Secret and Overt Information Acquisition in Financial Markets

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Abstract

We study the observability of investors’ information acquisition in financial markets. Two strategic effects arise when this observability improves: the pricing effect (concerning the interaction between investors and market makers) and the competition effect (concerning the interaction among investors). When the pricing effect prevails, investors would withhold their information-acquisition activity, and relative to unobservable information acquisition, investors tend to acquire less information. These patterns are reversed when the competition effect instead dominates. This information-production result has further consequences for market quality. Our analysis sheds new light on casual observations that funds voluntarily disclose corporate site visits, financial regulations such as those mandating the site-visit disclosures, and the practice of tracing investors’ digital footprints.

Keywords: Information acquisition, observability, corporate site visits, digital footprints, disclosure

JEL: D82, G14, G18

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1 Introduction

Information acquisition is a key decision for many investors in financial markets. One often overlooked characteristic of this action is its unobservability:¹ the standard treatment in the literature takes information acquisition as observable (e.g., Admati and Pfleiderer, 1988a,b; Holden and Subrahmanyam, 1992).² The tendency to ignore this unobservability implies that traditional characterizations of information acquisition in financial markets are at best incomplete and at worst misleading.

More importantly, the observability of investors’ information acquisition changes over time and varies across scenarios. This is to be expected given that financial markets are deeply shaped by regulations, technologies, and the interplay between them. For instance, while institutions are usually very secretive about their research activity, some funds are observed to voluntarily disclose their corporate site visits. Since July 2012, the Shenzhen Stock Exchange (SZSE) in China mandated all listed firms to disclose investors’ site visits within two trading days of the event, which effectively enhanced the transparency of information acquisition by investors. In addition to the regulatory changes, technological advances also change this observability. For example, investors’ digital footprints — such as their clicks on news links and their use of the United States Securities and Exchange Commission (SEC)’s Electronic Data Gathering, Analysis, and Retrieval (EDGAR) data filings — are now becoming more traceable.³

These observations motivate the following relevant questions: Why do some investors voluntarily disclose their otherwise secretive information-acquisition activity? How do these regula-

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¹Unobservability is the rule rather than exception in practice. A few examples make this point clear. The coverage and depth of in-house research within a fund are unknown to the public. The client base that purchases alternative data such as satellite images is often kept secret. Sophisticated investors often go to great lengths to conceal and erase their footprints, which makes their information acquisition even more secretive.

²Specifically, these standard models add a stage before a strategic trading game and solve the equilibrium by backward induction. This modeling technique implies that information-acquisition activities — or more specifically, the amount of money spent on information acquisition or equivalently the precision of the acquired information — are publicly observable to market participants in financial markets. The recent contributions by Banerjee and Breon-Drish (2020) and Rüdiger and Vigier (2020) are the very few exceptions that explicitly consider unobservable information acquisition in financial markets. However, we study different research questions. See the literature review for more discussions.

³See, for example, Ben-Rephael et al. (2017); Fedyk (2020); Li et al. (2018); Crane et al. (2020); Wu (2019); Chen et al. (2020); Gibbons et al. (2020).
tions and technologies affect investors’ information-acquisition and trading behavior? What are the consequences for aggregate financial market quality? Addressing these questions helps us understand how regulatory and technological developments are changing the nature of financial markets.

To answer these questions, we need a model that is flexible enough to include both the novel secret and the classic overt information acquisition. By changing the observability of information acquisition, we can gauge its effect and provide insights into related policy debates and empirical regularities. We also need to focus on an economy with large strategic investors, as in a competitive market with small investors (e.g., Grossman and Stiglitz, 1980; Hellwig, 1980; Verrecchia, 1982) the observability of information acquisition should not matter. The Kyle (1985) framework is a natural starting point. Although almost no models can sufficiently match the complex reality in modern financial markets, the stylized Kyle (1985) setting provides a good conceptual approximation to the functioning of the real markets, by modeling liquidity demand arising from information or hedging needs as well as liquidity supply provided by uninformed market makers.

We extend Kyle (1985) to consider multiple strategic investors. The main innovation of our model is to introduce a communication technology that may convey messages about investors’ information-acquisition activity to the public. If the message function perfectly reveals to the public an investor’s private signal precision, then the investor engages in overt information acquisition, as in the classic models. By contrast, if the message function does not reveal any information about the investor’s information precision, then the investor engages in secret information acquisition.

The message function can arise from various sources, depending on the specific application.

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4 In fact, large investors dominate asset ownership worldwide, making it important to study their behavior. For example, Kacperczyk, Nosal, and Sundaresan (2020) document that in many developed economies, institutional investors hold a large fraction of equities, and that the institutional ownership increases by 200% to 300% over the last 20 years. The authors also show that the holdings of the largest institutions, a measure of investor concentration, are significant in most markets.

5 We adopt the imperfect information approach as in Holmström (1982) and Stein (1989) to model secret investors’ hidden information acquisition. In terms of this approach, secret investors’ information acquisition is not observable to other market participants but in equilibrium the action is correctly conjectured by other market participants. This model technique is different from the incomplete information approach. See the literature review for a more detailed discussion.
contexts. We emphasize three sources in our paper. First, investors can voluntarily disclose their own information-acquisition activity. Second, due to regulations, investors’ information-acquisition activity can be mandated to be disclosed. Third, with the technology advancement, investors’ otherwise secretive information acquisition can become imperfectly observed. We will use two applications – investors’ corporate site visits and download requests at the SEC EDGAR system – to illustrate the three forms of message functions.

We start our analysis in Section 3 by contrasting secret with overt information acquisition. This analysis forms building blocks for our subsequent applications. The key to understanding the comparison is the equilibrium amount of information production. Our analysis identifies two types of strategic interactions regarding the observability of investors’ information acquisition: (i) the interaction between investors (as liquidity demanders) and market makers (as liquidity suppliers) (labeled the “pricing effect”) and (ii) the interaction among investors themselves as liquidity demanders (labeled the “competition effect”).

To isolate the interaction between the investor and market makers, Section 3.1 analyzes an economy with a monopolistic investor and finds that a secret monopolistic investor always acquires more information than an overt monopolistic investor. This is intuitive because the overt investor takes into account the strategic response of market makers and refrains from acquiring too much information. On observing that the overt investor acquires more information, market makers adjust the pricing schedule to better manage adverse selection risk, thereby reducing the investor’s profits.

Section 3.2 explores an oligopoly economy with multiple strategic investors so that both the pricing effect and the competition effect operate. Whether secret or overt information acquisition produces more precise information in equilibrium depends on the interplay between these two effects. The competition effect bears some similarity to what Hauk and Hurkens (2001) find in Cournot markets: as an overt investor acquires more information, rival investors are forced to trade less aggressively, which gives the initial investor a competitive advantage. The competition effect therefore encourages more information production when information acquisition is overt.
than when it is secret. Unlike in the monopoly economy, the pricing effect is ambiguous in this oligopoly economy. When the number of investors is small and/or when the cost of acquiring information (normalized by noise trading) is high, the pricing effect works in the same way as in the monopoly economy (but in the opposite way to the competition effect); that is, it encourages information production under secret information acquisition. Overall, only when this pricing effect prevails can the secret market continue to produce more precise information in equilibrium. Consequently, the secret market is associated with higher market efficiency (measured by the posterior precision about the asset fundamental conditional on asset prices) due to higher information production; it is also associated with lower market liquidity (measured by the inverse of the price impact of noise trading) due to the more severe adverse selection problem faced by market makers.

We next apply our model to study two important forms of information acquisition in financial markets. First, in Section 4, we study investors’ corporate site visits. We find that investors’ incentives to disclose their information-acquisition activity are also shaped by the pricing and competition effects. Specifically, in the economy with a monopolistic investor, the pricing effect always induces the investor to minimize disclosure about her information acquisition, resulting in secret information acquisition. This is because given the acquired information, more disclosures would make market makers believe that they are faced with a more informed investor and accordingly set steeper pricing schedule to the harm of the investor. In the economy with oligopolistic investors where both pricing and competition effects operate, the equilibrium disclosure hinges on the information-acquisition cost (normalized by noise trading). When the cost is high, investors only acquire coarse information, and their signals are distinct from each other’s. In this sense, each investor behaves like a local monopolist and the above documented pricing effect prevails, depressing investors’ disclosure incentives. By contrast, when the cost is low, investors share almost common information and compete intensively with each other. The dominant competition effect encourages disclosure because more disclosure induces other investors to believe that they are competing with a well informed investor and thus scale back in their
trading, thereby gaining the disclosed investor a competitive advantage. This explains why some funds may voluntarily disclose their otherwise secretive corporate site visits.

We also examine the effectiveness of the regulation that mandates the disclosure of investors’ corporate site visits, as seen in the Shenzhen Stock Exchange in China. Intuitively, such a mandate is effective only when investors would have kept their information acquisition secret. We find that when the mandate is effective, it causes investors to acquire less information, improves investors’ payoff and market liquidity, but worsens market efficiency.

Section 5 is devoted to our second application – investors’ download requests at the SEC EDGAR system. The SEC publicizing its EDGAR log files makes investors’ otherwise secretive digital footprints in information acquisition imperfectly observable. In this case, the message function perfectly communicates investors’ information acquisition with some probability, but otherwise reveals nothing. The effect of the observing probability on investors’ information acquisition, again, hinges on the pricing and competition effects. When the pricing effect dominates, a higher probability of observing investors’ digital footprints makes them acquire less information, which improves market liquidity but harms price efficiency. These patterns reverse when the competition effect instead dominates.

Related literature Our paper is related to the vast literature on information acquisition in financial markets. As noted before, the existing literature on large investors largely assumes that decisions relating to the acquisition of information are perfectly observable. We contribute to this literature by providing a parsimonious framework that allows secret information acquisition in financial markets and undertaking a systematic analysis of the effect of the observability of investors’ information acquisition. Our analysis suggests that in situations in which investors secretly collect information, traditional models based on overt information acquisition are likely to misestimate the amount of information produced in financial markets and even have qualitatively different implications for market quality.

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6See, for example, Grossman and Stiglitz (1980); Admati and Pfleiderer (1988a,b); Barlevy and Veronesi (2000); Van Nieuwerburgh and Veldkamp (2010); Kondor (2012); Mele and Sangiorgi (2015); Benhabib, Liu, and Wang (2019); Dávila and Parlatore (2019); Rüdiger and Vigier (2020).
Hauk and Hurkens (2001) investigate the secret acquisition of demand information in a duopoly Cournot market. They find that if information acquisition is overt, firms engage in excessive acquisition of information to compete with each other. We show that a similar competition effect emerges in a financial market in which investors compete as liquidity demanders. At the most basic level, this effect is reminiscent of the Stackelberg effect as an investor being observed to acquire more information can force competing investors to trade less aggressively. In addition to this competition effect, we show that financial markets also exhibit a novel pricing effect due to the interaction between investors as liquidity demanders and market makers as liquidity suppliers. This pricing effect has starkly different implications from those of the competition effect. Moreover, we also contribute by exploring the implications of the pricing and competition effects for investors’ disclosure incentives.

Mendelson and Tunca (2004) touch on the unobservability of investors’ information acquisition but they adopt different settings and focus on different research questions. Banerjee and Breon-Drish (2020) show that in a dynamic setting with a single trader, if there is a fixed-cost component to acquiring information and the trader discounts future profits, the unobservability of information acquisition can lead to the non-existence of an equilibrium with information acquisition. We complement these studies by (i) considering information acquisition by multiple strategic investors and exploring the interaction between the pricing and competition effects, (ii) presenting the existence and uniqueness of equilibria under secret information acquisition in a static setting and under a variable information-acquisition cost function, and (iii) studying investors’ disclosures about their information-acquisition activities. Rüdiger and Vigier (2020) investigate secret information acquisition in Glosten-Milgrom types of markets (Glosten and Milgrom, 1985). They do not consider the change in the observability of information acquisition but focus on investigating which players (investors or market makers) will choose to secretly acquire information in equilibrium. In our setting, investors can acquire information but market makers

7The serious study of unobservability in a game-theoretic framework can be dated back to Hart and Tirole (1990) and McAfee and Schwartz (1994). The issue of the transparency of agents’ information acquisition has been considered in other contexts such as an entry game (Hurkens and Vulkan, 2001) and auctions (Persico, 2000).
cannot do so; we also apply our model to understand the implications of changing information-acquisition observability.

Our paper also relates to the literature that studies uncertainty about strategic investors’ informedness (e.g., Chakraborty and Yılmaz, 2004; Altı, Kaniel, and Yoeli, 2012; Banerjee and Green, 2015; Back, Crotty, and Li, 2018; Dai, Wang, and Yang, 2019; Peress and Schmidt, 2019). Uncertainty over investors’ informedness comes from nature; in our model, there is no such uncertainty because in equilibrium market participants hold correct beliefs about strategic investors’ decisions on acquiring information. The literature therefore adopts a framework with incomplete information, while our analysis follows the corporate-finance literature (e.g., Holmström, 1982; Stein, 1989) and adopts a framework with imperfect information. Our parsimonious modeling approach can also be applied to other settings featuring secret information acquisition.

More broadly, our paper is related to the large literature on transparency in financial markets, which is central to many policy debates. Transparency can take many forms. It can be related to the quality of public signals (e.g., Banerjee, Davis, and Gondhi, 2018; Dugast and Foucault, 2018), to the disclosures by market participants (e.g., Admanti and Pfeiderer, 1991; Madhavan, 1995; Frenkel, Guttmann, and Kremer, 2020), to the ambiguity of certain types of investors’ investment strategies (e.g., Easley, O’Hara, and Yang, 2014). Specific to institutional investors, information acquisition and trading are two important activities that they are engaged in. Investor trading is mostly unobservable: institutions are only required to report their ownership quarterly in 13-F filings; more recently, the SEC is considering to raise the reporting threshold for institutional investment managers from $100 million to $3.5 billion, which can further reduce the trading observability.8 Because of this unobservability, many studies develop new methods to infer high-frequency institutional behavior (e.g., Froot, O’connell, and Seasholes, 2001; Campbell, Ramadorai, and Schwartz, 2009; Froot and Ramadorai, 2008). So far, this observability issue of information acquisition by investors is understudied and we contribute to the literature by conducting a systematic analysis of it.

8See www.sec.gov/comments/s7-08-20/s70820.htm for further detail.
2 A Model of Secret and Overt Information Acquisition in Financial Markets

Setup We consider a Kyle-type model (Kyle, 1985) with three dates \( t = 0, 1, 2 \). Figure 1 describes the timeline of the economy. There is a single risky asset with a date-2 liquidation value \( \tilde{v} \), where \( \tilde{v} \sim N(0, 1) \).\(^9\) The financial market operates on date 1. It is populated by three groups of agents: competitive market makers, noise traders, and \( J \) rational investors, where \( J \geq 1 \) is a positive integer. As usual, market makers provide liquidity by setting the price based on the weak market-efficiency rule. Noise traders and rational investors demand liquidity. Noise traders submit exogenous random market orders to meet their unmodeled liquidity needs. Rational investors trade on private information to earn profits. On date 0, rational investors engage in information acquisition and then there exists a communication technology that may reveal some information about investors’ information acquisition activities to the general public. This communication technology is the main innovation of our setup, and we will discuss it shortly in detail.

\begin{align*}
\text{\( t = 0 \)} & \quad \text{\( t = 1 \)} & \quad \text{\( t = 2 \)} \\
- \text{Investors simultaneously make information-acquisition decisions.} & \quad \text{Investors observe their acquired private information.} & \quad \text{The value of the asset is realized, and all agents consume.} \\
- \text{The communication technology reveals certain information about investors’ information-acquisition activity.} & \quad \text{Investors and noise traders submit order flows, and market makers set the price.} & \\
\end{align*}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{timeline.png}
\caption{Timeline}
\end{figure}

Information acquisition On \( t = 0 \), the \( J \) investors simultaneously make their information-acquisition decisions. Consider investor \( j \in \{1, \ldots, J\} \). Prior to trading on date 1, she can observe

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\(^9\)The normalization that \( \tilde{v} \) has a zero mean and a unit standard deviation is without loss of generality. Instead, if we assume \( \tilde{v} \sim N(\bar{v}, \sigma_v^2) \) (with \( \bar{v} \in \mathbb{R} \) and \( \sigma_v > 0 \)), then all of our results still hold as long as we reinterpret the information precisions as signal-to-noise ratios.
a private signal of the form:

\[ \tilde{y}_j = \tilde{v} + \tilde{e}_j, \]  

(1)

where \( \tilde{e}_j \sim N(0, h_j^{-1}) \) and \( \tilde{v} \) and \( \tilde{e}_j \) are mutually independent. Investor \( j \) will use signal \( \tilde{y}_j \) to guide her trade. The more precise the signal, the higher the trading profits she expects to obtain. Precision \( h_j \) can be acquired according to a linear cost function:

\[ C(h_j) = c \cdot h_j, \]  

(2)

where \( c \) is a positive constant. This linear information-acquisition technology is used for tractability.\(^\text{10}\) It can be justified as discrete sampling with a constant cost per observation and is commonly adopted in the literature (e.g., Verrecchia, 1982; Kim and Verrecchia, 1991; Hauk and Hurkens, 2001; Myatt and Wallace, 2012). Investor \( j \) chooses precision \( h_j \) to maximize her expected trading profits net of the information-acquisition cost.

**Communication technology** On \( t = 0 \), there exists a communication technology that can potentially reveal investors’ information acquisition activities to the public. We model this communication technology with a message function. Specifically, after investor \( j \) acquires information, all active agents (investors and market makers) observe a message \( m_j \) that may contain information about investor \( j \)’s precision \( h_j \). This message function \( m_j \) can arise from various sources, depending on the specific application contexts. In this paper, we emphasize three sources. First, investors can voluntarily disclose their information-acquisition activities to the public and thus, the messages are endogenously determined in equilibrium. Second, due to regulations, investors’ information-acquisition activities can be mandated to be disclosed. Third, the message function can also be exogenously determined by technologies. For instance, due to technology advance-
ment, investor $j$’s otherwise secretive information acquisition can be detected with certain probability. In Sections 4 and 5, we will consider two applications — investors’ corporate site visits and downloading requests at the SEC EDGAR website — to illustrate these three forms of message functions. In these applications, the economic interpretation of these message functions is more concrete. For instance, in the application of corporate site visits, the messages correspond to the numbers of site visits by financial institutions.

The notion of message functions also facilitates the definitions of overt and secret information acquisition. Specifically, if the message $m_j$ perfectly reveals to the public the precision $h_j$ of the private signal acquired by investor $j$, then investor $j$ engages in overt information acquisition. By contrast, if the message function does not reveal any information about investor $j$’s information precision $h_j$, then the investor engages in secret information acquisition. In Section 3, we will explore these two special cases of message functions, and the analysis in that section forms building blocks for our applications in Sections 4 and 5.

**Trading and pricing** Let $\tilde{p}$ denote the date-1 price of the risky asset in the financial market. Conditional on the acquired signal $\tilde{y}_j$ and the observed messages $\{m_1, ..., m_J\}$, investor $j$ places market order $\tilde{x}_j$ to maximize her expected trading profits as follows:

$$E[\tilde{x}_j (\tilde{v} - \tilde{p}) | \tilde{y}_j; m_1, ..., m_J].$$

(3)

As usual, noise traders place random order $\tilde{u}$, where $\tilde{u} \sim N(0, \sigma_u^2)$ (with $\sigma_u > 0$) and $\tilde{u}$ is independent of all other random shocks. Thus, the total order flows faced by market makers are:

$$\tilde{\omega} = \sum_{j=1}^{J} \tilde{x}_j + \tilde{u}. \quad (4)$$

Competitive market makers set price $\tilde{p}$ according to the weak-efficiency rule:

$$\tilde{p} = E[\tilde{v} | \tilde{\omega}; m_1, ..., m_J].$$

(5)
As usual, at the trading stage, we will consider linear equilibrium in which investors’ trading rules are linear in their private information $\tilde{y}_j$ and market makers’ pricing rule is linear in the total order flows $\tilde{\omega}$. Since the messages $\{m_1, \ldots, m_J\}$ are effectively public information, they will affect the coefficients in the trading and pricing rules.

## 3 Secret versus Overt Information Acquisition

In this section, we analyze and compare two benchmark economies: all investors engaging in secret information acquisition and all of them engaging in overt information acquisition. The analysis in this section serves two purposes. First, since we depart from the literature by proposing a tractable framework allowing for secret information acquisition, we want to introduce to readers the details of equilibrium derivations and illustrate transparently how secret information acquisition exactly differs from the classic overt information acquisition. Second, the analysis forms the building blocks for our subsequent applications in Sections 4 and 5.

### 3.1 Monopolistic Investor ($J = 1$)

We start the analysis by considering a monopolist-investor setting. In this setting, only the interaction between the investor and market makers prevails. Analyzing this setting allows for a stark and intuitive comparison between secret and overt information acquisition.

#### 3.1.1 Solve the Model

As there is only one investor, we drop the subscript $j$ and use $h$, $\tilde{y}$, and $\tilde{x}$ to denote the investor’s information precision, private signal, and order flow, respectively. The investor’s trading rule is given by $\tilde{x} = \alpha \tilde{y}$ and the market makers’ pricing rule is given by $\tilde{p} = \lambda \tilde{\omega}$. An equilibrium is characterized by a tuple $(h, \lambda, \alpha)$. We use $(h_s, \alpha_s, \lambda_s)$ and $(h_o, \alpha_o, \lambda_o)$ to denote the equilibrium variables in the secret and overt information-acquisition cases, respectively. We will simultaneously compute the equilibrium for the two cases.
On date 1, the investor takes the market makers’ pricing rule as given and computes her conditional expected trading profits in (3) as $E[\tilde{x} (\tilde{v} - \tilde{p}) | \tilde{y}; m] = -\lambda \tilde{x}^2 + \frac{h\tilde{y}}{1+h} \tilde{x}$. The first-order condition (FOC)$^{11}$ implies that the optimal trading strategy is given by $\tilde{x} = \alpha \tilde{y}$, with

$$\alpha = \frac{h}{2(1+h)\lambda}. \quad (6)$$

Market makers form beliefs $\hat{h}$ about the investor’s information precision based on the message $m$. Under secret information acquisition, the message does not reveal any information about the investor’s information precision. Since the market makers’ beliefs must be correct in equilibrium, $\hat{h}$ is fixed at the equilibrium precision level $h_s$. By contrast, under overt information acquisition, the message perfectly reveals to market makers the investor’s information precision, that is, $\hat{h} = h$. In both cases, given precision belief $\hat{h}$, market makers form conjectures $\hat{\alpha}$ about the investor’s optimal trading strategy according to (6): $\hat{\alpha} = \frac{h}{2(1+h)}$. Then, equipped with the conjectured $\hat{h}$ and $\hat{\alpha}$, market makers choose $\lambda$ according to the pricing rule as specified by (5), yielding $\lambda = \frac{\hat{\alpha}}{\hat{\alpha}^2 (1+1/h) + \sigma_u^2}$. Combining this equation with $\hat{\alpha} = \frac{h}{2(1+h)\lambda}$, we can compute market makers’ pricing rule as a function of their belief about the investor’s information precision as follows:

$$\lambda = \frac{\sqrt{h}}{2\sigma_u \sqrt{1 + h}}. \quad (7)$$

We now go back to date 0 to determine the investor’s optimal information-acquisition strategy. We can compute the investor’s unconditional expected profits as $\lambda E(\tilde{x}^2) = \lambda \alpha^2 (1 + 1/h)$. Using the expression of $\alpha$ in (6) and subtracting the information-acquisition cost, we obtain the investor’s expected net trading profits $\pi$ as a function of $(h, \lambda)$ as follows:

$$\pi(h, \lambda) = \frac{h}{4\lambda(1+h)} - c \cdot h \quad (8)$$

$^{11}$It is easy to confirm that the second-order condition (SOC) is negative so that this trading strategy indeed maximizes the investor’s profits. In the remainder of the paper, the SOC is always checked.
We can use equation (7) to replace $\lambda$ in the above equation and reexpress $\pi$ as a function of $(h, \hat{h})$:

$$\pi(h, \hat{h}) = \frac{h\sigma_u\sqrt{1 + \hat{h}}}{2(1 + h)\sqrt{h}} - c \cdot h.$$  \hfill (9)

As mentioned above, the difference between secret and overt information acquisition lies in the specification of $\hat{h}$. Under secret information acquisition, $\hat{h}$ is fixed at the equilibrium precision $h_s$. Thus, when maximizing the investor’s profits, we fix $\hat{h} = h_s$ in (9) and compute the first-order derivative with respect to $h$, yielding her optimal information precision: $h = \sqrt{\frac{\sigma_u^2}{2c}} \left(1 + \frac{1}{h_s} \right)^{1/4} - 1$. Setting $h = h_s$ completes the characterization of the equilibrium. In contrast, under overt information acquisition, information precision is observable and so $\hat{h} = h$. Now, the pricing rule (7) adjusts with $h$ and the investor anticipates this adjustment, which changes the investor’s information-acquisition incentive. Specifically, we first replace $\hat{h}$ with $h$ in equation (9) and then take total derivative with respect to $h$ to maximize the investor’s profits. Taken together, in the monopoly setting, the key difference between secret and overt information-acquisition incentives is whether the investor’s information precision choice can influence the market makers’ pricing rule $\lambda$ through influencing their belief $\hat{h}$.

Lemma 1. Suppose $J = 1$. Then:

(1) If information acquisition is secret, there exists a unique equilibrium. The equilibrium information precision $h_s$ is the unique solution to the following equation:

$$h_s(1 + h_s)^3 = \frac{\sigma_u^2}{4c^2}. \hfill (10)$$

The investor’s trading strategy is $\tilde{x} = \alpha_s \tilde{y}$ with $\alpha_s = \sigma_u \sqrt{h_s}. \sqrt{1 + h_s}$. The pricing rule is $\tilde{p} = \lambda_s \tilde{\omega}$ with $\lambda_s = \frac{1}{2}\sigma_u \sqrt{\frac{h_s}{1+h_s}}$.

(2) If information acquisition is overt, there exists a unique equilibrium. The equilibrium information precision $h_o$ is the unique solution to the following equation: $h_o(1 + h_o)^3 = \frac{\sigma_u^2}{16c^2}$. The investor’s
trading strategy is $\tilde{x} = \alpha_o \tilde{y}$ with $\alpha_o = \sigma_u \sqrt{\frac{h_o}{1 + h_o}}$. The pricing rule is $\tilde{p} = \lambda_o \tilde{\omega}$ with $\lambda_o = \frac{1}{2\sigma_u} \sqrt{\frac{h_o}{1 + h_o}}$.

3.1.2 Equilibrium Comparison

We now compare the equilibrium outcomes with secret and overt information acquisition. First, we can show that a secret monopolist investor always acquires more information than an overt monopolist investor does. The key difference between secret and overt information acquisition is that the overt investor takes into account the market makers’ response to her information acquisition. To illustrate, we use the chain rule on equation (8) to express the investor’s information-acquisition incentive as follows:

$$\frac{d\pi}{dh} = \frac{\partial \pi}{\partial h} + \frac{\partial \pi}{\partial \lambda} \frac{\partial \lambda}{\partial h},$$

where $\frac{\partial \pi}{\partial h} = \frac{1}{4\lambda(1+h)^2} - c, \frac{\partial \pi}{\partial \lambda} = -\frac{h}{4\lambda^2(1+h)^2} < 0$, and if the investor engages in overt information acquisition $\frac{\partial \lambda}{\partial h} = \frac{1}{4\sigma_u(1+h)\sqrt{h(1+h)}} > 0$ whereas if she engages in secret information acquisition $\frac{\partial \lambda}{\partial h} = 0$.

The first term in (11) captures the standard trade-off faced by an investor considering the acquisition of information; in terms of this trade-off, other things being equal (more specifically, $\lambda$ being fixed), more precise information helps the investor better forecast fundamental value $\tilde{v}$ but costs more resources. This trade-off is present in both the secret and overt information-acquisition cases.

The second term in (11) is an additional effect that only operates in the case of overt information acquisition. It comes from the strategic response of market makers to the investor’s information acquisition. We label it the pricing effect. Specifically, as the overt investor acquires more precise information and trades more aggressively, market makers expect that they will be more exposed to private information when providing liquidity, and their adverse-selection con-
cern worsens. In response, market makers set a higher \( \lambda \) (i.e., \( \frac{\partial \lambda}{\partial h} > 0 \)). Facing this steeper pricing schedule, the overt investor trades less aggressively and makes a lower profit (i.e., \( \frac{\partial \pi}{\partial \lambda} < 0 \)). The strategic response of market makers therefore reduces the overt investor’s incentive to acquire information (i.e., \( \frac{\partial \pi}{\partial \lambda} \frac{\partial \lambda}{\partial h} < 0 \)).\(^1\) Importantly, under secret information acquisition, this strategic effect is muted since market makers’ pricing rule \( \lambda \) is independent of the investor’s actual information acquisition.

We next examine two market-quality variables: market liquidity and market efficiency (price discovery). These two variables are central to regulatory debates and have been extensively studied in the literature (see the discussions in Easley, O’Hara, and Yang, 2016). We measure market liquidity using Kyle’s \( \lambda \): a more liquid market is associated with a smaller \( \lambda \). Intuitively, \( \lambda \) captures the price impact of uninformed noise trading and is thus negatively related to market depth. Note that in equilibrium, \( \lambda \) is connected to information precision \( h \) through equation (7) in both the overt and secret cases. Since \( h_s > h_o \), we have \( \lambda_s > \lambda_o \); that is, the secret market is less liquid than the overt market.

Market efficiency captures how much information is incorporated into the asset price. We measure market efficiency by the precision of the asset payoff conditional on its price; \( m \equiv \frac{1}{\text{Var}(\tilde{v} | \tilde{p})} \). Intuitively, when the price aggregates a great deal of information, the residual uncertainty of the fundamental \( \tilde{v} \) conditional on the price \( \tilde{p} \) is low and thus market efficiency is high. Given \( h \), we can show that market efficiency is determined as \( m = \frac{2(h+1)}{h+2} \), which is increasing in the signal precision \( h \). As \( h_s > h_o \), it immediately follows that \( m_s > m_o \). This means that the secret investor acquires more information than the overt investor, and thus the price in the secret market contains more information about the asset’s fundamental value.

Proposition 1 and Panel (a) of Figure 2 summarize the above discussions.

**Proposition 1** (Secret versus overt information acquisition in the monopoly). Suppose that \( J = 1 \). Relative to overt information acquisition, secret information acquisition leads to more precise information, lower market liquidity, and higher market efficiency. That is, \( h_s > h_o, \lambda_s > \lambda_o, \) and

\(^{12}\)This result can be reversed when there are multiple investors, a scenario that we revisit in next subsection (see Lemma 3).
\[ m_s > m_\sigma. \]

Figure 2: Secret versus overt information acquisition

3.2 Oligopolistic Investors \((J \geq 2)\)

In this section, we study the setting with oligopolistic investors \((J \geq 2)\). In this setting, we introduce the interaction among strategic investors and investigate its interplay with the pricing effect. The following proposition extends Proposition 1 to the general case with \(J \geq 2\). The computation parallels that in Section 3.1.1 and is thus relegated to the appendix.

Lemma 2. (1) If information acquisition is secret, there exists a unique equilibrium. The equilibrium information precision \(h_s\) is the unique solution to the following equation:

\[ h_s(1 + h_s)(2 + h_s + h_sJ)^2 = \frac{\sigma_u^2}{Jc^2}. \]  

(12)
The investor’s trading strategy is $\tilde{x}_j = \alpha_s \tilde{y}_j$ with $\alpha_s = \sigma_u \sqrt{\frac{h_s}{(1+h_s)r}}$. The pricing rule is $\tilde{p} = \lambda_s \tilde{\omega}$.

with $\lambda_s = \frac{\sqrt{Jh_s(1+h_s)}}{\sigma_u(2+(J+1)h_s)}$.

(2) If information acquisition is overt, there exists a unique equilibrium. The equilibrium information precision $h_o$ is the unique solution to the following equation:

$$h_o(1 + h_o)(2 + h_o)^2(2 + h_o + h_oJ)^4}{((6J^2 - J - 3)h_o^2 + (4J^2 + 10J - 8)h_o + 8J - 4)^2} = \frac{\sigma_u^2}{4J^3c^2}. \quad (13)$$

The investor’s trading strategy is $\tilde{x} = \alpha_o \tilde{y}$ with $\alpha_o = \sigma_u \sqrt{\frac{h_o}{(1+h_o)r}}$. The pricing rule is $\tilde{p} = \lambda_o \tilde{\omega}$

with $\lambda_o = \frac{\sqrt{Jh_o(1+h_o)}}{\sigma_u(2+(J+1)h_o)}$.

**Pricing effect versus competition effect** To understand the consequences of secret versus overt information acquisition, let us examine a representative investor $j$’s information-acquisition incentive. Given the equilibrium $h^*$ (which are $h_s$ and $h_o$ under secret and overt information acquisition respectively), in the oligopolistic setting, we can compute the expected net profit of investor $j$ as follows:

$$\pi_j(h_j, \alpha(\hat{h}_j), \lambda(\hat{h}_j); h^*) = \frac{\hat{h}_j \left[ 1 - (J - 1) \alpha(\hat{h}_j) \lambda(\hat{h}_j) \right]^2}{4\lambda(\hat{h}_j)(h_j + 1)} - c \cdot h_j, \quad (14)$$

where $\lambda(\hat{h}_j)$ is the price impact set by market makers and $\alpha(\hat{h}_j)$ is the trading intensity of other investors except $j$, which are specified as follows:

$$\lambda(\hat{h}_j) = \frac{\sqrt{\hat{h}_j^2(4 + (h^*)^2J + h^*(J + 3)) + \hat{h}_j(4 + 4h^*J + (h^*)^2(4J - 3)) + 4h^*(1 + h^*)(J - 1)}}{4 + 2h^*J + \hat{h}_j(4 + h^* + h^*J)} \sigma_u,$$

$$\alpha(\hat{h}_j) = \frac{(2 + h^*)\hat{h}_j}{4 + 2Jh^* + (4 + h^* + h^*J)\hat{h}_j} \lambda(\hat{h}_j).$$

Similar to the monopolistic case, under secret information acquisition ($h^* = h_s$), investor $j$’s message function does not reveal any information about her information acquisition, and
thus other agents hold belief $\hat{h}_j$ to be fixed at the equilibrium level, i.e., $\hat{h}_j = h_s$. Under overt information acquisition ($h^* = h_o$), through the message function both market makers and other investors can perfectly observe investor $j$’s information acquisition, i.e., $\hat{h}_j = h_j$, and investor $j$ well anticipates that $(\lambda, \alpha)$ will be affected by $h_j$.

We apply the chain rule to equation (14) and decompose investor $j$’s information-acquisition incentive as follows:

$$
\frac{d\pi_j}{dh_j} = \frac{\partial \pi_j}{\partial h_j} + \frac{\partial \pi_j}{\partial \lambda} \frac{\partial \lambda}{\partial h_j} + \frac{\partial \pi_j}{\partial \alpha} \frac{\partial \alpha}{\partial h_j}.
$$

Equation (15) extends equation (11) in the monopoly setting to the oligopoly setting. As in (11), the first term in (15) captures the standard trade-off in acquiring information: other things being equal (i.e., $\alpha$ and $\lambda$ being fixed), acquiring more precise information helps investor $j$ predict fundamental value $\tilde{v}$ and the rival investors’ demands but, at the same time incurs a higher cost. The second term in (15) is the pricing effect due to the strategic response of market makers, as we have discussed in (11) for the monopolist-investor setting (but as we will see shortly, here, the sign of the pricing effect is ambiguous, while it is always negative when $J = 1$). The third term is due to the strategic response of other investors. We label this effect the competition effect, which is an additional effect present in the oligopolistic-investor setting. The first effect is present in both the secret and overt information-acquisition cases, while the second and third effects only prevail in the overt information-acquisition case. As a result, whether overt information acquisition generates more precise information in equilibrium than secret information acquisition depends on the interaction of the latter two effects.

We can show that the competition effect is always positive (i.e., $\frac{\partial \pi_j}{\partial \alpha} \frac{\partial \alpha}{\partial h_j} > 0$). First, investor $j$’s profits decrease as her rival investors trade more aggressively (i.e., $\frac{\partial \pi_j}{\partial \lambda} < 0$). Second, as investor $j$ acquires more precise information, her competitors trade less aggressively on their own private information (i.e., $\frac{\partial \alpha}{\partial h_j} < 0$) because they expect that investor $j$ will trade more aggressively on
her more precise information. Taken together, we have $\frac{\partial \pi_j}{\partial \alpha} \frac{\partial \alpha}{\partial h_j} > 0$, so that the competition effect tends to strengthen an overt investor’s information-acquisition incentive.

The pricing effect is ambiguous in this extended oligopoly setting (i.e., $\frac{\partial \pi_j}{\partial \lambda} \frac{\partial \lambda}{\partial h_j}$ can be either positive or negative). We can show that $\frac{\partial \pi_j}{\partial \lambda} < 0$; that is, a steeper pricing schedule tends to lower investor $j$’s profits. However, we cannot determine the sign of $\frac{\partial \lambda}{\partial h_j}$, which makes the pricing effect ambiguous. Specifically, the pricing rule (5) implies:

$$\lambda = \frac{Cov(\tilde{v}, \tilde{\omega})}{Var(\tilde{\omega})} = \frac{\alpha_j + (J - 1) \alpha}{[(J - 1) \alpha + \alpha_j]^2 + \alpha^2 (J - 1) h_j^{-1} + \alpha^2_j h_j^{-1} + \sigma_u^2},$$

where the second equality follows from $\tilde{\omega} = \tilde{x}_j + \sum_{k \neq j}^J \tilde{x}_k + \tilde{u}$, where $\tilde{x}_j = \alpha_j \tilde{y}_j$, and $\tilde{x}_k = \alpha \tilde{y}_k$ (for $k \neq j$). In general, $\lambda$ is determined by two terms: $Cov(\tilde{v}, \tilde{\omega})$ and $Var(\tilde{\omega})$. An increase in $h_j$ affects these two terms through various channels. First, the increase directly lowers $Var(\tilde{\omega})$ through the error term in investor $j$’s private signal. Second, it raises investor $j$’s trading aggressiveness $\alpha_j$, which in turn can increase both $Cov(\tilde{v}, \tilde{\omega})$ and $Var(\tilde{\omega})$. Third, it reduces other investors’ trading aggressiveness $\alpha$ and hence both $Cov(\tilde{v}, \tilde{\omega})$ and $Var(\tilde{\omega})$. The interactions among these various channels generate an ambiguous sign of $\frac{\partial \lambda}{\partial h_j}$.

Lemma 3 suggests that the pricing effect is negative when the number $J$ of investors is small and/or when the ratio $c/\sigma_u$ is high. A small $J$ is close to the monopoly setting explored in the previous section; in this case, concerns over an adverse-selection problem causes market makers to steepen the pricing schedule in response to more informed investors, leading to a negative pricing effect. Now, suppose that $J$ is large. If $c/\sigma_u$ is low, then the low cost of information acquisition makes all investors acquire very precise signals in equilibrium so that they share almost common information and thus compete very aggressively. As a result, their aggressive trading reveals fundamental information to market makers, alleviating market makers’ concerns about adverse selection. By contrast, if $c/\sigma_u$ is high, then investors acquire very coarse signals with the result that their signals are distinct from each other’s. In this sense, each investor retains her own information advantage and behaves like a local monopolist, giving rise to adverse selection.
Lemma 3.  (i) The competition effect is positive. That is, $\frac{\partial \pi_j}{\partial \alpha} \frac{\partial \alpha}{\partial h_j} > 0$.

(ii) The pricing effect is ambiguous. In particular, at the equilibrium precision level $h_o$ of overt information acquisition, we have the following:

- For $J \leq 3$, $\frac{\partial \pi}{\partial \lambda} \frac{\partial \lambda}{\partial h_j} \bigg|_{h_j = h_o} < 0$.
- For sufficiently large $J$, $\frac{\partial \pi}{\partial \lambda} \frac{\partial \lambda}{\partial h_j} \bigg|_{h_j = h_o} < 0$ if and only if $c/\sigma_u > \frac{1}{4\sqrt{2}}$.

Secret versus overt information acquisition  Lemma 3 suggests that when $J$ is large and $c/\sigma_u$ is low, the pricing effect and the competition effect work in the same direction. In equilibrium, overt information acquisition thus leads to more precise information than does secret information acquisition. We can generalize this result and show that for any value of $J$, the equilibrium precision level of overt information is higher than that of secret information if $c/\sigma_u$ is lower than a threshold value. Panel (b1) of Figure 2 graphically illustrates this result for $J = 2$.

In this duopoly setting, the pricing effect works against the competition effect (see Lemma 3). Nonetheless, when $c/\sigma_u$ is low, investors choose to acquire very precise information, so that the competition among them is very fierce. This in turn implies that the competition effect is particularly strong and dominates the pricing effect.

This information-acquisition result has important consequences for market quality. Given precision level $h$, we can compute market efficiency in equilibrium as: $m = \frac{2 + h + hJ}{2 + h}$. Intuitively, market efficiency $m$ increases with information precision $h$. As a result, the comparison in terms of market efficiency mirrors the comparison in terms of information precision across both the secret and overt cases.

Market liquidity is still inversely measured by Kyle's $\lambda$. By Proposition 2, in both the secret and overt information acquisition cases, we have $\lambda = \frac{\sqrt{h(1 + h)}J}{\sigma_u(2 + (J + 1)h)}$. The way that information precision $h$ affects market liquidity $\lambda$ shares some similarities with the discussion in Lemma 3.

In Lemma 3, we vary one investor’s information precision $h_j$ and examine how market makers
adjust $\lambda$. Here, we vary the information precision of all investors. However, a similar result prevails. Specifically, when $J \leq 3$, $\lambda$ is increasing in $h$, i.e., $\frac{\partial \lambda}{\partial h} > 0$, whereas when $J > 3$, $\lambda$ is hump-shaped in $h$, that is, $\frac{\partial \lambda}{\partial h} > 0$ if and only if $h < \frac{2}{J-3}$. If we focus on the case of $J \leq 3$, then the comparison of $\lambda$ immediately follows from the comparison of $h$ across the secret and overt information-acquisition cases.

Panel (b) of Figure 2 and Proposition 2 summarize the above results. 

**Proposition 2** (Secret versus overt information acquisition in oligopoly). Define

$$\bar{h} \equiv \frac{4 - J + \sqrt{17J^2 - 20J + 4}}{4J^2 - 3J - 3} \quad \text{and} \quad \bar{c} \equiv \frac{1}{(2 + \bar{h} + \bar{h}J)\sqrt{\bar{h}(1 + \bar{h})J}}.$$

Then:

1. Overt investors acquire more precise information than secret investors if and only if $c/\sigma_u < \bar{c}$; that is, $h_o > h_s$ if and only if $c/\sigma_u < \bar{c}$.

2. An overt market is more informationally efficient than a secret market if and only if $c/\sigma_u < \bar{c}$; that is, $m_o > m_s$ if and only if $c/\sigma_u < \bar{c}$.

3. Suppose that $J \leq 3$. A secret market is more liquid than an overt market if and only if $c/\sigma_u < \bar{c}$; that is, $\lambda_o > \lambda_s$ if and only if $c/\sigma_u < \bar{c}$.

## 4 Site Visits: Voluntary and Mandatory Disclosure of Information Acquisition

In this section, we apply our model to study corporate site visits, an important source of information for investors in financial markets.\(^1\) We first introduce how to connect our model to

\(^{1}\text{Corporate site visits represent a valuable source of information for investors (e.g., Chen et al., 2021; Cicero et al., 2021; Solomon and Soltes, 2015). Funds that meet with firms have been shown to exhibit superior investment performance afterwards (e.g., Chen et al., 2021; Cicero et al., 2021; Solomon and Soltes, 2015). In particular, as emphasized by Chen et al. (2021), soft information such as corporate culture, employee morale and firm strategies that can only be collected through face-to-face interactions with firms remains valuable.}\)
this setting in Section 4.1 and then discuss voluntary and mandatory disclosure of information acquisition in Sections 4.2 and 4.3, respectively.

4.1 Institutional Background

Investors can voluntarily disclose their information-acquisition activity. One salient example is that funds voluntarily disclose their corporate site visits. For instance, in February 2020, a UK investment company Castlefield disclosed on its website the site visit it conducted earlier that year to Alumasc, a UK based supplier of premium building products.\footnote{See for detail https://www.castlefield.com/news-media/blog/site-visit-to-timloc-a-division-of-the-stock-alumasc/. For other examples of site visits, see https://www.youtube.com/watch?v=HddsefV7b5o in Israel and https://mp.weixin.qq.com/s/bRkjATzjRLqJqNPvW7-UbQ in China.} The fund emphasized that such costly visits helped it learn about this company: “taking the time to attend these visits often allows us to gain more insight into the processes and products involved in its operations and the potential to speak to other senior managers that do not typically meet with investors.” Importantly, the disclosed report focused on the content of this site visit, including an introductory presentation, facility tour, and lunchtime Q&A, rather than on the fund’s view about the company fundamentals. Moreover, such a report can be viewed as hard evidence for this specific corporate site visit as the fund expresses a very serious attitude towards the disclosed content, lending credibility to the disclosure.\footnote{At the end of the report, the investment company adds the following legal statement: “Information is accurate as at 04.02.2020. Opinions constitute the fund manager’s judgement as of this date and are subject to change without warning. The officers, employees and agents of CIP may have positions in any securities mentioned herein. This material may not be distributed, published or reproduced in whole or in part.”}

This voluntary-disclosure setting can be nested in our general Section 2 setup by considering an endogenously determined message function; that is, investors choose messages to maximize their expected profits. To build a tight conceptual connection between our model and the disclosed corporate site visits, consider the following setting. Suppose that each site visit by investor $j$, denoted by $l$, costs a fixed amount of resources $c$ and generates a signal $\tilde{v} + \tilde{z}_{j,l}$ about the company’s fundamental value $\tilde{v}$, where $\tilde{z}_{j,l} \sim \mathcal{N}(0, \sigma_z^2)$ (with $\sigma_z > 0$) and $(\tilde{v}, \tilde{z}_{j,l})$ are mutually independent. Then, $H_j$ visits can lead to a sufficient statistic $\hat{v} + \hat{e}_j$, where $\hat{e}_j \equiv \frac{1}{H_j} \sum_{l=1}^{H_j} \tilde{z}_{j,l} \sim$
$N(0,h^{-1}_j)$ and $h_j \equiv H_j\sigma_z^{-2}$. Releasing a number $M_j$ (where $M_j \leq H_j$) of site visits therefore yields a sufficient statistics $\tilde{v} + \tilde{e}_j'$, where

$$\tilde{e}_j' \equiv \frac{1}{M_j}\sum_{l=1}^{M_j} \tilde{z}_{j,l} \sim N(0, m_j^{-1}).$$

Naturally, an investor cannot disclose the site visits more than she actually conducts and thus the disclosure must not exceed the actual level of information acquisition, i.e., $m_j \leq h_j$. In addition, after observing investor $j$’s message $m_j$, other agents form a belief about the investor’s information-acquisition activities, and we denote this belief $\hat{h}_j$. Since the message needs to be supported by hard evidence, the belief must be at least as high as the disclosed level. That is, if investor $j$ reveals that she has conducted $M_j$ times of site visits, while other agents are still uncertain about the exact number of site visits, they must believe that the investor has at least a number $M_j$ of visits, and thus the precision of investor $j$’s private signal must be at least $m_j$ (i.e., $\hat{h}_j \geq m_j$).

In addition to voluntary disclosure of information acquisition, there exist regulations that mandate the disclosure of investors’ information-acquisition activities. For example, since July 2012, the Shenzhen Stock Exchange (SZSE) in China has required all listed firms to disclose investors’ site visits through a designated web portal within two trading days of the event. The requested information includes the date, location, and names and employer institutions of all participating individuals. This regulation was put forth in the same spirit of Regulation Fair Disclosure (RegFD) in the U.S.. Specifically, RegFD was designed to prevent selective disclosure by public companies. In spite of the passage of the RegFD, some investors continue to meet privately with executives and make more informed trading decisions subsequently (Solomon and Soltes, 2015), which suggests that the disclosure mandate on the timely release of material information may not be enough to ensure market participants’ fair access to information. Therefore, in this spirit the SZSE regulation can be viewed as one step further along the RegFD in revealing

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16 In our model, we assume that investor $j$ makes disclosure decisions after her information-acquisition decisions $h_j$ and before observing her private signal (see Figure 1). We confirm that investors’ optimal disclosure strategies regarding their information-acquisition activity are not affected even if they are allowed to make disclosure decisions after they observe their private signals but before their trading decisions.

17 See the investor relations section on the web portal: http://irm.cninfo.com.cn/szse/index_en.html and the third point in the Q&A on investor relations issued by the SZSE (http://www.csrc.gov.cn/pub/hunan/xxfw/tzzsyd/zqztz/201412/t20141222_265331.htm). In addition, the reports classify the meetings into various categories, such as site visits, analyst day meetings, online interactions, roadshows, and remote conference calls.
investors’ information-acquisition activity (not private information) in a timely manner.\footnote{The disclosure mandates in the SZSE provide a unique and valuable setting to observe investors information acquisition during corporate site visits. This setting has been exploited in several recent studies (e.g., Chen et al., 2021; Cheng et al., 2016, 2019; Cicero et al., 2021; Dong et al., 2019; Han et al., 2018).}

In our model, this disclosure mandate can be operationalized as follows: as every site visit needs to be disclosed, the disclosure of a total number $H_j$ of site visits is equivalent to the perfect observability of investor $j$’s information precision $h_j$, that is, $m_j = h_j$. In other words, in this mandatory-disclosure setting the message functions are mandated and fully reveal investors’ information acquisition.

### 4.2 Voluntary Disclosure of Information Acquisition

As common in endogenous signaling games (In and Wright, 2018), multiple equilibria might arise in our voluntary disclosure setting.\footnote{Our game differs from the typical signaling game in that the message sent by the investor conveys information about her endogenous information-acquisition choice. As such, the standard equilibrium refinement approaches such as the intuitive criterion (Cho and Kreps, 1987), the divinity criterion (Banks and Sobel, 1987), the Perfect Sequential Equilibrium (Grossman and Perry, 1986) and the undefeated equilibrium (Mailath et al., 1993) do not apply to our model.} Thus, analysing the equilibria of the voluntary disclosure game requires an assumption on how other agents revise their beliefs about investor $j$’s information acquisition, when receiving an unexpected (off-equilibrium) disclosure. Following the industrial organization literature (e.g., McAfee and Schwartz, 1994; Rey and Vergé, 2004), rather than choosing beliefs, we propose a criterion that acceptable beliefs should satisfy and that allows us to select beliefs.\footnote{Existing criteria are not readily applicable to our setting. For example, Hart and Tirole (1990) and O’Brien and Shaffer (1992) use passive beliefs and under passive beliefs the agents do not revise their beliefs when observing a disclosure different from what they expect in the candidate equilibrium. However, this should not hold in our setting because if investor $j$ makes a evidence-backed disclosure higher than the equilibrium level ($m_j > h^*$), other agents must update their beliefs to at least the disclosure level.} We call these beliefs “weakly-increasing beliefs” and formalize them as follows:\footnote{This definition is reminiscent of the monotonicity assumption commonly seen in the security design literature (e.g., Nachman and Noe, 1994; DeMarzo and Duffie, 1999), which requires that the payment for those who purchase the security is nondecreasing in the available firm cash flows.}

**Definition 1.** Under weakly-increasing beliefs, other agents’ beliefs about investor $j$’s information acquisition are weakly increasing in investor $j$’s disclosure; that is, $\hat{h}_j$ is weakly increasing in $m_j$. 

\[ 
\]
The weakly-increasing beliefs are compelling in our setting because any disclosure made by investor \( j \) is supported by hard evidence. Specifically, once investor \( j \) presents evidence about her information-acquisition activity (e.g., corporate site visits), the other agents must update their beliefs and believe that investor \( j \)’s actual information acquisition is at least as high as the disclosure level (i.e., \( \hat{h}_j \geq m_j \)). As such, when an investor presents more evidence (a higher disclosure level), it is reasonable to assume that other agents revise up their beliefs about the investor’s acquired information and hence the precision of her private information.

4.2.1 Monopolistic Investor (\( J = 1 \))

We start the analysis of voluntary disclosure by considering a monopolist-investor setting. As in Section 3.1, investor’s net expected trading profits \( \pi(h, \hat{h}) \) are given by equation (9). Taking derivative of \( \pi(h, \hat{h}) \) with respect to the belief \( \hat{h} \) yields the following

\[
\frac{\partial \pi(h, \hat{h})}{\partial \hat{h}} = -\frac{h\sigma_u}{4(1+h)\hat{h}^2} \sqrt{\frac{\hat{h}}{1+\hat{h}}} < 0.
\]

This inequality suggests that given any actual level \( h \) of information acquisition, the monopolist investor always wants the market makers’ beliefs \( \hat{h} \) about her signal precision the lowest the possible. Under weakly-increasing beliefs, to sustain such a belief, the investor tends to disclose less about her information acquisition. We specify that \( m = 0 \); that is, the monopolistic investor conceals her information acquisition in equilibrium.\(^{22}\) Closely related to the pricing effect, given the investor’s acquired information, more disclosure would induce market makers to believe that they are faced with a more informed investor and thus set steep pricing schedule to the harm of the investor. As such, nondisclosure can be the optimal strategy for the investor and this voluntary disclosure game boils down to the secret information acquisition. Overall, in the monopoly

\(^{22}\)One example set of belief that can sustain the secret information-acquisition equilibrium is as follows: if \( m \leq h_s \), \( \hat{h} = h_s \) and otherwise \( \hat{h} = \beta m \), where \( \beta \geq 1 \). Under this set of beliefs, all \( m \leq h_s \) can be supported in equilibrium. However, once we consider an infinitesimal disclosure cost, \( m = 0 \) becomes robustly the unique equilibrium message function. Also note that despite the indeterminate belief, allocation wise the equilibrium is unique; that is, the equilibrium amount of information acquisition is uniquely pinned down.
setting, the pricing effect induces not only less information production, but also less disclosure about information-acquisition activity.

**Proposition 3** (Voluntary disclosure – Monopolistic investor). Consider a monopolistic investor who can disclose her information-acquisition activity. There exists a unique perfect Bayesian equilibrium with weakly-increasing beliefs in which the investor engages in secret information acquisition and acquires private information of precision $h_s$, where $h_s$ is the unique solution to equation (10).

### 4.2.2 Oligopolistic Investors ($J \geq 2$)

This section studies the setting with oligopolistic investors ($J \geq 2$). As in Section 3.2, we introduce the interaction among strategic investors and investigate its interplay with the pricing effect. The following proposition extends Proposition 3 to the general case with $J \geq 2$ and characterizes the equilibrium under extreme conditions, and the numerical analysis in Figure 3 demonstrates its generality.

**Proposition 4** (Voluntary disclosure – Oligopolistic investors). Consider oligopolistic investors that can make disclosures about their information acquisition. There exists a unique perfect Bayesian equilibrium with weakly-increasing beliefs. Specifically,

1. When $c/\sigma_u$ is sufficiently high, all investors engage in secret information acquisition and acquire private information of precision $h_s$, where $h_s$ is the unique solution to equation (12).

2. When $c/\sigma_u$ is sufficiently low, all investors engage in overt information acquisition and acquire private information of precision $h_o$, where $h_o$ is the unique solution to equation (13).

Proposition 4 states that the equilibrium in the oligopolistic-investor voluntary disclosure setting crucially hinges on $c/\sigma_u$. Specifically, when $c/\sigma_u$ is sufficiently high, investors do not reveal their information-acquisition activity and the equilibrium in the voluntary disclosure game is the same as that with secret information acquisition. By contrast, when $c/\sigma_u$ is sufficiently low, all investors fully reveal their information-acquisition activity and the equilibrium becomes the same as in the classic models where information acquisition is overt.
To ease exposition, we fix the noise trading $\sigma_u$ and focus on the information-acquisition cost $c$. As in Section 3.2, we can express investor $j$’s expected net trading profit $\pi_j(h_j; h^*)$ as in equation (14). When the information-acquisition cost is sufficiently high, the derivative of $\pi_j(h_j, \hat{h}_j; h^*)$ with respect to the belief $\hat{h}_j$ can be shown to be negative, regardless of the level of investor $j$’s actual information acquisition. That is,

$$\lim_{c \to \infty} \frac{\partial \pi_j(h_j, \hat{h}_j; h^*)}{\partial \hat{h}_j} < 0,$$

which suggests that investor $j$ favors the lowest belief $\hat{h}_j$ about her information acquisition the possible. This is because with high cost, competition between the investors is mild as neither of them produces precise information. Now, the interaction between an investor and market makers becomes more of a concern and the equilibrium resembles that in the monopolistic-investor setting. Since an investor is very concerned that once the market makers know that she has acquired a great deal of information, market makers will dramatically adjust the pricing schedule, the investor attempts to minimize the disclosure about her information-acquisition activity. Therefore, despite the option to disclose their information acquisition, the equilibrium amount of acquired information in this voluntary disclosure setting remains the same as that with secret information acquisition, i.e., $h^* = h_s$.

By contrast, when the information-acquisition cost is sufficiently low (i.e., sufficiently low $c/\sigma_u$), investors would like to fully reveal her information acquisition. Mathematically, regardless of $h_j$, the derivative of $\pi_j(h_j, \hat{h}_j; h^*)$ with respect to $\hat{h}_j$ becomes positive,

$$\lim_{c \to 0} \frac{\partial \pi_j(h_j, \hat{h}_j; h^*)}{\partial \hat{h}_j} > 0.$$

With low information-acquisition costs, investors can acquire very accurate information so that the competition between them is very fierce. In this case, given their information acquisition, each investor is very keen to build up a competitive advantage through their disclosure strategy. We can specify that $m_j = h_j$; that is, the message function fully reveals investors’ information-
acquisition activity. Different from the pricing effect, the competition effect works through encouraging an investor to disclose her information acquisition: knowing that she has acquired a great deal of information can force her rival investors to scale back in their informed trading. As such, in equilibrium the investors acquire the same amount of information as in the classic overt-information-acquisition models, i.e., $h^* = h_o$.

Overall, the pricing effect prevails when $c/\sigma_u$ is high, leading to the secret case, whereas the competition effect dominates when $c/\sigma_u$ is low and results in the overt case. The disclosure strategy in turn affects investors’ information acquisition. We further confirm the intuition gleaned from Proposition 4 in the following numerical analysis. Figure 3 plots the equilibrium information-acquisition in a duopolistic-investor setting ($J = 2$) under weakly-increasing beliefs. Specifically, the shaded area indicates whether the equilibrium features secret or overt information acquisition; the solid line plots the equilibrium information acquisition and the dashed line plots the hypothetical information acquisition if investors adopt an otherwise disclosure strategy so that the observability changes. Consistent with Proposition 4, Figure 3 shows that when $c/\sigma_u$ is lower than a threshold, investors engage in overt information acquisition, whereas when $c/\sigma_u$ is higher than that threshold, investors engage in secret information acquisition.

Finally, going back to our motivating examples in which funds may voluntarily disclose their corporate site visits, our model offers the following insights. First, the common prior that funds are very secretive about their research activities works in our model when $c/\sigma_u$ is high. That is, when the cost of conducting site visits is very high (high $c$) or the firm is mainly traded by institutional investors (low $\sigma_u$), funds indeed go great lengths to hide their research activities, thereby avoiding the steep pricing schedule from market makers. However, when $c/\sigma_u$ is low, funds adopt an opposite disclosure strategy and fully disclose their research activities. In this way, the disclosed funds gain a competitive advantage by forcing the competing investors to trade less aggressively. This could potentially explain some funds’ voluntary disclosure of corporate site visits.

As in footnote 22, we can construct one example of belief that sustains the overt information-acquisition equilibrium as follows: $h_j = h_j$. Under this belief, the message function is determined as $m_j = h_j$. Again, despite many potential beliefs, the equilibrium is unique allocation wise.
Figure 3: Information acquisition under voluntary disclosure

4.3 Mandatory Disclosure of Information Acquisition

We now use our analysis to shed light on the effectiveness of the mandatory disclosure. If investors are willing to disclose information-acquisition activity on a voluntary basis, then mandatory disclosure is redundant. By contrast, if investors withhold their information acquisition, then mandatory disclosure can significantly affect the equilibrium outcomes and thus mandatory-disclosure regulations — such as the Shenzhen Stock Exchange’s disclosure mandate about site visits — are effective. The following proposition summarizes the results.

**Corollary 1** (Effectiveness of mandatory disclosure). Let $J \geq 2$.

1. When $c/\sigma_u$ is sufficiently low, mandatory disclosure of information-acquisition activities is ineffective.

2. When $c/\sigma_u$ is sufficiently high, mandatory disclosure of information-acquisition activities is effective. In addition, mandatory disclosure causes investors to acquire less information, improves investors’ payoff and market liquidity, but worsens market efficiency.
First, as shown in Proposition 4 and Figure 3, when $c/\sigma_u$ is low, investors voluntarily disclose their information-acquisition activity. Therefore, the regulations that mandate the disclosure of information acquisition is redundant, consistent with Part (1) of Corollary 1.

Second, Part (2) of Corollary 1 shows that when $c/\sigma_u$ is high, as investors would have chosen to withhold their information-acquisition activity if not mandated, the disclosure mandate will be effective in forcing investors’ otherwise secretive information acquisition to be overt. Moreover, investors tend to acquire less information after the mandate. This is because as argued above, for high $c/\sigma_u$ investors acquire so little information that the head-to-head competition between them is mild, thereby making the interaction with market makers the dominant concern for the investors, namely, the pricing effect prevails. Once information acquisition becomes observable, market makers will respond to investors’ more information acquisition by aggressively adjusting the pricing schedule. Therefore, investors tend to scale back in their information acquisition after the disclosure mandate. Moreover, because market makers are faced with less informed investors in equilibrium, market liquidity improves due to less severe adverse selection. Meanwhile, we caution a dark side of mandatory disclosure; mandatory disclosure can lower price efficiency. This consequence may be undesirable if price efficiency has real effects (e.g., company managers may learn information from asset prices to guide their real decisions; see the review article by Bond, Edmans, and Goldstein (2012)).

Finally, Part (2) of Corollary 1 also suggests that when $c/\sigma_u$ is sufficiently high, mandatory disclosure can weakly Pareto-improve the well-being of all financial market participants. Specifically, both rational investors and noise traders are better off, while market makers always make zero profits. Here, we use the expected loss $\lambda \sigma_u^2$ to negatively measure noise traders’ welfare to capture the idea that they are better off if they can realize their hedging or liquidity needs at a lower expected opportunity cost. This Pareto-improvement result might appear surprising, as one would expect that the financial market is a zero-sum game. Indeed, it is, albeit post-information acquisition (i.e., taking the information-acquisition cost as sunk). In our setting, investors are better off precisely because they save the information-acquisition cost under mandatory-disclosure
regulations.

5 Downloading at the SEC EDGAR Website

In this section, we apply our model to study investors’ downloading requests at the SEC EDGAR website. As in Section 4, we first introduce the institutional details and then our analysis.

5.1 Institutional Background

Investors’ otherwise secretive information-acquisition activity can also become more observable due to the technology advancement. This is particularly prevalent in the internet age in which market participants inevitably leave behind a trail of digital footprints. One notable example is investors’ potentially observable footprints at the SEC EDGAR system. In 1993 the SEC implemented the EDGAR system for the automated collection, validation, and forwarding of submissions by companies, which has constituted a very important source of information for investors given its authenticity, completeness, and timeliness.\(^\text{24}\) Around 2015, the SEC has begun to publicly provide the amount of traffic on the EDGAR data filings, including the visitor’s Internet Protocol (IP) address, and updated on a quarterly basis.\(^\text{25}\) To protect the privacy of the visitors, the SEC partially masks each IP address by only providing the first three of the four IP octets. However, with the data analysis technology, investors’ downloading requests at the EDGAR system can still become observable with some probability. For instance, the IP address “117.91.231.129” will be reported as “117.91.231.jff” in the SEC server log files, where the last octet “129” is randomly

\(^\text{24}\)While the EDGAR is by no means the only information source, sophisticated investors do benefit from acquiring information from it. Indeed, digital footprints at the EDGAR system have been frequently identified to be left by various market participants such as mutual funds and hedge funds (e.g., Cao et al., 2019; Chen et al., 2020; Crane et al., 2020; Iliev et al., 2020) and sell-side analysts (e.g. Gibbons et al., 2020). Moreover, compared with other information sources such as commercial data aggregators, the EDGAR contains more information that is not easy to be streamlined (e.g., off-balance-sheet items, footnotes; see Loughran and McDonald (2017)) on a more timely basis (Li et al., 2011).

\(^\text{25}\)The data contains information on the visitor’s Internet Protocol (IP) address, timestamp, the Central Index Key (CIK) used in SEC’s computer systems to identify filers, and accession numbers that uniquely match a particular company’s specific SEC filings; see https://www.sec.gov/dera/data/edgar-log-file-data-set.html for more details. The specific time for the availability of the log files is unknown, but according to Lee et al. (2015), one can infer that it should be around 2015.
encrypted as “[^f]” by the SEC. If investor $j$ happens to purchase entire blocks of IP addresses, e.g., “117.91.231.0”, “117.91.231.1”, ..., “117.91.231.255”, or own a big part of the block, then based on the first three octets, one can determine her identity and further analyze her information-acquisition behavior at the EDGAR system. Otherwise the identity of the downloading investors remains unobservable. Probably due to the decryption of the IP addresses, the SEC stopped to update the EDGAR log files and the data ends only in June 30, 2017.

To capture such imperfect observability, following the literature (e.g., Bagwell, 1995; Gavazza and Lizzieri, 2009), we suppose that at $t = 0$ after investor $j$’s information-acquisition decisions, other agents – including market makers and all the other investors – can perfectly observe investor $j$’s information-acquisition activity (and thus infer the precision of her private information) with a probability of $q \in [0, 1]$, and with the remaining probability $1 - q$ investor $j$’s information acquisition remains unobservable. That is, $m_j = h_j$ with probability $q$ whereas $m_j = 0$ with the remaining probability. We further assume that it is common knowledge among all agents whether investor $j$’s information-acquisition activity is observable or not. Note that when $q = 0$ this setting boils down to the secret information acquisition, whereas when $q = 1$ this becomes the overt information acquisition. In the EDGAR setting, the period in which the SEC regularly updates the EDGAR log files correspond to $q > 0$, whereas other periods feature with $q = 0$.

To further connect our model to this EDGAR setting, we note that the EDGAR downloading requests can approximate a real-time, ex-ante, signal about the precision of investors’ ultimate trading signal. First, the downloading requests (as a proxy for information acquisition) have been updated on a quarterly basis with a six-month delay (Chen et al., 2020, p.122) and can proceed the actual investor trading. For example, after checking insider-trading filings of the firm, institutions trade up to 150 trading days post the event, with around 50% of the trade occurring between 31 trading days to 150 trading days (Chen et al., 2020, Figures 2 and 3). This trading lag represents the deliberation required to develop long-term tradable signals from the EDGAR raw filings.\textsuperscript{26} Second, while for investors who collect information from the EDGAR system, the real

\textsuperscript{26}Some anecdotal evidence suggests the use of the EDGAR log files by hedge funds (Crane et al., 2020), which indicates the usefulness of analyzing other investors’ EDGAR downloading requests in informing one’s own trading.
costs are in analysing the files, rather than in downloading them, the observable downloading requests are found to be positively associated with investors’ ultimate trading signal in a meaningful way. For instance, Chen et al. (2020) find that the average tracked stock that an institution buys generates annualized alphas of over 12% relative to the purchase of an average non-tracked stock. Crane et al. (2020) show that hedge funds acquiring information from the EDGAR system subsequently exhibit 1.5% higher annualized returns than non-acquirers. While the observable download requests and investors’ performance may not be a one-to-one mapping, these pieces of evidence show that the former can be a good proxy for the latter.

5.2 Equilibrium Characterization under Imperfect Observability

We now characterize the equilibrium under imperfect observability. We focus on the symmetric Perfect Bayesian equilibrium and the following lemma characterizes the equilibrium information acquisition $h^*$, which is a “weighted-average” between the secret case where information acquisition is unobservable and the overt case where it becomes perfectly observable.

Lemma 4 (Imperfect observability of information acquisition). Suppose that investors’ information-acquisition activity is imperfectly observable. There exists a unique equilibrium in which investors acquire private information of precision $h^*$, where $h^*$ is the unique solution to the following equation:

$$
\frac{c}{\sigma_u} = q \cdot \frac{(6J^2 - J - 3)(h^*)^2 + 2(2J^2 + 5J - 4)h^* + 8J - 4}{2J(2 + h^*)(2 + h^* + Jh^*)^2\sqrt{Jh^*(1 + h^*)}} + (1 - q) \cdot \frac{1}{\sqrt{Jh^*(1 + h^*)(2 + h^* + Jh^*)}}.
$$

(17)

After characterizing the equilibrium in the economy with imperfectly observable information acquisition, one natural question is how an increasing probability of being observed affects investors’ information-acquisition behavior and market quality. The following proposition addresses this question. Figure 4 graphically illustrates this proposition and demonstrates its robustness for general values of $c/\sigma_u$. 

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Proposition 5 (Implications of increasing observability). When $c/\sigma_u$ is sufficiently low (resp. high), as $q$ increases, investors acquire more (resp. less) precise information, the market becomes more (resp. less) informationally efficient, and if $J \leq 3$, the market becomes less (resp. more) liquid.

Figure 4: Increasing observability of information acquisition

Proposition 5 is closely related to Proposition 2 since the imperfect observability case is a combination of secret and overt information acquisition. As evident in equation (15), when the observability probability $q$ increases, the economy shifts away from secret information acquisition ($q = 0$) but closer to overt information acquisition ($q = 1$), and the pricing and competition effects start to kick in. The dominant effect will determine the equilibrium amount of information production and which effect dominates hinges on $c/\sigma_u$.

When $c/\sigma_u$ is low, investors end up acquiring very accurate information on date 0 so that their competition in trading is fierce and each investor is very keen to build up a competitive advantage through information acquisition. In this case, the competition effect prevails, and as investors’ information acquisition becomes more likely to be observed, they tend to acquire more informa-
tion. By contrast, when $c/\sigma_u$ is high, investors couldn’t produce very precise information, which limits the competition between investors and diminishes the competition effect. Instead, the pricing effect dominates, i.e., the interaction with market makers becomes the dominant concern for investors. If an investor is observed to acquire a great deal of information, market makers will dramatically adjust the pricing schedule. Therefore, investors tend to acquire less information if the probability of being observed increases. The market-variable implications then naturally follow these information-production results.

6 Conclusion

Trading and information acquisition are the two major activities that investors undertake in financial markets. While trading is often viewed as unobservable, information acquisition is typically assumed to be observable in the literature. This observability assumption about information acquisition is, however, rather strong in lots of scenarios. We make theoretical contributions by considering a parsimonious framework that allows for secret information acquisition in financial markets. Under secret information acquisition, investors’ information-acquisition activities are not observable, but market participants hold correct beliefs about the equilibrium amount of these activities.

We further extend and apply our framework to understand two representative forms of investors’ information-acquisition activities – corporate site visits and download requests on the SEC EDGAR system, which sheds light on the implications of regulations and technologies for financial markets. We find that how the information-acquisition observability issues affect investors’ disclosure and information-acquisition incentives depends on the interplay between the pricing effect (the interaction between investors and market makers) and the competition effect (the interaction among investors). This information-acquisition result has further equilibrium consequences for market liquidity and market efficiency.
References


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Appendix: Proofs

Proof of Lemma 1 and Proposition 1

See the main text for the majority of the proof of Proposition 1. Here we only show that there exists a unique equilibrium in both the secret and overt cases. Let \( g(h) = h(1 + h)^3 \). Since \( g(0) = 0 \) and \( g(\infty) = \infty \), by intermediate value theorem, there exists a positive root to \( g(h_s) = \sigma_u^2/(4c^2) \) and \( g(h_o) = \sigma_u^2/(16c^2) \). Further, since \( g'(h) > 0 \) for \( h > 0 \), the root to the respective equations must be unique.

For Proposition 1, since \( h = h_o \) is the solution to \( g(h) = \sigma_u^2/(16c^2) \), \( h = h_s \) is the solution to \( g(h) = \sigma_u^2/(4c^2) \), and \( \sigma_u^2/(16c^2) < \sigma_u^2/(4c^2) \), we have \( h_o < h_s \). Based on the discussion in the main text, \( \lambda_s > \lambda_o \) and \( m_s > m_o \) immediately follow.

Proof of Lemma 2

The equilibrium computation parallels Section 3. Now we need to take into account the interactions among investors. In a symmetric linear equilibrium, all investors choose the same precision level \( h^* \), where \( h^* = h_s \) under secret information acquisition and \( h^* = h_o \) under overt information acquisition. Let other agents’ belief about investor \( j \)’s signal precision be \( \hat{h}_j \).

We first consider investor \( k \)’s optimal trading strategy, where \( k \neq j \) and \( k \in \{1, ..., J\} \). Denote investor \( j \)’s trading strategy \( \alpha_j \), investor \( k \)’s trading strategy \( \alpha_k \) and all other investors’ trading strategy \( \alpha \). Let market makers’ pricing rule be \( \lambda \). For investor \( k \), given her information-acquisition decision \( h_k = h^* \), her optimal trading strategy is characterized as follows:

\[
\max_{x_k} x_k \left( \frac{h^*}{1 + h^* \bar{y}_k} - \lambda \left( x_k + \alpha_j \frac{h^*}{1 + h^* \bar{y}_k} (J - 2) \alpha \frac{h^*}{1 + h^* \bar{y}_k} \right) \right).
\]

Solving it yields the optimal trading strategy for investor \( k \): \( x_k = \alpha_k \bar{y}_k \), where

\[
\alpha_k = \frac{h^* (1 - (\alpha_j + (J - 2)\alpha)\lambda)}{2(1 + h^*)\lambda}.
\]
Imposing $\alpha_k = \alpha$ yields

$$\alpha = \frac{h^*(1 - \alpha_j \lambda)}{(2 + h^*J)\lambda}. \quad (A1)$$

Similarly, for investor $j$, her trading problem is as follows:

$$\max_{x_j} x_j \left( \frac{h_j}{1 + h_j} \tilde{y}_j - \left( x_j + (J - 1)\alpha \frac{h_j}{1 + h_j} \tilde{y}_j \right) \lambda \right). \quad (A2)$$

Solving it yields the optimal trading strategy for investor $j$: $x_j = \alpha_j \tilde{y}_j$, where

$$\alpha_j = \frac{h_j (1 - (J - 1)\alpha \lambda)}{2(1 + h_j)\lambda}. \quad (A3)$$

Combining equations (A1) and (A3) and replacing $h_j = \hat{h}_j$, we obtain that based on the beliefs $\hat{h}_j$, other investors’ trading strategy and all other agents’ conjecture about investor $j$’s trading strategy are respectively as follows:

$$\alpha = \frac{(2 + \hat{h}_j)h^*}{\left(4 + 2Jh^* + (4 + h^* + h^*J)\hat{h}_j \right)\lambda}, \quad (A4)$$

$$\hat{\alpha}_j = \frac{(2 + h^*)\hat{h}_j}{\left(4 + 2Jh^* + (4 + h^* + h^*J)\hat{h}_j \right)\lambda}. \quad (A5)$$

For market makers, according to (5), their pricing rule is $\tilde{p} = \lambda \tilde{\omega}$, where

$$\lambda = \frac{(J - 1)\alpha + \hat{\alpha}_j}{((J - 1)\alpha + \hat{\alpha}_j)^2 + (J - 1)\frac{\alpha^2}{h^*} + \frac{\sigma^2}{h_j} + \sigma^2_u}.$$
Replacing $\alpha$ in (A4) and $\hat{\alpha}_j$ in (A5) and solving for $\lambda$ yields

$$\lambda = \frac{\sqrt{\hat{\alpha}_j^2(4 + (h^*)^2J + h^*(J + 3)) + \hat{\alpha}_j(4 + 4h^*J + (h^*)^2(4J - 3)) + 4h^*(1 + h^*)(J - 1)}}{\left(4 + 2h^*J + \hat{\alpha}_j(4 + h^* + h^*J)\right)\sigma_u}.$$  

(A6)

Next, replacing $\tilde{x}_j = \alpha_j\tilde{y}_j$ into (A2), where $\alpha_j$ is given by (A3), inserting (A4) and (A6) into (A2), taking expectation, and subtracting the information-acquisition cost, we obtain investor $j$’s expected net trading profit as follows:

$$\pi_j(h_j, \hat{\alpha}_j; h^*) = -c \cdot h_j$$

(A7)

$$+ \frac{h_j(1 + \hat{\alpha}_j)^2(2 + h^*)^2\sigma_u}{\left(1 + h_j\right)(4 + 2Jh^* + \hat{\alpha}_j(4 + h^* + h^*J))\sqrt{4h^*(1 + h^*)(J - 1) + \hat{\alpha}_j(4 - 3(h^*)^2 + 4Jh^*(1 + h^*)) + \hat{\alpha}_j^2(4 + h^*(3 + J + Jh^*))}} \sigma_u.$$  

Under secret information acquisition, replacing $h^* = h_s$ and $\hat{\alpha}_j = h_s$ in equation (A7) yields

$$\pi_j(h_j, h_s; h_s) = \frac{h_j(1 + h_s)^2\sigma_u}{\left(1 + h_j\right)(2 + h_s + h_sJ)\sqrt{h_s(1 + h_s)J}} - ch_j.$$  

(A8)

Maximizing the net profit with respect to $h_j$ and imposing $h_j = h_s$ yields the equilibrium information acquisition $h_s$, as characterized by equation (12).

Under overt information acquisition, replacing $h^* = h_o$ and $\hat{\alpha}_j = h_j$ in equation (A7) yields:

$$\pi_j(h_j, h_j; h_o) = -c \cdot h_j$$

(A9)

$$+ \frac{h_j(1 + h_j)^2\sigma_u}{\left(1 + h_j\right)(4 + 2Jh_o + h_j(4 + h_o + h_oJ))\sqrt{4h_o(1 + h_o)(J - 1) + h_j(4 - 3h_o^2 + 4Jh_o(1 + h_o)) + h_j^2(4 + h_o(3 + J + Jh_o))}}.$$  

Similarly, maximizing the net profit with respect to $h_j$ and imposing $h_j = h_o$ yields the equilib-
rium information acquisition $h_o$, as characterized by equation (13).

Next, in equilibrium $\lambda$ and $\alpha$ are connected to $h$ through equations (A4) and (A6), respectively. Setting $h_j = h_o = h$ in equations (A4) and (A6) yields

$$\lambda = \frac{\sqrt{h(1 + h)J}}{\sigma_u (2 + (J + 1)h)} \quad \text{and} \quad \alpha = \sigma_u \sqrt{\frac{h}{(1 + h)J}},$$

respectively. With the equilibrium information precision $h_s$ and $h_o$ we can derive $\alpha_s, \lambda_s, \alpha_o$ and $\lambda_o$ as specified in Proposition 2.

Finally, we confirm that the equilibrium information precision $h_s$ and $h_o$ are unique. Denote the left-hand-side (LHS) of equation (12) as $f_s(h)$. It is easy to show that $f_s(0) = 0$ and $f_s(+\infty) = +\infty$. Thus, by the intermediate value theorem, there exists a positive root to equation (12). Further, since $f_s(h)' > 0$, the root to equation (12) must be unique.

Similarly, denote the LHS of equation (13) as $f_o(h)$. As $f_o(0) = 0$ and $f_o(+\infty) = +\infty$, by the intermediate value theorem, there exists a positive root to equation (13). In addition, $f_o(h)$ is monotonically increasing in $h$ because

$$f_o'(h) = \frac{4h^5(6J^3 + 5J^2 - 4J - 3)}{(6J^2 - J - 3)h^2 + (4J^2 + 10J - 8)h + 8J - 4)^3} > 0.$$

The inequality follows because we can prove the coefficients of all polynomial terms about $h$ in the big bracket to be positive for $J \geq 2$. For instance, we look at the coefficient of the term $h^5$ and denote it as $f_{o5}(J)$, that is, $f_{o5}(J) = 4(6J^3 + 5J^2 - 4J - 3)$. Because $f_{o5}'(J) = 8(9J^2 + 5J - 2) > 0$ for all $J \geq 2$, we must have $f_{o5}(J) \geq f_{o5}(2) = 44 > 0$ for all $J \geq 2$. Similarly we can check all coefficients and $f_o'(h) > 0$ immediately follows. Therefore, the real root to equation (13) is
unique.

**Proof of Lemma 3**

Investor $j$’s expected profits are given by equation (14) where $\alpha$ and $\lambda$ are given by equations (A4) and (A6), respectively. Taking derivative of $\pi_j(h_j, \hat{h}_j; h^*)$ yields equation (15). Further,

\[
\frac{\partial \pi_j}{\partial h_j} = \frac{(1 - (J - 1)\alpha \lambda)^2}{4\lambda(1 + h_j)^2} - c, \\
\frac{\partial \pi_j}{\partial \lambda} = -\frac{(1 - (J - 1)^2\alpha^2 \lambda^2) h_j}{4\alpha^2(1 + h_j)} < 0, \\
\frac{\partial \pi_j}{\partial \alpha} = -\frac{(J - 1)(1 - (J - 1)\alpha \lambda) h_j}{2(1 + h_j)} < 0,
\]

Under secret information, it is obvious that $\frac{\partial \lambda}{\partial h_j} = 0$ and $\frac{\partial \alpha}{\partial h_j} = 0$. Next we discuss the case with overt information acquisition. It can be shown that

\[
\frac{\partial \lambda}{\partial h_j} = \frac{(2 + h_o)\left( (2 + h_o)^2 (J - 3)h_o^2 + 2(J + 3)h_o + 8 \right) h_j + (2(3J - 4)h_o^2 + 4h_o(J - 3) - 8)}{2\lambda \sigma_u^2 (h_j(h_oJ + h_o + 4) + 2h_oJ + 4)^3}, \\
\frac{\partial \alpha}{\partial h_j} = -\frac{\alpha^3(2 + h_o)^2(2 + 3h_j)}{2h^2\sigma_u^2(2 + h_j)^3} < 0.
\]

Note that $\frac{\partial \pi_j}{\partial h_j} < 0$ because based on equation (A3) we know that $1 - (J - 1)\alpha \lambda > 0$. At $h_j = h_o$, we obtain the following:

\[
\frac{\partial \lambda}{\partial h_j} \bigg|_{h_j=h_o} = \frac{(2 + h_o)(2 - (J - 3)h_o)}{2\sqrt{h_o(h_o + 1)(h_o + 2)J(h_oJ + h_o + 2)^2\sigma_u}}.
\]

When $J \leq 3$, it is obvious that $\frac{\partial \lambda}{\partial h_j} \big|_{h_j=h_o} < 0$ so that the pricing effect is positive. Further, as $J \rightarrow +\infty$, using equation (13) in Proposition 2, we know that $h_o \rightarrow 0$. We can show equation (13) approaches the following limit:

\[
\frac{4h_o(2 + h_o J)^4}{(4J^2h_o + 8J)^2} = \frac{\sigma_u^2}{4J^3c^2}.
\]
which can be simplified to the following: \( Jh_o(Jh_o + 2)^2 = \sigma_u^2/c^2 \). Note that \( \frac{\partial \lambda}{\partial h_j} \mid_{h_j = h_o} < 0 \) only if \( 2 - (J - 3)h_o < 0 \). This is equivalent to \( h_oJ > 2 \) when \( J \to \infty \), which suggests that \( c/\sigma_u > \frac{1}{4\sqrt{2}} \).

**Proof of Proposition 2**

We compare the equilibrium information acquisition across the overt and secret information acquisition. First, we show that when \( c \to 0 \), \( h_o > h_s \). Note that when \( c \to 0 \), \( h_o \to \infty \) and \( h_s \to \infty \). Equations (13) and (12) can be simplified as follows:

\[
\frac{c}{\sigma_u} = \frac{(6J^2 - J - 3)h_o^2 + o(h_o^2)}{2J \sqrt{J(J + 1)^2} h_s^2 + o(h_s^2)},
\]

\[
\frac{c}{\sigma_u} = \frac{1}{\sqrt{J(J + 1)} h_s^2 + o(h_s^2)}.
\]

It follows that

\[
h_o = \frac{\sigma_u}{c} \cdot \frac{6J^2 - J - 3}{2J \sqrt{J(J + 1)^2}} + o \left( \frac{1}{c} \right),
\]

\[
h_s = \frac{\sigma_u}{c} \cdot \frac{1}{\sqrt{J(J + 1)}} + o \left( \frac{1}{c} \right),
\]

where

\[
\frac{6J^2 - J - 3}{2J \sqrt{J(J + 1)^2}} > \frac{1}{\sqrt{J(J + 1)}}
\]

for any \( J \geq 2 \). Therefore, when \( c \to 0 \), \( h_o > h_s \).

Second, consider the case where \( c \to \infty \), which leads to \( h_o \to 0 \) and \( h_s \to 0 \). Again, equations (13) and (12) can be simplified as follows:

\[
\frac{c}{\sigma_u} = \frac{8J - 4 + o(1)}{16J \sqrt{h_o J} + o(\sqrt{h_o})},
\]

\[
\frac{c}{\sigma_u} = \frac{1}{2\sqrt{h_o J} + o(\sqrt{h_s})}.
\]
It follows that

\[
\sqrt{h_oJ} = \frac{8J - 4}{16J} \cdot \frac{\sigma_u}{c} + o\left(\frac{1}{c}\right),
\]

(A10)

\[
\sqrt{h_sJ} = \frac{\sigma_u}{c} + o\left(\frac{1}{c}\right).
\]

(A11)

Using the fact that \(\frac{\sigma_u}{c} + o\left(\frac{1}{c}\right) > \frac{8J - 4}{16J} \cdot \frac{\sigma_u}{c} + o\left(\frac{1}{c}\right)\), we can show that \(h_s > h_o\). Therefore, we prove that when \(c \to \infty\), \(h_s > h_o\).

Next, consider the function \(\Delta(c) = h_o - h_s\). We would like to find out the roots of \(\Delta(c) = 0\). Straightforward calculation suggests that \(\Delta(c) = 0\) holds if and only if \(h_o = h_s = \bar{h}\), where \(\bar{h}\) is given in Proposition 2. And this corresponds to a single solution \(c = \bar{c}\sigma_u\), where \(\bar{c}\) is given in Proposition 2. Using the intermediate value theorem, we prove that when \(c/\sigma_u > \bar{c}(c/\sigma_u < \bar{c})\), \(h_s > h_o(h_s < h_o)\).

The remainder of the proof about the comparison of market efficiency and market liquidity follows immediately from the discussion in the main text.

**Proof of Proposition 3**

Following the same derivation as in Section 3.1.1, the monopolistic investor’s expected net trading profit is given by (9). Taking derivative of \(\pi(h, \hat{h})\) with respect to \(\hat{h}\) yields

\[
\frac{\partial\Pi(h, \hat{h})}{\partial \hat{h}} = -\frac{h\sigma_u}{4(1+h)h^2} \sqrt{\frac{h}{1+h}} < 0.
\]

Under weakly increasing beliefs, this inequality suggests that regardless of the monopolistic investor’s actual information acquisition, the investor tends to disclose the lowest possible level of signal precision. We specify that \(m = 0\). As explained in footnote 22, there are two reasons to justify \(m = 0\). First, we do not consider disclosure cost in our model, but any infinitesimal disclosure cost can make \(m = 0\) the unique disclosure strategy adopted by the monopolistic investor. Second, for a set of weakly-increasing beliefs that can sustain the equilibrium, \(m = 0\) should remain as a robustly optimal disclosure strategy. Moreover, as will be shown shortly, under weakly increasing beliefs, allocation wise the equilibrium information acquisition is unique.

Next we solve for the investor’s optimal information-acquisition strategy. Since the monop-
olistic investor withholds her information acquisition, market makers only hold beliefs that the investor acquires the equilibrium level $h_s$ of information. Setting $\hat{h} = h_s$ in (9), the monopolistic investor’s expected net trading profit becomes $\pi(h, h_s) = \frac{h_s\sigma_u\sqrt{1+h_s}}{2(1+h)\sqrt{h_s}} - c h$. Maximizing the profit with respect to $h$ and then setting $h = h_s$ yields the optimal precision $h_s$, which is the unique solution to the equation (10). We also confirm the SOC: $\frac{\partial^2 \pi}{\partial h^2} = -\frac{\sigma_u}{(1+h)^3\sqrt{1+h_s}} < 0$.

Overall, given the existence of an equilibrium, a perfect Bayesian equilibrium with weakly-increasing beliefs must be the one in which the monopolistic investor engages in secret information acquisition and the equilibrium information precision is $h_s$.

Next, we specify one example of beliefs that can sustain the above characterized equilibrium. Specifically, the market makers’ beliefs about the investor’s information acquisition are as follows:

$$\hat{h} = \begin{cases} h_s & \text{if } m \leq h_s, \\ m & \text{otherwise.} \end{cases} \quad (A12)$$

It is easy to verify that this belief is weakly increasing in the investor’s disclosure. Here is the intuition of this belief. Recall that in the conjectured secret equilibrium, the monopolistic investor would like to withhold her information acquisition due to the pricing effect. When observing an off-equilibrium disclosure that is lower than the equilibrium level, market makers naturally believe that the investor understates her information acquisition, thereby sticking to the beliefs with the equilibrium level of information acquisition. However, when they observe an off-equilibrium disclosure (backed by hard evidence) that is higher than the equilibrium level, market makers must update their beliefs upwards ($\hat{h} \geq m$). Meanwhile, as they well understand that the investor tends to lower their beliefs to avoid the averse pricing effect, they believe that the investor makes the lowest possible disclosure and thus $\hat{h} = m$. We then verify that this set of beliefs can
indeed sustain the above equilibrium \( h_s \). Inserting (A12) into (9) we obtain that

\[
\pi(h, m) = \begin{cases} 
\frac{h \sigma_u \sqrt{1 + h}}{2(1 + h) \sqrt{h_s}} - c \cdot h & \text{if } m \leq h_s, \\
\frac{h \sigma_u \sqrt{1 + m}}{2(1 + h) \sqrt{m}} - c \cdot h & \text{otherwise}.
\end{cases}
\]

Given \( h \), maximizing the net profit yields the optimal disclosure \( m \leq h_s \), that is, the investor tends to understate her information acquisition than the equilibrium level. Then, following the same procedure as before, we can confirm that given the disclosure strategy, the investor will not deviate from \( h_s \) in information acquisition.

**Proof of Proposition 4**

We will show that if there exists a perfect Bayesian equilibrium with weakly-increasing beliefs, it must be unique. Following the same deviation as in the proof of Lemma 2, investor \( j \)'s expected net trading profit is given by (A7). We next discuss the two cases \( c \to \infty \) and \( c \to 0 \) separately.

**Case 1. Sufficiently high \( c \).** We conjecture that when \( c \) is sufficiently high, in equilibrium investors engage in secret information acquisition; that is, \( h^* = h_s \). Setting \( h^* = h_s \) in \( \pi_j(h_j, \hat{h}_j; h^*) \) as given by (A7), taking derivative of \( \pi_j(h_j, \hat{h}_j; h^*) \) with respect to the belief \( \hat{h}_j \), and imposing \( h_s \to 0 \) we obtain that

\[
\lim_{h_s \to 0} \frac{\partial \pi_j(h_j, \hat{h}_j; h_s)}{\partial \hat{h}_j} = -\frac{h_j(1 + \hat{h}_j)\sigma_u}{4(1 + h_j)(1 + \hat{h}_j)^{3/2}} < 0. \quad (A13)
\]

Further, based on (12), we know that \( \lim_{c \to \infty} h_s = 0 \). Coupled with (A13), we show that

\[
\lim_{c \to \infty} \frac{\partial \pi_j(h_j, \hat{h}_j; h_s)}{\partial \hat{h}_j} < 0. \quad (A14)
\]

That is, when \( c \) is sufficiently high, the lower the other agents’ belief about investor \( j \)'s signal precision, the higher is investor \( j \)'s profit. Therefore, regardless of her actual information ac-
quisition, when \( c \) is sufficiently high, investor \( j \) always wants the other agents’ belief about her signal precision to be as low as possible. Investor \( j \) can use her disclosure strategy to affect other agents’ beliefs. Since we focus on the weakly-increasing belief where \( \hat{h}_j \) is weakly increasing in investor \( j \)’s message \( m_j \), to achieve the lowest possible belief, investor \( j \) can set \( m_j = 0 \), that is, investor \( j \) withholds her information acquisition (we can justify that \( m_j = 0 \) based on the same reasons mentioned in the proof of Proposition 3).

Next, having shown that investor \( j \)’s \( m_j = 0 \) is sustained regardless of \( h_j \), we will prove that given this message function, investor \( j \) will not deviate from the equilibrium information-acquisition strategy \( h_s \). Since investor \( j \) does not reveal her signal precision, even if she deviates from the equilibrium information-acquisition strategy, other agents (other investors and market makers) still believe that her signal precision is maintained at \( h_s \), i.e., \( \hat{h}_j = h_s \). Replacing \( \hat{h}_j = h_s \) in (A7) we can compute investor \( j \)’s net trading profit as given by (A8). Taking derivative of the net trading profit with respect to \( h_j \) and imposing \( h_j = h_s \) yields the following:

\[
\frac{\partial \pi(h_j, h_s; h_s)}{\partial h_j} |_{h_j = h_s} = \frac{\sigma_u}{(2 + h_s + h_sJ)\sqrt{h_s(1 + h_s)J}} - c = 0,
\]

where the second equality follows equation (12). We also confirm that the second-order derivative of \( \pi(h_j, h_j; h_s) \) with respect to \( h_j \) is negative:

\[
\frac{\partial^2 \pi(h_j, h_s; h_s)}{\partial h_j^2} = -\frac{2(1 + h_s)^2\sigma_u}{(1 + h_j)^3(2 + h_s + h_sJ)\sqrt{h_s(1 + h_s)J}} < 0.
\]

Therefore, \( h_j = h_s \), where \( h_s \) is the unique solution to equation (12), indeed maximizes investor \( j \)’s expected profit net of the information-acquisition cost. In other words, investor \( j \) will not deviate from the equilibrium information-acquisition strategy.

Thus far, we’ve shown that given the existence of the equilibrium, the equilibrium with the weakly-increasing beliefs must be featured with unique information acquisition, which is characterized by equation (12). We now offer an example that illustrates that indeed there exists beliefs
to sustain such an equilibrium:

\[
\hat{h}_j = \begin{cases} 
  h_s & \text{if } m_j \leq h_s \\
  m_j & \text{otherwise}
\end{cases}
\]  

(A15)

Following similar arguments in the proof of Proposition 3, we can verify that this specific belief can sustain the characterized equilibrium.

Overall, we have proven that when \( c \) is sufficiently large, that all investors acquire private information of precision \( h_s \) is the unique equilibrium with weakly-increasing beliefs.

**Case 2. Sufficiently low \( c \).** We conjecture that when \( c \) is sufficiently low, in equilibrium investors engage in overt information acquisition; that is, \( h^* = h_o \). Setting \( h^* = h_o \) in (A7), taking derivative with respect to the belief \( \hat{h}_j \), and imposing \( h_o \to \infty \) yields the following:

\[
\lim_{h_o \to \infty} \frac{\partial \pi_j(h_j, \hat{h}_j; h_o)}{\partial \hat{h}_j} = \frac{h_j(1 + \hat{h}_j)\sigma_u \cdot \Psi_1(\hat{h}_j)}{2(1 + h_j)(\hat{h}_j + \hat{h}_j J + 2J)^2 \left( (J - 1)(3\hat{h}_j + 4) + \hat{h}_j(1 + \hat{h}_j J) \right)^{3/2}} > 0.
\]

The positive sign holds because

\[
\Psi_1(\hat{h}_j) = \hat{h}_j^2(4J^2 - 3J - 3) + \hat{h}_j(16J^2 - 21J + 1) + (16J^2 - 26J + 8) > 0,
\]

where the inequality holds because the discriminant \((16J^2 - 21J + 1)^2 - 4(4J^2 - 3J - 3)(16J^2 - 26J + 8) < 0 \) and when \( J \geq 2, 4J^2 - 3J - 3 > 0 \). That is, \( \Psi_1(\hat{h}_j) \) opens upward and does not interact with the x-axis.

Next, clearly, based on equation (13), if the right-hand side (RHS) diverges to \( \infty \) due to sufficiently low \( c \), \( h_o \) must diverge to \( \infty \) as well, i.e., \( h_o = O(c^{-1/4}) \), where the notation \( X_2 = O(X_1) \) means that \( \frac{X_2}{X_1} \) converges to a finite constant as \( c \to 0 \). Coupled with the result \( \lim_{h_o \to \infty} \frac{\partial \pi_j(h_j, \hat{h}_j; h_o)}{\partial \hat{h}_j} > \)
0, we know that
\[
\lim_{c \to 0} \frac{\partial \Pi_j(h_o, h_j, \hat{h}_j)}{\partial \hat{h}_j} > 0.
\]

That is, investor \(j\)'s expected profit is monotonically increasing in the belief \(\hat{h}_j\). Therefore, regardless of the actual information acquisition \(h_j\), investor \(j\) always wants other agents' belief \(\hat{h}_j\) about her information acquisition as high as possible. So, when choosing disclosure strategy, investor \(j\) would like the disclosure strategy that triggers the highest possible belief. Since we assume that \(\hat{h}_j\) is weakly increasing in \(m_j\), investor \(j\) must disclose the highest possible level of signal precision. We thus specify that investor \(j\) fully discloses her information acquisition, \(m_j = h_j\) (recall that \(m_j \leq h_j\)). Note that this specification does not affect the equilibrium information acquisition and remain robust across a set of weakly-increasing beliefs that can sustain the equilibrium. Overall, regardless of her actual information acquisition, investor \(j\) will not deviate from the full-disclosure strategy.

We next confirm that investor \(j\) will not deviate from the equilibrium information acquisition \(h_o\). Replacing \(\hat{h}_j = h_j\) and \(h^* = h_o\) in equation (A7) yields investor \(j\)'s expected net trading profit \(\pi_j(h_j, h_j; h_o)\) as given by (A9). Taking derivative of \(\pi_j(h_j, h_j; h_o)\) with respect to \(h_j\) and imposing \(h_j = h_o\) yields
\[
\frac{\partial \pi_j(h_j, h_o)}{\partial h_j} \bigg|_{h_j=h_o} = \frac{((6J^2 - J - 3)h_o^2 + (4J^2 + 10J - 8)h_o + 8J - 4) \sigma_u}{2J(2 + h_o)(2 + h_o + h_oJ)^2 \sqrt{h_o(1 + h_o)}} - c = 0,
\]
where the last equality follows from equation (13). The second-order derivative of \(\pi_j(h_j, h_j; h_o)\) with respect to \(h_j\) at \(h_j = h_o\) is
\[
\frac{\partial^2 \pi_j(h_j, h_j; h_o)}{\partial h_j^2} \bigg|_{h_j=h_o} = -\frac{(2 + h_o) \sigma_u \cdot \Psi_2(h_o)}{4J(2 + h_o + h_oJ)^3 (h_o(1 + h_o)(2 + h_o)J)^{3/2}} < 0.
\]
The negative sign follows because

\[ \Psi_2(h_o) = 64J - 48 + 16(8J^2 + 11J - 12)h_o \]
\[ + 4h_o^2(12J^3 + 105J^2 + 10J - 66) + 4h_o^3(52J^3 + 85J^2 - 43J - 36) \]
\[ + h_o^4(24J^4 + 184J^3 + 21J^2 - 102J - 27) + 4h_o^5J(6J^3 + 5J^2 - 4J - 3) > 0, \]

where the inequality follows because the coefficients of the constant term 64J - 48 and those of all \( h_o \) terms are positive when \( J \geq 2 \). Therefore, \( h_j = h_o \) indeed maximizes \( \pi_j(h_j, h_j; h_o) \), that is, investor \( j \) will not deviate from the equilibrium information acquisition \( h_o \).

Finally we note that one example of beliefs that can sustain the equilibrium is that \( \hat{h}_j = m_j \),

\[ (A16) \]

and it is easy verify that this is indeed the case.

**Proof of Corollary 1**

Most proof of the corollary follows Propositions 2 and 4 immediately. We here only prove that when \( c/\sigma_u \) is sufficiently high, investors’ expected net trading profits increase upon the disclosure mandate. Given an equilibrium level \( h \) of information acquisition, we can derive investors’ expected net trading profit as follows by replacing \( h^* = h_j = \hat{h}_j = h \) in equation (A7):

\[ \pi(h) = \frac{\sqrt{h(1+h)\sigma_u}}{\sqrt{J(2+h+hJ)}} - c \cdot h. \]

Taking derivative of \( \pi(h) \) with respect to \( h \) yields

\[ \frac{\partial \pi(h)}{\partial h} = \sigma_u \left( \frac{2 - h(J - 3)}{2(2 + h + hJ)^2 \sqrt{h(1+h)J}} - \frac{c}{\sigma_u} \right). \]

\[ (A17) \]

For sufficiently high \( c/\sigma_u \), based on equations (12) and (13) we know that both \( h_o \to 0 \) and \( h_s \to 0 \). Setting \( h \to 0 \) in (A17) yields \( \frac{\partial \pi(h)}{\partial h} = \sigma_u \left( \frac{1}{4J\sqrt{h}} - \frac{c}{\sigma_u} \right) \). Thus, we know that \( \frac{\partial \pi}{\partial h} < 0 \) if
and only if

\[ \sqrt{h} > \frac{1}{4J} \frac{\sigma_u}{c}. \] (A18)

Based on equations (A10) and (A11) in the proof of Proposition 2, we know that when \( c/\sigma_u \) is sufficiently high, \( \sqrt{h_o} = \frac{1}{2\sqrt{J}}(1 - \frac{1}{2J}) \cdot \frac{\sigma_u}{c} + o \left( \frac{1}{c} \right) > \frac{1}{4J} \frac{\sigma_u}{c} \) and \( \sqrt{h_s} = \frac{1}{\sqrt{J}} \cdot \frac{\sigma_u}{c} + o \left( \frac{1}{c} \right) > \frac{1}{4J} \frac{\sigma_u}{c} \), where the two inequalities follow because \( J > 2 \). Further, we’ve shown in Proposition 2 that for sufficiently high \( c/\sigma_u \), \( h_s > h_o \); coupled with \( \frac{\partial \pi}{\partial h} < 0 \) we know that \( \pi_s < \pi_o \); that is, from the secret to overt case, investors’ expected net trading profit improves.

**Proof of Lemma 4**

In the proof, we first derive investors’ optimal trading strategies and market makers’ pricing rule given \( h^* \). Then we solve for the equilibrium information acquisition \( h^* \) by considering possible deviation of a representative investor \( j \).

Consider investor \( j \). Given \( h^* \), the conditional expected profit of investor \( j \) is the following

\[ E[\tilde{x}_j(\tilde{v} - \tilde{p})] = \tilde{x}_j \left( \frac{h^*}{1 + h^* \tilde{y}_j} - \lambda \left( \tilde{x}_j + (J - 1) \alpha^* \frac{h^*}{1 + h^* \tilde{y}_j} \right) \right). \]

Taking all the other investors’ trading strategy as given, maximizing the profit yields investor \( j \)’s optimal trading strategy \( \tilde{x}_j = \alpha_j \tilde{y}_j \), where

\[ \alpha_j = \frac{h^* (1 - (J - 1) \lambda \alpha^*)}{2 \lambda (1 + h^*)}. \] (A19)

In equilibrium, investor \( j \) must adopt the equilibrium trading strategy as the other investors, namely, \( \alpha_j = \alpha^* \). Then, equation (A19) can be simplified to the following:

\[ \alpha^* = \frac{h^*}{(2 + h^* + Jh^*) \lambda}. \] (A20)
Following equation (5), we can compute the pricing rule as $\tilde{p} = \lambda \tilde{\omega}$, where

$$
\lambda = \frac{J\alpha^*}{(J\alpha^*)^2 + J\frac{(\alpha^*)^2}{h^*} + \sigma_u^2}.
$$

(A21)

Based on equations (A20) and (A21), we obtain the trading strategy and pricing rule along the equilibrium path as the following:

$$
\alpha^* = \frac{\sqrt{h^*\sigma_u}}{\sqrt{(1+h^*)J}} \text{ and } \lambda^* = \frac{\sqrt{h^*(1+h^*)J}}{(2+h^*+h^*J)\sigma_u}.
$$

(A22)

Next, we solve for the equilibrium information acquisition by considering the possible deviation by a representative investor $j$, where $j \in \{1, \ldots, J\}$. Suppose that investor $j$ deviates to acquiring private information of precision $h_j$. Then with a probability of $q$, the deviation is detected by all other agents (overt information acquisition) and with the remaining probability the deviation is not detected (secret information acquisition). We then discuss the two cases separately.

Case 1: overt information acquisition. When investor $j$’s deviation from $h^*$ to $h_j$ is observable, other investors and market makers will adjust their strategies accordingly. We need to solve for the off-equilibrium trading strategy of investor $j$, that of the other investors and the off-equilibrium pricing rule so to derive investor $j$’s expected net trading profits. Following the same procedure as we derived (A9) in the proof of Lemma 2, we obtain that under overt information acquisition, investor $j$’s expected net trading profit is as follows:

$$
\pi_{j, overt}^o (h_j) = \frac{(2+h^*)^2h_j(1+h_j)\sigma_u}{4+2h^*h_j(4+h^*+Jh^*)} - ch_j.
$$

(A23)

Case 2: secret information acquisition. When investor $j$’s deviation from $h^*$ to $h_j$ is not observable, other investors and market makers will retain their respective equilibrium trading
strategies and pricing rule as given by (A22). Following the same procedure as we derived (A8) in the proof of Lemma 2, we obtain that under overt information acquisition, investor $j$’s expected net trading profit is as follows:

$$
\pi_{j}^{\text{secret}}(h_j) = \frac{(h^* + 1)^2 h_j \sigma_u}{\sqrt{Jh^*(h^* + 1)(h_j + 1)(Jh^* + h^* + 2)}} - ch_j.
$$

(A24)

Overall, taking into account the two possible cases, investor $j$’s expected net trading profit is a weighted average of (A23) and (A24):

$$
\pi_j(h_j) = q\pi_j^{\text{overt}} + (1 - q)\pi_j^{\text{secret}}.
$$

Taking derivative of $\pi_j(h_j)$ with respect to $h_j$ and then setting $h_j = h^*$, we can solve for the equilibrium information acquisition $h^*$, which is the unique solution to equation (17).

**Proof of Proposition 5**

First, based on equation (17), when $c/\sigma_u \to 0$, $h^* \to \infty$. And equation (17) can be simplified to the following:

$$
\frac{c}{\sigma_u} = q \cdot \frac{(6J^2 - J - 3)(h^*)^2 + o((h^*)^2)}{2J\sqrt{J(J+1)^2(h^*)^4 + o((h^*)^4)}} + (1 - q) \cdot \frac{1}{\sqrt{J(J+1)(h^*)^2 + o((h^*)^2)}}.
$$

It follows that

$$
(h^*)^2 = \frac{\sigma_u}{c} \cdot \frac{1}{(J+1)\sqrt{J}} \left(q \cdot \frac{6J^2 - J - 3}{2J(J+1)} + 1 - q\right) + o\left(\frac{\sigma_u}{c}\right)
$$

Let $f(q) = q \cdot \frac{6J^2 - J - 3}{2J(J+1)} + 1 - q$. Then $f'(q) = \frac{4J^2 - 3J - 3}{2J(J+1)} > 0$ for $J \geq 2$. Therefore, $\frac{\partial h^*_j}{\partial q} > 0$ when $c/\sigma_u \to 0$.

Second, similarly, based on (17), when $c/\sigma_u \to \infty$, $h^* \to 0$, and the equation can be simplified
as follows:

\[
\frac{c}{\sigma_u} = q \cdot \frac{8J - 4 + o(1)}{16J \sqrt{h^*J} + o(\sqrt{h^*})} + (1 - q) \cdot \frac{1}{2\sqrt{h^*J} + o(\sqrt{h^*})},
\]

which yields that

\[
\sqrt{h^*} = \frac{\sigma_u}{c} \cdot \frac{1}{2\sqrt{J}} (1 - \frac{1}{2J} q) + o \left( \frac{\sigma_u}{c} \right).
\]

Then \(\frac{\partial h^*}{\partial q} < 0\) immediately follows.

The remainder of the proof about market efficiency and market liquidity immediately follows that in the proof of Proposition 2.