Signaling Character in Electoral Competition*

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Abstract

We study a one dimensional Hotelling-Downs model of electoral competition with the following innovation: a fraction of candidates have "character" and are exogenously committed to a campaign platform; this is unobservable to voters. However, character is desirable, and a voter's utility is a convex combination of standard policy preferences and her assessment a candidate's character. This structure generates a signaling game between strategic candidates and voters, since a policy platform not only affects voters' utilities directly, but also indirectly through inferences about a candidate's character. The model generates a number of predictions, starting with a failure of the median voter theorem. The results may help explain why candidates sometimes choose non-median platforms, and moreover, why a majority of voters can rationally vote for a non-centrist candidate.

Keywords: electoral competition, signaling, character, integrity, valence median voter theorem, policy divergence

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"*Americans believe Mr. Bush himself honestly believed Saddam was a threat ... [voters] can tell he is not doing it all by polls and focus groups ... You can agree or disagree with him, but it is hard to doubt his guts, his seriousness, and his commitment ... This is why in presidential elections character trumps everything.*"

— Wall Street Journal Editorial, April 22, 2004

1 Introduction

Throughout the 2000 U.S. presidential race, spokespeople for the George Bush campaign frequently alleged that opponent Al Gore would "say anything to get elected." The implicit suggestion is that George Bush believed in his position and would state it even if it hurt his chances of being elected. The interesting proposition that the Bush campaign took is that voters should vote against politicians who state popular positions, because such politicians lack character.

That career politicians might lack character is not new. Criticism of career politicians and insiders has been a frequent refrain in political campaigns. George Washington said "Few men have virtue to withstand the highest bidder." Calvin Coolidge said "Character is the only secure foundation of the state." Barry Goldwater used character ineffectively against Lyndon Johnson in 1964. In a series of television advertisements, a spokesman said "You must not give power to a man unless, above everything else, he has character. Character is the most important qualification the President of the United States can have." It is a historical irony that Goldwater's spokesman for presidential character was Richard Nixon.

The simple purpose of this paper is to investigate the effects of character—taken to be an exogenous characteristic of people—on policy selection and voting. The idea is that a politician with character prescribes to voters the position that the politician thinks is best. Politicians without character tell voters whatever is most likely to get the politician elected. Voters have preferences both over positions and over character. Voters try to infer character from the positions taken by candidates. Only the candidates without character are strategic, because the candidates with character state their exogenous best position.

One immediate consequence of character is that the median voter theorem cannot hold. To see why, suppose that all politicians without character choose the position of the median voter. Then even the slightest deviation from that position will inform the voters that the candidate has character, and hence insure the election of the candidate, who would be a politician with character almost at the median. This is by itself an important effect of character, because while the median voter theorem is robust in many theoretical models (see, for example, Banks and Duggan, 2003), it does not appear to be an empirically salient characteristic of many elections (Ansolabehere et al., 2001).

The theory we develop produces a unique equilibrium, in which candidates without character randomize. The distribution of their positions is related to the distribution of the positions of candidates with character, with a distortion toward the median. Thus, there is a centripetal force at work. Most elections result in a tie; only extreme candidates lose, and extremists are candidates with character. Tied elections arise because candidates without character randomize, randomization entails indifference, and in the political context, indifference entails ties.

Why do voters care about character? We appeal to a standard answer given in the literature: voters use it to partially assess the set of actions a candidate may take if elected. Politicians cannot commit to a full set of contingent actions, and so voters are necessarily unsure what an elected official will actually do once in office. At most, the candidate can commit to a handful of stated positions. As voters typically care about subtle and sometimes unobservable behavior by an elected official, voters will also care about attributes of the candidate beyond the candidate's stated policy dimension. The canonical class of such attributes are known as *valence* and are familiar characteristics of political theory (Stokes, 1963).[1] The crucial feature of valence characteristics is that they are valued by all voters—more of the trait is preferred to less—independent of ideological position. It is this interpretation we take when introducing our basic model. The novel aspect of our approach is that the position chosen by the candidate is interpreted by the voters as a signal about character.

In the latter part of the paper, we extend our theory to cover a richer set of voter preferences. In particular, this permits voters with differing ideal policy positions to value a candidate's character differently. Moreover, a voter's preference for character can vary with the stated position of the candidate. This allows us to endogenize the preference for character, stepping beyond mere valence interpretations. Suppose that an elected official takes actions of two kinds: observable or "in plain view", and unobservable or "out of sight". By their very nature, observable actions are committed to in the campaign platform; unobservable actions, on the other hand, cannot be committed to. A politician who has character says what he will actually do (on both dimensions), whereas a politician without character may do something very different on the unobservable dimension than what he promised and necessarily lives up to on the observable dimension. In this sense, it is natural to interpret the character trait here as that of *integrity*.[2] This setting generates an endogenous taste for character among voters, which will depend both on stated position of the candidate and a voter's own ideal position. We show that our main insight extends unchanged to such an environment.

Recent literature, such as Osborne and Slivinski (1996) and Besley and Coate (1997),

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[1] Not all valence attributes are about character, however. Although Stokes's (1963) original discussion of valence comports well with our notion here, more recent literature sometimes uses the term 'valence' to describe attributes such as "handshaking ability", charisma, and so forth. These traits are not what we have in mind.

[2] The Merriam-Webster dictionary definition of 'integrity' is "firm adherence to a code of especially moral or artistic values". Discussion of integrity often arises in politics, and in fact the word was the most-researched word on Merriam-Webster online by Americans in 2005. (Source: CNN `http://www.cnn.com/2005/US/12/10/top.word.ap.ap/.`)

considers candidates that cannot commit to positions at all. Lack of commitment should tend to make candidate character more important, strengthening the implications of our analysis. Some commitment is plausible, however, based on repeated game arguments of Alesina (1988) and Alesina and Spear (1988); empirical support for commitment appears in Poole and Rosenthal (1997). One implication of our analysis is that a candidate who changes their position toward the median may be perceived as lacking character, thus encouraging commitment to positions. That is, the possibility of character itself may enhance commitment.

We formulate our model in the simplest possible structure: the standard one-dimensional spatial model of electoral competition following Downs (1957), adapted from the spatial model of Hotelling (1929). Character itself is taken to be an exogenous characteristic of candidates. We think it is reasonable to consider that character is formed long before individuals choose to enter politics, and that character in the general population is in fact exogenous. However, character may play a role in the selection of candidates who run for office (e.g. Bernheim and Kartik, 2004), and a weakness of our analysis is that the process generating candidates is not modeled. In defense, we analyze the subgame of platform selection given an arbitrary candidate selection mechanism, and analysis of the subgame is necessary to investigate a more general model.

We find that the posterior probability that a candidate has character is higher the further the candidate is from the ideal position of the median voter. This is the feature of the equilibrium that leads to voter indifference. Voters think extremists usually mean what they say, while middle-of-the-road candidates are more likely to have simply said what voters want to hear. The beliefs are constructed to create indifference among candidates without character. A useful aspect of this construction is that it leads to a closed form for the density of positions of candidates without character, up to a single parameter that must be implicitly constructed. This construction would aid an attempt at empirical analysis, although the problems of quantifying the position space and beliefs about character necessary for empirical testing are daunting, indeed.

As the proportion of candidates with character diminishes to zero, the equilibrium platform distribution of those without character collapses on the median voter's ideal point. However, the support of these positions does not collapse. Interestingly, however, if almost all candidates have character, both the distribution and support of positions of candidates without character also collapses to the median. Thus, while there is a sense in which the median voter theorem holds at either extreme of the model, the mere presence of character has an echo on policy platforms even when it is very unlikely.

An important feature of the equilibrium of our model is that it is an ex-post equilibrium, which means that even after the candidates see each other's position, they do not regret their choice. Thus, it does not matter if the game is played sequentially or simultaneously, as the predicted behavior is invariant to timing. We find this a particularly appealing aspect of our mixed-strategy equilibrium. Moreover, the equilibrium is robust to the number of candidates running for office. Unlike the standard Hotelling-Downs model, our analysis extends unchanged to an arbitrary number of politicians.

The theory presented here provides a theoretical but intuitive grounding for a mechanism that is often believed to operate in real elections. As the opening quote from the Wall Street Journal suggests, the perception of not pandering to the public can be valuable to a politician. This provides a novel explanation for why candidates may select non-median platforms, and moreover, why voters may vote for such candidates, rather than simply selecting a centrist candidate. Applications of this principle abound. For example, it is widely agreed that in the 2004 U.S. presidential election, George W. Bush won despite choosing a platform that was well to the right of the center. His victory is often attributed to a belief amongst the public of his "conviction" in his policy position, in contrast to the perception of John Kerry. This is consistent with our theory; moreover, the model delivers this as rational behavior, and in particular, rational inferences by the public. However, our theory also makes the following point: it is not guaranteed that the non-centrist candidate truly possesses the desirable character trait; he could in fact be a strategic politician mimicking the behavior of those with character.

As another example, consider the case of the VLD (Flemish Liberal Party) in Belgium.[3] In 1994, the VLD committed to propose as its platform the policy preferred by the majority of the Belgian public, which it elicited through a public poll. It lost the election by a large margin to a less centrist party. In 1999, on the other hand, the VLD did not pursue this approach, and instead proposed a less centrist platform without polling the public. It won the election, beating more centrist parties. Why? The explanation here is that in simply pandering to the median voter in 1994, the VLD leaders signaled a lack of character. In contrast, in 1999, despite not being as desirable purely on the platform dimension, the new leaders of the VLD signaled that they would choose policies that *they* believed were best, and this trait was valued by voters.

As a final example, we recall a well-publicized statement made by senator John McCain during a 1999 Republican Primaries debate in Iowa: "I'm here to tell you the things that you dont want to hear..." McCain went on to denounce ethanol subsidies, which are widely popular in Iowa. From the perspective of appealing to voters on policy alone, this is perhaps puzzling; in fact, all the other Republican candidates either supported or expressed neutrality on this issue. However, when a campaign is interpreted as also signaling character, McCain' approach is straightforward to interpret. The quoted statement prefacing his position suggests that his goal was to convince voters that he would not merely choose platforms that appeal to them, but instead choose platforms that he truly believed in—and that such a trait should be appreciated by voters.

There are other papers in the literature that derive non-median-voter results. The closest in spirit to ours is the work of Callander (2004). He too presumes that voters value a trait about politicians that is unobservable but may be signaled in their campaign platform. The specific trait he considers is that of effort exerted in implementation of

[3] We owe this example to Carrillo and Castanheira (2002).

policy. Unlike in our model with character, all candidates in his model are fully strategic: they differ only in whether they are policy motivated (and exert high effort if elected) or office motivated (and exert low office if elected). While there are obvious similarities in motivation and in the results derived, there is an important conceptual difference between the notions of policy motivated candidates versus candidates with character as we define them. The former entails a preference about final policy outcomes; the latter entails a direct preference over one's own campaign platforms. We discuss this in more detail after introducing our model.

Signaling in electoral competition was first considered by Banks (1990). He develops a model where campaign promises are non-binding, and candidates implement their privately known preferred policy once elected. However, candidates suffer a cost of lying. Thus, the non-binding platforms can nevertheless serve as a signal of their preferred policy. Our model is different because platforms are binding, but some candidates—those with character—announce their truly preferred platforms. Callander and Wilkie (2003) extend Banks (1990) by allowing some candidates to suffer infinite lying costs, so that they simply announce their preferred policies. Nevertheless, they maintain that campaign promises are non-binding, rendering our analysis distinct.

The impact of valence on policy platforms was first formally studied by Ansolabehere and Snyder (2000), Aragones and Palfrey (2002), Groseclose (2001). These papers take a candidate's valence attribute as observable, hence there is no signaling element. Policy divergence stems from asymmetry of valence across candidates. Our divergence result arises even in a completely symmetric environment.

The rest of the paper is structured as follows. We present the basic model in the following Section. In Section 3, we derive the unique equilibrium and study a number of its implications. Section 4 discusses extensions of the theory. We conclude in Section 5. All formal proofs are collected in an Appendix.

2 The Model

The basic element of the model is a standard Hotelling-Downs one-dimensional policy location game. The set of policies is denoted $X = [0, 1]$. There is a continuum of voters (synonymous with citizens), each with single-peaked policy preferences on X. A voter is identified by her ideal point, $v \in X$, and we assume that her[4] policy preferences can be represented by a utility function $u(x, v)$ that is twice continuously differentiable, and satisfies $u_1(v, v) = 0$ and $u_{12}(x, v) > 0$ for all x.[5] A voter's ideal point is drawn from a

[4]Throughout, we use female pronouns for voters and male pronouns for candidates.

[5]This represents single-peaked preferences because for any $x \neq v$,

$$u(v, v) - u(x, v) = \int_x^v [u_1(z, v) - u_1(z, z)]\, dz = \int_x^v \int_z^v u_{12}(z, y) dy dz > 0$$

probability distribution with median $m \in (0,1)$. There are two candidates (synonymous with politicians), A and B, each of whom must commit to a platform, $x^i \in X$. After observing both candidates' platforms, each voter votes sincerely to maximize her expected utility.[6]

We now depart from the standard model by introducing *character* as follows. Character is a binary variable: candidate $i \in \{A, B\}$ either possess it $\left(c^i = 1\right)$ or does not $\left(c^i = 0\right)$. This is private information and drawn independently from a Bernoulli distribution with $\Pr\left(c^i = 1\right) = b > 0$. If a candidate i has character, his platform choice, x^i, is constrained to be the draw of a random variable that has a differentiable cumulative distribution function F with density $f(x) > 0$ for all $x \in X$. Hence, a candidate with character has no strategic choice to make, and we refer to such types as *non-strategic types*.[7] On the other hand, candidates without character only care about holding office, and hence strategically choose their platform to maximize their probability of being elected.

Voters care about character in addition to policy: a voter v's expected utility from a candidate i with platform x^i is denoted $U(x^i, v)$, where

$$U(x^i, v) \equiv \lambda \Pr\left(c^i = 1 \middle| x^i\right) + u\left(x^i, v\right)$$

Thus, $\lambda > 0$ is the relative weight attached to the character of a politician by voters. Note that the inference about a politician's character depends upon his chosen platform. The standard Hotelling-Downs model is a special case of our model when either $b = 0$ or $\lambda = 0$.

Some remarks are in order about our modeling choices, and further literature connections. First, candidates without character in our model are purely office-motivated and fully strategic. Formally, candidates with character are non-strategic and can be thought of as "crazy" types following Kreps et al. (1982). However, this interpretation is strained, and we prefer to think of them as fully rational, despite our choice of terminology. They idea is that candidates with character suffer (infinite) disutility from proposing a platform they do not "believe in". As discussed in the introduction, our assumption that citizens care about a politician's character is meant to capture the idea that citizens have not only policy preferences, but also care about some unobservable characteristic about politicians that is correlated with their willingness to campaign on platforms that are not their true policy preferences. Indeed, our model can be thought of as a reduced form for the following: each candidate i has a policy, p^i, drawn from the cdf F, that he thinks is "right". He then find out whether he has character in which case he effectively must choose platform $x^i = p^i$, perhaps due to a preference for not pandering. If he does not have character, he strategically choose platform x^i to maximize probability of being elected. Now, suppose

[6] Sincere voting is fully rational in this setting; it only serves to eliminate trivial equilibria such as all voters voting for one candidate, and no-one deviating because they are powerless to change the outcome.

[7] As will be clear from the analysis, what formally matters is not that there actually be positive probability that a candidate is non-strategic, but only that the electorate believe this to be case.

character types will resist special interests if elected, whereas non-character or flexible types will fall prey to them. Voters get a disutility of λ from an elected politician who deals with special interests. In this environment, a voter has an instrumental reason to prefer politicians who are non-strategically choosing their platform, all else equal.

The non-strategic candidates in our model are not policy motivated in the usual sense the term is used in (Wittman, 1977; Calvert, 1985). The utility for policy motivated candidates depends on the final policy outcome and not directly on their individual policy platform; whereas candidates with character in our model care directly the platform they propose, regardless of the final policy outcome. In a sense, character candidates are *procedurally* motivated. To take another perspective, policy motivated candidates have unlimited ability to compromise in platform, and will do so if it results in a more desirable final policy outcome; character candidates are limited in their ability to compromise on their platform. Fiorina (1999, esp. fn. 10 on p. 9) makes this distinction.

We have assumed that the prior probability of having character is the same for both candidates, and the distribution of platform conditional on character is also the same. Thus, there is no ex-ante asymmetry between the candidates. This contrasts with the literature on observable valence asymmetry between candidates (e.g. Aragones and Palfrey, 2002). In Section 4, we extend our model to cover ex-ante differences between the candidates.

Since candidates with character are non-strategic, we refer to a candidate's strategy as his behavioral rule conditional on *not* having character, i.e. conditional on being strategic. A strategy for candidate $i \in \{A, B\}$ is represented by a cumulative distribution function (cdf), G^i; for technical convenience, we restrict attention to strategies that can be written as the sum of absolutely continuous and discrete distributions.[8] If G^i has a density, we denote it g^i. As this is a signaling game, voter beliefs about a candidate's character are critical. Let $\phi^i(x)$ be the posterior probability that i has character given his platform choice of x.

It is convenient to define $\mu(x) \equiv u(x, m)$, so that $\mu(x)$ is the median voter's policy utility from platform x. Given the posterior belief of character and the platform of candidate i, the median voter's expected utility should this candidate be elected is

$$\alpha^i\left(x|\phi^i\right) = \lambda\phi^i(x) + \mu(x)$$

Where there is no risk of confusion, we typically suppress the dependence of α^i on ϕ^i to reduce notation. Sincere voting implies that candidate i wins and candidate $j \neq i$ loses if $\alpha^i\left(x^i\right) > \alpha^j\left(x^j\right)$. When the two expressions are equal, the election is tied, and we assume that each candidate gets elected with probability $\frac{1}{2}$. Nothing rests on this rule for breaking ties; all that matters is that tie-breaking is non-degenerate. Given the beliefs ϕ^A and ϕ^B, voter behavior is completely pinned down (except perhaps for a measure 0 set of voters), hence we are not explicit about it in what follows.

[8] By the Lebesgue decomposition theorem for the Real line, this is only a restriction insofar as it precludes a strategy from having a singular component without mass points.

Our solution concept is that of (weak) perfect Bayesian equilibrium (Fudenberg and Tirole, 1991). This requires that σ^A and σ^B respectively maximize the probability of being elected for each strategic candidate given voter beliefs ϕ^A and ϕ^B, and that ϕ^A and ϕ^B be consistent with Bayes Rule.[9]

3 Signaling Character

3.1 The Unique Equilibrium

Due to the symmetry in the model, a candidate must win with positive probability if strategic. To see this, observe that a candidate can always play the same strategy as his opponent. If he does so, he loses with probability one only if voters believe that for all platforms in the support of this strategy, he is less likely to have character than his opponent. Given that both candidates have the same prior likelihood of character, $b > 0$, and the same distribution over platforms with character, F, this cannot be the case.

The only way that strategic candidates win with positive probability and yet maximize their probability of winning is if every platform that they choose offers the same value to the median voter (and all platforms not chosen offer no greater value). Call this value α^*. Since there are no mass points in the platforms of the candidates with character, strategic candidates will not use mass points either. To see this, note that the Bayes' update on the probability a candidate has character is zero at any mass point in the strategy of the strategic candidates. Thus at any mass point in the strategy, there is a choice for the strategic candidate with a nearby policy platform, but a positive likelihood of having character. Consequently, the distribution of positions chosen by the candidates will be continuous. This means that we can write the Bayes' update as

$$\phi^i(x) = \frac{bf(x)}{bf(x) + (1-b)\,g^i(x)}$$

Since within the support of their strategies, both strategic candidates offer value α^*, we conclude that

$$\alpha^* = \mu(x) + \lambda \frac{bf(x)}{bf(x) + (1-b)\,g^i(x)}$$

This equation says that for any choice actually taken by a candidate, the utility offered to the median voter, which is the sum of the platform's direct value and the Bayes' updated beliefs about the candidate's likely character, is a constant. This equation can be

[9] More precisely, each ϕ^i must be a regular conditional distribution derived from the distributional strategy (Milgrom and Weber, 1985) induced by F and G^i.

solved for the density of platform choices, which turns out to be

$$g^i(x) = g^*(x) = \max\left\{0, \frac{bf(x)}{1-b}\left[\frac{\lambda}{\alpha^* - \mu(x)} - 1\right]\right\} \tag{1}$$

This is a symmetric expression, so g doesn't depend on i. Moreover, the value of α^* is determined by the requirement that g^* be a density, and hence integrate to one. It is readily shown that $\mu(m) < \alpha^* < \mu(m) + \lambda$, since a candidate can always offer a value of at least $\mu(m)$ by choosing platform m, and can offer value of no greater than $\mu(m) + \lambda$.

An important aspect of this construction is that the median voter is indifferent to every position taken by strategic candidates. Thus, even knowing their opponent's platform, a strategic candidate has no incentive to revise his position. This property means that the equilibrium is an *ex-post equilibrium*, and in this setting, it has two important implications. First, the outcome is not sensitive to the timing of the game: the same behavior remains an equilibrium whether one candidate announces first, or second, or both announce platforms simultaneously. Second, all elections between strategic candidates result in a tie, even though candidates have distinct positions.

It turns out that, with substantially more work, it can be shown that this equilibrium is unique. This conclusion is summarized in Theorem 1, whose formal proof is in the Appendix.

Theorem 1. *There is a unique equilibrium. It is an ex-post equilibrium where both candidates use the same strategy, G^*, with density*

$$g^*(x) = \max\left\{0, \frac{bf(x)}{1-b}\left[\frac{\lambda}{\alpha^* - \mu(x)} - 1\right]\right\}$$

where $\alpha^* \in (\mu(m), \mu(m) + \lambda)$ is the unique constant such that $\int_x g^*(x)dx = 1$.

Therefore, both candidates mix in equilibrium, using a continuous density. Our interpretation of mixed strategies follows the Bayesian view of opponents' conjectures, originating in Harsanyi (1973). That is, a candidate's mixed strategy need not represent him literally randomizing over platforms; instead, it represents the uncertainty that the *other candidate and the electorate* have about his pure strategy choice. Each candidate could be playing a pure strategy which depends upon an auxiliary variable that is his private information.

One implication of Theorem 1 that is worth emphasizing is that the median voter theorem (MVT) does not hold in our model. Note that in the current setting, the appropriate version of the MVT is that ex-ante, each candidate chooses platform m with probability $1 - b > 0$, viz. with the probability of being strategic.

Corollary 1. *The MVT fails. In the unique equilibrium, the ex-ante probability that either candidate chooses platform m is 0.*

3.2 Properties

We now derive various implications of Theorem 1. The first is a simple observation.

Fact 1. *If a candidate is strategic, he wins with probability at least as large as if he had character.*

This is an immediate consequence of the fact that a strategic candidate provides the median voter with utility α^*, whereas a candidate with character provides the same utility if his platform falls within the support of the equilibrium strategy, G^*, and strictly less utility if his platform falls outside the support.

Since α^* is the expected utility of the median voter, comparative statics on α^* immediately become comparative statics on the utility of the median voter. The comparative statics on α^* are readily computed from the equation

$$\int_x \max\left\{0, \frac{bf(x)}{1-b}\left[\frac{\lambda}{\alpha^*(b,f,\mu,\lambda)-\mu(x)}-1\right]\right\} dx = 1 \tag{2}$$

which implicitly defines α^* as a function of b, f, μ, and λ. We use this and the equilibrium construction from the previous section to discuss various implications of our theory. Denote by $\phi^*(x)$ the posterior held by voters upon seeing platform x, in equilibrium.

Fact 2. $\frac{g^*(x)}{f(x)}$ *is strictly increasing for* $x < m$*, and strictly decreasing for* $x > m$*.* $\phi^*(x)$ *is strictly decreasing for* $x < m$*, and strictly increasing for* $x > m$*.*

This property of $\frac{g^*(x)}{f(x)}$ is immediate from inspection of (1), because μ is single-peaked around m. That the reverse is true for ϕ^* then follows because we can write the Bayes' update as

$$\phi^*(x) = \frac{b}{b+(1-b)\frac{g^*(x)}{f(x)}}$$

Thus, strategic candidates skew their positions toward the median voter's preferred position. Moreover, voters believe that candidates near the median voters preferred policy are less likely to have character, and the further is the chosen position, the more likely the candidate is to have character. This leaves the median voter indifferent, provided the positions are in the support of g^*. Outside the support of g^*, the voters are certain the candidate has character but the candidate loses nonetheless, because the advantage of a certainty of character is unable to overcome the disadvantage of an extreme position. Thus, the model accommodates extreme positions losing with certainty and distinct moderate positions resulting in ties.

Fact 3. α^* *is increasing in* b *and* λ*.*

This directly follows from inspection of (2). That ex-ante welfare, α^*, increases in b is relatively trivial, since voters value character, and b is the ex-ante probability of a candidate having character. The rationale behind α^* increasing in the weight placed on character, λ, is more subtle. As λ rises, $\frac{g^*(x)}{f(x)}$ flattens out, and strategic candidates look more like candidates with character. This casts the effect of λ on utility in an interesting light. The flattening of $\frac{g^*(x)}{f(x)}$ means that the value of the platform offered by strategic candidates falls as rises. Moreover, the likelihood that a candidate has character hasn't changed. Thus, the value offered by strategic candidates falls and their prevalence remains unchanged. The behavior and prevalence of candidates with character hasn't changed. Thus, the overall value of the system seems to have fallen. The apparent paradox is resolved by noting that the total value of the candidates with character has risen because character is valued more highly.

Fact 4. *The support of g^* is increasing in λ.*

By (1), the support of g^* is given by the solutions to $\lambda + \mu(x) \geq \alpha^*$. Thus, whether the support increases in λ reduces to the question of whether $\frac{\partial \alpha^*}{\partial \lambda} < 1$. This is indeed the case, since by differentiating (2), we see that

$$\int_{\{x|\lambda+\mu(x)\geq\alpha^*\}} \frac{bf(x)}{1-b} \frac{1}{[\alpha^* - \mu(x)]^2} \left[\alpha^* - \mu(x) - \lambda \frac{\partial \alpha^*}{\partial \lambda}\right] dx = 0$$

Since $\alpha^* \leq \lambda + \mu(x)$ for all x in the support of g^*, the above equality cannot hold unless $\frac{\partial \alpha^*}{\partial \lambda} < 1$. Thus, the support of g is indeed increasing in λ. This means that as character becomes more important, the likelihood of extremists, with and without character, being elected rises. The intuition is that when policy becomes less important relative to character, candidates with extreme platforms can still win. If character is sufficiently important, so that is λ is sufficiently large, g^* will have full support. In this case, any election is tied.

Fact 5. *As $\lambda \to \infty$, $g^*(x) \to f(x)$ for all x.*

We defer the argument to the the Appendix. Intuitively, as the weight on character diverges, the gain from being perceived as having character increases for each candidate. Thus, in the limit, strategic candidates fully mimic the distribution of those with character.

Another comparative static considers a change in the platform of candidates with character. Suppose that f is replaced with h, and that $h(x) - f(x)$ is increasing for $x < m$, and decreasing for $x > m$. That is, candidates with character are more likely to come from the center under h than than under f.

Fact 6. *α^* is higher under density h than under density f.*

The proof is in the Appendix. The idea is simple: strategic candidates mimic the distribution of those with character, but skew their play towards the median voter's preferred policy. If the distribution of platforms of candidates of character is more concentrated towards the center, then both strategic and non-strategic candidates' play become more desirable to the median voter in policy terms. Thus, α^* goes up.

For our last comparative statics, we consider the limits as b converges to 0 or 1.

Fact 7. *As $b \to 1$, $Supp(g^*) \to \{m\}$. As $b \to 0$, $Supp(g^*) \to \{x | \mu(x) + \lambda \geq \mu(m)\}$, but G^* converges to an atom on m.*

The proof is in the Appendix. If most candidates have character, so that b is close to one, there is little advantage to a strategic candidate of signaling character by position since the prior is so strong. Consequently, the support of the distribution of the strategic candidate's strategy collapses to m. This case replicates the standard model and the MVT emerges at this limit. In contrast, if there are few candidates with character, and b is close to zero, the distribution collapses on m but the support of the distribution converges to the set $\{x|\mu(x) + \lambda \geq \mu(m)\}$. This is as large as the support can get for a given λ. Thus, while almost all strategic candidates locate very near the middle, the possibility of strategic candidates a long way from the middle remains in the limit as candidates without character vanish. The possibility of candidates with character then has an echo in the model, even when the probability of such candidates goes to zero. Note that in such a case, ex-post, an elected candidate may have a position far from the middle.

4 Discussion

In this section, we discuss various interpretations and extensions of our theory.

4.1 Tied Elections

As already noted, our simple model results in all elections between strategic candidates being tied. Moreover, if the weight on character, λ, is large enough, then all elections end in ties. Nevertheless, we do not wish to suggest that elections in real life typically end in ties.[10] This is a consequence of the assumption we made that candidates have no uncertainty about the electorate, and in particular, about the median voter's location, m. Consider an extended model where candidates share a common belief about the median voter's location, but the uncertainty is only resolved ex-post after positions have been chosen. Our analysis carries through unchanged, but it would now be the case that after uncertainty has been resolved at the last stage, ties do not generally occur.[11] However, the

[10] However, the number of elections that do end in ties for all practical purposes is intriguing; see Talor and Yidrim (2005), for example.

[11] Note that it remains true that neither candidate has an incentive to deviate from his strategy after observing the other candidates' position, but this property would not hold true once the median voter's location is revealed or observed. On a related point, adding private information for the candidates about the location of the median voter would complicate matters as usual.

ex-ante probability of winning when two strategic candidates compete against each other remains one half. Indeed, this is an unavoidable and not unreasonable feature of any (symmetric) model with strategic office-motivated candidates. Note, in particular, that it also applies to the standard Downsian model.

4.2 Broader Interpretations of Character

Under some conditions, our model can be given a richer interpretation at the cost of predictive precision, in the following sense.[12] The main hypothesis we posed is that there is an unobservable trait among politicians that is valued by voters and is also negatively correlated with the willingness to pander to the public in order to get elected. Assume instead that while there is an unobservable trait that is valued by voters, it has nothing to do with willingness to pander. For example, the trait may be competence: some candidates are competent, some are not. All candidates are purely office-motivated and fully strategic. Suppose voters conjecture that competent candidates are playing the strategy F, which recall is the (exogenous) distribution chosen by non-strategic types in our model, whereas incompetent candidates are playing the strategy G, which recall is the (endogenous) distribution chosen by strategic types in our model. If the preference weight on competence (λ) is large enough such that the equilibrium of our model has a full support G, then the median voter is indifferent upon observing any platform. But then, candidates are indifferent over platform choices, and it is in fact an equilibrium for the competent ones to play G and the incompetent ones to play F.

The benefit of this perspective is that our policy divergence equilibrium is rationalized under much broader interpretations of unobservable traits, viz. any observable but desirable trait, even if it has nothing to do with willingness to pander for office. However, there is are two caveats: first, it only applies if the preference weight on the unobservable trait is sufficiently large; second, and perhaps more importantly, unlike in our model with non-strategic types, uniqueness of equilibrium will not hold here. In particular, a "median voter equilibrium" also exists, where all candidates choose the median voter's platform. This is sustained in equilibrium by the out-of-equilibrium beliefs that when any non-median platform is observed, the candidate must not possess the desirable trait (e.g., must be incompetent).

4.3 Ex-ante Asymmetry

Suppose that the prior likelihood of having character can differ across candidates, and moreover, the distribution of policies conditional on character can also differ. This may be appealing when thinking of candidates as representing different political parties, for example. We now index b and f as b^i and f^i for each $i \in \{A, B\}$; the setup is otherwise unchanged. Call this the *asymmetric model.*

[12]We thank Dino Gerardi for this suggestion.

Theorem 2. *In the asymmetric model,*

1. *There is an ex-post equilibrium where candidate* $i \in \{A, B\}$ *uses the strategy,* $\hat{G}^i$, *with density*

$$\hat{g}^i(x) = \max\left\{0, \frac{b^i f^i(x)}{1-b^i}\left[\frac{\lambda}{\hat{\alpha}^i - \mu(x)} - 1\right]\right\} \quad (3)$$

where $\hat{\alpha}^i \in (\mu(m), \mu(m) + \lambda)$ *is a constant.*

2. *If* $\hat{\alpha}^A = \hat{\alpha}^B$, *then the above is the unique equilibrium.*

3. *If* $\hat{\alpha}^i > \hat{\alpha}^j$, *then in any equilibrium, candidate* i *wins with probability* 1 *when strategic.*

The first part of Theorem 2 is a trivial extension of the existence portion of Theorem 1. As before, the constant $\hat{\alpha}^i$ is unique and determined by the requirement that $\int_x \hat{g}^i(x)\,dx = 1$. As was the case earlier, the median voter's expected utility from electing candidate i is $\hat{\alpha}^i$, for any observed platform that is in the support of strategic i's strategy in this equilibrium. The difference with the base (symmetric) model is that it will no longer generally be true that $\hat{\alpha}^A = \hat{\alpha}^B$. Therefore, one of the candidates may win with probability 1 whenever he is a strategic type. For example, if $f^A = f^B$ but $b^A > b^B$, then when playing the strategies $\hat{G}^A$ and $\hat{G}^B$, candidate A always wins so long as the chosen platform is in the support of G^A.

That one of the candidates may win with probability 1 when there is an ex-ante asymmetry is not all too surprising. Such a property of the Downsian model is well-known when one candidate has an observable valence advantage (Aragones and Palfrey, 2002; Groseclose, 2001). The effect of asymmetric b^i's (or f^i's) is similar in our model, since it endows one candidate with an ex-ante advantage. As is the approach taken in that literature, extending our setting to one where there is uncertainty over the median voter's location—as we have already outlined earlier—would yield the possibility that even with ex-ante asymmetry, neither strategic candidate wins with probability 1.

Part 2 of Theorem 2 says that if the constellation of parameters do in fact generate the same $\hat{\alpha}$, then the equilibrium identified in Part 1 is unique. Thus, the result subsumes Theorem 1. When $\hat{\alpha}^A \neq \hat{\alpha}^B$, then the equilibrium identified in Part 1 of Theorem 2 may not be unique. Intuitively, say that candidate B is at an ex-ante disadvantage relative to candidatex A, and will lose with probability 1 when both are strategic types in the equilibrium of Part 1. Then, candidate B may be indifferent over multiple losing strategies when strategic, and this can lead to a multiplicity of equilibria. However, the multiplicity is inessential in terms of whether candidate A wins when strategic. This is the content of Part 3 of Theorem 2.

4.4 Richer or Endogenous Preferences for Character

We now enrich the preferences for character thus far considered. Let utility for a voter with ideal point v facing a candidate i with policy x^i be given by

$$U(x^i, v) \equiv \lambda(x^i, v) \Pr\left(c^i = 1 \middle| x^i\right) + u\left(x^i, v\right)$$

Here, the weight placed on character need not be constant—which was the assumption before—but instead can depend upon both the candidate's platform and a voter's ideal point. One natural motivation for such a specification is to capture the idea that politicians take actions of two kinds: one observable ("in plain view") and one unobservable ("out of sight"). The $u(\cdot,\cdot)$ component of a voter's utility represents the utility over observable actions that have been committed to by the politician during the electoral process. The $\lambda(\cdot,\cdot)$ component represents the voter's utility over the unobservable actions that the politician will take in office. If the politician has character, then his position on the unobservable dimension will be the same as what he committed to on the observable action, since those with character say what they will actually do. If he does not have character, however, he might do something very different on the unobservable dimension that what he promised (and necessarily lives up to) on the observable dimension. That $\lambda(\cdot,\cdot)$ can vary over policies and over voter ideal points allows for the possibility that different voters care differently about whether a candidate keeps his word behind the scenes, and moreover, this can depend on the position a candidate promised in different ways to voters. For example, a voter with ideal point $v = 1$ may prefer a candidate with platform $x^i = 0$ to *not* have character and thus likely do something different on the unobservable dimension than what was promised. On the other, the same voter may prefer a candidate with $x^i = 1$ to in fact have character, thus guaranteeing that he will take the same policy position on the unobservable dimension. This preference ordering over character can be reversed for a voter with ideal point $v = 0$.

The function $u(x, v)$ is exactly the same as earlier. We make the following additional assumptions. Denote $W(x, v) \equiv u(x, v) + \lambda(x, v)$. Assume that $W(x, v)$ is twice continuously differentiable; $W_{12}(x, v) > 0$ for all x; and for each v, there exists some $\hat{x}(v)$ such that $W_1(\hat{x}, v) = 0$. Thus, $W(\cdot, v)$ is single-peaked around $\hat{x}(v)$. The key assumption is that that the median over $\hat{x}(v)$ is identical to m, which recall is the median of the v's and lies in $(0, 1)$. This is satisfied, for example, if $\lambda(x, v)$ is a constant, or if it is also single-peaked around $x = v$; but it allows for more general cases as well. Let $\lambda^m(x) \equiv \lambda(x, m)$; we require that $\lambda^m(m) > 0$, so that character is valuable.

The assumption on candidates are as before in the base ex-ante symmetric setup. Call this the *rich preferences model*. The following result shows that our earlier logic carries over without change.

Theorem 3. *The rich preferences model has a unique equilibrium. It is an ex-post equi-*

librium where both candidates use the same strategy, $\overline{G}$, with density

$$\overline{g}(x) = \max\left\{0, \frac{bf(x)}{1-b}\left[\frac{\lambda^m(x)}{\overline{\alpha} - \mu(x)} - 1\right]\right\}$$

where $\overline{\alpha} \in (\mu(m), \mu(m) + \lambda^m(m))$ is the unique constant such that $\int_x \overline{g}(x)dx = 1$.

A proof is omitted, since it parallels that of Theorem 1 closely.

4.5 Multiple Candidates

Nothing in our analysis of either the base model or the above extensions depends on there only being two candidates. Since the key feature of equilibrium is that the median voter is indifferent over all platforms in the support of G and strictly prefers these to any platform outside the support, our findings readily extend to any number of candidates.[13] That is, in the base symmetric model, for example, even if there are $n \geq 2$ candidates, each candidate when strategic plays the equilibrium strategy G. The median voter is indifferent over any candidate whose platform is in the support of G, and therefore, the ex-ante probability of any candidate winning is $\frac{1}{n}$.

This presents a striking contrast with the standard Hotelling-Downs model, where equilibrium changes drastically from two candidates to more than two candidates. Typically, pure strategy equilibria do not exist, mixed strategy equilibria often do not exist, and even if they do, they are untractable (Osborne, 1993). Our theory provides some insight into why even if there are two dominant candidates in an election, both of whom choose platforms relatively close to the median, there may well be "fringe" candidates with quite extreme platforms, who have essentially no chance of winning: the former are politicians who may or may not possess character—the closer they are to median, the less likely they are to; the latter are politicians who the public can be sure do in fact possess character.[14]

5 Conclusion

"In a president, character is everything." — Peggy Noonan

This paper develops a theory of character in elections. The two key assumptions we make are that some candidates may have character and do not strategically choose policy platforms to simply maximize the probability of getting elected, and voters value character in addition to campaign promises. Character quashes the median voter theorem, as strategic candidates pretend to have the positions of candidates with character. Elections

[13]While this intuition relies on the symmetric model, the result applies generally. Proofs are omitted since they follow straightforwardly from the proofs of Theorems 1, 2, and 3 respectively.

[14]We say "essentially no chance" to allow for some uncertainty about the median voter's location. This also permits the equilibrium to be robust to a (small) cost of running for office.

between strategic candidates are tied in the symmetric version of our model. As character becomes more important to voters, the behavior of strategic candidates grows further away from the ideal policy of the median voters and closer to the behavior of candidates with character.

In discussing extensions to richer preferences, we sketched a model of endogenous preference for character, where character entails following the promised policy in all circumstances, even when the choice is unobservable to voters. Voters who care about the unmonitored behavior will naturally care about the character of candidates. However, our treatment of this setting is very stylized, and it would be useful to model this in a more detailed way.

On the politicians' side, certainly our assumption that character is "all or nothing" is stark. While we do think that a population of agents should include both extremes of the character spectrum we have modeled, it would also be interesting to consider intermediate character types, such as those who are willing to pander but face a cost of doing so.[15] In addition, the introduction of policy motivated candidates would enrich the theory, if policy motivated candidates with character do what they promised even out of sight of voters, whereas policy motivated candidates without character follow their own preferences whenever possible.

We have modeled the proportion of candidates possessing character as an exogenous parameter, but in fact candidates usually run for higher office only after a complex winnowing process that includes serving for lower offices. Does this process tend to favor candidates with character or strategic candidates? In the model, the probability that a candidate with character is elected is no more, and maybe strictly less, than the probability that a strategic candidate is (see Fact 1). This suggests that the winnowing process may favor strategic candidates. However, if candidates with character are more likely to be re-elected or advance to higher office—perhaps because a candidate's type may be discovered with some probability once in office—then character may actually increase through the political hierarchy. A richer analysis is needed to illuminate the dynamics of candidate selection, but we view our model as a useful starting point.

[15]This is reminiscent of Banks (1990), but it is important to emphasize that his model is one without policy commitment.

Appendix: Proofs

To prove Theorem 1, some preliminaries are needed. Note that due to the continuum of policy locations, various statements about optimal strategies will be subject to "almost all" qualifiers; we suppress such caveats unless essential.

The first Lemma shows that a candidate must win with positive probability when strategic.

Lemma A.1. *In any equilibrium, the strategic type of a candidate wins with positive probability; thus, ex-ante, both candidates win with positive probability.*

Proof. Without loss of generality, it suffices to show that for some set of Y of positive G^A-measure, $\phi^A(x) \leq \phi^B(x)$ for G^A-a.e. $x \in Y$. (Because then by concentrating mass on Y, strategic B can win with positive probability.) This is immediate if G^A has atoms because $\phi^A(x) = 0$ for any x that G^A has an atom at, so suppose that G^A is atomless, hence absolutely continuous with density g^A. Let X^B be the set of all non-atomic points of G^B; since G^B can have only a countable number of atoms, X^B has full G^A-measure and there is a density of G^B, denoted g^B, on X^B. Then for a.e. $x \in X^B$, $\phi^i(x) = \frac{bf(x)}{bf(x)+(1-b)g^i(x)}$, and there must be a set $Y \subseteq X^B$ with the desired properties; otherwise $1 = \int_{X^B} g^A(x)dx < \int_{X^B} g^B(x)dx \leq 1$, a contradiction. □

Now we show that both candidates must play mixed strategies.

Lemma A.2. *In any equilibrium, neither candidate plays a pure strategy.*

Proof. By way of contradiction, suppose there is an equilibrium where candidate j is playing a pure strategy of choosing platform $\hat{x}$. This implies that $\phi^j(\hat{x}) = 0$ and $\phi^j(x) = 1$ for all $x \neq \hat{x}$. It suffices to argue that j must win probability 1, since this is a contradiction with Lemma A.1. First note that if G^i has an atom on x, then $\alpha^i(x) = \mu(x) < \lambda + \mu(m)$. Moreover, for any $x \neq m$, $\alpha^i(x) < \lambda + \mu(m)$. Now consider j choosing a platform $m + \varepsilon$ ($m + \varepsilon \neq \hat{x}$) with small $\varepsilon > 0$. Then $\alpha^j(m + \varepsilon) = \lambda + \mu(m + \varepsilon) > \alpha^i(x)$ for all x outside an ε-neighborhood of m and those x inside the neighborhood that G^i has atoms at. It follows that by picking ε arbitrarily small, j can win with probability arbitrarily close to 1. Thus, if j does not win with probability 1, he has a profitable deviation. □

We now define an **ex-post** equilibrium, which is an equilibrium such that even if a candidate observed his opponent's platform before choosing his own, he would have no incentive to deviate from his prescribed strategy.

Definition 1. An equilibrium with cdf's G^A and G^B is an *ex-post equilibrium* if the probability that candidate $i \in \{A, B\}$ wins is the same for all realizations of x^A and x^B in the support of G^A and G^B respectively.

Given Lemma A.1, it is straightforward that a pair of cdf's $\left(G^A, G^B\right)$ constitutes an *ex-post equilibrium* strategy profile if and only if for each $i \in \{A, B\}$:

$$\begin{aligned} \alpha^i\left(x^i\right) &= \alpha^j\left(x^j\right) \text{ for all } x^i \in Supp[G^i] \text{ and } x^j \in Supp[G^j] && \text{(A-1)} \\ \alpha^i\left(y\right) &\leq \alpha^i\left(x\right) \text{ for all } x \in Supp[G^i] \text{ and } y \notin Supp[G^i] && \text{(A-2)} \end{aligned}$$

The powerful result we derive is that every equilibrium is an ex-post equilibrium.

Lemma A.3. *Any equilibrium is an ex-post equilibrium.*

Proof. The following notation will be used:

$$\begin{aligned} \overline{x^i} &\in \arg\max_x \alpha^i(x|\phi^i) \\ \overline{\alpha^i} &\equiv \max_x \alpha^i(x|\phi^i) \\ \underline{x^i} &\in \arg\min_x \alpha^i(x|\phi^i) \\ \underline{\alpha^i} &\equiv \min_x \alpha^i(x|\phi^i) \end{aligned}$$

First, we prove that condition (A-1) must hold in any equilibrium. It suffices to show that for $i \in \{A, B\}$, $\overline{\alpha^i} = \underline{\alpha^i}$, and $\overline{\alpha^A} = \overline{\alpha^B}$. The latter is straightforward: if not, wlog say $\overline{\alpha^A} > \overline{\alpha^B}$, then playing $\overline{x^A}$ guarantees election for A, contradicting equilibrium mixing and Lemma A.1. Similarly, $\underline{\alpha^A} = \underline{\alpha^B}$. Now, if $\overline{\alpha^i} > \underline{\alpha^i}$, then $\underline{x^i}$ loses at least against $\overline{x^j}$, whereas $\overline{x^i}$ would win with probability $\frac{1}{2}$ against $\overline{x^j}$ and moreover against anything that $\underline{x^i}$ either ties or beats, so it is not optimal for $\underline{x^i}$ to be in the support of i's strategy.

Now, we prove necessity of condition (A-2). If condition (A-2) does not hold, then given condition (A-1), one of the candidates can profitably deviate to winning the election with probability 1, since he only wins with lower probability in equilibrium by Lemma A.1. □

The following Lemma formally shows that strategic candidates do not use mass points in their strategies.

Lemma A.4. *In any equilibrium, both G^A and G^B are atomless.*

Proof. Suppose that G^i has an atom on $\hat{x}$. Then $\alpha^i(\hat{x}) = \mu(\hat{x})$. Since there can be only a countable number of atoms, we can find a small $\varepsilon \neq 0$ such that

$$\alpha^i(\hat{x} + \varepsilon) = \lambda\phi^i(\hat{x} + \varepsilon) + \mu(\hat{x} + \varepsilon) > \mu(\hat{x})$$

If $\hat{x} + \varepsilon \in Supp[G^i]$, condition (A-1) is violated; if $\hat{x} + \varepsilon \notin Supp[G^i]$, condition (A-2) is violated. Either way, G^i cannot be an equilibrium strategy. □

Accordingly, any equilibrium strategy, G^i, has a density, g^i, and by Bayes rule,

$$\phi^i(x) = \frac{bf(x)}{bf(x) + (1-b)\, g^i(x)}$$

The following Lemma shows that the support of a candidate's strategy must be in an interval containing the median voter's ideal policy.

Lemma A.5. *In any equilibrium, for $i \in \{A, B\}$, Supp$[G^i]$ is an interval that contains m.*

Proof. First we argue that the support is an interval. Suppose not for player i. Then there exist $x > y > z$ such that $g^i(x) > 0$, $g^i(y) = 0$, and $g^i(z) > 0$. This implies that $\phi^i(x) < 1$, $\phi^i(y) = 1$, and $\phi^i(z) < 1$. If $y \geq m$, it follows that $\alpha^i(y) > \alpha^i(z)$; if $y \leq m$ then $\alpha^i(y) > \alpha^i(x)$. Either case contradicts condition (A-2) for an ex-post equilibrium.

Next, we show that the interval must contain m. If it didn't, wlog say it is $[l, h]$ with $h < m$, then $\phi^i(m) = 1$, hence for any $x \in [l, h]$,

$$\alpha^i(m) = \lambda + \mu(m) > \lambda \phi^i(x) + \mu(x) = \alpha^i(x)$$

contradicting condition (A-2) for an ex-post equilibrium. □

The above lemmata in hand, we can now proceed with the proof of Theorem 1.

Proof of Theorem 1 on page 9.

(Existence) We first prove that both players playing G^* is an ex-post equilibrium for some constant α^*. It is easy to verify that given G^*, the posterior belief is the following function ϕ^* for both candidates (so we drop the superscripts indexing candidates):

$$\phi^*(x) = \begin{cases} \frac{\alpha^* - \mu(x)}{\lambda} & \text{if } g^*(x) > 0 \\ 1 & \text{if } g^*(x) = 0 \end{cases}$$

Accordingly,

$$\alpha(x|\phi^*) = \lambda \phi^*(x) + \mu(x) = \begin{cases} \alpha^* & \text{if } g^*(x) > 0 \\ \lambda + \mu(x) & \text{if } g^*(x) = 0 \end{cases}$$

Noting from (1) that $g^*(x) = 0$ requires $\alpha^* - \mu(x) \geq \lambda$, one sees that conditions (A-1) and (A-2) for an ex-post equilibrium are indeed satisfied. It only remains to verify that there is a constant α^* which makes g^* a density. Since $g^*(x) \geq 0$ for all $x \in X$, we need to only check that $\int_X g^*(x)\, dx = 1$ for some α^*.

Define

$$\gamma(x, \alpha) = \max\left\{0, \frac{bf(x)}{1-b}\left[\frac{\lambda}{\alpha - \mu(x)} - 1\right]\right\}$$

so that $g^*(x) = \gamma(x, \alpha^*)$. We now prove that there is a unique constant α^* such that $\int_X \gamma(x, \alpha^*)\, dx = 1$. First observe that γ is continuous in α. Next, note that if $\alpha \geq \lambda + \mu(m)$, then $\lambda \leq \alpha - \mu(x)$ for all $x \in X$, hence $\gamma(x, \alpha) = 0$ for all $x \in X$ and $\int_X \gamma(x, \alpha)\, dx = 0 < 1$. On the other hand, for $\alpha = \mu(m)$ and small $\varepsilon > 0$,

$$\begin{aligned}
\int_X \gamma(x, \mu(m))\, dx &\geq \int_{m-\varepsilon}^{m+\varepsilon} \gamma(x, \mu(m))\, dx \\
&= \int_{m-\varepsilon}^{m+\varepsilon} \max\left\{0, \frac{bf(x)}{1-b}\left[\frac{\lambda}{\mu(m) - \mu(x)} - 1\right]\right\} dx \\
&\approx \frac{bf(m)}{1-b}\left[\int_{m-\varepsilon}^{m+\varepsilon} \frac{\lambda}{\mu(m) - \mu(x)} dx - 2\varepsilon\right] \\
&\approx \frac{bf(m)}{1-b}\left[\int_{m-\varepsilon}^{m+\varepsilon} \frac{\lambda}{-\frac{1}{2}\mu''(m)(x-m)^2} dx - 2\varepsilon\right] \\
&\approx \infty
\end{aligned}$$

where the penultimate line follows from $\mu'(m) = u_1(m, m) = 0$ and Taylor expansion, and the last line from the fact that $\mu''(m) = u_{11}(m, m) \leq 0$. By the Intermediate Value Theorem, there is a value of α, call it $\alpha^* \in (\mu(m), \mu(m) + \lambda)$, that satisfies $\int_X \gamma(x, \alpha^*)\, dx = 1$. It is straightforward that there cannot be any other value of α such that $\int_X \gamma(x, \alpha)\, dx = 1$ since $\gamma(x, \alpha)$ is strictly decreasing in α for any x such that $\gamma(x, \alpha) > 0$.

(Uniqueness) Now we prove that both players playing G^* is the unique equilibrium. Denote the support of G^* by $[l^*, h^*] \supsetneq \{m\}$. Suppose there is another (necessarily ex-post, by Lemma A.3) equilibrium where a candidate i plays $G^i \neq G^*$. By Lemmas A.4 and A.5, G^i has a density g^i with support $[l, h] \supsetneq \{m\}$. Since $G^i \neq G^*$ and their supports have non-empty intersection, there must be some non-degenerate interval, $Y \subseteq [l, h]$, such that $g^i(x) > g^*(x)$ for all $x \in Y$. Then $\alpha^i(x) > \alpha^*(x)$ for all $x \in Y$, and ex-postness implies $\alpha^i(x) > \alpha^*(x)$ for all $x \in [l, h]$. Consequently,

$$g^i(x) > g^*(x) \text{ for all } x \in [l, h] \tag{A-3}$$

Since g^i and g^* are both densities, either $l > l^*$ or $h < h^*$ (or both). Suppose $l > l^*$ (the argument is analogous for $h < h^*$). By its construction, $g^*(x) > 0$ for all x in a small neighborhood of l. A contradiction ensues with (A-3) if we show that $g^i(x) \to 0$ as $x \to l$. To prove this, observe that $\phi^i(x) = 1$ for all $x < l$, and hence $\alpha^i(x|\phi^i) = \lambda + \mu(x)$ for all $x < l$. By condition (A-2) for ex-postness, and using Bayes rule, it must be that for any $\varepsilon > 0$, there is a $\delta > 0$ such that $g^i(x) < \varepsilon$ for all $x \in (l, l + \delta)$. $\square$

Proof of Fact 5 on page 11. Observe that for any $x \in Supp\left[G^*\right]$, when $\lambda \approx \infty$,

$$\begin{aligned} g^*(x) &= \frac{bf(x)}{1-b}\left[\frac{\lambda}{\alpha^* - \mu(x)} - 1\right] \\ &= \frac{bf(x)}{1-b}\left[\frac{1}{\frac{\alpha^*}{\lambda} - \frac{\mu(x)}{\lambda}} - 1\right] \\ &\approx \frac{bf(x)}{1-b}\left[\frac{\lambda}{\alpha^*} - 1\right] \end{aligned}$$

The proof is completed by noting from (1) that $\frac{\lambda}{\alpha^*} \to \frac{1}{b}$ as $\lambda \to \infty$. $\square$

Proof of Fact 6 on page 11. Let the value α_f^* refer to the the utility generated by f and α_h^* refer to that generated by h. Since $\frac{\lambda}{\alpha^* - \mu(x)} - 1$ is decreasing as x moves away from the median, m, $h(x) - f(x)$ and $\frac{\lambda}{\alpha^* - \mu(x)} - 1$ are positively correlated. If the support of g^* is the whole policy space,

$$\int \frac{b(h(x) - f(x))}{1-b}\left[\frac{\lambda}{\alpha_f^* - \mu(x)} - 1\right] dx > \int \frac{b(h(x) - f(x))}{1-b} dx \int \left[\frac{\lambda}{\alpha_f^* - \mu(x)} - 1\right] dx = 0$$

and thus

$$\int_x \frac{bh(x)}{1-b}\left[\frac{\lambda}{\alpha_f^* - \mu(x)} - 1\right] dx > 1$$

It follows that $\alpha_h^* > \alpha_f^*$. $\square$

Proof of Fact 7 on page 12. For the case of $b \to 1$, inspection of (1) shows that α^* increases in b without no bound. Since $Supp(g^*) = \{x|\lambda + \mu(x) \geq \alpha^*\}$, as α^* increases, the support shrinks to $\{m\}$. (Note that m is always in the support; see Lemma A.5.)

As $b \to 0$, inspection of (1) shows that $\alpha^* \to \mu(m)$. Thus indeed $Supp(g^*) \to \{x|\mu(x) + \lambda \geq \mu(m)\}$. To see that G^* converges in distribution to point-mass on m, observe that for as $b \to 0$, for any $\varepsilon > 0$, both $\int_0^{m-\varepsilon} \max\left\{0, \frac{bf(x)}{1-b}\left[\frac{\lambda}{\alpha^* - \mu(x)} - 1\right]\right\} dx \to 0$ and $\int_{m+\varepsilon}^1 \max\left\{0, \frac{bf(x)}{1-b}\left[\frac{\lambda}{\alpha^* - \mu(x)} - 1\right]\right\} dx \to 0$. $\square$

Proof of Theorem 2 on page 14. The first part is almost identical to the existence portion of Theorem 1, hence omitted. To prove the second and third parts, two intermediate claims are needed. For $i \in \{A, B\}$, let $\hat{\alpha}^i$ denote the constant defined by $\int_X \hat{g}^i(x)\, dx = 1$.

Claim 1: $\hat{G}^i$ is the unique solution to the following program:

$$\max_{G^i, \phi^i}\left[\min_{x \in Supp(G^i)} \lambda\phi^i(x) + \mu(x)\right] \text{ s.t. } \phi^i \text{ being a posterior given } G^i \quad \text{(A-4)}$$

Proof: Let $\tilde{G}^i$ be a solution to program (A-4). We argue that $\tilde{G}^i = \hat{G}^i$. The support of $\tilde{G}^i$ must be contained in the support of $\hat{G}^i$; otherwise by the construction of $\hat{G}^i$, it is immediate that $\tilde{G}^i$ cannot be a solution to (A-4). Clearly then $\tilde{G}^i$ has no atoms; hence it has a density. But then if there is an interval on which $\tilde{g}^i(x) > \hat{g}^i(x)$, there must be an interval on which $\tilde{g}^i(x) < \hat{g}^i(x)$, for the densities to integrate to 1. This contradicts $\tilde{G}^i$ being a solution to (A-4). ‖

Claim 2: $\hat{G}^i$ is the unique solution to the following program:

$$\min_{G^i,\phi^i}\left[\max_x \lambda\phi^i(x) + \mu(x)\right] \text{ s.t. } \phi^i \text{ being a posterior given } G^i \tag{A-5}$$

Proof: Let $\breve{G}^i$ be a solution to program (A-5). We argue that $\breve{G}^i = \hat{G}^i$. Suppose not. Let $\breve{\alpha}^i \equiv \max_x \lambda\breve{\phi}^i(x) + \mu(x)$, where $\breve{\phi}^i$ is a posterior that is a solution to program (A-5). We have $\breve{\alpha}^i < \hat{\alpha}^i$. If $\breve{G}^i$ has mass outside the support of $\hat{G}^i$, then there must be an interval inside the support of $\hat{G}^i$ on which $\breve{G}^i$ has a density $\breve{g}^i(x) < \hat{g}^i(x)$, which contradicts $\breve{\alpha}^i < \hat{\alpha}^i$. So the support of $\breve{G}^i$ is within that of $\hat{G}^i$. But then, since the distributions are not the same by hypothesis, there must be an interval on which $\breve{G}^i$ has a density $\breve{g}^i(x) < \hat{g}^i(x)$, which contradicts $\breve{\alpha}^i < \hat{\alpha}^i$. ‖

Proof of Part (2) of the Theorem: Suppose that $\hat{\alpha}^A = \hat{\alpha}^B$ and let (G^A, G^B) be an equilibrium. By Claim 2, some platform for A provides a utility of at least $\hat{\alpha}^A$; by Claim 1, some platform in the support of G^A provides a utility weakly less than $\hat{\alpha}^A$. The same applies to player B. Since $\hat{\alpha}^A = \hat{\alpha}^B$, we conclude that both players win with positive probability when strategic. By the same logic as Lemma A.3, it follows that the equilibrium must be ex-post, and all platforms in the support of each G^i must provide the same utility. So now suppose towards contradiction that without loss of generality, $G^A \neq \hat{G}^A$. Claim 1 implies that some platform in the support of G^A provides utility strictly less than $\hat{\alpha}^A$, which by ex-postness extends to all platforms in the support. But Claim 2 implies than B has a platform that provides utility at least that of $\hat{\alpha}^B = \hat{\alpha}^A$, implying that B must win with probability 1 if strategic, a contradiction.

Proof of Part (3) of the Theorem: If $\hat{\alpha}^i > \hat{\alpha}^j$ then for any equilibrium (G^i, G^j), something in the support of G^j gives weakly less utility than $\hat{\alpha}^j$ (Claim 1) and some platform for i gives gives weakly more utility than $\hat{\alpha}^i$ (Claim 2). So i wins with positive probability when strategic. If j wins with positive probability when strategic, then by the same logic as Lemma A.3, it follows that the equilibrium must be ex-post, and all platforms in the support of both distributions G^i and G^j must provide the same utility. But then, i has a profitable deviation to some platform outside the support of G^i, a contradiction. □

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