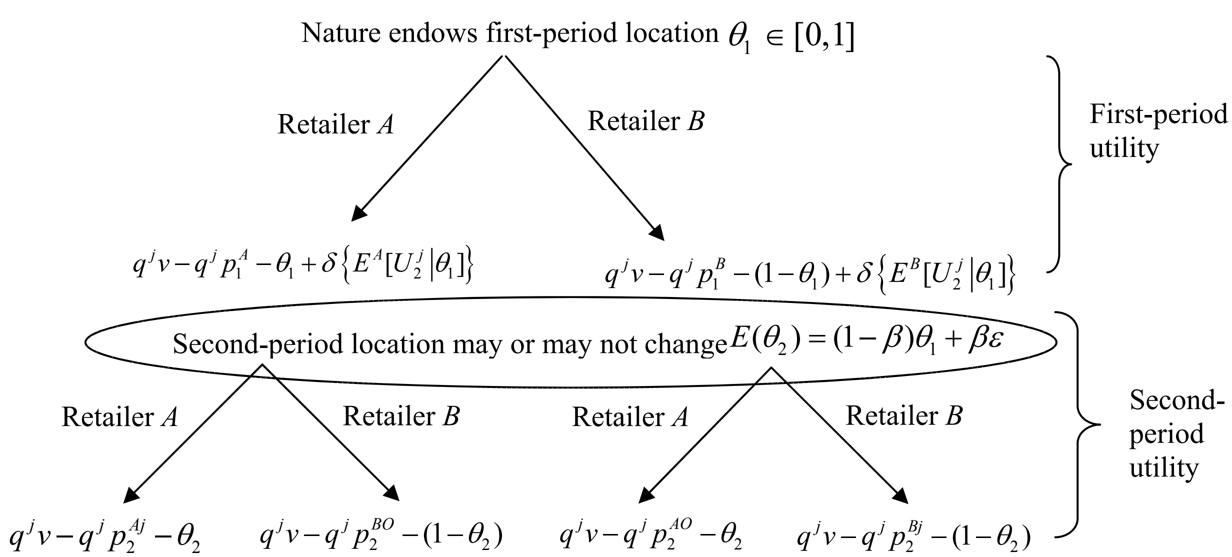
In Figure 4b, we present the case when β is high, such that retailers reward own best customers. An H-type customer (H_1), who bought from retailer A in the first period but gets a shock in the second period and, therefore, moves closer to retailer B, still buys from retailer A in the second period. Whereas an L-type customer (L_1), who bought from retailer A in the first period but receives a shock that makes him even closer to the retailer A, may switch to retailer B

in the second period. In effect, the observed level of switching among high types is lower relative to their customer mobility level, in that they still repurchase from the same retailer even though they are much closer to the alternative; in contrast, the observed level of switching among low-type customers is greater than their customer mobility level.

4.3. First Period

To solve for first-period prices, it is useful to describe the consumer's decision tree over two periods, and how consumer's future consideration affects the first period decision, as we show in Figure 5. The forward-looking consumer solves a dynamic program in the first period that takes into account the probabilities of second-period location and the prices offered by both retailers. Because these prices are the outcome of a dynamic strategic game played by the retailers, solving for the equilibrium retailer and consumer strategies requires embedding the consumer's dynamic programming problem within a dynamic strategic game that involves one price for each retailer in the first period and three prices (for own high and low types and for the competitor's customers) for each retailer in the second period.

Figure 5: Consumer Decision Tree



Let retailer A's first-period price be p_1^A and retailer B's first-period price be p_1^B. If the first-period prices lead to a cut-off $\tilde{\theta}_1^j$, the marginal consumer of type j, where $j \in \{L, H\}$ with location $\tilde{\theta}_1^j$, must be indifferent between buying a product from *A* or *B* in period 1. In other words, the consumer compares the utility of purchasing from either retailer *A* or retailer *B*, recognizing that his or her location may change due to customer mobility. Consumers are forward looking, so they rationally anticipate the consequence of their first-period choice in terms of the prices they will receive in the second period. Thus, the following equation holds for the marginal consumer:

$$q^j v - q^j p_1^A - \tilde{\theta}_1^j + \delta\left\{E^A[U_2^j \,|\theta_1 = \tilde{\theta}_1^j]\right\} = q^j v - q^j p_1^B - (1-\tilde{\theta}_1^j) + \delta\left\{E^B[U_2^j \,|\theta_1 = \tilde{\theta}_1^j]\right\}, \quad (5)$$

where $\delta < 1$ is the discount rate, and $E^A[U_2^j \,|\theta_1 = \tilde{\theta}_1^j] = E_2^{Aj}$, $E^B[U_2^j \,|\theta_1 = \tilde{\theta}_1^j] = E_2^{Bj}$ represents the expected second-period utility that the "marginal" consumer derives from purchasing from retailer *A* or *B* in the first period, respectively. Thus,

$$\begin{aligned} E_2^{Aj} &= E^A[U_2^j \,|\theta_1 = \tilde{\theta}_1^j] = \int_{\theta_2=0}^{\tilde{\theta}_2^{Aj}} \Pr(\theta_2 \,|\theta_1 = \tilde{\theta}_1^j) \times \left(q^j v - q^j p_2^{Aj} - \theta_2\right) d\theta_2 \\ &\quad + \int_{\theta_2=\tilde{\theta}_2^{Aj}}^{1} \Pr(\theta_2 \,|\theta_1 = \tilde{\theta}_1^j) \times \left(q^j v - q^j p_2^{BO} - (1-\theta_2)\right) d\theta_2, \end{aligned} \quad (6)$$

$$\begin{aligned} E_2^{Bj} &= E^B[U_2^j \,|\theta_1 = \tilde{\theta}_1^j] = \int_{\theta_2=0}^{\tilde{\theta}_2^{Bj}} \Pr(\theta_2 \,|\theta_1 = \tilde{\theta}_1^j) \times \left(q^j v - q^j p_2^{AO} - \theta_2\right) d\theta_2 \\ &\quad + \int_{\theta_2=\tilde{\theta}_2^{Bj}}^{1} \Pr(\theta_2 \,|\theta_1 = \tilde{\theta}_1^j) \times \left(q^j v - q^j p_2^{Bj} - (1-\theta_2)\right) d\theta_2, \end{aligned} \quad (7)$$

where $\tilde{\theta}_2^{Aj}$ and $\tilde{\theta}_2^{Bj}$ represent the second-period cut-off locations of *j*-type customers who purchase from retailer *A* and *B* in the first period, respectively.

The expected second-period utility in equations (6) and (7) has two components in each case. For example, the first term in equation (6), $\int_{\theta_2=0}^{\tilde{\theta}_2^{Aj}} \Pr(\theta_2 \,|\theta_1 = \tilde{\theta}_1^j) \times \left(q^j v - q^j p_2^{Aj} - \theta_2\right) d\theta_2$, represents the case in which a second-period location (θ_2) is such that the first-period marginal consumer decides to repurchase from retailer *A* at its repeat purchase price. The second term, $\int_{\theta_2=\tilde{\theta}_2^{Aj}}^{1} \Pr(\theta_2 \,|\theta_1 = \tilde{\theta}_1^j) \times \left(q^j v - q^j p_2^{BO} - (1-\theta_2)\right) d\theta_2$, represents the case in which the second-period location is such that the first-period marginal consumer decides to purchase from retailer *B* with its poaching price. Similarly, we can see both repeat purchase case and poaching case constitute equation (7).

From equation (5), it follows that the marginal first-period customer of type *j* is

$$\tilde{\theta}_1^j = \frac{\left\{1 + q^j p_1^B - q^j p_1^A + \delta\left(E^A[U_2^j \,|\theta_1 = \tilde{\theta}_1^j] - E^B[U_2^j \,|\theta_1 = \tilde{\theta}_1^j]\right)\right\}}{2}. \tag{8}$$

Note that $p_2^{AO}, p_2^{BO}, p_2^{Aj}, p_2^{Bj}$, which are embedded in E_2^{Aj}, E_2^{Bj}, are all functions of both $\tilde{\theta}_1^L$ and $\tilde{\theta}_1^H$. Hence, E_2^{AL}, E_2^{BL}, E_2^{AH}, and E_2^{BH} are again functions of both $\tilde{\theta}_1^L$ and $\tilde{\theta}_1^H$.

We now solve for the first-period equilibrium prices and overall profits.[15] Retailer *A*'s and *B*'s overall net present values of profits, as viewed from period 1, are given respectively by:

$$\begin{aligned} \Pi^A &= (p_1^A)\cdot\left(q\tilde{\theta}_1^H + \tilde{\theta}_1^L\right) + \delta\Pi_2^A, \\ \Pi^B &= (p_1^B)\cdot\left(q(1-\tilde{\theta}_1^H) + (1-\tilde{\theta}_1^L)\right) + \delta\Pi_2^B. \end{aligned} \tag{9}$$

Taking first-order conditions with respect to prices and solving for prices and profits, we find a symmetric pure strategy equilibrium in which both retailers charge the same prices (see Appendix).

$$p_1^A = p_1^B = \frac{(1+q) - 2\delta\left(\Omega^H\left(-\frac{\partial\tilde{\theta}_1^H}{\partial p_1^A}\right) + \Omega^H\left(-\frac{\partial\tilde{\theta}_1^L}{\partial p_1^A}\right)\right)}{2\left(q\left(-\frac{\partial\tilde{\theta}_1^H}{\partial p_1^A}\right) + \left(-\frac{\partial\tilde{\theta}_1^L}{\partial p_1^A}\right)\right)}, \tag{10}$$

where

$$\Omega_L = \frac{\partial\Pi_2^{AA}}{\partial p_2^{BO}}\frac{dp_2^{BO}}{d\tilde{\theta}_1^L} + \frac{\partial\Pi_2^{AB}}{\partial p_2^{BL}}\frac{dp_2^{BL}}{d\tilde{\theta}_1^L} + \frac{\partial\Pi_2^{AB}}{\partial p_2^{BH}}\frac{dp_2^{BH}}{d\tilde{\theta}_1^L} + \frac{\partial\Pi_2^{AA}}{\partial\tilde{\theta}_1^L} + \frac{\partial\Pi_2^{AB}}{\partial\tilde{\theta}_1^L},$$

$$\Omega_H = \frac{\partial\Pi_2^{AA}}{\partial p_2^{BO}}\frac{dp_2^{BO}}{d\tilde{\theta}_1^H} + \frac{\partial\Pi_2^{AB}}{\partial p_2^{BL}}\frac{dp_2^{BL}}{d\tilde{\theta}_1^H} + \frac{\partial\Pi_2^{AB}}{\partial p_2^{BH}}\frac{dp_2^{BH}}{d\tilde{\theta}_1^H} + \frac{\partial\Pi_2^{AA}}{\partial\tilde{\theta}_1^H} + \frac{\partial\Pi_2^{AB}}{\partial\tilde{\theta}_1^H}.$$

Therefore, the equilibrium profits are

[15] With no customer switching and uniform distribution of customer preferences on the Hotelling line, Fudenberg and Tirole (2000) solve for the first period prices using an explicit solution strategy, simply plugging back the primitives (the first period prices) into the second period profit functions because E_2^{Aj}, E_2^{Bj} turn out to be linear functions of $\tilde{\theta}_1^L$ and $\tilde{\theta}_1^H$. However, with customer switching, such an explicit solution strategy is not feasible because E_2^{Aj} and E_2^{Bj} are highly nonlinear functions that involve integrals with boundaries ($\tilde{\theta}_2^{Aj} = \frac{1+(q^j p_2^{BO} - q^j p_2^{Aj})}{2}$, $\tilde{\theta}_2^{Bj} = \frac{1-(q^j p_2^{Bj} - q^j p_2^{AO})}{2}$) that are themselves linked to $\tilde{\theta}_1^L$ and $\tilde{\theta}_1^H$ (since $p_2^{Aj}, p_2^{Bj}, p_2^{AO}, p_2^{BO}$ are again functions of $\tilde{\theta}_1^L$ and $\tilde{\theta}_1^H$. See equations 6 and 7). We therefore develop an indirect solution strategy that avoids explicitly solving for $\tilde{\theta}_1^L$ and $\tilde{\theta}_1^H$. Lemma 2 shows how to estimate forward-looking price elasticity of the high and low types to first period prices and then combines it with an application of the envelope theorem to solve for first period equilibrium prices.

$$\Pi^{\text{BBP}} = \Pi^A = \Pi^B$$
$$= \begin{cases} \left(\dfrac{(1+q) - 2\delta\left(\Omega^H \left(-\frac{\partial \tilde{\theta}_1^H}{\partial p_1^A} \right) + \Omega^L \left(-\frac{\partial \tilde{\theta}_1^L}{\partial p_1^A} \right) \right)}{2\left(q\left(-\frac{\partial \tilde{\theta}_1^H}{\partial p_1^A} \right) + \left(-\frac{\partial \tilde{\theta}_1^L}{\partial p_1^A} \right) \right)} \right) \left(\frac{1+q}{2} \right) + \delta \left(\frac{1}{72\beta} + \frac{72 + \beta\left(80q^2 + 88q + 40(1+q)\beta - (1+q)^2 \beta - 32 \right)}{144\beta\left(2 + (q^2-1)\beta \right)} \right) & \text{if } \beta \geq \overline{\chi}(q), \\ \left(\dfrac{(1+q) - 2\delta\left(\Omega^H \left(-\frac{\partial \tilde{\theta}_1^H}{\partial p_1^A} \right) + \Omega^L \left(-\frac{\partial \tilde{\theta}_1^L}{\partial p_1^A} \right) \right)}{2\left(q\left(-\frac{\partial \tilde{\theta}_1^H}{\partial p_1^A} \right) + \left(-\frac{\partial \tilde{\theta}_1^L}{\partial p_1^A} \right) \right)} \right) \left(\frac{1+q}{2} \right) + \delta \left(\frac{(q^2+1)(116 - (52 - 17\beta)\beta) + 2q(22-\beta)(2+\beta)}{144(q^2+1)(2-\beta)} \right) & \text{if } \beta < \underline{\chi}(q). \end{cases} \tag{11}$$

We now compare these profits obtained by BBP with a model that does not use customer purchase information. The comparison should help us understand the benefits, if any, of BBP.

The model that does not use customer purchase information reduces to a static pricing model. In each period, the two retailers maximize their current profits. In this static pricing scenario, both retailers would charge the static price (p_1^s), which maximizes the static profit functions in each period, such that

$$\Pi_1^A = (p_1^A) \cdot \left(q\left(\tfrac{1 - qp_1^A + qp_1^B}{2} \right) + \left(\tfrac{1 - p_1^A + p_1^B}{2} \right) \right) \text{ and}$$
$$\Pi_1^B = (p_1^B) \cdot \left(q\left(\tfrac{1 - qp_1^B + qp_1^A}{2} \right) + \left(\tfrac{1 - p_1^B + p_1^A}{2} \right) \right). \tag{12}$$

Taking the first-order conditions and solving for prices, the equilibrium price in the static case is given by $p_1^{\text{No BBP}} = p_1^A = p_1^B = \frac{1+q}{1+q^2}$, and the per period profit is $\Pi_1^A = \Pi_1^B = \left(\frac{(1+q)^2}{2(1+q^2)} \right)$. Therefore, the discounted net present value of the total profit across two periods for both retailers is:

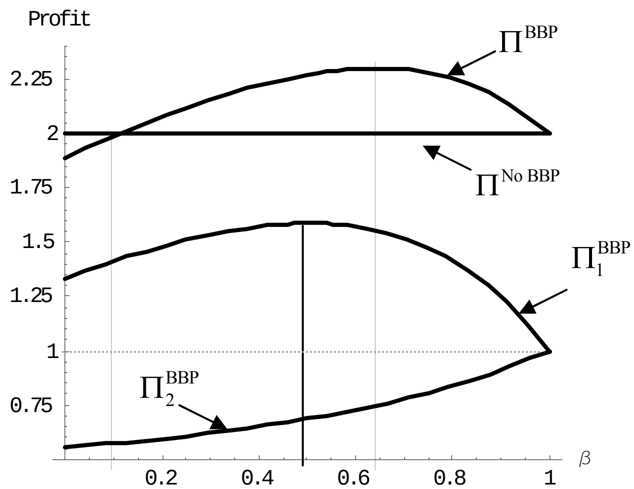
$$\Pi^{\text{No BBP}} = \Pi^A = \Pi^B = \left(\tfrac{(1+q)^2}{2(1+q^2)} \right) (1+\delta). \tag{13}$$

We begin the profit discussion by first considering two special cases in which we consider (1) only customer mobility and (2) only customer heterogeneity in purchase quantity. These cases can be obtained by setting $q = 1$ and $\beta = 0$, respectively, in the general model. By doing these, we can isolate the true effect of these features. Next, we describe the general profit result with both customer heterogeneity and customer mobility to clarify how these two key features interact.

4.3.1. Only Customer Mobility, No Customer Heterogeneity in Purchase Quantity

The case without customer heterogeneity may be captured by setting $q = 1$ in the model, such that all customers buy one unit of the product. As Proposition 1(a) states, when $q = 1$, it is always optimal for firms to charge the lowest price to the competitor's customers, irrespective of the level of customer mobility (because the cut-off level is $\beta \le \underline{\chi}(1) = 1$ in this case). Figure 6 shows the total profits across the first and second periods with BBP (behavior-based pricing) and no BBP, as a function of β with a discount rate $\delta \approx 1$. The graph also shows the split in total BBP profits across the first and second periods. The per period profits without BBP is close to 1 with $\delta \approx 1$; and the total profits over two periods without BBP is close to 2.

Figure 6: Profitability of BBP when $q = 1$



Note that Fudenberg and Tirole (2000)'s model can be considered as the special case of $\beta = 0$ and $q = 1$ in our model, and they find that profit from BBP is lower than profit without BBP. We confirm their finding and show firms are certainly worse off with BBP when $\beta = 0$. Firms compete aggressively in the second period to poach competitor customers using the customers' past purchase information, and, therefore, the second period profits decrease when $\beta = 0$. However, the consumer's dynamic consideration of future price reduces demand elasticity in the first period, enabling firms to raise first-period prices and profits. The increased profit from the first period does not offset the reduced profit from the second period, and therefore, firms are worse off with BBP when $\beta = 0$.

Interestingly, the total profits take on an inverted U-shaped curve when we extend our analysis to the case of general customer mobility of $\beta \in [0,1]$. To understand the intuition behind this result, we need to decompose the total profits into profits from the first and second periods, as shown in Figure 6. First, we note that the second-period profit increases monotonically with β, because customer mobility softens competition. More precisely, firms do not wish to compete aggressively for customers who could be on their own turf naturally through customer mobility. Hence, they do not offer aggressive prices to customers who may prefer their own product anyway. Therefore, both the poaching price and retention prices increase (see Figure 3) in β. This competition-softening effect, similar to the "mis-targeting" effect identified by Chen et al. (2001), raises second-period prices and profit.[16]

The effect of customer mobility on first-period profit is more subtle, including an indirect effect due to the consumer's consideration of future price. Consumers recognize that future prices will increase (both second-period repeat purchase and poaching prices increase in β), so their choices become less sensitive to changes in the first-period price. The lower price sensitivity shifts first-period price upward as β increases.[17]

However, a countervailing direct effect exerts downward pressure on first-period prices. As β increases, the link between customers' choices in the first and second periods weakens, because consumer preferences become less correlated, and price elasticity in the first period is less affected by what happens in the second period. The direct effect interacts with the indirect effect and effectively weakens the upward pressure of the indirect effect. At the extreme, when $\beta = 1$, demand in the two periods becomes independent, and price elasticity increases to short-run elasticity without BBP; in turn, profits with and without BBP become identical.

In summary, the net effect on first-period price (upward pressure from indirect effect and downward pressure from direct effect) leads to an inverted U-shaped curve for first-period profits (and total profits). Therefore, the total equilibrium profit is maximized at $\beta = 0.6435$, increasing for $\beta \in [0, 0.6435]$ and decreasing for $\beta \in [0.6435, 1]$. Not surprisingly, at $\beta = 1$, the

[16] Unlike Chen et al.'s mis-targeting effect, which shows an inverted U shape with respect to mis-targeting probability, our competition softening effect monotonically increases with customer mobility β. The inverted U shape of the total profit from BBP arises from consumers' forward looking behavior in the first period; this is clear from the shape of the first period profit curve (Figure 6). In contrast, Chen et al. is a static model. Thus the similarity of the inverted U-shape profitability is superficial; the underlying mechanism is different.

[17] More precisely, the indirect effect of consumer's future consideration affects the first-period price in two ways: (1) second-period high repeat purchase price decreases price elasticity in the first period because customers know they will be ripped off by the same firm in the second period (*ratchet effect*), and (2) second-period low poaching price increases price elasticity in the first period, causing a downward pressure on prices. As β increases, both poaching and repeat purchase prices increase, and the correct expectation of high prices makes consumers less price sensitive in the first period (higher repeat purchase price further decreases price elasticity; higher poaching price weakens upward pressure on price elasticity). Therefore, the indirect effect through consumer's dynamic consideration makes first-period demand less elastic, causing firms to increase prices and profits in the first period as β increases.

total profits from BBP are equal to the profits without BBP because there are no linkages between demand in the first and second periods.

4.3.2. No Customer Mobility, Only Customer Heterogeneity in Purchase Quantity

When there is no customer mobility (which can be seen by setting $\beta = 0$ in the model), we know from Proposition 1(a) that retailers offer the lowest price to their competitor's customers; therefore, $p_2^{iO} \leq p_2^{iH} \leq p_2^{iL}$. In other words, it is not optimal for firms to reward own customers when customer mobility does not exist. However, the more surprising result indicates that when heterogeneity in purchase quantities is sufficiently high, the total profit with BBP can be greater than profits without BBP.

Proposition 2. Suppose no customer mobility ($\beta = 0$). If heterogeneity in purchase quantities is sufficiently high (q > 5), both retailers increase their profits under behavior-based targeted pricing.

Proof. See Appendix.

Thus, BBP can increase both firms' profits without any customer mobility, even under symmetric competition. It manifests the importance or usefulness of using the customers' data when the extent of heterogeneity in customer value is great enough. We should note that this result does not occur simply because of heterogeneity in quantities and the ability to distinguish between high- and low-type customers. What is critical is that information about customer types is asymmetric with competition, because each firm only knows about its own customer types, not those of its competitor.

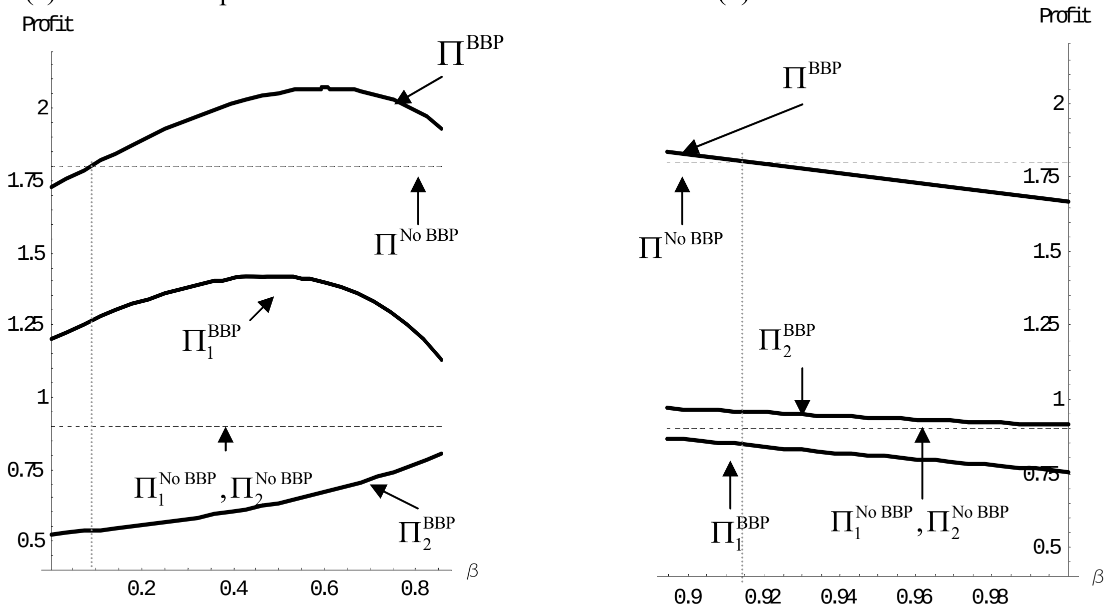
The intuition for this result is as follows: Because the price offered to high types is lower than that for low types, firms recognize that poaching disproportionately brings in *L*-type relative to *H*-type customers, and therefore, they do not compete intensively to attract the competitor's customers in equilibrium. In other words, with asymmetric information, the poacher faces a lemon's problem for attracting the competitor's customers, the most valuable of which are well protected from the incumbent retailer because of asymmetric customer information. This asymmetric informational advantage enables firms to price discriminate among their high- and low-type customers without competitors eroding profits; in other words, it works to shield both firms' profits from competition.

4.3.3. Both Customer Mobility and Customer Heterogeneity in Purchase Quantity

We next consider the full model by relaxing both the $q = 1$ and $\beta = 0$ assumptions. Specifically, we consider the $q = 2$ case to allow for customer heterogeneity, which helps build the intuition for the customer heterogeneity case with the least complexity.

Figure 7: Profits when $q = 2$

(a) Reward Competitors's Customers (b) Reward Own Customers



In Figure 7, we show the total profits with and without BBP when $q = 2$. As anticipated from Proposition 1, there are two regimes based on the pricing strategy in period 2. When customer mobility is low ($\beta < \underline{\chi}(2)$ = 0.857), firms reward competitors' customers by offering them the best prices; when customer mobility is high $(\beta \geq \overline{\chi}(2) \approx 0.894)$, firms reward their own best customers by offering them the best prices. The key takeaway is that we find that behavior based pricing can be profitable under both the "reward competitor customer" and "reward own customer" regimes. Specifically, profit with BBP is greater than profits without BBP in the range $\beta \in (0.115, 0.857)$ in the "reward competitor customer" regime (Figure 7a). Similarly in the range, $\beta \in (0.894, 0.917)$, profit with BBP is greater than that without BBP, in the "reward own customer" regime (Figure 7b).

Summarizing the above discussion, we can state our third proposition:

Proposition 3 (Profitability of BBP). **(a) Reward Competitor's Customers in Second Period:** When customer mobility is low ($\beta < \underline{\chi}(2) = 0.857$), the total profit with BBP is greater than profits without BBP if customer mobility is in the range $\beta \in (0.115, 0.857)$.

(b) Reward Own Customers in Second Period: When customer mobility is sufficiently high $(\beta \geq \overline{\chi}(2) = 0.894)$, the total profit with BBP is greater than profits without BBP when customer mobility is in the range $\beta \in (0.894, 0.917)$.

It is worth comparing profits from the first and second period relative to profits in a static model without BBP. We see a stark reversal in profit patterns over time between the reward own and competitor customer regimes. In the reward competitor customer regime, we see a decreasing profit pattern over two periods: profits in the first period are above static levels, while profits in second period are below. This profit ordering reverses under the reward own customers regime so that we see an increasing profit pattern over time – the first period profits are below the static levels, while the second period profits are above.

The intuition for the profits under the "reward competitors' customer" regime follows directly from the discussion on the direct and indirect effects of customer mobility in Section 4.3.1 (only mobility) and the discussion of information asymmetry in Section 4.3.2 (only heterogeneity). In essence, forward looking consumers anticipate higher future prices as mobility increases, making them less price sensitive in the first period. Hence, this raises first period price and profits relative to the static level. In the second period, firms compete hard to poach competitors' customers relative to the static level; however, the information asymmetry about the high type customers reduces the extent of price competition. While customer poaching (as in Fudenberg and Tirole 2000) explains the profit pattern over time – higher first period profit and lower second period profit, the combination of customer mobility and information asymmetry (new features in our model) explains why profits can be higher with BBP.

On the other hand, the price and profit pattern over time in the reward own customer regime is similar to switching cost models (Farrell and Klemperer 2007, Klemperer 1995) where firms charge price below static levels in the first period and then raise their prices in the second period once consumers are locked in. Similarly, firms compete hard to attract more customers in the first period to gain an information advantage on customer types. By using this information advantage, firms can more effectively lock-in their high type customers by rewarding them in the second period.[18] This lock-in intuition explains the profit pattern over time – lower first period profit and higher second period profit. Moreover, we find that firms can be better off with BBP. Why? This is because of customer mobility. The intensity of competition in the first period is softened due to imperfect lock-in arising from customer mobility. As customer mobility prevents perfect lock-in of the customers (particularly the *H*-type), it mitigates the firms' incentives to attract customers in the first period. Therefore, gains from locked-in customers in the second period are not completely negated by the intense competition for locking in customers in the first period.

As customer mobility gets closer to 1 (independent preference), firms seek to maximize their market share in the first period only to take advantage of the vertical information asymmetry about customer types in the second period (the same adverse selection intuition in the pure heterogeneity case in Section 4.3.2 applies here). This competition for information advantage leads to a prisoner's dilemma in the first period, which in turn lowers prices and profits. Hence, overall profits are also lower. The threshold mobility at which the benefit from information asymmetry in the second period is dominated by the resulting increased competition in the first period is 0.917 when q=2. Note, however, that when the value of information asymmetry can be greater (as can be seen below in Section 4.3.4), this threshold rises and rewarding own customers is a profitable strategy in a wider range of customer mobility.

We elucidate the implications in propositions 1 and 3 with a numerical example in Table 2 for alternative levels of customer mobility. When there is no intrinsic mobility ($\beta = 0$) or when mobility is extreme such that preferences for firms are completely uncorrelated ($\beta = 1$), BBP reduces profits. When $\beta = 0.5$, below the threshold of 0.894, it is optimal to reward competitor's customers. At $\beta = 0.9$ above the threshold of 0.894, it is optimal to reward one's own best customers.

*** Table 2 ***

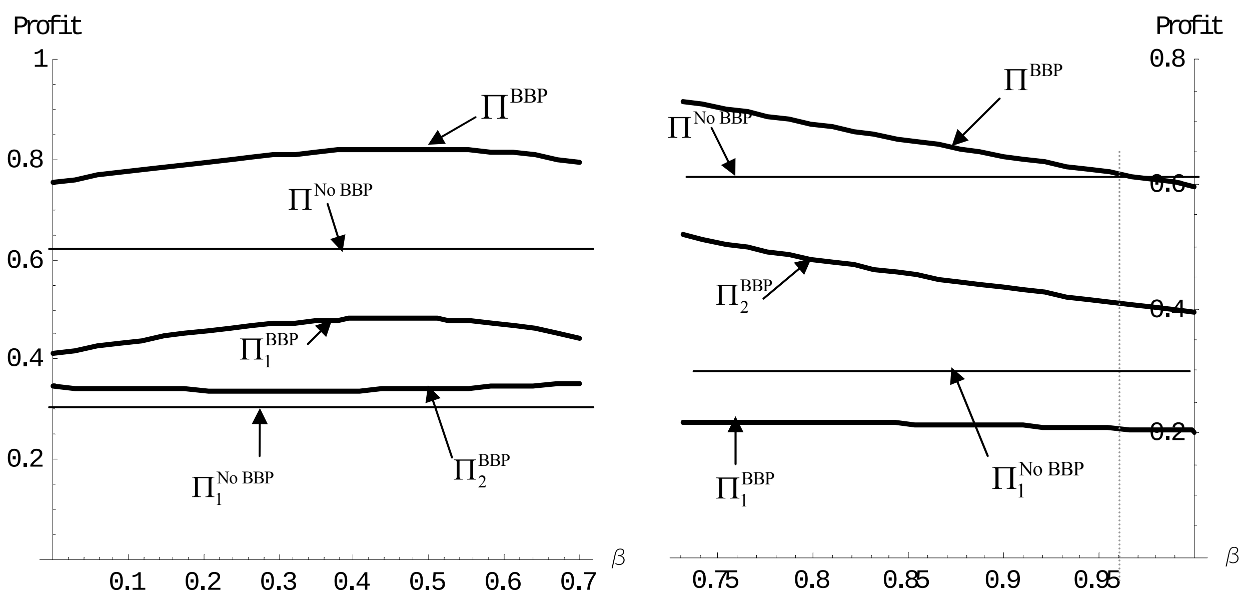
[18] In our model, the optimal prices will be such that per-period profits will be higher for the high type (that is $q \times p_2^{AH} > \left(\frac{1+q}{2}\right) \times p_2^{AO} > p_2^{AL}$ for all β and $q \geq 2$). The high types are indeed most valuable customers for firms and firms gain much more by retaining them. In other words, the cost to retain high types does not exceed the benefit of retaining them. Furthermore, it is easy to see that second period prices all higher than the first period price $p_2^{AH} > p_1^{A}$.

4.4. Nesting the 80/20 Rule: Unequal Size of High and Low Volume Consumers

To ease exposition and analysis, we have so far considered the case where high and low volume customers are equal in number. One concern is whether our results are robust when we allow for a smaller share of high volume consumers to dominate profitability as suggested by the 80/20 rule. We therefore consider the general case where the proportions of high and low volume consumers are given by $\alpha(>0)$ and $1-\alpha$ respectively. This nests the special case of the 80/20 rule when $\alpha = 0.2$ and $q = 16$.[19]

Figure 8: Profits when $\alpha = 0.2$ and $q = 16$

(a) Reward Competitors's Customers (b) Reward Own Customers



While the complete analysis of the general case is available in a technical appendix from the authors, we summarize the results for the 80/20 in Figure 8. Insights can be drawn by comparing against the corresponding Figure 7 for the basic model. First, we find that profits from BBP are greater for a wider range of customer mobility. Specifically, below the threshold of $\beta = 0.731$, where firms find it optimal to reward competitor's customers, BBP is always profitable even when there is no customer mobility, $\beta = 0$. This is consistent with the no mobility case in Section 4.3.2. since q is sufficiently large here. Thus, customer acquisition efforts are profitable when customer heterogeneity follows the 80/20 rule. Above the threshold of $\beta = 0.731$, rewarding

[19] The q that solves for the 80/20 rule is obtained by solving $0.2q = 0.8(0.2q + 0.8)$.

own customers is optimal. However, the range in which BBP is profitable is larger $\beta \in (0.731, 0.961)$. This shows that BBP is almost always profitable when customer heterogeneity follows the 80/20 rule in the range $\beta \in (0, 0.961)$.

This result suggests that the key results of the basic model are not only robust when markets follow the 80/20 rule, they become even stronger. In particular, when markets follow the 80/20 rule, BBP is almost always profitable and the region of customer mobility in which own best customers are rewarded is large.

5. Conclusion

5.1. Summary of Results

Table 3 summarizes the key pricing and profit results as a function of the two key market characteristics, namely, customer heterogeneity in purchase quantity and customer mobility.

Table 3: Summary of Results

		Customer Mobility	
		Low	Sufficiently High
Heterogeneity in Quantity (information advantage about current customers)	Low	**Prices**: Reward competitor customers (Proposition 1a) **Profits**: Behavior-based pricing less profitable (Proposition 3a)	**Prices**: Reward competitor customers (Proposition 1a) **Profits**: Behavior-based pricing more profitable (Proposition 3a)
	Sufficiently High	**Prices**: Reward competitor customers (Proposition 1a) **Profits**: Behavior-based pricing more profitable (Proposition 2)	**Prices**: Reward current high-type customers (Proposition 1b) **Profits**: Behavior-based pricing more profitable unless switching is extreme (Proposition 3b)

When neither heterogeneity in purchase quantity nor customer mobility exists, consistent with existing theoretical models, it is not optimal to reward current customers, and BBP is less profitable. However, the profit result deviates from existing models when we introduce either heterogeneity in purchase quantity or customer mobility. With sufficiently high heterogeneity (as with the 80/20 rule), BBP becomes more profitable, regardless of whether customer mobility exists. With sufficient customer mobility, BBP becomes more profitable when customer quantity is heterogeneous or not. Thus, BBP can improve profits under competition, even when consumers and firms are *strategic and forward looking* if either heterogeneity in customer quantities or customer mobility exists.

The rewarding own (high-type) customer strategy is effective only when both sufficient heterogeneity in purchase quantities and sufficiently high customer mobility exist. In the absence of customer mobility, current customers already revealed their relatively high preference for their incumbent firm and there is limited danger that they will switch to the competition. So, it is never optimal to offer a better price to them. Still, the threat of switching is not sufficient to give current customers a better price; if customers are identical in their purchase quantities (lifetime value), offering better prices to competitor's customers always (weakly) dominates offering better prices to current customers.

Hence, our extended model reconciles the apparent conflict between practitioners' optimism and analysts' skepticism about using customer's past purchase information by nesting both existing analytical results and practitioner intuition. Both heterogeneity in customer value and the threat of customer mobility are characteristics of a wide variety of markets, so understanding the extent of such heterogeneity and customer mobility offers critical information about the value of using BBP and arriving at the right balance between customer acquisition and retention.

Heterogeneity in customer values is the norm in many markets. According to an American Express executive (quoted by Peppers and Rogers 1993 p. 108), best customers outspend others by 16 to 1 in retailing, 13 to 1 in the restaurant business, 12 to 1 in airlines, 5 to 1 in the hotel/motel industry. In the car rental industry, the best customers outspend by 35 to 1 (Peppers and Rogers 2002). Many case studies of such markets typically revolve around rewarding and retaining the most valuable customers; as with banking (Narayanan 2002; Selden and Colvin 2004), casinos (Lal, 2004), and retirement service providers (Rosenthal et al. 2006). In contrast, in markets such as magazines and software where individual consumers are unlikely to buy multiple units of the product, we expect low levels of value heterogeneity. Indeed most customers are likely to get better prices when they subscribe fresh through a discount magazine seller (customer acquisition) than by renewing the offer provided by the magazine.[20]

Customer mobility can also vary across markets. We expect markets with contractual relationships (banking, financial services, phone service, magazine subscriptions) to have lower

[20] For instance, Business Week offers an annual renewal subscription rate of $65 for its subscribers, while the introductory rate for the magazine at the Business Week Website is $40. The cover price is $124.75.

churn than markets with non-contractual relationships (retailing, catalog retailing, airlines, hotels etc). Even with contractual relationships, one that has an explicit renewal requirement might be open to greater switching because there is a specific time point at which such a decision needs to be made. Leases are an interesting example, of contractual relationships with an explicit switching requirement. Firms make more money on leases, and such customers purchase more frequently and are among a car manufacturer's most valuable customers. Most car manufacturers have an explicit and substantial lease loyalty bonus for a consumer renewing a lease; car dealers are also more lenient about fines for excess wear and tear for those who renew their leases.

Our analysis provides a basis for answers to the questions we raise in the opening WLC Bank scenario. We expect a bank's customers to be very heterogeneous in their value to the bank and follow the 80–20 rule (e.g., Narayanan 2002; Selden and Colvin 2004) but less mobile due to the contractual nature of the banking relationship, that does not require periodic renewal. Based on our results, BBP should increase WLC's profits, but the bank should offer the best prices to its competitor's customers.[21] Therefore, it appears optimal to occasionally let even a highly valuable customer like Bob move to the competition because competitor's customers get better rates. We conclude that WLC's decisions are consistent with our model predictions.

5.2. Limitations and Suggestions for Further Research

Certain limitations of our model suggest interesting directions for further research. We focus only on customer heterogeneity in quantities and customer mobility, which enables us to isolate the critical characteristics that help reconcile practitioner intuition and current analytical results. However, several additional issues might be explored. We do not allow consumers to split their purchases across different firms or even across periods[22], which would make it harder for firms to infer the customer's true potential and their share of customer wallet. Thus, in contrast to our model, in which the first period purchases help firms unambiguously identify high- and low-type consumers, the inference would be only probabilistic if firms have share of wallet information Share of wallet information weakens the extent of information asymmetry in the model (see Du and Kamakura (2008) as an example for how a firm may infer share of wallet).

[21] This practice to offer lower introductory rate is very common in banking industry. Bailey and Kilman (1998) report that 60% of offers to the competitor's customers include very low introductory rates.

[22] Consumers may strategically time their purchases. This could be related to the sales force literature where in response to quotas and bonuses, sales people sometimes shift their effort and book sales towards the end of the year to qualify for a bonus (Steenburgh 2008). We thank an anonymous reviewer for raising this issue.

Many third parties also sell estimates of customer spending potential, based on observed characteristics of the household (e.g., zip code, demographics), but this information is far from perfect. Therefore, even with third-party information, observed purchase histories still provide the information asymmetry critical to the profitability of BBP. We, therefore, believe our findings are robust even with the introduction of noisy potential information, though it might weaken the information asymmetry. A systematic analysis of how share of wallet and potential estimates may affect information asymmetry, and their resulting effect on BBP, would offer an important next step both from an empirical and a theoretical perspective.

Moreover, though we focus on BBP, firms also differentiate customers through differentiated services such as better service, faster check-in (Acquisti and Varian 2005, Zhang 2008)) and advertising (Iyer et al. 2005). Our focus here is restricted to price cuts; additional research should investigate the effect of other strategic variables such as services and advertising.

In our model, we treat the firms' positioning (location) as exogenously given and assume maximal differentiation on the Hotelling line. We believe the issue of whether firm positioning would change once they adopt behavior based price discrimination and how to choose their locations is clearly an interesting issue and we leave it for future research.[23]

We also abstract away from modeling second-degree price discrimination in this paper. The second degree price discrimination problem in a competitive setting with customer behavior information is a technical challenge that has not yet been solved generally and awaits further research (Stole 2007). We therefore leave the issue of combining behavior-based and second-degree price discrimination as an important, but challenging area of future research.

Finally, our theoretical model provides empirically testable hypotheses for further research. We find that BBP is most likely effective when there is sufficient heterogeneity in customer quantities and a threat of customer mobility. We believe empirical research across categories could ascertain whether these predictions are empirically valid. We, therefore, hope our model serves as an impetus for further theoretical and empirical research in BBP.

[23] The question as to what may happen if firms are located within the [0,1] line is a very interesting one. While a formal analysis of this issue is beyond the scope of this paper, one can intuitively see the direction in which this assumption would lead to. Customers to the left of firm A and customers to the right of firm B are virtually captive to their respective firms. As the captive market size increases, firms need to reward one's own customers goes even further down, because they are captive to the firm. Therefore, we are more likely to reward competitor's customers. We thank an anonymous reviewer for suggesting that we clarify the assumption of firm location at the ends of the Hotelling line.

Appendix

Proof of Lemma 1 (Value of information).

We first address retailer *A*. Let z be the first-period market share. Then, $\Pr[\theta_2 \le z | \theta_1 \le z] \ge \Pr[\theta_2 > z | \theta_1 \le z] \Leftrightarrow (1-\beta)+\beta z \ge \beta(1-z)$. In turn, $\Pr[\theta_2 \le z | \theta_1 \le z] - \Pr[\theta_2 > z | \theta_1 \le z] \ge 0 \Leftrightarrow \beta \le \frac{1}{2(1-z)}$. Therefore, it always holds when $\beta \le \frac{1}{2(1-z)}$. In particular, when $z \ge \frac{1}{2}$, the inequality always holds for $\forall \beta \in (0,1]$. Next, let $\bar{z} = 1 - z$ be retailer *B*'s first-period market share and substitute it into equation (2). We get the exactly same result. **Q.E.D.**

Proof of Proposition 1 (Reward own customers or competitor's customers).

(a) We start with the case $\beta < \underline{\chi}(q) = \frac{2(3-q+2q^2)}{3+q+4q^2}$, where $\underline{\chi}(q) < \overline{\chi}(q)$, which ensures that $\tilde{\theta}_1^L > \tilde{\theta}_2^{AL}$ and $\tilde{\theta}_1^H > \tilde{\theta}_2^{AH}$, in equilibrium. Again, using equations (1) and (2), we know that $\Pr^{AH} = \left(\frac{1-\beta}{\tilde{\theta}_1^H} + \beta\right)\left(\frac{1+q(p_2^{BO} - p_2^{AH})}{2}\right)$, $\Pr^{AL} = \left(\frac{1-\beta}{\tilde{\theta}_1^L} + \beta\right)\left(\frac{1+p_2^{Bo} - p_2^{AL}}{2}\right)$, $\Pr^{BH} = \left(\frac{1-\beta}{1-\tilde{\theta}_1^L} + \beta\right)\left(\frac{1-q(p_2^{BH} - p_2^{Ao})}{2}\right)$, and $\Pr^{BL} = \left(\frac{1-\beta}{1-\tilde{\theta}_1^L} + \beta\right)\left(\frac{1-p_2^{BL} + p_2^{AO}}{2}\right)$. Similar to $\beta \ge \overline{\chi}(q)$, we solve the first-order conditions. When $\tilde{\theta}_1^j = \frac{1}{2}$, second-period prices are $p_2^{AH} = p_2^{BH} = \frac{1}{2q} + \frac{(2+\beta)(1+q)}{6(q^2+1)(2-\beta)}$, $p_2^{AL} = p_2^{BL} = \frac{1}{2} + \frac{(2+\beta)(1+q)}{6(q^2+1)(2-\beta)}$, and $p_2^{AO} = p_2^{BO} = \frac{(2+\beta)(1+q)}{3(q^2+1)(2-\beta)}$, which confirms that $\tilde{\theta}_1^L > \tilde{\theta}_2^{AL}$ and $\tilde{\theta}_1^H > \tilde{\theta}_2^{AH}$ when $q > 1$ and $\beta < \underline{\chi}(q)$. The second-period profits are $\Pi_2^A = \Pi_2^B = \frac{(q^2+1)(116-(52-17\beta)\beta)+2q(22-\beta)(2+\beta)}{144(q^2+1)(2-\beta)}$.

Next, we notice that $1 > \overline{\chi}(q) = \frac{2q^2-q-6+\sqrt{4q^4-4q^3+25q^2+24q}}{4q^2+q-3} > \frac{2q^2-q-15}{4q^2+q-3} = \frac{2q+5}{4q+1} > \frac{1}{2}$ for all $q \ge 1$. We also have $p_2^{iO} \le p_2^{iL} \Leftrightarrow \frac{(2+\beta)(1+q)}{12+6(q^2-1)\beta} \le \frac{1}{2} \Leftrightarrow 2(1+q) - 6 \le (3q+1)(q-4)\beta$. The inequality always satisfies for all $q \ge 1$ when $\beta = \frac{1}{2}$. Since $\overline{\chi}(q)$ is monotonically increasing in β and $\chi(q) > \frac{1}{2}$, the inequality always satisfies for $\forall \beta \ge \overline{\chi}(q)$. In addition, $p_2^{iH} \le p_2^{iO} \Leftrightarrow \frac{2-\beta}{2q\beta} \le \frac{(2+\beta)(1+q)}{12+6(q^2-1)\beta}$. Note that $\overline{\chi}(q)$ is the unique cut-off value of $\beta \in [0,1]$**,** such that $p_2^{iH} = p_2^{iO}$. The derivative of (LHS) with respect to β is $-\frac{1}{q\beta^2} < 0$, and the derivative of (RHS) with respect to β is $-\frac{(1+q)(q^2-2)}{3(2+(q^2-1)\beta)^2} < 0$ for $q > 1$. Thus, both p_2^{iH}, p_2^{iO} are monotonically decreasing in β. Moreover, when $\beta = 1$, $p_2^{iH} \le p_2^{iO} \Leftrightarrow \frac{1}{q} \le \frac{(1+q)}{2+(q^2-1)} \Leftrightarrow 1 \le q$. By monotonicity and because $p_2^{iH} \le p_2^{iO}$ at $\beta = 1$, we know that when $q > 1$ and $\beta \in \left[\overline{\chi}(q), 1\right]$, $p_2^{iH} \le p_2^{iO}$. Note that when $q = 1$, the inequality does not hold for all $\beta \in [0,1]$. However, the equality $p_2^{iH} = p_2^{iO}$ only holds when $\beta = 1$ in this case. Therefore, $p_2^{iH} \le p_2^{iO} \le p_2^{iL}$ when $\beta \in \left[\overline{\chi}(q), 1\right]$ and $q > 1$.■

(b) We solve for $\beta \ge \overline{\chi}(q)$, where $\overline{\chi}(q) = \frac{2q^2-q-6+\sqrt{4q^4-4q^3+25q^2+24q}}{4q^2+q-3}$, which ensures that in equilibrium, the market is $\tilde{\theta}_1^L > \tilde{\theta}_2^{AL}$ and $\tilde{\theta}_1^H \le \tilde{\theta}_2^{AH}$. From equations (1) and (2), we know that $\Pr^{AH} =$

$(1-\beta)+\beta\left(\frac{1+q(p_2^{BO}-p_2^{AH})}{2}\right)$, $\Pr^{AL}=\left(\frac{1-\beta}{\tilde{\theta}_1^L}+\beta\right)\left(\frac{1+p_2^{Bo}-p_2^{AL}}{2}\right)$, $\Pr^{BH}=(1-\beta)+\beta(\frac{1-q(p_2^{BH}-p_2^{Ao})}{2})$, and $\Pr^{BL}=\left(\frac{1-\beta}{1-\tilde{\theta}_1^L}+\beta\right)(\frac{1-p_2^{BL}+p_2^{AO}}{2})$. Plugging these into equation (3), we obtain the second-period prices by solving the retailers' first-order conditions: $p_2^{AH}=\frac{2-\beta}{2q\beta}+\frac{(2+\beta)\bar{\theta}_1+(1-\beta)(2\tilde{\theta}_1^L-1)}{6(1-\beta+\beta\Lambda)}$, $p_2^{AL}=\frac{1}{2}+\frac{(2+\beta)\bar{\theta}_1+(1-\beta)(2\tilde{\theta}_1^L-1)}{6(1-\beta+\beta\Lambda)}$, $p_2^{AO}=\frac{3(1+q)-(2+\beta)\bar{\theta}_1-(1-\beta)(q+2\tilde{\theta}_1^L)}{3(1+q^2\beta-\beta\Lambda)}$, $p_2^{BH}=\frac{2-\beta}{2q\beta}+\frac{3(1+q)-(2+\beta)\bar{\theta}_1-(1-\beta)(q+2\tilde{\theta}_1^L)}{6(1+q^2\beta-\beta\Lambda)}$, $p_2^{BL}=\frac{1}{2}+\frac{3(1+q)-(2+\beta)\bar{\theta}_1-(1-\beta)(q+2\tilde{\theta}_1^L)}{6(1+q^2\beta-\beta\Lambda)}$, and $p_2^{BO}=\frac{(2+\beta)\bar{\theta}_1+(1-\beta)(2\tilde{\theta}_1^L-1)}{3(1-\beta+\beta\Lambda)}$, where $\bar{\theta}_1=q\tilde{\theta}_1^H+\tilde{\theta}_1^L$ and $\Lambda=q^2\tilde{\theta}_1^H+\tilde{\theta}_1^L$. Note that the first-period symmetric equilibrium have equal market share, and both retailers charge the same price in the pure strategy equilibrium. Specifically, when $\tilde{\theta}_1^j=\frac{1}{2}$, the second-period prices are $p_2^{AH}=p_2^{BH}=\frac{2-\beta}{2q\beta}+\frac{(2+\beta)(1+q)}{12+6(q^2-1)\beta}$, $p_2^{AL}=p_2^{BL}=\frac{1}{2}+\frac{(2+\beta)(1+q)}{12+6(q^2-1)\beta}$, and $p_2^{AO}=p_2^{BO}=\frac{(2+\beta)(1+q)}{6+3(q^2-1)\beta}$. We confirm that $\tilde{\theta}_1^L>\tilde{\theta}_2^{AL}$ and $\tilde{\theta}_1^H\le\tilde{\theta}_2^{AH}$ when $q>1$ and $\beta\ge\overline{\chi}(q)$. Furthermore, the equilibrium second-period profits are $\pi_2^A=\pi_2^B=\frac{1}{72\beta}+\frac{72+\beta\left(80q^2+88q+40(1+q)\beta-(1+q)^2\beta-32\right)}{144\beta\left(2+(q^2-1)\beta\right)}$.

Next, we verify that $p^{iL}-p^{iO}=\frac{1}{2}-\frac{(2+\beta)(1+q)}{6(q^2+1)(2-\beta)}\ge 0\Leftrightarrow 2(3q^2-q+2)\ge(3q^2+q+4)\beta$. It is obvious that $(3q^2+q+4)\beta\le(3q^2+q+4)$ and $2(3q^2-q+2)>(3q^2+q+4)\Leftrightarrow 3q(q-1)\ge 0$. Therefore, $p^{iL}\ge p^{iO}$ for all $\beta\in[0,1]$ and $q\ge 1$. We only need to show that $p^{iO}\le p^{iH}\Leftrightarrow\frac{(2+\beta)(1+q)}{3(q^2+1)(2-\beta)}\le\frac{1}{q}$. The (LHS) is monotonically increasing in $\beta\in[0,1]$, because the derivative of (LHS) with respect to β is $\frac{4(1+q)}{3(1+q^2)(2-\beta)^2}>0$. Moreover, $\beta=\frac{2(3-q+2q^2)}{3+q+4q^2}$ is the unique cut-off value of β, such that $p_2^{iH}=p_2^{iO}$. For all $\beta\le\frac{2(3-q+2q^2)}{3+q+4q^2}$, $\frac{(2+\beta)(1+q)}{3(q^2+1)(2-\beta)}$ is decreasing as β approaches 0, whereas (RHS) is constant, $\frac{(2+\beta)(1+q)}{3(q^2+1)(2-\beta)}\le\frac{1}{q}\Leftrightarrow p^{iO}\le p^{iH}$. **Q.E.D.**

Derivation of first-period price.

We follow Fudenberg and Tirole's (2000) proof strategy based on the envelope theorem.

Because consumers are distributed along the Hotelling line, $\tilde{\theta}_1^j$ is the demand from type *j* consumers for retailer *A*, and $1-\tilde{\theta}_1^j$ is the demand from type *j* consumers for retailer *B*. By symmetry, we expect that $p_1^A=p_1^B$, and $\tilde{\theta}_1^j=\frac{1}{2}$. In other words, marginal customers of both types *j* in the first period reside at the center, and therefore, the two retailers split demand equally in the first period. We subsequently confirm that this symmetric outcome is an equilibrium, but the level of the first-period equilibrium price depends on the elasticity of demand to a change in price. Because consumers are forward looking, elasticity is affected by consumer expectations about prices in the second period. In addition, the high- and low-type consumers face different prices in the second period, so the price elasticity of the two types in the first period differs, and the optimal first-period prices require retailers to balance the effects of a change in price on the demand of the two types of customers.

Applying the implicit function theorem, we arrive at the following lemma:

Lemma 2. The first-period price sensitivities for *L*- and *H*-type consumers are

$$\frac{d\tilde{\theta}_1^L}{dp_1^A} = -\frac{F_{\theta H}^H - q\cdot F_{\theta H}^L}{F_{\theta L}^L F_{\theta H}^H - F_{\theta H}^L \cdot F_{\theta L}^H} \quad \text{and} \quad \frac{d\tilde{\theta}_1^H}{dp_1^A} = -\frac{q\cdot F_{\theta L}^L - F_{\theta L}^H}{F_{\theta L}^L F_{\theta H}^H - F_{\theta H}^L \cdot F_{\theta L}^H},$$

where $F_{\theta L}^L = 2-\delta\left(\frac{\partial E^{AL}}{\partial\tilde{\theta}_1^L} - \frac{\partial E^{BL}}{\partial\tilde{\theta}_1^L}\right)$, $F_{\theta L}^H = -\delta\left(\frac{\partial E^{AH}}{\partial\tilde{\theta}_1^L} - \frac{\partial E^{BH}}{\partial\tilde{\theta}_1^L}\right)$, $F_{\theta H}^L = -\delta\left(\frac{\partial E^{AL}}{\partial\tilde{\theta}_1^H} - \frac{\partial E^{BL}}{\partial\tilde{\theta}_1^H}\right)$, and $F_{\theta H}^H = 2-\delta\left(\frac{\partial E^{AH}}{\partial\tilde{\theta}_1^H} - \frac{\partial E^{BH}}{\partial\tilde{\theta}_1^H}\right)$.

Proof: We define the implicit functions from equation (8) as follows:

$$F^L(p_1^A,\tilde{\theta}_1^L,\tilde{\theta}_1^H) = 2\tilde{\theta}_1^L - \left\{1+p_1^B - p_1^A + \delta\left(E^{AL}(\tilde{\theta}_1^L,\tilde{\theta}_1^H) - E^{BL}(\tilde{\theta}_1^L,\tilde{\theta}_1^H)\right)\right\} = 0, \text{ and}$$

$$F^H(p_1^A,\tilde{\theta}_1^L,\tilde{\theta}_1^H) = 2\tilde{\theta}_1^H - \left\{1+qp_1^B - qp_1^A + \delta\left(E^{AH}(\tilde{\theta}_1^L,\tilde{\theta}_1^H) - E^{BH}(\tilde{\theta}_1^L,\tilde{\theta}_1^H)\right)\right\} = 0.$$

We differentiate both equations and rearrange them as follows:

$$\frac{d\tilde{\theta}_1^L}{dp_1^A} = -\frac{\left(\frac{\partial F^L}{\partial p_1^A} + \frac{\partial F^L}{\partial\tilde{\theta}_1^H}\cdot\frac{d\tilde{\theta}_1^H}{dp_1^A}\right)}{\frac{\partial F^L}{\partial\tilde{\theta}_1^L}}, \text{ and } \frac{d\tilde{\theta}_1^H}{dp_1^A} = -\frac{\left(\frac{\partial F^H}{\partial p_1^A} + \frac{\partial F^H}{\partial\tilde{\theta}_1^L}\cdot\frac{d\tilde{\theta}_1^L}{dp_1^A}\right)}{\frac{\partial F^H}{\partial\tilde{\theta}_1^H}}.$$

Solving these two equations simultaneously, we obtain the following results:

$$\frac{d\tilde{\theta}_1^L}{dp_1^A} = -\frac{\left(F_p^L - \frac{F_{\theta H}^L}{F_{\theta H}^H}\cdot F_p^H\right)}{\left(F_{\theta L}^L - \frac{F_{\theta H}^L\cdot F_{\theta L}^H}{F_{\theta H}^H}\right)} = -\frac{F_{\theta H}^H F_p^L - F_{\theta H}^L\cdot F_p^H}{F_{\theta H}^H F_{\theta L}^L - F_{\theta H}^L\cdot F_{\theta L}^H} \text{ and } \frac{d\tilde{\theta}_1^H}{dp_1^A} = -\frac{\left(F_p^H - \frac{F_{\theta L}^H}{F_{\theta L}^L}\cdot F_p^L\right)}{\left(F_{\theta H}^H - \frac{F_{\theta H}^L\cdot F_{\theta L}^H}{F_{\theta L}^L}\right)} = -\frac{F_{\theta L}^L F_p^H - F_{\theta L}^H\cdot F_p^L}{F_{\theta L}^L F_{\theta H}^H - F_{\theta H}^L\cdot F_{\theta L}^H},$$

Using $F_p^L = \frac{\partial F^L}{\partial p_1^A} = 1, F_p^H = \frac{\partial F^H}{\partial p_1^A} = q$, we recognize that $F_{\theta L}^L = \frac{\partial F^L}{\partial\tilde{\theta}_1^L} = 2-\delta\left(\frac{\partial E^{AL}}{\partial\tilde{\theta}_1^L} - \frac{\partial E^{BL}}{\partial\tilde{\theta}_1^L}\right)$, $F_{\theta L}^H = \frac{\partial F^H}{\partial\tilde{\theta}_1^L} = -\delta\left(\frac{\partial E^{AH}}{\partial\tilde{\theta}_1^L} - \frac{\partial E^{BH}}{\partial\tilde{\theta}_1^L}\right)$, $F_{\theta H}^L = \frac{\partial F^L}{\partial\tilde{\theta}_1^H} = -\delta\left(\frac{\partial E^{AL}}{\partial\tilde{\theta}_1^H} - \frac{\partial E^{BL}}{\partial\tilde{\theta}_1^H}\right)$, and $F_{\theta H}^H = \frac{\partial F^H}{\partial\tilde{\theta}_1^H} = 2-\delta\left(\frac{\partial E^{AH}}{\partial\tilde{\theta}_1^H} - \frac{\partial E^{BH}}{\partial\tilde{\theta}_1^H}\right)$. ■

Lemma 2 demonstrates how price sensitivity in the first period is affected by the forward-looking behavior of consumers. When consumers are not forward looking, price sensitivity can be obtained by setting $\delta = 0$. In particular, *H*-type consumers are more price sensitive than are *L*-types ($\left|\frac{\partial\tilde{\theta}_1^H}{\partial p_1^A}\right| > \left|\frac{\partial\tilde{\theta}_1^L}{\partial p_1^A}\right|$), which is consistent with empirical findings (Draeger 2000; Kim and Rossi 1994).

It is convenient to rewrite equation (11) for firm *A*'s overall profits using the functions π_2^{AA}, π_2^{AB} to represent its second-period profit from its own previous customers and from retailer *B*'s previous customers, respectively:

$$\Pi^A = (p_1^A - c)\cdot\left\{q\tilde{\theta}_1^H + \tilde{\theta}_1^L\right\} + \delta\Big\{\Pi_2^{AA}\left(p_2^{Ai}(p_1^A,p_1^B), p_2^{Bi}(p_1^A,p_1^B), \tilde{\theta}_1^L(p_1^A,p_1^B), \tilde{\theta}_1^H(p_1^A,p_1^B)\right)$$
$$+\Pi_2^{AB}\left(p_2^{Ai}(p_1^A,p_1^B), p_2^{Bi}(p_1^A,p_1^B), \tilde{\theta}_1^L(p_1^A,p_1^B), \tilde{\theta}_1^H(p_1^A,p_1^B)\right)\Big\}, \quad \text{(A-10)}$$

where $\Pi_2^{AA}\left(p_2^{Ai}, p_2^{Bi}, \tilde{\theta}_1^L, \tilde{\theta}_1^H\right) = (p_2^{AH})q\tilde{\theta}_1^H \Pr^{AH}\left(p_2^{AH}, p_2^{BO}, \tilde{\theta}_1^H\right) + (p_2^{AL})\tilde{\theta}_1^L \Pr^{AL}\left(p_2^{AL}, p_2^{BO}, \tilde{\theta}_1^L\right)$,

$\Pi_2^{AB}\left(p_2^{Ai}, p_2^{Bi}, \tilde{\theta}_1^L, \tilde{\theta}_1^H\right) = (p_2^{AO})\left\{q(1-\tilde{\theta}_1^H)\left(1-\Pr^{BH}\left(p_2^{AH}, p_2^{BO}, \tilde{\theta}_1^H\right)\right) + (1-\tilde{\theta}_1^L)\left(1-\Pr^{BL}\left(p_2^{AL}, p_2^{BO}, \tilde{\theta}_1^L\right)\right)\right\}$.

Note that $\Pr^{AH} = \Pr[\theta_2 \leq \frac{\left[1+q(p_2^{BO}-p_2^{AH})\right]}{2} \Big| \theta_1 \leq \tilde{\theta}_1^H]$ and $\Pr^{BH} = \Pr[\theta_2 > \frac{\left[1+q(p_2^{BH}-p_2^{AO})\right]}{2} \Big| \theta_1 > \tilde{\theta}_1^H]$ are functions of $p_2^{AH}, p_2^{BO}, \tilde{\theta}_1^H$; in addition, $\Pr^{AL} = \Pr[\theta_2 \leq \frac{\left[1+(p_2^{BO}-p_2^{AL})\right]}{2} \Big| \theta_1 \leq \tilde{\theta}_1^L]$ and $\Pr[\theta_2 > \frac{\left[1+(p_2^{BL}-p_2^{AO})\right]}{2} \Big| \theta_1 > \tilde{\theta}_1^L]$ are functions of $p_2^{AL}, p_2^{BO}, \tilde{\theta}_1^L$.

Because retailer *A*'s own second-period prices are set to maximize *A*'s second-period profit, we can use the envelope theorem ($\frac{\partial \Pi_2^{AA}}{\partial p_2^{Ai}} = 0$, $\frac{\partial \Pi_2^{AB}}{\partial p_2^{AO}} = 0$) to write the first-order conditions for this maximization as:

$$\begin{aligned}
&\left(q\tilde{\theta}_1^H + \tilde{\theta}_1^L\right) + (p_1^A)\left(q\frac{\partial \tilde{\theta}_1^H}{\partial p_1^A} + \frac{\partial \tilde{\theta}_1^L}{\partial p_1^A}\right) \\
&+\delta\left[\left\{\frac{\partial \Pi_2^{AA}}{\partial p_2^{BO}}\frac{dp_2^{BO}}{d\tilde{\theta}_1^H} + \frac{\partial \Pi_2^{AB}}{\partial p_2^{BL}}\frac{dp_2^{BL}}{d\tilde{\theta}_1^H} + \frac{\partial \Pi_2^{AB}}{\partial p_2^{BH}}\frac{dp_2^{BH}}{d\tilde{\theta}_1^H} + \frac{\partial \Pi_2^{AA}}{\partial \tilde{\theta}_1^H} + \frac{\partial \Pi_2^{AB}}{\partial \tilde{\theta}_1^H}\right\}\frac{\partial \tilde{\theta}_1^H}{\partial p_1^A}\right. \\
&\left.+\left\{\frac{\partial \Pi_2^{AA}}{\partial p_2^{BO}}\frac{dp_2^{BO}}{d\tilde{\theta}_1^L} + \frac{\partial \Pi_2^{AB}}{\partial p_2^{BL}}\frac{dp_2^{BL}}{d\tilde{\theta}_1^L} + \frac{\partial \Pi_2^{AB}}{\partial p_2^{BH}}\frac{dp_2^{BH}}{d\tilde{\theta}_1^L} + \frac{\partial \Pi_2^{AA}}{\partial \tilde{\theta}_1^L} + \frac{\partial \Pi_2^{AB}}{\partial \tilde{\theta}_1^L}\right\}\frac{\partial \tilde{\theta}_1^L}{\partial p_1^A}\right] = 0.
\end{aligned} \tag{A-11}$$

Then, the first-order condition of equation (12) at $\tilde{\theta}_1^H = \tilde{\theta}_1^L = \frac{1}{2}$ simplifies to

$$p_1^A = p_1^B = \frac{(1+q) - 2\delta\left(\Omega^H\left(-\frac{\partial \tilde{\theta}_1^H}{\partial p_1^A}\right) + \Omega^H\left(-\frac{\partial \tilde{\theta}_1^L}{\partial p_1^A}\right)\right)}{2\left(q\left(-\frac{\partial \tilde{\theta}_1^H}{\partial p_1^A}\right) + \left(-\frac{\partial \tilde{\theta}_1^L}{\partial p_1^A}\right)\right)}, \tag{12}$$

where $\Omega_L = \frac{\partial \Pi_2^{AA}}{\partial p_2^{BO}}\frac{dp_2^{BO}}{d\tilde{\theta}_1^L} + \frac{\partial \Pi_2^{AB}}{\partial p_2^{BL}}\frac{dp_2^{BL}}{d\tilde{\theta}_1^L} + \frac{\partial \Pi_2^{AB}}{\partial p_2^{BH}}\frac{dp_2^{BH}}{d\tilde{\theta}_1^L} + \frac{\partial \Pi_2^{AA}}{\partial \tilde{\theta}_1^L} + \frac{\partial \Pi_2^{AB}}{\partial \tilde{\theta}_1^L}$ $\Omega_H = \frac{\partial \Pi_2^{AA}}{\partial p_2^{BO}}\frac{dp_2^{BO}}{d\tilde{\theta}_1^H} + \frac{\partial \Pi_2^{AB}}{\partial p_2^{BL}}\frac{dp_2^{BL}}{d\tilde{\theta}_1^H} + \frac{\partial \Pi_2^{AB}}{\partial p_2^{BH}}\frac{dp_2^{BH}}{d\tilde{\theta}_1^H} + \frac{\partial \Pi_2^{AA}}{\partial \tilde{\theta}_1^H} + \frac{\partial \pi_2^{AB}}{\partial \tilde{\theta}_1^H}$.

Q.E.D.

Proof of Proposition 2.

We apply a rewarding competitor customer strategy scenario, because $\beta = 0 < \underline{\chi}(q)$ for all q>1. Therefore, equation (3) can be rewritten as:

$$\begin{aligned}
\Pi_2^A &= (p_2^{AH})q\left(\tfrac{1+q(p_2^{BO}-p_2^{AH})}{2}\right) + (p_2^{AL})\left(\tfrac{1+p_2^{BO}-p_2^{AL}}{2}\right) + (p_2^{AO})\left\{q\left(\tfrac{1+q(p_2^{BH}-p_2^{AO})}{2} - \tilde{\theta}_1^H\right) + \left(\tfrac{1+p_2^{BL}-p_2^{AO}}{2} - \tilde{\theta}_1^L\right)\right\}, \\
\Pi_2^B &= (p_2^{BH})q\left(\tfrac{1-q(p_2^{AO}-p_2^{BH})}{2}\right) + (p_2^{AL})\left(\tfrac{1-p_2^{AO}+p_2^{BL}}{2}\right) + (p_2^{BO})\left\{q\left(\tilde{\theta}_1^H - \tfrac{1+q(p_2^{BO}-p_2^{AH})}{2}\right) + \left(\tilde{\theta}_1^L - \tfrac{1+p_2^{BO}-p_2^{AL}}{2}\right)\right\}.
\end{aligned} \tag{A-15}$$

The first-order conditions for retailers' maximization yield $p_2^{AH} = \frac{3+q(2q-1)+4q\bar{\theta}_1}{6q(1+q^2)}$, $p_2^{AL} = \frac{2+q(3q-1)+4\bar{\theta}_1}{6(1+q^2)}$, $p_2^{AO} = \frac{3(1+q)-4\bar{\theta}_1}{3(1+q^2)}$, $p_2^{BH} = \frac{3(q-1)+6(1+q^2)-4q\bar{\theta}_1}{6q(1+q^2)}$, $p_2^{BL} = \frac{3(1+q)+3(1+q^2)-4\bar{\theta}_1}{6(1+q^2)}$, and $p_2^{BO} = \frac{4\bar{\theta}_1-(1+q)}{3(1+q^2)}$, where $\bar{\theta}_1 = q\tilde{\theta}_1^H + \tilde{\theta}_1^L$.

In turn, firms *A*'s and *B*'s overall profit functions can be rewritten as:

$$\begin{aligned}
\Pi^A &= (p_1^A - c)\cdot\left\{q\tilde{\theta}_1^H + \tilde{\theta}_1^L\right\} + \delta\left[\tfrac{49}{72} + \tfrac{62q+80\bar{\theta}_1\left(\bar{\theta}_1-(1+q)\right)}{72(1+q^2)}\right], \\
\Pi^B &= (p_1^B - c)\cdot\left\{q(1-\tilde{\theta}_1^H) + (1-\tilde{\theta}_1^L)\right\} + \delta\left[\tfrac{49}{72} + \tfrac{62q+80\bar{\theta}_1\left(\bar{\theta}_1-(1+q)\right)}{72(1+q^2)}\right].
\end{aligned} \tag{A-17}$$

By using Lemma 1 and equation (12), we obtain $p_1^A = p_1^B = \frac{(3+\delta)(1+q)}{3(1+q^2)}$. We get the profit result directly from plugging prices into the equation, $\Pi^A - \Pi^s = \frac{41\delta}{72} + \frac{36(1+q)^2+46q\delta}{72(1+q^2)} - \frac{(1+q)^2(1+\delta)}{2(1+q^2)} = \frac{(q-5)(5q-1)\delta}{72(1+q^2)} > 0$ if $q > 5$. **Q.E.D.**

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Table 1: Terminology

q^j	Purchase quantity of customer type $j \in \{L, H\}$, where $q^L = 1$, $q^H = q$.
p_1^i	Retailer $i \in \{A, B\}$'s first-period price to all consumers.
p_2^{iL}	Retailer $i \in \{A, B\}$'s second-period price to its *L*-type customers.
p_2^{iH}	Retailer $i \in \{A, B\}$'s second-period price to its *H*-type customers.
p_2^{iO}	Retailer $i \in \{A, B\}$'s second-period price to its *competitor's* customers.
β	Probability that consumer location changes in period 2, $\beta \in [0,1]$
ε	If consumer location changes, consumer location in period 2, $\varepsilon \sim U[0,1]$
θ_t	Consumer's preference or geographical location at period $t \in \{1, 2\}$. In particular, $\theta_2 = \varepsilon$ with prob β and $\theta_2 = \theta_1$ with prob 1-β
$\tilde{\theta}_1^j$	First-period threshold for customer type $j \in \{L, H\}$, such that all consumers of type *j* with $\theta \le \tilde{\theta}_1^j$ purchase from retailer *A* and the rest purchase from retailer *B*
$\tilde{\theta}_2^{Aj}$	Second-period threshold for customer type $j \in \{L, H\}$, who was on retailer *A*'s turf in the first period, such that all consumers of type *j* with $\theta \le \tilde{\theta}_2^{Aj}$ repeat purchase from retailer *A* and the rest switch to *B*.
$\tilde{\theta}_2^{Bj}$	Second-period threshold for customer type $j \in \{L, H\}$, who was on retailer *B*'s turf in the first period, such that all consumers of type *j* with $\theta \ge \tilde{\theta}_2^{Bj}$ repeat purchase from retailer *B* and the rest switch to *A*.
$\Pr^{Aj}$	Probability of repeat purchasing in second period from retailer *A* for customer type $j \in \{L, H\}$ who was in *A*'s turf in the first period
$\Pr^{Bj}$	Probability of repeat purchasing in second period from retailer *B* for customer type $j \in \{L, H\}$ who was in *B*'s turf in the first period
Π_t^i	Profit of retailer $i \in \{A, B\}$ in period *t*. Total profit for retailer *i* is $\Pi^i = \Pi_1^i + \delta\Pi_2^i$.
δ	Discount rate.
$E^A[U_2^j \mid \theta_1 = \tilde{\theta}_1^j]$	Expected second-period utility that marginal customer of type $j \in \{L, H\}$ gets when he or she purchases from retailer *A*. A marginal customer is indifferent between purchasing a product from *A* and *B* in the first period.
$E^B[U_2^j \mid \theta_1 = \tilde{\theta}_1^j]$	Expected second-period utility that marginal customer gets when he or she purchases from retailer B.

	$\beta = 0$	$\beta = 0.5$	$\beta = 0.9$	$\beta = 1$
1st period price	$p_1^A = \frac{4}{5}$	$p_1^A = \frac{125}{132}$	$p_1^A = 0.575$	$p_1^A = \frac{13597}{27000} \simeq 0.504$
2nd period prices	$p_2^{AH} = \frac{7}{20}$ $p_2^{AL} = \frac{3}{5}$ $p_2^{AO} = \frac{1}{5}$	$p_2^{AH} = \frac{5}{12}$ $p_2^{AL} = \frac{2}{3}$ $p_2^{AO} = \frac{1}{3}$	$p_2^{AH} = \frac{1039}{1692} \simeq 0.614$ $p_2^{AL} = \frac{38}{47} \simeq 0.809$ $p_2^{AO} = \frac{29}{47} \simeq 0.617$	$p_2^{AH} = \frac{11}{20}$ $p_2^{AL} = \frac{4}{5}$ $p_2^{AO} = \frac{3}{5}$
Total Profit	$\Pi^{\text{BBP}} = \Pi_1 + \Pi_2$ $= \frac{6}{5} + \frac{21}{40}$ $= \frac{69}{40} = 1.725$	$\Pi^{\text{BBP}} = \Pi_1 + \Pi_2$ $= \frac{125}{88} + \frac{61}{96}$ $\simeq 2.056$	$\Pi^{\text{BBP}} = \Pi_1 + \Pi_2$ $= 0.862 + 0.966$ $\simeq 1.83$	$\Pi^{\text{BBP}} = \Pi_1 + \Pi_2$ $= \frac{13597}{18000} + \frac{73}{80}$ $\simeq 1.67$
Profit without BBP	$\Pi^{\text{No BBP}} = 1.8$	$\Pi^{\text{No BBP}} = 1.8$	$\Pi^{\text{No BBP}} = 1.8$	$\Pi^{\text{No BBP}} = 1.8$
Comparison	$p_2^{AO} < p_2^{AH} < p_2^{AL}$ $\Pi^{\text{No BBP}} > \Pi^{\text{BBP}}$	$p_2^{AO} < p_2^{AH} < p_2^{AL}$ $\Pi^{\text{No BBP}} < \Pi^{\text{BBP}}$	$p_2^{AH} < p_2^{AO} < p_2^{AL}$ $\Pi^{\text{No BBP}} < \Pi^{\text{BBP}}$	$p_2^{AH} < p_2^{AO} < p_2^{AL}$ $\Pi^{\text{No BBP}} > \Pi^{\text{BBP}}$
	Punish Own Customers BBP **reduces profits**	**Punish** Own Customers BBP **increases profits**	**Reward** Own Customers BBP **increases profits**	**Reward** Own Customers BBP **reduces profits**

Table 2: Numerical Illustration ($\alpha = \frac{1}{2}, q = 2$)