

INFORMATION, INSTITUTIONS AND CONSTITUTIONAL ARRANGEMENTS

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“The future is purchased by the present; it is not possible to secure distant or permanent happiness but by the forbearance of some immediate gratification.” SAMUEL JOHNSON

1. INTRODUCTION

Imagine two agents, each employees of separate divisions of a firm, who interact each period to allocate a surplus to be shared by their respective divisions. The subordinates are hired by their respective divisions for two periods at a fixed salary. Each subordinate’s performance in the surplus allocation task in the two periods is then assessed, and he is either reappointed or fired. One subordinate, however, was hired in period $t - 1$ and the other in period t . The former, therefore, comes up for renewal at the end of period t , while the latter does not face a renewal decision until the end of period $t + 1$. Thus, the agents are of different “types” — one EARLY (and thus not up for renewal this period) and one LATE (up for renewal this period). The types are determined by time-varying characteristics, not immutable properties of the individual agents.

Many organizations possess this *staggered-term* feature in which strategic interaction among agents incorporates a time-specific quality to their conditions of employment. Firms with employees who interact over time for whom renewal or promotion decisions are made at different dates constitute one such class. In the present paper we will use as a running example another prominent class of organization — staggered-term legislatures. Upper chambers of many national and provincial legislatures possess this feature. The United States Senate, for example, consists of 100 senators, each elected for six years and then eligible for reelection. They are partitioned into three approximately equal classes (subject only to the constraint that the two senators from any particular state cannot be in the same class), with one-third of the senators coming up for (re)election every two years.

Strategic interaction among the agents is enriched in interesting ways by staggered terms. One complication that we emphasize is related to information. Agents come and go, as some fail to have their employment renewed, obscuring exactly who knows what about the past history of the organization and, in particular, about past patterns of strategic interaction. The ability of contemporaneous agents to condition their actions on this history is

confounded as a result. Another complication has to do with the manner in which assessments are carried out by principals. If a supervisor or an electoral constituency assesses past performance in time-dependent ways — for example, weighing recent performance more heavily than performance further in the past — then agents will have type-dependent motivations each period; the incentives of EARLY types will differ from those of LATE types.

This paper explores dynamic strategic interaction in principal-agent relationships. Equilibrium behavior is affected by the timing of agent assessment (with reward by reappointment or punishment by termination) and the time-dependent way in which principals go about the task of assessing performance. After exploring the game among agents, we step back and analyze how the principals design the game in the first instance. The world we study is one in which principals cannot commit *ex ante* to reappoint agents when the latter come up for renewal. Nor can agents commit to one another to behave in particular ways over time. The equilibria we identify, therefore, are self-enforcing and satisfy a particular form of sequential rationality.

In the next three sections we develop a baseline dynamic model involving two agents engaged each period in a divide-the-cake exercise. Each agent has the same fixed term and the same compensation per term, but a distinct start date and hence a distinct renewal date. Principals decide whether to renew or terminate their respective agents probabilistically on the basis of past performance in delivering cake to them. We examine two information regimes — one in which the full history of play in the organization is commonly known and one in which that is not the case. The strategic interaction — the game form so to speak — is taken as exogenously fixed. Then, in section 5, we relax this restriction, inquiring how principals, in a “constitutional moment,” would determine the features of subsequent strategic interaction among their respective agents. We also explore the robustness of such constitutional decisions to renegotiation (“constitutional amendments”). In section 6 we show how our baseline model of two principals and their respective agents extends to multiple principals each with multiple agents and, in section 7, we briefly develop an analytical description of the US Senate as a quintessential staggered-term organization. After summing up the conclusions our analysis permits in section 8, we show in section 9 how our approach can be extended in a variety of ways. Section 10 connects our contribution to the existing literature, where we also discuss the main assumptions and features of our model. Intuitions for our theoretical propositions are provided in the body of the paper, but proofs are placed in the appendix.

The contributions of this paper revolve around how information affects strategic interaction, agent welfare, and principal welfare, shedding new light on the role of *transparency*. We find that transparency as a solution to moral hazard problems — the conventional interpretation in the literature — is not the only purpose it serves. In our models transparency is the means by which long-term relationships among *agents* are sustained; indeed, it may not even matter whether strategic interaction among agents is transparent to their principals.

We also shed light on the political economy of constitutional moments and how the institutional subgame created at that time structures the interactions of preferences, procedures, and information.

2. THE BASELINE MODEL

2.1. The Structure. We consider an infinitely-lived legislative body consisting of two legislators, each elected from a separate electoral district. A legislator's term in office consists of two periods followed by the possibility of reelection — there are no term limits. Furthermore, legislators from the two districts have staggered terms of office. This means that at the end of each period only one legislator comes up for reelection.

The legislature is founded in period -1 , when the two principals determine procedural rules and informational features (in a manner specified in subsection 2.5). At this constitutional moment, there are no legislators (agents) present.

The legislature starts operating from period 0 onwards. In each period $t \in \{0, 1, 2, 3, \dots\}$, the two legislators will be of different “types”, in the following sense. In any period $t \geq 1$, one of the legislators was reelected, or elected for the first time, at the beginning of period t . This legislator is therefore in the first period of his two-period term in office. The other legislator was reelected, or elected for the first time, at the beginning of period $t - 1$. He is currently in the second (and last) period of his two-period term in office. For expositional convenience, we denote the former type of legislator by EARLY and the latter type by LATE.¹

In period 0, matters are a little different as this is the first period of operation of the legislature. In this period, a legislator from each district is elected. Given that the terms of office are staggered, one of them is randomly selected to be the EARLY type (and thus he begins a two-period term in office), while the other is selected to be the LATE type (and thus this legislator comes up for reelection at the end of this very period having served a one-period term in office).²

The period- t EARLY legislator is the period- $(t + 1)$ LATE legislator. At the end of period t , the period- t LATE legislator faces his electorate. He is either reelected or replaced by a challenger.³ If reelected, then he becomes the period- $(t + 1)$ EARLY legislator; otherwise the challenger, a newly minted legislator, is the period- $(t + 1)$ EARLY legislator. Our model

¹Note that “types” refer to period-dependent characteristics of legislators, and not, as is standard in economic theory, to some immutable characteristic.

²The US Senate operated exactly like this. In its opening session in 1790, the method of lots was employed to distribute senators in this staggered-term legislature across types, subject to the constraint that both of a state's senators could not be of the same type (cf. XXXX).

³Challengers are not modelled as players in our framework. It is implicitly assumed that a challenger exists and that he has not previously served in the legislature. This means that a legislator who fails to get reelected cannot be a future challenger; he withdraws from legislative politics in that eventuality. Apart from these differences in legislative experience, there are no other differences between a challenger and a legislator seeking reelection.

of elections is described in subsection 2.2. Although the legislators are not in different generations as such, as is the case in standard OLG models, our structure is similar in a few important respects to such models, especially in terms of the relative incentives of the two types of legislators.

The policy context in each period concerns the sharing of an economic surplus. We stylize this as the allocation of cake (or pork) between the two districts. In each period t , the period- t EARLY and LATE legislators negotiate over the partition of a unit-size cake (the pork barrel). The bargaining procedure (which in particular embodies the distribution of proposal power between them) is described in subsection 2.3. If an agreement is struck, then the agreed shares of the cake flow to the districts. The legislators receive no direct benefit from any portion of this cake. A legislator simply receives a fixed payoff $b > 0$ in each term he serves in office. Any share of the cake that he negotiates for his district, however, may help his reelection prospects.

The structure and timing is summarized in Figure 1. We now turn to a description of elections, bargaining, information and the principals' problem (where we also discuss the "renegotiation" issue mentioned in Figure 1).

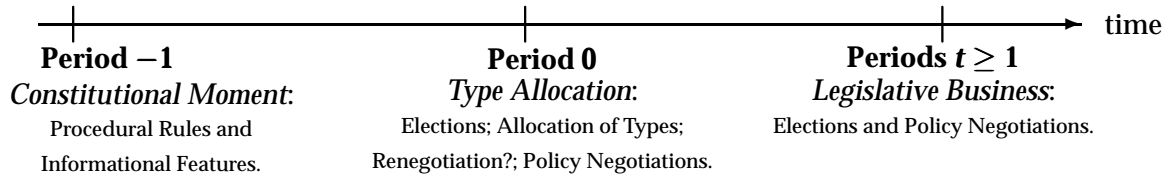


FIGURE 1. Structure and Timing.

2.2. Elections. The likelihood of a legislator being reelected depends on a variety of factors. Even when such factors are taken into account, some uncertainty about the election outcome remains. Let Π denote the probability that an arbitrary legislator (in an arbitrary period) is reelected. Our first idea is that voters care about the legislator's past performance in office when deciding whether or not to reelect him. We formalize this idea by positing that Π depends on the amounts of cake he obtained for his constituents during his most recent two-period term in office. With a slight abuse of notation, we write this as $\Pi(x_E, x_L)$, where x_E and x_L are the amounts of cake obtained by the legislator when EARLY and LATE, respectively, in his most recent two-period term of office.⁴

It is natural to assume that receiving more cake does not make the voters worse off, and thus does not decrease a legislator's chances of getting reelected. However, it may be that for some increases, the chances are unaffected. Hence:

⁴Note that for the period-0 LATE legislator, $x_E = 0$, by definition. In fact, the probability of reelection function for the period-0 LATE legislator could, in principle, differ from $\Pi(0, x_L)$. However, because it does not affect our results, for expositional convenience we assume that his probability of reelection is indeed $\Pi(0, x_L)$.

Assumption 1 (Weak Monotonicity). *The probability Π that a legislator is reelected is non-decreasing in each of its two arguments.*

The second central idea we adopt about the election outcome is the notion that voters engage in a particular form of retrospective assessment. Known in the psychological literature as the recency effect, voters ask “What have you done for me lately?” which we abbreviate with the acronym *WHYDFML* (pronounced whid’fiml). We adopt an especially weak form of recency effect: the probability of reelection is higher if a legislator receives the entire surplus LATE rather than EARLY. Hence:

Assumption 2 (Weak Recency Effect). *The probability Π that a legislator is reelected satisfies $\Pi(0, 1) \geq \Pi(1, 0)$.*

Our third and final assumption about Π imposes a restriction between the maximum and minimum feasible reelection probabilities, requiring that $\Pi(0, 0)$ not be too small unless $\Pi(1, 1)$ is large:

Assumption 3. *The probability of reelection Π satisfies $\Pi(0, 0) \geq 1 - \Pi(1, 1)$.*

Note that Assumptions 1 and 3 imply that $\Pi(1, 1) \geq 0.5$. In summary, our model of elections is characterized entirely by the probability of reelection function Π satisfying A1-A3.

2.3. Bargaining. The procedure that determines the negotiated partition of the unit-size cake comprises the organizational structure of the legislature, pinning down the allocation of power (proposal power in particular) between the two legislators. Our framework abstracts from many of the details of real institutions through which power is derived (such as committees), capturing the allocation of power in a simple manner. We posit a random proposer, “take-it-or-leave-it-offer” format. With probability $\theta \in [0, 1]$ the EARLY legislator makes an offer of a partition of the unit-size cake to the LATE legislator, and with the complementary probability $1 - \theta$ it is the LATE legislator who makes an offer to the EARLY legislator. If the offer is accepted, agreement is struck. But if the offer is rejected, then bargaining terminates, no agreement is reached, and no cake is obtained (in the period in question) by either district. We adopt the convention that an offer designates the share going to the proposer. It is therefore sometimes convenient to use the word “demand” rather than “offer”.

The probability θ captures the relative proposal power of the legislators. If it equals one-half, then power is not type-contingent; otherwise it is. If $\theta = 0$ then the LATE legislator has all the power, while the exact opposite is the case if $\theta = 1$. Each type of legislator has some power if $0 < \theta < 1$. As θ increases more power is vested in the EARLY legislator. We adopt the following regularity assumption:

Assumption 4 (Tie-Breaking).

(i) *When indifferent between accepting or rejecting an offer, a legislator accepts it.*

(ii) *When indifferent between making one of several offers, a legislator selects the offer which allocates the largest share of the cake to him.*

For future reference, it may be noted that the expected payoff to a legislator who is re-elected on each occasion with a constant probability $\pi \in (0, 1)$ equals $b/(1 - \pi)$. Notice that, without loss of generality, we do not endow legislators with a discount factor.

2.4. Information. How much information and what kind of information does any legislator have in any given period? The issue is especially pertinent here since every two periods a legislator faces reelection, and with positive probability he is replaced by a newly minted legislator. While the legislature is an infinitely-lived body, operating over an indefinite number of periods, legislators come and go. As such a legislator may not know the full history at any given period. We will entertain both of the possible kinds of informational structures, which we now define.

Assumption 5 (Perfect Information). *Any legislator in any period has full knowledge of the entire history of play.*

The perfect information assumption could alternatively be stated as follows: for any t , the actions taken in period t are known by legislators in each and every subsequent period. Given this, we relax the perfect information assumption as follows:

Assumption 6 (Imperfect Information).

- (i) *For any t there exists a $T \geq t + 2$ such that legislators in period T and onwards do not know of the actions taken by the legislators in periods $s \leq t$.*
- (ii) *For any t , legislators in period $t + 1$ do know of the actions taken in period t .*

A6(i) would be implied, for example, if legislators have finite memory, the length of which could vary across legislators. A6(ii) ensures that in any period, the two legislators know the amount of cake received by LATE in the previous period.

An altogether different kind of information concerns what a legislator knows about the game form, the payoffs and various parameters. Throughout this paper we adopt the *complete information* assumption: there is common knowledge amongst all legislators about the game itself, including whether there is perfect information (A5 holds) or imperfect information (A6 holds).

2.5. Principals' Problem. At the constitutional moment in period -1 , the principals jointly choose the allocation of proposal power (value of θ) and the informational structure (perfect information or imperfect information). These are institutionalized through appropriate constitutional mechanisms, which determine legislative procedures and rules.⁵ They are

⁵An example relevant to informational structure: The U.S. constitution requires each chamber of the legislature to “keep a Journal of its Proceedings, and from time to time publish the same...” (Article I, Section 5).

selected to optimize over the principals' joint interests. Let $u_i(c)$ denote the per-period utility obtained by the principal from district i ($i = 1, 2$) when her consumption is c in the period in question, and let $\delta_i < 1$ denote the per-period discount factor used by her to discount future utility. We assume that u_i is strictly increasing and strictly concave in c .

The principals' joint expected payoffs take into account that in period 0 it will be randomly determined (with equal probability) as to which district's legislator is to be the EARLY type and which is to be the LATE type. Furthermore, these payoffs depend on the equilibrium outcome of the game once the legislature starts operating at the start of period 0, which in general vary according to the selected procedural rules and information structure. These equilibria are derived in sections 3 and 4 according to whether there is perfect information or imperfect information. Section 5 then studies the principals' period -1 problem. We will also ask whether the selected procedural rules and information structure are renegotiation-proof at the beginning of period 0, after the veil is lifted as to which district's legislator is EARLY and which is LATE.

This completes the description of our baseline model, which is a stochastic game with a countably infinite number of agents, but only two agents are active in any one period, and the number of periods for which an agent is active is determined endogenously.⁶

3. INTERGENERATIONAL COOPERATION UNDER PERFECT INFORMATION

In this section we study the subgame perfect equilibria (SPE henceforth) of our baseline model with Assumption 5 and for an arbitrary value of θ . Given this informationally rich environment, the focus of this section is in addressing the following question: Can *agent-optimal* outcome paths be sustained in *any* SPE? Roughly speaking (a precise definition is stated below), an "agent-optimal" outcome path is a path of play of our stochastic game that maximizes the legislators' joint expected payoffs.

We establish that an agent-optimal outcome path is a SPE path for any allocation of bargaining power and for any probability of reelection function satisfying our three mild assumptions, A1–A3. Legislators credibly sustain agent-optimal outcome paths via intergenerational cooperation, and with the credible threat of inflicting *maximal* punishments on deviators. It is because of the latter that the allocation of bargaining power is irrelevant, as is the magnitude of the reelection probability associated with the agent-optimal outcome path (subject to A3).

The analysis in this section rests crucially on the assumption that legislators have perfect information about the history of play, since the equilibria constructed here require that to be the case. In summary, then, the key message of this section can be put as follows: *When*

⁶Our stochastic game falls outside of the classes of stochastic games studied in the current literature (see, for example, Friedman (1986), Fudenberg and Tirole (1990), and Dutta (1995)). Thus, we cannot appeal to or apply results from that literature. However, some of our main results are derived using methods and ideas borrowed from that literature and from the theory of infinitely-repeated games.

legislators possess perfect information, the structure of legislative institutions is irrelevant, as is the nature of the reelection probability function (subject to A1–A3).

Our analysis is based on the insightful methodology laid down by Abreu (1988). The key to it is to derive the “optimal penal code”, that is, the *worst* SPE payoffs to each and every legislator. Section 3.1 is devoted to their derivation. Our main result is then established in section 3.2. It is derived on the fundamental observation, first made by Abreu, that an outcome path is a SPE path if and only if it is sustained in a SPE in which any unilateral deviation from it by a player immediately moves play into that player’s worst SPE.

3.1. Worst Punishments. In any Nash equilibrium, each legislator’s expected payoff is no less than his minmax payoff. This is the worst possible expected payoff that all other legislators can hold him down to. It involves them rejecting all offers and demanding the whole cake. Hence, a legislator’s minmax payoff is $b/[1 - \Pi(0, 0)]$, which, given the symmetric nature of our stochastic game, is the same for each and every legislator (see Lemma 1).

The worst SPE for an arbitrary legislator can be conveniently defined by two paths Q^1 and Q^2 , and two transition rules T^1 and T^2 . If the initial path is Q^i ($i = 1, 2$), then any legislator from district i is held to his minmax payoff.

The path Q^i : Each and every legislator from district i always (i.e., in any period, for any history, and whether he is EARLY or LATE) offers the whole cake to the legislator from district j ($j \neq i$), and accepts all offers. Each and every legislator from district j always demands the whole cake, and only accepts an offer that allocates to him the whole cake.

The transition rule T^i : If, when play is on path Q^i , a legislator from district j accepts an offer in which he is allocated less than the whole cake, then immediately (from the start of the next period onwards) play switches to path Q^j . For any other deviation on path Q^i , play remains on this path.

Lemma 1 (The Worst SPE). *Given Assumptions 1–3 and 5, the strategies implicitly defined by the pair of paths (Q^1, Q^2) and the pair of transition rules (T^1, T^2) are subgame perfect. If the initial path is Q^i , then the expected payoff in this SPE to any legislator from district i ($i = 1, 2$) is his minmax payoff, i.e., $b/[1 - \Pi(0, 0)]$.*

Proof. In the appendix. □

The proof of Lemma 1 involves checking that no legislator can undertake a profitable deviation from either of the two paths. For example, Assumptions 2 and 3 respectively help ensure that a EARLY legislator and a LATE legislator from district i cannot profitably deviate from path Q^j when offered almost all (but not the whole) of the cake.

Lemma 1 has defined two extremal SPE according to whether the initial path is Q^1 or Q^2 . The former is the worst SPE path for any legislator from district 1, while the latter is the worst SPE path for any legislator from district 2. Notice that when a legislator from district i is meted out maximal punishment (he never gets any cake), the legislator from the other

district (who is doing the punishing) likes it as he obtains the whole cake. Interestingly, these extremal equilibria exist for any feasible value of θ ; i.e., the worst punishment paths are sustainable as SPE paths irrespective of the structure of the legislative institutions.

3.2. Incentive-Compatible, Agent-Optimal Outcomes. Given the underlying symmetric and stationary structure of our stochastic game, the legislators' joint expected payoffs are maximized with an outcome path in which in each period the partition of the unit-size cake is contingent on at most the type of the legislator who is randomly selected to make the offer (but is otherwise independent of time and history). Fix, therefore, such an arbitrary outcome path, $Q(k_E, k_L)$: in each period, the legislator who is selected to propose demands a share $k_E \in [0, 1]$ if he is EARLY, and a share $k_L \in [0, 1]$ if he is LATE, and the demand is accepted. The expected payoff $P(k_E, k_L)$ to an arbitrary legislator at the beginning of any period when he is EARLY from this outcome path is $P(k_E, k_L) = b/[1 - \Omega(k_E, k_L)]$, where⁷

$$(1) \quad \Omega(k_E, k_L) = \theta^2 \Pi(k_E, 1 - k_E) + \theta(1 - \theta) \Pi(k_E, k_L) + \\ (1 - \theta)\theta \Pi(1 - k_L, 1 - k_E) + (1 - \theta)^2 \Pi(1 - k_L, k_L).$$

An *agent-optimal* outcome path, $Q(k_E^*, k_L^*)$, is characterized by a pair of numbers (k_E^*, k_L^*) which maximizes $P(k_E, k_L)$ (or, equivalently, the expected probability of reelection, $\Omega(k_E, k_L)$) over the set of all $(k_E, k_L) \in [0, 1] \times [0, 1]$.⁸

We now establish that any agent-optimal outcome path is a SPE path. Following Abreu (1988), this result is achieved by constructing a SPE in which any unilateral deviation from the (initial) path $Q(k_E^*, k_L^*)$ by a legislator from district i immediately moves play onto the path Q^i (which is the worst SPE for such a legislator).

Proposition 1 (Agent-Optimal SPE). *Given Assumptions 1–3 and 5, the agent-optimal outcome path $Q(k_E^*, k_L^*)$, defined above, is a SPE path. The expected payoff to any legislator in any agent-optimal outcome path is*

$$P(k_E^*, k_L^*) = \frac{b}{1 - \Omega(k_E^*, k_L^*)} \quad \text{with} \quad \Omega(k_E^*, k_L^*) = \max_{(k_E, k_L) \in [0, 1] \times [0, 1]} \Omega(k_E, k_L)$$

where $\Omega(k_E, k_L)$ is defined in (1).

Proof. In the appendix. □

⁷The expression for this expected probability is made up of four terms corresponding to four possible outcomes. Each such outcome is determined by the realizations in each of the two periods of who the proposer is. For example, with probability θ^2 , in each of the two periods it is the EARLY legislator who is selected. That means that the legislator in question gets to propose when EARLY but not when LATE, and so in this eventuality his probability of reelection is $\Pi(k_E, 1 - k_E)$.

⁸Existence is guaranteed since Π is bounded and the feasible set is compact. Depending on the properties of Π , there may however exist more than one agent-optimal outcome path, but they will all, by definition, generate the same reelection probability and expected payoff.

Proposition 1 establishes that an agent-optimal outcome path is a SPE path for any allocation of bargaining power and for any reelection probability function satisfying the three mild assumptions, A1–A3. The sustainability of this outcome path as a SPE path does not hinge either on the structure of the legislative institutions or on the magnitude of the reelection probability associated with it (subject only to the implication of A3 that $\Omega(k_E^*, k_L^*)$ cannot be too small unless $\Pi(1, 1)$ is large). All this is made possible because the agent-optimal outcome path is being sustained by the optimal penal code (i.e., the worst SPE).

The existence of these equilibria requires legislators to possess perfect information about the history of play (Assumption 5 holds). In short, then, *the structure of legislative institutions is irrelevant for the existence of an agent-optimal SPE in a setting with perfect information, as is the nature of the reelection probability function (subject to A1–A3).*

It should however be noted that since $\Omega(k_E, k_L)$ depends on θ and Π , so in general will the agent-optimal outcome (k_E^*, k_L^*) and the legislator’s agent-optimal expected payoff $P(k_E^*, k_L^*)$. But without imposing additional structure on the probability of reelection function Π , it is not possible in general to establish any results concerning that dependence. Before we continue with our main argument, in the next section, we nonetheless establish two results about the agent-optimal outcome when Π is concave. We first show that if Π is concave, then the agent-optimal outcome and the legislator’s expected payoff are independent of θ :

Corollary 1. *If Π is concave, then the agent-optimal outcome $(k_E^*, k_L^*) = (x^*, 1 - x^*)$, where x^* maximizes $\Pi(x, 1 - x)$ over $x \in [0, 1]$, and the expected payoff to any legislator in any agent-optimal outcome path is $b/[1 - \Pi(x^*, 1 - x^*)]$.*

Proof. In the appendix. □

Thus, if Π is concave, then the agent-optimal outcome path involves the legislators allocating, in each period, x^* to EARLY and $1 - x^*$ to LATE irrespective which type makes the offer (hence the irrelevance of the value of θ). We now show that with a slightly stronger recency effect than what is captured in Assumption 2, it follows that $x^* < 0.5$:

Corollary 2. *The agent-optimal outcome, defined by x^* in Corollary 1, is strictly less than one-half if $\Pi(0, 1) > \Pi(0.5, 0.5)$.*

Proof. In the appendix. □

The inequality in Corollary 2 entails a strengthening of recency bias. Together with concavity of Π , it implies A2. In words it says not only is it better to get the whole cake when LATE rather than EARLY, it is also better to get the whole cake when LATE than half a cake each period. When this stronger condition of recency holds, then there will be a strict “back loading” of the cake in order to answer the principal’s query, “what have you done for me lately?”

4. INSTITUTIONAL RELEVANCE UNDER IMPERFECT INFORMATION

We now study our baseline model with imperfect information about the history of play (Assumption 6). We establish, as the main result of this section, that *in any pure-strategy equilibrium, legislators use Markov strategies*. Thus, none of the equilibria described in section 3 hold here, and intergenerational cooperation is unsustainable in equilibrium. We then characterize the pure-strategy equilibria (necessarily in Markov strategies) and establish the following key message: *with imperfect information, the structure of legislative institutions matters, as does the nature of the reelection probability function*. This conclusion is to be contrasted with the opposite conclusion when legislators possess perfect information.

There are no proper subgames in the baseline model with A6. As such we cannot use the SPE concept. But, as is now well-established, it is desirable to work nonetheless with a solution concept which embodies the general notion of sequential rationality, which is the central element of the SPE concept. In the context of our stochastic game, the sequential rationality concept requires that in any period t and for any *observed* history, each legislator's actions are ex-post optimal (i.e., they maximize his expected payoff *from that period onwards*). We define a sequentially rational, symmetric pure strategy equilibrium (henceforth equilibrium) to be a pure strategy, adopted by all legislators, which is sequentially rational.⁹ We now state the main result of this section:

Proposition 2 (Structure of Equilibria with Imperfect Information). *Given Assumptions 4 and 6, any pure-strategy equilibrium is a Markov pure strategy.*

Proof. In the appendix.¹⁰ □

This remarkable and unexpected result implies that with imperfect information about the history of play, there cannot exist equilibria in which a legislator uses a non-Markov (history dependent) strategy; that is, any strategy in which a legislator conditions his current actions on payoff-irrelevant past actions cannot be part of an equilibrium. This means that none of the equilibria described in section 3 can hold here. Indeed, intergenerational cooperation is unsustainable in equilibrium when Assumption 6 holds.

Given Proposition 2, the set of pure-strategy equilibria of our baseline model with imperfect information is identical to the set of pure-strategy equilibria in Markov strategies. The following proposition characterizes the unique such equilibrium.

⁹Note that since (by A6(ii)) the legislators in period t know the amount of cake the period- t LATE legislator obtained in period $t - 1$ (which comprises the payoff-relevant bits of the history at the beginning of period t), we do not need to invoke any beliefs regarding past actions in defining and implementing this equilibrium concept. For example, we do not need to employ the relatively more complex sequential equilibrium concept.

¹⁰The proof is based on a method taken from the insightful contribution by Bhaskar (1998), who studies a version of Samuelson's OLG model with imperfect information. We discuss the game-theoretic OLG literature, including Bhaskar (1998), in section 10.

Proposition 3 (Unique Markov Equilibrium, ME). *If the probability of reelection Π satisfies Assumptions 1 and 4, then the following strategy, adopted by all legislators, is the unique ME. A legislator accepts any offer when EARLY and any offer when LATE. When making an offer, either when EARLY or when LATE, he demands the whole unit-size cake. The expected payoff to a legislator when EARLY associated with this equilibrium is*

$$P(1, 1) = \frac{b}{1 - \Omega(1, 1)}$$

where $\Omega(1, 1)$ is obtained by setting $k_E = k_L = 1$ in (1).

Proof. In the appendix. □

Notice that the equilibrium expected payoff to a legislator $P(1, 1)$ depends on θ (through $\Omega(1, 1)$). The following corollary provides a characterization of the value of θ which maximizes $P(1, 1)$:

Corollary 3. *Assume that Π satisfies Assumption 2. Let $\hat{\theta}$ denote the value of θ which maximizes the expected payoff $P(1, 1)$ of a legislator in the unique ME. If $\Pi(0, 1) \geq [\Pi(1, 1) + \Pi(0, 0)]/2$ then $\hat{\theta} = 0$; Otherwise $0 < \hat{\theta} < 0.5$, with $\hat{\theta}$ decreasing in the difference $\Pi(0, 1) - \Pi(1, 0)$.*

Proof. In the appendix. □

Thus, if legislators have imperfect information about the history of play, then they would like the legislative institutions to be such that a relatively greater amount of bargaining power is allocated to LATE legislators (and in some cases all the bargaining power). With perfect information, legislators are less concerned with institutional structure but that is far from the case with imperfect information. This point can be seen quite starkly in the case when Π is concave and $\Pi(0, 1) > \Pi(0.5, 0.5)$: With perfect information, legislators are indifferent to the value of θ (Corollary 1), but with imperfect information, they would like to allocate all power to LATE legislators (Corollary 3; under the assumptions on Π in this case, the inequality stated in the corollary is satisfied and hence $\hat{\theta} = 0$).

5. CONSTITUTIONAL ARRANGEMENTS

Given the results derived in sections 3 and 4, we are now ready to study the principals' problem in period -1 of selecting the institutional and informational structures of the legislature. We will establish three main results, the first two are the subject of subsections 5.2 and 5.3, while the third is established in subsection 5.4.

First, the principals want to ensure — by building appropriate record-keeping mechanisms — that legislators (agents) have *perfect information* about the history of actions taken in the legislature. Transparency of agent actions *amongst the agents* is all important for the principals because it enables agents to hold each other to account. This key insight is shown to be fairly robust (including renegotiation-proof).

Second, under some conditions, the principals are indifferent to the institutional structure. Given that agents will have perfect information about the history of play (the first result in this section), principals do not care how bargaining power is allocated between agents. But there are conditions under which a specific institutional structure (combined with agents possessing perfect information about history) would deliver the best outcome for the principals.

Our third main result concerns environments in which principals are unable to create the mechanisms under which legislators would have perfect information. As such, principals select the best allocation of bargaining power recognizing that play will subsequently be determined by the equilibrium described in Proposition 3. We show that in these circumstances, if the principals have identical preferences then they are indifferent to the allocation of bargaining power, but if these differ then this is not the case.

5.1. The First-Best. We begin by laying down some preliminary concepts, and establish the *first-best* outcome. This is the outcome path that maximizes the principals' joint interests when the partitions of the cakes are chosen *directly* by them — that is, in the absence of the fundamental feature of representative democracies, which underpins this paper, that the partitions of the cakes are delegated to negotiations in the legislative body amongst the elected legislators. The first-best serves as the ideal from the principals' perspective.

Let $U_E^i \equiv U_E^i(x, y)$ and $U_L^i \equiv U_L^i(x, y)$ respectively denote the present discounted expected payoffs to the principal from district i depending on whether her period-0 legislator is EARLY or LATE, and given that in each period $t \geq 0$ the agreed allocations are $(x, 1 - x)$ and $(1 - y, y)$ respectively according to whether it is EARLY or LATE who makes the offer. For each $i = 1, 2$, U_E^i and U_L^i respectively satisfy the following Bellman equations:

(2)

$$U_E^i(x, y) = [\theta u_i(x) + (1 - \theta)u_i(1 - y)] + \delta_i [\theta u_i(1 - x) + (1 - \theta)u_i(y)] + \delta_i^2 U_E^i, \quad \text{and}$$

(3)

$$U_L^i(x, y) = [\theta u_i(1 - x) + (1 - \theta)u_i(y)] + \delta_i [\theta u_i(x) + (1 - \theta)u_i(1 - y)] + \delta_i^2 U_L^i.$$

Straightforward computations show that the expected value is:

$$(4) \quad U_P^i(x, y) \equiv \frac{U_E^i(x, y) + U_L^i(x, y)}{2} = \frac{\theta [u_i(x) + u_i(1 - x)] + (1 - \theta) [u_i(y) + u_i(1 - y)]}{2(1 - \delta_i)}.$$

Notice that since u_i is strictly concave, U_P^i is maximized at $(x, y) = (0.5, 0.5)$. This makes sense: since each principal has strictly concave preferences, she prefers to smooth out her consumption over time. The *first-best, or ideal solution for both principals is the same, and it involves splitting each period's unit-size cake down the middle, half for district 1 and half for district 2.* Alas, the partition of the cakes are not determined directly by them. Rather it is determined by legislators through their actions in the legislative body. They may, however, be constrained constitutionally in various ways. Indeed, while the principals do not choose the allocations of the cakes, we can ask which institutional and informational

structures would maximize their joint expected payoffs subject to the constraint that once these are chosen, the amounts of cake that they respectively receive are determined by the legislators' equilibrium behaviour (as described in Propositions 1–3). We now proceed to answer this question.

5.2. Principal-Optimality Behind The Veil. We first study the determination of the choice variables $\theta \in [0, 1]$, and perfect or imperfect information (A5 or A6), in period -1 , made in ignorance of which district's period-0 legislator will be EARLY and which will be LATE. It follows from Propositions 1–3 that the equilibrium ex ante (behind the veil) expected payoffs to principal i with imperfect information is $U_P^i(1, 1)$, and with perfect information is $U_P^i(k_E^*, k_L^*)$. Since u_i is strictly concave, the latter strictly exceeds the former. Hence we have our first main result of this section:

Proposition 4 (Agent Transparency). *Given Assumptions 1–4, and with the constitutional choices being made behind the veil, principals from both districts strictly prefer to have a legislative body structured in such a way that legislators possess perfect information (A5) about the history of play rather than imperfect information (A6).*

This means that the principals will want to require appropriate record-keeping mechanisms to ensure that the history of actions taken in the legislature are always available for inspection by any *legislator*. Whether they are also available for inspection by other parties (eg., by the principals themselves, or the media) is not an issue of any relevance. Here, what matters is *transparency of legislators' actions amongst the legislators to enable legislators to hold each other to account*. This notion of transparency differs from standard notions of transparency in principal-agent relationships that are concerned with the extent to which the agent's action is known to the principals or other third parties to enable them to hold the agent to account.¹¹ Our result (relevant, of course, only in multi-agent environments) concerns the notion of *agents holding each other to account*.

Next we turn to the institutional structure as parameterized by θ that maximizes the joint ex ante expected payoffs of the principals. Given Proposition 4, this would be the value of θ that solves the following maximization problem:

$$(5) \quad U_P^* \equiv \max_{\theta \in [0,1]} [U_P^1(k_E^*, k_L^*) + U_P^2(k_E^*, k_L^*)].$$

Without additional assumptions on Π , it is in general not possible to solve this problem. In general, $U_P^i(k_E^*, k_L^*)$ is potentially sensitive to θ , and hence one or both principals may have a strict preference for a specific value of θ . The set of solutions of (5) may be a proper subset of $[0, 1]$, and possibly there may be a unique θ which maximizes the principals' joint ex ante expected payoffs. In that case, principals not only will want to ensure that legislators can hold each other to account (Proposition 4), but also will design the legislature in such a way

¹¹See, for example, Besley (2005) and Prat (2005).

that bargaining power is allocated appropriately. However, when Π is concave it is easy to solve this problem. In that case, k_E^* is independent of θ , and that $k_L^* = 1 - k_E^*$ (cf. Corollary 1). Hence, it can be verified that this implies for both $i = 1, 2$, $U_P^i(k_E^*, k_L^*)$ is independent of θ . Consequently, we have:

Proposition 5 (Institutional Irrelevance). *Given Assumptions 1–4, Π concave, and constitutional choices made behind the veil, both principals do not care about the institutional structure. Any $\theta \in [0, 1]$ maximizes their joint ex ante expected payoffs.*

Thus, combining Propositions 4 and 5, we have established that if Π is concave, then principals want to ensure that legislators can hold each other to account, no more no less; they do not care about the allocation of bargaining power.¹² Taking the principle of delegation as fundamental — that the allocations of the cakes are decided by legislators in the legislative body — the best that the principals can do in terms of maximizing their joint ex ante expected payoffs (when Π is concave) is to design the legislature in such a way that legislators are able to monitor each others' actions in order to generate the required information to punish deviating legislators, and hence sustain intergenerational cooperation.

5.3. Renegotiation-Proofness and Ex Post Optimality. In this subsection, we address the following two different but related questions concerning the nature of the institutional and informational structures that maximize the principals' joint *ex post* expected payoffs. By “ex post”, it is meant *after the veil is lifted* (i.e., immediately after it is randomly determined which district's period-0 legislator is EARLY and which is LATE):

- Renegotiation-Proofness. Can the two principals mutually benefit from *amending* the informational and/or institutional structure of the legislative body, which was selected behind the veil in period -1 ?
- Ex Post Optimality. If the constitutional choices are determined immediately after the veil is lifted (and not behind the veil), then what values would be selected by the two principals?

Letting district i 's period-0 legislator be the one randomly selected to be EARLY and district j 's LATE (where $i, j = 1, 2$ with $i \neq j$), it follows that the principals' joint ex post expected payoffs with imperfect information and perfect information are respectively $U_E^i(1, 1) + U_L^j(1, 1)$ and $U_E^i(k_E^*, k_L^*) + U_L^j(k_E^*, k_L^*)$. The latter joint expected payoff is strictly greater than the former (since u_1 and u_2 are strictly concave), and hence Proposition 4 is robust:

Proposition 6. *Given Assumptions 1–4:*

- (i) *If the constitutional choices are made after the veil is lifted, then principals strictly prefer to have*

¹²We noted in footnote 5 that the U.S. Constitution (Article I, Section 5) requires record-keeping by each chamber. It is also relevant to observe that it does not prescribe other features of the legislative organization. In particular, also in Article I, Section 5, it states that “Each House may determine the Rules of its Proceedings, punish its Members for disorderly Behaviour, and with the Concurrence of two thirds, expel a Member.”

a legislative body structured in such a way that legislators possess perfect information (A5) about the history of play rather than imperfect information (A6).

(ii) If the constitutional choices are made behind the veil, then the selected informational structure (perfect information) is renegotiation-proof (i.e., the principals cannot mutually benefit from amending it once the veil is lifted).

What about the institutional structure? Is the ex ante, principal-optimal institutional structure renegotiation-proof? What is the ex post, principal-optimal institutional structure? For reasons discussed after the statement of (5), it is in general not possible to answer these questions without additional assumptions on Π , except that when Π is concave then a result parallel to Proposition 5 can be established here as well.

5.4. The Constraint of Imperfect Information. Suppose for some reason principals cannot create the mechanisms needed to provide legislators with perfect information about the history of play. Thus, they are restricted to selecting the value of θ only, and recognizing that play will then proceed according to the unique equilibrium of the baseline model with imperfect information (stated in Proposition 3). Which value of θ maximizes the principals' joint ex-ante payoffs? Will such a choice of the institutional structure be renegotiation-proof (after the veil is lifted)? Which value of θ would maximize their ex post joint payoffs? We now address these questions.

The expected payoff to principal i — i.e., the principal from district i ($i = 1, 2$) — behind the veil for any value of $\theta \in [0, 1]$ and given that play subsequently proceeds according to the unique equilibrium (Proposition 3) is

$$U_P^i(1, 1) = \frac{u_i(1) + u_i(0)}{2(1 - \delta_i)}.$$

Hence, it immediately follows that any value of $\theta \in [0, 1]$ maximizes the principals' joint ex ante expected payoffs. Thus, behind the veil, in period -1 , the principals are indifferent over the institutional structure of the legislature. They can either leave it to the legislators to select this (as occurred in the context of the US Senate), or themselves select a particular institutional structure (which we denote by θ^*). In the former case, Corollary 3 states the specific value of θ that would be selected by the legislators (allocating most, and under some circumstances all, power to LATE legislators). In the latter case, it then becomes of interest to know whether or not the selected choice of institutional structure θ^* is renegotiation-proof (amongst the principals), once the veil is lifted and it is known which district's period-0 legislator is allocated to be the EARLY type and which the LATE type. We now address that issue.

Suppose district 1's period-0 legislator is EARLY and district 2's period-0 legislator is LATE. Hence, from (2) and (3), the ex post expected payoffs to principals 1 and 2 for an

arbitrary θ are respectively

$$U_E^1(1, 1) = \theta \left[\frac{u_1(1) - u_1(0)}{1 + \delta_1} \right] + \frac{u_1(0) + \delta_1 u_1(1)}{1 - \delta_1^2} \quad \text{and}$$

$$U_L^2(1, 1) = \theta \left[\frac{u_2(0) - u_2(1)}{1 + \delta_2} \right] + \frac{u_2(1) + \delta_2 u_2(0)}{1 - \delta_2^2}.$$

Note that U_E^1 is strictly increasing in θ while U_L^2 is strictly decreasing in θ . So, ex-post, there is a conflict of interest between the principals over the institutional structure. Hence, the ex-ante selected value of θ , namely θ^* , is renegotiation-proof unless there exists an alternative value of θ which produces a strictly higher joint ex post expected payoff than θ^* does. Adding, we obtain that $U_E^1(1, 1) + U_L^2(1, 1) = A\theta + B$ where

$$A = \left[\frac{u_1(1) - u_1(0)}{1 + \delta_1} \right] - \left[\frac{u_2(1) - u_2(0)}{1 + \delta_2} \right].$$

If the principals have identical utility functions and identical discount factors, then $A = 0$ and hence the ex ante selected institutional structure, θ^* , is renegotiation-proof. But now suppose that the principals have identical utility functions, but different discount factors. This means that the sign of A equals the sign of $\delta_2 - \delta_1$, which is non-zero. The following proposition follows immediately:

Proposition 7. *Assume that principals are restricted to select the institutional structure only, accepting that legislators will face an imperfect information environment. Further, assume that principals have identical utility functions but different discount factors. Then, the ex ante optimal, renegotiation-proof institutional structure (which is also the ex post optimal one) is one in which all the bargaining power is allocated either to the EARLY or to the LATE type of legislator. It will be the former if the principal of the period-0 LATE legislator is relatively more patient than the principal of the period-0 EARLY legislator. Symmetrically, it will be the latter if the principal of the period-0 EARLY legislator is relatively more patient than the principal of the period-0 LATE legislator.*

This result has potential implications for constitutional reform during the later life of the legislature. Assuming that a principal's discount factor can change through time, as the subjective rate of time preference is influenced by factors such as wealth, income and market conditions, it is possible that at a later period it may be mutually beneficial for the principals to change the allocation of bargaining power.

6. MULTI-PERSON LEGISLATURES

6.1. Preliminaries. We have developed a baseline model of staggered-term principal-agent relationships in the previous four sections. This baseline model examines the simplest case — two agents who come up for renewal at different dates based on retrospective assessment of their performance by their respective principals. We derived equilibrium results

under two informational regimes, and then stepped back to ask how principals would structure interactions in the first instance.

Our results extend naturally to more general circumstances as we demonstrate in the present section. The baseline model partitioned agents in each period into two classes, EARLY and LATE. Now we consider the general case of M classes.¹³ In period 0 each agent is randomly assigned a class. The agent in class 1 serves for M periods before facing a renewal decision. The agent in class i serves $M - i + 1$ periods ($i = 2, 3, \dots, M - 1$). The agent in the last class serves a single period before facing renewal. Each renewed or newly appointed agent serves a full M -period term. If an agent is in class $i < M$ in period t , then he is in class $i + 1$ in period $t + 1$. An agent in class M in period t is in the final period of his M -period term; hence if he is reelected at the end of period t , then he is in class 1 in period $t + 1$.

As in the baseline model, agent renewal is based on performance during the periods of his service. The bargaining is, as before, over a unit-size cake each period. However, now we assume a decision rule which may range from simple majority rule up to unanimity, according to which a decision is taken to accept a proposal on the table.

Somewhat counterintuitively, we first consider four or more agents, showing how our main results from the baseline model extend to these cases. We separate out the three-agent case which we take up last. The reason for this treatment has to do with the issue of *pivotalness*. We had shown in the two-agent case with perfect information that an agent-optimal outcome can be sustained as a SPE through an optimal penal code (worst SPE). We develop a similar result in the case of four or more agents. To illustrate, consider four agents, perfect information, and simple majority rule (hence the support of three agents is required to accept a proposed division of the cake). Suppose behavior is on the agent-optimal path when a unilateral deviation occurs by agent i , moving play onto his worst punishment path, Q^i . Suppose i , suffering under this worst-SPE punishment, makes an especially attractive offer to the agent in his last period. This is tempting to that agent. But since — given the SPE concept which requires equilibrium actions to be immune to profitable *unilateral* deviations — all other agents are expected to continue playing their equilibrium actions associated with the path Q^i (which involves them rejecting the deviant offer), the tempted agent will not be pivotal, so succumbing to the temptation is not profitable. It is because of this non-pivotalness that establishing the existence of worst SPE punishment paths for the case of four or more agents is straightforward. The three-agent case requires separate treatment precisely because, on the punishment path Q^i , if i makes an attractive proposal to another agent, that agent *would* be pivotal under simple majority rule, and so he might well be tempted to defect, making the existence of worst SPE punishment paths a more challenging exercise. These distinctions will become clear in the development below.

¹³We assume one agent per class, but then comment on how our results extend to n_i agents in the i th class.

The structure and timing of our extended baseline model to multi-person legislatures is as in the baseline model, illustrated in Figure 1. In the next subsection, we study the extended baseline model with M principals and M agents, where each agent (legislator) represents a distinct principal (electoral district) and is in a class on his own; a full term of office is M periods. This perfectly symmetric set-up allows us to focus attention on the novel element of pivotalness (or otherwise) that arises when $M > 2$. The asymmetric set-up in which each principal i ($i = 1, 2, \dots, M$) can have n_i agents represent it in the legislative body and for them to be in potentially different classes raises further issues (such as whether or not principal i can distinguish the performances of his n_i agents) and as such we take up this more general case in subsection 6.3.

6.2. The Perfectly Symmetric Extended Framework. At the beginning of period 0, M legislators are elected, one from each of the separate electoral districts, and randomly allocated into the M classes. The probability that a legislator is reelected is $\Pi(x_1, x_2, \dots, x_M)$ (or simply $\Pi(\underline{x})$), where $x_t \in [0, 1]$ is the amount of cake that he delivered to his principal in period t of his most recent term in office ($t = 1, 2, \dots, M$).¹⁴ For the time being we make no assumptions on Π as these will differ according to whether $M \geq 4$ or $M = 3$. Let θ_i denote the probability with which a legislator from class i is selected to make an offer of the unit-size cake, where $\theta_i \in [0, 1]$ and $\sum_{i=1}^M \theta_i = 1$. An offer is a partition of the cake amongst the M legislators. If it is accepted by q or more legislators, then the cake is partitioned according to the proposed offer; otherwise no principal receives any cake in the period in question (where $q \geq (N/2) + 1$ if N is even, and $q \geq (N + 1)/2$ if N is odd).

As in the baseline model, each agent receives a fixed payoff of $b > 0$ for each term of office, and principal i 's utility from per-period consumption c is $u_i(c)$, where u_i is strictly increasing and strictly concave.

We first show that when legislators have perfect information about the history of play, then any agent-optimal outcome path can be sustained in a SPE for any class-contingent distribution of bargaining power. Given the symmetric and stationary structure of this extended baseline model, an agent-optimal outcome path is one in which in each period, the legislator who is randomly selected to propose, offers a partition \hat{x}^i if he belongs to class i ($i = 1, 2, \dots, M$), where these M vectors maximize a legislator's expected probability of reelection.¹⁵

Proposition 8 (Perfect Information in Multi-Person Legislatures). *Assume that the legislators have perfect information in the perfectly symmetric, multi-person legislative body with M legislators; and that Π is non-decreasing in each of its M arguments.*

(a) *If $M \geq 4$, then the agent-optimal outcome is a SPE.*

¹⁴Note that for a period-0 legislator put in class $i \geq 2$, $x_t = 0$ for $t = i - 1, \dots, 1$, by definition.

¹⁵If Π is concave, then (like in the two-agent case; cf. Corollary 1), an agent-optimal outcome is characterized by a single, class-independent, partition of the unit-size cake, denote it by \hat{x} , which maximizes the probability of reelection $\Pi(\hat{x})$.

(b) If $M = 3$, then the agent-optimal outcome is a SPE provided Π satisfies the following three inequalities: $\Pi(0, 0.5, 0.5) \geq \Pi(1, 0, 0)$, $\Pi(0.5, 0, 0.5) \geq \Pi(0.5, 1, 0)$ and

$$\frac{\Pi(0.5, 0.5, 0)}{\Pi(0.5, 0.5, 1)} \geq \frac{1 - \Pi(0.5, 0.5, 0.5)}{1 - \Pi(0, 0, 0)}.$$

Proof. In the appendix. □

The key aspect of the proof involves the construction of M worst SPE punishment paths, which extend those used in section 3 for the two-agent case. Notice that when $M \geq 4$, the existence of the agent-optimal SPE is obtained under extremely mild conditions on Π . We only require that Π be non-decreasing (which extends Assumption 1). But when $M = 3$, we require Π also to satisfy three inequalities (as stated in Proposition 8); the first two capture recency effects while the third has implications for the magnitudes of the reelection probabilities. The reason for this non-trivial difference has already been noted above in subsection 6.1 (arising due to the issue of pivotalness), and can be seen formally in the proof of this proposition. We note that in either case the agent-optimal outcome is being sustained as a SPE for *any* class-contingent distribution of bargaining power (just like in the two-agent case), i.e., for any feasible $\underline{\theta} = (\theta_1, \theta_2, \dots, \theta_M)$.

We now turn attention to the case when agents have imperfect information — Assumption 6(i) is extended by setting the critical value of $T > t + M$, and A6(ii) is extended by requiring that legislators in period $t + 1$ know the actions taken in the preceding M periods. In this imperfect information case, the results of section 4 extend straightforwardly for any $M \geq 3$ (it is no longer necessary to treat the $M = 3$ case distinctly):

Proposition 9 (Imperfect Information with Multi-Person Legislators). *If Π is nondecreasing, and Assumption 4 and the extended version of Assumption 6 hold, then the following strategy, adopted by all legislators, is the unique equilibrium. A legislator, irrespective of the class he is in, always accepts any offer and always demands the whole unit-size cake.*

Proof. In the appendix. □

Given Propositions 8 and 9, it is now straightforward to extend the arguments of section 5 and establish that those results of section 5 extend to the M -principal, M -agent case. The assumption that each principal has strictly concave preferences plays, as before, a key role. The first-best, or ideal solution for each principal is the same, and it involves splitting each period's unit-size equally amongst them (i.e., in each period, each principal is allocated a share $1/M$ of the cake). But this is not directly achievable since cake allocations are determined by the legislators. The main results are as follows.

The principal-optimal outcome behind the veil has the fundamental feature that principals require appropriate record-keeping mechanisms to ensure that legislators have perfect information. This feature is renegotiation-proof and ex post optimal. These two results extend Propositions 4 and 6. Of course the assumptions required for them differ according

to whether $M = 3$ or $M \geq 4$. Besides the assumptions required for Propositions 8 and 9 above, we require that principals' have strictly concave preferences. With respect to the principal-optimal institutional structure, just like for the two-principal, two-agent case, it is in general not possible to say much without additional assumptions. For example, if Π is concave, then the conclusion of Proposition 5 carries over.

6.3. Asymmetric Multi-Person Legislators. We now briefly discuss the more general dynamic multi-principal, multi-agent framework in which principal i ($i = 1, 2, \dots, M$) is represented in the legislature by n_i legislators (agents) who are potentially in different classes. One key novel issue that this raises is whether or not the principal can distinguish the per-period performances of her $n_i > 1$ agents. If all that the principal observes is the aggregate amount of cake delivered to her district in any period, but does not observe the contributions of each of its agents, then this complicates the analysis. It brings into play free-rider and team considerations. These issues are outside the scope of this paper, and so we adopt the assumption that the principal can observe exactly what each legislator delivered in each period, what political scientists would call a reliable "credit-claiming" technology. This assumption may be applicable in some contexts but not in others. With this assumption, the probability of reelection of any legislator can, once again, be conditioned on his performance during his term in office.

There are some other issues that this asymmetric framework raises that are absent from the analysis conducted thus far. For example, although legislators are assumed to be identical (have identical preferences and identical probability of reelection function Π), the sizes of the M classes would differ in each period, and this would mean that the outcome path that maximizes the legislators' joint payoffs would be asymmetric and relatively more complex than in the symmetric case. If the size of each class is the same, then the arguments and conclusions reported in subsection 6.2 carry over with minor modifications to the assumptions required for them to hold. But with unequal class sizes the arguments would have to be somewhat altered.

7. AN APPLICATION TO THE U.S. SENATE

The United States Senate, like many of the world's upper chambers, is a staggered-term legislature. Each state sends two senators to the Senate (by the selection of the state legislature until the beginning of the twentieth century and by popular election since then). Beginning with the ten initial states which selected senators in May 1789 (the three remaining of the original thirteen sending senators shortly thereafter), random assignment was employed to establish the staggered-term arrangement. Each of the ten states was randomly assigned to two of three groups, with the proviso that no state could be twice assigned to the same group. Then each of these groups was randomly assigned a class. A designated senator from each state in class 1 must vacate his seat after two years, a senator in class 2 after four years, and those in class 3 after six years. A vacated seat is then

filled by the state's (s)electorate, so that a third of the seats were filled every two years and each newly filled seat had a term of six years. The original twenty senators did not divide evenly into the three classes — one class was a senator short. When the next admitted state sent two senators, there would be a double-random drawing, the first to determine which of the two senators would be assigned to the “short” class, and the second to determine which class the other senator would join. Now there are two “short” classes, so that the next newest state's two senators would simply be randomized between these two classes. This assignment sequence would repeat as new states entered the union. In this manner a staggered-term legislative chamber was implemented in accord with constitutional provision, with each class approximately equal. Today, fifty states supply two senators each with approximately one-third facing reelection every two years.¹⁶

In the modern era politicians are held accountable in the electoral arena by their respective constituents. They are judged on criteria as diverse as their personal characteristics (some of which are observable — race, gender, charisma, wealth — and some of which are not — competence, honesty, intelligence), their ideology (as measured by their voting records, their speeches, their association with legislative products or special interests), their party, and their position in the legislature (seniority, committee assignments, chairmanships, party and institutional leadership posts). In our model, we abstract from these features, treating senators as essentially identical agents, distinguished only by their location in the electoral cycle and their success in the divide-the-cake distributive politics game.¹⁷

Despite abstracting from the empirical richness of a real legislative body like the US Senate, we believe we have captured some of its essential features, as the following stylized facts suggest:

- Aside from the various appropriations that are mandated by permanent legislation or that accompany new legislation, about 15% of each of the thirteen separate appropriations bills that is passed each fiscal year consists of expenditures devoted to *earmarks*. These are specific line items designating a particular project and an expenditure amount in a particular location (e.g., \$3.5 million to expand the visitors center in Yosemite National Park in California). An appropriations subcommittee (the Interior subcommittee, for example, has jurisdiction over expenditures in national parks) will be inundated with as many as 2500 or 3000 such requests from

¹⁶Upper chambers in most national and subnational legislatures employ four-year terms with half the legislators up for election every two years. However, India's upper house, the Rajya Sabha, is a staggered-term legislative body, with one-third of its members elected every two years. A term of office is six years. It differs from the contemporary US Senate in two ways — members are elected from Indian states by their respective state legislatures (like the pre-twentieth century US Senate), and the number of members from a state depends on state population (like most lower houses around the world).

¹⁷The factors mentioned above may well affect a senator's bargaining ability. We leave this as a direction for future research.

senators in a fiscal year, each request a claim on a small piece of the earmarks budget. (The roughly \$10 billion Interior appropriation will have approximately \$1.5 billion worth of earmarks for example). Thus, in the context of each appropriations subcommittee bill, there is an active “divide-the-earmarks-budget” game. From conversations with subcommittee staff (personal interviews by author), some clear criteria emerge by which winners and losers in a given fiscal year are determined: Priority is given first to the requests of senators on the subcommittee, then senators on the full committee, and finally senators “in cycle” (by which is meant senators in the last two years of their six year term). Since committee assignment is effectively a proxy for underlying demand for earmarks in a committee’s jurisdiction, those not on the (sub)committee, but in cycle, are effectively given special treatment. The practice of tilting toward those in cycle is a recognition, if not an institutionalization, of the *WHYDFML* logic.

- The allocation of earmark funding is consistent with the equilibrium outcomes we would expect in a world with recency effects. That politicians believe they are subject to retrospective assessment, and that this assessment is indeed biased toward recent performance, is an established stylized fact in the legislative scholarship field. It is exemplified by the (probably apocryphal) story of the Kentucky senator who is shocked to learn that one of his reliable constituents might not support him in the upcoming election. In response to the surprise expressed by the senator, the constituent concedes, “Sure you got my brother that agricultural subsidy. And you got my grandson admitted to the US Military Academy. And you even got my factory a small military contract.” But, he goes on to ask, “what have you done for me lately?”
- Because earmarks are thought to be wasteful, often not even passing a simple benefit-cost test, there are always “reformers” in the legislature who seek to eliminate them from the appropriations process. Indeed, most of the professional staffs of appropriations subcommittees dislike earmarks for this reason (plus the fact that they are a time-intensive nuisance). In the 1970s a bold attempt was made by Senator James Buckley of New York. When an appropriations bill came to the floor of the Senate, Buckley introduced amendments to strike 50 pork-barrel projects, one from each state. Although these amendments were intended to symbolize the profligacy and waste of earmarking, only the amendment striking the project from New York passed, causing the *New York Times* to ask “Why is New York State handicapped by being the only state with only one U.S. senator?” We would characterize Buckley’s attempt as an off-the-equilibrium path deviation that elicited a punishment.

These stylized facts portray a U.S. Senate engaging in distributive politics along the lines we have described in our theoretical development — overlapping terms, retrospective assessment, recency bias, and punishment of those seeking to overturn equilibrium practices. In addition to these facts, we would call attention to the informational features governing

Senate deliberation which, we claim, are consistent with our analysis. The Constitution requires, first, the keeping and publishing of a journal, a daily record of Senate proceedings that records bills and motions introduced, the speeches and other deliberative acts of the senators, and finally their votes. Increasingly, information about activities off the floor of the Senate, in its committee and subcommittee rooms, is part of the official record. Second, and perhaps more remarkable, the Constitution permits senators to make their own procedural arrangements. Thus, the constitutional founders imposed a record-keeping requirement to enhance the availability and reliability of the historical record, but imposed no other constraints on the operation of the legislature. That is, they encouraged transparency among agents but expressed little interest in fine-tuning procedural parameters (like θ).

Having offered stylized evidence of the relevance of our approach to real-world organizations, we conclude this section by briefly illustrating the way in which our theoretical development applies to the U.S. Senate. The present U.S. Senate has three classes (I, II, and III), with each of its members going through three stages, called EARLY, MIDDLE, and LATE. Respective class sizes are a permutation of $\{33, 33, 34\}$. We will assume here that $n_I = 34$ and $n_{II} = n_{III} = 33$. Each principal — say, the fifty state median voters — has two agents who are members of $\{I, II\}$, $\{I, III\}$, or $\{II, III\}$. We assume that each principal can distinguish which of her agents is responsible for which slices of cake.¹⁸ Thus the utility to the principal at t is $u(x_i^t + x_j^t)$ where $i, j \in \{I, II, III\}$ are the classes of her agents. Since u is concave, the first-best outcome involves for each principal to obtain $1/50$ of the cake each period. This could be produced by each agent obtaining $1/100$ of the cake each period. But $\Pi(1/100, 1/100, 1/100)$ will not in general maximize the probability of reelection for an agent judged retrospectively by a recency-biased principal. Indeed, if $(x_I^*, x_{II}^*, x_{III}^*)$ is the agent-optimal outcome, some forms of recency bias will imply $x_I^* \leq x_{II}^* \leq x_{III}^*$. Assuming that transparency prevails so that agent information is rich, then $(x_I^*, x_{II}^*, x_{III}^*)$ will be a SPE and we should observe “backloading” of pork-barrel projects over the three periods of an agent’s term. Since there are two agents per state (principal), this means that states will receive a time-stream of payoffs equal to (a permutation of) $x_I^* + x_{II}^*$, $x_I^* + x_{III}^*$, and $x_{II}^* + x_{III}^*$ — the former a “lean” period and the latter two “fat” periods.¹⁹

¹⁸If information is perfect and transparency means not only that agents can condition on one another’s past history but also that the principal can determine which agent did what at any time t , then we may be warranted in assuming the principal’s ability to differentiate. In real life, the credit-claiming technology by which agents attempt to convince their principal that they deserve credit is the *press release*. When the facts about earmarks are made public by an Appropriations subcommittee, one often observes a race between a state’s two senators to get off press releases to hometown media outlets. In these they claim credit for various of the state’s projects funded in the appropriations bill.

¹⁹Interestingly, since members of the lower chamber face election every period (that is, every two years), a state delegation of lower-chamber members will be quite pleased in relatively fat years and quite distraught in the lean ones. This is a source of bicameral tension not commonly noted in the literature on congressional politics.

8. CONCLUSIONS

9. EXTENSIONS

10. DISCUSSION AND RELATED LITERATURE

Our approach departs from several conventions in the literature, so we provide further elaboration and justification on three issues — probabilistic election, retrospective assessment, and recency bias — while at the same time relating our approach to some of the existing literature.

10.1. Probabilistic election. In our approach agent reappointment is probabilistic. Exemplars of the agency-theoretic approach to legislative organization — Barro (1973), Ferejohn (1986, 1999) — utilize uncertainty in a very different way. For them, an uncertain state of the world affects the productivity of an agent’s effort for his principal’s welfare. Thus, if x measures agent effort and ξ is the state (with “good” states having larger values), then the payoff to the principal is weakly monotonic in $x\xi$. The state (ξ) is unknown *ex ante* to both principal and agent, but comes to be known to the agent by the time he must make his (costly) effort decision (x). Although neither the state nor agent effort is observable to the principal (voter), she nevertheless makes a *deterministic* decision about whether to renew the agent’s contract based on past performance. Employing a *cut-off rule*, she retains the agent if $x\xi$ exceeds a threshold (optimally determined, and committed to, *ex ante* in light of the commonly known probability distribution over ξ), and replaces him otherwise. She cannot distinguish the agent’s effort from the state of the world because these elements are pooled into an undifferentiated performance result. The optimal rule set by the principal is deterministic: reappoint if performance exceeds a cut-off value; replace otherwise. Therefore, at the time the agent chooses his effort, he knows with certainty whether it is worthwhile for him to bear the cost of enough effort so that, pooled with the state of the world (then known to him), the performance it yields will exceed the principal’s cut-off requirement. If he concludes that it is not worthwhile, then he will do nothing and not be reappointed. Uncertainty in these models enters only in the determination of the principal’s optimal cut-off rule. Once the cut-off threshold is set, the principal’s choice on agent retention is deterministic and this is fully appreciated by the agent at the time he selects his effort level.

In our model, in contrast, the legislative agent is not able to resolve uncertainty surrounding his reappointment prospects entirely. At the end of his two-period term, his reelection depends upon past performance but in no completely discernable way (apart from weak monotonicity and recency). Many factors confound the relationship between performance and reappointment. For example, voters may participate probabilistically. The choices of those who do participate may be stochastic. The characteristics of an opponent (unmodeled in most approaches in this literature) may not be known *ex ante*. Put differently, the agent does not know the identity of the decisive voter who will determine

a renewal decision. There are, in short, myriad reasons why an agent enters his reappointment phase with some randomness in his fate unresolved. In our model the best he can do is take actions that increase the likelihood of reappointment.

Thus, we characterize the agent's optimization problem in terms of a probability-of-reelection function, Π . We have made relatively weak assumptions about this function (A1-A3). A1, it should be noted, is compatible with the notion of performance thresholds so that the accumulation of cake beyond some fixed amount need not strictly improve reelection probabilities. In the limit, when $\Pi = 1$ for performance exceeding an upper threshold, our formulation approaches the spirit of cut-off formulations. Indeed, cut-off formulations are special cases of our probabilistic formulation.²⁰ As is evident in the development in earlier sections, many existence results depend on nothing stronger than the three assumptions we make. In order to provide substantive characterizations, however, we often must restrict ourselves to more specific classes of Π , e.g. a probability of reelection function concave in its arguments.

We have implicitly assumed, by not subscripting Π , that agents share identical probability-of-reelection functions. In effect, this means that principals draw their respective reappointment choices from the same probability distribution. This is unrealistically restrictive in contexts in which wealth and tastes are likely to vary across districts. We have acknowledged this in indexing the utility function and discount parameter of principals. This and the preceding observation emphasize the need to elaborate more detailed micro-foundations for Π .

10.2. Retrospective assessment. As in most of the literature, our approach depends upon retrospective assessment of agents by principals.²¹ Although this contrasts with Downsian approaches to elections in which the promise of future performance is the coin of the realm (prospective voting), retrospective voting models are now the conventional approach to this subject. There is both an empirical and a theoretical literature on elections in which retrospective assessment is taken as unexceptional. In addition to Barro and Ferejohn mentioned above, theoretical contributions by Austen-Smith and Banks (1989), Banks and Sundaram (1998), Dixit (1995), and Maskin and Tirole (2004) adopt, without comment,

²⁰A slightly different kind of cut-off rule is the *threshold contract* described in Gersbach and Liessem (2005). A threshold contract stipulates the minimum performance level an agent must attain in order to be eligible for renewal. This is also a special case of our probabilistic formulation.

²¹It should also be noted that, along with the rest of the literature, we stipulate *representative* democracy as the institutional arrangement. That is, principals cannot provide cake for themselves, nor can they command it of their agents. Moreover, we take as axiomatic that policy-making in (democratic) legislative bodies has the fundamental characteristic that no agent has the power simply to grab whatever share of the economic surplus he likes. Policies enacted in legislatures require the support (approval) of at least one legislator besides the legislator who proposes the policy (in a legislature with two or more members). The maximal power of a single legislator is the authority to be the sole proposer of policy. This arrangement is to be contrasted, for example, with the model in Dixit, Grossman and Gul (2000) in which in each period each player, if he is the proposer, unilaterally decides how much of the economic surplus to grab for himself (a bit like a dictator) in light of the prospect that he may not be the proposer in a future period.

a retrospective point of view as axiomatic. Theirs are primarily moral hazard frameworks in which retrospective assessments by principals provide ex ante incentives for agents to perform.²² Fearon (1999) in contrast raises selection issues, suggesting that elections are chiefly about selecting agent qualities — for example, preferences concordant with those of principals or immutable qualities like competence, ability, and honesty. Retrospective assessment for Fearon involves the acquisition of information by principals about agent type, allowing principals to update beliefs and reappoint those agents more likely than potential replacements to possess appropriate attributes. Our model is more in the moral hazard tradition in which the reelection probability reflects incentives for agents to perform in a manner desired by principals. Departures from the wishes of principals may arise, and this is one of the main points of our analysis, because of institutional features of elections — particularly staggered terms that provide agents with time-dependent considerations at odds with the objectives of principals.

There is a large empirical literature in political science on retrospective assessment. The *locus classicus* of this literature is Fiorina (1981). Kramer (1971) was one of the earliest to demonstrate a retrospective effect, showing for the US that stronger economic performance in the year of an election translated into a greater share of the national vote for the congressional candidates of the incumbent president's party. Tufte (1978) and others experiment with weighted averages of multiple years worth of past economic performance to operationalize a retrospective effect. Kiewiet and Udall (1998), on the basis of new data, re-estimate a number of different specifications, demonstrating a very robust retrospective effect. As in the theoretical literature, there is very little attention paid to justifying or explaining retrospective assessments; researchers are content to demonstrate the existence of such effects. As Kahneman, Wakker, and Sarin (1997, 389) observe, "...retrospective evaluation of outcomes is a cognitive activity in which people routinely engage, much as they engage in grammatical speech or in deductive reasoning."

Our approach fits right into this literature. Our results are most easily explained in a moral hazard context, but a selection interpretation also is consistent with our approach. In the former interpretation Π incentivizes an agent to deliver cake in light of his principal's intertemporal preferences; it represents the likelihood of agent reward conditional on his performance. In the latter interpretation Π constitutes the principal's belief that the agent type is best suited to pursuing her interests; past agent performance provides the principal the means by which to formulate her beliefs.

10.3. Recency effects. Recency effects in the context of retrospective assessment are commonly noted in the literature. Levitt (1996), for example, finds that "...[US] senators give twice as much weight to (median) constituent preferences in the year before elections as

²²A recent model of retrospective voting, formalizing earlier arguments of the American political scientist V.O. Key, may be found in Bendor, Kumar, and Siegel (2005).

compared to four years or more before elections.” We assume a very weak version of recency bias in Assumption 2. (We discuss other versions shortly.) It would be valuable to have some microfoundations for this.

In our model principals with concave preferences for the consumption of cake *ex ante* prefer, subject to discounting, a relatively smooth distribution over the two periods of an agent’s term. If, *ex post*, a principal’s actions display a recency bias, as is evident in the empirical literature, then it is conceivable there is some incentive rationale for her acting this way. Perhaps a principal elicits better responses from an agent when she acts as if she gives greater weight to more recent performance relative to more distant performance. Our results cast doubt on this belief since (if Π is concave) a recency bias will encourage agents to prefer extraordinary performance (getting more cake) in later periods, even if it means foregoing performance success earlier. This attenuates the smooth intertemporal provision of cake, contrary to what is preferred by principals.

An alternative consideration, suggested to one of the authors by David Laibson (personal communication), emphasizes information extraction as the foundation for recency bias (in the spirit of selection models of principal-agent relations). Recent information is viewed, rightly or wrongly, as more informative about agent type. Related to this, recent information may be more easily remembered (whether more reliably informative or not) or may be given more prominence by the media. Sarafidis (2004) presents a model of memory (of a principal) from which it may be deduced that particular patterns of information transmission from agent to principal — sometimes reflecting a recency effect, sometimes not — are optimal for the agent.

Perhaps the most famous psychological experiments on recency bias are Kahneman’s colonoscopy studies (Kahneman, Fredrickson, Schreiber, and Redelmeier 1993, Redelmeier and Kahneman 1996, Redelmeier, Katz, and Kahneman 2003). There it was discovered that patient memories of unpleasant medical procedures are highly correlated with peak periods of unpleasantness on the one hand, and the experience in the most recent periods on the other. In particular, they found that a number of periods of relatively less unpleasant experience tacked on to the end of the procedure reduced patient retrospective assessment of the unpleasantness, even if these additional less unpleasant experiences actually lengthened the time of the procedure.

Thus, while we do not have a good micro-theoretical rationale for recency bias, we have several useful conjectures and considerable empirical and experimental evidence for the bias. One of the interesting effects of recency bias is that discounting by voter-principals is effectively the reverse of ordinary economic discounting. At the outset of an agent’s multiple periods of incumbency, he can anticipate that principals, at the time of a renewal

decision, will discount the agent's first period performance at least as heavily as the second, the second at least as heavily as the third, and so on, discounting the most recent performance the least.²³

Finally, we note that there are a variety of ways to specify a recency effect theoretically. Assumption 2 is, as noted, a very weak requirement of a recency bias. Another version, mentioned in the discussion of Corollary 3, has $\Pi(0, 1) > \Pi(0.5, 0.5)$ — in effect, moving half the cake in a smooth distribution from the early period to the late increases the renewal likelihood. Perhaps the strongest version would be one which requires that $\Pi(x_E - \epsilon, x_L + \epsilon) > \Pi(x_E, x_L)$ for any (x_E, x_L) and any $0 < \epsilon < x_E$. That is, a strong version of recency bias is one in which the probability of reappointment is strictly increasing in *any* reallocation of cake from an early period to a late period. Clearly, repeated application of this definition yields Assumption 2. It turns out that this requirement imposes strong restrictions on Π .²⁴

APPENDIX

PROOF OF LEMMA 1. To establish that the proposed strategies comprise a SPE, we need only to check that no legislator can benefit from any one-shot, unilateral deviation from path Q^i ($i = 1, 2$). It is straightforward to see that any one-shot, unilateral deviation from the path Q^i by any legislator from district i does not increase his expected payoff.

Now consider a one-shot, unilateral deviation by a legislator from district j ($j \neq i$) from path Q^i . If, in any period, he either demands less than the whole cake, or rejects the offer of the whole cake, then he is worse off in that period and there is no effect on his continuation expected payoff. We now come to the final, but critical deviation: Suppose, in some period, a district j legislator considers, while on path Q^i , accepting an offer which gives him a share $x < 1$. His expected payoffs from accepting (i.e., deviating) and from rejecting (i.e., conforming) such an offer depend on whether in the period in question he is EARLY or LATE. If he is EARLY, then he will reject the offer if and only if his payoff from rejecting is greater than or equal to his payoff from accepting, i.e.,

$$b + \Pi(0, 1) \left[\frac{b}{1 - \Pi(1, 1)} \right] \geq b + \Pi(x, 0) \left[\frac{b}{1 - \Pi(0, 0)} \right].$$

Assumptions 1 and 2 imply that this inequality is satisfied. If he is LATE in period $t \geq 1$, then he will reject the offer if and only if

$$b + \Pi(1, 0) \left[\frac{b}{1 - \Pi(1, 1)} \right] \geq b + \Pi(1, x) \left[\frac{b}{1 - \Pi(0, 0)} \right].$$

²³This suggests that the combination of ordinary economic discounting and recency bias produces inconsistent time preferences: principals have a bias for the present over the past. This parallels the hyperbolic discounting literature (Laibson 1997) in which players have dynamically inconsistent time preferences because of self-control problems: a principal has a bias for the present over the future.

²⁴To illustrate this point suppose that Π is an increasing function of $\beta v(x_E) + v(x_L)$, where $\beta < 1$, and v is differentiable and increasing. The strong version of recency bias holds if and only if for any (x_E, x_L) , $v'(x_L) > \beta v'(x_E)$. This, in turn, holds only if $\min_x v'(x) > \beta \max_x v'(x)$. But this means that either v is linear, or if non-linear then β is sufficiently small (i.e., the premium on cake obtained when LATE is sufficiently large).

This inequality is satisfied for any possible $x < 1$ if and only if

$$\frac{\Pi(1, 0)}{\Pi(1, 1)} \geq \frac{1 - \Pi(1, 1)}{1 - \Pi(0, 0)}.$$

Assumptions 1 and 3 imply that this inequality is satisfied. A3 implies (making use of A1) that $\Pi(1, 1)^2 - \Pi(0, 0)^2 \geq \Pi(1, 1) - \Pi(0, 0)$. This, in turn, implies that $\Pi(0, 0)/\Pi(1, 1) \geq [1 - \Pi(1, 1)]/[1 - \Pi(0, 0)]$. The desired conclusion follows since (by A1) $\Pi(1, 0) \geq \Pi(0, 0)$.²⁵

PROOF OF PROPOSITION 1. Consider the strategies implicitly defined by the following three paths and three transition rules. The initial path is an agent-optimal path $Q(k_E^*, k_L^*)$. The other two paths are Q^1 and Q^2 , and two of the transition rules are T^1 and T^2 , all of which are associated with the worst SPE (cf. Lemma 1). The third transition rule, denoted by $T(k_E^*, k_L^*)$, concerns transitions from $Q(k_E^*, k_L^*)$: If, when on path $Q(k_E^*, k_L^*)$, a legislator from district i ($i = 1, 2$) either makes or accepts a deviant offer, then immediately (before the next decision node) play moves on to path Q^i .²⁶

Given Lemma 1, the proposition follows once we establish that no legislator can benefit by a one-shot, unilateral deviation from the agent-optimal path $Q(k_E^*, k_L^*)$. The following four steps, which establish that, can be easily verified. First, given Assumption 1, each legislator's payoff from accepting the demand associated with the path $Q(k_E^*, k_L^*)$ is no less than rejecting it. Second, given Assumption 1, each legislator cannot profit from making a deviant demand (either when he is EARLY or when he is LATE). Third, given Assumptions 1 and 2, when a deviant offer is received by EARLY, he cannot profit from accepting it. Fourth, the same is true when a deviant offer is received by LATE if and only if

$$\frac{\Pi(z, 0)}{\Pi(z, 1)} \geq \frac{1 - \Pi(1, 1)}{1 - \Pi(0, 0)},$$

where $z \in [0, 1]$ denotes the amount of cake he obtained in the preceding period, when he was EARLY. Assumptions 1 and 3 imply that this inequality is satisfied. As shown in the proof of Lemma 1, A3 implies (making use of A1) that $\Pi(0, 0)/\Pi(1, 1) \geq [1 - \Pi(1, 1)]/[1 - \Pi(0, 0)]$. The desired conclusion follows since (by A1) $\Pi(z, 0)/\Pi(z, 1) \geq \Pi(0, 0)/\Pi(1, 1)$.

PROOF OF COROLLARY 1. Fix an arbitrary pair $(k_E, k_L) \in [0, 1] \times [0, 1]$. Rewrite $\Omega(k_E, k_L)$ as follows:

$$\Omega(k_E, k_L) = \theta[\theta\Pi(k_E, 1 - k_E) + (1 - \theta)\Pi(k_E, k_L)] + (1 - \theta)[\theta\Pi(1 - k_L, 1 - k_E) + (1 - \theta)\Pi(1 - k_L, k_L)].$$

Since (by the hypothesis of the corollary) Π is concave, it follows that

$$\Omega(k_E, k_L) \leq \theta\Pi(k_E, \theta(1 - k_E) + (1 - \theta)k_L) + (1 - \theta)\Pi(1 - k_L, \theta(1 - k_E) + (1 - \theta)k_L).$$

Using concavity again, it follows that $\Omega(k_E, k_L) \leq \Pi(\hat{x}, 1 - \hat{x})$, where $\hat{x} = \theta k_E + (1 - \theta)(1 - k_L)$. We have thus established that for any $(k_E, k_L) \in [0, 1] \times [0, 1]$ there exists an $x \in [0, 1]$ such that

²⁵If he is LATE in period 0, then he will reject the offer if and only if $\Pi(0, 0)/\Pi(0, 1) \geq [1 - \Pi(1, 1)]/[1 - \Pi(0, 0)]$. This is slightly different, but it is satisfied by essentially the same argument.

²⁶A deviant offer made by EARLY is a demand $x \neq k_E^*$; and a deviant offer made by LATE is a demand $x \neq k_L^*$.

$\Pi(\mathbf{x}, 1 - \mathbf{x}) \geq \Omega(k_E, k_L)$. Hence, it immediately follows that

$$\max_{0 \leq x \leq 1} \Pi(\mathbf{x}, 1 - \mathbf{x}) \geq \max_{(k_E, k_L) \in [0,1] \times [0,1]} \Omega(k_E, k_L),$$

which establishes the corollary.

PROOF OF COROLLARY 2. We argue by contradiction. Thus suppose that $x^* \geq 0.5$. Since Π is concave,

$$\Pi(0.5, 0.5) \geq \left[\frac{1}{2x^*} \right] \Pi(x^*, 1 - x^*) + \left[1 - \frac{1}{2x^*} \right] \Pi(0, 1).$$

Since (by definition) $\Pi(x^*, 1 - x^*) \geq \Pi(0.5, 0.5)$, and (by the hypothesis of the corollary) $\Pi(0, 1) > \Pi(0.5, 0.5)$, it follows that $\Pi(0.5, 0.5) > \Pi(0.5, 0.5)$, a contradiction.

PROOF OF PROPOSITION 2. Fix an arbitrary pure-strategy equilibrium, and fix an arbitrary period $t \geq 1$. We will show that the equilibrium actions in period t are conditioned on at most z_t (the amount of cake obtained by the period- t LATE legislator in period $t - 1$), but on no other bits of observed history, which then establishes the proposition. The argument involves induction.

First, note that A6(i) implies that there exists a $T \geq t + 2$ such that the equilibrium actions in any period from and including period T onwards cannot be conditioned on the actions taken in any period before and including $t - 1$. Second, we establish the following inductive step:

Fix an arbitrary period s , where $s \geq t + 1$. If the equilibrium actions in any period from and including period $s + 1$ onwards are not conditioned on the actions taken in any period before and including period $t - 1$, then the same is true of the equilibrium actions in period s .

Proof of inductive step. Since $s \geq t + 1$, none of the actions in any period before and including period $t - 1$ directly affects the payoffs of any legislator in period s . Given this and the hypothesis of the inductive step, it follows that the equilibrium expected payoff to a legislator from period s onwards does not depend on the actions in any period before and including $t - 1$. Let h_{t-1} and h'_{t-1} denote two different histories till the end of period $t - 1$ that are observable to an arbitrary legislator in period s . Furthermore, let h denote a history of actions observed by the arbitrary legislator between and including periods t and $s - 1$. Hence, two different observed histories at the beginning of period s are (h_{t-1}, h) and (h'_{t-1}, h) . The equilibrium expected payoffs to this arbitrary legislator from period s onwards will be the same following either observed history (for any set of period s actions and given the equilibrium pure-strategy). Hence, given Assumption 4, the legislator's equilibrium actions in period s following these two observed histories are the same. This completes the proof of the inductive step.

Hence, it now follows from the principle of mathematical induction that the equilibrium actions in any period from and including period $t + 1$ are not conditioned on the actions taken in any period before and including period $t - 1$. The desired conclusion follows immediately.

PROOF OF PROPOSITION 3. We first show that the strategy described in the proposition, when adopted by all legislators, is the unique stationary Markov equilibrium, and then we establish the non-existence of non-stationary Markov equilibria. This then establishes the proposition.

A stationary Markov pure strategy for a legislator is made up of two numbers, k_E and k_L , and two functions, f_E and f_L : k_i denotes the legislator's demand when he is type i , and $f_i : [0, 1] \rightarrow \{\text{"Accept"}, \text{"Reject"}\}$ such that $f_i(x)$ denotes whether the legislator accepts or rejects the demand x when he is type i , where $i = E, L$ (E stands for EARLY and L stands for LATE). Fix an arbitrary stationary Markov equilibrium, and let W denote the expected payoff associated with this equilibrium to the legislator when he is EARLY at the beginning of any period (before the proposer is randomly selected). We first establish the following result:

Claim 1. If the probability of reelection Π satisfies Assumption 1, then a legislator accepts any offer when EARLY and any offer when LATE.

Proof of Claim 1. To establish this claim, we need to show that the legislator, when EARLY and when LATE, respectively, accepts any demand $x \in [0, 1]$ made by the proposer. It follows from the *One-Shot Deviation Principle* that the legislator, when EARLY, accepts a demand $x \in [0, 1]$ if and only if $H_E(x) \geq H_E(1)$, where $H_E(x) = b + [\theta\Pi(1 - x, y_E) + (1 - \theta)\Pi(1 - x, y_L)]W$, where

$$y_E = \begin{cases} 1 - k_E & \text{if } f_L(k_E) = \text{"Accept"} \\ 0 & \text{if } f_L(k_E) = \text{"Reject"} \end{cases} \quad \text{and} \quad y_L = \begin{cases} k_L & \text{if } f_E(k_L) = \text{"Accept"} \\ 0 & \text{if } f_E(k_L) = \text{"Reject"} \end{cases}.$$

Assumption 1 implies that for any $x \in [0, 1]$, $H_E(x) \geq H_E(1)$. Hence, this means that $f_E(x) = \text{"Accept"}$ for all $x \in [0, 1]$. A legislator, when LATE, accepts an offer $x \in [0, 1]$ if and only if $H_L(x) \geq H_L(1)$, where $H_L(x) = b + \Pi(z, 1 - x)W$, and z is the amount of cake received by LATE in the previous period. Assumption 1 implies that for any $x \in [0, 1]$, $H_L(x) \geq H_L(1)$. Hence, this means that $f_L(x) = \text{"Accept"}$ for all $x \in [0, 1]$. This completes the proof of Claim 1.

Given Claim 1, it follows from the One-Shot Deviation Principle that the pair (k_E, k_L) satisfy the following conditions:

$$k_E \in \arg \max_{x \in [0, 1]} \left[b + [\theta\Pi(x, 1 - k_E) + (1 - \theta)\Pi(x, k_L)]W \right] \quad \text{and} \quad k_L \in \arg \max_{x \in [0, 1]} \left[b + \Pi(z, x)W \right],$$

where z is the amount of cake earned by the LATE legislator a period earlier. That is,

$$k_E \in \arg \max_{x \in [0, 1]} \left[\theta\Pi(x, 1 - k_E) + (1 - \theta)\Pi(x, k_L) \right] \quad \text{and} \quad k_L \in \arg \max_{x \in [0, 1]} \left[\Pi(z, x) \right].$$

Assumptions 1 and 4(b) thus imply that $(k_E, k_L) = (1, 1)$ is the unique solution.

PROOF OF COROLLARY 3. Maximizing $P(1, 1)$ over θ is equivalent to maximizing $\Omega(1, 1)$ over θ . Differentiating $\Omega(1, 1)$ w.r.t. θ , we obtain:

$$\frac{\partial \Omega(1, 1)}{\partial \theta} = 2\alpha\theta + \beta, \quad \text{where}$$

$$\alpha = \Pi(1, 0) + \Pi(0, 1) - \Pi(1, 1) - \Pi(0, 0) \quad \text{and} \quad \beta = \Pi(1, 1) + \Pi(0, 0) - 2\Pi(0, 1).$$

First, we consider the case in which $\beta > 0$. Since $\alpha + \beta = \Pi(1, 0) - \Pi(0, 1)$ is strictly negative (by Assumption 2), it follows that $\alpha < 0$. Now note that $2\alpha + \beta = -\beta + 2[\Pi(1, 0) - \Pi(0, 1)]$, which is strictly negative (by Assumption 2 and since, by hypothesis, $\beta > 0$). Finally note that since $\alpha < 0$, it follows that $\Omega(1, 1)$ is strictly concave in θ . Putting these results together, it follows that $\Omega(1, 1)$ is increasing in θ over the interval $[0, \hat{\theta})$, decreasing over the interval $(\hat{\theta}, 1]$ and achieves a maximum

at $\theta = \hat{\theta}$, where $\hat{\theta} = -\beta/2\alpha$. The second part of the corollary follows immediately since $\hat{\theta} \in (0, 0.5)$, and since $\hat{\theta}$ is increasing in $\Pi(1, 0)$ and decreasing in $\Pi(0, 1)$.

Now consider the case in which $\beta \leq 0$. We break our argument into three cases. First suppose that $\alpha < 0$. This immediately implies that $\Omega(1, 1)$ is maximized at $\theta = 0$ (since $\Omega(1, 1)$ is in this case decreasing and strictly concave in θ). Now suppose that $0 < \alpha < -\beta/2$. In this case $\Omega(1, 1)$ is also maximized at $\theta = 0$ (since $\Omega(1, 1)$ in this case is decreasing but strictly convex in θ). Finally suppose that $\alpha > -\beta/2$. In this case $\Omega(1, 1)$ is strictly convex in θ , decreasing in θ over the interval $(0, \tilde{\theta})$, achieves a minimum at $\theta = \tilde{\theta}$ and is increasing over the interval $(\tilde{\theta}, 1]$, where $\tilde{\theta} = -\beta/2\alpha$. This implies that $\Omega(1, 1)$ achieves a maximum either at $\theta = 0$ or at $\theta = 1$. The desired conclusion follows immediately, since (by Assumption 2) $\Omega(1, 1)$ takes a higher value at $\theta = 0$ than at $\theta = 1$.

PROOF OF PROPOSITION 8. Given the symmetric and stationary structure of the extended baseline model, the legislators' joint expected payoffs are maximized with an outcome path in which in each period the partition of the unit-size cake is contingent on at most the class of the legislator who is randomly selected to make an offer. Fix such an arbitrary outcome path, $Q(x)$, where $x = (\underline{x}^i)_{i=1}^M$ are M partitions of the unit size cake: in each period, with probability θ_i the legislator from class i ($i = 1, 2, \dots, M$), who is selected to propose, offers \underline{x}^i , which is accepted by all the other legislators. The expected payoff to an arbitrary legislator at the beginning of his M -period term in office from this outcome path is $P(x) = b/[1 - \Omega(x)]$, where

$$\Omega(x) = \sum_{i_1=1}^M \sum_{i_2=1}^M \dots \sum_{i_M=1}^M [\theta_{i_1} \theta_{i_2} \dots \theta_{i_M}] \Pi(x_1^{i_1}, x_2^{i_2}, \dots, x_M^{i_M})$$

where for each $t \in \{1, 2, \dots, M\}$, the symbol $i_t \in \{1, 2, \dots, M\}$ denotes the class of the legislator who is selected to propose in the t -th period of the legislator's term in office. An agent-optimal outcome path, $Q(\hat{x})$, is characterized by the M , $(M - 1)$ -dimensional vector \hat{x} which maximizes $P(x)$ (or equivalently $\Omega(x)$). We now proceed to establish (by construction) the existence of a SPE that sustains the agent-optimal outcome path. As in the two-agent case (and following Abreu (1988)), we construct a SPE defined by an initial path (which is the agent-optimal outcome path $Q(\hat{x})$), M district-contingent worst punishment paths, and $M + 1$ transition rules (one for each of the $M + 1$ paths):

Path Q^i ($i = 1, 2, \dots, M$): Each and every legislator from district i always (i.e., in any period, for any history, and whatever his class) offers y_i and accepts any offer, where y_i is the partition of the unit-size cake in which he gets no cake, and every other legislator gets a share $1/(M - 1)$. Every other legislator always makes the offer y_i and only accepts the offer y_i .

Transition rule T^i ($i = 1, 2, \dots, M$): If, when play is on path Q^i , a legislator from district j either makes a deviant offer (i.e., does not offer y_j) or accepts a deviant offer (i.e., accepts an offer that differs from y_j), then immediately play switches to path Q^j . For any other deviation from path Q^i , play remains on this path.

Transition rule \hat{T} : If, when on path $Q(\hat{x})$, a legislator from district i ($i = 1, 2, \dots, M$) either makes or accepts a deviant offer, then immediately play moves on to path Q^i .

We first establish that no legislator can make a profitable unilateral, one-shot deviation from path Q^i ($i = 1, 2, \dots, M$). It is straightforward to see that any unilateral, one-shot deviation from Q^i by a legislator from district i does not increase his expected payoff. Now consider a one-shot, unilateral deviation by a legislator from district j ($j \neq i$) from path Q^i . If, in any period, he either makes a deviant offer, or rejects the offer y_i , then he is worse off in that period and his continuation expected payoff decreases (as play will have moved onto path Q^j).

We now come to the final, but critical deviation: Suppose, in some period, a district j legislator considers, while on path Q^i , accepting a deviant offer $y \neq y_i$. We divide the argument according to whether $M \geq 4$ or $M = 3$. First suppose $M \geq 4$. In that case, since he is not pivotal (the deviant offer will be rejected in equilibrium by all the other players), whether he unilaterally deviates (accepts the deviant offer) or conforms (and rejects it), the amount of cake he obtains in this period is the same (i.e., zero) but his continuation payoffs differ since in the former case play moves onto path Q^j while in the latter case play stays on path Q^i . The assumption that Π is nondecreasing in each of its arguments implies that the legislator in question is strictly better off conforming and rejecting the deviant offer.

Now suppose $N = 3$. In this case he would be pivotal, and he may have an incentive to accept the deviant offer (if it is sufficiently attractive). There will be three incentive-compatibility conditions that need to hold (corresponding to the three possible classes to which the legislator belongs when he considers the deviant offer) in order for it to be the case that he would reject the deviant offer. Since the most attractive deviant offer is one in which he is offered the whole cake, the three conditions corresponding to when he is in class 1 (EARLY), class 2 (MIDDLE), or class 3 (LATE) are respectively

$$\begin{aligned} b + \Pi(0, 0.5, 0.5) \left[\frac{b}{1 - \Pi(0.5, 0.5, 0.5)} \right] &\geq b + \Pi(1, 0, 0) \left[\frac{b}{1 - \Pi(0, 0, 0)} \right] \\ b + \Pi(0.5, 0, 0.5) \left[\frac{b}{1 - \Pi(0.5, 0.5, 0.5)} \right] &\geq b + \Pi(0.5, 1, 0) \left[\frac{b}{1 - \Pi(0, 0, 0)} \right] \\ b + \Pi(0.5, 0.5, 0) \left[\frac{b}{1 - \Pi(0.5, 0.5, 0.5)} \right] &\geq b + \Pi(0.5, 0.5, 1) \left[\frac{b}{1 - \Pi(0, 0, 0)} \right]. \end{aligned}$$

The conditions on Π stated in the proposition imply that these three incentive constraints are satisfied. Finally, using similar arguments to those used here to establish that the punishment path Q^j is subgame perfect, it is easy to verify that the agent-optimal path $Q(\hat{x})$ is subgame perfect. This then concludes the proof of the proposition.

PROOF OF PROPOSITION 9. It is trivial to verify that the stationary strategy described in the proposition, when adopted by all legislators is an equilibrium. It is straightforward to extend the arguments in the proofs of Propositions 2 and 4 and establish that there does not exist any other equilibrium (by first establishing that in any pure-strategy equilibrium, a legislator uses a Markov pure strategy, and then establishing that the only Markov equilibrium is the one described in the proposition).

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