

Capital Market Prices, Management Forecasts, and Earnings

Management*

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Abstract

The paper studies a manager's optimal earnings forecasting strategy and optimal accruals management policy in a setting where both the mean and the variance of the distribution generating the firm's cash flows are unknown. The paper shows that the equilibrium price of the firm is a function of the manager's forecast, the firm's reported earnings, and the squared error in the manager's earnings forecast. The model in the paper contains several predictions, including: (i) the manager manipulates accruals to reduce his forecast error at the earnings announcement date; the less persistent the firm's cash flows the stronger is the manager's incentive to reduce his forecast error by accruals manipulation; (ii) the firm's stock price is more sensitive to the firm's actual earnings announcement than to the manager's forecast, and (iii) controlling for the level of reported earnings and the magnitude of the earnings surprise, the firm's price is higher when it has a positive surprise at the earnings announcement date than when it has a negative surprise.

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1 Introduction

This paper contains a model of management earnings forecasts and accruals manipulation. The goal of the paper is to study managers' propensity to bias their forecasts of earnings and to manipulate accruals in a parsimonious model, and also to evaluate the capital market's equilibrium reactions to management forecasts and earnings releases. In the model, a manager's incentives to manage accruals and distort his earnings forecast are linked through the manager's attempt to influence his firm's market value. I contrast the predictions of the model with empirical results regarding discretionary accruals and management forecasts. Many of the predictions of the model are consistent with established empirical findings. For example, the model provides one potential explanation for why managers care about the error in their earnings forecasts and why capital markets value firms with positive earnings surprises at a premium even after controlling for the level of reported earnings.

Unlike most analytical models of earnings management or discretionary accruals, the model assumes that investors are uncertain about the process generating both the mean and the variance of the firm's (present value of) cash flows.¹ This seems to be the realistic case: in practice, investors are uncertain about many aspects of the distribution of a firm's cash flows.² Their knowledge of both the mean and variance of this distribution is affected by both the firm's management's earnings forecast and the firm's earnings report. Investors use the information they receive about the firm's earnings to update their beliefs about the unknown mean and variance of cash flows. The manager has incentives to manipulate his earnings forecast and earnings report so that investors perceive the firm's expected cash flows to be high and the variance of cash flows to be low. The model also contains a parameter that affects the persistence of the firm's cash flows over time. By changing this parameter, I evaluate how the persistence and predictability of cash flows influence the manager's reporting behavior, as well as how capital market participants assess the firm's value.

The model has five main predictions. First, it predicts that the relation between reported earnings and

¹Most analytical models assume that means of distributions are unknown but variances are known. Exceptions are Subramanyam (1996), Penno (1996), and Kirschenheiter and Melumad (2002).

²"Cash flows" as referred to here and throughout the paper is short-hand for the present value of future cash flows.

equity prices is non-linear.³ The firm's market price is demonstrated to be a function of the manager's forecast, the actual earnings report, and the squared management earnings forecast error. The market price decreases in the absolute value of the manager's forecast error because investors perceive the firm's cash flows to be riskier if the manager's forecast error is large in magnitude. The weight investors attach to the manager's forecast error in the pricing function decreases in their risk-aversion.

Second, the manager manipulates accruals to reduce the earnings surprise, i.e. his earnings forecast error, at the earnings announcement date. The less persistent the firm's cash flows, the stronger is the manager's incentive to reduce his forecast error. When managers adhere to this equilibrium reporting strategy, the capital market values firms with positive earnings surprises at a premium. That is, controlling for the level of reported earnings and the magnitude of the earnings surprise, firms with earnings exceeding their forecast will enjoy a higher capital market valuation than firms that disappoint investors by reporting earnings below their forecast.

Third, the model predicts that the manager biases his forecast downward such that the average forecast error (measured as the difference between reported earnings and the manager's forecast) is positive. The capital market induces the manager to issue a forecast that he is likely to beat, i.e. a "pessimistic" forecast, because the market values firms whose reported earnings exceed their managers' forecasts at a premium compared to firms whose reported earnings fall short of their managers' forecasts.

Fourth, the model predicts that the sensitivity of a firm's stock price to the earnings forecast is smaller than to the actual earnings announcement and that the difference between those two response coefficients varies systematically with the persistence of cash flows. It is intuitive that investors are more sensitive to earnings reports than to earnings forecasts given the manager's tendency to reduce earnings surprises at the earnings announcement date by selecting discretionary accruals appropriately. It is also intuitive that the higher the persistence of cash flows, the stronger stock prices react to inferred first period cash flows at both the forecast release date and the earnings announcement date. Anticipating the manager's propensity

³While this paper focuses on the effect of unknown precision on the earnings-return relation, there exist other factors that may also cause the earnings-return relation to be nonlinear. For instance, nonlinearity of the relation between price and earnings may be due to the nonsustainability of negative earnings (Hayn 1995) or due to investors' limited liability (Fischer and Verrecchia 1997).

to reduce his forecast error, investors rationally revise their beliefs about expected cash flows more if cash flows are less persistent. Together, these facts imply that the difference between the stock price response coefficient at the earnings announcement date and the stock price response coefficient at the forecast release date is increasing in the persistence parameter when the persistence of cash flows is either negative or slightly positive, and is decreasing in the persistence parameter for large positive values of persistence.

Finally, the model predicts that the expected stock price reaction to earnings surprises increases as the cost to the manager of manipulating accruals declines. This finding initially seems counterintuitive. It becomes intuitive once one recognizes that by lowering the cost of accruals manipulation, the manager increases the amount he will understate the surprise at the earnings announcement date. The latter follows because of his incentives to manipulate reported earnings so that investors perceive the variance of cash flows to be low.

There is empirical evidence consistent with several of the predictions of the model. For instance, the prediction that capital markets value firms with positive earnings surprises at a premium even after controlling for the level of reported earnings is broadly consistent with evidence provided in Soffer, Thiagarajan, and Walther (2000). Soffer et al. show that, controlling for the combined news released at the earnings preannouncement and announcement dates, the total excess return over the time period that contains both the earnings preannouncement and the earnings announcement is higher for firms that have a positive surprise at the earnings announcement. In a related experimental study Tan, Libby, and Hunton (2002) analyze how analysts react to firms' earnings preannouncement strategies. Similar to the results of Soffer, Thiagarajan, and Walther (2000), they find that lower earnings preannouncements lead to higher forecasts after the earnings announcement date. That is, holding reported earnings news constant, Tan et al. find that analysts' forecasts of earnings increase when the earnings preannouncement understates the amount of positive news and exaggerates the amount of negative news. To the extent that analyst forecasts are related to market values, the experimental findings in Tan et al. support the prediction of the model that investors value firms with positive surprises at a premium.

Another prediction of the model that has been confirmed empirically is the prediction that capital mar-

kets “place more weight” on news at the earnings announcement date (as measured by the price response coefficient) than on news at the forecast release date. Pownall, Wasley, and Waymire (1993) show that the mean and median stock price response coefficients are larger at the earnings announcement date than at the forecast release date, consistent with the model’s predictions. A possible explanation for the different stock price response coefficients is that earnings affect investors’ beliefs to a greater extent than forecasts because earnings are a more precise signal of future cash flows than forecasts. The model presented here puts forward an alternative explanation for this difference in stock price response coefficients. Since the manager attempts to reduce his forecast error by taking discretionary accruals, investors are highly sensitive to any earnings surprise at the announcement date. Also, consistent with the model’s predictions that managers care about their forecast errors, Kasznik (1999) finds that managers manipulate earnings upwards by taking positive discretionary accruals if reported earnings fall short of their own forecast.

All of the model’s predictions discussed above arise as a direct consequence of investors’ uncertainty about the variance of the firms’ cash flows. If one were to change the model so that investors knew the variance of cash flows, the model would predict that a firm’s stock price is independent of both the manager’s forecast and the error in that forecast. In that case, only the manager’s earnings report would influence the firm’s stock price. This highlights the importance of taking into account that the variance of cash flows is unknown.

Some recent papers model the effect of unknown precision on reporting and capital market prices. Jorgensen and Kirschenheiter (2003) derive managers’ equilibrium strategies for voluntarily disclosing information about the risk of the firm’s future cash flows in a setting where the manager privately learns the idiosyncratic variance of the firm’s cash flows and can disclose his private information at a fixed cost. Hughes and Pae (2004) consider a manager who privately learns the precision of a signal of the firm’s liquidating value and then chooses whether to disclose that precision. Hughes and Pae show that the manager discloses the precision if the precision is high (low) and the signal lies above (below) prior expectations. A distinguishing feature of the present model is that the manager does not directly disclose the variance of the firm’s cash flows (the manager might not know the precision himself). Instead, the manager issues an earnings

forecast and report and investors use this information to draw inferences about the variance of the firm's cash flows.

Subramanyam (1996) presents a model in which the precision of a signal about firm value obtained by the capital market is uncertain. This results in the relation between the signal (e.g. precision of reported earnings) and stock returns being nonlinear. When the random variable describing the precision has a gamma distribution, Subramanyam shows that returns are an “S-shaped” function of reported earnings, consistent with the nonlinear earnings-returns relation documented in Freeman and Tse (1992). Kirschenheiter and Melumad (2002) extend Subramanyam by giving the manager some discretion in reporting his firm's earnings. They show that both income-smoothing and “big bath” behavior characterize the manager's reporting strategy in equilibrium. In Penno (1996), management privately chooses the precision of a signal in a model where the management receives two signals about the firm's liquidating dividend. Penno shows that management's preferred precision for the second signal depends on the realization of the first signal and that there are equilibria in which investors successfully infer management's precision choice based on the realized value of the first signal.

The paper proceeds as follows. The model is presented in section 2. Section 3 describes the equilibrium forecasting, reporting and pricing rules. Section 4 describes additional characteristics of the manager's optimal forecasting and reporting strategies as well as the resulting stock price reaction and contrasts the predictions of the model with existing empirical evidence. Section 5 extends the model in several ways. Section 6 concludes. The appendix contains all proofs.

2 Model

This section outlines a two period model of management forecasts and accruals manipulation when both the mean and the variance of the firm's cash flows are unknown. The sequence of events is the following. The firm's production technology generates stochastic cash flows over two periods.⁴ The realized values of these

⁴The stochastic cash flows generated by the firm's production technology in the two periods need not to be realized over just these two periods. Rather, the cash flows might occur in future periods even though they are attributable to the firm's

cash flows are not directly observable to anybody outside the firm. However, the initial owner-manager obtains private information about the firm's first period cash flows that he may reveal to future investors by means of initially issuing an earnings forecast and subsequently reporting the firm's earnings. The manager's incentives to distort his forecast and reported earnings are linked through his attempt to maximize the firm's selling price net of his personal cost of forecast and earnings manipulation. At the end of the first period, the initial owner-manager sells the firm to outside investors. His motivation to sell the firm is taken to be exogenous.

During the first period the manager privately observes the expected value, μ_1 , of the firm's first period cash flows, \tilde{x}_1 . Based on the signal μ_1 , he issues a forecast, MF , of first period earnings. The manager is not confined to tell the truth and may issue a forecast that differs from his perception of the expected first period cash flows, μ_1 . However, if he does so he incurs a personal cost. The manager's personal cost of issuing a forecast that deviates from the observed signal, μ_1 , is $\frac{c_1}{2} (MF - \mu_1)^2$ where c_1 is a positive constant.⁵ At the end of the first period the manager privately observes the firm's first period cash flows, x_1 , and issues an earnings report, R . Again, the manager may issue a report that differs from actual first period cash flows, x_1 . The manager's personal cost associated with manipulating reported earnings is $\frac{c_2}{2} (R - x_1)^2$ for some positive constant c_2 . Since the initial owner-manager sells the firm at the end of the first period after he releases the earnings report but before the firm's first period cash flows become publicly observable the manager seeks to maximize the price of the firm at the end of the first period net of his personal cost from manipulating the forecast, MF , and the earnings report, R .

The second generation of shareholders that buys the firm at the end of the first period is entitled to the cash flows from both periods, \tilde{x}_1 and \tilde{x}_2 . The selling price of the firm is the expected value of the sum of $\tilde{x}_1 + \tilde{x}_2$ adjusted for a risk-premium factor. That is, given the public information set Ω , the selling price of

activities in the first and second periods. For simplicity, I assume that the discount rate is zero and I refer to the present value of cash flows attributable to the firm's first and second period activities as "first period cash flows" and "second period cash flows", respectively.

The present value of cash flows in a period can also be viewed as the unmanaged economic earnings of that period. It is common in the literature to assume that unmanaged earnings are priced by the capital market; see, e.g., Dye (1988), Fischer and Verrecchia (1997) and Sankar and Subramanyam (2001).

⁵The cost function can be generalized to $\frac{c_1}{2} (MF - \mu_1)^2$ for $MF \leq \mu_1$ and $c(MF, \mu_1) \geq 0$ for $MF > \mu_1$. The results also remain unchanged if the manager's personal cost of issuing a forecast is $\frac{c_1}{2} (MF - \tilde{x}_1)^2$. The stylized cost function might capture manager's litigation and reputation costs.

the firm is $P(\Omega) = E[\tilde{x}_1 + \tilde{x}_2 | \Omega] - \gamma \text{Var}[\tilde{x}_1 + \tilde{x}_2 | \Omega]$ where the parameter $\gamma > 0$ determines the magnitude of the risk-premium.⁶ γ captures investors' mean-variance trade-off and can be viewed as a measure of investors' risk-aversion. The firm's first and second period cash flows, \tilde{x}_1 and \tilde{x}_2 , are distributed normally with mean $\tilde{\mu}_1$ and $\tilde{\mu}_2$, respectively. The precision (inverse of variance) of \tilde{x}_1 is denoted by $\tilde{\tau}$. \tilde{x}_2 has the same precision as \tilde{x}_1 . The mean parameters in both periods, $\tilde{\mu}_1$ and $\tilde{\mu}_2$, are random variables themselves. Let $E[\tilde{\mu}]$ and σ_μ^2 denote the expected value and variance of the marginal distribution of $\tilde{\mu}_n$, $n = 1, 2$. For the moment, no restrictions on the marginal distribution of $\tilde{\mu}_n$ besides the existence of the first and second moment are required. The persistence parameter ρ links both realized and expected cash flows over time. That is, $\tilde{\mu}_2 = \rho\tilde{\mu}_1 + \tilde{\eta}_{2\mu}$ and $\tilde{x}_2 = \rho\tilde{x}_1 + \tilde{\eta}_2$ where $\tilde{\eta}_{2\mu}$ and $\tilde{\eta}_2$ are independent of all other random variables in the model.⁷ The persistence parameter ρ is a known constant, $\rho \in (-1, 1)$, and captures to what extent first period cash flows predict the level of second period cash flows. In the special case of $\rho = 0$, the cash flows \tilde{x}_1 and \tilde{x}_2 are independent random variables, and cash flows innovations are transitory.

Note that, while the extent to which second period cash flows reflect the level of first period cash flows is allowed to vary with the persistence parameter ρ , the precision $\tilde{\tau}$ stays the same – though unobservable – for both periods. $\tilde{\tau}$ may stay constant over time because the variability of cash flows is an inherent characteristic of a firm's production technology and/or market situation and those conditions do not change from one period to the next. The prior distribution of $\tilde{\tau}$ is $g(\tilde{\tau})$ where $g(\cdot)$ denotes a gamma distribution with shape-parameter $\alpha > 1$, scale-parameter $\beta > 0$, and location parameter zero.⁸ Moreover, the precision, $\tilde{\tau}$, is distributed independently of all other random variables of the model. Figure (1) summarizes the sequence of events in the model. The beginning of the first period is labeled $t = 0$, the time when the manager issues his forecast during the first period is $t = 1$, and the end of the first period when the manager reports earnings is $t = 2$.

The manager's forecasting and reporting rules, denoted $MF(\cdot)$ and $R(\cdot)$ respectively, maximize the

⁶Verrecchia (1983) and Melumad, Weyns, and Ziv (1999) make similar assumptions about firms' stock prices.

⁷ $E[\tilde{\eta}_{2\mu}] = E[\tilde{\eta}_2] = (1 - \rho)E[\tilde{\mu}]$, $\text{Var}[\tilde{\eta}_{2\mu}] = (1 - \rho^2)\sigma_\mu^2$ and $\text{Var}[\tilde{\eta}_2] = (1 - \rho^2)(\sigma_\mu^2 + \frac{1}{\tilde{\tau}})$. Appendix A presents a more detailed specification of the distributional assumptions (p. A1).

⁸The density $g(\tau)$ is given by $g(\tau) = \frac{\tau^{\alpha-1} \exp(-\beta\tau)}{\beta^{-\alpha} \Gamma(\alpha)}$ where $\Gamma(\cdot)$ denotes the complete gamma function.

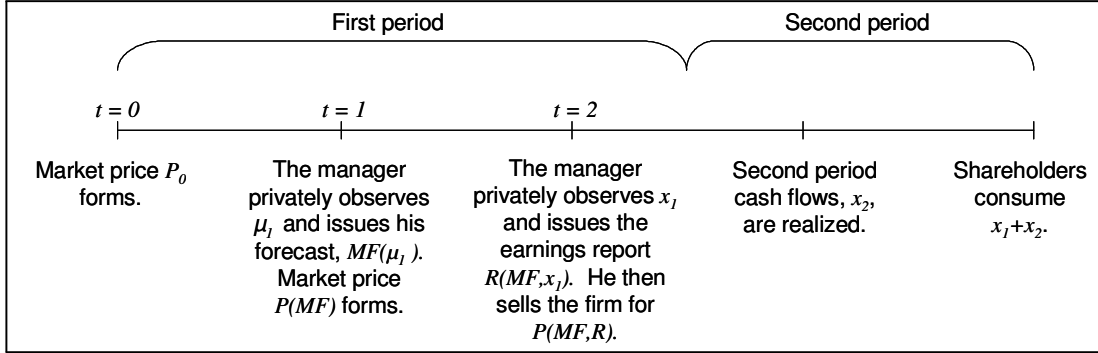


Figure 1: Timeline

expected price of the firm at the end of the first period net of the manager's cost from manipulating earnings forecast and report. Capital market prices, $P(\cdot)$, at each point in time equal the conditional expected value of cash flows, $x_1 + x_2$, less a risk-premium that is proportional to the variance of $x_1 + x_2$. In equilibrium, the forecasting and reporting rules the manager chooses are consistent with the forecasting and reporting rules investors expect the manager to choose. The equilibrium is formally defined as follows.

Definition 1 *An equilibrium consists of a forecasting rule $MF(\cdot)$, a reporting rule, $R(\cdot)$, and a capital market pricing rule $P(\cdot)$ such that:*

- (i) *Given the pricing function $P(\cdot)$, the manager's private information, μ_1 and x_1 , and forecast MF , reported earnings for the first period, $R(x_1, MF)$, maximize*

$$P(R, MF) - \frac{c_1}{2} (MF - \mu_1)^2 - \frac{c_2}{2} (R - x_1)^2;$$

- (ii) *Given the reporting rule, $R(\cdot)$, and the pricing function, $P(\cdot)$, and the manager's private information μ_1 , the forecasting rule $MF(\mu_1)$ maximizes*

$$E \left[P(R(\tilde{x}_1, MF), MF) - \frac{c_1}{2} (MF - \mu_1)^2 - \frac{c_2}{2} (R(\tilde{x}_1, MF) - \tilde{x}_1)^2 \middle| \mu_1 \right];$$

(iii) Given the forecasting rule, $MF(\cdot)$, and the reporting rule, $R(\cdot)$, the capital market prices $P(\Omega)$ satisfy

$$P(\Omega) = E[\tilde{x}_1 + \tilde{x}_2 | \Omega] - \gamma \text{Var}[\tilde{x}_1 + \tilde{x}_2 | \Omega];$$

where Ω is the publicly available information at the point the capital market price forms, and $\Omega \in \{\{\}, \{MF\}, \{MF, R\}\}$.

The definition of equilibrium is straightforward. In part (i), the manager takes the capital market pricing rule as given and decides on the optimal earnings report given the forecast he issued earlier as well as his private information, μ_1 and x_1 . The optimal reporting rule maximizes the firm's market value net of the costs of distorting the earnings report. In part (ii), the manager selects his forecast optimally, anticipating both the pricing rule and his subsequent optimal earnings report. In part (iii), the capital market price of the firm is set consistent with the manager's forecasting and reporting rule. As mentioned above, I assume that the signals released by the manager are the only information available to the capital market in the first period. At the beginning of the first period the investors have not yet received any information by the manager and hence the capital market price is conditioned on their prior beliefs. After the manager makes his forecast, MF , the capital market updates its beliefs about the first and second period cash flows. Similarly, at the end of the first period, the price is a function of both the forecast and the earnings report, taking the manager's optimal forecasting and reporting rules as given. The final price, $P(R, MF)$, is the price that enters the manager's objective function.

3 Equilibrium

The following Proposition characterizes the manager's optimal forecasting and reporting rule and the associated capital market equilibrium prices.⁹

⁹For the equilibrium in Proposition 1 to exist investors must not be too risk-averse. Appendix B discusses the reasons for the equilibrium in Proposition 1 to break down if investors are highly risk-averse and also derives an alternative degenerate equilibrium that exists irrespective of investors' risk-preferences.

Proposition 1 If $\gamma \leq \frac{c_2(2\alpha-1)}{8(1-\rho^2)}$ there is an equilibrium defined as follows.

(i) The manager's optimal reporting strategy is

$$R(x_1, MF) = r_0 + r_1 MF + r_2 x_1$$

where the coefficients are given by

$$\begin{aligned} r_2 &= \frac{1}{2} \left(1 + \frac{\sqrt{c_2 - \frac{8\gamma(1-\rho^2)}{2\alpha-1}}}{\sqrt{c_2}} \right) \\ r_1 &= 1 - r_2 \\ r_0 &= (1 + \rho) \left(\frac{1}{r_2 c_2} + \frac{r_1^2}{r_2 c_1} \right). \end{aligned}$$

(ii) The manager's optimal forecasting rule is

$$MF(\mu_1) = m_0 + \mu_1$$

and $m_0 = -(1 + \rho) \frac{r_1}{r_2 c_1}$.

(iii) The pricing function $P(R, MF)$ takes the form

$$P(R, MF) = p_0 + p_1 MF + p_2 R + p_3 \left(\frac{R - (r_0 + r_1 MF)}{r_2} - MF + m_0 \right)^2$$

where the pricing coefficients are given by

$$\begin{aligned} p_3 &= \frac{-\gamma(1-\rho^2)}{2\alpha-1} \\ p_2 &= \frac{1+\rho}{r_2} \\ p_1 &= 1 + \rho - p_2 \\ p_0 &= (1-\rho) E[\tilde{\mu}] - \gamma(1-\rho^2) \left(\frac{2\beta}{2\alpha-1} + \sigma_\mu^2 \right) - \frac{p_1^2}{c_1} - \frac{p_2^2}{c_2}. \end{aligned}$$

(iv) The capital market prices at $t = 0$ and $t = 1$ are given by

$$\begin{aligned} P_0 &= 2E[\tilde{\mu}] - 2\gamma(1 + \rho) \left(\sigma_\mu^2 + \frac{\beta}{\alpha - 1} \right) \\ P(MF) &= (1 + \rho) \left(MF - \frac{p_1}{c_1} \right) + (1 - \rho) E[\tilde{\mu}] - \gamma(1 + \rho) \left(\frac{2\beta}{\alpha - 1} + (1 - \rho) \sigma_\mu^2 \right). \end{aligned}$$

Since the vector (R, MF) is informationally equivalent to (x_1, μ_1) in equilibrium, the price at the end of the first period equals $P(x_1, \mu_1) = x_1 + E[\tilde{x}_2|x_1, \mu_1] - \gamma Var[\tilde{x}_2|x_1, \mu_1]$.¹⁰ Using the distributional assumption about the firm's cash flows, $P(x_1, \mu_1)$ yields

$$P(x_1, \mu_1) = (1 + \rho)x_1 + (1 - \rho)E[\tilde{\mu}] - \gamma(1 - \rho^2) \left(\frac{2\beta + (x_1 - \mu_1)^2}{2\alpha - 1} + \sigma_\mu^2 \right). \quad (1)$$

Rewriting the above equation in terms of MF and R yields the pricing equation as in part (iii) of Proposition 1. It follows that the price is quadratic in reported earnings and that the coefficient on the quadratic term (p_3) is negative. Thus, the stock price at the end of the first period is decreasing in reported earnings for large positive earnings surprises. The economic interpretation for that result is that the risk-averse investors associate a high riskiness of future cash flows with current earnings surprises that are large in magnitude. While an increase in reported earnings always has a positive effect for a given forecast on the market's inferred value of expected cash flows, for extreme positive surprises the increase in risk-premium will dominate the positive mean effect. Consequently, the firm's market valuation will decrease in reported earnings for extreme positive surprises. For large negative surprises, however, the firm's market valuation will always increase in reported earnings, because the updated beliefs about the riskiness of the firm's cash flows decreases the firm's market value even further as the magnitude of the surprise increases.

The above discussion emphasized the importance of the earnings surprise in the price formation process attributable to the assumption of unknown precision. The following Corollary restates the pricing equation of part (iii) in Proposition 1 in terms of reported earnings, the manager's forecast and the forecast error

¹⁰Section 5 presents a modified setup for which the arising equilibrium is such that (R, MF) is no longer informational equivalent to (μ_1, x_1) .

where the forecast error, or earnings surprise, is defined as the difference between reported earnings and the manager's forecast.

Corollary 1 *Let $FE = R - MF$. Then, in the equilibrium of Proposition 1*

$$P(R, MF) = \pi_0 + \pi_1 MF + \pi_2 R + \pi_3 FE^2$$

where the coefficients are given by $\pi_0 = p_0 + \frac{p_3}{r_2^2}(r_0 - r_2 m_0)^2$, $\pi_1 = p_1 + \frac{2p_3}{r_2^2}(r_0 - r_2 m_0)$, $\pi_2 = p_2 - \frac{2p_3}{r_2^2}(r_0 - r_2 m_0)$, and $\pi_3 = \frac{p_3}{r_2^2}$.

Corollary 1 shows that the price at the end of the first period can be written as a linear combination of forecast, report and squared forecast error. The coefficient on the squared forecast error is negative, reflecting a penalty or risk-premium that is proportional to the squared surprise at the earnings announcement date, consistent with the intuition presented following Proposition 1. The functional form of the risk-premium is due to the expected variance term in equation (1), which is quadratic in the difference between x_1 and μ_1 .

4 Implications

The equilibrium in Proposition 1 addresses several aspects of the relation among forecasts, earnings reports, earnings persistence and stock price reactions, that have been studied in the empirical literature. Each of the following corollaries discusses a specific property or implication of the equilibrium in Proposition 1 in more detail and provides a link to related empirical findings. The first three corollaries describe the manager's optimal reporting and forecasting strategies.

Investors use the information they receive about the firm's first period earnings to update their beliefs about both expected cash flows and the variance of cash flows. This provides the manager with incentives to manipulate reported earnings so that investors perceive expected cash flows to be high and the variance of cash flows to be low. The following Corollary describes how the manager's reporting function reflects his dual reporting incentives.

Corollary 2 *In the equilibrium of Proposition 1,*

(a) *expected first period reported earnings exceed expected first period cash flows, $E[R(\tilde{x}_1, MF(\tilde{\mu}_1))] > E[\tilde{x}_1]$; and*

(b) *the optimal reporting rule, $R(x_1, MF) = r_0 + r_1 MF + r_2 x_1$, satisfies $r_1, r_2 \in (0, 1)$ and $r_1 + r_2 = 1$.*

Investors make inferences about first period cash flows based on reported earnings. Persistence of cash flows leads investors to also update their beliefs about second period expected cash flows when they observe first period reported earnings. Since investors perceive total cash flows to be higher if first period reported earnings are higher, the manager uses his discretion to report first period earnings that on average exceed first period cash flows. This reporting behavior is established in part (a) of Corollary 2.

As discussed earlier, the pricing implication of reported earnings is twofold. Based on reported earnings, investors draw inferences not only about the level but also about the riskiness of future cash flows. Since the capital market attaches a risk-premium to forecast errors (see Corollary 1), the manager has a natural tendency to reduce his forecast error. Part (b) of Corollary 2 states that the manager reduces his forecast error by reporting a weighted average of forecast and cash flows (plus a constant) as first period earnings. Basing reported earnings on a weighted average of forecast and cash flows rather than on cash flows alone essentially “smooths” the way earnings news are reflected in management forecast and reported earnings.¹¹ As a result, the variance of reported earnings is smaller than the variance of cash flows.

Rewriting the manager’s reporting rule as a function of first period cash flows, x_1 , and the difference between his forecast and first period cash flows, $MF - x_1$, further illustrates the manager’s tendency to report earnings that are close to his forecast: $R(x_1, MF) = r_0 + x_1 + r_1(MF - x_1)$. If his forecast exceeds first period cash flows, x_1 , the manager reports higher earnings than if his forecast falls short of first period cash flows. In that way, the unknown riskiness of cash flows causes reported earnings to be a function of management’s forecast and, equivalently, the discretionary element in reported earnings to depend on the

¹¹Graham, Harvey, and Rajgopal (2005) document that managers smooth earnings because they believe that investors demand a lower risk-premium if the earnings path is smooth.

forecast.¹²

In equilibrium, discretionary accruals (i.e. the difference between reported earnings, R , and first period cash flows, x_1) is always income increasing if the forecast error (measured as the difference between reported earnings, R , and manager's forecast, MF) is negative. This follows because the manager's incentives to manipulate the investors' perception of the level and the variability of the firm's cash flows are aligned: manipulating reported earnings upward increases the level of cash flows inferred by investors and also decreases the manager's forecast error. However, this is not the case if first period cash flows exceed the manager's forecast. While manipulating reported earnings upward still increases reported income it also increases the manager's forecast error. As a consequence the manager needs to trade off stock price increases due to an increase in the perceived level of cash flows against stock price increases due to a reduction in the perceived riskiness of cash flows. Which effect is dominating depends on the magnitude of the difference between cash flows and forecast. If the difference is large, the variance effect dominates and the manager reports earnings that fall short of first period cash flows to reduce his forecast error. If the difference is small a further reduction does not have a significant effect on the perceived variability of cash flows and hence the manager prefers to manipulate reported earnings upward.

The above discussion leads to two additional predictions: (i) discretionary accruals and the manager's forecast error are negatively correlated and (ii) reported earnings on average exceed cash flows if the manager's forecast error is negative. If the manager's forecast error is positive, the model does not make any predictions about the sign of the average difference between reported and cash flows. This is again a consequence of the manager's contradictory incentives to manipulate earnings when first period cash flows exceed his own forecast. Because of these opposing incentives, the model predicts that the magnitude of discretionary element in reported earnings is smaller when the forecast error is positive than when the forecast error is negative.¹³

To the extent that discretionary accruals in the empirical literature can proxy for the discretionary ele-

¹²The relation between voluntary management forecasts and earnings management has also been studied by Dutta and Gigler (2002). In contrast to the model presented here, Dutta and Gigler study the relation between forecasting and reporting in a moral-hazard setting.

¹³The formal proofs of these two predictions are included in the proof of Corollary 2.

ment in reported earnings in this model, some of the above predictions have already been tested empirically. Kasznik (1999) provides empirical evidence on the relation between managers' forecast errors and discretionary accruals. He finds that mean and median discretionary accruals are positive if reported annual earnings miss the management forecast. If reported earnings exceed the management forecast, then both mean and median discretionary accruals are insignificantly different from zero. While this is largely consistent with the prediction of the relation between earnings management and management forecast described above, it is also consistent with alternative explanations discussed in Kasznik (1999). For instance, reputational concerns or potential legal actions may also cause managers to take discretionary accruals that reduce their own forecast errors. The asymmetry in the findings with respect to the sign of the forecast error may be due to those reputational or legal costs being larger for overestimates than for underestimates.

The degree to which the manager understates the surprise at the earnings announcement date by taking appropriate manipulative action depends on the cost of discretionary accruals and the benefits of making the firm's cash flows appear less risky. The sensitivity of reported earnings to first period cash flows, r_2 , reflects this cost-benefit trade-off. Corollary 3 provides related comparative statics.

Corollary 3 *In the equilibrium of Proposition 1, r_2 , the sensitivity of reported earnings to first period cash flows,*

- (a) *increases in the cost of manipulating the report, c_2 ;*
- (b) *decreases in investors' risk-aversion coefficient, γ , and increases in α ; and*
- (c) *decreases in the persistence parameter ρ for $\rho < 0$ and increases in ρ for $\rho > 0$.*

When choosing reported earnings the manager trades off the pricing implication of reported earnings and his personal cost from manipulating earnings. In equilibrium, the sensitivity of reported earnings to x_1 reflects the trade-off between signaling low riskiness (low sensitivity) and the manager's personal costs from taking manipulative actions. From this it follows that the equilibrium sensitivity is strictly less than one. This was established in part (a) of Corollary 2. The more expensive it is for the manager to manipulate

his report the closer his report will reflect actual cash flows. It follows that r_2 is increasing in c_2 with $\lim_{c_2 \rightarrow \infty} r_2 = 1$. This is established in part (a) of Corollary 3.

As investors become less risk-averse, the manager has less incentives to understate the variance of the firm's cash flows and hence the sensitivity of reported earnings to first period cash flows will be greater in equilibrium (part b of Corollary 3). To see this consider the extreme case where investors are risk-neutral and hence do not care about the riskiness of future cash flows ($\gamma = 0$). In this case $p_3 = 0$ and $R = \frac{p_2}{c_2} + x_1$ so that the sensitivity of R with respect to x_1 equals 1 while part (a) of Corollary 2 establishes that $r_2 < 1$ in equilibrium. Closely related to this comparative statics result concerning investors' risk-aversion is a comparative static result concerning investors' ex-ante beliefs about the firm's variance of cash flows. A higher value of the parameter α corresponds to a lower variance of cash flows as perceived by investors at the beginning of the first period, as well as investors' posterior beliefs being less sensitive to information observed throughout the period. The latter effect diminishes the manager's incentives to manipulate reported earnings, resulting in a higher value of r_2 for higher values of α .

Finally, part (c) of Corollary 3 considers the effect of persistence on the sensitivity of reported earnings to first period cash flows. The persistence and variance of the firm's cash flows are linked in the following way: The better investors are able to predict future cash flows the lower the remaining uncertainty of cash flows investors face when buying the firm and the lower the manager's incentives to manipulate reported earnings in a way that understates the firm's variance of cash flows. To what extent investors can predict future cash flows depends on the persistence of the firm's cash flows. If $\rho = 1$ or $\rho = -1$ investors could perfectly predict second period cash flows and the remaining uncertainty would be zero. In that case, firm's end-of-period stock price would not contain any risk premium, and investors' perception of the firm's variance of cash flows would be irrelevant. Consequently, the manager would not spend any resources on managing reported earnings in a way that made cash flows appear less risky. If $\rho = 0$, first period cash flows do not provide any information about expected second period cash flows. This provides the manager with maximum incentives to manage reported earnings so that investors perceive the firm's cash flows as less risky. Clearly, it is irrelevant whether cash flows are predictable because they are positively or negatively

correlated across periods. Hence, we can restate the result in part (c) of Corollary 3 as r_2 being increasing in the magnitude of the persistence parameter, $|\rho|$.

The pricing implication of reported earnings not only affects the manager's reporting strategy but also how equilibrium reporting and forecasting rules compare in terms of expected values.

Corollary 4 *In the equilibrium of Proposition 1, the expected forecast error, $E[FE] = E[R - MF]$, is*

- (a) *strictly positive;*
- (b) *increasing in the investors' risk-aversion coefficient γ ; and*
- (c) *decreasing in α .*

The manager's general tendency to overstate earnings leads to reported earnings that are on average larger than the manager's forecast. Hence, the forecast appears to be pessimistic even though it is fully rational relative to the manager's costs and benefits. In Corollary 3 we have seen that reported earnings are less sensitive to cash flows when investors are highly risk-averse or when the prior variance of cash flows is high (low values of α). Conversely, rational investors are very sensitive to earnings surprises when the sensitivity of reported earnings to cash flows is low because they rationally anticipate that managers have taken manipulative actions to conceal their news. As a consequence stock prices react strongly to earnings surprises providing managers with large incentives to overstate reported earnings.

The effect of the persistence of cash flows on forecast bias is ambiguous because persistence affects the manager's incentives to manipulate earnings in two ways. On the one hand higher values of ρ increase the manager's incentives to overstate earnings because of the inferences investors draw about second period expected cash flows. On the other hand, higher magnitudes of ρ reduce the manager's incentives to manipulate reported earnings such as to minimize surprises at the announcement date (as established in Corollary 3 part c). Since this in turn causes stock prices to be less sensitive to earnings surprises, the manager's incentives to increase reported earnings are smaller for larger magnitudes of ρ . For negative values of ρ the direction of the two effects coincide. The more negative the persistence of the firm's cash flows, the smaller is the expected equilibrium forecast error. However, for positive values of ρ the two forces are in opposition

to each other. Depending on the magnitude of ρ as well as other parameter values the expected forecast error can be either increasing and decreasing in ρ for $\rho > 0$.

The predicted pessimistic bias of the manager's forecast is consistent with management guiding analysts' and stock market expectation downward in order to obtain a positive surprise at the earnings announcement date as suggested by several empirical studies on analyst forecasts (e.g. Matsumoto 2002 and Richardson, Teoh, and Wysocki 2001). Yet, empirical evidence on the bias of management point and range estimates of future earnings is mixed. Choi and Ziebart (2000) analyze the bias in management forecasts as a function of the forecast horizon. The authors find that mean and median forecasts with a forecast horizon of up to three months are pessimistic while forecasts with a horizon of more than seven months appear optimistic. McNichols (1989) finds no evidence of bias in management forecasts and other studies such as Penman (1980) and Feng (2004) document an optimistic bias in management earnings forecasts.

Given any forecast MF , the expected forecast error is $r_0 - r_2 m_0 = \frac{p_2}{c_2} - \frac{p_1}{c_1}$ and the variance of the forecast error is $\frac{r_2^2 \beta}{\alpha - 1}$. The distribution of forecast errors is not normal since the precision is unknown.¹⁴ However, it is still smooth, unimodal, and symmetrically distributed around the mean. A pessimistic bias in the forecast implies that the frequencies of (small) negative forecast errors is smaller than the frequency of (small) positive forecast errors. Higher frequencies of small positive forecast errors over small negative forecast errors have been documented in the empirical literature for a variety of thresholds including analyst consensus forecasts and past earnings realizations (Hayn 1995, Burgstahler and Dichev 1997, Degeorge et al. 1999, and Abarbanell and Lehavy 2003). Feng (2004) provides evidence of a similar pattern for earnings surprises relative to point and range management forecasts. However, there is also empirical evidence that the frequency of small positive forecast errors does not just exceed the frequency of small negative forecast errors, but that the distribution of forecast errors is discontinuous around those thresholds. Since the model predicts the distribution of forecast errors to be smooth, an explanation for the empirical finding of a discontinuity around zero is beyond the scope of this model.

¹⁴The distribution of forecast errors is given by $f(FE|MF) = f(FE) = \frac{\Gamma(\alpha + \frac{1}{2})}{\Gamma(\alpha)(2\pi)^{1/2}} * \frac{r_2^{2(\alpha+1/2)} \beta^\alpha}{(\frac{1}{2}(FE - r_0 + r_2 m_0)^2 + r_2^2 \beta)^{\alpha+1/2}}$.

While Corollary 2, 3, and 4 characterized the equilibrium earnings report and forecast, the following Corollary discusses the implications of the manager's optimal forecasting and reporting strategies on the stock market reaction to the news released at the forecast and announcement date.

Corollary 5 *In the equilibrium of Proposition 1,*

- (a) *the average stock market sensitivity to news released at the earnings announcement, $E \left[\frac{\partial P(R, MF)}{\partial R} \right]$, is larger than the stock market sensitivity to news contained in the management forecast, $\frac{\partial P(MF)}{\partial MF}$;*
- (b) *the difference in stock market sensitivity, $E \left[\frac{\partial P(R, MF)}{\partial R} \right] - \frac{\partial P(MF)}{\partial MF}$, is increasing in the investors risk-aversion coefficient γ and decreasing in α and the cost of manipulating earnings c_2 ; and*
- (c) *the difference in stock market sensitivity, $E \left[\frac{\partial P(R, MF)}{\partial R} \right] - \frac{\partial P(MF)}{\partial MF}$, increases in ρ if $\rho < 0$ or $\rho \in (0, \frac{1}{3})$ and $\gamma < \frac{(2\alpha-1)c_2}{8(1-\rho^2)} \frac{(1+\rho)(1-3\rho)}{(1-\rho)^2}$ and decreases in ρ if $\rho > \frac{1}{3}$ or $\rho \in (0, \frac{1}{3})$ and $\gamma > \frac{(2\alpha-1)c_2}{8(1-\rho^2)} \frac{(1+\rho)(1-3\rho)}{(1-\rho)^2}$.*

Consistent with the prediction of Corollary 5, empirical evidence seems to suggest that the market “places more weight” on news at the earnings announcement as measured by the price change relative to the news contained in the earnings forecast. In particular, Pownall, Wasley, and Waymire (1993) show that the mean and median stock price response coefficients are larger at the earnings announcement date than at the forecast release date. One potential explanation for the difference in stock price response coefficients are systematic differences in the precision of the signal relative to the precision of the investors' beliefs at the forecast release date versus the earnings announcement date.

In the model presented here the reason for the different stock price response coefficients is not related to the precision of the signals and investor beliefs but rather to the fact that the news at the announcement date are understated because of the manager's incentive to make the firm's cash flows appear less risky. At the time the manager releases his forecast, the stock market learns the mean of the firm's first period cash flows. This merely causes a shift in the expected value of x_1 but has no implication for the perceived riskiness of future cash flows. Hence, the forecasting rule increases in the manager's private signal, μ_1 , with a slope of one. Consequently, the market response coefficient with respect to the forecast at the time the forecast is released is $\frac{\partial P(MF)}{\partial MF} = 1 + \rho$ reflecting the information in the manager's forecast on the sum of expected

first and second period cash flows. Since investors rely on reported earnings to update their beliefs about the variance of future cash flows, the sensitivity of reported earnings to the manager's private information is less than one ($r_2 < 1$ as established in Corollary 2). To infer the expected total cash flows the market uses $p_2 = \frac{1+\rho}{r_2}$ as a multiple on reported earnings. Moreover, in equilibrium, the average stock market reaction to news released at the earnings announcement is also $E \left[\frac{\partial P(R,MF)}{\partial R} \right] = E \left[p_2 + \frac{2p_3}{r_2} (\tilde{x}_1 - \mu_1) \right] = p_2 > 1 + \rho$. Hence, the average stock market reaction to news released at the earnings announcement is larger than the stock market reaction to news contained in the forecast. The more risk-averse investors are the greater the benefits of understating the riskiness of the firm's cash flows distribution and hence the higher p_2 , while investors' risk-aversion does not affect the marginal price reaction at the forecast release date.

The next result established in Corollary 5 relates the difference in price response coefficients to the cost of manipulating earnings. As it becomes less costly for the manager to manipulate reported earnings, stock prices react more strongly to surprises at the earnings announcement date. This might seem counter-intuitive. However, recall that the manager uses his manipulative actions to reduce the earnings surprise because of his incentives to manipulate reported earnings so that investors perceive the variance of cash flows to be low. Since investors correctly anticipate the manager's reporting strategy, the stock price reaction to earnings surprises increases as the cost of the manager to manipulate earnings declines.

The relative size of the coefficients and the fact that the market reacts significantly to the sign and magnitude of the news released at the forecast date is consistent with the empirical evidence provided in Pownall, Wasley, and Waymire (1993) who find that the mean and median coefficients are positive but significantly lower than one. The model predicts the earnings response coefficients to be greater than zero and the forecast response coefficient to take on values between 0 and 2 depending on the persistence of cash flows. If ρ is negative the size of the coefficients predicted by the model is roughly consistent with the empirical evidence in Pownall et al. Part (c) of Corollary 5 describes how the stock price response coefficients at the forecast release date and the earnings announcement date vary with persistence. A change in the persistence of cash flows has two effects on the stock market reaction. First, stock market reactions at the forecast release date and the earnings announcement date become more sensitive to forecasts and reported

earnings if the persistence increases because of the inferences investors draw about second period expected cash flows. Second, the stock market reaction at the earnings announcement date becomes less sensitive to reported earnings if the magnitude of the persistence increases because of the manager's incentives to understate the earnings surprise (see Corollary 3 part c). Together, the two effects cause the difference between the marginal stock price reactions at the earnings announcement date and at the forecast release date to increase in ρ for $\rho < 0$ and to decrease in ρ for $\rho > \frac{1}{3}$. If $\rho \in (0, \frac{1}{3})$ then other parameter values also affect the sign of the comparative statics.

Building on the results established in the preceding corollaries we can describe the stock market reaction at the earnings announcement date as a function of the magnitude and the sign of the surprise. Letting $\Delta > 0$ denote the absolute value of the surprise, a positive surprise implies $MF = R - \Delta$ and the price following such a surprise is $P(R, R - \Delta)$. If the surprise is negative, $R + \Delta$ and $P(R, R + \Delta)$ denote the forecast and the price following a negative surprise respectively.

Proposition 2 *In the equilibrium of Proposition 1,*

- (a) *the stock market price at the end of the first period is on average decreasing in the forecast, MF , holding reported earnings constant ($p_1 < 0$);*
- (b) *holding the level of reported earnings and the magnitude of the earnings surprise constant, the stock market price at the end of the first period is higher following a positive surprise than following a negative surprise, i.e. $P(R, R - \Delta) - P(R, R + \Delta) > 0$; and*
- (c) *holding the level of reported earnings and the magnitude of the earnings surprise constant, the stock price premium associated with a positive surprise compared to a negative surprise, $P(R, R - \Delta) - P(R, R + \Delta)$, increases in the magnitude of the earnings surprise, Δ .*

Since investors are risk-averse, the manager has incentives to manipulate the earnings report in a way that cash flows appear less risky. This causes reported earnings to be a weighted average of the forecast and first period cash flows where both weights, r_1 and r_2 , are positive. From this it follows that – holding reported earnings constant – investors will infer higher first period cash flows from a lower forecast. As a

result, the firm's market price at the end of the first period is on average decreasing in the manager's forecast. In the present model, total excess returns, $\frac{P(R, MF) - P_0}{P_0}$, behave the same as stock prices at the end of the period, $P(R, MF)$, and hence the findings in Proposition 2 can be restated in terms of returns. Part (a) of Proposition 2 predicts that returns measured over a period that includes both forecast release and earnings announcement decrease in the manager's forecast. Similarly, part (b) states that a firm earns excess returns if its earnings surprise is positive compared to if its earnings surprise is negative, holding reported earnings and the magnitude of the surprise constant. Finally, part (c) predicts the excess return to be increasing in the magnitude of the surprise.

While it is not true in general that the stock price at the end of the first period is decreasing in the forecast for any forecast error, Proposition 2 shows that controlling for the magnitude of the surprise (= forecast error) and reported earnings, the price following a positive surprise, $P(R, R - \Delta)$, is always higher than the price following a negative surprise, $P(R, R + \Delta)$ where Δ denotes the magnitude of the earnings surprise, $|R - MF|$. The intuition for this kind of path dependence is the following. With respect to the price component that reflects the firm's expected cash flows, a lower forecast is uniformly desirable. That is, since $p_1 < 0$, $E[\tilde{x}_1 + \tilde{x}_2 | MF, R] = p_1 MF + p_2(R - r_0) + (1 - \rho) E[\tilde{\mu}]$ declines in MF . But, this is not true for the risk component of market price, $-\gamma Var[\tilde{x}_1 + \tilde{x}_2 | R, MF] = p_3 \left(\frac{R - (r_0 + r_1 MF)}{r_2} - MF + m_0 \right)^2 + p_0 + p_2 r_0 - (1 - \rho) E[\tilde{\mu}]$, since MF affects this risk component of prices non-monotonically. However, the pessimistic forecast bias causes the market to infer a higher variance of underlying cash flows if the forecast exceeds reported earnings by an amount Δ (negative earnings surprise) than if the forecast falls short of reported earnings by the same amount (positive earnings surprise). Hence, holding the magnitude of the surprise constant, a positive surprise is always preferred.

The relation between returns, forecasts or earnings preannouncements and reported earnings has been studied empirically. Soffer, Thiagarajan, and Walther (2000) provide evidence that controlling for the magnitude of the total news released (measured as the difference between reported earnings and consensus analysts' forecast prior to the release of the management earnings preannouncement) the total excess return over the time period that contains both earnings preannouncement and earnings announcement is lower for firms

that have negative surprises at the earnings announcement. That is, firms with negative total news have lower excess returns if they announced only a fraction of their total news at the earnings preannouncement and have another negative surprise at the earnings announcement date relative to firms that have the same amount of total news but exaggerated their negative news at the earnings preannouncement and hence have a positive surprise at the earnings announcement. Moreover, Soffer et al. find that the opposite holds for firms with positive total news. In that case, the excess return is higher if firms understate their positive news at the earnings preannouncement and, again, have a positive surprise relative to firms that announce all of their positive news at the earnings preannouncement. Hence, in both cases, for positive as well as negative total news, the excess return is higher if the forecast is lower and the surprise at the announcement is positive. This seems broadly consistent with the prediction in part (a) of Proposition 2.¹⁵

5 Extensions

This section presents extensions to the base model. The extensions address two aspects of the base model: (1) in the equilibrium of the base model, (R, MF) is informationally equivalent to (μ_1, x_1) and (2) the base model assumed only part of the variance of cash flows is random. After introducing the modified model setup, I present the equilibrium and discuss how the results presented above are sensitive to these extensions.

In the base model, the capital market has been able to perfectly infer the manager's private information about the first period cash flows from his forecast and earnings report. The first modification addresses this aspect by introducing a noise term to the cost function of manipulating reported earnings. In particular, I assume that the cost of discretionary accruals is $\frac{c_2}{2} (R - x_1 - \varepsilon_1)^2$ where $\tilde{\varepsilon}_1 \sim N(0, \frac{\tau}{k})$ and $\frac{\tau}{k}$ denotes the precision of $\tilde{\varepsilon}_1$ for some constant $k > 0$. Given τ , $\tilde{\varepsilon}_1$ is independently distributed of all other random variables. This approach of introducing noise into the cost function in order to prevent users of the released report to infer the private information of the agent that prepares the report has been used by Dye and Sridhar

¹⁵However, the model does not reflect some key elements of the setting in Soffer et al. since the authors select their sample in a way to reduce the likelihood that managers received significant additional information after the earnings preannouncement by requiring the earnings preannouncement being made no more than two weeks before the end of the quarter.

(2004).¹⁶ With this modified cost function, the manager is unable to fully reveal his private knowledge about first period cash flows to the second generation of shareholders. Rather, investors will be able to infer only the noisy signal $x_1 + \varepsilon_1$ from observing the manager's forecast and earnings report. As a consequence, the vector (R, MF) is no longer informationally equivalent to the manager's private information (μ_1, x_1) .

The second generalization of the model addresses investors' prior knowledge about the variance of cash flows. The base model presumed that only part of the variance of the firm's cash flows is unknown. In the base model, the variance of the firm's first period cash flows that gets resolved before the manager's forecast is released (that is the variance of $\tilde{\mu}_1$) is constant and common knowledge: $\tilde{\mu}_1$ is distributed with known mean, $E[\tilde{\mu}]$, and known variance, σ_μ^2 . Only the variance of the firm's first period cash flows that gets resolved after the manager's forecast is released (that is the variance of \tilde{x}_1 given μ_1) is random: $\tilde{x}_1|_{\mu_1}$ is distributed normally with mean μ_1 and unknown precision $\tilde{\tau}$. There is no obvious economic justification for the assumption that part of the variance of cash flows is unknown while another part is known.

The modified setup presumes that the total variance of first period cash flows, $Var(\tilde{x}_1)$, is random and equals $\frac{1}{\tilde{\tau}}$. Part of the total variance of first period cash flows gets resolved before the manager's forecast is released and the other part gets resolved only after the forecast is issued. I denote the fraction of the total variance of cash flows that gets resolved before the manager's forecast is released with q where q is a known constant and $q \in (0, 1)$. Accordingly, $(1 - q)$ of the total variance of cash flows gets resolved only after the forecast is issued. To reflect this assumption, I assume that $\tilde{\mu}_1$ is distributed normally with mean $E[\tilde{\mu}]$ and variance $\frac{q}{\tilde{\tau}}$ and that the firm's cash flows, \tilde{x}_1 , given μ_1 are distributed normally with mean μ_1 and variance $\frac{1-q}{\tilde{\tau}}$. The following contrasts the modified assumptions about the variance of cash flows with the

¹⁶In the present model, noise is introduced to the cost function related to managing reported earnings but not to the cost function related to manipulating the manager's forecast. Introducing noise in the cost function related to manipulating the forecast causes the observations based on which investors update their beliefs to be correlated. This renders the updating process intractable in the context of this model. Also, note that the manager's communication is restricted such that he cannot reveal ε_1 .

assumptions in the base model.

$$\begin{aligned} \text{base model:} \quad & \text{Var}(\tilde{x}_1) = \text{Var}(\tilde{\mu}_1) + \text{Var}(\tilde{x}_1|\mu_1) = \sigma_\mu^2 + \frac{1}{\tilde{\tau}}; \\ \text{extension:} \quad & \text{Var}(\tilde{x}_1) = \text{Var}(\tilde{\mu}_1) + \text{Var}(\tilde{x}_1|\mu_1) = \frac{q}{\tilde{\tau}} + \frac{1-q}{\tilde{\tau}}. \end{aligned}$$

From this it follows that in the modified setup the unconditional distribution of \tilde{x}_1 given τ is a normal distribution with mean $E[\tilde{\mu}]$ and variance $\frac{q}{\tau} + \frac{1-q}{\tau} = \frac{1}{\tau}$. Also, the fraction of the total variance that gets resolved prior to the release of the manager's forecast yields $\frac{\text{Var}(\tilde{\mu}_1)}{\text{Var}(\tilde{x}_1)} = \frac{q/\tau}{1/\tau} = q$ and the fraction of the total variance that gets resolved only after the release of the forecast is $\frac{\text{Var}(\tilde{x}_1|\mu_1)}{\text{Var}(\tilde{x}_1)} = \frac{(1-q)/\tau}{1/\tau} = 1 - q$. In this way, q might be interpreted as a measure of forecast timing where time is measured relative to when uncertainty gets resolved. Following this interpretation a smaller value of q refers to a forecast that is issued earlier in the first period.

As in the base model, the persistence parameter ρ links both realized and expected cash flows over time. To simplify notation, in the following I set $E[\tilde{\mu}] = 0$. The random precision $\tilde{\tau}$ continues to be the same across both periods and is distributed gamma with shape parameter $\alpha > 1$ and scale parameter $\beta > 0$.¹⁷ The timeline of the model remains unchanged (see figure 1).

In equilibrium, it will be the case that forecast and earnings report combined reveal μ_1 and a noisy signal of the first period cash flows, $x_1 + \varepsilon_1$. If the precision τ was known, the remaining uncertainty at the end of the first period would be $\text{Var}[\tilde{x}_1 + \tilde{x}_2|\mu_1, x_1 + \varepsilon_1, \tau] = \left[(1 + \rho)^2 \frac{(1-q)k}{1-q+k} + (1 - \rho^2) \right] \frac{1}{\tau}$. For notational convenience, I let $\left[(1 + \rho)^2 \frac{(1-q)k}{1-q+k} + (1 - \rho^2) \right]$ be denoted by θ . Then the risk-premium of the capital market price at the end of the first period when the firm is sold will be directly proportional to θ and $E\left[\frac{1}{\tilde{\tau}} \mid \mu_1, x_1 + \varepsilon_1\right]$. The following Proposition characterizes the optimal forecasting and reporting rule and consistent capital market prices that arise under the modifications introduced above.

Proposition 3 *If $\gamma \leq \frac{\alpha}{4\theta} \min\{qc_1, (1 - q + k)c_2\}$ then there is an equilibrium defined as follows.*

¹⁷A formal description of the distributional assumptions is also included in Appendix A, p. A11.

(i) The manager's optimal reporting strategy at $t = 2$ is

$$R(x_1, \varepsilon_1, MF) = r_0 + r_1 MF + r_2(x_1 + \varepsilon_1)$$

where the coefficients are given by

$$\begin{aligned} r_2 &= \frac{1}{2} \left(1 + \frac{\sqrt{c_2 - \frac{4\gamma}{\alpha} \frac{\theta}{1-q+k}}}{\sqrt{c_2}} \right) \\ r_1 &= \frac{1 - r_2}{\frac{1}{2} \left(1 + \frac{\sqrt{c_1 - \frac{4\gamma}{\alpha} \frac{\theta}{q}}}{\sqrt{c_1}} \right)} \\ r_0 &= (1 + \rho) \left(\frac{1}{r_2 c_2} + \frac{r_1^2}{r_2 c_1} \right). \end{aligned}$$

(ii) The manager's optimal forecasting rule at $t = 1$ is

$$MF(\mu_1) = m_0 + m_1 \mu_1$$

$$\text{where } m_1 = \frac{1-r_2}{r_1} \text{ and } m_0 = \frac{1+\rho}{m_1 c_1} \left(1 - \frac{1-q}{1-q+k} \frac{1}{r_2} \right).$$

(iii) The pricing function $P(R, MF)$ takes the form

$$P(R, MF) = p_0 + p_1 MF + p_2 R + p_3 \left(\frac{R - (r_0 + r_1 MF)}{r_2} - \frac{MF - m_0}{m_1} \right)^2 + p_4 \left(\frac{MF - m_0}{m_1} \right)^2$$

where the pricing coefficients are given by

$$\begin{aligned}
p_4 &= \frac{-\gamma \theta}{2\alpha} \frac{1}{q} \\
p_3 &= \frac{-\gamma \theta}{2\alpha} \frac{1}{1-q+k} \\
p_2 &= (1+\rho) \frac{1-q}{1-q+k} \frac{1}{r_2} \\
p_1 &= \frac{1+\rho-p_2}{m_1} \\
p_0 &= -\gamma \theta \frac{\beta}{\alpha} - \frac{p_1^2}{c_1} - \frac{p_2^2}{c_2}.
\end{aligned}$$

(iv) The capital market prices at $t = 0$ and $t = 1$ are given by

$$\begin{aligned}
P_0 &= -\frac{2\gamma\beta}{\alpha-1} (1+\rho) \\
P(MF) &= (1+\rho) \frac{MF-m_0}{m_1} - \gamma \left[(1+\rho)^2 (1-q) + (1-\rho^2) \right] \frac{2\beta + \frac{1}{q} \left(\frac{MF-m_0}{m_1} \right)^2}{2\alpha-1}.
\end{aligned}$$

The strong similarities of the equilibrium in Proposition 3 and the equilibrium in Proposition 1 are apparent despite considerable modifications in the setup. Most notable differences are the additional squared terms in both the price at $t = 1$ and the price at $t = 2$. Unlike in the original setting, investors can now update their beliefs about the firm's variance of cash flows at the forecast release date. The more the forecast deviates from its expected value, the higher investors perceive the volatility of firm's cash flows to be. This is reflected in the investors' posterior beliefs about the firm's variance of cash flows at $t = 1$ which is given by $E \left[\frac{1}{\tau} \mid MF \right] = \frac{2\beta}{2\alpha-1} + \frac{1}{q(2\alpha-1)} \left(\frac{MF-E[MF(\tilde{\mu}_1)]}{m_1} \right)^2$ and increases in the magnitude of the difference between the manager's forecast and its expected value.

At the end of the period, after the manager has released the firm's reported earnings, investors have two observations based on which they can update their beliefs about the firm's variance of cash flows: (i) the amount by which the forecast differs from its expected value and (ii) the amount by which reported earnings differ from their expected value taking into account the manager's forecast. The following equation shows

how the two observations affect investors' posterior beliefs about the expected value of $\tilde{\tau}^{-1}$.¹⁸

$$\begin{aligned} E \left[\frac{1}{\tilde{\tau}} \middle| MF, R \right] &= \frac{\beta}{\alpha} + \frac{1}{2\alpha(1-q+k)} \left(\frac{R - E[R(MF, \tilde{x}_1) | MF]}{r_2} \right)^2 + \frac{1}{2\alpha q} \left(\frac{MF - E[MF(\tilde{\mu}_1)]}{m_1} \right)^2 \\ &= \frac{\beta}{\alpha} + \frac{1}{2\alpha(1-q+k)} \left(\frac{R - (r_0 + r_1 MF)}{r_2} - \frac{MF - m_0}{m_1} \right)^2 + \frac{1}{2\alpha q} \left(\frac{MF - m_0}{m_1} \right)^2 \end{aligned}$$

Stock prices reflect investors' perception of the firm's variance of cash flows because investors are risk-averse. This provides the manager with incentives to understate the surprise at both disclosure dates, the forecast release date and the earnings announcement date. As a result, the forecast differs from its expected value by a smaller amount than the manager's private signal, μ_1 , differs from $E[\tilde{\mu}_1]$. Similarly, the manager reports earnings that differ from $E[R(\tilde{x}_1 + \tilde{\varepsilon}_1, MF) | MF]$ by a smaller amount than the garbled version of his private information, $x_1 + \varepsilon_1$, differs from its expected value, $E[\tilde{x}_1 + \tilde{\varepsilon}_1 | \mu_1]$. The following Corollary describes these disclosure strategies.

Corollary 6 *In the equilibrium of Proposition 3, m_1 and r_2 are*

- (a) *less than one ($m_1 < 1$ and $r_2 < 1$);*
- (b) *decreasing in γ and increasing in α ; and*
- (c) *decreasing in ρ if $\rho < \frac{(1-q)k}{1-q+qk}$ and increasing in ρ if $\rho > \frac{(1-q)k}{1-q+qk}$.*

The manager's intention to make the firm appear less risky by understating the surprise at the forecast release date and the earnings announcement date is reflected in $\frac{\partial MF(\mu_1)}{\partial \mu_1} = m_1 < 1$ and $\frac{\partial R(MF, x_1 + \varepsilon_1)}{\partial (x_1 + \varepsilon_1)} = r_2 < 1$. This compares to $m_1 = 1$ and $r_2 < 1$ in the original setting when μ_1 did not provide a signal about unknown variance of cash flows and the manager lacked a reason to downplay the surprise in the forecast. Part (b) of Corollary 6 parallels the results of part (b) of Corollary 3. As investors become more risk-averse or the prior variance of cash flows is larger, the manager has greater incentives to manipulate his disclosure in ways that make the firm's cash flows appear less volatile resulting in lower values of m_1 and r_2 .

¹⁸In the original setting of Proposition 1 only the second observation, the amount by which reported earnings differ from their expected value given the manager's forecast, reveals information about the unknown component of the variance of cash flows. The first observation, the amount by which the forecast deviates from its expected value, does not provide any information about the precision $\tilde{\tau}$ in the original setting because the variance of $\tilde{\mu}_1$ is known and constant (σ_μ^2).

Finally, part (c) describes the effect of a change in persistence of cash flows on manager's disclosure strategies. The persistence of cash flows determines the extent to which investors can predict second period cash flows based on the information obtained in the first period. The residual uncertainty investors face at the end of the first period, $Var(\tilde{x}_1 + \tilde{x}_2 | R, MF)$, therefore depends on the persistence of cash flows. A change in ρ has two effects on the remaining uncertainty at the end of the first period. First, an increase in the magnitude of ρ makes x_2 more predictable based on x_1 . However, if $k > 0$ then reported earnings do not perfectly reveal first period cash flows. This leads to the second effect of a change in ρ . The higher the persistence ρ the more positively correlated are (the forecast errors of) x_1 and x_2 . As a result, the sum of x_1 and x_2 becomes less predictable based on forecast and reported earnings. To illustrate this effect consider the limiting cases of $\rho = -1$ and $\rho = 1$. If ρ equaled -1 then innovations of cash flows would fully reverse in the second period making the sum of first and second period cash flows perfectly predictable whether the manager's disclosures reveal x_1 with or without error. If, however, $\rho = 1$, then first period innovations of cash flows would be fully persistent, i.e. $x_2 = x_1$ and investors' are not able to perfectly predict the sum of first and second period cash flows based on the manager's forecast and reported earnings if $k > 0$.

Because of these two effects the uncertainty investors face at the end of the first period increases in ρ for $\rho < \rho^*$ and decreases in ρ for $\rho > \rho^*$ for $\rho^* = \frac{(1-q)k}{1-q+qk} > 0$. Since the uncertainty investors face determines the manager's incentives to manipulate his disclosure in ways that make the firm's cash flows appear less volatile, m_1 and r_1 decrease in ρ for $\rho < \rho^*$ and increase in ρ for $\rho > \rho^*$. To reconcile this result with part (c) of Corollary 3 consider the case of first period cash flows being perfectly revealed by reported earnings ($k = 0$). Then part (c) of Corollary 6 states that m_1 and r_2 increase in the magnitude of ρ . This is consistent with part (c) of Corollary 3.

Proposition 3 states that reported earnings are a linear combination of MF and $(x_1 + \varepsilon_1)$ and that the manager's forecast is a deterministic function of his private signal μ_1 . Therefore, investors can infer μ_1 and a noisy signal, $x_1 + \varepsilon_1$, of first-period cash flows from observing the manager's forecast and earnings reports. In equilibrium, the process of garbling the manager's private information affects not only the manager's disclosure strategies but also the equilibrium stock price reactions to those disclosures. Corollary 7 shows

how the “noise” in the earnings report affects the stock price reaction at the earnings announcement date.

Corollary 7 *In the equilibrium of Proposition 3,*

- (a) p_4 always decreases in k ; and
- (b) if $k > (1 - q) \frac{2\rho - q(1 + \rho)}{2 - q(1 + \rho)}$ then p_1 and p_3 increase in k and p_2 decreases in k .

A higher value of k reduces the value of the report R in communicating information about firm’s cash flows to investors. If the value of the earnings report in communicating information is limited, investors rely more on the manager’s forecast when updating their beliefs about the firm’s variance of cash flows. That is, investors react more strongly to deviations from the manager’s forecast (p_4 is more negative) for higher values of k (part *a* of Corollary 7). Conversely, if the value of the earnings report in revealing information to investors is limited, investors rely less on deviations of the earnings report from its expected value when updating their beliefs about the firm’s variance of cash flows. However, this is only true for larger values of k because an increase in the level of noise in reported earnings also increases the uncertainty investors are exposed to when buying the firm. As a result, investors put more weight on signals about the firm’s variance of cash flows (part *b* of Corollary 7).

The coefficients p_1 and p_2 are the average marginal price response coefficients to forecast and earnings report respectively. They are affected by the noise in the earnings report because the noise affects the extent the manager manipulates his disclosures to hide surprises. If the manager manipulates his forecast and report to a greater extent, investors rationally anticipate the manager’s disclosure strategies and react stronger to surprises at the forecast release date and the earnings announcement date.

The setup of the model was constructed such that the variance of cash flows in each period is τ^{-1} – independent of how much of the total variance gets resolved prior versus post the forecast release date, i.e. independent of the value of q . However, if reported earnings reveal only a noisy signal of x_1 , a change in q will affect the residual uncertainty investors face at the end of the first period, $Var(\tilde{x}_1 + \tilde{x}_2 | R, MF)$. As q increases a smaller portion of the information gets garbled because most of the variance of cash flows has been resolved prior to the forecast release. From this it follows that $Var(\tilde{x}_1 + \tilde{x}_2 | R, MF)$ decreases

in q as long as $k > 0$. If the manager is able to communicate his private information without any noise ($k = 0$), then a change in q has no effect on the residual uncertainty, $Var(\tilde{x}_1 + \tilde{x}_2 | R, MF)$. Setting $k = 0$ therefore has the desirable effect that for a given τ a change in q does not influence the residual uncertainty of the firm's cash flows. Speaking loosely, if $k = 0$ then a change in q affects only *when* information gets released but not *how much* information is reflected in manager's disclosures. For this reason, I present the comparative statics results with respect to q for the case of $k = 0$.

Corollary 8 *Assume $k = 0$. In the equilibrium of Proposition 3*

- (a) p_3 and r_2 decrease in q while p_4 and m_1 increase in q ; and
- (b) $\exists q^*$ s.t. $E[FE] = \frac{p_2}{c_2} - \frac{p_1}{c_1}$ decreases in q if $q < q^*$ and $E[FE]$ increases in q if $q > q^*$.

At the end of the first period, the stock price reaction to both reported earnings and the manager's forecast depend on q . If a higher fraction of the variance of cash flows gets resolved prior to the release of the forecast, investors expect the manager's forecast to deviate on average more from its expected value and therefore attach a smaller risk-premium to forecast surprises (p_4 increases in q , i.e. p_4 becomes less negative as q increases). As p_4 becomes less negative the manager has less incentives to manipulate his forecast to reduce the forecast surprise, making the forecast more sensitive to his private information (higher value of m_1). Since a change in q has exactly the opposite effect on investors' reactions to earnings surprises than it has on investors' reaction to forecast surprises, p_3 and r_2 decrease in q .

As previously noted, q might be interpreted as a measure of forecast timing, with the caveats that q is exogenous to the model and measures the time relative to the arrival of information about the firm's cash flows. Keeping the qualifications about interpreting q in mind, part (b) of Corollary 8 suggests that the manager's expected forecast error depends on the timing of the forecast. While the expected forecast error initially decreases, it increases towards the end of the period. In an empirical study, Choi and Ziebart (2000) provides evidence that the bias of management forecasts in deed depends on the forecast horizon. However, in contrast to the prediction of the model, Choi and Ziebart provide evidence that the manager's forecast bias is lower for longer forecast horizons which would correspond to $E[FE]$ being monotonically increasing

in q .

As discussed above, the main implication of the total variance of cash flows being random is that it provides the manager with incentives to manipulate his forecast so that investors perceive the firm's cash flows as less risky. Because investors rationally anticipate the manager's manipulative action, the stock price response coefficient at the forecast release date is on average higher in the equilibrium of Proposition 3 than in the equilibrium of Proposition 1. As a result, it is no longer the case that the average stock price response coefficient to reported earnings will always be larger than the average stock price response coefficient to the manager's forecast. The following Corollary states the unique condition under which this finding continues to hold.

Corollary 9 *Assume $k = 0$. In the equilibrium of Proposition 3, the average stock market sensitivity to reported earnings is larger than the average stock market sensitivity to the management forecast if and only if $\frac{c_1}{c_2} > \frac{1-q}{q}$.*

The economic intuition of the result in Corollary 9 rests upon the manager trading off manipulation costs against the benefits of making the firm's cash flows appear less risky. To see this consider the link between average stock market reactions and the sensitivity of earnings forecast and earnings report to the news μ_1 and x_1 , respectively. The average stock market sensitivity to reported earnings is $p_2 = \frac{1+\rho}{r_2}$ (where r_2 is the sensitivity of the earnings report to cash flows, x_1) and the average stock market sensitivity to the management forecast is $E\left[\frac{\partial P(MF)}{MF}\right] = \frac{1+\rho}{m_1}$ (where m_1 is the sensitivity of the forecast to μ_1). The sensitivity of the forecast to the signal μ_1 ($\frac{\partial MF(\mu_1)}{\partial \mu_1} = m_1$) increases in the cost of manipulating the forecast, c_1 , and decreases in the benefits of understating the surprise component as captured in the forecast's effect on the stock price risk-premium, $-p_4$. Likewise, the sensitivity of the earnings report to first period cash flows x_1 ($\frac{\partial R(MF, x_1)}{\partial x_1} = r_2$) increases in the cost of manipulating the report, c_2 , and decreases in the benefits of understating the earnings surprise as captured in the impact of the earnings report on the risk-premium, $-p_3$. If the cost-benefit ratio of making the firm appear less risky is higher at the forecast release date than at the earnings announcement date, the manager will conceal his news to a lesser extent when issuing his forecast than when issuing the earnings report ($m_1 > r_2$). As a consequence, the capital market will react

more strongly to surprises at the earnings announcement date than to surprises at the forecast release date.

6 Conclusion

This paper develops an analytical model of an owner-manager who discloses a forecast of his firm's earnings during the period and who releases a possibly manipulated value of the firm's earnings at the end of the period. The paper derives predictions about the manager's incentives to bias his forecast and to manage the firm's accruals as well as the capital market's equilibrium reaction to the manager's disclosures. In this model, disclosures provide information about firm value to future investors. A key feature of the model is that the variance of the firm's cash flows is unknown to the capital markets, and constant over time. Investors can therefore learn about both the mean and variance of future cash flows from the earnings forecast the manager releases and the reported value of the firm's earnings. Since the manager attempts to maximize the firm's capital market value net of his personal costs of forecast management and accruals manipulation, his optimal forecasting and reporting strategies are affected by, and affect, the inferences investors make about both the expected value and variance of the firm's cash flows.

I have contrasted the predictions of the model regarding discretionary accruals and management forecasts with existing empirical results. Several of the predictions of the model are consistent with established empirical results. One empirical result that the model does not explain is the rarity of management earnings forecasts. Addressing the frequency, as well as the timing, of management forecasts would constitute interesting extensions of the present model.

Appendix A

Specification of distributional assumptions for the equilibrium in Prop. 1

First period cash flows are given by

$$\tilde{x}_1 = \tilde{\mu}_1 + \tilde{v}_1$$

where $E[\tilde{\mu}_1] = E[\tilde{\mu}]$ and $Var(\tilde{\mu}_1) = \sigma_\mu^2$ and $\tilde{v}_1 \sim N(0, \frac{1}{\tau})$. Similarly, $\tilde{x}_2 = \tilde{\mu}_2 + \tilde{v}_2$. To reflect the persistence of cash flows, $\tilde{\mu}_2$ and \tilde{v}_2 are given by

$$\tilde{\mu}_2 = \rho\tilde{\mu}_1 + \tilde{\eta}_{2\mu}$$

$$\tilde{v}_2 = \rho\tilde{v}_1 + \tilde{\eta}_{2v}$$

where $E[\tilde{\eta}_{2\mu}] = (1 - \rho)E[\tilde{\mu}]$ and $Var[\tilde{\eta}_{2\mu}] = (1 - \rho^2)\sigma_\mu^2$ and $\tilde{\eta}_{2v} \sim N(0, \frac{1-\rho^2}{\tau})$. Given τ , $\tilde{\mu}_1, \tilde{v}_1, \tilde{\eta}_{2\mu}$, and $\tilde{\eta}_{2v}$ are distributed independently. The marginal distribution on $\tilde{\tau}$ is Gamma with shape-parameter α , scale-parameter β , and location parameter 0. Using this setup we can rewrite \tilde{x}_2 as

$$\begin{aligned} \tilde{x}_2 &= \rho\tilde{\mu}_1 + \tilde{\eta}_{2\mu} + \rho\tilde{v}_1 + \tilde{\eta}_{2v} \\ &= \rho\tilde{x}_1 + \tilde{\eta}_2 \end{aligned}$$

where $\tilde{\eta}_2 = \tilde{\eta}_{2\mu} + \tilde{\eta}_{2v}$. From this it follows that the prior expected value and variance of \tilde{x}_2 and $\tilde{\mu}_2$ are the same as the prior expected value and variance of \tilde{x}_1 and $\tilde{\mu}_1$ respectively.

$$E[\tilde{\mu}_2] = \rho E[\tilde{\mu}] + (1 - \rho) E[\tilde{\mu}] = E[\tilde{\mu}]$$

$$Var(\tilde{\mu}_2) = \rho^2 \sigma_\mu^2 + (1 - \rho^2) \sigma_\mu^2 = \sigma_\mu^2$$

$$E[\tilde{x}_2] = E[\tilde{\mu}_2] + E[\rho\tilde{v}_1 + \tilde{\eta}_{2v}] = E[\tilde{\mu}_2] = E[\tilde{\mu}]$$

$$Var(\tilde{x}_2|\tau) = Var(\tilde{\mu}_2 + \tilde{v}_2|\tau) = Var(\tilde{\mu}_2|\tau) + Var(\rho\tilde{v}_1 + \tilde{\eta}_{2v}|\tau) = \sigma_\mu^2 + \frac{\rho^2}{\tau} + \frac{1 - \rho^2}{\tau} = \sigma_\mu^2 + \frac{1}{\tau}$$

Proof of Proposition 1

Conjectured equilibrium

$$P(R, MF) = p_0 + p_1 MF + p_2 R + p_3 \left(\frac{R - (r_0 + r_1 MF)}{r_2} - MF + m_0 \right)^2 \quad (2)$$

$$R(x_1, MF) = r_0 + r_1 MF + r_2 x_1 \quad (3)$$

$$MF(\mu_1) = m_0 + \mu_1 \quad (4)$$

First I solve the manager's reporting problem at the end of the first period. Then, I solve for the optimal forecast. Finally, I derive the pricing function $P(R, MF)$ taking the conjectured reporting and forecasting rule as given and show that it is consistent with the conjectured pricing function in (2).

At the end of the first period ($t = 2$) the manager solves the following problem

$$\max_R P(R, MF) - \frac{c_1}{2} (MF - \mu_1)^2 - \frac{c_2}{2} (R - x_1)^2$$

where $P(R, MF) = p_0 + p_1 MF + p_2 R + p_3 \left(\frac{R - (\hat{r}_0 + \hat{r}_1 MF)}{\hat{r}_2} - MF + \hat{m}_0 \right)^2$. The hats above the coefficients indicate that these are the conjectured coefficients the capital market uses when valuing the company at the end of the first period. Solving for R yields the following first order condition.

$$\begin{aligned} 0 &= p_2 + \frac{2p_3}{\hat{r}_2} \left(\frac{R - (\hat{r}_0 + \hat{r}_1 MF)}{\hat{r}_2} - MF + \hat{m}_0 \right) - c_2 (R - x_1) \\ R &= \frac{p_2 \hat{r}_2^2 + 2p_3 (-(\hat{r}_0 + \hat{r}_1 MF) - \hat{r}_2 MF + \hat{r}_2 \hat{m}_0) + c_2 \hat{r}_2^2 x_1}{c_2 \hat{r}_2^2 - 2p_3} \end{aligned} \quad (5)$$

The second order condition to the manager's optimization problem at $t = 2$ yields $\frac{2p_3}{\hat{r}_2^2} - c_2 < 0$ which is always satisfied if $p_3 < 0$ as it will be the case in equilibrium. Equation (5) shows that the manager's optimal reporting strategy (assuming that the pricing rule and optimal forecast are given by (2) and (4) respectively) is linear in x_1 and MF as conjectured in (3). In equilibrium, the optimal report in (5) has to equal the

$R(x_1, MF)$ as in (3) for all possible values of (x_1, MF) . Equating coefficients yields

$$\begin{aligned} r_0 &= \frac{p_2 \hat{r}_2^2 + 2p_3 (-\hat{r}_0 + \hat{r}_2 \hat{m}_0)}{c_2 \hat{r}_2^2 - 2p_3} \\ r_1 &= -\frac{2p_3 (\hat{r}_1 + \hat{r}_2)}{c_2 \hat{r}_2^2 - 2p_3} \\ r_2 &= \frac{c_2 \hat{r}_2^2}{c_2 \hat{r}_2^2 - 2p_3} \end{aligned}$$

In equilibrium, we also have that the conjectured coefficients equal the true coefficients, i.e. $r_0 = \hat{r}_0, r_1 = \hat{r}_1,$ and $r_2 = \hat{r}_2,$ and hence

$$r_0 = \frac{2p_2 + m_0 \sqrt{c_2^2 + 8p_3 c_2} - c_2 m_0}{2c_2} \quad (6)$$

$$r_1 = \frac{1}{2} \left(1 - \frac{\sqrt{c_2 + 8p_3}}{\sqrt{c_2}} \right) \quad (7)$$

$$r_2 = \frac{1}{2} \left(1 + \frac{\sqrt{c_2 + 8p_3}}{\sqrt{c_2}} \right) \quad (8)$$

With the manager's optimal reporting strategy given we can now solve the manager's forecasting problem at $t = 1$.

$$\max_{MF} E \left[P(R(\tilde{x}_1, MF), MF) - \frac{c_1}{2} (MF - \mu_1)^2 - \frac{c_2}{2} (R(\tilde{x}_1, MF) - \tilde{x}_1)^2 \middle| \mu_1 \right];$$

where the manager anticipates his optimal reporting strategy $R(MF, x_1) = r_0 + r_1 MF + r_2 x_1$ at $t = 2$ with the coefficients $r_0, r_1,$ and r_2 given by (6) – (8). The pricing function the manager takes as given is the same as at $t = 2$. Using the equilibrium conditions for the reporting strategy $r_0 = \hat{r}_0, r_1 = \hat{r}_1,$ and $r_2 = \hat{r}_2$ this simplifies to

$$\begin{aligned} &E \left[p_0 + p_1 MF + p_2 (r_0 + r_1 MF + r_2 \tilde{x}_1) + p_3 (\tilde{x}_1 - MF + \hat{m}_0)^2 \middle| \mu_1 \right] \\ &- \frac{c_1}{2} (MF - \mu_1)^2 - \frac{c_2}{2} E \left[(r_0 + r_1 MF + r_2 \tilde{x}_1 - \tilde{x}_1)^2 \middle| \mu_1 \right] \end{aligned}$$

The first order condition for MF yields

$$\begin{aligned} p_1 + p_2 r_1 - 2p_3 (E[\tilde{x}_1|\mu_1] - MF + \hat{m}_0) - c_1 (MF - \mu_1) - c_2 r_1 E[r_0 + r_1 MF + (r_2 - 1)\tilde{x}_1|\mu_1] &= 0 \\ p_1 + p_2 r_1 - 2p_3 (\mu_1 - MF + \hat{m}_0) - c_1 (MF - \mu_1) - c_2 r_1 (r_0 + r_1 MF + (r_2 - 1)\mu_1) &= 0 \end{aligned}$$

The second order condition is $2p_3 - c_1 - c_2 r_1^2 < 0$ which is always satisfied if $p_3 < 0$ (as it will be the case in equilibrium). From (7) – (8) we know that $r_1 + r_2 = 1$. Then, solving the first order condition for $MF(\mu_1)$ yields

$$MF = \mu_1 + \frac{p_1 + p_2 r_1 - 2p_3 \hat{m}_0 - c_2 r_1 r_0}{-2p_3 + c_1 + c_2 r_1^2}$$

As conjectured the optimal forecast is linear in μ_1 with a slope coefficient of 1. In equilibrium, the optimal forecast above has to equal the forecasting conjectured by the capital market for all possible values of μ_1 .

Equating intercept coefficient yields

$$m_0 = \frac{p_1 + p_2 r_1 - 2p_3 \hat{m}_0 - c_2 r_1 r_0}{-2p_3 + c_1 + c_2 r_1^2}$$

In equilibrium, it also has to be the case that $m_0 = \hat{m}_0$ and hence we can solve for m_0 . Some algebraic simplification yields $m_0 = \frac{p_1}{c_1}$. So, the manager's forecasting strategy is $MF(\mu_1) = \mu_1 + \frac{p_1}{c_1}$. We can use this result to compute the intercept of the manager's reporting strategy in (6).

$$r_0 = \frac{p_2}{c_2} + \frac{\sqrt{c_2 + 8p_3} - \sqrt{c_2}}{2\sqrt{c_2}} \frac{p_1}{c_1} = \frac{p_2}{c_2} - r_1 \frac{p_1}{c_1}$$

Finally, we can use the optimal forecasting and reporting rules to derive consistent pricing coefficients. In equilibrium

$$P(R, MF) = E[\tilde{x}_1 + \tilde{x}_2 | R, MF] - \gamma Var[\tilde{x}_1 + \tilde{x}_2 | R, MF] \quad (9)$$

where

$$\begin{aligned}
P(R, MF) &= p_0 + p_1 MF + p_2 R + p_3 \left(\frac{R - (r_0 + r_1 MF)}{r_2} - MF + m_0 \right)^2 \\
&= p_0 + p_1 \left(\mu_1 + \frac{p_1}{c_1} \right) + p_2 \left(\frac{p_2}{c_2} + \mu_1 + r_2 (x_1 - \mu_1) \right) + p_3 (x_1 - \mu_1)^2
\end{aligned}$$

To compute the RHS of (9) we need expressions for the conditional expected value and variance of the liquidating dividend, $\tilde{x}_1 + \tilde{x}_2$. From the manager's forecasting and reporting strategy it follows that the vector (MF, R) is informationally equivalent to (μ_1, x_1) . Hence, $E[\tilde{x}_1 + \tilde{x}_2 | R, MF] = E[\tilde{x}_1 + \tilde{x}_2 | \mu_1, x_1] = x_1 + \rho x_1 + (1 - \rho) E[\tilde{\mu}]$ and $Var[\tilde{x}_1 + \tilde{x}_2 | R, MF] = Var[\tilde{x}_2 | \mu_1, x_1]$.

$$\begin{aligned}
Var[\tilde{x}_2 | \mu_1, x_1] &= Var[\rho \tilde{x}_1 + \tilde{\eta}_{2\mu} + \tilde{\eta}_{2v} | \mu_1, x_1] \\
&= Var[\tilde{\eta}_{2\mu} | \mu_1, x_1] + Var[\tilde{\eta}_{2v} | \mu_1, x_1] \\
&= (1 - \rho^2) \sigma_\mu^2 + E[Var[\tilde{\eta}_{2v} | \tau] | \mu_1, x_1] \\
&= (1 - \rho^2) \sigma_\mu^2 + E\left[\frac{1 - \rho^2}{\tilde{\tau}} \middle| \mu_1, x_1\right]
\end{aligned}$$

To compute the second summand first note that if $\tilde{\tau}$ is distributed gamma with shape-parameter $\hat{\alpha}$ and scale-parameter $\hat{\beta}$ then

$$\begin{aligned}
E\left[\frac{1}{\tilde{\tau}}\right] &= \int_0^\infty \frac{1}{\tau} g(\tau) d\tau = \int_0^\infty \frac{\tau^{\hat{\alpha}-2} \exp(-\hat{\beta}\tau)}{\hat{\beta}^{-\hat{\alpha}} \Gamma(\hat{\alpha})} d\tau \\
&= \frac{\Gamma(\hat{\alpha} - 1) \hat{\beta}^{-(\hat{\alpha}-1)}}{\hat{\beta}^{-\hat{\alpha}} \Gamma(\hat{\alpha})} = \frac{\Gamma(\hat{\alpha} - 1) \hat{\beta}}{\Gamma(\hat{\alpha})} = \frac{\Gamma(\hat{\alpha} - 1)}{(\hat{\alpha} - 1) \Gamma(\hat{\alpha} - 1)} \hat{\beta} \\
&= \frac{\hat{\beta}}{\hat{\alpha} - 1}
\end{aligned}$$

where the last equality follows from integrating $\Gamma(\hat{\alpha})$ by parts. Since $\tilde{x}_1 \sim N(\mu_1, \frac{1}{\tilde{\tau}})$, $g(\tau | x_1, \mu_1)$ is distributed gamma with $\hat{\alpha} = \alpha + \frac{1}{2}$ and $\hat{\beta} = \beta + \frac{1}{2}(x_1 - \mu_1)^2$ it follows that $E[\frac{1}{\tilde{\tau}} | x_1, \mu_1] = \frac{\hat{\beta}}{\hat{\alpha} - 1} =$

$\frac{\beta + \frac{1}{2}(x_1 - \mu_1)^2}{\alpha - \frac{1}{2}} = \frac{2\beta + (x_1 - \mu_1)^2}{2\alpha - 1}$ (DeGroot 1970) and the RHS of (9) yields

$$\begin{aligned} & E[\tilde{x}_1 + \tilde{x}_2 | R, MF] - \gamma \text{Var}[\tilde{x}_1 + \tilde{x}_2 | R, MF] \\ &= x_1 + \rho x_1 + (1 - \rho) E[\tilde{\mu}] - \gamma(1 - \rho^2) \left(\sigma_\mu^2 + \frac{2\beta + (x_1 - \mu_1)^2}{2\alpha - 1} \right). \end{aligned}$$

Equating the intercept and the coefficients for x_1 , μ_1 , and $(x_1 - \mu_1)^2$ respectively for equation (9) yields

$$\begin{aligned} p_2 &= \frac{1 + \rho}{r_2} \\ p_1 &= 1 + \rho - p_2 = (1 + \rho) \left(1 - \frac{1}{r_2} \right) = -(1 + \rho) \frac{r_1}{r_2} \\ p_3 &= \frac{-\gamma(1 - \rho^2)}{2\alpha - 1} \\ p_0 &= (1 - \rho) E[\tilde{\mu}] - \gamma(1 - \rho^2) \left(\sigma_\mu^2 + \frac{2\beta}{2\alpha - 1} \right) - \frac{p_1^2}{c_1} - \frac{p_2^2}{c_2} \end{aligned}$$

At $t = 1$ the capital market price is

$$\begin{aligned} P(MF) &= E[\tilde{x}_1 + \tilde{x}_2 | MF] - \gamma \text{Var}[\tilde{x}_1 + \tilde{x}_2 | MF] \\ &= (1 + \rho) \left(MF - \frac{p_1}{c_1} \right) + (1 - \rho) E[\tilde{\mu}] - \gamma \left((1 + \rho)^2 \text{Var}[\tilde{x}_1 | MF] + \text{Var}[\tilde{\eta}_{2\mu} + \tilde{\eta}_{2v} | MF] \right) \\ &= (1 + \rho) \left(MF - \frac{p_1}{c_1} \right) + (1 - \rho) E[\tilde{\mu}] - \gamma \left(\frac{2(1 + \rho)\beta}{\alpha - 1} + (1 - \rho^2) \sigma_\mu^2 \right) \end{aligned}$$

At $t = 0$ the capital market price is

$$P_0 = E[\tilde{x}_1 + \tilde{x}_2] - \gamma \text{Var}[\tilde{x}_1 + \tilde{x}_2] = 2E[\tilde{\mu}] - 2\gamma(1 + \rho) \left(\sigma_\mu^2 + \frac{\beta}{\alpha - 1} \right) \blacksquare$$

Proof of Corollary 1

Letting $FE = R - MF$ and using the fact that in equilibrium $r_1 + r_2 = 1$ we can rewrite the pricing equation.

$$\begin{aligned} P(R, MF) &= p_0 + p_1 MF + p_2 R + p_3 \left(\frac{R - (r_0 + r_1 MF)}{r_2} - MF + m_0 \right)^2 \\ &= p_0 + p_1 MF + p_2 R + \frac{p_3}{r_2^2} \left(FE^2 - 2(r_0 - r_2 m_0) FE + (r_0 - r_2 m_0)^2 \right) \end{aligned}$$

and hence $P(R, MF) = \pi_0 + \pi_1 MF + \pi_2 R + \pi_3 FE^2$ where the coefficients are given by $\pi_0 = p_0 + \frac{p_3}{r_2^2} (r_0 - r_2 m_0)^2$, $\pi_1 = p_1 + \frac{2p_3}{r_2^2} (r_0 - r_2 m_0)$, $\pi_2 = p_2 - \frac{2p_3}{r_2^2} (r_0 - r_2 m_0)$, and $\pi_3 = \frac{p_3}{r_2^2}$. ■

Proof of Corollary 2

Part (a). Using $r_1 + r_2 = 1$ (see Proposition 1)

$$\begin{aligned} E[R(\tilde{x}_1, MF(\tilde{\mu}_1))] - E[\tilde{x}_1] &= r_0 + r_1(m_0 + E[\tilde{\mu}]) + r_2 E[\tilde{\mu}] - E[\tilde{\mu}] \\ &= \frac{p_2}{c_2} - r_1 \frac{p_1}{c_1} + r_1 \frac{p_1}{c_1} = \frac{p_2}{c_2} > 0 \end{aligned}$$

Part (b). I show that $r_2 \in (\frac{1}{2}, 1)$ for all parameter values.

$$\begin{aligned} r_2 &= \frac{1}{2} \left(1 + \frac{\sqrt{c_2 + 8p_3}}{\sqrt{c_2}} \right) > \frac{1}{2} \Leftrightarrow \frac{\sqrt{c_2 + 8p_3}}{\sqrt{c_2}} > 0 \\ r_2 &= \frac{1}{2} \left(1 + \frac{\sqrt{c_2 + 8p_3}}{\sqrt{c_2}} \right) < 1 \Leftrightarrow \sqrt{c_2} > \sqrt{c_2 + 8p_3} \end{aligned} \tag{10}$$

where (10) follows from $p_3 < 0$. $r_1 + r_2 = 1$ follows immediately from Proposition 1. ■

Part (i): Discretionary accruals and forecast error are negatively correlated.

$$\begin{aligned} \widetilde{DA} &= R(MF(\tilde{\mu}_1), \tilde{x}_1) - \tilde{x}_1 = r_0 + r_1 m_0 + r_1 (\tilde{\mu}_1 - \tilde{x}_1) = \frac{p_2}{c_2} - r_1 \tilde{v}_1 \\ \widetilde{FE} &= R(MF(\tilde{\mu}_1), \tilde{x}_1) - MF(\tilde{\mu}_1) = \frac{p_2}{c_2} - \frac{p_1}{c_1} + r_2 (\tilde{x}_1 - \tilde{\mu}_1) = \frac{p_2}{c_2} - \frac{p_1}{c_1} + r_2 \tilde{v}_1 \\ \text{Cov}(\widetilde{DA}, \widetilde{FE}) &= -r_1 r_2 \text{Var}(\tilde{v}_1) < 0. \end{aligned}$$

Part (ii): Expected discretionary accruals are positive if the forecast error is negative. Using the fact that $r_1 > 0$ we have

$$E \left[\widetilde{DA} \mid \widetilde{FE} < 0, \tau \right] = \frac{p_2}{c_2} - r_1 E \left[\widetilde{v}_1 \mid \widetilde{v}_1 < -\frac{1}{r_2} \left(\frac{p_2}{c_2} - \frac{p_1}{c_1} \right), \tau \right] = \frac{p_2}{c_2} + \frac{r_1}{\tau} \frac{\phi \left(-\frac{\sqrt{\tau}}{r_2} \left(\frac{p_2}{c_2} - \frac{p_1}{c_1} \right) \right)}{\Phi \left(-\frac{\sqrt{\tau}}{r_2} \left(\frac{p_2}{c_2} - \frac{p_1}{c_1} \right) \right)} > 0$$

$$E \left[\widetilde{DA} \mid \widetilde{FE} > 0, \tau \right] = \frac{p_2}{c_2} - r_1 E \left[\widetilde{v}_1 \mid \widetilde{v}_1 > -\frac{1}{r_2} \left(\frac{p_2}{c_2} - \frac{p_1}{c_1} \right), \tau \right] = \frac{p_2}{c_2} - \frac{r_1}{\tau} \frac{\phi \left(-\frac{\sqrt{\tau}}{r_2} \left(\frac{p_2}{c_2} - \frac{p_1}{c_1} \right) \right)}{1 - \Phi \left(-\frac{\sqrt{\tau}}{r_2} \left(\frac{p_2}{c_2} - \frac{p_1}{c_1} \right) \right)}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ denote the probability density function and the cumulative density function for the standard normal distribution respectively. (To see that $E \left[\widetilde{DA} \mid \widetilde{FE} > 0, \tau \right]$ can be both positive and negative consider $\gamma \rightarrow 0$ and $\gamma \rightarrow \gamma_{\max}$ and $\rho \rightarrow -1$). If $E \left[\widetilde{DA} \mid \widetilde{FE} > 0, \tau \right] > 0$ then

$$\left| E \left[\widetilde{DA} \mid \widetilde{FE} > 0, \tau \right] \right| < \left| E \left[\widetilde{DA} \mid \widetilde{FE} < 0, \tau \right] \right|$$

$$\frac{p_2}{c_2} - \frac{r_1}{\tau} \frac{\phi \left(-\frac{\sqrt{\tau}}{r_2} \left(\frac{p_2}{c_2} - \frac{p_1}{c_1} \right) \right)}{1 - \Phi \left(-\frac{\sqrt{\tau}}{r_2} \left(\frac{p_2}{c_2} - \frac{p_1}{c_1} \right) \right)} < \frac{p_2}{c_2} + \frac{r_1}{\tau} \frac{\phi \left(-\frac{\sqrt{\tau}}{r_2} \left(\frac{p_2}{c_2} - \frac{p_1}{c_1} \right) \right)}{\Phi \left(-\frac{\sqrt{\tau}}{r_2} \left(\frac{p_2}{c_2} - \frac{p_1}{c_1} \right) \right)}$$

If $E \left[\widetilde{DA} \mid \widetilde{FE} > 0, \tau \right] < 0$ then

$$\left| E \left[\widetilde{DA} \mid \widetilde{FE} > 0, \tau \right] \right| < \left| E \left[\widetilde{DA} \mid \widetilde{FE} < 0, \tau \right] \right|$$

$$\frac{r_1}{\tau} \frac{\phi \left(-\frac{\sqrt{\tau}}{r_2} \left(\frac{p_2}{c_2} - \frac{p_1}{c_1} \right) \right)}{1 - \Phi \left(-\frac{\sqrt{\tau}}{r_2} \left(\frac{p_2}{c_2} - \frac{p_1}{c_1} \right) \right)} - \frac{p_2}{c_2} < \frac{p_2}{c_2} + \frac{r_1}{\tau} \frac{\phi \left(-\frac{\sqrt{\tau}}{r_2} \left(\frac{p_2}{c_2} - \frac{p_1}{c_1} \right) \right)}{\Phi \left(-\frac{\sqrt{\tau}}{r_2} \left(\frac{p_2}{c_2} - \frac{p_1}{c_1} \right) \right)}$$

$$\frac{r_1}{\tau} \phi \left(-\frac{\sqrt{\tau}}{r_2} \left(\frac{p_2}{c_2} - \frac{p_1}{c_1} \right) \right) \frac{2\Phi \left(-\frac{\sqrt{\tau}}{r_2} \left(\frac{p_2}{c_2} - \frac{p_1}{c_1} \right) \right) - 1}{\left[1 - \Phi \left(-\frac{\sqrt{\tau}}{r_2} \left(\frac{p_2}{c_2} - \frac{p_1}{c_1} \right) \right) \right]} < 0 < \frac{2p_2}{c_2}$$

which follows from $\frac{p_2}{c_2} - \frac{p_1}{c_1} > 0$ (see proof of Corollary 4). ■

Proof of Corollary 3

Part (a). From (8) and $p_3 < 0$ it follows that $\frac{\partial r_2}{\partial c_2} = \frac{1}{2} \frac{\frac{\sqrt{c_2}}{2\sqrt{c_2+8p_3}} - \frac{\sqrt{c_2+8p_3}}{2\sqrt{c_2}}}{c_2} = \frac{-8p_3}{4c_2\sqrt{c_2^2+8c_2p_3}} > 0$.

Part (b). From (8) $\frac{\partial r_2}{\partial \gamma} = \frac{\partial r_2}{\partial p_3} * \frac{\partial p_3}{\partial \gamma}$ where $\frac{\partial p_3}{\partial \gamma} = -\frac{1-\rho^2}{2\alpha-1} < 0$ and $\frac{\partial r_2}{\partial p_3} = \frac{1}{4} \frac{8}{\sqrt{c_2^2+8c_2p_3}} > 0$. Hence, $\frac{\partial r_2}{\partial \gamma} < 0$.

Similarly, $\frac{\partial r_2}{\partial \alpha} > 0$ because $\frac{\partial p_3}{\partial \alpha} = \frac{2\gamma(1-\rho^2)}{(2\alpha-1)^2} > 0$.

Part (c). $\frac{\partial r_2}{\partial \rho} = \frac{\partial r_2}{\partial p_3} \frac{\partial p_3}{\partial \rho} = \frac{\partial r_2}{\partial p_3} \frac{2\gamma\rho}{2\alpha-1}$. Since $\frac{\partial r_2}{\partial p_3} > 0$ it follows that $\text{sign}\left(\frac{\partial r_2}{\partial \rho}\right) = \text{sign}(\rho)$. ■

Proof of Corollary 4

Part (a). With $r_0 = \frac{p_2}{c_2} - r_1 \frac{p_1}{c_1}$, $MF = \mu_1 + \frac{p_1}{c_1}$, and $r_1 + r_2 = 1$ the forecast bias yields

$$\begin{aligned} E[FE] &= E[r_0 + r_1 MF + r_2 x_1 - MF] \\ &= \frac{p_2}{c_2} - r_1 \frac{p_1}{c_1} + (r_1 - 1) \left(\mu_1 + \frac{p_1}{c_1} \right) + r_2 \mu_1 = \frac{p_2}{c_2} - \frac{p_1}{c_1} \end{aligned}$$

Making use of the result in Corollary 2 part (a) that $r_2 < 1$ it follows that $p_1 = 1 + \rho - p_2 = (1 + \rho) \left(1 - \frac{1}{r_2} \right) < (1 + \rho) \left(1 - \frac{1}{1} \right) = 0$. Moreover, $p_2 > 0$. Hence, $E[FE] > 0$.

Part (b). The investors' risk-aversion enters the manager's forecasting and reporting strategy only through the magnitude of the pricing coefficient p_3 . Using $p_2 = \frac{1+\rho}{r_2}$

$$\frac{\partial E[FE]}{\partial p_3} = \frac{\partial}{\partial p_3} \left[\frac{p_2}{c_2} - \frac{1+\rho-p_2}{c_1} \right] = \frac{\partial p_2}{\partial p_3} \left[\frac{1}{c_2} + \frac{1}{c_1} \right] = \frac{\partial p_2}{\partial r_2} \frac{\partial r_2}{\partial p_3} \left[\frac{1}{c_2} + \frac{1}{c_1} \right] = -\frac{1+\rho}{r_2^2} \frac{\partial r_2}{\partial p_3} \left[\frac{1}{c_2} + \frac{1}{c_1} \right] < 0$$

where the inequality follows from $\frac{\partial r_2}{\partial p_3} > 0$ as established in Corollary 2. Since $\frac{\partial p_3}{\partial \gamma} < 0$ it follows that $\frac{\partial E[FE]}{\partial \gamma} > 0$.

Part (c). Following the same line of argument, $\frac{\partial E[FE]}{\partial \alpha} = \frac{\partial p_2}{\partial r_2} \frac{\partial r_2}{\partial p_3} \left[\frac{1}{c_2} + \frac{1}{c_1} \right] \frac{\partial p_3}{\partial \alpha} < 0$. ■

Proof of Corollary 5

Part (a). The average difference in stock market reaction is $E[p_2 + p_3(\tilde{x}_1 - \tilde{\mu}_1)] - \frac{\partial P(MF)}{\partial MF} = p_2 - (1 + \rho) = (1 + \rho) \left(\frac{1}{r_2} - 1 \right)$. From part (a) of Corollary 2 we know that $r_2 < 1$ and hence $(1 + \rho) \left(\frac{1}{r_2} - 1 \right) > 0$.

Part (b). Part (b) of Corollary 3 establishes that $\frac{\partial r_2}{\partial \gamma} < 0$, $\frac{\partial r_2}{\partial \alpha} > 0$ and $\frac{\partial r_2}{\partial c_2} > 0$. Hence, the average difference in stock market reaction, $(1 + \rho) \left(\frac{1}{r_2} - 1 \right)$, is increasing in γ and decreasing in α and c_2 .

Part (c). We want to show that

$$\frac{\partial}{\partial \rho} \left(p_2 - \frac{\partial P(MF)}{\partial MF} \right) \begin{cases} > 0 & \text{if } \rho < 0 \\ > 0 & \text{if } \rho \in (0, \frac{1}{3}) \text{ and } \gamma < \frac{(2\alpha-1)c_2}{8(1-\rho^2)} \frac{(1+\rho)(1-3\rho)}{(1-\rho)^2} \\ < 0 & \text{if } \rho \in (0, \frac{1}{3}) \text{ and } \gamma > \frac{(2\alpha-1)c_2}{8(1-\rho^2)} \frac{(1+\rho)(1-3\rho)}{(1-\rho)^2} \\ < 0 & \text{if } \rho > \frac{1}{3}. \end{cases}$$

Taking the derivative of p_2 with respect to ρ yields

$$\begin{aligned} \frac{\partial p_2}{\partial \rho} &= \frac{\partial}{\partial \rho} \frac{1+\rho}{r_2} = \frac{1}{r_2} - (1+\rho) \frac{1}{r_2^2} \frac{\partial r_2}{\partial \rho} = \frac{1}{r_2} - (1+\rho) \frac{1}{r_2^2} \frac{\partial r_2}{\partial p_3} \frac{\partial p_3}{\partial \rho} \\ &= \frac{1}{r_2} + 2(r_2 - 1) \frac{1}{r_2} \frac{1}{(2r_2 - 1)} \frac{\rho}{1-\rho} \end{aligned}$$

and

$$\frac{\partial}{\partial \rho} \left(p_2 - \frac{\partial P(MF)}{\partial MF} \right) = \frac{\partial}{\partial \rho} (p_2 - (1+\rho)) = \frac{\partial p_2}{\partial \rho} - 1 = \frac{1}{r_2(2r_2 - 1)} \left[-2r_2^2 + r_2 \frac{3-\rho}{1-\rho} - \frac{1+\rho}{1-\rho} \right]$$

We know that $r_2 \in (\frac{1}{2}, 1)$. Hence, $\frac{1}{r_2(2r_2-1)} > 0$. Since $r_2 < 1$ and $p_3 < 0$ it follows that for $\rho < 0$

$\frac{\partial}{\partial \rho} (p_2 - (1+\rho)) > 0$. Solving $\left[-2r_2^2 + \frac{3-\rho}{1-\rho} r_2 - \frac{1+\rho}{1-\rho} \right] = 0$ for r_2 yields

$$r_2 = \frac{-\frac{3-\rho}{1-\rho} \pm \sqrt{\left(\frac{3-\rho}{1-\rho}\right)^2 - 8\frac{1+\rho}{1-\rho}}}{-4} = \frac{3-\rho \pm (1-3\rho)}{4(1-\rho)}$$

or $r_2 \in \left\{ \frac{1+\rho}{2(1-\rho)}, 1 \right\}$. Hence, $\left[-2r_2^2 + \frac{3-\rho}{1-\rho} r_2 - \frac{1+\rho}{1-\rho} \right] > 0$ iff $r_2 \in \left(\frac{1+\rho}{2(1-\rho)}, 1 \right)$ and $\rho < \frac{1}{3}$ (note that $\rho < \frac{1}{3}$ implies that $\frac{1+\rho}{2(1-\rho)} < 1$) and $\left[-2r_2^2 + \frac{3-\rho}{1-\rho} r_2 - \frac{1+\rho}{1-\rho} \right] < 0 \forall r_2 \in (\frac{1}{2}, 1)$ if $\rho > \frac{1}{3}$. $r_2 > \frac{1+\rho}{2(1-\rho)}$ is equivalent to

$$\begin{aligned} \frac{1}{2} \left(1 + \frac{\sqrt{c_2 - \frac{8\gamma(1-\rho^2)}{2\alpha-1}}}{\sqrt{c_2}} \right) &> \frac{1+\rho}{2(1-\rho)} \\ \frac{(1+\rho)(1-3\rho)(2\alpha-1)c_2}{(1-\rho)^2} &> \gamma \end{aligned}$$

Note that $\frac{(1+\rho)(1-3\rho)}{(1-\rho)^2} < 1$ (otherwise it would not be a restriction to the condition $\gamma < \frac{(2\alpha-1)c_2}{8(1-\rho^2)}$ in Proposition

1). ■

6.1 Proof of Proposition 2

Part (a). Since $r_1, r_2 \in (0, 1)$ and $p_1 = -(1 + \rho) \frac{r_1}{r_2}$ it follows that $p_1 < 0$.

Part (b). Let $P(R, R + \Delta)$ be the stock price after a negative surprise Δ at the earnings announcement:

$$\begin{aligned} P(R, R + \Delta) &= p_0 + p_1(R + \Delta) + p_2R + p_3 \left(\frac{R - (r_0 + r_1(R + \Delta))}{r_2} - R - \Delta + m_0 \right)^2 \\ &= p_0 + (1 + \rho)R + p_1\Delta + p_3 \left(-\frac{r_0 + \Delta}{r_2} + m_0 \right)^2 \end{aligned}$$

Controlling for the magnitude of surprise (Δ) and the level of reported earnings, the difference in market value following a positive surprise ($MF = R - \Delta$) and a negative surprise ($MF = R + \Delta$) is

$$\begin{aligned} &P(R, R - \Delta) - P(R, R + \Delta) \\ &= \left[p_0 + (1 + \rho)R - p_1\Delta + p_3 \left(-\frac{r_0 - \Delta}{r_2} + m_0 \right)^2 \right] - \left[p_0 + (1 + \rho)R + p_1\Delta + p_3 \left(-\frac{r_0 + \Delta}{r_2} + m_0 \right)^2 \right] \\ &= -2p_1\Delta - 4\frac{p_3}{r_2^2}\Delta \left(\frac{p_2}{c_2} - \frac{p_1}{c_1} \right) > 0 \end{aligned}$$

where the inequality follows from $p_1, p_3 < 0$ and $p_2, r_2 > 0$.

Part (c).

$$\frac{\partial P(R, R - \Delta) - P(R, R + \Delta)}{\partial \Delta} = -2p_1 - 4\frac{p_3}{r_2^2} \left(\frac{p_2}{c_2} - \frac{p_1}{c_1} \right) > 0. \blacksquare$$

Specification of distributional assumptions for the equilibrium in Prop. 3

First period cash flows are given by

$$\tilde{x}_1 = \tilde{\mu}_1 + \tilde{v}_1$$

where $\tilde{\mu}_1 \sim N\left(0, \frac{q}{\tau}\right)$ and $\tilde{v}_1 \sim N\left(0, \frac{1-q}{\tau}\right)$ where both $\frac{q}{\tau}$ and $\frac{1-q}{\tau}$ denote variances. Similarly, $\tilde{x}_2 = \tilde{\mu}_2 + \tilde{v}_2$.

To reflect the persistence of cash flows $\tilde{\mu}_2$ and \tilde{v}_2 are given by

$$\tilde{\mu}_2 = \rho\tilde{\mu}_1 + \tilde{\eta}_{2\mu}$$

$$\tilde{v}_2 = \rho\tilde{v}_1 + \tilde{\eta}_{2v}$$

where $\tilde{\eta}_{2\mu} \sim N\left(0, \frac{(1-\rho^2)q}{\tau}\right)$ and $\tilde{\eta}_{2v} \sim N\left(0, \frac{(1-\rho^2)(1-q)}{\tau}\right)$. Given τ , $\tilde{\mu}_1$, \tilde{v}_1 , $\tilde{\eta}_{2\mu}$, and $\tilde{\eta}_{2v}$ are distributed independently. The marginal distribution on $\tilde{\tau}$ is Gamma with shape-parameter α , scale-parameter β , and location parameter 0. Again, we can rewrite \tilde{x}_2 as $\tilde{x}_2 = \rho\tilde{x}_1 + \tilde{\eta}_2$ where $\tilde{\eta}_2 = \tilde{\eta}_{2\mu} + \tilde{\eta}_{2v}$. From this it follows that the prior expected value and variance of \tilde{x}_2 and $\tilde{\mu}_2$ are the same as the prior expected value and variance of \tilde{x}_1 and $\tilde{\mu}_1$ respectively.

$$E[\tilde{\mu}_2] = \rho * 0 + (1 - \rho) * 0 = 0$$

$$Var(\tilde{\mu}_2|\tau) = \rho^2 \frac{q}{\tau} + (1 - \rho^2) \frac{q}{\tau} = \frac{q}{\tau}$$

$$E[\tilde{x}_2] = E[\tilde{\mu}_2] + E[\rho\tilde{v}_1 + \tilde{\eta}_{2v}] = E[\tilde{\mu}_2] = E[\tilde{\mu}] = 0$$

$$Var(\tilde{x}_2|\tau) = Var(\tilde{\mu}_2 + \tilde{v}_2|\tau) = Var(\tilde{\mu}_2|\tau) + Var(\rho\tilde{v}_1 + \tilde{\eta}_{2v}|\tau) = \frac{q}{\tau} + \rho^2 \frac{1-q}{\tau} + (1 - \rho^2) \frac{1-q}{\tau} = \frac{1}{\tau}$$

Proof of Proposition 3

Let $y_1 = x_1 + \varepsilon_1$. The posterior distribution of $\tilde{\tau}$ after observing μ_1 is Gamma $(\hat{\alpha}, \hat{\beta}) = \left(\alpha + \frac{1}{2}, \beta + \frac{\mu_1^2}{2q}\right)$ and

the posterior distribution of $\tilde{\tau}$ after observing (μ_1, y_1) is Gamma $(\hat{\hat{\alpha}}, \hat{\hat{\beta}}) = \left(\alpha + 1, \beta + \frac{\mu_1^2}{2q} + \frac{1}{2} \frac{(y_1 - \mu_1)^2}{1-q+k}\right)$.

Also, $E\left[\frac{1}{\tilde{\tau}} \mid \mu_1, y_1\right] = \frac{\hat{\hat{\beta}}}{\hat{\hat{\alpha}} - 1} = \frac{\beta + \frac{\mu_1^2}{2q} + \frac{1}{2} \frac{(y_1 - \mu_1)^2}{1-q+k}}{\alpha}$. The conjectured equilibrium is

$$P(R, MF) = p_0 + p_1 MF + p_2 R + p_3 \left(\frac{R - (\hat{r}_0 + \hat{r}_1 MF)}{\hat{r}_2} - \frac{MF - \hat{m}_0}{\hat{m}_1} \right)^2 + p_4 \left(\frac{MF - \hat{m}_0}{\hat{m}_1} \right)^2 \quad (11)$$

$$R = r_0 + r_1 MF + r_2 y_1 \quad (12)$$

$$MF = m_0 + m_1 \mu_1 \quad (13)$$

Solving for the equilibrium follows similar steps as the proof for Proposition 1 and is omitted for the sake of space. A detailed proof is available upon request. ■

Proof of Corollary 6

Part (a). Follows from $m_1 = \frac{1}{2} \left(1 + \frac{\sqrt{c_1 + 8p_4}}{\sqrt{c_1}} \right)$ and $r_2 = \frac{1}{2} \left(1 + \frac{\sqrt{c_2 + 8p_3}}{\sqrt{c_2}} \right)$ and $p_3, p_4 < 0$.

Part (b). Follows from $\frac{\partial m_1}{\partial p_4} > 0$ and $\frac{\partial r_2}{\partial p_3} > 0$ and $\frac{\partial p_i}{\partial \gamma} < 0, \frac{\partial p_i}{\partial \alpha} > 0$ for $i = 3, 4$.

Part (c). Note that $\frac{\partial m_1}{\partial \rho} = \frac{\partial m_1}{\partial p_4} \frac{\partial p_4}{\partial \rho}$ and $\frac{\partial r_2}{\partial \rho} = \frac{\partial r_2}{\partial p_3} \frac{\partial p_3}{\partial \rho}$

$$\begin{aligned} \frac{\partial \theta}{\partial \rho} &= 2(1+\rho) \frac{(1-q)k}{1-q+k} - 2\rho = 2 \frac{(1-q)k}{1-q+k} + 2\rho \frac{(1-q)k - (1-q+k)}{1-q+k} = \frac{2(1-q)}{1-q+k} \left[k - \rho \left(1 + \frac{q}{1-q} k \right) \right] \\ \frac{\partial p_3}{\partial \rho} &= -\frac{\gamma}{2\alpha} \frac{\theta'(\rho)}{1-q+k} \\ \frac{\partial p_4}{\partial \rho} &= -\frac{\gamma}{2\alpha} \frac{\theta'(\rho)}{q} \end{aligned}$$

Also, $\frac{\partial \theta}{\partial \rho} > 0 \Leftrightarrow \frac{(1-q)k}{1-q+k} > \rho$. ■

Proof of Corollary 7

Part (a). θ is always increasing in k : $\frac{\partial \theta}{\partial k} = (1+\rho)^2 (1-q) \frac{1-q+k-k}{(1-q+k)^2} = \frac{(1+\rho)^2 (1-q)^2}{(1-q+k)^2} > 0$. Hence, $\frac{\partial p_4}{\partial k} = -\frac{\gamma}{2\alpha} \frac{\theta'(k)}{q} < 0$.

Part (b). Assume $k > (1-q) \frac{2\rho - q(1+\rho)}{2 - q(1+\rho)}$. Then,

$$\begin{aligned} \frac{\partial p_3}{\partial k} &= -\frac{\partial}{\partial k} \frac{\gamma}{2\alpha} \frac{\theta(k)}{1-q+k} = \frac{\gamma(1+\rho)(2-q(1+\rho))}{2\alpha(1-q+k)^3} \left[k - (1-q) \frac{2\rho - q(1+\rho)}{2 - q(1+\rho)} \right] > 0 \\ \frac{\partial r_2}{\partial k} &= \frac{2}{\sqrt{c_2}\sqrt{c_2 + 8p_3}} \frac{\partial p_3}{\partial k} > 0 \\ \frac{\partial p_2}{\partial k} &= -(1+\rho) \frac{1-q}{(1-q+k)^2} \frac{1}{r_2} - (1+\rho) \frac{1-q}{1-q+k} \frac{\partial r_2}{\partial k} < 0 \\ \frac{\partial p_4}{\partial k} &= -\frac{\gamma}{2\alpha} \frac{\theta'(k)}{q} < 0 \\ \frac{\partial m_1}{\partial k} &= \frac{2}{\sqrt{c_1}\sqrt{c_1 + 8p_4}} \frac{\partial p_4}{\partial k} < 0 \\ \frac{\partial p_1}{\partial k} &= \frac{-m_1 \frac{\partial p_2}{\partial k} - (1+\rho - p_2) \frac{\partial m_1}{\partial k}}{m_1^2} = -\frac{1}{m_1} \left[\frac{\partial p_2}{\partial k} + p_1 \frac{\partial m_1}{\partial k} \right] > 0. \blacksquare \end{aligned}$$

Proof of Corollary 8

Part (a). For $k = 0$ we have $\theta = 1 - \rho^2$, $p_3 = \frac{-\gamma}{2\alpha} \frac{1-\rho^2}{1-q}$, and $p_4 = \frac{-\gamma}{2\alpha} \frac{1-\rho^2}{q}$. Then, $\frac{\partial p_3}{\partial q} = -\frac{\gamma}{2\alpha} \frac{1-\rho^2}{(1-q)^2} < 0$,

$$\frac{\partial p_4}{\partial q} = \frac{\gamma}{2\alpha} \frac{1-\rho^2}{q^2} > 0, \quad \frac{\partial m_1}{\partial q} = \frac{1}{2\sqrt{c_1}} \frac{8}{2\sqrt{c_1+8p_4}} \frac{\partial p_4}{\partial q} > 0, \quad \text{and} \quad \frac{\partial r_2}{\partial q} = \frac{1}{2\sqrt{c_2}} \frac{8}{2\sqrt{c_2+8p_3}} \frac{\partial p_3}{\partial q} < 0.$$

Part (b). CLAIM: Assume $k = 0$. Then, $\exists q^* \in (q_{\min}, q_{\max})$ s.t. that $\frac{\partial E[FE]}{\partial q} \leq 0$ if $q < q^*$ and $\frac{\partial E[FE]}{\partial q} \geq 0$ if $q > q^*$.

First, note that the equilibrium restriction $\gamma \leq \frac{\alpha}{4(1-\rho^2)} \min\{c_1 q, c_2(1-q)\}$ essentially puts restrictions on q for a given value of γ . We need $q \in [q_{\min}, q_{\max}] = \left[\frac{4\gamma}{\alpha} \frac{1-\rho^2}{c_1}, 1 - \frac{4\gamma}{\alpha} \frac{1-\rho^2}{c_2} \right]$. Next, I consider the case when $q = q_{\min}$. Then $c_1 + 8p_4 = 0$ and hence $m_1 = \frac{1}{2}$ and $p_1 = 2(1 + \rho - p_2)$. Since $(1 + \rho - p_2) < 0$ it follows that $\left. \frac{\partial E[FE]}{\partial q} \right|_{q=q_{\min}} < 0$. In the case when $q = q_{\max}$, $c_2 + 8p_3 = 0$ and hence $r_2 = \frac{1}{2}$, $p_2 = 2(1 + \rho)$, and $p_1 = -\frac{1+\rho}{m_1}$. From this it follows that $\left. \frac{\partial E[FE]}{\partial q} \right|_{q=q_{\max}} > 0$. Finally, we want to show that there is a unique local optimum of $E[FE]$ on $[q_{\min}, q_{\max}]$ and that the local optimum is a minimum. Rewriting the first derivative yields $\frac{\partial E[FE]}{\partial q} = \left(\frac{1}{c_2} + \frac{1}{c_1} \frac{1}{m_1} \right) \frac{\partial p_2}{\partial q} + \frac{1}{c_1} \frac{1+\rho-p_2}{m_1^2} \frac{\partial m_1}{\partial q}$. Evaluating the second derivative at a local optimum shows that the second derivative at any local optimum is negative. From this it follows that $E[FE]$ is convex in q at any local optimum. Hence, any local optimum must be a minimum therefore $E[FE]$ is decreasing in q for $q < q^*$ and increasing for $q > q^*$ where q^* is the local minimum on $[q_{\min}, q_{\max}]$. ■

Proof of Corollary 9

Average stock price sensitivity at forecast release date: $E \left[\frac{\partial P(MF)}{\partial MF} \right] = E \left[\frac{1+\rho}{m_1} - \gamma \frac{2q+1}{q} \frac{\beta + \frac{q}{m_1} \left(\frac{MF-m_0}{m_1} \right)}{\alpha - \frac{1}{2}} \right] = \frac{1+\rho}{m_1}$. Average stock price sensitivity at earnings announcement date: $E \left[\frac{\partial P(R,MF)}{\partial R} \right] = p_2 = \frac{1+\rho}{r_2}$. Solving $E \left[\frac{\partial P(MF)}{\partial MF} \right] < E \left[\frac{\partial P(R,MF)}{\partial R} \right]$ yields $\frac{c_1}{c_2} > \frac{1-q}{q}$. ■

Appendix B

Both equilibria in Proposition 1 and 3 require that investors' risk-aversion is relatively low. To provide an intuition for why those equilibria break down when investors are highly risk-averse, I return to the simpler, original setting as in Proposition 1. There, the first derivative to the manager's optimization problem when

choosing the optimal reporting policy at $t = 2$ is

$$p_2 + \frac{2p_3}{\hat{r}_2} \left(\frac{R - (\hat{r}_0 + \hat{r}_1 MF)}{\hat{r}_2} - MF + \hat{m}_0 \right) - c_2 (R - x_1) = 0$$

From the implicit function theorem it follows that

$$\frac{dR}{dx_1} = -\frac{c_2}{\frac{2p_3}{\hat{r}_2^2} - c_2}$$

In equilibrium $\frac{dR}{dx_1} = \hat{r}_2$. Then, the fixed point problem $\frac{dR}{dx_1}(\hat{r}_2) = \hat{r}_2$ has a solution only if $\frac{c_2}{2} \geq -4p_3$ (which is equivalent to $\gamma \leq \frac{c_2(2\alpha-1)}{8(1-\rho^2)}$). If $\frac{c_2}{2} < -4p_3$ then for any value of \hat{r}_2 conjectured by the market, it is optimal for the manager to choose a lower sensitivity of his earnings report to first period cash flows (i.e. $\frac{dR}{dx_1} < \hat{r}_2$). While this is potentially more costly in terms of discretionary accruals, a lower slope of reported earnings with respect to realized cash flows decreases the perceived riskiness of cash flows in the second period. If the investors are highly risk-averse the lower risk-premium exceeds the owners cost from choosing discretionary accruals. The cutoff for high risk aversion at which the equilibrium of Proposition 1 collapses is relative to the prior beliefs about the riskiness of the firm's cash flows. The higher α , the shape parameter of the gamma distribution of τ , the higher is the expected value of the precision of the firm's cash flows and the less severe is the updating process based on any realization of cash flows as inferred from the manager's reported earnings.

A natural candidate for an equilibrium if $\frac{c_2}{2} < -4p_3$ is $\hat{r}_2 = 0$. If r_2 is zero in equilibrium then reported earnings do not contain any information beyond MF and hence investors are unable to update their beliefs regarding the riskiness of the firm's cash flows. In such an equilibrium, the capital market price does not depend on reported earnings. However, having reported earnings be independent of realized cash flows is costly for the manager as long as $c_2 > 0$. Hence, for $r_2 = 0$ to be an equilibrium, there have to be off-equilibrium beliefs in place that impose a penalty for reporting anything other than the deterministic function of MF that is expected in equilibrium. The penalty has to be large enough to cover the cost of discretionary accruals that the manager is forced to take such that earnings do not depend on the realization

of cash flows, x_1 . Because of the assumption of normally distributed cash flows the discretionary accruals are potentially infinite and hence the off-equilibrium penalty has to be infinite, too. This is reflected in the following degenerated equilibrium with $R(x_1, MF) = R(MF) = MF$.

$$\begin{aligned} MF(\mu_1) &= \frac{1+\rho}{c_1+c_2} + \mu_1 \\ R(x_1, MF) &= MF \\ P(R, MF) &= \begin{cases} p_0 + p_1 MF & \text{if } R = MF \\ -\infty & \text{otherwise} \end{cases} \end{aligned}$$

where $p_1 = 1 + \rho$ and $p_0 = (1 - \rho) E[\tilde{\mu}] - \frac{(1+\rho)^2}{c_1+c_2} - \gamma(1+\rho)^2 \left(\frac{2\beta}{\alpha-1} + (1-\rho)\sigma_\mu^2 \right)$. Note that even though the capital market price does not depend on reported earnings in equilibrium, the infinite penalty for $R \neq MF$ is needed to prevent the manager to report $R = x_1$ which would eliminate any costs from discretionary accruals. Since the capital market cannot commit to not using the information revealed by the earnings report, the pricing function $P(R, MF) = p_0 + p_1 MF$ would be inconsistent with Bayesian updating if reported earnings equaled x_1 .

Proof. Let $R(MF) = r_0 + r_1 MF$ and $MF = m_0 + \mu_1$. Since in equilibrium no information about the realization of x_1 is revealed, $p_2 = p_3 = 0$. For the reporting strategy to be part of an equilibrium it has to minimize the cost of discretionary accruals, i.e. $\hat{r}_0 + \hat{r}_1 MF = \mu_1$. Then at $t = 1$ the manager solves

$$\max_{MF} p_0 + p_1 MF - \frac{c_1}{2} (MF - \mu_1)^2 - \frac{c_2}{2} E \left[(\hat{r}_0 + \hat{r}_1 MF - x_1)^2 \mid \mu_1 \right]$$

and hence

$$\begin{aligned} p_1 - c_1 (MF - \mu_1) - c_2 (\hat{r}_0 + \hat{r}_1 MF - \mu_1) &= 0 \\ MF &= \frac{p_1 + (c_1 + c_2) \mu_1 - c_2 \hat{r}_0}{c_1 + c_2 \hat{r}_1} \end{aligned}$$

With $\hat{r}_0 + \hat{r}_1 MF = \mu_1$ this yields $r_0 = 0$, $r_1 = 1$, and $m_0 = \frac{p_1}{c_1+c_2}$. The equilibrium price, $P(MF)$ has to

equal

$$\begin{aligned}
E[\tilde{x}_1 + \tilde{x}_2|MF] - \gamma \text{Var}[\tilde{x}_1 + \tilde{x}_2|MF] &= (1 + \rho)\mu_1 + (1 - \rho)E[\tilde{\mu}] - \gamma \left((1 + \rho)^2 \text{Var}[\tilde{x}_1|\mu_1] + \text{Var}[\tilde{\eta}_2] \right) \\
&= (1 + \rho) \left(MF - \frac{1 + \rho}{c_1 + c_2} \right) + (1 - \rho)E[\tilde{\mu}] - \gamma(1 + \rho)^2 \left(\frac{2\beta}{\alpha - 1} + (1 - \rho)\sigma_\mu^2 \right)
\end{aligned}$$

and hence $P(MF) = p_0 + p_1 MF$ where $p_0 = (1 - \rho)E[\tilde{\mu}] - \frac{(1 + \rho)^2}{c_1 + c_2} - \gamma(1 + \rho)^2 \left(\frac{2\beta}{\alpha - 1} + (1 - \rho)\sigma_\mu^2 \right)$ and

$p_1 = 1 + \rho$. ■

References

- Abarbanell, J. and R. Lehavy (2003). Biased forecasts or biased earnings? the role of reported earnings in explaining apparent bias and over/underreaction in analysts' earnings forecasts. *Journal of Accounting and Economics* 36, 105–146.
- Burgstahler, D. and I. Dichev (1997). Earnings management to avoid earnings decreases and losses. *Journal of Accounting and Economics* 24, 99–126.
- Choi, J.-H. and D. A. Ziebart (2000). A reexamination of bias in management earnings forecasts. Working paper University of Illinois at Urbana-Champaign.
- DeGeorge, F., J. Patel, and R. Zeckhauser (1999, January). Earnings manipulations to exceed thresholds. *Journal of Business* 71(1), 1–34.
- DeGroot, M. H. (1970). *Optimal Statistical Decisions*. New Jersey: Wiley Classics Library Edition.
- Dutta, S. and F. Gigler (2002). The effect of earnings forecasts on earnings management. *Journal of Accounting Research* 40, 631–655.
- Dye, R. A. (1988). Earnings management in an overlapping generations model. *Journal of Accounting Research* 26, 195–235.
- Dye, R. A. and S. S. Sridhar (2004). Reliability-relevance trade-offs and the efficiency of aggregation. *Journal of Accounting Research* 42, 51–88.

- Feng, M. (2004). *Why Do Managers Meet or Slightly Beat Earnings Forecast in Equilibrium?* Ph. D. thesis, University of Michigan.
- Fischer, P. E. and R. E. Verrecchia (1997). The effect of limited liability on the market response to disclosure. *Contemporary Accounting Research* 14, 515–541.
- Freeman, R. and S. Tse (1992). A nonlinear model of security price responses to unexpected earnings. *Journal of Accounting Research* 30, 185–109.
- Graham, J. R., C. R. Harvey, and S. Rajgopal (2005). The economic implications of corporate financial reporting. *Journal of Accounting and Economics* 40, 3–73.
- Hayn, C. (1995). The information content of losses. *Journal of Accounting and Economics* 28, 125–153.
- Hughes, J. S. and S. Pae (2004). Voluntary disclosure of precision information. *Journal of Accounting and Economics* 37, 261–289.
- Jorgensen, B. N. and M. T. Kirschenheiter (2003). Discretionary risk disclosure. *The Accounting Review* 78, 449–470.
- Kaszniak, R. (1999). On the association between voluntary disclosure and earnings management. *Journal of Accounting Research* 37, 57–81.
- Kirschenheiter, M. and N. D. Melumad (2002). Can "big bath" and earnings smoothing co-exist as equilibrium financial reporting strategies? *Journal of Accounting Research* 40, 761–796.
- Matsumoto, D. (2002). Management's incentives to avoid negative earnings surprises. *The Accounting Review* 77, 483–515.
- McNichols, M. (1989). Evidence of informational asymmetries from management earnings forecasts and stock returns. *The Accounting Review* 64, 1–27.
- Melumad, N. D., G. Weyns, and A. Ziv (1999). Comparing alternative hedge accounting standards: Shareholders' perspective. *Review of Accounting Studies* 4, 265–292.
- Penman, S. H. (1980). An empirical investigation of the voluntary disclosure of corporate earnings forecasts.

- Journal of Accounting Research* 18, 132–160.
- Penno, M. (1996). Unobservable precision choices in financial reporting. *Journal of Accounting Research* 34, 141–149.
- Pownall, G., C. Wasley, and G. Waymire (1993). The stock price effects of alternative types of management earnings forecasts. *The Accounting Review* 68, 896–912.
- Richardson, S., S. H. Teoh, and P. D. Wysocki (2001, August). The walkdown to beatable analysts' forecasts: The roles of equity issuance and insider trading incentives. MIT Sloan Working Paper No. 4221-01.
- Sankar, M. R. and K. R. Subramanyam (2001). Reporting discretion and private information communication through earnings. *Journal of Accounting Research* 39, 365–386.
- Soffer, L., S. R. Thiagarajan, and B. L. Walther (2000). Earnings preannouncement strategies. *Review of Accounting Studies* 5, 5–26.
- Subramanyam, K. (1996). The pricing of discretionary accruals. *Journal of Accounting and Economics* 22, 249–281.
- Tan, H., R. Libby, and J. Hunton (2002). Analysts' reactions to earnings pre-announcements strategies. *Journal of Accounting Research* 40, 223–246.
- Verrecchia, R. E. (1983). Discretionary disclosure. *Journal of Accounting and Economics* 5, 179–194.