Liquidity and Market Crashes

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Abstract

In this paper, we develop an equilibrium model for stock market liquidity and its impact on asset prices when participation in the market is costly. We show that, even when agents' trading needs are perfectly matched, costly participation prevents them from synchronizing their trades, which gives rise to the endogenous need for liquidity. Moreover, the endogenous liquidity need, when it occurs, is one-sided and of significant magnitude. In particular, it is dominated by excessive selling which leads to market crashes in the absence of any aggregate shocks. As a result, endogenous liquidity needs gives rise to negative skewness and fat-tails in stock returns.

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1 Introduction

It is well recognized that liquidity is of critical importance to the stability and the efficiency of the financial market (see, for example, CGFS (1999)). Yet, there is little consensus about exactly what liquidity is, what determines it, and how it affects asset prices.[1] Market frictions have been considered as important determinants of liquidity and asset prices. But the precise nature of this link is not well understood due to the complexity in analyzing the interactions among diverse market participants under market frictions.[2]

In this paper, we study how market frictions lead to market participants' need for liquidity and how such a need can destabilize asset prices. We focus our attention on a specific form of market frictions, the cost to participate in the market.[3] Participation costs prevent all agents from being in the market at all times. Their infrequent presence in the market causes non-synchronization in their trades even when their underlying trading needs are perfectly matched. Non-synchronized trades give rise to order imbalances and the need for liquidity. We show that this endogenous liquidity need, when it arises, generates excess sell orders of significant sizes. Consequently, it leads to large price drops in the absence of any aggregate shock to the fundamentals. This result suggests that, in the presence of market frictions, endogenous surges in the need for liquidity can cause market crashes.

Two elements are essential for liquidity to be economically important: the need to trade and the cost to trade. In the absence of any trading needs, there is no need for liquidity. In the absence of any cost to trade, agents will trade in the market at all times in response to exogenous shocks. Although the equilibrium price does adjust to equate demand and supply, the market is perfectly liquid in the sense that the price reflects the "fair value" of the asset. Actual markets do not function in this "gigantic town meeting" style, as Grossman and Miller

[1] See for, example, Keynes (1936) and Hicks (1962). More recent work include Allen and Gale (1994), Diamond and Dybvig (1983), Grossman and Miller (1988), Holmstrom and Tirole (1998), Kiyotaki and Moore (1997), and Kyle (1985). See also, Brunnermeier and Pedersen (2005), Cochrane (2005), and Hodrick and Moulton (2005).

[2] The literature on the impact of market frictions on liquidity and asset prices is extensive. The theoretical work include Amihud and Mendelson (1986), Aiyagari and Gertler (1991), Constantinides (1986), Glosten and Milgrom (1985), Grossman and Miller (1988), Heaton and Lucas (1996), Hirshleifer (1988), Huang (2003), Kyle and Xiong (2001), Lo, Mamaysky, and Wang (2004), Orosel (1998), Pagano (1989), Stoll (1985, 1989), and Vayanos (1998, 2004). Recent empirical work on the impact of liquidity on asset prices include Acharya and Pedersen (2004), Amihud (2002), Brennan and Subrahmanyam (1996), Chordia, Roll, and Subrahmanyam (2000), and Pastor and Stambaugh (2003).

[3] See, for example, Brennan (1975), Chatterjee and Corbae (1992), Hirshleifer (1988), and Merton (1987) for discussions of the importance of participation costs.

(1988) call it, where all potential buyers and sellers are present at all times and trades are conducted to balance the full demand and supply. Costs prevent potential participants from constantly being in the market. At any given instant, only a subset of traders are present in the market. When a trader arrives at the market, he only faces a "partial" demand/supply. Adjustments in prices fail to attract all potential buyers and sellers or to synchronize their trades. It is this non-synchronization in trading that gives rise to the need for liquidity, which in turn affects asset prices.

To formalize this intuition, we start with an economy in which potential traders receive idiosyncratic shocks to their total risk exposure. Those who receive positive shocks to their risk exposure are potential sellers while those receiving opposite shocks are potential buyers. They desire to trade in the market to offset their idiosyncratic shocks. Since idiosyncratic shocks always sum to zero across all potential traders, their trading needs are perfectly matched. In absence of participation costs, all traders are in the market at all times. Their trades, driven by off-setting shocks to their risk exposure, are fully synchronized. Sell orders are always accompanied by the same amount of buy orders. In this case, there is no need for liquidity and asset prices depend only on the "fundamentals". The idiosyncratic trading needs of individual traders have no impact on prices.

In the presence of participation costs, however, potential traders will stay out of the market when expected gains from trading are small. They participate only when they are far away from their desired positions. The infrequent participation in the market has two consequences. First, the gains from trading are in general different different even between traders with offsetting trading needs. This immediately leads to the non-synchronization in their trades when they decide to participate. The non-synchronization in trades then gives rise to order imbalances in the market, which lead to the need for liquidity and corresponding price deviations. Second, with infrequent trading, potential traders cannot always offset their idiosyncratic shocks through trading. Having to bear some idiosyncratic risks, they become effectively more risk averse. As a result, their desired stock holdings decrease when hit by idiosyncratic shocks. This effect is independent of the realizations of idiosyncratic shocks. Given that their new desired holdings are lower than their initial holdings, potential traders who receive positive idiosyncratic shocks (i.e., potential sellers) becomes further away from their desired positions than those who receive negative shocks (i.e., potential buyers). The gains from trading are larger for potential sellers than for potential buyers. As a result, potential sellers are always more eager to enter the market than potential buyers, even though their idiosyncratic shocks perfectly match and they face the same participation costs. This asymmetry in participation between buyers and sellers implies the order imbalance, when it arises, is always skewed toward sell orders. Consequently, the stock price has to decrease in order to attract the market makers to absorb the excess sell orders.

Furthermore, we show that the order imbalance and the need for liquidity are highly nonlinear in the idiosyncratic shocks that drive agents' trading needs. When the magnitude of idiosyncratic shocks is moderate, gains from trading are relatively small for all traders. As a result, they all stay out of the market and there is no need for liquidity. Only for large enough idiosyncratic shocks, gains from trading exceed participation costs and potential traders may decide to enter the market. Thus, the order imbalance and the need for liquidity, when they occur, are often large in magnitude, causing the price to drop discretely in absence of any shocks to the fundamentals. As a result, "liquidity crashes," which are market crashes driven purely by liquidity, can arise.

Most of the existing literature on liquidity examines how market frictions affect the provision of liquidity and asset prices, given the need for liquidity.[4] For example, Grossman and Miller (1988) consider the role of market makers in providing liquidity and reducing price volatility, taking as given the non-synchronization in trades. Such an approach ignores the fact that it is the same market frictions that give rise to the need for liquidity in the first place. Our analysis shows that endogenous liquidity need can lead to different predictions for its impact on prices and market efficiency.

In this regard, our paper shares the same spirit with Allen and Gale (1994), who consider the ex-ante participation decisions of agents with different future liquidity needs. They show that the ex-ante optimal level of participation can be inadequate ex post when the realized liquidity need is much larger than expected, causing additional volatility in prices. We focus more on the dynamic aspect of liquidity by allowing traders to make their participation decisions after observing new shocks to their trading needs over time. Thus, we are able to study how the need for liquidity rises endogenously in response to new idiosyncratic shocks to the traders. As we show, the properties of the endogenous liquidity need (e.g., one-sided

[4]See, for example, Amihud and Mendelson (1980), Grossman and Miller (1988), Ho and Stoll (1981), and Huang (2003). In the market micro-structure literature, which has liquidity as one of its central focus, the need for liquidity, as described by the order flow process, is often taken as given. See, for example, Glosten and Milgrom (1985), Kyle (1985), and Stoll (1985). Admati and Pfleiderer (1988), however, do allow the need for liquidity to be influenced by equilibrium.

and fat tailed) can be quite different from those assumed for exogenous liquidity shocks.[5]

Our model is closely related to the model of Lo, Mamaysky, and Wang (2004), who consider the impact of fixed transactions costs on trading volume and the level of asset prices. They show that, in a continuous-time stationary setting, gains from trading is in general asymmetric between traders with offsetting shocks when trading is infrequent. Such an asymmetry naturally leads to non-synchronization in trades, the need for liquidity, and price deviations. In order to focus on the impact of trading costs on price levels, they avoid order imbalances by allowing the participation cost to be allocated endogenously so that the trades of different market participants are always synchronized in equilibrium. As we show in this paper, it is the order imbalance that leads to changes in liquidity needs and the instability in asset prices.

By modelling costly participation, we capture an important aspect of the actual market. But the market structure we use, with the continuous presence of market makers, still takes the form of a centralized exchange. When the costs for such a market structure are significantly large, we may have to consider alternative market structures such as over-the-counter markets (see, e.g., Duffie, Gârleanu, and Pedersen (2004, 2005)).

The paper proceeds as follows. Section 2 describes the basic model. Section 3 solves for the intertemporal equilibrium of the economy. In Section 4, we analyze how individual traders' participation decisions give rise to the need for liquidity and the properties of equilibrium liquidity needs. In Section 5, we examine how the endogenous need for liquidity affects asset prices. Section 6 concludes. The appendix provides the proofs.

2 The Model

We construct a parsimonious model that captures two important factors in analyzing liquidity, the need to trade and the cost to constantly participate in the market. For simplicity, we consider a discrete-time, infinite-horizon setting.

[5]By making trading needs perfectly matched between the traders, liquidity is purely associated with the non-synchronization in their trades. When trading needs are not perfectly matched, there is also an aggregate shift in demand, which is the case considered by Allen and Gale (1988) as well as Campbell, Grossman, and Wang (1993), Campbell and Kyle (1993), and Kyle and Xiong (2001), among others. In this case, the distinction between shocks to liquidity and risk (and/or preference) becomes less clear.

2.1 Economy

A Asset Market

There is a stock traded in a competitive asset market. The stock yields a risky dividend D_t at time t, where $t = 0, 1, 2, \ldots$ Dividends are i.i.d. normally distributed with a mean of $\bar{D}$ and volatility of σ_D. Let P_t denote the ex-dividend stock price at time t. In addition, there is a short-term riskless bond, which yields a constant interest rate of $r > 0$ per period.

B Agents

At $t = 0, 1, 2, \ldots$, a set of agents are born who live for one period. Agents born at t are referred to as generation t. They are born with initial wealth W_t, which they use to purchase stocks from the previous generation and to invest in risk-free bonds. They sell all their assets for consumption at time $t + 1$.

Each generation consists of two types of agents who face different endowments and trading costs. As described below, agents' heterogeneity in endowments will gives rise to their trading needs in our model.[6] The first type of agents, denoted by m, are "market makers." They have no inherent trading needs, but are present in the market at all times, ready to trade with others. The second type of agents are "traders" who have inherent trading needs. Although they are free to trade upon birth and death, i.e., at t and $t + 1$ for generation t, they face additional costs to trade in the market between t and $t + 1$. Furthermore, traders consist of two equal subgroups, denoted by a and b. The population weight of the market makers and the traders are μ and 2ν, respectively.

The total supply of the stock is $(\mu + 2\nu)\,\bar{\theta}$ shares (i.e., $\bar{\theta}$ per capita). In addition, each agent i of generation t receives a non-traded payoff N^i_{t+1} at the end of his life-span, which is given by

$$N^i_{t+1} = \lambda^i Z\, n_{t+1}, \quad i = m, a, b \tag{1}$$

where Z and n_{t+1} are mutually independent, normal random variables with mean of zero and volatility of σ_z and σ_n, respectively, λ^i is a binomial random variable drawn independently

[6] Heterogeneity in endowment is merely a device to introduce the need to trade for risk-sharing as in Wang (1994) and Lo, Mamaysky, and Wang (2004). Other forms of heterogeneity can also generate risk-sharing trading needs, such as difference in preferences (e.g., Dumas (1992) and Wang (1996)) or beliefs (e.g., Detemple and Murthy (1994)). Our modelling choice is mainly motivated by simplicity and tractability.

for each agent within his group, where

$$\lambda^m = 0, \quad \lambda^a = -\lambda^b = \begin{cases} 1, & \text{probability } \lambda \\ 0, & \text{probability } 1-\lambda. \end{cases} \tag{2}$$

Here we have suppressed the time subscript for λ^i and Z for brevity. Thus, market makers receive no non-traded payoff, while a fraction λ of traders within each trader group receives non-traded payoffs. Since $\lambda^a = -\lambda^b$, the two groups of traders receive perfectly offsetting non-traded payoffs. By construction, we have

$$\sum_{i=a,b,m} N^i_{t+1} = 0. \tag{3}$$

The non-traded payoff is assumed to be correlated with the stock dividend D_{t+1}. In particular, we let $n_{t+1} = D_{t+1} - \bar{D}$.[7]

In the absence of risks from non-traded payoffs, all agents are identical and there will be no trading needs among them. However, in the presence of non-traded risks, traders who receive them want to trade in order to share these risks. In particular, given the correlation between the non-traded payoff and the stock payoff, they want to adjust their stock positions in order to hedge their non-traded risk. Thus, traders' idiosyncratic risk exposures give rise to their inherent trading needs.

Since the non-traded risks sum to zero as in (3), the traders' underlying trading needs are perfectly matched. If all traders are present in the market at all times, a seller is always matched with a buyer and there is perfect synchronization in their trades. If, however, only some traders are present at a given time, trades may not be always synchronized and the need for liquidity arises.

For tractability, we assume that all agents have a utility function of constant absolute risk aversion over their terminal wealth. The utility function for generation-t agents is

$$\mathrm{E}\left[-e^{-\alpha W^i_{t+1}}\right], \qquad i = a, b, m \tag{4}$$

where W^i_{t+1} denotes agent i's terminal wealth. For the model to be well defined, we require

$$\alpha^2 \sigma_D^2 \sigma_z^2 < 1. \tag{5}$$

[7]We only need to assume that the correlation between n_{t+1} and D_{t+1} to be non-zero. The qualitative nature of our results are independent of the sign and the magnitude of the correlation. To fix ideas, we simply set it to 1.

C Trading Costs

All agents can trade in the market at no cost at the beginning and the end of their life-span. That is, agents of generation t can trade in the market at t and $t+1$ without cost. In addition, market makers can also trade at no cost at any time between t and $t+1$. The traders, however, face a fixed cost $c \geq 0$ if they want to trade between t and $t+1$. This form of transactions costs can be associated with the costs of being in a market by setting up a trading operation, gathering and incorporating the information into trading activities.

D Time Line

We now describe in detail the timing of events and actions. At t, agents of generation t are born. They purchase shares of the stock from the old generation and construct their optimal portfolio θ_t^i, $i = a, b, m$. Market equilibrium at t determines P_t.

After t, traders learn if they will be exposed to any idiosyncratic risks (i.e., their draws of λ^i). Those who will ($\lambda^i \neq 0$) also observe a signal S about its potential magnitude Z:

$$S = Z + u \tag{6}$$

where u is the noise in the signal, normally distributed with a mean of zero and a variance of $\sigma_u^2 > 0$. For future convenience, we denote by X the expectation of Z conditional on signal S and $\sigma_{\hat{z}}^2$ the conditional variance. We then have

$$X \equiv \mathrm{E}[Z|S] = \frac{\sigma_z^2}{\sigma_z^2 + \sigma_u^2}\, S, \quad \sigma_{\hat{z}}^2 \equiv \mathrm{Var}[Z|S] = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_z^2}\, \sigma_z^2. \tag{7}$$

Under normality, X is a sufficient statistic for signal S. Thus, we will use X to denote these traders' information about the magnitude of their idiosyncratic risks.

After learning about their idiosyncratic risk, traders face the choice of staying out of the market (until their terminal date) or paying a cost c to enter the market. Those who choose to enter the market will then trade among themselves as well as with market makers. For simplicity, we assume that by the time of trading they also observe the realization of Z. Afterwards, there is no more need to trade; all agents hold their portfolios until $t+1$ and then liquidate for consumption.

Except their sequencing, the exact times for receiving idiosyncratic risks, entering the market and then trading are not important. To fix ideas, we assume that signal X and entry decisions occur right after t (i.e., at t_+) while trading occurs at $t + 1/2$. This specification has the advantage of carrying forward interests more easily.[8]

A trader's choice to enter the market depends on his draw of λ^i and the signal X on the magnitude of the idiosyncratic risk if $\lambda^i \neq 0$. Let η^i be the discrete choice variable of trader i ($i = a, b$) for whether to enter the market, where $\eta^i = 1$ denotes entry and $\eta^i = 0$ denotes no entry. Among group i traders ($i = a, b$), we use $\omega^{i,NL}$ to denote the fraction of those with $\lambda^i = 0$ (i.e., receiving no idiosyncratic shocks) and $\omega^{i,L}$ to denote the fraction of those with $\lambda^i \neq 0$, respectively, who choose to enter the market. We also use $\theta^i_{t+1/2}(\eta^i)$ to denote the number of stock shares agents i ($i = m, a, b$) holds after trading at date $t + 1/2$. Of course, $\theta^i_{t+1/2}(\eta^i = 0) = \theta^i_t$.

Summarizing the description above, Figure 1 illustrates the time-line of the economy.

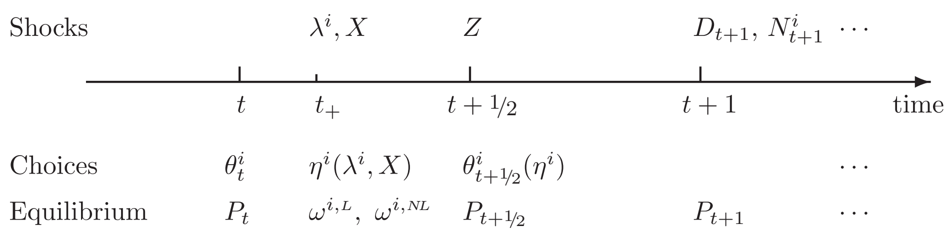

Figure 1: The time line of the economy.

For agent i, his terminal financial wealth, denoted by F^i_{t+1}, is

$$F^i_{t+1} = (W_t - \eta^i c^i)R^2 + \theta^i_t \left(P_{t+1/2} - RP_t\right) R + \theta^i_{t+1/2}(\eta^i) \left(D_{t+1} + P_{t+1} - RP_{t+1/2}\right) \qquad (8)$$

where $R = (1+r)^{1/2}$ is the gross interest rate for each $1/2$ period, $c^i = c$ for $i = a, b$ and $c^i = 0$ for $i = m$. His total wealth of at date $t + 1$ is then given by

$$W^i_{t+1} = F^i_{t+1} + N^i_{t+1} \qquad (9)$$

where N^i_{t+1} is the income from the non-traded asset in (1).

2.2 Definition of Equilibrium

The equilibrium of the economy requires three conditions. First, taking prices as given, all agents optimize with respect to their participation and trading decisions. Second, agents' participation reaches an equilibrium. Third, the stock market clears. Given the repetitive

[8]In principle, trading can occur anytime between t and $t + 1$. We avoid the potential coordination issue among traders by assuming that they trade only once right after observing the full magnitude of their exposures.

nature of each generation, we only need to focus on the equilibrium over the life-span of one generation, say, generation t.

At time t, generation-t is born and they purchase the shares from the previous generation. Since the total supply of the stock is $(\mu + 2\nu)\,\bar{\theta}$, the equilibrium stock price P_t is determined by the market clearing condition

$$\mu\,\theta^m_t + \nu\,(\theta^a_t + \theta^b_t) \;=\; (\mu + 2\nu)\,\bar{\theta}. \tag{10}$$

After t, traders from each trader group learn about whether or not they are exposed to idiosyncratic risks and they decide whether to pay a cost to enter the market. A participation equilibrium is reached if either all traders within the same group make identical participation decisions, i.e., $\omega^{i,j} = 0$ or 1, $i = a, b$ and $j = L$, NL, or they are indifferent between participating or not at an interior value $\omega^i_j \in (0, 1)$.

At time $t + 1/2$, only market makers and the participating traders are in the market. Ignoring non-participating traders, we let $\theta^{i,j}_{t+1/2} \equiv \theta^i_{t+1/2}\big(\eta^i(\lambda^i) = 1\big)$ $(i = a, b)$ to denote the stock holding at $t+1/2$ of participating traders from each group, with and without idiosyncratic shock $(\lambda^i = 0, 1)$, respectively. The clearing of the stock market at $t + 1/2$ requires

$$\mu\,\theta^m_{t+1/2} + \nu \sum_{i=a,b} \left[\lambda\,\omega^{i,L}\theta^{i,L}_{t+1/2} + (1-\lambda)\omega^{i,NL}\theta^{i,NL}_{t+1/2}\right] = \mu\,\theta^m_t + \nu \sum_{i=a,b} \left[\lambda\,\omega^{i,L} + (1-\lambda)\omega^{i,NL}\right]\theta^i_t \tag{11}$$

which determines the stock market equilibrium at $t + 1/2$.

At time $t+1$, the economy repeats itself: A new generation is born; the current generation sells their stock holdings and consume their total wealth as given in (9); the stock market clears as the new generation purchases all the stock shares. Given the stationary nature of the economy, we consider a stationary equilibrium in which

$$P_{t+1} = P_t. \tag{12}$$

It is worth noting that we assumed a constant interest rate and thus do not require the bond market to clear.

2.3 Equilibrium with Costless Participation

Before solving for the equilibrium, we describe the special case in which participation costs are zero for all agents. This case serves as a benchmark when we examine the impact of participation costs on liquidity and stock prices. If $c^i = 0\ \forall\ i = a, b, m$, all traders and market makers will be in the market at all times and $\omega^{i,L} = \omega^{i,NL} = 1 \ \forall \ i = a, b$. The equilibrium price and agents' equilibrium stock holdings are:

$$\begin{aligned} &P_t = \bar{P} \equiv \tfrac{1}{r}\left(\bar{D} - \alpha\sigma_D^2\,\bar{\theta}\right), \qquad && \theta_t^i = \bar{\theta} \\ &P_{t+1/2} = RP_t, && \theta_{t+1/2}^i = \bar{\theta} - \lambda^i Z \end{aligned} \tag{13}$$

where $t = 0, 1, 2, \ldots$ and $i = a, b, m$.

In this case, stock prices, P_t and $P_{t+1/2}$, are determined by the stock's expected future dividends $\bar{D}$, the dividend risk σ_D, and the aggregate (per capita) risk exposure $\bar{\theta}$. We call these the "fundamentals" of the stock. In particular, prices do not depend on the idiosyncratic risk exposure $\lambda^i Z$. For traders exposed to non-traded risks, their stock holding equal the per capita shock share $\bar{\theta}$ plus an additional component $\lambda^i Z$, which reflects their hedging demand to offset the exposure to non-traded risk. It is important to note that these traders' underlying trading needs are perfectly matched ($\lambda^a = -\lambda^b$), so are their trades when they are all in the market. In this case, the market is perfectly liquid in the sense that order flows have no price impact. There is no need for liquidity and market makers perform no role (their holdings stay at $\theta^m = \bar{\theta}$).

3 Equilibrium

We now solve for the equilibrium when participation is costly in three steps. First, taking the stock price at time $t+1$ and agents' initial stock holdings and participation decisions at t as given, we solve for the stock market equilibrium at $t+1/2$. Next, we solve for individual agents' participation decisions and the participation equilibrium, given the market equilibrium at $t + 1/2$ and their initial stock holdings at t. Then, we solve for the market equilibrium at time t. Using the condition $P_{t+1} = P_t$, we finally obtain the full stationary equilibrium of the economy.

In the first two steps (Sections 3.1-3.3), we will assume that traders who receive no idiosyncratic shocks ($\lambda^i = 0$) will stay out of the market until the the end of their horizon, that is, $\omega^{i,NL} = 0$, $i = a, b$. We then only consider those traders who do receive shocks and solve for their participation decisions, the participation equilibrium, and the market equilibrium at $t + 1/2$. In these subsections, unless stated otherwise, traders refer only to those with $\lambda^i \neq 0$ and participation weights, $\omega^a = \omega^{a,L}$ and $\omega^b = \omega^{b,L}$, refer to fractions of them who choose to participate. In the last step (Section 3.4), we include all traders and confirm that in equilibrium those who receive no idiosyncratic shocks indeed choose not to participate in the market.

3.1 Market Equilibrium at $t + 1/2$

At $t + 1/2$, agents' initial stock holdings and their participation decisions, i.e., $\{\theta, \omega\}$, are given, where $\theta \equiv (\theta^a_t, \theta^b_t, \theta^m_t)$ denotes agents' stock holdings at t and $\omega \equiv (\omega^a, \omega^b)$. Since our analysis will focus on generation t, we have omitted the time subscript t of θ and ω for brevity. Together with Z, $\{\theta, \omega\}$ defines the state of the economy at $t + 1/2$. Two variables are of particular importance in describing the market condition:

$$\hat{\theta} \equiv \frac{\mu\theta^m + \lambda\,\nu(\omega^a\theta^a + \omega^b\theta^b)}{\mu + \lambda\,\nu(\omega^a + \omega^b)}, \qquad \delta \equiv \frac{\lambda\,\nu}{\mu + \lambda\,\nu(\omega^a + \omega^b)}\,(\omega^a - \omega^b) \tag{14}$$

where $\hat{\theta}$ gives the per capita stock supply in the market (brought by participating agents) and δ measures the difference in participation between the two trader groups. Since the participation equilibrium at t depends on the information X about the non-traded risk, ω^a and ω^b and thus $\hat{\theta}$ and δ are in fact functions of X.

Given the market condition $\hat{\theta}$ and δ and the magnitude of idiosyncratic risks Z, we can solve for the market equilibrium at $t + 1/2$.

Proposition 1. *Let P_{t+1} be the equilibrium price at time $t+1$. Given $\{\theta, \omega\}$, the equilibrium stock price at $t + 1/2$ is*

$$P_{t+1/2} = \frac{1}{R}\left(\bar{D} + P_{t+1} - \alpha\sigma^2_D\,\hat{\theta} - \alpha\sigma^2_D\,\delta Z\right) \tag{15}$$

and the equilibrium stock holding of participating agent i is

$$\theta^i_{t+1/2} = \hat{\theta} + \delta Z - \lambda^i Z, \qquad i = a, b, m. \tag{16}$$

When $\delta = 0$, the participation of the two groups of traders is symmetric. The participating agents' holdings are equal to the per capita holding $\hat{\theta}$ minus the hedging demand $\lambda^i Z$. Since $\lambda^a = -\lambda^b$, there is a perfect match between the buy and sell orders among traders, and the equilibrium price is not affected by the idiosyncratic shock Z. This situation is reminiscent of the benchmark case when participation is costless.

When $\delta \neq 0$, the participation of the two groups of traders is asymmetric. The quantity δZ measures the excess exposure (per capita) to the non-traded risk due to the asymmetric participation of traders. In this case, the optimal holding in (16) has an extra term δZ for all participating agents since they equally share this additional source of risk. The idiosyncratic shock Z now affects the equilibrium price. Thus in our model, even though traders face offsetting shocks, asymmetry in their participation can give rise to a mismatch in their trades and cause the price to change in response to these shocks.

Here, we have taken traders' participation and the resulting δ and $\hat{\theta}$ as given. In the next subsection, we show that when individual participation decisions are made endogenously, asymmetric participation occurs as an equilibrium outcome.

3.2 Optimal Participation Decision

Given the market equilibrium at $t + 1/2$ and the signal X for future idiosyncratic shocks, we now solve the participation equilibrium at t. First, taking as given the participation decision of others and price $P_{t+1/2}$, we derive the optimal participation policy of an individual trader. Next, we find the competitive equilibrium for traders' participation decisions.

For trader i, let J_P and J_{NP} denote his utility after he chooses to participate or not to participate, respectively. We have

$$J_P(\theta^i; \lambda^i, X; \hat{\theta}, \delta) \;=\; \mathrm{E}\left[\max_{\theta^i_{t+1/2}} \mathrm{E}_{t+1/2}\left[-e^{-\alpha W^i_{t+1}}\right] \;\middle|\; \lambda^i, X;\; \eta^i = 1\right] \tag{17a}$$

$$J_{NP}(\theta^i; \lambda^i, X; \hat{\theta}, \delta) \;=\; \mathrm{E}\left[-\, e^{-\alpha W^i_{t+1}} \;\middle|\; \lambda^i, X;\; \eta^i = 0,\; \theta^i_{t+1/2} = \theta^i\right] \tag{17b}$$

where θ^i denotes his initial stock holding, λ^i and X define his exposure to the non-traded risk, $\hat{\theta}$ and δ define the market condition, and $\mathrm{E}[\cdot|X]$ denotes the expectation conditional on X. His net gain from participation can be defined by the corresponding certainty equivalence gain in wealth:

$$g(\theta^i; \lambda^i, X; \hat{\theta}, \delta) = -\frac{1}{\alpha} \ln \frac{J_P(\cdot)}{J_{NP}(\cdot)}. \tag{18}$$

The minus sign on the right-hand-side adjusts for the fact that $J_P(\cdot)$ and $J_{NP}(\cdot)$ are negative. The optimal decision for trader i is to participate if and only if the net gain from participating is positive. The following proposition describes the optimal participation policy for an individual trader.

Proposition 2. *For trader i with initial stock holding θ^i, idiosyncratic shock $\lambda^i \neq 0$ and X,* *and under market condition $\hat{\theta}$ and δ, his net gain from participation is*

$$g(\theta^i;\lambda^i,X;\hat{\theta},\delta) = g_1(\theta^i;\lambda^i,X;\hat{\theta},\delta) + g_2(\lambda^i;\delta) - R^2c^i \tag{19}$$

where

$$g_1(\cdot) = \frac{\alpha\sigma_D^2(1-k\,\lambda^i\delta)^2}{2(1-k)[1-k+k\,(1-\lambda^i\delta)^2]}\left(\theta^i - \hat{\theta}^i\right)^2 \tag{20a}$$

$$g_2(\cdot) = \frac{1}{2\alpha}\ln\left[1 + (1-\lambda^i\delta)^2 k/(1-k)\right] \tag{20b}$$

and

$$k \equiv \alpha^2\sigma_D^2\sigma_{\hat{z}}^2 \tag{21a}$$

$$\hat{\theta}^i \equiv \frac{1-k}{1-k\,\lambda^i\delta}\,\hat{\theta} - \frac{1-\lambda^i\delta}{1-k\,\lambda^i\delta}\,\lambda^i X. \tag{21b}$$

He chooses to participate if and only if $g(\cdot) > 0$.

The net gain from participation consists of three terms, $g_1(\cdot)$, $g_2(\cdot)$ and $-R^2c^i$. The first term, $g_1(\cdot)$, represents the expected gain from trading given the current signal X on non-traded risks. This term depends on trader i's initial holding θ^i, the per capita stock supply of all participating agents $\hat{\theta}$, and the expected idiosyncratic risk, $\lambda^i X$. It is important to note that as long as the initial holding θ^i is different from $\frac{1-k}{1-\lambda^i k\,\delta}\hat{\theta}$, this term is not symmetric between the two trader groups. As we will see below, the asymmetry in trading gains gives rise to asymmetric participation decisions. The second term, $g_2(\cdot)$, captures the expected gain from trading to offset future shocks to non-traded risks. This term depends on the market condition δ and the variation in future trading needs, which is captured by k. The last term, $-R^2c^i$, simply reflects the cost of participation.

Given the gain from participation, the optimal participation decision becomes very intuitive. Since both $g_1(\cdot)$ and $g_2(\cdot)$ are always positive, the gain $g(\cdot)$ is always positive when the participation cost is smaller than the gain from offsetting futures shocks, i.e., when $c \leq R^{-2}g_2(\cdot)$. Trader i always participates in this case, independent of signal X. The more interesting case is when $c > R^{-2}g_2(\cdot)$ and trader i chooses to participate only if the expected gain $g_1(\cdot)$ from trading against his current expected exposure is sufficiently large. Note that $g_1(\cdot)$ is zero when his current holding θ^i is equal to $\hat{\theta}^i$. Thus, we can interpret $\hat{\theta}^i$ as trader i's desired stock holding after observing his idiosyncratic risk, given by λ^i and X. In this case, a trader chooses to participate when his holding θ^i is sufficiently far away from the desired position $\hat{\theta}^i$.

3.3 Participation Equilibrium

Given traders' optimal participation decisions, we now solve for the participation equilibrium. In the absence of participation costs, traders with positive idiosyncratic shocks ($\lambda^i X > 0$) expect to sell the stock while traders with negative shocks ($\lambda^i X < 0$) expect to buy. Depending on the realization of $\lambda^i X$, each group of traders become either potential sellers or buyers. For example, when $X > 0$, group-a traders become potential sellers ($\lambda^a X > 0$) while group-b traders become potential buyers ($\lambda^b X < 0$). When $X < 0$, the opposite is true. Given the symmetry between the cases for $X > 0$ and $X < 0$, we state our results in this subsection only for the case of $X > 0$, that is, when group-a traders are potential sellers. All the results hold for the case of $X < 0$ by switching the index of a and b.

In order to solve for ω^a and ω^b in equilibrium, we substitute the expression of $\hat{\theta}$ and δ in (14) into the definition of $g(\cdot)$ and define a function of participation gain for group-a and b traders respectively,

$$g^a(\omega^a, \omega^b) \equiv g(\theta^a; \lambda^a, X; \hat{\theta}, \delta), \qquad g^b(\omega^a, \omega^b) \equiv g(\theta^b; \lambda^b, X; \hat{\theta}, \delta). \tag{22}$$

In general, participation gain depends on agents' initial positions. We only consider the situation in which their initial holdings satisfy the following condition:

$$|\theta^i - \theta^m| \leq \min\left\{\frac{\mu\,\sigma_{\hat{z}}}{\mu + \lambda\nu},\ k\,\theta^m\right\}, \qquad i = a, b. \tag{23}$$

We verify later that this condition is satisfied in equilibrium (see Theorem 1).

Since traders' trading needs are the same within the group and offsetting between groups, a trader's gain from participation decreases as more traders from his own group participates but increases as more traders from the other group participates. Hence, we have the following lemma.

Lemma 1. *When traders' initial stock holdings satisfy (23), the gain from participation $g^a(\omega^a, \omega^b)$ for group-a traders decreases with ω^a and increases with ω^b, while the opposite is true for group-b traders' gain $g^b(\omega^a, \omega^b)$.*

The fact that the gain from trading depends on the participation of other traders reflects the externality of traders' participation decisions. This externality is an important driving force in the determination of participation equilibrium.

The following proposition describes the participation equilibrium.

Proposition 3. *When agents initial stock holdings satisfy (23), there exists a unique participation equilibrium. Let*

$$\hat{s}^a = \begin{cases} 0, & \text{if } g^a(0,0) \le 0 \\ 1, & \text{if } g^a(1,0) \ge 0 \\ s^a, & \text{otherwise} \end{cases} \quad \text{and} \quad \hat{s}^b = \begin{cases} 0, & \text{if } g^b(1,0) \le 0 \\ 1, & \text{if } g^b(1,1) \ge 0 \\ s^b, & \text{otherwise} \end{cases}$$

where s^a and s^b are the solutions to $g^a(s^a, 0) = 0$ and $g^b(1, s^b) = 0$, respectively. The equilibrium is fully specified as follows:

A. If $g^a(1, \hat{s}^b) \ge 0$, then $\omega^a = 1$ and $\omega^b = \hat{s}^b$.

B. If $g^a(1, \hat{s}^b) < 0$ and $g^b(\hat{s}^a, 0) \le 0$, then $\omega^a = \hat{s}^a$ and $\omega^b = 0$.

C. Otherwise, $\omega^a, \omega^b \in (0,\, 1)$ and solve both $g^a(\omega^a, \omega^b) = 0$ and $g^b(\omega^a, \omega^b) = 0$.

Moreover, $\omega^a \ge \omega^b$.

Cases A and B describe two polar cases when we have corner solutions, either all potential sellers participate (case A) or no buyers do (case B). Case A corresponds to the situation in which trading gains for sellers are overwhelming so that they will all enter the market, irrespective of what buyers do. The presence of a large number of sellers increases the trading gain for buyers. Thus, in this case some buyers may also choose to participate. Case B corresponds to the situation in which not all sellers will participate but independent of what they do the net trading gains for buyers remains negative. In this case, some sellers choose to participate but no buyers do. Case C corresponds to the intermediate case when we have an interior solution. In this case, participation of each group depends on the degree of participation of the other group.

Proposition 3 confirms that there are always more sellers entering the market than buyers in equilibrium. Additional sellers bring excess sell orders to the market and the need for liquidity, which is provided by market makers.

3.4 Full Equilibrium of the Economy

We now turn to the solution to the full equilibrium of the economy. We start by computing the value function for all agents at time t, including traders who receive no idiosyncratic risks. First, we observe that the indirect utility function, J_P or J_{NP}, defined in (17) is valid for each agent i ($i = a, b, m$) conditional on his own λ^i, X, given his initial stock holding θ^i_t. Next, for a trader with $\lambda^i \neq 0$, his unconditional value function becomes

$$J^{L}(\theta^i_t;\theta_t) = \mathrm{E}\left[\max\{J_{P}(\theta^i_t;\lambda^i,X;\hat{\theta},\delta),\ J_{NP}(\theta^i_t;\lambda^i,X;\hat{\theta},\delta)\}\ \Big|\ \lambda^i \neq 0\right]. \tag{24}$$

and for a trader with $\lambda^i = 0$, who does not observe X, his value function is

$$J^{NL}(\theta^i_t;\theta_t) = \max\left\{\mathrm{E}\left[J_{P}(\theta^i_t;\lambda^i,X;\hat{\theta},\delta)\ \Big|\ \lambda^i = 0\right],\ \mathrm{E}\left[J_{NP}(\theta^i_t;\lambda^i,X;\hat{\theta},\delta)\ \Big|\ \lambda^i = 0\right]\right\} \tag{25}$$

where $\hat{\theta}$ and δ are defined in (14), which depend on the equilibrium participation ratio ω^a and ω^b in Proposition 3 and thus are are functions of X (and θ_t), and $\mathrm{E}[\cdot]$ denotes expectation over X.[9] The ex-ante utility of any trader before any information regarding idiosyncratic shocks can then be defined as a weighted average of J^L and J^{NL}:

$$J^{i}(\theta^i_t;\theta_t) = \lambda\, J^{L}(\theta^i_t;\theta_t) + (1-\lambda)\, J^{NL}(\theta^i_t;\theta_t), \quad i = a, b. \tag{26}$$

Finally, for market makers, the ex-ante utility is simply

$$J^{m}(\theta^m_t;\theta_t) = \mathrm{E}\left[J_{P}(\theta^m_t;\lambda^m,X;\hat{\theta},\delta)\ \Big|\ \lambda^m = 0,\ c^i = 0\right]. \tag{27}$$

To solve for the full equilibrium of the economy, we first take P_{t+1} as given to derive the equilibrium price P_t and stock holding θ_t, then we impose the stationarity condition (12) (i.e., $P_{t+1} = P_t$) to derive the full equilibrium. In addition, we need to confirm that in equilibrium, traders receiving no idiosyncratic shocks optimally choose stay out of the market, i.e.,

$$\mathrm{E}\left[J_{P}(\theta^i_t;\lambda^i,X;\hat{\theta},\delta)\ \Big|\ \lambda^i = 0\right] \leq \mathrm{E}\left[J_{NP}(\theta^i_t;\lambda^i,X;\hat{\theta},\delta)\ \Big|\ \lambda^i = 0\right]. \tag{28}$$

The following proposition describes the condition that defines the equilibrium.

Proposition 4. *A stationary equilibrium of the economy is determined by the set of prices and stock holdings $\{P_t, \theta_t\}$ that solves the agents' optimality condition at t*

$$0 = \frac{\partial}{\partial \theta^i_t}\, J^{i}(\theta^i_t;\theta_t), \quad i = a, b, m, \tag{29}$$

the market clearing condition (10), the stationarity condition (12), and satisfies conditions (23) and (28).

---

[9] For J^{NL}, the expectation is taken over X before the maximization because a trader with $\lambda^i = 0$ does not observe X and makes his participation decisions independent of the realization of X.

Equation (29) is agents' first order condition for optimal portfolio choice at t before they receive any idiosyncratic shocks.

We can solve the equilibrium explicitly when the probability of idiosyncratic shock λ is small as shown in the appendix, which leads to the following theorem:

Theorem 1. *When the probability of idiosyncratic shock λ is small, there exists a stationary equilibrium as described by Proposition 4.*

For arbitrary λ, we have to solve the equilibrium numerically.

4 Limited Participation and the Need for Liquidity

The equilibrium under costly participation shows two striking features. First, despite the fact that the two groups of traders have perfectly matching trading needs, their actual trades are not synchronized when participation in the market is costly. The non-synchronization in their trades gives rise to the need for liquidity in the market. A group of traders may bring their orders to the market while traders with off-setting trading needs are absent, creating an imbalance of orders. The stock price adjusts in response to the order imbalance in order to induce market makers to provide liquidity and to accommodate the orders. As a result, the price of the stock not only depends on the fundamentals (i.e., its expected future payoffs and total risk), but also depends on idiosyncratic shocks that market participants face. Second, despite the symmetry between shocks to potential buyers and sellers, the order imbalance observed in the market tends to be asymmetric and is on average dominated by sell orders. Thus, the endogenous liquidity need typically takes the form of excessive selling, which causes the price to tank. In the next two sections, we examine in more detail these results and the economic intuition behind them.

4.1 Gains from Trading and Individual Participation Decisions

We start with the individual participation decision, taking as given the initial holding, the idiosyncratic shocks, and the equilibrium participation in the market. A trader bases his participation decision on the trade-off between the cost to be in the market and the gain from trading. Our key result regarding the trading gain is stated below.

Result 1. *The gain from trading is in general different between buyers and sellers even when their trading needs are perfectly matched.*

We start with the simple situation when the market participation rate is symmetric, i.e., $\omega^a = \omega^b$ and $\delta = 0$. The gain from participating in the market for trader i is $g_1(\theta^i; \lambda^i, X; \hat{\theta}, 0) + g_2(\lambda^i; 0) - R^2 c^i$. Since $g_2(\cdot)$ and c^i are identical for both trader groups, we only need to focus on $g_1(\cdot) = g_1(\theta^i; \lambda^i, X; \hat{\theta}, 0)$, which is fully determined by the distance between current stock holding θ^i and the desired holding $\hat{\theta}^i$. In particular, in this case we have $\hat{\theta}^i = (1-k)\hat{\theta} - \lambda^i X$ and

$$g_1(\theta^i; \lambda^i, X; \hat{\theta}, 0) = \frac{\alpha\sigma_D^2}{2(1-k)} \left[\theta^i - (1-k)\hat{\theta} + \lambda^i X\right]^2, \quad i = a, b. \tag{30}$$

The trading gain is symmetric between the two groups of traders, who have oppsite $\lambda^i X$, only when $\theta^i = (1-k)\hat{\theta}$. When λ is small, the equilibrium θ^i is close to $\bar{\theta}$, and hence $\theta^i > (1-k)\hat{\theta}$. Thus, the trading gain is different for the two trader groups.

It is important to recognize that Result 1 is a general phenomenon when trading is costly. When the traders can trade without cost, they will constantly maintain the optimal position and the gains from trading is always symmetric for small deviations from the optimal position. Let $u(\theta)$ denote the utility from holding θ, and θ^* be the optimal holding. Then, $u'(\theta^*) = 0$. For a small deviation $x = \theta - \theta^*$ from the optimum, the gain from trading is given by $u(\theta^*) - u(\theta^* + x) \simeq -u''(\theta^*)\, x^2/2$, which is the same for an opposite deviation $-x$. Thus, at the margin, traders with offsetting shocks or trading needs have the same gain from trading. This is no longer the case when trading is costly. Facing a cost, traders no longer trade constantly. They only trade when the deviation from the optimal is sufficiently large. As Figure 2 illustrates, the trading gain is no longer symmetric for finite deviations from the optimum since in general $u(\theta^*) - u(\theta^* + x) \neq u(\theta^*) - u(\theta^* - x)$. Hence, the gains from trading become different between traders with perfectly matching trading needs.

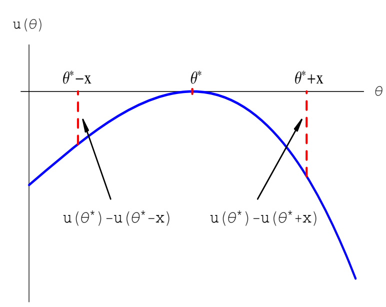

Figure 2: Utility gain from trading is asymmetric when trading is costly.

In our specific setting, we can go beyond simply confirming the above general result. In particular, we can make directional predictions regarding the relative size of trading gains for buyers and sellers.

Result 2. *When the probability of idiosyncratic shock λ is small, in equilibrium sellers always enjoy larger gains from trading than buyers.*

To understand this result, we continue our previous example with $\delta = 0$. From (30), it is apparent that $g_1(\cdot)$ is higher for potential sellers (who have ($\lambda^i X > 0$) than buyers (who have $\lambda^i X < 0$). Figure 3 illustrates the changes in the desired stock holding before and after traders observe whether or not they receive an idiosyncratic shock (λ^i) and its expected magnitude (X). The solid lines represent the desired stock holdings. A trader i starts with an initial holding θ_t^i ($i = a, b$) before receiving any information on his idiosyncratic risk. Then, he learns whether or not he is exposure to the idiosyncratic risk, i.e., his draw of λ^i. If he is not exposed, i.e., $\lambda^i = 0$, he stays out of the market and keeps his initial holding.[10] If instead he is exposed, i.e., $\lambda^i \neq 0$, even without learning the actual sign or the magnitude of the shock X, the trader's preferred stock holding changes to $(1-k)\,\hat{\theta}$, given by the dashed line in Figure 3, which is lower than θ^i, his initial holding. It is worth pointing out that this new preferred holding level is independent of the sign of his idiosyncratic shock, that is, whether $\lambda^i = 1$ or -1. Moreover, even if the expected idiosyncratic shock is zero, $X = 0$, the desired holding changes to $(1-k)\,\hat{\theta}$ and the gain from trading is not zero, as (30) shows.

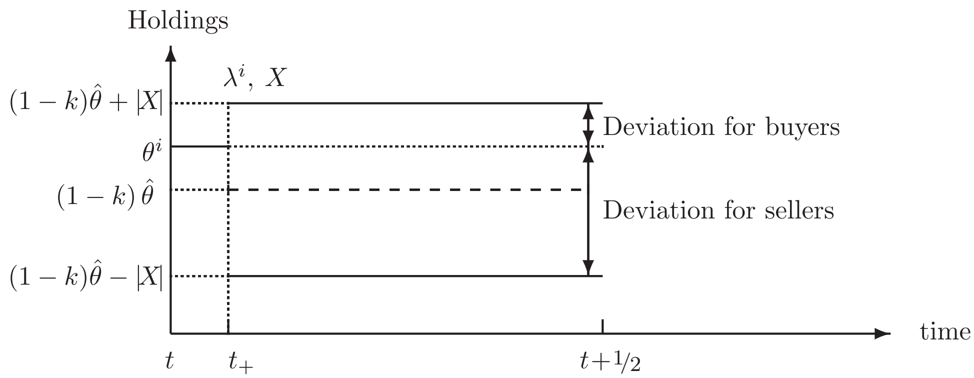

Figure 3: Traders' desired stock holdings before and after observing liquidity signals.

The desired holding for the trader has a straightforward interpretation: He chooses to hold $(1-k)$ times the per-capita stock supply in the market. Since $k > 0$, the trader prefers an overall stock exposure that is lower than the per-capita supply even though he has the

[10] In this case, his preferred stock holding will actually increase. Without observing X, as stated in Theorem 1, the expected gain from participation remains negative and he will not enter the market.

same utility function as the rest of agents in the market. The reason is the following. The cost of participation prevents the trader from trading in the market at all times while the market makers face no cost and can always trade. As a result, the trader has to bear some idiosyncratic risk, at least sometimes. This extra risk effectively reduces the risk tolerance of the trader and lowers his desired stock exposure relative to market makers.[11] The percentage reduction in the trader's desired position, captured by k, is proportional to the level of the remaining uncertainty in his idiosyncratic risk exposure.

Finally, taking into account the sign and magnitude of his non-traded risk $\lambda^i X$ further increases or decreases his desired stock holding to $\hat{\theta}^i = (1-k)\,\hat{\theta} - \lambda^i X$. In particular, a potential buyer, who receives a negative shock to his risk exposure, has $\lambda^i X < 0$ and a desired holding of $(1-k)\,\hat{\theta}+|X|$, while a potential seller, who receives a positive shock, has $\lambda^i X > 0$ and a desired holding of $(1-k)\,\hat{\theta}-|X|$. Figure 3 shows that the desired holdings, given by the top and bottom solid lines, respectively, deviate further from the initial position, given by the dotted line, for a potential seller than for a potential buyer. Obviously, the gain from trading is higher for the seller.

The main intuition behind the above result is as follows. Since traders choose their initial holdings before they learn whether or not they will receive idiosyncratic shocks, they rationally choose a high initial holding if they expect a low probability of ever receiving a shock. However, once they are hit with shocks, their initial holding level becomes too high given the possibility of bearing some un-hedged risk. Irrespective of the sign of his idiosyncratic shock, he prefers to decrease his stock exposure. Obviously, potential sellers who have received additional positive exposure is further away from the desired holding level than the potential buyers. As a result, sellers enjoy larger gains from trading.[12]

Intuitively, one expects that the asymmetry in gains from trading can lead to asymmetric participation between the traders. In particular, since potential sellers always have higher gains from trading than potential buyers in our setting, we further expect that sellers are more likely to participate in the market than buyers. We verify these intuition in the next subsection.

[11] The result that traders become effectively more risk averse with un-hedged idiosyncratic risks is clearly preference dependent. Kimball (1993) shows that it is true for "standard risk aversion, " which is defined as a class of utility function that exhibits both DARA and decreasing absolute prudence.

[12] In a setting similar to ours, Lo, Mamaysky, and Wang (2004) show that even in continuous-time the gain from trading is asymmetric around the optimal holding due to the fact that traders only trade infrequently.

4.2 Non-Synchronized Trading and the Need for Liquidity

Given the individual trader's entry policy, we now examine the participation equilibrium, which is stated in Proposition 3. From the proposition, it is clear that in general, $\omega^a \neq \omega^b$. That is, even with perfectly matching trading needs, the traders fail to synchronize their trades under costly participation. Whenever the participation is asymmetric, there is a mismatch in the buy and sell orders in the market. This order imbalance then creates the need for liquidity. Thus, we have the following result.

Result 3. *In equilibrium, participation can be asymmetric among traders even when their trading needs are perfectly matched, giving rise to non-synchronization in their trades and the endogenous need for liquidity.*

Figures 4 shows the equilibrium participation decisions as functions of the idiosyncratic shock X. Panel (a) reports the fraction ω^i of traders within group i who choose to participate. The dotted line plots ω^a and the dashed line plots ω^b. Panel (b) reports the difference in participation ratio between the two groups of traders δ, defined in equation (14), as a function of X. When $X > 0$, group-a traders are potential sellers and group-b traders are potential buyers. Consistent with our earlier intuition, more sellers are participating than buyers as ω^a is always above ω^b in this region. In particular, when X is not too far from zero, $\omega^a > 0$ and $\omega^b = 0$, that is, no group-b traders choose to participate because the benefit from trading is too small, and only a fraction of group-a traders participates. This corresponds to Case B in Proposition 3. As X increases, the gains from trading increases for both groups and both ω^a and ω^b increases. In particular, for medium levels of X, ω^b becomes positive and ω^a reaches 1. When X is sufficiently large, $\omega^a = \omega^b = 1$. That is, the gain from trading dominates the cost for both groups of traders and they all choose to participate. This corresponds to Case A in Proposition 3. When $X < 0$, group-a traders become potential buyers and group-b traders become potential seller. All the above results flip. In fact, ω^b is simply the mirror image of ω^a around the vertical axis, reflecting the fact that traders a and b face opposite idiosyncratic shocks. Interestingly, neither ω^a nor ω^b are symmetric about 0, consistent with the fact that a trader's gain from trading is asymmetric between positive and negative idiosyncratic shocks.

It is worth pointing out that in this example, we do not observe any interior solutions, i.e., Case C in Proposition 3. This is not a coincidence. Although interior solutions can occur (for other parameter values), in general interior solutions are rare. The reason is that more participation of potential sellers generally enhance the participation gain for potential buyers and vice versa. As a result, more buyers will be lured into the market, which in turn encourages more sellers to participate. Hence, starting with an interior participation level, increasing participation for both groups generally improves utility for both groups of traders. An equilibrium is reached when all sellers and enough buyers are in the market so that the gain from further entry by buyers diminishes to zero. The interior solution in Case C is possible when traders start with large enough initial stock holdings and the competition for market makers' risk sharing capacity overwhelms their gain from offsetting each other's idiosyncratic risks.

(a) Participation Rate ω^a and ω^b (b) Difference in Participation Rate δ

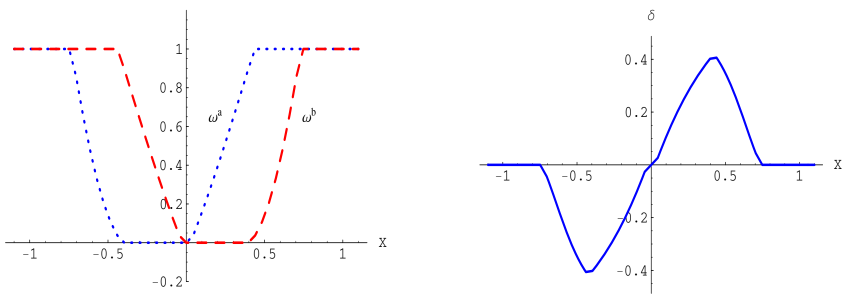

Figure 4: Equilibrium Participation. The figure plots the equilibrium participation rate for the two trader groups for different values of idiosyncratic shock X. Panel A reports the equilibrium fraction of group i traders who choose to participate, where the dotted line refers to trader a (ω^a) and the dashed line refers to trader b (ω^b). Panel B reports the difference in participation decisions, $\delta = \lambda\nu(\omega^a - \omega^b)/[\mu + \lambda\nu(\omega^a + \omega^b)]$. Other parameters are set at the following values: $\bar{\theta} = 1$, $\alpha = 4$, $r = 0.05$, $\bar{D} = 0.36$, $c = 0.09$, $\sigma_D = 0.3$, $\sigma_z = 0.7$, $\sigma_u = 0.7$, $\mu = 1$, $\nu = 5$, and $\lambda = 0.15$.

Penal B of Figure 4 shows that the normalized difference between ω^a and ω^b is always positive when $X > 0$, indicating that more group-a traders are participating. Since they are potential sellers when $X > 0$, the aggregate order imbalance is skewed towards sell orders. Similarly, when $X < 0$, δ is always negative, indicating more group-b traders are participating. Since group-b traders are potential sellers when $X < 0$, the order imbalance is again skewed towards sell orders. In summary,

Result 4. *When the probability of idiosyncratic shock λ is small, potential sellers always participate more in the market than potential buyers in equilibrium. The aggregate order imbalance always takes the form of an excess supply.*

5 Liquidity and Equilibrium Stock Price

Our analysis above suggests that self interest fails to coordinate agents' participating decisions and synchronize their trades even when their trading needs perfectly match. This non-synchronization in trades gives rise to imbalances in asset demand and the need for liquidity. Such exogenous order imbalances are the starting point of Grossman and Miller (1988) and market microstructure models like Ho and Stoll (1981) and Glosten and Milgrom (1985). In our model, by explicitly modelling the motives and the costs to be in the market, we endogenously derive the order imbalance. In particular, we show that, in absence of any aggregate shocks, the order imbalance often takes the form of an excess supply. This directional prediction leads to interesting implications on equilibrium prices, which we now turn our attention to.

By construction, the equilibrium stock price is stationary over time at the beginning of each generation, $P_{t+1} = P_t = P$. And it fluctuates in-between each generation as a function of the idiosyncratic shocks. From (15), the intermediate price consists of two components: the risk-adjusted fundamental value, $R^{-1}\big(\bar{D} + P - \alpha\sigma_{D}^{2}\,\hat{\theta}\big)$, and the liquidity component,

$$\tilde{p} \equiv -(\alpha\sigma_{D}^{2}/R)\,\delta Z. \tag{31}$$

We call the first term "the fundamental value" since it determines the stock price when the expected future idiosyncratic risk (signal X) is zero. It simply equals the expected future payoffs (dividend plus resale price) minus a risk premium. The liquidity component, on the other hand, captures the premium related to market illiquidity. It is non-zero only when agents anticipate future idiosyncratic shocks since the symmetry between trader groups implies that $\delta = 0$ when $X = 0$. Moreover, it is proportional to the per-capita order imbalance, driven by the asymmetric participation between buyers and sellers. Since our purpose here is to understand the endogenous nature of order imbalances and its impact on asset prices, we will focus our discussion on the liquidity component.

To understand the properties of the liquidity component $\tilde{p}$, we first solve the stationary equilibrium described in Theorem 1. Then we derive the equilibrium prices at time $t + 1/2$ by substituting in the stationary equilibrium price and optimal θ_t^i and θ_t^m. The $\tilde{p}$ in (31) depends on the difference in market participation rate δ, which is determined after observing the signal X, and the realized idiosyncratic shock Z, which equals to the signal X plus some noise, $Z - X \sim N(0, \sigma_{\hat{z}}^2)$. The noise term makes the liquidity component more dispersed without qualitatively changing the liquidity effect. In Figure 5, we average out the noise term and report the expected liquidity component conditional on the signal X,

$$\hat{p} = \mathrm{E}[\,\tilde{p}\,|\,X\,] = -(\alpha\sigma_D^2/R)\,\delta X. \tag{32}$$

We plot $\hat{p}$ as a function of X in panel A, and plot its probability distribution in panel B.

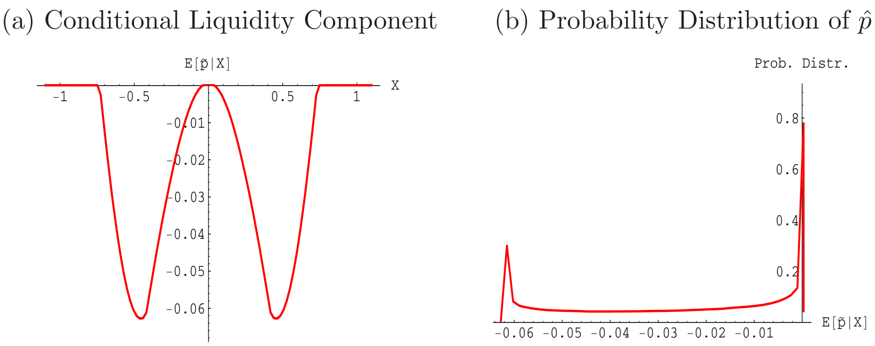

Figure 5: **The conditional liquidity component in price, $\hat{p} = \mathrm{E}[\tilde{p}|X]$.** The conditional liquidity component $\hat{p}$ is defined in (32) and captures the price movement in excess of the "fundamentals." Panel (a) plots $\hat{p}$ as a function of the signal X. Panel (b) plots the probability density function of $\hat{p}$, except at the point 0 where the value corresponds to the total probability mass at the point (since the density function should be infinity at the point). For ease of exposition, except at the point of $\hat{p} = 0$, we scale down the probability density function by a factor of 200 in the plot. Other parameters are set at the following values: $\bar{\theta} = 1$, $\alpha = 4$, $r = 0.05$, $\bar{D} = 0.36$, $c = 0.09$, $\sigma_D = 0.3$, $\sigma_z = 0.7$, $\sigma_u = 0.7$, $\mu = 1$, $\nu = 5$, and $\lambda = 0.15$.

When traders face no participation costs, they all participate in the market and $\delta = 0$. There is no need for liquidity. By (31), the liquidity component $\tilde{p}=0$. The stock price equals the fundamental value and does not depend on the idiosyncratic shocks individual traders face. In the presence of participation costs, partial participation leads to non-synchronized trades among traders, and $\delta \neq 0$. There is a need for liquidity. The stock price has to adjust to attract the market makers to provide the liquidity and to accommodate the trades. In general, $\hat{p} \neq 0$ and the stock price becomes dependent on the idiosyncratic shocks of individual traders.

As Result 4 states, potential sellers are more willing to enter the market to sell the stock. Thus, the order imbalance, as captured by $-\delta X$, is negative, which leads to a negative $\hat{p}$. It is important to note that the sign of $\hat{p}$ is independent of the sign of X. In other words, it does not depend on the distribution of idiosyncratic shocks among the traders. Therefore, we have the following result:

Result 5. *The impact of liquidity needs always decreases asset prices.*

The magnitude of the liquidity effect on price depends on the signal X. Figure 5(a) plots $\hat{p}$ against X. First, we note that the liquidity effect on the price is always negative, as mentioned before. Second, the impact of liquidity on the stock price is highly non-linear in X, the idiosyncratic shocks to the traders. In particular, for small values of X, gains from trading are small for all traders and they do not enter the market. As a result, there is no need for liquidity and price equals the fundamental. For large values of X, gains from trading are sufficiently large for all traders and they all enter the market. As a result, their traders are synchronized and there is no need for liquidity. The stock price also equals the fundamental. For intermediate ranges of X, the gains from trading are large enough for some traders to enter the market, but not for all traders. It is in this case when trades are non-synchronized and liquidity is needed in the market, which will in turn affect the stock price. As Figure 5 shows, the price impact of liquidity reaches the maximum for a certain magnitude of the idiosyncratic shock.

The result that the price impact of liquidity need is one-sided and highly non-linear arises from the fact that liquidity needs are endogenous in our model. In most of the existing models of liquidity, such as Grossman and Miller (1988), liquidity needs are exogenously specified. Consequently, its price impact is linear in the exogenous liquidity needs and symmetrically distributed. Our analysis shows that modelling the liquidity needs endogenously is important to understand the impact of liquidity on prices. After all, it is the same economic factor, namely, the cost to participate in the market, that drives both the liquidity needs of the traders and the liquidity provision of market makers.

The non-linearity in the price impact of liquidity leads to another interesting result: large and frequent price movements in absence of any aggregate shocks. Figure 5(b) plots the probability distribution of $\hat{p}$. As a benchmark, when participation is costless, the idiosyncratic shock X does not affect stock prices and there is no liquidity effect. The distribution is simply a delta-function at zero. When traders face costs to participate in the market, however, the stock price also depends on the idiosyncratic shock X. Moreover, even though the underlying idiosyncratic shocks that drive the individual traders' trading needs are normally distributed, their price impact as measured by $\hat{p}$ is always negative and has a fat-tailed distribution. Aside from a non-zero probability mass at the origin, the distribution peaks at a finite and negative value, reflecting the fact that liquidity becomes important and affects the price for a range of finite shocks. Moreover, the impact of liquidity gives rise to the possibility of a large price movement in absence of any shocks to the fundamentals of the stock. Since such a price movement is associated with a large imbalance in trades and a surge of liquidity needs, it is a market crash driven purely by liquidity needs. We call it "liquidity crashes." Summarizing the results above, we have the following:

Result 6. *The impact of liquidity can lead to "liquidity crashes" in which large price drops occur in the absence of any shocks to the fundamentals.*

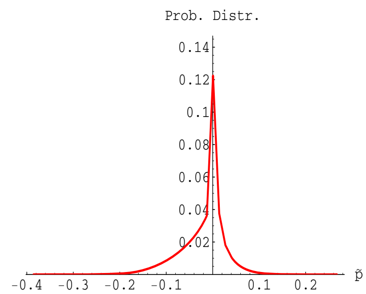

Figure 6: **The unconditional liquidity component in price $\tilde{p}$.** The liquidity component in price $\tilde{p}$ is defined in (31) and captures the price movement in excess of the "fundamentals". We plot the probability density function of p. For ease of exposition, we scale down the probability density function by a factor of 200 in the plot. Other parameters are set at the following values: $\bar{\theta} = 1$, $\alpha = 4$, $r = 0.05$, $\bar{D} = .36$, $c = 0.09$, $\sigma_D = 0.3$, $\sigma_z = 0.7$, $\sigma_u = 0.7$, $\mu = 1$, $\nu = 5$, and $\lambda = 0.15$.

The above discussion focuses on $\hat{p}$, which gives the expected impact of liquidity need on the stock price conditional on X, the signal on future idiosyncratic shock. The actual price at $t + 1/2$, as given in (31), will depend on Z, the actual realization of idiosyncratic shock. Although the behavior of $\tilde{p}$ is qualitatively similar to that of $\hat{p}$, its distribution is slightly different from that of $\hat{p}$ due to the additional noise in the realized idiosyncratic shock. Figure 6 plots the unconditional distribution of $\tilde{p}$.

Table 1: **Unconditional moments of the liquidity component $\tilde{p}$ in stock price.**

St. dev.	Skewness	Kurtosis
0.047	-1.033	5.078

From the figure, we observe that the distribution exhibits negative skewness and fat tails. To confirm this observation, we report in Table 1 the moments of the liquidity component $\tilde{p}$ (for the particular set of parameters used in the figures). Note that in the absence of liquidity effect, the return distribution will simply be a delta function at zero. If we were to include any news on the fundamentals (i.e., future dividends), which are assumed to be normally distributed, the return would simply be normal. Hence, we have the following result.

Result 7. *The impact of liquidity can lead to negative skewness and fat tails in asset prices.*

6 Conclusion

In this paper, we show that frictions such as participation costs can induce non-synchronization in agents' trades even when their trading needs are perfectly matched. Each trader, when arriving at the market, faces only a partial demand/supply of the asset. The mismatch in the timing and the size of trades creates temporary order imbalances and the need for liquidity, causing asset prices to deviate from the fundamentals. Purely idiosyncratic idiosyncratic shocks can affect prices, introducing additional price volatility. Moreover, the price deviations tend to be highly skewed and of large sizes. In particular, the shortage of liquidity always causes the price to decrease and when it happens, the price tends to drop significantly, resembling a crash due to a sudden surge in liquidity needs.

A few additional comments are in order. First, the significance of the need for liquidity and its impact in our model clearly depends on the magnitude of the partipation cost and idiosyncratic shocks. Although most of potential buyers and sellers do not constantly participate in the market, it is difficult to directly measure the cost for doing so. However, there is cumulating empirical evidence suggesting that such costs are non-trival, even for fairly liquid stocks.[13] The amount of volume and order in balance observed in the market suggests that the magnitude of idiosyncratic shocks can be significant. Second, our analysis takes as given the population weight of market makers, which determines the amount of liuqidity they can provide and thus the equilibrium impact of liquidity needs. As shown in Huang and Wang (2005), the population weight of market makers can be endogenized. In particular, they assume that all agents can either pay a low cost ex ante to become a market maker or a high cost ex post when trading needs arise. They show that typically only a small fraction of agents will choose to become market makes. In light of their analysis, we can interpret the relative population weight of market makers and traders as an equilibrium outcome. Third, in our model the idiosyncratic shocks are transitory. Thus, when a liquidity crash occurs, the stock price tanks but eventually recovers. The possibility of such a price

[13]See, for example, Scholes (1972), Shleifer (1986) and Coval and Stafford (2005).

pattern might seem puzzling since it seems to leave profitable opportunities. However, this is not so given the costs. With a small probability for such an event to happen, it only attracts a small number of market makers with even a small cost for being a market maker. For others, the significant cost to jump in on the spot prevents them from taking advantage of the opportunities. Fourth, in our setting the cost to jump into the market on the spot does impose an upper bound on the potential impact of liquidity on prices. But, this is true only in the absence of aggregate shocks as we assumed in the model. In the presence of aggregate shocks, the potential impact of engogenous liquidity needs on prices becomes unbounded.

A Appendix

Proof of Proposition 1

Given P_{t+1}, participating agent i maximizes his expected utility over his terminal wealth W^i_{t+1} given in (9), which is obtained by integrating over the distribution of D_{t+1} given $\theta^i_{t+1/2}$:

$$\max_{\theta^i_{t+1/2}} -e^{-\alpha\left[R^2(W_t-c^i)+R\,\theta^i_t\left(P_{t+1/2}-RP_t\right)+\theta^i_{t+1/2}\left(\bar{D}+P_{t+1}-RP_{t+1/2}\right)-\frac{1}{2}\alpha\sigma^2_D(\theta^i_{t+1/2}+\lambda^i Z)^2\right]}. \tag{A1}$$

His optimal holding is calculated by solving the first order condition with respect to $\theta^i_{t+1/2}$,

$$\theta^i_{t+1/2} = \frac{1}{\alpha\sigma^2_D}(\bar{D} + P_{t+1} - RP_{t+1/2}) - \lambda^i Z, \qquad i = a, b, m.$$

Solving the market clearing condition (11) yields the equilibrium price $P_{t+1/2}$. Using δ and $\hat{\theta}$ defined in (14) yields the expression of $P_{t+1/2}$ in the proposition. The optimal holding in the proposition is obtained by substituting the equilibrium price $P_{t+1/2}$ back into (A1).

Proof of Proposition 2

First, we substitute the equilibrium price $P_{t+1/2}$ and holding $\theta^i_{t+1/2}$ back into (A1), and integrate over the distribution of Z conditional on X to derive the (indirect) utility function J_P in (17a) for the participating traders,

$$J_P(\cdot) = -\frac{1}{\sqrt{1-k+k\,(1-\lambda^i\delta)^2}}\, e^{-\alpha\left[R^2(W_t-c^i)+\theta^i_t\left(\bar{D}+P_{t+1}-R^2P_t\right)-\frac{\alpha\sigma^2_D}{2(1-k)}(\theta^i_t+\lambda^i X)^2+g_1(\cdot)\right]} \tag{A2}$$

where $g_1(\cdot)$ and k are defined in (20) and (21). Next, we calculate the value function for non-participating traders J_{NP} in (17b) by integrating over D_{t+1} and Z conditional on X,

$$J_{NP}(\cdot) = -\frac{1}{\sqrt{1-k}}\, e^{-\alpha\left[R^2W_t+\theta^i_t\left(\bar{D}+P_{t+1}-R^2P_t\right)-\frac{\alpha\sigma^2_D}{2(1-k)}(\theta^i_t+\lambda^i X)^2\right]}. \tag{A3}$$

Finally, we substitute J_P and J_{NP} into (18) to derive the gains from participation. Obviously, trader i chooses to participate in the market if and only if $g(\cdot) > 0$.

Proof of Lemma 1

Given the definition of $g(\cdot)$ in (19), we compute its partial derivative with respect to ω^a and ω^b. Note that both $g_1(\cdot)$ and $g_2(\cdot)$ depend on ω^a and ω^b through δ and $\hat{\theta}$ only. Define

$\delta^i \equiv \lambda^i \delta$ and $d^i \equiv 1-k+k(1-\delta^i)^2$. Following (22), let $g^i \equiv g(\theta^i_t; \lambda^i, X; \hat{\theta}, \delta)$. Then,

$$\frac{\partial g^i}{\partial \omega^j} = \left(\frac{\partial g_1}{\partial \delta^i} + \frac{\partial g_2}{\partial \delta^i}\right) \frac{\partial \delta^i}{\partial \omega^j} + \frac{\partial g_1}{\partial \hat{\theta}} \frac{\partial \hat{\theta}}{\partial \omega^j}, \quad j = a, b$$

where

$$\begin{aligned}
\frac{\partial g_1}{\partial \delta^i} &= -\frac{\alpha \sigma_D^2 (1-k\,\delta^i)}{(d^i)^2} \left[k\,\delta^i \theta^i_t + k\,(1-\delta^i)\hat{\theta} + \lambda^i X\right] \left(\theta^i_t - \frac{1-k}{1-k\,\delta^i}\,\hat{\theta} + \frac{1-\delta^i}{1-k\,\delta^i}\,\lambda^i X\right) \\
\frac{\partial g_2}{\partial \delta^i} &= -\frac{(1-\delta^i)\,k}{\alpha d^i} \\
\frac{\partial \delta^i}{\partial \omega^j} &= \lambda^i\,(\lambda^j - \delta)\hat{\lambda}, \qquad \hat{\lambda} \equiv \frac{\lambda\nu}{\mu + \lambda\nu(\omega^a + \omega^b)} \\
\frac{\partial g_1}{\partial \hat{\theta}} &= -\frac{\alpha \sigma_D^2 (1-k\,\delta^i)}{d^i} \left(\theta^i_t - \frac{1-k}{1-k\,\delta^i}\,\hat{\theta} + \frac{1-\delta^i}{1-k\,\delta^i}\,\lambda^i X\right) \\
\frac{\partial \hat{\theta}}{\partial \omega^j} &= \hat{\lambda}\,(\theta^i_t - \hat{\theta}).
\end{aligned}$$

We can further simplify the expression for $\partial g^i / \partial \omega^j$ by considering the cases of $j = i$ and $j \neq i$ separately. In particular, if $j = i$, then $\lambda^i \lambda^j = 1$ and

$$\frac{\partial g^i}{\partial \omega^i} = -\frac{\alpha \sigma_D^2 \hat{\lambda}\,(1-\delta^i)^2}{(d^i)^2} \left[\lambda^i X + k\,\hat{\theta} + \frac{1-k\,\delta^i}{1-\delta^i}(\theta^i_t - \hat{\theta})\right]^2 - \frac{k\hat{\lambda}\,(1-\delta^i)^2}{\alpha d^i}.$$

From (14), δ increases in ω^a and decreases in ω^b. Hence, $\delta \in [-\bar{\delta}, \bar{\delta}]$, where $\bar{\delta} = \frac{\lambda\nu}{\mu+\lambda\nu} < 1$, and so is δ^i. As a result, $\partial g^i / \partial \omega^i < 0$. If $j \neq i$, then $\lambda^i \lambda^j = -1$ and

$$\begin{aligned}
\frac{\partial g^i}{\partial \omega^j} &= \frac{\alpha \sigma_D^2 \hat{\lambda}\,(1-\delta^2)}{(d^i)^2} \left[\lambda^i X + k\,\hat{\theta} + \frac{(1+k)\,\delta^i - 2k\,\delta^2}{1-\delta^2}\,(\theta^i_t - \hat{\theta})\right]^2 \\
&\quad + \frac{\alpha \sigma_D^2 \hat{\lambda}}{1-\delta^2} \left[\frac{k\,(1-\delta^2)^2}{\alpha^2 \sigma_D^2 d^i} - (\theta^i_t - \hat{\theta})^2\right].
\end{aligned}$$

Since $\delta^2 \in [0, \bar{\delta}^2]$, $(1-\delta^i)^2 \in [0, 1+\bar{\delta}^2]$, and $k = \alpha^2 \sigma_D^2 \sigma_{\hat{z}}^2 \in [0, 1]$, we have

$$\frac{k\,(1-\delta^2)^2}{\alpha^2 \sigma_D^2 d^i} \geq \frac{\sigma_{\hat{z}}^2\,(1-\bar{\delta}^2)^2}{1-k+k\,(1+\bar{\delta})^2} > \sigma_{\hat{z}}^2\,(1-\bar{\delta})^2.$$

On the other hand, $\hat{\theta}$ in (14) is a weighted average of θ^i_t and θ^m_t with weights in-between 0 and 1. We have

$$(\theta^i_t - \hat{\theta})^2 \leq (\theta^i_t - \theta^m_t)^2 < \left(\frac{\mu\,\sigma_{\hat{z}}}{\mu + \lambda\nu}\right)^2 = \sigma_{\hat{z}}^2 (1-\bar{\delta})^2$$

where the second inequality is due to condition (23). Thus, $\partial g^i / \partial \omega^j > 0$ for $j \neq i$, proving the lemma.

Proof of Proposition 3

The following lemma is useful:

Lemma 2. *When traders' initial stock holdings satisfy (23), under symmetric participation, sellers always enjoy larger gains from trading than buyers, i.e.,* $g^a(\omega, \omega) \geq g^b(\omega, \omega)\ \forall\ \omega \in [0, 1]$.

The proof is as follows. When $\omega^a = \omega^b$, $\delta = 0$ and $g(\cdot)$ in (19) reduces to

$$g(\theta_t^i; \lambda^i, X; \hat{\theta}, 0) = \frac{\alpha \sigma_D^2}{2(1-k)} \left[\theta_t^i - (1-k)\,\hat{\theta} + \lambda^i X\right]^2 - \frac{1}{2\alpha} \ln(1-k) - R^2 c^i.$$

Hence,

$$g^a(\omega^a, \omega^b) - g^b(\omega^a, \omega^b) = \frac{\alpha \sigma_D^2}{1-k} \left[\theta_t^i - (1-k)\,\hat{\theta}\right] (\lambda^a - \lambda^b) X.$$

If group-a traders are sellers and group-b are buyers, then $\lambda^a X \geq 0 \geq \lambda^b X$. Since $\hat{\theta}$ is weighted average of θ_t^i and θ_t^m, $\theta_t^i - (1-k)\,\hat{\theta} \geq \theta_t^i - (1-k)\,\theta_t^m > 0$, where the last inequality comes from (23). Hence, $g^a(\omega^a, \omega^b) \geq g^b(\omega^a, \omega^b)$ when $\omega^a = \omega^b$, which is the lemma.

Now we prove Proposition 3. First, from Lemma 1, we know that $g^a(\omega^a, 0)$ is a monotonically decreasing function of ω^a. If $g^a(0, 0) > 0 > g^a(1, 0)$, then there exists an $s^a \in (0, 1)$ that solves $g^a(s^a, 0) = 0$. Similarly, $g^b(1, \omega^b)$ is monotonically decreasing in ω^b and $g^b(1, 0) > 0 > g^b(1, 1)$ guarantees that the solution $s^b \in (0, 1)$. Hence, $\hat{s}^a,\ \hat{s}^b \in [0, 1]$.

We now consider the three possible cases, which are exhaustive. In case A, there are three subcases depending on the value of $\hat{s}^b$: First, if $\hat{s}^b = 0$, then $g^b(1, 0) \leq 0 \leq g^a(1, 0)$. The market condition $\omega^a = 1$ and $\omega^b = 0$ is the most favorable for buyers and the least favorable for sellers. Yet the gain from participation is still positive for potential sellers and negative for potential buyers. Hence, $\omega^a = 1$ and $\omega^b = 0$ is the solution. Second, if $\hat{s}^b = 1$, then $g^b(1, 1) \geq 0$ and Lemma 2 implies $g^a(1, 1) \geq 0$ as well. Hence, all traders participate and $\omega^a = \omega^b = 1$. And third, if $\hat{s}^b = s^b \in (0, 1)$, then $g^b(1, \hat{s}^b) = 0$. The condition $g^a(1, \hat{s}^b) \geq 0$ confirms that sellers enjoy positive gains in this case. Hence, at equilibrium participation $\omega^a = 1$ and $\omega^b = \hat{s}^b$, trader a enjoys positive gain and trader b is indifferent between participating or not.

In case B, there are only two subcases depending on the value of $\hat{s}^a$. Note that $\hat{s}^a = 1$ is not feasible under the condition $g^a(1, \hat{s}^b) < 0$, since $g^a(1, 0) \leq g^a(1, \hat{s}^b) < 0$ according to Lemma 1, while $\hat{s}^a = 1$ requires $g^a(1, 0) \geq 0$. The first subcase is when $\hat{s}^a = 0$. Then

$g^a(0,0) \leq 0$. Since $g^b(0,0) < g^a(0,0)$ by Lemma 2, $\omega^a = \omega^b = 0$ is the only solution. The second subcase is when $\hat{s}^a = s^a \in (0,1)$ and solves $g^a(\hat{s}^a, 0) = 0$. At $\hat{s}^a$, trader a is indifferent between participating or not. The condition $g^b(\hat{s}^a, 0) \leq 0$ confirms that trader b does not want to participate when $\omega^a = \hat{s}^a$. Hence, $\omega^a = \hat{s}^a$ and $\omega^b = 0$ in equilibrium.

In case C, the condition is that $g^a(1, \hat{s}^b) < 0 < g^b(\hat{s}^a, 0)$. Similar to case B, the condition $g^a(1, \hat{s}^b) < 0$ still rules out the possibility that $\hat{s}^a = 1$. In addition, $g^b(\hat{s}^a, 0) > 0$ rules out the possibility that $\hat{s}^b = 0$, since $0 < g^b(\hat{s}^a, 0) < g^b(1, 0)$ by Lemma 1. Similarly, we can rule out $\hat{s}^a = 0$ and $\hat{s}^b = 1$. Hence, the condition in case C reduces to $g^a(1, s^b) < 0 < g^b(s^a, 0)$. Note that $g^a(s^a, 0) = 0$ and $g^b(s^a, 0) > 0$ implies that in equilibrium, $\omega^b > 0$. To prove this, assume by contradiction that $\omega^b = 0$. Then at the optimal $\omega^a = s^a$, trader a is indifferent while trader b can gain from participating. Thus $\omega^b = 0$ cannot be the equilibrium. Similarly, $g^a(1, s^b) < 0$ and $g^b(1, s^b) = 0$ implies $\omega^a < 1$ in equilibrium. Lemma 2 guarantees that both ω^a and ω^b are interior solutions. Both traders need to be indifferent between participating or not, i.e., $g^a(\omega^a, \omega^b) = 0$ and $g^b(\omega^a, \omega^b) = 0$. The monotonicity of g^a and g^b functions ensures the existence of a solution in this case. Finally, to prove that $\omega^a \geq \omega^b$, we assume by contradiction that $\omega^a < \omega^b$. Then $0 = g^a(\omega^a, \omega^b) > g^a(\omega^a, \omega^a) > g^b(\omega^a, \omega^a) > g^b(\omega^a, \omega^b) = 0$, yielding a contradiction. Note that the first inequality is because $g^a(\omega^a, \omega^b)$ increases in ω^b, and the last is because $g^b(\omega^a, \omega^b)$ decreases in ω^b. The middle inequality is because of Lemma 2.

Proof of Proposition 4

We first calculate the value function for traders with $\lambda^i = 0$. Conditional on the signal X, the utility if they choose to participate is:

$$J_P(\lambda^i = 0) = -\frac{1}{\sqrt{1+k\,\delta^2}} e^{-\alpha\left[R^2(W_t - c^i) + \theta_t^i\left(\bar{D} + P_{t+1} - R^2 P_t\right) - \frac{1}{2}\alpha\sigma_D^2 {\theta_t^i}^2 + \frac{1}{2}\frac{\alpha\sigma_D^2}{1+k\,\delta^2}(\theta_t^i - \hat{\theta} - \delta X)^2\right]}. \tag{A5}$$

If they choose not to participate, the utility is

$$J_{NP}(\lambda^i = 0) = -e^{-\alpha\left[R^2 W_t + \theta_t^i\left(\bar{D} + P_{t+1} - R^2 P_t\right) - \frac{1}{2}\alpha\sigma_D^2 {\theta_t^i}^2\right]}. \tag{A6}$$

By substituting in the participation and market equilibrium from Propositions 1 and 3, and integrating over X, we can derive the unconditional value function J^L and J^{NL} in (24) and (25). Hence, the ex-ante value function $J^i(\cdot)$ in (26) is well defined for all traders. Moreover, the utility conditional on X for the market maker is the same as $J_P(\lambda^i = 0)$

except for his initial holding θ_t^m and cost $c^m = 0$:

$$J_P^m(X) = -\frac{1}{\sqrt{1+k\,\delta^2}} e^{-\alpha\left[R^2 W_t + \theta_t^m\left(\bar{D}+P_{t+1}-R^2 P_t\right) - \frac{1}{2}\alpha\sigma_D^2 \theta_t^{m\,2} + \frac{\alpha\sigma_D^2}{2(1+k\delta^2)}(\theta_t^m - \hat{\theta} - \delta X)^2\right]} . \tag{A7}$$

Integrating over X then yields the ex-ante utility $J^m(\cdot) = \mathrm{E}[J_P^m(X)]$.

With one market clearing condition 10), stationarity condition (12), and three first order conditions in (29), we have five equations in total and five unknowns, i.e., $\{\theta_t^a, \theta_t^b, \theta_t^m, P_t, P_{t+1}\}$. A solution to the system gives a full equilibrium of the economy.

Proof of Theorem 1

When traders face no idiosyncratic shocks, i.e., $\lambda = 0$, it is easy to show that traders never participate whenever $c^i > 0$, that is, $\omega^i = 0 \;\forall\; i = a, b$. The equilibrium prices are determined by market makers as representative agents, and are identical to those derived in (13). Equilibrium holdings of the stock for all agents are always equal to the per capita supply $\bar{\theta}$.

Proposition 4 describes conditions for an equilibrium. The ex ante symmetry between the two groups of traders implies that $J^a = J^b$ and $\theta_t^a = \theta_t^b$. For simplicity, we use index i to denote traders a or b. Substituting in the stationarity condition ($P_{t+1} = P_t$) directly, we are left with three variables to solve for in equilibrium $\{P_t, \theta_t^i, \theta_t^m\}$ from three equilibrium conditions: two first order conditions (29) for agents i and m, respectively, and one market clearing condition (10).

For small λ, we expand the solution to equilibrium in λ to the first order:

$$P_t = \bar{P} + P_\lambda \lambda + o(\lambda) \tag{A8a}$$

$$\theta_t^i = \bar{\theta} + \theta_\lambda^i \lambda + o(\lambda) \tag{A8b}$$

$$\theta_t^m = \bar{\theta} + \theta_\lambda^m \lambda + o(\lambda) \tag{A8c}$$

where $\bar{P}$ is defined in (13) and $o(\lambda)$ denotes terms of higher order of λ. We then solve the equilibrium up to the first order of λ.

Given (A8), we first solve δ and $\hat{\theta}$ to the first order of λ. Note that both $\mu\,\sigma_{\hat{z}}/(\mu + \lambda\nu)$ and $k\,\theta^m$ in condition (23) are of order $O(1)$ while $\theta_t^i - \theta_t^m = \lambda(\theta_\lambda^i - \theta_\lambda^m)$ are of order $O(\lambda)$, where $O(\cdot)$ denotes terms of the same order. Condition (23) is satisfied when λ is small (i.e., to the first order of λ) and Proposition 3 holds. In particular, the trading gain in (19) can be simplified to

$$g^i(\cdot) = -\frac{1}{2\alpha}\ln(1-k) + \frac{\alpha\sigma_D^2}{2(1-k)}(k\bar{\theta} + \lambda^i X)^2 + O(\lambda)$$

Thus, trader i participates iff $g^i(\cdot) > 0$, which occurs iff $X > X_+^i$ or $X < X_-^i$, where

$$X_\pm^i = -\lambda^i\, k\,\bar{\theta} \pm h + O(\lambda)$$

and

$$h \equiv \begin{cases} \frac{1}{\alpha\sigma_D}\sqrt{(1-k)[2\alpha c^i R^2 + \ln(1-k)]} & \text{if } 2\alpha c^i R^2 + \ln(1-k) \geq 0 \\ 0 & \text{if } 2\alpha c^i R^2 + \ln(1-k) \geq 0 \end{cases}$$

Since δ and $\hat{\theta}$ depends on ω^i only through term $\lambda\omega^i$, we can ignore all $O(\lambda)$ terms for the calculation of ω^i. The equilibrium participation in Proposition 3 can be simplified to

$$\begin{cases} \omega^a = \omega^b = 1, & \delta = 0, & \text{if } X \leq -k\,\bar{\theta} - h \\ \omega^a = 0,\ \omega^b = 1, & \delta = -\bar{\delta}, & \text{if } -k\,\bar{\theta} - h < Xleq - |k\,\bar{\theta} - h| \\ \omega^a = \omega^b = 1, & \delta = 0, & \text{if } -|k\,\bar{\theta} - h| < X \leq |k\,\bar{\theta} - h| \text{ and } k\,\bar{\theta} > h \\ \omega^a = \omega^b = 0, & \delta = 0, & \text{if } -|k\,\bar{\theta} - h| < X \leq |k\,\bar{\theta} - h| \text{ and } k\,\bar{\theta} < h \\ \omega^a = 1,\ \omega^b = 0, & \delta = \bar{\delta}, & \text{if } |k\,\bar{\theta} - h| < X < k\,\bar{\theta} + h \\ \omega^a = \omega^b = 1, & \delta = 0, & \text{if } X \geq k\,\bar{\theta} + h. \end{cases} \tag{A9}$$

Since $\bar{\delta}$ is of order $O(\lambda)$, so is δ. The following equation linearizes δ and $\hat{\theta}$:

$$\delta(\theta_t^i, \theta_t^m, X) = \delta_\lambda(X)\lambda \tag{A10a}$$

$$\hat{\theta}(\theta_t^i, \theta_t^m, X) = \bar{\theta} + \frac{\mu\theta_\lambda^m + \lambda\,\nu(\omega^a + \omega^b)\theta_\lambda^i}{\mu + \lambda\,\nu(\omega^a + \omega^b)}\ la + o(\lambda) = \bar{\theta} + \theta_\lambda^m\lambda. \tag{A10b}$$

Using (A8) and (A10) and the definition of $\bar{P}$ in (13), the first order condition for market makers can be written as

$$\begin{aligned} 0 = \frac{\partial J^m}{\partial\theta_t^m} &= \mathrm{E}\left[-\alpha J_P^m(X)\left(\bar{D} + P_{t+1} - R^2 P_t - \alpha\sigma_D^2\theta_t^m - \frac{\alpha\sigma_D^2}{1 + k\delta^2}(\hat{\theta} - \theta_t^m + \delta X)\right)\right] \\ &= \mathrm{E}\left[-\alpha J_0\, e^{\alpha r\bar{\theta}P_\lambda\,\lambda + o(\lambda)}\left[-(rP_\lambda + \alpha\sigma_D^2\theta_\lambda^m + \alpha\sigma_D^2\delta_\lambda X)\lambda + o(\lambda)\right]\right] \\ &= \mathrm{E}\left[\alpha J_0\left(rP_\lambda + \alpha\sigma_D^2\theta_\lambda^m + \alpha\sigma_D^2\delta_\lambda X\right)\right]\lambda \end{aligned}$$

where $J_P^m(X)$ is defined in (A7), and $J_0 \equiv -e^{-\alpha\left(R^2 W_t + \frac{1}{2}\alpha\sigma_D^2\bar{\theta}^2\right)}$. Or equivalently,

$$rP_\lambda + \alpha\sigma_D^2\,\theta_\lambda^m + c_1 = 0 \tag{A11}$$

where

$$c_1 \equiv \frac{\nu}{\mu}\,\alpha\sigma_D^2\sigma_x\sqrt{\frac{2}{\pi}}\left(e^{-h_1^2}-e^{-h_2^2}\right), \qquad h_1 \equiv \frac{k\,\bar{\theta}-h}{\sqrt{2}\,\sigma_x}, \quad h_2 \equiv \frac{k\,\bar{\theta}+h}{\sqrt{2}\,\sigma_x}$$

and $\sigma_x^2 = \frac{\sigma_z^4}{\sigma_z^2+\sigma_u^2}$. Since $h_1 \le h_2$, we know that $c_1 \ge 0$.

We now consider the first order condition for trader i. First, we verify that traders with $\lambda^i = 0$ never participates. Combining (A5) and (A6), the gain from participation for these traders becomes:

$$g_{NL}(\theta_t^i;\theta) = -\frac{1}{\alpha}\ln\,\frac{\mathrm{E}[J_P(\lambda^i=0)]}{J_{NP}(\lambda^i=0)} = -\frac{1}{\alpha}\ln\mathrm{E}\left[\frac{e^{-\frac{1}{2}\frac{\alpha^2\sigma_D^2}{1+k\delta^2}(\theta_t^i-\hat{\theta}-\delta X)^2}}{\sqrt{1+k\delta^2}}\right] - R^2\,c^i \qquad \text{(A12)}$$

where $\mathrm{E}[\cdot]$ is taken with respect to X. Given (A8) and (A10) and the fact that $\theta_t^i-\hat{\theta}$ and δ are both of the order $O(\lambda)$, we have

$$g_{NL}(\theta_t^i;\theta) = O(\lambda) - R^2 c^i$$

which is negative as long as c^i is finite and λ is small enough. Thus, $J^{NL} = \mathrm{E}[J_{NP}(\lambda^i=0)]$. A trader's first order condition can be written as

$$\begin{aligned} 0 &= \lambda\,\frac{\partial J^L}{\partial\theta_t^i} + (1-\lambda)\,\frac{\partial J^{NL}}{\partial\theta_t^i} \\ &= \lambda\,\frac{\partial \mathrm{E}[J_{NP}]}{\partial\theta_t^i} + \lambda\,\frac{\partial \mathrm{E}[\mathbf{1}_{\{g(\cdot)>0\}}\,(J_P-J_{NP})]}{\partial\theta_t^i} + (1-\lambda)\,\frac{\partial J^{NL}}{\partial\theta_t^i} \end{aligned} \qquad \text{(A13)}$$

where $J^{NL} = \mathrm{E}[J_{NP}(\lambda^i=0)]$ is given in (A6), J_P and J_{NP} are defined in (A2) and (A3), and $g(\cdot)$ is the trading gain in (19).

$$\begin{aligned} \lambda\,\frac{\partial \mathrm{E}[J_{NP}]}{\partial\theta_t^i} &= \lambda\,\frac{\alpha\left(\bar{D}+P_{t+1}-R^2P_t-\frac{\alpha\sigma_D^2\theta_t^i}{1-\bar{k}}\right)}{\sqrt{1-\bar{k}}}\,e^{-\alpha\left[R^2W_t+\theta_t^i\left(\bar{D}+P_{t+1}-R^2P_t\right)-\frac{1}{2}\frac{\alpha\sigma_D^2}{1-\bar{k}}\theta_t^{i\,2}\right]} \\ &= \lambda\,J_0\,\alpha\,c_2 + o(\lambda^2), \quad c_2 \equiv \frac{\bar{k}\,\alpha\sigma_D^2\,\bar{\theta}}{(1-\bar{k})^{3/2}}\,e^{\frac{1}{2}\frac{\bar{k}}{1-\bar{k}}\alpha^2\sigma_D^2\bar{\theta}^2} \end{aligned}$$

where $\bar{k} = \alpha^2\sigma_D^2\sigma_z^2$ captures the total uncertainty in idiosyncratic shocks. Since $J_P = J_{NP}$

when $g(\cdot)=0$, for the second term in (A13), we have

$$
\begin{aligned}
&\lambda\,\frac{\partial \mathrm{E}[\mathbf{1}_{\{g(\cdot)>0\}}\,(J_P-J_{NP})]}{\partial\theta_t^i} = \lambda\,\mathrm{E}\left[\mathbf{1}_{\{g(\cdot)>0\}}\frac{\partial(J_P-J_{NP})}{\partial\theta_t^i}\right]\\
=\;& \mathrm{E}\left[-\mathbf{1}_{\{g(\cdot)>0\}}\,J_0\,\frac{\alpha^2\sigma_D^2\,(k\,\bar\theta+X)}{(1-k)^{3/2}}\;e^{\frac{\alpha^2\sigma_D^2}{2(1-k)}(k\,\bar\theta^2+2\bar\theta X+X^2)}\lambda+o(\lambda)\right]\\
=\;& -J_0\,\alpha\,c_2c_3\,\lambda+o(\lambda)
\end{aligned}
$$

where

$$
\begin{aligned}
c_3 &\equiv \sqrt{\frac{1-\bar k}{2\pi(1-k)}}\frac{\sigma_x}{\bar k\,\bar\theta}\left(e^{-h_3^2}-e^{-h_4^2}\right)-\frac{1}{2}\left(\mathrm{Erf}(h_3)+\mathrm{Erf}(h_4)-2\right)\\
h_3 &\equiv \frac{h(1-\bar k)-\bar k(1-k)\bar\theta}{\sqrt{2(1-k)(1-\bar k)}\,\sigma_x},\quad h_4\equiv\frac{h(1-\bar k)+\bar k(1-k)\bar\theta}{\sqrt{2(1-k)(1-\bar k)}\,\sigma_x}.
\end{aligned}
$$

Note that $\frac{\partial c_3}{\partial h}=-\frac{h}{\sqrt{2\pi}\,\bar k\,\sigma_x\bar\theta}\left(\frac{1-\bar k}{1-k}\right)^{3/2}(e^{-h_3^2}-e^{-h_4^2})\le 0$ (since $h_3\le h_4$.) Since $h\ge 0$, and $c_3=1$ when $h=0$, we have $c_3\le 1$. For the third term in (A13), we have

$$
\begin{aligned}
(1-\lambda)\,\frac{\partial J^{NL}}{\partial\theta_t^i} &= (1-\lambda)\,\alpha\left(\bar D+P_{t+1}-R^2P_t-\alpha\sigma_D^2\theta_t^i\right)\;e^{-\alpha\left[R^2W_t+\theta_t^i\left(\bar D+P_{t+1}-R^2P_t\right)-\frac{1}{2}\alpha\sigma_D^2{\theta_t^i}^2\right]}\\
&= J_0\,\alpha(rP_\lambda+\alpha\sigma_D^2\,\theta_\lambda^i)\,\lambda.
\end{aligned}
$$

Hence, the first order condition for trader i reduces to

$$
rP_\lambda+\alpha\sigma_D^2\,\theta_\lambda^i+c_2(1-c_3)=0. \tag{A14}
$$

Finally, the market clearing condition (10) is equivalent to

$$
\mu\,\theta_\lambda^m+2\nu\,\theta_\lambda^i=0. \tag{A15}
$$

Solving system (A11),(A14), and (A15), we derive the linear stationary equilibrium

$$
P_\lambda=-\frac{\mu\,C_1+2\,\nu\,c_2(1-c_3)}{r\,(\mu+2\nu)} \tag{A16a}
$$

$$
\theta_\lambda^i=\frac{\mu\,(c_1-c_2(1-c_3))}{\alpha\sigma_D^2\,(\mu+2\nu)} \tag{A16b}
$$

$$
\theta_\lambda^m=-\frac{2\,\nu\,(c_1-c_2(1-c_3))}{\alpha\sigma_D^2\,(\mu+2\nu)} \tag{A16c}
$$

Since $c_1\ge 0$, $c_2\ge 0$, and $c_3\le 1$, the price P_λ is always negative.

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