Diversification and its Discontents:
Idiosyncratic and Entrepreneurial Risk in the Quest for
Social Status.*

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Abstract

Incorporating preference for social status into a simple model of portfolio choice helps
to explain a range of qualitative and quantitative stylized facts about the heterogeneity in
asset holdings among U.S. households. I specify preferences for status as a function of a
household’s relative position in the cross-sectional wealth distribution. In the model, investors
hold undiversified portfolios in equilibrium, suggesting, in particular, a possible explanation for
the apparently small premium for undiversified entrepreneurial risk - “private equity premium
puzzle.” Consistent with empirical evidence, the wealthier households own a disproportionate
share of risky assets, particularly private equity, and experience more volatile consumption
growth. The calibrated model generates sufficient wealth mobility to help explain the transition
dynamics of the U.S. wealth distribution without excessive consumption growth volatility.

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1 Introduction

Diversification and risk-sharing are fundamental principles of modern finance and macroeconomics. However, empirical evidence suggests that household portfolios are poorly diversified, with many people reporting substantial holdings of a single stock, often of their own employer or a prominent local company. For the wealthiest households, large shares in closely held businesses constitute a particularly important source of idiosyncratic risk. Surprisingly, from a standpoint of portfolio theory, entrepreneurship does not appear to be well compensated: e.g., find that returns on undiversified entrepreneurial investment are no higher than the average return on publicly traded equity despite the greater risk. They refer to this phenomenon as the “private equity premium puzzle.” The apparent implication of this evidence is that many investors are willing to take poorly rewarded risks, despite the availability of superior investment opportunities, such as public equity that earns a large risk premium.

In the present paper, I attempt to rationalize these facts within a simple portfolio choice framework in which households can optimally choose their level of exposure to idiosyncratic risk. The key ingredient of my model is the utility derived from social status, defined as the individual household’s relative standing in the cross-sectional distribution of wealth. I build on the insight of that the relative wealth concerns generated by such preferences create a wedge in people’s attitudes towards aggregate risk and idiosyncratic risk, since, by definition, the former affects the entire wealth distribution, while the latter only affects individual wealth. In my model, investors’ desire to “get ahead of the Joneses” is sufficiently strong to overcome the desire to smooth consumption across states of nature, leading them to fear individual-specific risk less than aggregate risk.

Such “getting ahead of the Joneses” behavior is present when an individual’s marginal utility of wealth rises with the ratio of individual wealth to per capita reference wealth level. This condition is intuitive: it means that the rich care more about their relative position than do the

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1 See for a survey of the evidence on household portfolio choice. Some of the earliest evidence of poorly diversified household portfolios was documented by . Most recently, measure the extent of underdiversification using data on portfolio composition of Swedish households.

2 It is certainly not surprising that entrepreneurial risk is not fully diversified, since there is potentially a trade-off between risk-sharing and incentives. For example, find evidence of agency costs affecting entrepreneurs’ holdings of business equity.

3 reaches similar conclusions by analyzing the earnings differentials between self-employment and paid employment.
poor, for whom consumption is a more pressing concern than status. This property also implies that status-seeking investors view economy-wide “bad times” (states of nature in which aggregate wealth is low relative to individual wealth) as a chance to move up in society. Therefore they value a marginal dollar of wealth more highly in “bad times” than in “good times,” holding their own wealth fixed. Consequently, investors dislike assets that pay off poorly in the bad states of the economy, even if individual wealth is not highly correlated with the aggregate. If households have investment opportunities that are not highly correlated with the returns of an “average investor,” such as private business projects, the heightened aversion to aggregate risk acts as a “hedging demand,” leading them to reduce their portfolios’ exposure to the public equity market and to load more heavily on individual-specific risk.

The “getting ahead of the Joneses” feature arises in my model of social status preferences if the curvature of utility over consumption is greater than the curvature of preferences with respect to relative wealth, specifically, if the former is “more concave” than the latter. In the model the curvature of preferences over relative wealth is controlled, in particular, by the shape of the cross-sectional distribution of wealth. The wealth distribution itself arises from aggregation of individual households’ consumption and investment choices. I use the empirical distribution of wealth in the U.S. to impose this restriction on the calibrated model.

My model has the following implications. Quantitatively, while it is difficult to measure the extent of diversification of household portfolios across the wealth distribution empirically, the model can match both the overall levels of risk-taking and the shares of household wealth concentrated in a single risky asset that are observed in the U.S. data. Qualitatively, the richer households (but not necessarily the richest) have a larger fraction of their wealth invested in individual-specific idiosyncratic assets, such as private equity, consistent with the available evidence. They also consume a smaller fraction of their wealth than do the poor and experience greater consumption growth volatility.

As both a test and an application of the model, I evaluate its ability to explain the empirically observed dynamics of household wealth. Undiversified idiosyncratic risk manifests itself in the dramatic variation of household wealth both across the population and over time. The cross-sectional distribution of earnings, consumption, and especially asset holdings in the U.S. is extremely concentrated: the richest 10% of the population together own over 60% of the to-
tal capital stock (?). At the same time, there is a high degree of social mobility. A median household has a greater than 3% chance of moving to the top wealth decile after ten years, and a roughly 2% chance of falling into the bottom decile, with only a 33% probability of staying in the middle quintile (?).

My model is able to explain much of the variation in wealth holdings across households and over time. In the model, individuals choose to hold substantially more idiosyncratic risk than predicted by standard preference specifications. Under-diversification results in a dramatic dispersion of accumulated wealth among ex ante identical households, as well as high rates of transition between wealth quantiles. In the upper tail of the wealth distribution the simulated model can quantitatively match the empirical transition dynamics between percentile groups that are not explained by labor earnings and other life-cycle-induced variation. At the same time, the artificial data generated by the model preserve the high concentration of wealth in the top percentiles of the distribution. The particular success in explaining the wealth dynamics in the upper end of the wealth distribution should not come as a surprise, since my model is designed to generate demand for idiosyncratic risk among the wealthier households (e.g. business owners).

The success of the social status model in explaining the cross-sectional wealth dynamics complements the existing literature on wealth inequality. That literature analyzes variation in household wealth by modelling consumption and savings behavior over the life-cycle in the face of uninsurable labor income risk, a feature that is absent from my model. Standard macroeconomic models of the wealth distribution, which are at the core of this literature, have difficulty producing empirically accurate magnitudes of wealth mobility, as well as the concentration of wealth at the top of the wealth distribution. The model of ?, one of the most successful to date in explaining wealth inequality, implies almost twice as much persistence for the middle wealth deciles as is present in the data. It is not surprising that these models fail to generate substantial wealth mobility. Following ?, they typically impose uninsurable labor income risk, carefully matching its statistical properties to the data, but abstract from other (potentially endogenous) sources of idiosyncratic risk by limiting households’ investment opportunities. In fact, in the data consumers appear to be insured relatively well against most exogenous idiosyncratic income shocks, such as a temporary job loss or sickness (?). At the same time, however, there is substantial cross-sectional dispersion in wealth accumulated before retirement among households with similar earnings his-
tories, even after controlling for various life-time shocks and heterogeneity in asset allocations (?). This unexplained heterogeneity in wealth holdings suggests a potential role for idiosyncratic risk exposure of individual portfolios (?), which I explore in this paper.

My model shares some features with other successful recent models of the wealth distribution. One of the main difficulties encountered by the standard models is that the dramatic wealth concentration at the top of the distribution is largely driven by the fact that the rich accumulate assets faster than the poor. This is largely due to the former’s high propensity to save and to invest in risky assets (see ? and ? for surveys of the literature). I address the consumption/saving, portfolio choice, and entrepreneurial risk exposure decisions jointly. I show that the social status model is able to replicate qualitatively the stylized facts about the heterogeneity in household saving and capital accumulation. Wealthy households consume less and, consequently, save more as a fraction of their assets than poor ones. This difference is especially dramatic for the oldest consumers, for whom such behavior is the most at odds with the standard life-cycle intuition. The rich also allocate a larger share of their portfolios to risky assets, an in particular in private business ventures. ? models wealth concentration by appealing to a “joy of giving” bequest motive, which helps generate a saving profile that is increasing in wealth. ? and ? show that incorporating undiversified entrepreneurial risk helps explain wealth concentration and inequality. The former model entrepreneurial choice but abstract from other investment opportunities, while the latter imposes exogenous portfolio returns with large uncompensated idiosyncratic risk. In a similar fashion, ? advocates the role of entrepreneurial investment risk in explaining much of the wealth concentration and mobility.

In addition to calibrating a quantitative version of the social status model, I also discuss several intuitive qualitative extensions that help broaden its empirical support. When the “getting ahead of the Joneses” effect is sufficiently strong to produce non-concave utility of wealth and, consequently, risk-loving behavior, investors are willing to hold portfolios highly concentrated in assets that do not earn risk premia. If all investors have access to the same assets there must exist a way to ensure that different investors overweight different stocks, for such behavior to be consistent with equilibrium. Thus, I provide a novel interpretation of the tendency among many investors to overweight geographically or otherwise “familiar” stocks (“local bias”): “familiarity” provides a coordination device for investors seeking idiosyncratic risk exposure in the public stock
market. If some investors have a (real or perceived) informational advantage at investing in familiar stocks then this mechanism is consistent with status-induced non-diversification even in the absence of risk-seeking.

The remainder of the introductory section places this paper in the context of economic literature on social externalities. Section 2 develops the model of social status preferences and describes its qualitative implications for portfolio allocation. Section 3 defines the equilibrium concept and describes the numerical solution of the calibrated model. Section 4 introduces empirical evidence and evaluates the model’s ability to explain the quantitative features of both the aggregate and the household-level data. Section 5 addresses sensitivity of the model’s implications to alternative market structures, as well as other extensions. Section 6 concludes.

1.1 Social Status and Portfolio Choice

The notion that at higher levels of income people value the “social esteem” brought on by their wealth more than the consumption of goods and services that this higher wealth can buy goes back at least to Adam Smith (see ?, p. 70). ? argue that marginal utility of wealth rises as people move to a higher “social class.” They apply this intuition to motivate their famous concave-convex-concave utility function, which explains simultaneous demands for lotteries and insurance. ? formalizes their intuition by specifying a utility function which is concave in consumption but convex in status, defined as the percentile rank in the wealth distribution. Empirically, the intuition that higher status might increase marginal utility of wealth is consistent with a stronger effect of relative income on subjective well-being for richer people found by ?. This property of the marginal utility is key for my model.

There are several reasons to consider individual wealth rank as the measure of social status that enters preferences. In the psychology literature, range-frequency theory has been applied to understanding the effect of social comparisons on personal happiness and well-being (e.g. ?). Some economists have applied the latter approach to understanding the evidence that income rank, as well as both its absolute value and that relative to the average, are important components of workers’ wage-satisfaction (e.g. ?). Others have suggested micro-foundations for relative wealth

\[4\text{ Much of the economics literature on social comparisons is centered around the related notions of “conspicuous consumption,” introduced by ?, and “relative income” of ?. Another strand of literature appeals to “capitalist spirit” introduced by ?; in the latter context, external reference wealth levels can be interpreted as ways of measuring “moral self-worth.”} \]
concerns. ? show that social status corresponds to the wealth rank if there exists either an explicit market for status or at least a freely traded “status good.” ? and ? model preferences over wealth rank as arising endogenously due to non-market mediated interactions, such as marriage and similar matching contexts. In a similar vein, ? and ? motivate relative wealth concerns with competition for inelastically supplied goods. I do not take a stand on whether relative-wealth preferences are “hard-wired” or endogenous. My model of preferences can be viewed as a reduced-form version of preferences defined explicitly over status, or over a type of consumption good for which relative position is “instrumental.” ? discusses various economic interpretations of socially-dependent preferences, such as “fundamental” vs. “instrumental.”

In finance, preferences featuring social externalities have already been applied to understanding the lack of diversification of household portfolios.5 In contrast to my model, much of this literature emphasizes “herding” and “conformism” effects of interpersonal preferences (e.g. ? and ?). ? argue that the external habit formation model is able to explain the apparent tendency of investors to prefer assets local to their community and to avoid foreign assets (the so-called “home bias puzzle”). ? show that preference for a “local good” can generate undiversified portfolios in equilibrium, with households in each community tilting their portfolios toward community-specific assets. These kinds of models, however, are not able to explain holding large amounts of purely idiosyncratic risk, which is likely to be an important component of the “private equity premium puzzle.” In fact, most of the herding results rely on an assumption that there exists a group of “entrepreneurs” who are exogenously bound to hold a disproportionate share of local assets in their portfolios. Interestingly, ? find no evidence that direct community effects of this type are driving undiversification. It is certainly possible that there are other local peer effects that do not operate through the relative wealth channel: for example, ? and ? find empirical evidence consistent with social interactions affecting individuals’ stock market participation and holdings of own-company stock, respectively.

5Other attempts at explaining the apparent lack of diversification include models based on non-expected utility preferences, such as cumulative prospect theory (?) and rank-dependent utility (?), as well as on model misspecification and learning costs (?). ? and ? provide evidence of undiversification which, they argue, is consistent with explanations based on “familiarity”. Some forms of undiversification are consistent with anticipatory utility and optimism - see ? and ?. Overconfidence is also cited in explaining entrepreneurial behavior (e.g. ?). Among the proposed rational explanations of low average payoffs to entrepreneurship are real options-based models, such as ? and ?. While the illiquidity of private business investments might deepen the private equity premium puzzle (e.g. see ?), it can also provide a potentially attractive commitment mechanism for agents with time-inconsistent preferences (e.g. ??).
2 Individual investment decisions

2.1 Preferences over consumption and social status

I consider a continuum of households, indexed by $i \in A \subset \mathbb{R}$, with the total mass of 1 under the associated measure $\mu$. Each has a finite lifetime of $T$ active periods in which consumption and portfolio decisions are made and one terminal period in which terminal wealth (and status conferred by it) is consumed. The specification of preferences extends those proposed to a multiperiod environment (and consider related portfolio problems that do not feature undiversification). Households’ preferences are separable in consumption and social status, which is given by the household’s position (percentile rank) in the cross-sectional distribution of household wealth scaled by the per capita wealth $\tilde{F}_t$:

$$\tilde{F}_t(x) = \mu \left( i : \frac{W_i^t}{\bar{W}_t} \leq x \right), \quad \text{where} \quad \bar{W}_t = \int_A W_i^t d\mu(i). \quad (2.1)$$

so that they maximize

$$E_t \left\{ \sum_{s=t}^T \beta^{s-t} \left[ \left( C_s^t \right)^{1-\gamma} - 1 \right] + \eta \bar{W}_t^{1-\gamma} \tilde{F}_s \left( \frac{W_i^s}{\bar{W}_s} \right)^{\zeta} \right\} \quad (2.2)$$

The first term in the period utility is the standard power utility over consumption; the second term is the utility derived from social status. As a function of relative position (percentile rank) it is concave when $0 < \zeta < 1$ and convex when $\zeta > 1$. The parameter that controls the relative importance of consumption and status is $\eta > 0$. It is multiplied by an average wealth term, $\bar{W}_t^{1-\gamma}$ in order to ensure the invariance of preferences to changes in aggregate wealth (otherwise, as ? shows, it holds only if utility over consumption is logarithmic).

For integer values of $\zeta \geq 1$ this preference specification admits an additional interpretation. Suppose that relative wealth is important because it awards a fixed prize to the winner of a “status game”: an individual who is richer than $\zeta$ randomly drawn members of the population in the given period. Since the probability of winning the prize equals $\tilde{F}_t \left( \frac{W_i^t}{\bar{W}_t} \right)^{\zeta}$, the lifetime expected utility coincides with (2.2). In particular, for large $\zeta$, the prize is conferred on the wealthiest member of the population.

In addition, preferences over status that enter the household’s utility separably can be viewed
as a form of bequest motive, which is consistent with the “evolutionary” view of status and social norms proposed by ?. ? argues that a preference for wealth as a luxury good is key to explaining the heterogeneity in portfolio composition across households, in particular the fact that the rich save more as a fraction of their wealth than the poor and that they hold a much larger share of risky assets (including entrepreneurial ventures) in their portfolios. One interpretation of Carroll’s preferences is the “joy of giving” bequest motive which is operative only for the very rich households (see also ?). As ? and ? show, bequest motives are indeed important for understanding the dramatic differences in wealth accumulation between the rich and the poor. While I do not explicitly assume that preferences for status are derived from a bequest motive, I am able to capture the notion of accumulated wealth as a luxury good since in the model the marginal utility of wealth declines more slowly than the marginal utility of consumption.

2.2 Risk attitudes and getting ahead of the Joneses

Status concerns can generate underdiversification in individual portfolios via two related but distinct channels. The first one is the greater aversion to aggregate than idiosyncratic risk. The second is a pure risk-seeking (gambling) motive. In this paper I focus primarily on the former channel. While it can be compounded by the latter, the two can exist independently.

Risk preferences are determined by the marginal value of wealth across states of nature. Let \( V_i(W_i^t, \bar{W}_t; \bar{F}_t) \) denote the indirect utility (value) function of wealth for consumer \( i \) at time \( t \) (i.e. the maximized objective function in (2.2)). Under the definition of preferences (2.2) marginal utility of wealth is positive, while the derivative with respect to aggregate reference wealth is negative. This is intuitive since, holding individual wealth fixed, an increase in per capita wealth reduces the individual’s relative status and utility (such effects are often referred to in the literature as exhibiting “jealousy” - ?). The individual’s risk preferences are controlled by the partial derivatives of the marginal utility of wealth with respect to the state variables: own wealth and per capita wealth in the economy (in the following discussion I abstract from other state variables that might be of interest as they are not featured in the calibrated model). The former, denoted by \( V_{WW}^i \), represents aversion to all wealth gambles. The latter, \( V_{W\bar{W}} \), captures the attitude towards gambles that are correlated with aggregate wealth.\(^6\) As usual, when \( V_{WW} < 0 \) the consumer is risk

\(^{6}\)I assume throughout this discussion that the cumulative wealth distribution is a twice-differentiable function in order to ensure existence of the appropriate derivatives of the implied utility of wealth.
averse and when $V_{WW} > 0$ risk seeking. Similarly, when $V_{WW} < 0$ the consumer dislikes aggregate risk (in addition to its contribution to overall wealth risk), and conversely when $V_{WW} > 0$ the consumer seeks additional exposure to aggregate risk, relative to a no-status benchmark.

The property that marginal value of wealth is decreasing in aggregate wealth ($V_{WW} < 0$) that can be termed “getting ahead of the Joneses” (it is called “running away from the Joneses” by ?) captures the idea that an increase in aggregate wealth, holding individual wealth fixed, lowers the marginal utility of wealth. This is in contrast to a “keeping up with the Joneses” feature ($V_{WW} > 0$) that raises marginal utility when the aggregate reference level is high (e.g. ?). The intuition for the former type of preferences that drives my results is that higher relative status raises marginal utility of wealth (?, ?, also see discussion in the introductory section 1.1 above). Since status is an increasing function of the ratio of own wealth to reference wealth, as defined in (2.1), a decrease in aggregate wealth raises some people’s status, making them better off, but also raising their marginal utility of wealth. The latter effect causes them to avoid assets that pay off poorly in such states.

What drives the desire to “get ahead of the Joneses” in the social status model? The crucial feature that delivers this effect is the difference in aversions over consumption risk and over relative wealth risk. The former is controlled by the standard consumption curvature parameter $\gamma$, where as the latter is determined by a combination of the status curvature $\zeta$ and the shape of the wealth distribution, as well as the status weight $\eta$ which determines the relative importance of the two types of risk aversion. In particular, if investors are less averse to relative wealth shocks than towards consumption variability, marginal value of wealth increases with relative (but not necessarily absolute) wealth, generating the desired effect ($V_{WW} < 0$). This can happen if either the wealth distribution is not “too concave” relative to the power utility function of consumption, or if preferences are convex in status ($\zeta > 1$). Appendix A provides an example of this effect that relies solely on the curvature of the wealth distribution relative to the power utility of consumption, when utility is linear in status ($\zeta = 1$).

The overall allocation of assets to securities that bear aggregate risk would thus be determined by a combination of overall risk aversion and the “hedging demand” for insurance against fluctuations in per capita wealth. In particular, risk averse individuals with “keeping up with the Joneses” preferences might require less compensation for bearing aggregate wealth risk than for
purely idiosyncratic risk (e.g. see ?). Conversely, those with low risk aversion (or risk-seeking) over pure wealth gambles might coincide with low (or negative) allocation to aggregate assets even in the face of a high risk premium. The latter feature of “getting ahead of the Joneses” preferences is consistent with the view of the aggregate equity premium that emphasizes low individual risk aversion to idiosyncratic gambles (e.g. see discussion in ? and ?).

“Getting ahead of the Joneses” does not necessarily coincide with risk seeking over pure wealth gambles. As shown (by way of example) in Appendix A, for investors to be risk seeking over pure wealth gambles at least one of the following must hold:

a) the distribution of relative wealth is convex (at least locally), i.e. there exists a region of the state space over which $F'' > 0$ - e.g. ?

b) preferences are convex in status ($\zeta > 1$) - e.g. ?

Both of these conditions can contribute to generating marginal utility of wealth that is decreasing in the aggregate reference level. However, neither is required. In particular, for a given concave wealth distribution there exist parameter combinations such that $\zeta \leq 1$ yet $V_{WW}^i(W^i, \bar{W}; \tilde{F}) < 0$. It is required in this case that the wealth distribution is not “too concave” relative to the power utility function of consumption. Therefore, intuitively, getting ahead of the Joneses behavior arises when investors are less risk averse with respect to fluctuations in relative wealth than with respect to absolute wealth variation.

2.3 Technology and market structure: aggregate vs. idiosyncratic risk

I model aggregate and idiosyncratic risk symmetrically as different assets available to investors, following ? who consider entrepreneurs’ portfolio choice and capital structure decisions jointly. This is in contrast to much of the existing literature. Portfolio choice models with agent-specific idiosyncratic risk commonly assume that its “amount” is exogenously fixed, usually in the form of a stream of labor income. Conversely, models of entrepreneurial choice (e.g. ?) usually abstract from the composition of financial portfolios.

A wide variety of investment opportunities provide a choice between aggregate and idiosyncratic risk, which poses a modelling challenge. Thus I limit the set of assets available to the households for the sake of tractability. In the model, every household can invest in three linear
technologies with returns given by vector \( \mathbf{R} = [R^f, R^a, R^i] \) which follows a Markov process with transition density \( \varphi \). These investment opportunities are:

- riskless storage technology with return \( R^f \)
- common risky technology (“public equity”) with return \( R^a \)
- idiosyncratic risky technology (“private equity”) with return \( R^i \), which is individual-specific (and independent across agents).

The specification of the investment opportunities considered here captures the idea that investors might be able to choose the combination of aggregate and idiosyncratic risk optimally. This type of investment decision is meant to encompass human capital (career choice) as well as entrepreneurial investment (hence, with some abuse of terminology, I label it “private equity”). Market incompleteness (i.e., agents cannot invest in each others’ idiosyncratic asset) is important in that it allows purely idiosyncratic assets to have positive expected returns in excess of the risk-free rate. Finally, given a continuum of firms, undiversified portfolios of publicly-traded firms could be described using these technologies, provided that a coordination device exists matching households with firms (since if a large number of households outweighs the same stock, its share in the market portfolio and therefore covariation with aggregate wealth increases). I discuss the implications of the social status model in an environment with multiple public securities in section 5.1.3.

2.4 Undiversification: an example

The following simple example captures the main qualitative feature of the relative status preferences: the different attitudes towards idiosyncratic and aggregate risk.

**Example 1.** Consider a one-period case (\( T = 1 \)) with logarithmic utility of consumption (\( \gamma = 1 \)) and linear utility of status (\( \zeta = 1 \)). Assume that the future wealth distribution \( \tilde{F} \) is exogenously fixed so that relative wealth is distributed uniformly on \( [0, s_{\text{max}}] \).

In this one-period case there is no consumption-saving decision. Utility is logarithmic over consumption, which is equal to terminal wealth, and linear in status, which is given by terminal
wealth relative to the per capita wealth. Let \( \hat{\eta} = \frac{\eta}{s_{\text{max}}} \); then agent \( i \)'s optimization problem is

\[
\max E \left[ \log (W^i) + \hat{\eta} \min \left( \frac{W^i}{W}, s_{\text{max}} \right) \right].
\]

Suppose the distribution of returns faced by agent \( i \) is such that for any feasible portfolio combination \( \Pr \left[ \frac{W^i}{W} > s_{\text{max}} \right] = 0 \) (otherwise the assumption on \( \hat{\eta} \) could be violated). Then standard first-order conditions yield an Euler equation for asset returns:

\[
E \left[ (R^a - R^i) \left( 1 + \hat{\eta} \frac{1}{W^i} \right) \right] = 0.
\]

Under the assumption that \( \text{cov} (R^i, \tilde{W}) = 0 \), and assuming that expected returns on both assets are the same, i.e. \( E [R^a - R^i] = 0 \), we have

\[
\hat{\eta} \text{cov} \left( R^i, \frac{1}{W^i} \right) + \text{cov} \left( R^a, \frac{1}{W^i} \right) = 0.
\]

If the common asset \( R^a \) is in positive net supply, it is positively correlated with aggregate wealth (in fact, \( \tilde{W} \) is a linear function of \( R^a \)). Thus \( \text{cov} \left( R^a, \frac{1}{W} \right) \) is negative and, therefore,

\[
\text{cov} \left( R^a, \frac{1}{W^i} \right) > \text{cov} \left( R^i, \frac{1}{W^i} \right),
\]

implying that

\[
\text{cov} \left( R^a, W^i \right) < \text{cov} \left( R^i, W^i \right).
\]

This means that a “status-conscious” investor is optimally exposed to more idiosyncratic risk and less exposed to aggregate risk than a neoclassical investor (if \( \eta = 0 \) we have \( \text{cov} (R^a, W^i) = \text{cov} (R^i, W^i) \)). The magnitude of the wedge in attitudes towards aggregate and idiosyncratic risk depends on the strength of preference for social status, controlled by parameter \( \eta \) and the covariance between aggregate risk and per capita wealth, as well as the shape of the wealth distribution. In particular, the greater the degree of wealth dispersion, controlled in this case by \( s_{\text{max}} \), the lower the effect of status on portfolio composition. This effect of wealth dispersion can be understood as a substitution effect, since the more spread out wealth distribution means that attaining higher status becomes costlier. *Ceteris paribus*, it requires greater wealth accumulation
(or higher variance) and thus makes status less appealing relative to pure consumption. Therefore, higher values of $\eta$ are required in order to produce a similar status effect in economies with higher wealth dispersion.

Note that I have not shown that the assumption of uniform wealth distribution is consistent with aggregation of optimal portfolios. I address this issue in a more general setting of the quantitative model in Section 3.1. A similar model that is consistent with a suitable notion of equilibrium is developed by $\text{?}$, who consider the case of quadratic utility and \textit{ex ante} identical wealth endowments. They prove the corresponding result for the case of convex preferences over status. Similarly, $\text{?}$ argues that risk-loving preferences over status are required in order to generate demand for fair gambles. As is clear from this example, convexity is not necessary to capture, at least qualitatively, the idea that status concerns can tilt an investor’s portfolio away from the common asset and towards an idiosyncratic asset. This is because the idiosyncratic asset in the above example (as well as in $\text{?}$) is not a fair gamble - it earns a positive average excess return. The preferences above exhibit risk aversion, since (for all but the richest individual) $V_{WW} = -W^{-2} < 0$, while they also imply getting away from the Joneses, since $V_{W\bar{W}} = -\hat{\eta}\bar{W}^{-2} < 0$. Convex preferences over status (or convex wealth distribution) are needed to generate risk-loving behavior ($V_{WW} > 0$), which is necessary in order to explain undiversified portfolios of publicly traded securities, since idiosyncratic risk associated with such investments should earn no risk premium in the absence of arbitrage.

Convexity might be important for the model’s ability to generate predictions that are quantitatively relevant, even for the private equity returns that earn a risk premium. If that is the case, the assumption that the only source of idiosyncratic risk is given by a non-tradeable asset can be potentially restrictive. Zero-sum lotteries and other similar financial contracts that can generate idiosyncratic risk through either intrinsic uncertainty (e.g., breaking up the market portfolio into undiversified portfolios consisting of a few individual stocks or entering derivative contracts contingent on idiosyncratic events) or extrinsic uncertainty such as “sunspots” (\textit{?}, ?). The existing literature on nonconvexities induced by status concerns assumes the existence of fair lotteries but abstracts from any other assets and investment technologies (e.g. ? and ?), or else assumes away lotteries and relies on nontradeable personal investment technologies (e.g. ?). An ideal model might incorporate both. However, this would complicate the solution considerably. In the
main body of this paper I follow the latter approach, since personal investment projects are more likely to be empirically relevant than either lotteries (and gambling in general) or complicated trading strategies. In practice, it might be difficult for individuals to implement purely zero-cost idiosyncratic-risk strategies due to the heavy taxation and restrictions on lottery-type activities and the various costs associated with participating in derivatives markets. I show that the main qualitative results are robust to alternative market arrangements in Section 5.

2.5 Optimal policies

Each household $i$ aged $a_i^t$ at time $t$ solves the following recursive problem:

$$V_i^t(W_i^t, \bar{W}_t, a_i^t; I_t) = \max_{C,a} \left\{ \frac{(C_i^t)^{1-\gamma} - 1}{1-\gamma} + \eta W_i^{1-\gamma} \tilde{F}_{t+1} \left( \frac{W_i^t}{\bar{W}_t} \right)^\zeta + \beta E \left[ V_i^t(W_{t+1}^t, \bar{W}_{t+1}, a_{t+1}^t; I_{t+1}) | I_t \right] \right\},$$

subject to the resource constraint

$$W_{t+1}^i = (W_i^t - C_i^t) a_i^t R_{t+1}.$$

Since I rule out contemporaneous lotteries that could be used to eliminate the nonconvexity each period, the agents cannot influence their current-period status, which is determined by their beginning-of-period wealth endowment. Consequently, standard dynamic programming arguments can be applied to analyzing the problem quantitatively. Optimal consumption and investment policies that are solutions to the dynamic programming problem (2.3) generally depend on the wealth distribution $\tilde{F}$ and its evolution over time. For a generic distribution, the optimal strategies can be characterized as follows.

1. At low levels of wealth, the behavior of a status-conscious investor is close to that of a neoclassical investor with coefficient of relative risk aversion $\gamma$. This is because as $W_i^t$ tends to zero the status component of the period utility function $\tilde{F}\zeta$ goes to zero as well.\(^7\)

   Therefore, the objective function behaves like the power utility function over consumption.

2. At high levels of wealth the importance of the status contribution to utility declines as

\(^7\)As long as the equilibrium distribution of wealth is not “too concentrated” at zero, i.e. $\lim_{x \to 0} \tilde{F}(x) < \infty$. 

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\( W^i_t \to \infty \). Since empirical distributions of wealth have compact support, there always exists a \( W^\text{Max}_t \) such that for all \( W^i_t > W^\text{Max}_t \) social status is equal to 1, the maximum. Therefore, the closer an individual is to be ranked the highest, the higher the probability that an increase in wealth will leave status unaltered. Consequently, the lower sensitivity of the marginal utility of wealth to the relative wealth position reduces the incentive to deviate from the well-diversified benchmark portfolio that is optimal for a neoclassical investor.\(^8\)

It follows that we can expect the under-diversification effect induced by status preferences to be the strongest in the middle of the wealth distribution. This prediction appears to have been anticipated by ?: “Intermediate income groups might be expected to hold relatively large share of their assets in moderately speculative common stocks and to furnish a disproportionate fraction of entrepreneurs.” (p. 302). Note, however, that the “intermediate” region can include very rich households. At what levels of wealth does the status effect abate is a quantitative question, without an \textit{a priori} theoretical benchmark. The declining effect of status on risk attitudes at high wealth levels is however an important feature of the model. The fact that the utility of status is always bounded from above by one allows me to consider preferences that are convex in status without the undesirable consequence of unlimited gambling by risk-loving agents that can be present in a model with a global non-convexity.

### 3 Quantitative model

The risk attitudes of status-conscious investors depend critically on the shape of the reference distribution of wealth. Thus, in order to evaluate the quantitative implications of these preferences the distribution needs to be matched to the data. Moreover, the wealth distribution in the model is endogenous and must be consistent with individual optimization in equilibrium. In this section I describe the equilibrium concept associated with the dynamic version of the model, as well as the computational strategy employed in solving for an approximate equilibrium numerically.

\(^8\)Note that the effect of status-seeking on the consumption-saving behavior does not decline at high wealth levels. On the contrary, the higher utility level associated with the greater relative wealth acts as a negative rate of time preference, implying that the saving rates are monotonically increasing in wealth.
3.1 Equilibrium Wealth Distribution

In order to close the model I need to solve for the wealth distribution jointly with the consumption and investment policies. The model economy is populated by $T + 1$ overlapping generations indexed by age $a$. For simplicity I assume that the average wealth is the same across generations. After agents die their bequeathed wealth is considered “consumed”. This is without loss of generality, since in every period the newly born agents receive wealth endowments that follow the same distribution as the wealth of the old agents.

Aggregation of optimal choices of households given an “initial” (time $t$) wealth distribution $\tilde{F}_t$ yields the equilibrium wealth distribution in the aggregate state $\tilde{I}$

$$\tilde{F}_{t+1}(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ \frac{(W^i - C^i) a'_i(R_{t+1})}{W_{t+1}} \right] \varphi \left( R^i \mid \tilde{I}; I_t \right) dR \left( \tilde{I} \right) d\mu \left( i \right),$$

where $R^i$ is the return on the $i$'th agent’s idiosyncratic asset, so that the ex-post return for the agent with relative wealth $x$ at time $t + 1$ must be

$$R(x) = \left( \frac{xW^i_{t+1}}{W^i_t - C^i_t} - R^a \left( \tilde{I} \right) a^a - R^f \left( 1 - a^a - a^i \right) \right) / a^i,$$

and the average wealth is

$$\bar{W}_{t+1} \left( \tilde{I} \right) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ \frac{(W^i - C^i) a'_i(R_{t+1})}{W_{t+1}} \right] \varphi \left( R^i \mid \tilde{I}; I_t \right) dR \left( \tilde{I} \right) d\mu \left( i \right).$$

The main difficulty in solving the portfolio problem with social status is that, even in the setting where asset prices are set exogenously, preferences over wealth depend on the entire distribution of wealth, which is endogenous. I require that the wealth distribution perceived by agents is consistent with the one resulting from their optimal choices. This leads to a notion of equilibrium in consumption and portfolio strategies. Let the “expected” or perceived future wealth distribution used by the agents to form expectations of their status next period be denoted $F_{t+1}$. Household optimization implies a mapping from the probability measure $\lambda$ over the perceived future distributions $F_{t+1}$ to the space of probability measures over realized future wealth distributions, $\pi \left( \{ \tilde{F}_{t+1} \} \right)$, conditional on the current wealth distribution $\tilde{F}_t$ and the other state
variables:

\[ \Upsilon \left( \tilde{F}_t, I_t, X_t \right) : \lambda \rightarrow \pi \left( \left\{ \tilde{F}_{t+1} \right\} \right) \]

A measurable map from the current state to the space of probability measures over wealth distributions and all the other state variables \((\tilde{F}, I, X) \in S\) gives a law of motion (transition) operator

\[ T : (\tilde{F}_t, I_t, X_t) \rightarrow \mathcal{P}(S). \]

**Definition 1.** A recursive status/investment equilibrium consists of

- household optimal policies \([c^*, a]\), given expected distribution \(F_{t+1}\)

- law of motion operator \(T\)

- probability measure \(\lambda\) which is consistent with the law of motion:

\[ \lambda = T \left\{ \left( \tilde{F}, I, X \right) \right\} \bigg|_\pi \]

- a subset \(S' \subset S\) such that for all states \((\tilde{F}, I, X) \in S', \lambda\) is a fixed point of \(\Upsilon \left( \tilde{F}, I, X \right)\):

\[ \Upsilon \left( \tilde{F}, I, X \right) \lambda = \lambda \]

In general, there is no guarantee that such an equilibrium exists, or that it is unique. This equilibrium concept is related to the notion of a *Stable Wealth Distribution* introduced by ?, who prove existence in a one-shot setup where zero-sum gambles are the only source of uncertainty. Unlike the latter case, the equilibrium defined above does not require that households turn down any fair gambles. This difference reflects the fact that I do not allow households access to complete contingent claims markets or contemporaneous lotteries. The latter assumption is relaxed in Section 5 in order to show that it does not effect the results substantially. ? extend the model of ? to a deterministic dynamic environment by ruling out convexity in status. For a general analysis of time-homogeneous Markov equilibria in a variety of settings, including incomplete markets and overlapping-generations economies, see ?.
3.1.1 Dynamic optimization and scale-invariance

For this purpose it is convenient to restate the problem in a way that exploits scale-independence. Let

$$\tilde{c}_t^i = \frac{C_t^i}{W_t}, s_t^i = \frac{W_t^i}{W_t}, G_{t+1} = \frac{\bar{W}_{t+1}}{W_t}. \quad (3.1)$$

Then the value function (2.3) above can be written as

$$V(W_t^i, \bar{W}_t, a_t^i; I_t) = v(s_t^i, a_t^i; I_t)\bar{W}_t^{1-\gamma}, \quad (3.2)$$

where the scale-invariant function $v(s_t^i, a_t^i; I_t)$ solves the corresponding recursive problem (see appendix B).

If the derived utility of wealth is not concave everywhere, the inflexion points also scale up with aggregate wealth, since for the derivatives of the value function with respect to wealth we have

$$V_W(W_t^i, \bar{W}_t, a_t^i; I_t) = v_s(s_t^i, a_t^i; I_t)\bar{W}_t^{-\gamma}$$

$$V_{WW}(W_t^i, \bar{W}_t, a_t^i; I_t) = v_{ss}(s_t^i, a_t^i; I_t)\bar{W}_t^{-\gamma-1}$$

The local curvature of the value function (i.e. the relative risk aversion coefficient) is invariant to changes in aggregate wealth.

$$RRA(s_t^i, a_t^i; I_t) = -\frac{W_t^iV_{WW}}{V_W} = -\frac{s_t^i v_{ss}}{v_s}. \quad (3.3)$$

3.2 Numerical solution

The equilibrium notion introduced above implies that the state space, which includes the space of wealth distributions, is potentially infinite-dimensional. Thus finding such an equilibrium in practice is infeasible. Instead it is useful to consider a much more restricted set of approximate "steady-state" equilibria in which the households’ optimization problem is solved as if the distribution of relative wealth was constant over time, i.e. the wealth distribution $F$ is such that (with
some abuse of notation denoting by $\delta_G$ the probability measure assigning the full mass of 1 to the function $G$):

$$\lambda = T|_\pi (F) = \delta_F$$

This modification of the problem allows to reduce the relevant state space dramatically. Of course, it is not necessarily innocuous, since one cannot guarantee that the wealth distribution does not shift over time, and in fact it does, provided sufficient heterogeneity in asset holdings. Thus, one needs to ensure that the optimal policies obtained under the assumption of a fixed wealth distribution are sufficiently close to those obtained when the endogenous perturbations in the distribution are accounted for. I achieve this approximation by using the steady-state distribution that arises under the ergodic measure. Let $\mu$ be a probability measure associated with the wealth distribution $\tilde{F}$; then the steady-state distribution is given by the measure $\mu^*$ such that

$$\mu^* = \lim_{j \to \infty} T^j|_\pi \mu.$$  

The numerical approximation procedure therefore consists of solving the individual optimization problems, updating the resulting steady-state wealth distribution and the law of motion of aggregate wealth by simulating the optimal policies forward until the average wealth distribution converges. Such iterations are repeated until both the steady-state wealth distribution and the law of motion for aggregate wealth converge. This approach is related to the now standard method of solving dynamic general equilibrium models with heterogeneous agents introduced by ?. In order to reduce the dimensionality of the state space they suggest using forecasting regressions to capture the dynamics of the first few moments of the wealth distribution. In the social status model, a small set of moments is not sufficient to describe the dependence of optimal policies on the wealth distribution. This is because the curvature of the wealth distribution at each point determines risk preferences of agents under the social status model, whereas in standard models wealth distribution enters preferences only indirectly through the investment opportunity set. While I focus on approximating the equilibrium wealth distribution around a “steady state”, instead of approximating it with another time-varying distribution with simpler dynamics, as in ?, both approaches admit similar interpretations. In particular, while they are purely computational devices used to obtain approximate solutions, they both can be thought of as allowing for a mild
form of bounded rationality on the part of the agents, who use a subset of a large investment opportunity set to make their consumption and portfolio decisions. Moreover, my solution does require agents to forecast the rate of growth of aggregate wealth. However, in addition, it requires the agents to make sure they do know the entire shape of the (detrended) wealth distribution, if only at the steady state. This is similar to the refinement of the Krussel-Smith method by ?.

Further details of the computational procedure are provided in the appendix.

3.3 Parametrization

I solve the model for \( T = 7 \) periods so that each period corresponds to a 10-year investment horizon. Thus, if the youngest agents enter the model at age 20 then the last decision-making period corresponds to the age of 80. Table I lists the prespecified parameters of the investment opportunity set as well as the benchmark values of preference parameters. The unconditional moments of the stock return and the risk-free rate (i.e., 10-year bond yield) are chosen to match the U.S. data, at annualized values (for corresponding logarithmic returns) of 11 and 5 percent, respectively. The risk-free rate is assumed constant. The equity returns are assumed to be i.i.d. and uncorrelated with each other. I assume that the expected excess return on the idiosyncratic asset is no higher than the public equity premium, consistent with the findings of ?. I assume that the standard deviation of idiosyncratic project/private equity return is three times as high as that of the public equity, which is similar to the volatility of publicly traded individual stocks (see ?). This implies annualized standard deviations of public and private equity logarithmic returns of 15 and 45 percent, respectively. ? and ? consider similar volatility levels in calibrating entrepreneurial project hurdle rates.

I assume a discrete two-state distribution for the public equity return. I let idiosyncratic states follow a lognormal distribution and use Gauss-Hermite quadrature (with 10 nodes along the idiosyncratic dimension) to evaluate expectations.\(^9\) ? find that entrepreneurs face an approximately 10% chance of losing their entire investment within the first ten years of operation. I find that allowing for a positive probability of a “failure state,” in which idiosyncratic return is equal to zero, does not alter the results substantially, except for reducing the allocation to private equity by the poorest households (whose preferences do not differ substantially from standard power

\(^9\)See ? for a general discussion of numerical integration.
utility). The initial wealth distribution used as a starting point for the iterative procedure is calibrated using the percentiles of the U.S. wealth distribution from the 2001 Survey of Consumer Finances.

The preference parameters are calibrated to match the empirical facts about household portfolio allocation as described in the next section.

4 Evaluation: model vs. data

In this section I evaluate the ability of the social status model to explain quantitative as well as qualitative features of the data. I solve the model for a range of parameter values and then analyze its predictions for individual consumption and portfolio allocations as well as simulated macroeconomic aggregates. The data sources used are: Survey of Consumer Finances (SCF), for information on households asset holdings and the cross-sectional distribution of wealth, and Panel Study of Income Dynamics (PSID), for the evolution of household wealth over time. The details on the use of these datasets are described in the appendix. The estimates of consumption growth volatility of stockholders (and non-stockholders) are from and are based on the household consumption data from the Consumer Expenditure Survey (CEX).

4.1 Calibrating the model

The rate of time preference is set to 0.97 throughout. The benchmark values of preference parameters are chosen as to match the two key features of the data - risk-taking and portfolio concentration - while ensuring that there is no risk-seeking behavior. The latter requirement suggests $\zeta = 1$, which leaves two free parameters: $\gamma$ and $\eta$. The empirical measures of risk-taking and diversification are averages taken over the subsample of households that report positive holdings of one of the following: directly held individual stocks; private business; investment real estate. My empirical analog of “private equity” in the model is the single largest asset from the above list owned by a household. Risky assets also include, besides those described above, stock held through mutual funds, corporate bonds, and various other similar securities. See Appendix D for details of data construction.

The following combination of parameters allows the model to closely match the average allocation to risky assets and the average share of asset invested in “private equity”: consumption
curvature $\gamma = 5$ and status weight $\eta = 10000$. The latter number might appear large, although one must keep in mind that it multiplies status levels between zero and one, which are potentially small in magnitude compared to utility derived from consumption (which is unbounded from below).

Table II demonstrates the fit of the benchmark calibration. The calibrated social status model replicates the average shares of total assets allocated to risky assets and, in particular, to “private equity” almost exactly: the former is just under 40 percent in both data and model, whereas the latter is about 22 percent. The fit for the share of risky assets concentrated in the idiosyncratic asset is somewhat worse: 63 percent in the data versus 77 percent in the model. Nevertheless, the magnitudes are sufficiently similar to conclude that the model can broadly match the level of risk taking and the degree of portfolio concentration simultaneously.

4.2 Portfolio allocation in the cross-section

In order to further evaluate the model’s ability to explain portfolio allocation decisions I consider the variation in the degree of portfolio concentration across the wealth distribution. I present summary statistics of model-generated portfolio allocations across the wealth distribution alongside their empirical counterparts obtained from the SCF in table 6.

For both the empirical and the model quantities I again calculate the share of assets invested in the single largest risky holding. Since there is no housing in the model but in the data housing is a major component of household assets I use two definitions of assets: one that includes housing and one that does not. Both the model and the data exhibit upward sloping profile of private equity portfolio concentration. In the data, the fraction of households that own an idiosyncratic asset with a known value rises tenfold from 5% in the bottom quartile to 60% in the top decile. Conditional on owning such asset, the poorest quarter of households invest on average about one fifth of assets in it. The allocations to idiosyncratic asset increase with wealth, reaching about 30 percent in the top decile. When non-housing assets are used the share pattern is slightly nonlinear, rising to over a third in the middle of the distribution and then decreasing slightly in the top decile. In the model the variation in private equity allocations is of the same sign, but is more pronounced quantitatively. In the bottom quartile of the distribution only just over 10 percent of wealth is invested in the idiosyncratic asset, whereas this share rises to almost 40

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percent in the top decile. The fact that the model generates greater concentration of assets in the idiosyncratic security at the top of the wealth distribution than is apparent in the data should be interpreted with caution. In the data, the risky assets other than the largest asset are likely to also carry substantial amounts of idiosyncratic risk (e.g., in the data an average stock holding constitutes about one fifth of risky assets, which means that many such portfolios are concentrated in a few individual stocks). Therefore, as a measure of under-diversification the share of assets devoted to the largest one is downwardly biased.

The prediction that allocations to private equity increase with wealth could be consistent other models with wealth in the utility function, such as the “spirit of capitalism” model of ?, and even with more standard models that feature decreasing relative risk aversion, e.g. due to nonhomothetic utility functions. Motivated by models of this class ? find that college students from wealthier families choose riskier careers, while ? examines the household consumption data from the CEX and finds that richer households have higher consumption volatility than the poorer ones. Both of these findings are consistent with the predictions made here. However, these standard models predict that household financial portfolio are well diversified. The distinguishing feature of the social status model is that it is able to capture heterogeneity in risk taking and under-diversification simultaneously. The bottom panel of table 6 demonstrates what the allocations to private equity would be in a standard power utility model with different risk aversion coefficients. In particular, it shows that even for risk aversion as low as $\gamma = 3$ the share devoted to the idiosyncratic asset is only 14 percent, just about the lowest of the allocations observed in the data. Lowering risk aversion even further might bring the amount of private equity investment up slightly, but at a cost of generating implausibly high amount of risk taking and highly levered portfolios.

4.3 Short-selling

The calibrated model implies that at the high wealth levels households invest more than 100 percent of their risky asset into private equity. This prediction is not surprising in light of the fact that “getting ahead of the Joneses” implies a motive for hedging away aggregate risk. If the status preference is strong enough, this hedging demand creates short positions in public equity. This prediction does not appear to be borne out in the data, where the shares of private
equity as a fraction of risky (not total) assets either rise slightly or decline with wealth, especially for the business-owning households. The latter fact might not be surprising if private business investments are lumpy and liquidity-constrained entrepreneurs are forced to devote larger shares of their wealth to their businesses than they would without the constraints. Since the model abstracts from these frictions we cannot expect it to explain the declining share of private equity (see section 5 for the discussion of how this evidence could be reconciled with the social status model). The fact that the model does not explain empirically observed underdiversification at the bottom of the wealth distribution might not be a very serious drawback, however. As ? find, although richer households are better diversified than the poor, their tendency to hold riskier portfolios raises the economic importance of underdiversification. Thus, if “investment mistakes” made by the poor are less costly than those made by the rich, the latter are more puzzling to an economist.

One way to reconcile the predictions of the social status model with the empirical fact that few entrepreneurs have significant short positions in public equity would be to rule out such positions outright. To this end I solve a version of the model with a no-short-sales constraint. The constrained model produces essentially the same average allocation to private equity, but a substantially higher average allocation to total risky assets of 59 percent (not reported in the table), since public equity position are restricted to be nonegative. It is not necessarily the case that the prediction of negative allocations to public equity is counterfactual, especially for entrepreneurs, if private business investment risk is negatively correlated with public stock market returns. The degree to which individual project risk covaries with the aggregate risk and can conceivably be controlled by the entrepreneur, instead of being exogenously fixed to be purely idiosyncratic, as in the model. This can be achieved either through the choice of financial structure (e.g. as described in ?) or by exploiting production technologies that allow substitution between states of nature (as described in ?). Indeed, there is some evidence that the nontradeable components of wealth, such as human capital and private equity, are exposed to aggregate shocks that are negatively correlated with per capita wealth. As ? argue, the return on (aggregate) human capital is negatively correlated with the public stock market returns. Thus entrepreneurs, whose human capital is a substantial part of their total wealth, might have a built-in hedge against aggregate wealth risk.
4.4 Wealth distribution and social mobility

While it is well known that the distribution of household wealth in the U.S. is extremely wide and highly concentrated, there is also a substantial amount of cross-sectional wealth mobility over time. I estimate 10-year transition probabilities of wealth deciles following [1] using data from the Panel Study of Income Dynamics (PSID). They estimate transition probabilities using the PSID wealth supplements over the period 1984-1994. I update their estimates with data from the 1999 supplement. I adjust the estimated transition rates to limit the influence of measurement error and, most importantly, to remove the effects of labor income shocks as well as of the life-cycle accumulation/decumulation of assets that are absent from my model, in order to provide an appropriate benchmark for evaluating the model’s predictions. Details of this estimation can be found in the appendix.

Table IV displays the fractions of households in a given wealth group that are moving upwards (into a higher percentile group), moving downwards, and staying in the same group after a ten-year period. I divide the households into the following wealth quantiles based on their wealth rank: the lower half of the distribution; the 50th through the 90th percentile; the 90th through the 95th percentile; the 95th through the 99th percentile; and the top 1 percent of the wealth distribution. I use a finer grid of wealth percentiles at the top of the distribution because households in the highest brackets own a disproportionate share of total wealth, including stock market wealth and private equity. Moreover, my model is designed primarily to explain investment returns accruing to these richer families, warranting closer attention to their wealth dynamics.

As shown in the table IV (top panel), the empirical wealth distribution displays a great deal of mobility, both between the upper and the lower halves, and, in particular, within the upper end of the distribution. Of the richest 1 percent of households, one third fall behind their peers over the course of ten years, once the life-cycle effects and exogenous shocks are accounted for. Among the poorer half of the wealth distribution, just over 10 percent make it into the upper half. Note that mobility at the top of the distribution requires greater variance of wealth, since the wealth brackets between percentiles are wider. This suggests that the high degree of mobility, especially at the top of the wealth distribution, is largely due to heterogeneity in investment returns, rather than merely measurement error.

In order to evaluate the model’s ability to explain these empirical patterns of wealth mobility
I generate artificial data from the calibrated model. I use the numerical solutions for optimal consumption and investment policies that are used in calibrating the model to calculate the transition matrix for the wealth deciles via simulation. The quantitative features of the transition distribution are summarized in table IV alongside the empirical estimates.

The social status model is able to generate patterns of social mobility that very closely mimic those in the data for the top four deciles of the wealth distribution. The artificial data produced by the benchmark calibrated model ($\gamma = 5$, $\eta = 10000$, $\zeta = 1$) can match the persistence of wealth rank for the richest 1 percent of households, with one third of them falling behind over a 10-year period. The model is also quite successful at explaining the transition dynamics for the next two richest groups (95th through 99th percentile and 90th through the 95th percentile). For the former it overstates the persistence by over 10%, but matches the rate of upward mobility observed in the data (at about 5 percent). For the latter quantile the model closely matches the rate of downward mobility at about 45 percent but understates upward mobility by 10 percent. For the lower half of the wealth distribution the model does not generate as much mobility as is observed in the data, with only 4 percent moving into the upper half (compared with almost 11 percent in the data). This is not surprising, since the allocation to private equity (and risky assets in general) is much lower for these households (both in the model and in the data). Section 5.5 discusses various ways in which the model could be extended to match this aspect of the data more closely.

As demonstrated in the subsequent panel of table IV, relaxing the short-selling constraint does not affect ability of the model to explain wealth transitions substantially. However, both the unconstrained and the constrained calibrations of the social status model are in stark contrast with the predictions of the standard portfolio choice model with no status concerns.

In comparison with the predictions of my model, a portfolio choice model with standard preferences does not generate nearly as much wealth mobility and is therefore not able to match the data. For the top wealth percentile the probability of staying in the same group implied by the standard model is 90 percent, and for all of the other groups it does not fall much below 80 percent. This is not surprising, since standard preferences imply that investors hold well-diversified portfolios. Consequently, given the calibration of asset returns used here, the optimal portfolios contain little idiosyncratic risk, implying low cross-sectional wealth mobility.
4.5 Wealth concentration

Although I use the empirical wealth distribution as a starting guess in solving the model, the equilibrium solution does not necessarily coincide with it. Making sure the discrepancy between the two is not too large is therefore an important diagnostic of the model’s empirical success.

I assess the extent to which the model-generated population exhibits the extreme concentration of wealth at the right tail of the distribution that is present in the U.S. data. Table VI compares a set of quantities reflecting wealth concentration in the U.S. SCF data with those produced by the model. The statistics are the Gini coefficient, which is a standard measure of wealth inequality, as well as the shares of aggregate wealth held by the richest 1% of households, the richest 10%, 0.1%, and the top 0.01%. I also report a Pareto power law tail exponent fitted to the top 1% of the distribution. It is well known that the behavior of the extreme upper tail of the wealth distribution is described well by a power law \( \Pr(W > x) \propto x^{-\alpha} \), where \( \alpha \) is the tail exponent, typically estimated to be in the range \([1, 3]\). In 2001 the SCF estimate of the exponent is 1.8. As is typical, the distribution is highly concentrated - the top 0.01% of households control almost 3 percent of total net worth and the top 10% own over 60 percent of the total wealth.

The model generated wealth distribution statistics are fairly close to the empirical quantities. The main discrepancy is that the model tends to generate higher concentration of wealth in the top 10% (between 65 and 84 percent of the total), with concentration at the very top 0.01% varying between about 1 and 5 percent of the total wealth. The tail exponents are very similar as well - between 1.5 and 2. The benchmark no-status model with the same parameter values generates slightly less wealth concentration than is present in the data and than the status model does, due to less pronounced “over-saving” motive among the rich and less risk-taking. This result is consistent with the argument of that the heavy-tailed Pareto-type wealth distributions are due to heterogeneity of individual investment returns. Note that unlike I do not assume that investment returns themselves follow a power law. This is not necessary, since idiosyncratic lognormally-distributed shocks can lead to power-law tail behavior asymptotically (?).

4.6 Saving and consumption dynamics

The model generates considerable heterogeneity in saving rates. The optimal consumption-wealth ratios reported in table V show that the richest 10% of the households consume a much smaller
function of their wealth than the poorest half, and consequently save more. The youngest households at the bottom of the wealth distribution consume 45 percent of their initial wealth (over a 10-year period), as do power utility households. The richest 10 percent of the young (e.g., 20-year olds) consume only 11 percent. The difference is even more dramatic for the old households: the poorest 25 consume 60 percent of their wealth in the second-to-last period of their lifetime (i.e. at age 80), while the richest 5 percent still consume about 12 percent, thus leaving a disproportionately large amount of wealth for the last period. This prediction of the model is consistent with the stylized empirical observation that the rich elderly do not dissave as predicted by the standard life-cycle model (e.g. see ?). The intuition for the high saving rate among the very rich is that the future status utility acts to offset the discounting effect of $\beta$ by putting more weight on the utility of future wealth relative to present consumption (?, ? discuss the “oversaving” effects generated by relative wealth concerns). Quantitatively, the magnitudes of consumption-wealth ratios might appear quite large.

Does the social status model imply unrealistically volatile consumption growth? Table VII compares the standard deviations (annualized, in percentage points) of average logarithmic consumption growth across wealth groups generated by the model for a range of parameter values with those from the U.S. data. The latter are based on estimates for aggregate U.S. consumption as well as the estimates obtained using micro data from the Consumer Expenditure Survey (CEX) (see ?). These comparisons should be viewed with some caution, since the model numbers are based on 10-year periods, where as consumption data is based on quarterly consumption growth observations (I use estimates for quarterly consumption growth over 1 year and 5 year periods). Using consumption data aggregated over 10 years might be problematic due to the short sample available. Still, since aggregate consumption growth is close to a random walk at the annual frequency, this problem might not be too severe.

In both cases (data and model) the reported standard deviations of consumption growth reflect only systematic consumption risk due to group averaging. The reason I compare only the undiversifiable components of consumption volatility is that measuring idiosyncratic volatility empirically is a daunting task, and standard off-the-shelf estimates are still lacking. Thus I opt to comparing reliable empirical estimates of systematic consumption volatility provided by ? with their model

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10 The wealth accumulated by age 90, can be considered either consumed over another 10-year period, or else bequeathed, if we assume that the terminal date is at the end of the last active period.
counterparts. Besides the economy-wide aggregate I consider, in the data, consumption growth of stockholders and non-stockholders, as well as the richest third of stockholders and, in the model, the bottom, middle, and top thirds of the wealth distribution. When preferences are linear in status ($\zeta = 1$) the generated volatility of consumption growth is about 11 percent, no higher than under the neoclassical portfolio choice model with the same utility curvature $\gamma = 5$. Constraining the share of risky assets invested in private equity to be no greater than one increases consumption growth volatility, since investors are not able to hedge away the systematic risk exposure. Aggregate consumption growth volatility in this case is almost 13 percent. Qualitatively, group consumption volatility is increasing in wealth, which appears consistent with empirical evidence.

While the social status model substantially overstates the volatility of aggregate consumption, it produces a better trade-off between generating high cross-sectional wealth mobility and smooth consumption growth than standard neoclassical models. Producing a comparable degree of social mobility within a model with standard preferences would require a very low relative risk aversion, and consequently an implausibly high amount of risk-taking in the economy. The latter would lead to a substantially higher consumption growth volatility than that generated by the social status model.

5 Extensions

In this section I discuss several extensions of the “getting ahead of the Joneses” model of social status. I argue that the main predictions about underdiversification and wealth mobility are not highly sensitive to the specific assumptions about asset markets (as well as the status “market”) made in order to obtain quantitative solutions. I further show that relaxing some of these assumptions might help broaden the applicability of the model for explaining empirical phenomena.

5.1 Asset market structure

Throughout the analysis above I have restricted the investment opportunity set to 3 types of assets: risk-free, aggregate public equity (with returns modelled as a binomial process) and individual-specific “private” equity. A potential objection is that the model’s predictions hinge on the set of assets being limited and might not be robust to introduction of other assets such as zero-sum gambles between agents, contingent claims and multiple common assets. Indeed, such assets
could be used by agents to generate idiosyncratic risk and, especially in the case of non-concave derived utility of wealth, might potentially crowd out investment in individual-specific projects. The objective of this discussion is twofold. First, I show that restricting the status curvature parameter \( \zeta = 1 \) and calibrating the wealth distribution to the data indeed produces preferences that exhibit aversion to pure wealth gambles, and, therefore, the quantitative results above are not sensitive to introducing fair bets. Second, I argue that expanding the set of technologies in this way does not alter the main qualitative and quantitative predictions of the model, but it does help extend the model’s intuition to empirically relevant forms of non-diversification.

In this section I consider the one-period version of the model to simplify analysis, so that households maximize

\[
V^i_0(W^i_0, \bar{W}_0; \tilde{F}_0) = \max_{\alpha} E \left\{ \left( W^i_0 \right)^{1-\gamma} - 1 \right\} + \eta \bar{W}^{1-\gamma} \tilde{F} \left( \frac{W^i_0}{\bar{W}} \right)^\zeta \right\}
\]

(5.1)

where \( W^i_0 \) is initial wealth and the initial wealth distribution is characterized by \( (\bar{W}_0, \tilde{F}_0) \).

There is little prior information that can be used in determining what values of status preference parameters are reasonable. In this section I allow a wide range of values, between 0 and 40 for the curvature parameter \( \zeta \).

5.1.1 Lotteries and gambles: calibration

In order to make sure that the parameter values used in calibrating the quantitative model do not produce implausible amounts of risk-taking, I evaluate them using a series of simple gambles presented in table VIII. Panel A of the table lists the gambles: their payoffs, in dollar amounts, probabilities, and resulting expected payoffs (net present values). Panel B displays the responses of power-utility consumer with risk aversion coefficient \( \gamma = 5 \) and varying initial wealth levels to these payoffs, where “+” means the gamble is accepted and “-” means it is declined. All zero or negative NPV gambles are declined, as is the very skewed, low NPV gamble 5. Positive NPV gambles with large loss probabilities are only accepted by rich individuals (wealth of $1 million and higher).

Introducing preferences for status does increase the propensity for gambling. Setting status weight \( \eta = 1000 \) without introducing convexity over status \(( \zeta = 1 \)\), as in panel C of table VIII raises the uptake of positive-NPV gambles among the less wealthy individuals. An average person
(i.e. total wealth of slightly less than $500000) is willing to take an equal chance of losing $50000 and winning $60000, while the same gamble with the upside of $75000 is taken by even those with half as much wealth. Fair gambles are taken only by the very rich (since the wealth distribution is approximately linear locally at high wealth levels), and negative-NPV lotteries are declined at all wealth levels. Introducing increasing marginal utility of status (\(\zeta = 4\) as in panel C) shifts the propensity to take fair bets from the rich to the middle and lower-middle class (increasing \(\eta\) plays only a secondary role). Interestingly, gamble 5 that has a low but positive NPV as well as high positive skewness is not taken at any wealth level. Increasing convexity of utility over status to \(\zeta = 40\) (panel E) creates demand for this gamble among most of the intermediate-wealth as well as the wealthiest individuals. Also, those with close to average wealth are willing to take gamble 6 that has most lottery-like payoffs (with extremely small chance of a large winning and a small negative NPV). Thus for large values of the status weight \(\eta\), high degree of risk-seeking over status (\(\zeta = 40\)) is able to explain lottery participation better than the intermediate value \(\zeta = 4\). Overall, status preferences calibrated using parameter values discussed above appear to generate plausible risk-taking behavior.

5.1.2 Gambling-proof equilibrium

Allowing for contemporaneous lotteries or side-bets, i.e. gambles that agents can enter after aggregate uncertainty is realized, is equivalent to considering ex-ante gambles with payoff distributions that are state-contingent (in what follows I will use the term “lotteries” to describe all such gambles). Then the maximization problem (5.1) can be solved in two steps. First consider the pure lottery problem

\[
V^i(W^i, \bar{W}; \bar{F}) = \max_{\varphi} \mathbb{E} \left[ \tilde{V} \left( \tilde{W}^i, \tilde{W}; \tilde{F} \right) \right], \quad \text{where} \quad \tilde{V} \left( \tilde{W}^i, \tilde{W}; \tilde{F} \right) = \frac{\left( \tilde{W}^i \right)^{1-\gamma} \cdot 1 - 1}{1 - \gamma} + \eta \tilde{W}^{1-\gamma} \tilde{F} \left( \frac{\tilde{W}^i}{\tilde{W}} \right)^\zeta, \quad \text{subject to} \quad W^i = \int \tilde{W}^i \varphi \left( \tilde{W}^i; W^i \middle| \bar{W}; \bar{F} \right) d\tilde{W}^i
\]
and where the family of probability distributions with densities $\varphi \left(\cdot, W^i\right)$ describes the fair gambles over wealth conditional on the realization of aggregate uncertainty. Second, the portfolio choice problem becomes

$$V^i(W^i_0, \tilde{W}_0; \tilde{F}_0) = \max_\alpha E \left[V^i(W^i, \bar{W}; \bar{F})\right]$$

subject to the resource constraint

$$W^i = W^i_0[\alpha R^i + (1 - \alpha) R^a].$$

I ignore the risk-free rate in this specification of the problem for simplicity. Solving both parts of the household is not feasible, since the strategy space of all admissible fair lotteries is very large. However, this two-stage formulation of the problem implies that portfolio choice itself is not effected by the potential non-concavities in the objective, since they are eliminated at the second stage via lotteries. This is because

$$V^i(W^i, \bar{W}; \bar{F}) \in \text{Bdcon}\left\{(w, v)| w \in (0, \infty): v \leq \bar{V}\left(w, \bar{W}; \bar{F}\right)\right\}$$

and, in particular, if the objective function (5.2) is itself concave then no fair gambles are accepted (e.g. see [11] for an extensive discussion and proof of this result).

Therefore, we can use this fact to solve for optimal portfolio allocations (second stage). In particular, suppose that the equilibrium wealth distributions with and without lotteries coincide. Then the objective function in (5.1) equals the objective in (5.4), i.e. $V^i(W^i, \bar{W}; \bar{F})$. Computationally, the problem can be solved as follows:

1. find the equilibrium distribution without lotteries by solving (5.1) that would yield an equilibrium wealth distribution $F^*$

2. obtain a guess for the function $V^i(W^i, \bar{W}; \bar{F})$ by computing the convex hull of the objective in (5.2) using $F^*$ in place of the equilibrium distribution $\bar{F}$

3. solve (5.4) using this guess, obtain new endogenous wealth distribution $F^{**}$

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4. repeat step 2 using $F^{**}$ and iterate steps 2 and 3 until convergence $\implies$ obtain equilibrium wealth distribution $\tilde{F}$

5. verify that $\tilde{F}$ is close to the original distribution $F^*$

The assumption that the two distributions coincide is certainly not always tenable; in particular, it will be violated in the cases in which introducing lotteries alters the optimal portfolio allocations significantly. However, in the cases where non-concavity in the objective function is not the driving force behind underdiversification introducing lotteries will not alter the allocations substantially and we can expect the two distributions to be fairly close to each other. In particular, consider the case $\gamma = 3$, $\zeta = 4$, $\eta = 10000$. Figure 1 (panel A) plots the optimal allocations to private equity (simulated for 10000 agents) that correspond to the two equilibrium wealth distributions (with and without lotteries, depicted using solid and dash-dotted lines, respectively). We can see that the patterns are very similar, in both cases featuring a pronounced hump shape profile of private equity holdings over the range of relative wealth. The portfolio shares are under 20% for the bottom two deciles of the wealth distribution, increasing in the middle range up to almost 100% in deciles 6 and 7 and then falling to as low as 10% in the top few percentiles. Indeed, the simulated wealth distributions are very close, both having Gini coefficient of 0.75.

However, while this is a typical result for low values of status curvature, it does not hold generally. In particular, for values of $\zeta$ as high as 400 convexity becomes very important. Consequently, introducing lotteries alters portfolio allocations substantially yielding different wealth distributions that do not satisfy the above assumption. In some cases, however, introducing lotteries does not lower but increase allocations to private equity, at least for some agents. The case in point is $\gamma = 5$, $\zeta = 40$, $\eta = 10000$, shown in Figure 1 (panel B). While the general hump-shaped pattern of portfolio shares is similar, it appears that for the lower-wealth agents on the left side of the hump introducing fair gambles shifts the demand for individual-specific risk upwards, not downwards, like it does for the riche households. This shift is likely due to the slightly different shape of the wealth distributions that prevail in two cases (while Gini coefficients are virtually the same, the second moment of the gambling-proof distribution is slightly higher).

It is important to note that while the portfolio allocation results obtained in the status model might not always be gambling-proof, the central argument advanced here still stands. That is, demand for social status can alter risk preferences in a way consistent with agents endogenously
Choosing higher exposure to idiosyncratic risk than predicted by neoclassical models. Whether through exposure to individual-specific entrepreneurial or human capital risk or through artificially manufactured gambles and trading strategies (both of which are empirically plausible) the model generates cross-sectional wealth mobility that could potentially contribute to our understanding of the empirical distribution of wealth.

5.1.3 Multiple common assets

Consider the one-period problem (5.1) above with a following set of assets: the individual specific asset \( R^i \) as above and two publicly tradable risky assets \( R^{a1} \) and \( R^{a2} \). Let both of the public return be independent of each other and have a two-state distribution, so that there are four aggregate states. I calibrate these returns so that they are identically distributed and that the moments of the “market” return

\[
R^m = \frac{1}{2} R^{a1} + \frac{1}{2} R^{a2}
\]

coincide with those of the aggregate equity return used throughout.

The resource constraint is

\[
W^i = W_0^i [\alpha^i R^i + (1 - \alpha^i)(\alpha^1 R^{a1} + (1 - \alpha^1) R^{a2})],
\]

so that the optimal portfolio allocation consists of \((\alpha^i, \alpha^1)\) that maximizes the expected utility above.

I follow the same computational approach to solving the model as above with one modification. Since both of the public equity returns are identical, ties may arise in finding the optimal allocation if the “mean-variance efficient” portfolio combination of two public returns (one half in each) is not optimal for a given agent. I interpret such ties as mixed strategies. In simulating the resulting wealth distribution I assign equal probability to allocations that tie for the optimum for a given agent.

Figure 2 presents optimal portfolio allocations across wealth percentiles for the case \( \gamma = 5, \eta = 10^5, \zeta = 40 \). Is shown in Panel A, for most of the wealth levels the mean-variance efficient portfolio of public equity (50 percent in each stock) is optimal. However, for a small range at the top of the wealth distribution, roughly between the 90th and the 97th percentile, the pattern
on optimal holdings bifurcates, reaching an extreme long-short position with over 140% of the public equity portfolio invested one stock, with a 50% chance of any investor overweighing either one of them. The equilibrium patterns of portfolio holdings are very similar for higher values of coefficients, such as $\eta = 10^7$, $\zeta = 400$. When convexity in status utility is less pronounced (e.g. $\zeta = 4$ or lower) there is no bifurcation in shares of common assets, instead there are pure strategy equilibria where the public equity portfolios are mean-variance efficient for all investors.

What drives this result? As is apparent from Panel B, the allocation to private equity is hump-shaped over wealth distribution, as described above. In particular, for the upper half of the wealth distribution the individual-specific asset dominates, reaching almost 100% of the total portfolio and relegating the two publicly traded stocks to a tiny fraction of wealth. Given the convexity of utility over status, some of these agents might be able to satisfy their appetite for gambling with the individual-specific asset, since investing 100% or more in it would imply a positive probability of zero (or negative) consumption and therefore infinitely negative utility. The alternative way of generating idiosyncratic volatility is by overweighing one of the common assets, as depicted in figure 2. Note that it is critical that an equal (wealth-weighted) number of agents overweighs either security. Otherwise the total wealth portfolio is tilted towards one or the other common asset and the equilibrium breaks down, as investors try to load up on the asset that is least correlated with the aggregate wealth, thus making it more correlated with it.

How can we interpret the mixed strategy equilibrium described above? The notion of agents randomizing between stocks in their quest for the properly undiversified portfolio might or might not be an appealing description of actual investor behavior. Another possibility is reinterpreting the mixed-strategy symmetric equilibrium as an asymmetric equilibrium in pure strategies, where randomization over portfolio allocations is replaced with randomization over agents. Such equilibria are potentially realistic provided there exists a coordination device that helps select investors into groups overweighing one or the other asset. Such a coordination device might be naturally present in the form of varied amounts of information that agents receive about different investment opportunities. There is now a substantial amount of evidence that investors tend to overweigh shares of companies they are more familiar with - often those of their employer or a locally headquartered firm (e.g. see ?, ?, ?, ?, ?, etc.). Geographic proximity might give rise to information-based abnormal returns (e.g. ?, ?). Even if does not, investors’ familiarity
with local stocks can potentially provide a coordination device for achieving an optimal degree of under-diversification in equilibrium.

This interpretation is consistent with evidence of portfolios tilted more strongly towards idiosyncratic risks more highly correlated those already borne by investors (e.g., via their human capital). For example, \( ? \) find that instead of hedging away their labor income risk, many investors pick portfolio combinations that amplify it. \( ? \) finds that employees of stand-alone firms are more likely to overweigh their company stock than employees of conglomerates, which are presumably more diversified and thus carry less idiosyncratic risk. This evidence is consistent with the prediction that investors whose ability to gain enough exposure to individual-specific risk is constrained (perhaps by the inability to leverage their human capital) supplement it by investing in similar traded assets. Is this empirical observation due simply to the lack of investor sophistication? Learning about the potentially disastrous consequences of holding large fraction of retirement wealth in own company stock does not appear to lead employees to significantly reduce their holdings of their employer’s stock in 401(k) accounts, as documented by \( ? \). This fact suggests that a preference-based approach, such as the one advanced here, might prove more useful for explaining such apparently anomalous behavior than one based purely on lack of education and sophistication.

To what extent do the results reported here generalize to a more realistic environment with a large number of publicly traded assets? In such an environment it might be easier to take bets on purely idiosyncratic risk by overweighing a single security. As shown above, undiversified portfolios of public securities can be optimal only for investors whose preferences over status are sufficiently convex. This is because publicly traded idiosyncratic risk should not earn a risk premium by a standard arbitrage argument, while private equity investments do due to information frictions. \( ? \) and \( ? \) report evidence that at least some of the individuals with concentrated stock portfolios are able to earn abnormal returns on their investments, possibly owing to their ability to exploit market inefficiencies or, more likely, private information. Whether real or perceived, an ability to earn abnormal returns on certain securities provides a natural coordination device for undiversified public equity investments. It could also potentially make such positions more attractive for investors who are not risk-seeking with respect to wealth rank. Note, however, that abnormal returns (real or perceived) are not a sufficient explanation of undiversification by themselves, since
even with abnormal returns concentrated portfolios have higher volatility and lower Sharpe ratios than better diversified ones.

I conclude that the predictions of the social status model are, at least in principle, consistent with the seemingly disparate forms of investor undiversification that are observed in the data.

5.1.4 Contingent claims

Another way in which investors can effect their relative standing in the wealth distribution is by engaging in dynamic trading or through the use of options or other derivative securities written on aggregate shock realizations. The quantitative model above features a binomial aggregate stock price process, which is potentially too restrictive in that it allows only trivial derivative contracts. One way to relax this assumption would be to allow intermediate dynamic trading of the public securities; another would be to allow trading in contingent claims; both require a richer space of common equity payoffs. Here I present an example of the latter extension with the simplest generalization of the stock price process: trinomial.

Consider the one-period period version of the model (5.1) where investors have access to a full set of contingent claims on the three aggregate states, as well as the individual-specific asset. Since there are only three aggregate states, the complete set of contingent claims can be implemented by using the public equity return, the riskless asset, and one additional derivative security (e.g., a call or a put option). Since the option is not redundant, its price can be determined in equilibrium by imposing a market-clearing condition (e.g., zero net supply of options). Specifically, quoting the time \( t + 1 \) stock price and strike price \( K \) in the units of the time \( t \) price, let the option pay-off at time \( t + 1 \) be \( (R_{t+1}^A - K)^+ \), so that the time \( t \) price of the option (also in the units of the stock price) is

\[
C_t = E_t \left[ m_{t+1} \left( R_{t+1}^A - K \right)^+ \right],
\]

where \( m_{t+1} \) is the stochastic discount factor that prices \( R_{t+1}^A \) and the risk-free rate. This discount factor is in general not unique, but can be pinned down by a choice of \( C_t \). I solve for the option price (together with the equilibrium asset allocations and the wealth distribution) on the interval
the good-deal bounds of \( ? \).\(^{12}\) These bounds are much tighter than the
standard arbitrage bounds and thus simplify the search for equilibrium.

I calibrate the public equity return to match the state by state returns of the “market portfolio”
of the previous section. The riskless asset and the individual-specific asset are calibrated as before
(without the bankruptcy state). I set the strike price equal to the middle realization of the stock
return, which is greater than 1, so that the option is out of the money. The Sharpe ratio of the
public market portfolio is equal to 1.18 in this example; I impose the standard constraint that the
maximum attainable Sharpe ratio \( h_m \) is twice as high (e.g. \( ? \)). Then the option price bounds are
\( C_{Max}^t \approx 0.09 \) and \( C_{Min}^t \) is just above zero. The preference parameters are \( \gamma = 5 \), \( \eta = 10^4 \), \( \zeta = 1 \).
As expected, low values of \( C_t \) generate demand for the option, especially for the agents in the
middle of the wealth distribution where the preference for idiosyncratic risk is most pronounced;
conversely, for values closer to the upper bound, the demand is negative. Neither of these cases
is consistent with equilibrium. First, the market-clearing condition that net supply/demand for
the option is zero is not satisfied because the sign of the demand is always the same across wealth
levels. Second, when the demand for either long or short option positions is strong, the dynamics
of average wealth is affected in a way that offsets the desire to hold options. For example, if
net option position is positive, the rate of aggregate wealth growth is extremely high in the high
market return state. This makes holding a long call option position unattractive for those seeking
idiosyncratic risk exposure, since it becomes highly correlated with the aggregate. Shorting the
call (or synthesizing a put option) is also unattractive in the presence of the individual-specific
asset, since the low call option price makes for an inferior risk-return trade-off. Thus iterating
on the solution generates too low a demand for options in the subsequent step, resulting in
oscillations that do not converge to an equilibrium in a reasonable time. Moreover, the impact
of option trading on the optimal holdings of private equity is small. The peak allocations to the
individual-specific asset, around 75% without options in this calibration, are reduced by about

\(^{12}\)Specifically,

\[
C_{Min}^t = \min_m E_t \left[ m_{t+1} (R_{t+1}^A - K)^+ \right],
\]

subject to \( E_t \left[ m_{t+1} R_{t+1}^A \right] = 1 \), \( E_t \left[ m_{t+1} \right] = 1/R_f \), \( m_{t+1} > 0 \) and \( \sigma (m_{t+1}) \leq h_m / R_f \),

and similarly for \( C_{Max}^t \). The first two constraints state that the stochastic discount factor prices both the public
equity return and the risk-free rate, the positivity constraint rules out arbitrage opportunities, and the last constraint
prevents Sharpe ratios that are so large as to allow “good deals.”
5-10 percent in the cases when demand for options is substantially positive or negative.

The only case where the solution does converge to an equilibrium is when the optimal allocation to options is (approximately) zero. This happens for $C_t = 0.016$ (this option price is slightly higher than the equilibrium price for the benchmark case $\gamma = 5, \eta = 0$ which is equal to .014.). It is possible that for other values of there exist equilibria with different wealth distributions. Still, for those equilibria to feature substantial holdings of options that could alter the model’s predictions about private equity allocations the curvature of the wealth distribution needs to be sufficiently variable to generate both positive and negative demands for the option. For higher levels of status weight and convexity parameters the demand for option increases. Consequently, the equilibrium option price that clears the markets is higher. For example, in the case $\gamma = 5, \eta = 10^4, \zeta = 4$ the equilibrium price approximately equals 0.027, or almost twice as high as in the benchmark case above.

In addition, it is possible that either asymmetric equilibria or mixed-strategy symmetric equilibria of the type discussed in section 5.1.3 also exist. It is likely that the conclusions of that section will translate to this case as well: introducing additional assets will have only a second-order effect on holdings of private equity.

### 5.2 Unobserved wealth and “conspicuous consumption”

Throughout the discussion above I assumed that social status is assigned to individuals/households based on their total wealth rank. However, household wealth is generally not public information, and hence for status to be truly interpersonal (and not just an internal benchmark measure of “self-worth”) others must be able to infer one’s wealth based on observable characteristics. Naturally, individuals can signal their wealth through consumption of “conspicuous” goods (such as expensive cars, designer clothing, jewelry, and other luxuries, as well as some charitable contributions). In fact, much of the literature on social externalities focuses on relative consumption and not wealth comparisons, in part because the former is likely to be better observed than the latter (e.g. ?, ?, ?, ?, ?, ?, and ?). In order for the conspicuous consumption signal to be revealing the amount consumed by the rich households must be sufficiently high to deter the poor from emulating them and thus making the signal uninformative. This standard intuition of costly signalling leads to “overconsumption” of visible goods in equilibrium, especially by the wealthier households (?, ?).
It is therefore likely that the need to signal status through conspicuous consumption will mitigate the “oversaving” effect of pure relative wealth concerns discussed in section 4.6.

Consider a modification of the social status model in which wealth is not observable, but consumption is. Then social status is assigned to each individual by the public based on their wealth inferred from the consumption signal $C_i^t = \tilde{C}(W_i^t)$, e.g. as in [7]. Let this public inference of individual wealth be given by function $\Psi(C_i^t)$. In a separating equilibrium it must equal true wealth: $\Psi(\tilde{C}(W_i^t)) = W_i^t$. In order to be incentive compatible, the conspicuous consumption policy must solve the individual optimization problem

$$
\tilde{V}^i(W_i^t, \tilde{W}_i^t, a_i^t; I_t) = \max_{\tilde{C}, a} \left\{ \frac{\tilde{C}^{1-\gamma} - 1}{1 - \gamma} + \eta \tilde{W}^{1-\gamma} \tilde{F} \left( \Psi(\tilde{C}) \right)^\zeta + \beta E \left[ \tilde{V}^i(W_{t+1}^i, \tilde{W}_{t+1}^i, a_{t+1}^i; I_{t+1}) \right| I_t \right\},
$$

to all of the standard constraints as well. Assuming differentiability of the status assignment function as well as of the value and policy functions the consumption first-order condition for this problem is

$$
\tilde{C}(W_i^t)^{-\gamma} + \eta \zeta \tilde{W}_i^t^{-\gamma} \tilde{F}' \tilde{F}^{\zeta-1} = \beta E \left[ \tilde{V}^i(W_{t+1}^i, \tilde{W}_{t+1}^i, a_{t+1}^i; I_{t+1}) R_{t+1}^W \right| I_t],
$$

where $R_{t+1}^W$ is the return on the optimal financial portfolio. Thus in equilibrium the optimal consumption policy must solve

$$
\tilde{C}(W_i^t)^{-\gamma} + \eta \zeta \frac{\tilde{W}_i^t^{-\gamma} \tilde{F}' \tilde{F}^{\zeta-1}}{C'(W_i^t)} = \beta E \left[ \tilde{V}^i(W_{t+1}^i, \tilde{W}_{t+1}^i, a_{t+1}^i; I_{t+1}) R_{t+1}^W \right| I_t].
$$

The second term above that distinguishes the conspicuous consumption from the observed wealth benchmark is positive. This implies that the share of wealth that goes to consumption in each period is higher under the unobservable wealth model than under the standard social status model, for all but the poorest households. A similar result is established rigorously by [7]; they also show that the conspicuous consumption effect on expenditures increases with wealth.

What would be the quantitative implications of the social status model with conspicuous consumption? Solving the latter model explicitly does not appear feasible, due to the additional constraint that the optimal conspicuous consumption policy is a solution to a differential equation built into the dynamic maximization problem. Instead I approximate its behavior by imposing
an ad hoc constraint on the consumption-wealth ratios to be greater than some fixed bound and then solve the social status model in the same way as before, but with this additional constraint. This exogenous bound is meant to capture the intuition that in the separating equilibrium the rich must consume substantially more to deter mimicking by the less affluent. I set the bound at 40 percent (i.e. about 4 percent on an annual basis) which is a reasonable lower bound for the wealthy households (it is unlikely that the rich, especially the elderly, consume a much smaller a fraction of their wealth, even in the presence of a bequest motive). Since the constraint is binding only for these households it captures the intuition that conspicuous consumption effects are more pronounced for the richer households.

Constraining wealth accumulation by forcing the rich households to consume a greater share of their wealth each period leads to slightly more risk-taking, with standard deviations of portfolio returns rising on average by about 3-4 percentage points. Overall, however, the predictions of the model with conspicuous consumption for portfolio allocations are quantitatively very similar to those in the benchmark model (I do not report them here). The main differences are in the dynamics of household wealth and consumption. Constraining the consumption-wealth ratios leads to an increase in wealth mobility. As shown in the bottom panel of table IV, the fractions of simulated households moving up and down from each of the top five deciles of the wealth distribution almost exactly match their empirical counterparts, and for the two middle decile the fit is fairly close. At the same time, the constraint has a smoothing effect on consumption growth. As shown in table VII, the standard deviations of log consumption growth averaged over the richest one third of model households matches the empirical estimate of about 5.3 percent annually, and the volatility of economy-wide average consumption growth matches the empirical estimate for all stock-owning households at about 4.9 percent. Thus, imposing the constraint on consumption expenditures that is meant to capture the effect of “conspicuous consumption” produces a remarkable success for the social status model in explaining consumption dynamics. The reason is that the imposed constraint smooths the household-level variation in consumption-wealth ratios over time.
5.3 **Asset pricing implications**

The focus on entrepreneurs and the fact that most of both aggregate and idiosyncratic risk in the model economy is borne by the wealthy suggests potential links to the empirical literature on asset pricing and heterogeneous consumers. It has been shown that consumption-based asset pricing models that place greater weight on the wealthy households fare better empirically at explaining aggregate asset pricing facts (e.g. ?, ?, ?, ?, and ?).

I have solved the model above as a portfolio-choice problem with exogenous asset returns. Nevertheless, given the endogenously derived dynamics of aggregate consumption growth, it is interesting to consider potential implications of the social status model for aggregate asset pricing. The low volatility of aggregate consumption growth that can be generated by the status model sheds light on the discussion of the equity premium puzzle of ? in the context of heterogeneous agents models with incomplete markets. Following ?, ? show that it is possible to reconcile smooth and i.i.d. aggregate consumption growth with high (and time-varying) equity premium under incomplete markets provided that individual consumers experience permanent idiosyncratic income shocks with counter-cyclical volatility ( ? and ? present quantitative examples of such models). ? show that the latter is a necessary requirement. ? argue that such dynamics of idiosyncratic income shocks are consistent with the data. It is however uncertain whether persistence and volatility in idiosyncratic labor income risk is sufficient to match both the asset pricing and the aggregate consumption quantities with reasonable levels of risk aversion (?, ?).

My model does not rely on exogenous idiosyncratic income, but it is able to produce volatile individual consumption growth that averages to generate smoother aggregate consumption growth. Moreover, these consumption growth dynamics are consistent with high aggregate equity premium since agents experience permanent idiosyncratic shocks coming from the realizations of individual-specific return process. This fact suggests that, empirically, portfolio undiversification might be at least as important as labor income risk for understanding the interaction between idiosyncratic risk and aggregate asset returns. Linking the undiversification prediction of the social status model to equilibrium asset prices empirically requires measurement of marginal utility not only of consumption but also social status and is an interesting venue for future research.
5.4 Status: “local” vs. “global”

In calibrating my model I assume that the reference group for determining each individual’s status is the entire U.S. population. I make this choice largely in pursuit of parsimony. This is a common assumption in the finance and macroeconomics literature (e.g. ? , ? , ? , ? , ? , ? and ?). However, most empirical evidence of social externalities is based on “local” peer groups. For example, ? uses data from artificially created census areas with an average size of 127,000 inhabitants, while ? assumes city-level reference groups.

The apparent importance of local peer effects is not inconsistent with the view that people care about their rank on a larger scale. An important feature of socially-dependent preferences is that an individual’s peer group is itself, in large part, endogenous. One’s geographic location, place of employment and social circle are outcomes of individual choice, at least in the long run. Wealthier people tend to live in wealthier neighborhoods and associate with other affluent people; even though they could more easily attain higher “local” status in a poorer community, they often choose to give it up in favor of a higher ”global” status conferred by belonging to a higher social class. It is the latter type of status that ? refer to as motivation for the non-concave utility. Since I am interested in explaining the dynamics of the U.S. wealth distribution, the nation-level reference group is appropriate (I also rule out international comparisons). I avoid trade-offs between being “first in village” and “second in Rome” (e.g. see ?), since wealth percentile in the aggregate distribution is an increasing function of wealth both “locally” and “globally”.

5.5 Empirical issues

5.5.1 Participation

In my analysis of portfolio choice under social status preference above I have assumed that all investors are participate in the public stock market and can own a share of a private business or similar asset. Clearly, this is not true in the data, and modelling participation decisions is important for understanding the empirical evidence. However, the empirically observed heterogeneity in participation rates is largely consistent with the predictions of my model, even though the latter abstracts from participation decisions per se. In the SCF data (table 6) the rates of participation in both public and private equity markets are both increasing in household wealth. Under a fifth of all households in the bottom quartile of the wealth distribution own any risky assets (other
than housing), and just one percent of households in the bottom quartile have a private business interest (or other similar investment). These rates increase dramatically with wealth. In the top decile of the wealth distribution virtually everyone owns some risky assets and almost 70% of the households have both public and private equity.

How can the stylized facts on (non-)participation in private and public equity markets be reconciled with the social status model? Despite a long history as a focus of economic analysis, entrepreneurship and, in particular, the decision to enter a private business, is not yet well understood by economists. Constraints on entrepreneurial investment in the form of minimal capital requirements cannot by themselves explain the positive relationship between business ownership and wealth: ? find that wealth itself does not predict entry into entrepreneurship, except for the top 5 percent of the distribution, and that the liquidity constraints, while potentially important, do not explain entry rates either. The positive relationship between business ownership and wealth could be generated mechanically. Successful entrepreneurs become richer, but do not always divest themselves of their businesses, perhaps due to information asymmetries. However, as ? argues, the concentration of wealth in the hands of entrepreneurs is mostly due to their high saving rates, not merely due to high incomes. ? find that the high saving rates of business owners are not explained by precautionary motive, even though the latter do have riskier incomes than non-entrepreneurs.

The fact that the rich are more likely to be entrepreneurs (and vice versa) might by itself lend support to the social status model. Suppose that the strength of preference for social status varies across people. Then those with stronger concern for social status are not only more likely to become entrepreneurs, but also save more and thus accumulate wealth faster than others, thus generating the observed patterns of ownership rates. Moreover, provided such heterogeneity in preferences one could also expect the concentration of households’ financial portfolios to be most pronounced among entrepreneurs, especially if their ability to take on idiosyncratic risk through their business is limited. This prediction appears to be borne out in the data: ? report that the portfolios of entrepreneurs are on average less diversified than those of non-entrepreneurs.

What about stock market participation? While there is no agreed-upon explanation of why so many households do not own any stocks, it is apparent that they do so in the face of some type of participation or information costs. ? argue that fixed costs of stock market participation
combined with recursive preferences and uninsured labor income risk explain the patterns of (non-)
participation. \textsuperscript{7} finds empirical evidence supporting the importance of fixed transactions costs. In either case, in the presence of participation costs, increasing demand for risky assets in wealth implies that stock market participation rates are likely to rise with wealth as well.

Thus the fraction of empirical portfolios invested in risky assets (also reported in table 6) largely confirms the predictions of the social status model, at least for the households which are marginal in both public and private equity markets. If housing is included among risky assets, the share of risky assets is monotonically increasing in relative wealth. For the young households it ranges from under 50\% in the bottom quartile to over 80\% in the top 10 percent of the distribution, where as for the old the corresponding figures are 40 to 75 percent, respectively. Excluding housing from the definition of risky assets naturally reduces this share, except for the poorest households. Thus, the monotonic pattern is preserved for the old households, with the fraction invested in risky assets rising from 24 to almost 60 percent, but is broken for the young. For the young in the bottom quartile of wealth the risky share is the same 50\% as with housing, dropping to about 34\% in the third quartile, and rising again to above 50\% in the top decile. This pattern suggests that the poorest of the young households who own private businesses or other lumpy illiquid assets might be prevented from owning homes, perhaps due to indivisibility and liquidity constraints. At higher wealth levels the ability to invest in housing appears to crowd out other risky investments.

5.5.2 Undiversification among the poor

One salient feature of the data that is not explained by the social status model is that, empirically, undiversification (measured, in particular, as the degree of portfolio concentration in the largest risky asset) peaks in the bottom of the wealth distribution. This observation is as robust as it is intuitive. \textsuperscript{8} find that lower-income (as well as younger and less educated) investors are more likely to have highly concentrated stock portfolios. \textsuperscript{9} finds that the same types of investors appear to prefer “lottery-like” stocks, i.e. those with high idiosyncratic risk and a small probability of a very large payoff with a large probability of a small loss. This evidence parallels the well-known fact that poor investors are much more likely to participate in negative expected return lotteries. Some of this evidence could probably be explained by either education (i.e. lack of
understanding of diversification, or even of the most basic probabilistic concepts) or by a gambling motive that is or is not related to social status. The demand for idiosyncratic risk among the poor can be generated by the social status model if the initial wealth distribution is unimodal (see ?), a feature I do not model in this paper. Such an explanation would be consistent with the adverse risk-return tradeoff of participating in gang crime documented by ?. Otherwise, demand for lotteries (although not necessarily purely idiosyncratic risk) at low wealth levels can be due to indivisibilities in consumption goods or investment opportunities coupled with liquidity constraints (e.g. as in ?, ?, ?, or ?).

It is possible to reconcile the evidence on greater underdiversification of stock portfolios, as well as gambling, at the bottom of the wealth distribution rather than in the middle, with the social status model. Wealthier (and better educated) households might have better investment opportunities than the poor, in particular they might have entrepreneurial projects that dominate the strategy of holding a single stock. Unlike pure gambling, which has zero or negative expected returns, private equity has positive expected returns. It is likely that these returns are higher than those on some of the individual stocks held in a typical undiversified portfolio, and also a potentially longer right tail - the idiosyncratic “upside” that allows one to move up in the wealth distribution. In fact this argument is consistent with the finding of ? that higher income households are more likely to participate in lotteries that have larger prizes.

Finally, human capital is a large component of total wealth for the rich households, and, given its inherently large idiosyncratic risk is likely to be the biggest idiosyncratic asset in a household’s portfolio. Its riskiness might be sufficient to satisfy the investor’s demand for idiosyncratic variance, leading to a more diversified financial portfolio than that of a similar investor with less human capital. Testing this hypothesis empirically requires detailed data on both labor income and portfolio allocation, and is an interesting venue for future research.

6 Discussion and Concluding Remarks

In this paper I address the limited diversification of household portfolios together with the apparent lack of a premium for undiversified entrepreneurial risk by considering the investment choices of individuals who exhibit a preference for social status. The assumption that marginal utility of wealth increases with relative status leads investors to optimally hold undiversified portfolios
in equilibrium. This feature of the model suggests that at least some of the empirically observed cross-sectional dispersion in accumulated wealth can be understood using a simple portfolio-based approach that allows the amounts of both aggregate and idiosyncratic risk in the economy to be determined endogenously. Thus it supports the argument of ? who emphasizes the role of individual choice and, in particular, risk preferences in shaping the distribution of income and wealth.

The model also has potential implications for the study of investment and, consequently, economic growth. Most macroeconomic theory is predicated on the assumption that the demand for diversification leads households to pool and share their idiosyncratic risks. Perfect risk sharing is prevented, however, by the incompleteness of insurance markets due to asymmetric information and limited enforcement of contracts. Such market imperfections impose costs on society in the form of foregone investment opportunities, due to the inability of agents to share idiosyncratic risk of individual projects. Preference for social status can potentially mitigate this problem, since it can lead investors to take on more undiversified idiosyncratic risk than predicted by the standard theory, unleashing greater entrepreneurial investment and spurring economic growth. This intuition is similar to the argument of ? that evolutionary forces favor agents who are less averse to idiosyncratic than to aggregate risks, since the former are “diversified” at the macro-level, while the latter are not. I provide an example of how status-generated “overinvestment” in individual-specific projects can be socially optimal in economies with limited risk-sharing in ?. This possibility appears consistent with the evidence of ? that companies with concentrated founding-family ownership are less, not more, diversified, than other firms, contrary to the predictions of standard theories, such as ?. ? and ? make a related argument that “oversaving” generated by social status concerns can help overcome negative externalities arising from technological spillovers, and therefore lead to optimal economic growth.

A link between preferences with relative status concerns and economic growth could help explain the divergent patterns of entrepreneurship and economic development across countries. ? suggest that in societies in which the distribution of status is exogenously fixed (and thus does not correspond to the relative wealth rank) one should observe less risk-taking. The more or less rigid social norms can arise endogenously as evolutionary outcomes (e.g. “wealth-is-status” vs. “aristocratic equilibria” in ?). Resulting differences in cultural and social norms can be at least as
important as differences in economic policies in explaining the variation in the pace of economic growth around the world.
Appendix

A Aggregate wealth and marginal utility

Consider the one-period case with the utility of wealth given by

\[ V^i(W^i, \bar{W}; \tilde{F}) = \left( \frac{(W^i)^{1-\gamma} - 1}{1-\gamma} + \eta W^{1-\gamma} \tilde{F} \left( \frac{W^i}{\bar{W}} \right)^\zeta \right) \]

Then, assuming that the cumulative wealth distribution function \( \tilde{F} \) is continuously differentiable, the marginal utility of wealth is

\[ V^i_{W^i}(W^i, \bar{W}; \tilde{F}) = (W^i)^{1-\gamma} + \eta \zeta W^{1-\gamma} \tilde{F} \left( \frac{W^i}{\bar{W}} \right)^\zeta \tilde{F}' \left( \frac{W^i}{\bar{W}} \right) > 0; \]

similarly, the marginal contribution of average wealth to the individual utility is

\[ V^i_{\bar{W}}(W^i, \bar{W}; \tilde{F}) = (1-\gamma) \eta W^{\gamma-1} \tilde{F} \left( \frac{W^i}{\bar{W}} \right)^\zeta - \eta \zeta W^i W^{\gamma-1} \tilde{F} \left( \frac{W^i}{\bar{W}} \right)^\zeta \tilde{F}' \left( \frac{W^i}{\bar{W}} \right) < 0 \text{ for } \gamma \geq 1. \]

The cross-partial derivative is

\[ V^i_{W^i\bar{W}}(W^i, \bar{W}; \tilde{F}) = -\gamma \zeta \eta W^{\gamma-1} \tilde{F} \left( \frac{W^i}{\bar{W}} \right)^\zeta \tilde{F}' \left( \frac{W^i}{\bar{W}} \right) \]

\[ -\eta \zeta (\zeta - 1) W^i \bar{W}^{\gamma-2} \tilde{F} \left( \frac{W^i}{\bar{W}} \right)^\zeta - 2 \tilde{F}' \left( \frac{W^i}{\bar{W}} \right)^2 \]

\[ -\eta \zeta W^i \bar{W}^{\gamma-2} \tilde{F} \left( \frac{W^i}{\bar{W}} \right)^\zeta \tilde{F}'' \left( \frac{W^i}{\bar{W}} \right) \]

\[ = -\eta \zeta W^{\gamma-1} \tilde{F} \zeta \left( \gamma \tilde{F}' + (\zeta - 1) W^i \tilde{F}'' + W^i \tilde{F}'' \right) \]

This quantity is negative as long as the term in the parentheses is positive. In particular, this is true when \( \zeta \geq 1 \) and \( \tilde{F}'' > 0 \). However, the latter requirement that the wealth distribution be convex is not realistic, especially in the right tail of the distribution, since most empirical wealth distributions are not globally convex. Still, as long as the magnitude of the second derivative is not “too large” then there will exist a value of utility curvature \( \gamma \) such that the sum of the terms in the parentheses is positive, even for \( \zeta = 1 \). Otherwise, a large value of the status curvature \( \zeta \) might be required.
Most empirical wealth distributions are well described by “power laws” at high wealth levels. Indeed, suppose that for relative wealth $\frac{W_i}{\bar{W}}$ greater than some $s^*$ the density of the wealth
distribution can be approximated by

$$\tilde{F}' \left( \frac{W_i}{\bar{W}} \right) \sim \kappa \left( \frac{W_i}{\bar{W}} \right)^{-\alpha - 1}$$

for some $\kappa$ and $\alpha$. Then we have

$$\gamma \tilde{F}' \left( \frac{W_i}{\bar{W}} \right) + \left( \frac{W_i}{\bar{W}} \right) \tilde{F}'' \left( \frac{W_i}{\bar{W}} \right) \approx \kappa \left( \frac{W_i}{\bar{W}} \right)^{-\alpha - 1} (\gamma - (\alpha + 1)) > 0$$

as long as $(\gamma - (\alpha + 1)) > 0$, which implies $V_{iW}^i (W_i, \bar{W}; \tilde{F}) < 0$ for all $\zeta \geq 1$. The power law
exponent $\alpha$ is commonly estimated to be in the range $[1, 3]$; therefore, the above inequality holds
approximately for $\gamma \geq 4$. Thus this example shows that the parameter values required to generate
the “getting ahead of the Joneses” effect ($V_{iW}^i (W_i, \bar{W}; \tilde{F}) < 0$) are likely to be reasonable.

The second partial derivative is

$$V_{iW}^i (W_i, \bar{W}; \tilde{F}) = -\gamma W^{-\gamma - 1} + \eta \zeta W^{-\gamma - 1} F^{\zeta - 1} \left( F'' + (\zeta - 1) \frac{\tilde{F}''}{F} \right)$$

$$= -W^{-\gamma - 1} \left[ \gamma - \eta \zeta F^{\zeta - 1} \left( F'' + (\zeta - 1) \frac{\tilde{F}''}{F} \right) \right]$$

For this quantity to be positive it is necessary (although not sufficient) that either the wealth
distribution is locally convex ($F'' \left( \frac{W_i}{\bar{W}} \right) > 0$) or the marginal utility of status is increasing ($\zeta > 1$).
Note that neither is required for (although can lead to) having $V_{iW}^i (W_i, \bar{W}; \tilde{F}) < 0$.

**B Bellman equation with scale-invariance**

**Proposition B-1.** The dynamic program (2.3) is equivalent to

$$v_i(s_t, a_t) = \max_{\theta, \alpha} \left\{ \frac{(c_t s_t)^{1-\gamma}}{1 - \gamma} - 1 + \beta E_t \left[ \left( v_i(s_{t+1}, a_{t+1}) + \eta \tilde{F}_{t+1} (s_{t+1})^{\zeta} \right) G_{t+1}^{\zeta} \right] \right\}. \quad (B-1)$$

**Proof.** Proceed by backward induction: start with
The model is solved by iterating on the following steps:

1. **Maximization** of agents’ utility

\[
V^i(W^i_T, \bar{W}_T, T; I_T) = \frac{(W^i_T)^{1-\gamma} - 1}{1-\gamma} + \eta W^i_{T-1} \bar{F}_T \left( \frac{W^i_T}{W_T} \right)^\zeta \\
\equiv W^i_{T-1} \left( \frac{s^i_s}{1-\gamma} + \eta \bar{F}_T (s^i_s)^\zeta \right) \\
\triangleq W^i_{T-1} \left[ v^i_T(s^i_s, T) + \eta \bar{F}_T (s^i_s)^\zeta \right]
\]

and

\[
V^i(W^i_{T-1}, \bar{W}_{T-1}, T-1; I_t) = \max_{C, a} \left\{ \frac{(C^i_{T-1})^{1-\gamma} - 1}{1-\gamma} + \eta W^i_{T-1} \bar{F}_{T-1} \left( \frac{W^i_{T-1}}{W_{T-1}} \right)^\zeta \right\} \\
+ \beta E \left[ V^i(W^i_T, \bar{W}_T, T; I_t) \big| I_{T-1} \right] \\
= \max_{C, a} \left\{ \frac{(C^i_{T-1})^{1-\gamma} - 1}{1-\gamma} + \eta W^i_{T-1} \bar{F}_{T-1} \left( s^i_{T-1} \right)^\zeta \right\} \\
+ \beta E \left[ W^i_{T-1} \left[ v^i_T(s^i_s, T) + \eta \bar{F}_T (s^i_s)^\zeta \right] \big| I_{T-1} \right] \\
\equiv \max_{C, a} \left\{ \frac{(C^i_{T-1})^{1-\gamma} - 1}{1-\gamma} + \eta W^i_{T-1} \left( s^i_{T-1} \right)^\zeta \right\} \\
+ \beta E \left[ W^i_{T-1} \left[ v^i_T(s^i_s, T) + \eta \bar{F}_T (s^i_s)^\zeta \right] \big| I_{T-1} \right] \times W^i_{T-1} \\
\triangleq W^i_{T-1} \left[ v^i_T(s^i_s, T-1) + \eta \bar{F}_{T-1} (s^i_{T-1})^\zeta \right].
\]

Therefore, for any \(a^i_t\) we have

\[
v^i_t(s^i_s, a^i_t)W^i_t = W^i_t \times \max_{C, a} \frac{(C^i_s)^{1-\gamma} - 1}{1-\gamma} + \beta E_t \left[ \left( v^i_t(s^i_s, a^i_t) + \eta \bar{F}_{t+1} (s^i_{t+1})^\zeta \right) G^i_{t+1} \right],
\]

which is equivalent to (B-1) \(\Box\)

**Corollary B-2.** Households’ optimal consumption and investment policies do not depend on aggregate wealth.

**C Computational Algorithm**

The model is solved by iterating on the following steps:

1. **Maximization** of agents’ utility
2. Simulation of asset returns and the resulting wealth distribution

**Maximization**

The normalized Bellman equation (B-1) is solved by backward induction. The continuous space of endogenous state variable (agent-specific relative wealth $s^i_t$) is discretized using a grid with 60 points (logarithmically spaced, so that the grid is denser in the lower relative wealth region, where most of the agents are). For each age and idiosyncratic wealth state, optimal consumption and portfolio choices are found using simple grid search, which is the most robust (albeit costly) approach in the presence of nonconvexities. Future status is interpolated from the expected future wealth distribution. I use shape-preserving Hermite interpolation for the wealth distribution and for the next period’s value function (for the young agents) in the s direction in order to prevent internodal oscillations\(^{13}\).

**Simulation**

At each iteration for each age and aggregate state I draw a large number (10000 for each age group) relative wealth levels from the initial wealth distribution and interpolate the optimal consumption and portfolio policies from the solutions found in step 1 using linear interpolation. I then simulate idiosyncratic returns for all of the agents and estimate the resulting “empirical” distribution (EDF) of relative wealth in each of the aggregate states. I iterate this step forward until the average EDF converges. Projecting the resulting series future average wealth on the simulated sequence of aggregate returns is used to update the initial guess for the law of motion of aggregate wealth growth. The updated guess is used in the next iteration to solve the portfolio problem along with the new EDF which is used as the new expected future wealth distribution.

The iterations are repeated until the simulated steady-state EDF and the law of motion converge (state by state). I verify that the resulting optimal policies are invariant to small perturbations around the steady-state distribution to ensure that the solution is consistent with rational expectations.

\(^{13}\)Piecewise-cubic Hermite polynomial interpolation (PCHIP) is implemented in the MATLAB curve-fitting toolbox.
D Data description and estimation procedures

Asset holdings: Survey of Consumer Finances (SCF)

I use the 2001 SCF public dataset available from the Federal Reserve Board of Governors. The survey is representative of the U.S. population and is designed to oversample the wealthy households. Each household is represented in the dataset by 5 replicates (implicates) constructed in order to compensate for omitted information about households’ assets, etc; thus, there are 22210 observations produced from the 4442 households actually surveyed. Weights are provided to allow aggregation to population totals. For a detailed discussion of 2001 SCF see, e.g. ?.

The survey contains detailed information on household demographics, income, and asset holdings. I use the following conventions to define the value of the two main components of household risky assets, “public equity” and “private equity”. “Risky assets” are assumed to be comprised of both public equity and private equity (as defined in the appendix), and also to include corporate and foreign bonds (although their exclusion does not alter the results); I also consider the definition that includes owner-occupied housing as one of the risky assets.

Public equity includes directly held stocks plus managed assets such as mutual funds (except money market funds), retirement plans, annuities, trusts, thrifts, etc. For the purposes of calculating the households’ “public equity” investments the following convention is used in regard to these managed assets: full value if described as mostly invested in stock, 1/2 value if described as split between stocks/bonds or stocks/money market, 1/3 value if split between stocks/bonds/money market, etc.

Private equity includes the estimated market value of the households’ stakes in private business(es) and/or farm(s), plus loans from household to the business(es), minus loans from business to household, plus value of personal assets used as collateral; it also includes the market value of investment real estate, as well as other financial assets that are likely to be illiquid and/or undiversified, such as oil/gas/mineral leases or investments; association or exchange membership; futures contracts, stock options, hedge funds; royalties, patents; non-publicly traded stock, stock with restricted trading rights.

I define “largest risky asset” to be the largest of the following: market value of a private business interest; value of an investment real estate property; value of “other risky asset”; value of equity if concentrated in a single stock; average size of a stock holding for households holding
individual stocks (total value of stocks divided by the number of stocks); value of owner-occupied housing when the latter is included in the definition of risky assets.

In estimating the cross-sectional distribution of wealth I rank households on their total assets (instead of net worth) since in the model human wealth is potentially a component of total wealth, while in the data it is not. Although net worth and total assets are highly correlated, a number of individuals with high assets (as well as other characteristics correlated with human wealth, such as income and education) also have large debt (especially mortgage debt). This puts them into lower percentiles of net worth than individuals with the same level of assets but less debt and potentially lower human capital. Thus, sorts based on assets should better capture the total wealth ranking, although results based on net worth are very similar.

**Wealth mobility: Panel Study of Income Dynamics (PSID)**

I use the PSID wealth supplements for the years 1984, 1989, 1994 and 1999. In order to obtain estimates of wealth transitions over 10-year periods I track individuals who are heads of households in 3 successive observations that span a 10-year period. This results in a sample of 2608 households. I only include households with positive net worth in all 4 observations, which reduces the sample to 1973. This restriction simplifies estimation of growth rates of wealth across households and over time but does not affect the results otherwise. Further restricting the sample to male-headed households, as is often done in the literature due to the difficulties posed by changing head-of-household status for women who either marry or divorce, does not affect the results.

The measure of wealth is net worth (total assets minus total liabilities). Following ? I use the beginning-of-period sampling weights (i.e., those for 1984 and 1989 supplements) to compute averages. I consider households that answer the question whether they own stocks, mutual funds or IRAs (farms/proprietary businesses and real estate other than primary residence) affirmatively in any of the 3 successive observations to be stock-owning (business-owning) in estimating transitions for the 10 year period spanned by those observations.

I use artificial observations designed to limit the extent to which measurement error in wealth might bias the estimates of transition rates due to spurious volatility. These observations are obtained by averaging the first and the second pairs of observations: $W_{86.5}^i = \frac{1}{2} (W_{84}^i + W_{89}^i)$, $W_{96.5}^i = \frac{1}{2} (W_{94}^i + W_{99}^i)$. The transition probabilities are computed for the single implied period,
from mid-1986 to mid-1996 as a fraction of households from a given quantile that move to a target quantile after a 10-year period.

An important driver of wealth mobility that is not present in my model is uninsurable variation in labor income. Wealth mobility can be greatly effected by the life-cycle accumulation (and decumulation) of assets due to the fact that labor income cannot be capitalized in the beginning of working life and instead is converted into financial wealth slowly over time. Since my model abstracts from non-tradeable labor income and, in particular, life-cycle effects, using the raw estimated transition probabilities might be misleading. In order to estimate wealth transition probabilities adjusted for the life-cycle effects I use cross-sectional regressions for both time periods to predict growth rates of household wealth:

\[
\ln W_{t+10}^i - \ln W_t^i = a_0 + a_w \ln W_t^i + a_z Z_{t+10} + \epsilon_{t+10}^i
\]

The life-cycle variables included in the vector of controls \(Z\) are a quadratic in age (in order to capture both life-cycle accumulation and decumulation), change in marital status, and change in family size, as well as a variable indicating whether the head of household was in bad health in 1986. The labor income-related variables are based on labor earnings of “head” and “wife” for the period 1979-1999 (when available), split into two ten-year periods. “Permanent” income is estimated by averaging income over a ten-year period. Included in the vector of controls are the change in family labor income between 1983 and 1993, growth in permanent labor income, and the coefficient of variation of family income over the entire 1979-1999 period. I use the residuals from these regressions to generate artificial end-of-period wealth observations (e.g. as in ?).
**Table I: Calibration**

Technology parameters (annualized using log returns):

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free Rate</td>
<td>$R_f$</td>
</tr>
<tr>
<td>Public Equity Risk Premium</td>
<td>$E(R^a) - R_f$ 6%</td>
</tr>
<tr>
<td>Public Equity Return Volatility</td>
<td>$\sigma(R^a)$ 15%</td>
</tr>
<tr>
<td>Private Equity Risk Premiums</td>
<td>$E(R^i) - R_f$ 6%</td>
</tr>
<tr>
<td>Private Equity Return Volatility</td>
<td>$\sigma(R^i)$ 45%</td>
</tr>
<tr>
<td>Probability of good aggregate state</td>
<td>$\Pr{R^a &gt; R_f}$ $\frac{2}{3}$</td>
</tr>
</tbody>
</table>

Preference parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curvature of Status Utility</td>
<td>$\zeta$ 1</td>
</tr>
<tr>
<td>Subjective Discount Factor</td>
<td>$\beta$ 0.9710</td>
</tr>
<tr>
<td>Curvature of Consumption Utility</td>
<td>$\gamma$ 5</td>
</tr>
<tr>
<td>Status Utility Weight</td>
<td>$\eta$ 10000</td>
</tr>
</tbody>
</table>

**Table II: Data vs. model: average portfolio shares**

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risky/total assets</td>
<td>38</td>
<td>39</td>
</tr>
<tr>
<td>Private equity/risky assets</td>
<td>63</td>
<td>77</td>
</tr>
<tr>
<td>Private equity/total assets</td>
<td>22</td>
<td>23</td>
</tr>
</tbody>
</table>

Benchmark social status model: $\gamma = 5$, $\eta = 10000$, $\zeta = 1$

Data: Survey of Consumer Finances (2001)

“Private” equity is the single largest risky asset (e.g. average directly held stock, private business or investment real estate); only households with non-zero amounts included.
Table III: Portfolio concentration: model vs. data

**Data: portfolio share of largest risky asset (SCF)**

<table>
<thead>
<tr>
<th>Wealth percentile</th>
<th>0-25</th>
<th>25-50</th>
<th>50-75</th>
<th>75-90</th>
<th>90-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Households own, %</td>
<td>5</td>
<td>12</td>
<td>25</td>
<td>39</td>
<td>60</td>
</tr>
<tr>
<td>Share of total assets, %</td>
<td>18</td>
<td>21</td>
<td>20</td>
<td>23</td>
<td>26</td>
</tr>
<tr>
<td>Share of non-housing assets, %</td>
<td>21</td>
<td>30</td>
<td>34</td>
<td>34</td>
<td>32</td>
</tr>
</tbody>
</table>

**Model: $\gamma = 5$, $\eta = 10000$, $\zeta = 1$**

<table>
<thead>
<tr>
<th>Wealth percentile</th>
<th>0-25</th>
<th>25-50</th>
<th>50-75</th>
<th>75-90</th>
<th>90-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private equity, % of total assets</td>
<td>11</td>
<td>15</td>
<td>30</td>
<td>35</td>
<td>38</td>
</tr>
</tbody>
</table>

**No-status portfolio model: $\eta = 0$, $\zeta = 0$**

<table>
<thead>
<tr>
<th>Risk aversion, $\gamma$</th>
<th>7</th>
<th>5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private equity, % of total assets</td>
<td>7</td>
<td>9</td>
<td>14</td>
</tr>
</tbody>
</table>
Table IV: Wealth transition probabilities

<table>
<thead>
<tr>
<th>Wealth quantile</th>
<th>Bottom half</th>
<th>50-90</th>
<th>90-95</th>
<th>95-99</th>
<th>Top 1 percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Move down</td>
<td>0.00</td>
<td>0.17</td>
<td>0.40</td>
<td>0.48</td>
<td>0.35</td>
</tr>
<tr>
<td>Stay</td>
<td>0.86</td>
<td>0.76</td>
<td>0.34</td>
<td>0.46</td>
<td>0.65</td>
</tr>
<tr>
<td>Move up</td>
<td>0.14</td>
<td>0.07</td>
<td>0.26</td>
<td>0.06</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Model: $\gamma = 5$, $\eta = 10000$, $\zeta = 1$ (status benchmark)**

<table>
<thead>
<tr>
<th>Wealth quantile</th>
<th>Bottom half</th>
<th>50-90</th>
<th>90-95</th>
<th>95-99</th>
<th>Top 1 percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Move down</td>
<td>0.00</td>
<td>0.06</td>
<td>0.41</td>
<td>0.32</td>
<td>0.33</td>
</tr>
<tr>
<td>Stay</td>
<td>0.95</td>
<td>0.88</td>
<td>0.40</td>
<td>0.61</td>
<td>0.67</td>
</tr>
<tr>
<td>Move up</td>
<td>0.05</td>
<td>0.05</td>
<td>0.18</td>
<td>0.07</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Model: $\gamma = 5$, $\eta = 10000$, $\zeta = 1$ with no short-selling**

<table>
<thead>
<tr>
<th>Wealth quantile</th>
<th>Bottom half</th>
<th>50-90</th>
<th>90-95</th>
<th>95-99</th>
<th>Top 1 percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Move down</td>
<td>0.00</td>
<td>0.05</td>
<td>0.39</td>
<td>0.29</td>
<td>0.29</td>
</tr>
<tr>
<td>Stay</td>
<td>0.96</td>
<td>0.90</td>
<td>0.45</td>
<td>0.64</td>
<td>0.71</td>
</tr>
<tr>
<td>Move up</td>
<td>0.04</td>
<td>0.05</td>
<td>0.16</td>
<td>0.06</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Model: $\gamma = 5$, $\eta = 0$, $\zeta = 0$ (no status concerns)**

<table>
<thead>
<tr>
<th>Wealth quantile</th>
<th>Bottom half</th>
<th>50-90</th>
<th>90-95</th>
<th>95-99</th>
<th>Top 1 percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Move down</td>
<td>0.00</td>
<td>0.03</td>
<td>0.14</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>Stay</td>
<td>0.97</td>
<td>0.95</td>
<td>0.78</td>
<td>0.87</td>
<td>0.90</td>
</tr>
<tr>
<td>Move up</td>
<td>0.03</td>
<td>0.02</td>
<td>0.08</td>
<td>0.02</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Model: $\gamma = 5$, $\eta = 10000$, $\zeta = 1$, with “conspicuous consumption” constraint**

<table>
<thead>
<tr>
<th>Wealth quantile</th>
<th>Bottom half</th>
<th>50-90</th>
<th>90-95</th>
<th>95-99</th>
<th>Top 1 percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Move down</td>
<td>0.00</td>
<td>0.09</td>
<td>0.36</td>
<td>0.24</td>
<td>0.22</td>
</tr>
<tr>
<td>Stay</td>
<td>0.92</td>
<td>0.86</td>
<td>0.50</td>
<td>0.71</td>
<td>0.78</td>
</tr>
<tr>
<td>Move up</td>
<td>0.08</td>
<td>0.04</td>
<td>0.14</td>
<td>0.05</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Table V: Consumption as a share of wealth, per 10-year period, by age

<table>
<thead>
<tr>
<th>Wealth percentile</th>
<th>0-25</th>
<th>25-50</th>
<th>50-75</th>
<th>75-90</th>
<th>90-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 years old</td>
<td>45</td>
<td>43</td>
<td>31</td>
<td>20</td>
<td>11</td>
</tr>
<tr>
<td>50 years old</td>
<td>45</td>
<td>45</td>
<td>34</td>
<td>20</td>
<td>12</td>
</tr>
<tr>
<td>80 years old</td>
<td>60</td>
<td>58</td>
<td>40</td>
<td>25</td>
<td>12</td>
</tr>
</tbody>
</table>

Table VI: Top wealth shares

<table>
<thead>
<tr>
<th></th>
<th>10%</th>
<th>1%</th>
<th>0.1%</th>
<th>0.01%</th>
<th>Pareto exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>64.6</td>
<td>29.2</td>
<td>9.3</td>
<td>2.9</td>
<td>1.8</td>
</tr>
<tr>
<td>(\gamma = 5, \eta = 0, \zeta = 0)</td>
<td>60.7</td>
<td>21.9</td>
<td>6.1</td>
<td>1.0</td>
<td>2.0</td>
</tr>
<tr>
<td>(\gamma = 5, \eta = 10000, \zeta = 1)</td>
<td>65.0</td>
<td>20.4</td>
<td>7.1</td>
<td>2.5</td>
<td>2.0</td>
</tr>
<tr>
<td>(\gamma = 5, \eta = 10000, \zeta = 4)</td>
<td>83.5</td>
<td>34.9</td>
<td>8.9</td>
<td>1.9</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Table VII: Consumption growth volatility

<table>
<thead>
<tr>
<th></th>
<th>Stockholders</th>
<th>Non-stockholders</th>
<th>Top third stockholders</th>
<th>NIPA aggregate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-year</td>
<td>3.55</td>
<td>1.41</td>
<td>4.09</td>
<td>1.26</td>
</tr>
<tr>
<td>5-year</td>
<td>4.96</td>
<td>2.28</td>
<td>5.28</td>
<td>1.71</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Bottom third</th>
<th>Middle</th>
<th>Top third</th>
<th>Aggregate</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma = 5, \eta = 10000, \zeta = 1), status benchmark</td>
<td>10.92</td>
<td>11.67</td>
<td>11.25</td>
<td>11.28</td>
</tr>
<tr>
<td>(\gamma = 5, \eta = 10000, \zeta = 4)</td>
<td>12.62</td>
<td>9.73</td>
<td>11.08</td>
<td>11.11</td>
</tr>
<tr>
<td>(\gamma = 5, \eta = 0, \zeta = 0)</td>
<td>11.96</td>
<td>12.36</td>
<td>12.66</td>
<td>12.27</td>
</tr>
<tr>
<td>(\gamma = 5, \eta = 10000, \zeta = 1), w. consp. consumption</td>
<td>4.48</td>
<td>6.61</td>
<td>6.50</td>
<td>5.74</td>
</tr>
<tr>
<td>(\gamma = 5, \eta = 10000, \zeta = 1), with failure, consp.</td>
<td>3.61</td>
<td>5.85</td>
<td>5.34</td>
<td>4.94</td>
</tr>
</tbody>
</table>

Annualized standard deviations of log consumption growth, in percent.
Table VIII: Calibration: gambles

Panel A: gambles offered

<table>
<thead>
<tr>
<th>Gamble</th>
<th>Payoffs, $</th>
<th>Probabilities of payoffs</th>
<th>NPV, $</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-50000.00</td>
<td>50000.00</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>50000.00</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>2</td>
<td>-50000.00</td>
<td>60000.00</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>75000.00</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>3</td>
<td>-50000.00</td>
<td>5000.00</td>
<td>0.50</td>
</tr>
<tr>
<td>4</td>
<td>-5000.00</td>
<td>1000000.00</td>
<td>0.995</td>
</tr>
<tr>
<td>5</td>
<td>-5000.00</td>
<td>1000000.00</td>
<td>0.005</td>
</tr>
<tr>
<td>6</td>
<td>-5.00</td>
<td>1000000.00</td>
<td>0.9999999</td>
</tr>
</tbody>
</table>

Panel B: $\gamma = 5, \eta = 0$

<table>
<thead>
<tr>
<th>Gamble</th>
<th>Initial wealth ($ millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>- - - - - - - -</td>
</tr>
<tr>
<td>2</td>
<td>- - - + + + + +</td>
</tr>
<tr>
<td>3</td>
<td>- - + + + + + + +</td>
</tr>
<tr>
<td>4</td>
<td>- + + + + + + + +</td>
</tr>
<tr>
<td>5</td>
<td>- + + + + + + + + +</td>
</tr>
<tr>
<td>6</td>
<td>- + + + + + + + + +</td>
</tr>
</tbody>
</table>

Panel C: $\gamma = 5, \eta = 1000, \zeta = 1$

<table>
<thead>
<tr>
<th>Gamble</th>
<th>Initial wealth ($ millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>- - - - + + - -</td>
</tr>
<tr>
<td>2</td>
<td>- - + + + + + + +</td>
</tr>
<tr>
<td>3</td>
<td>- + + + + + + + +</td>
</tr>
<tr>
<td>4</td>
<td>- + + + + + + + +</td>
</tr>
<tr>
<td>5</td>
<td>- + + + + + + + +</td>
</tr>
<tr>
<td>6</td>
<td>- + + + + + + + +</td>
</tr>
</tbody>
</table>

Panel D: $\gamma = 5, \eta = 10000, \zeta = 4$

<table>
<thead>
<tr>
<th>Gamble</th>
<th>Initial wealth ($ millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>- + + + - - - -</td>
</tr>
<tr>
<td>2</td>
<td>- + + + + + + + +</td>
</tr>
<tr>
<td>3</td>
<td>- + + + + + + + +</td>
</tr>
<tr>
<td>4</td>
<td>- + + + + + + + +</td>
</tr>
<tr>
<td>5</td>
<td>- + + + + + + + +</td>
</tr>
<tr>
<td>6</td>
<td>- - + + + + + +</td>
</tr>
</tbody>
</table>

Panel E: $\gamma = 5, \eta = 10000, \zeta = 40$

<table>
<thead>
<tr>
<th>Gamble</th>
<th>Initial wealth ($ millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>- - - - + + - -</td>
</tr>
<tr>
<td>2</td>
<td>- - - + + + + + +</td>
</tr>
<tr>
<td>3</td>
<td>- - - + + + + + +</td>
</tr>
<tr>
<td>4</td>
<td>- - - + + + + + +</td>
</tr>
<tr>
<td>5</td>
<td>- - + + + + + + +</td>
</tr>
<tr>
<td>6</td>
<td>- - - + + + + + +</td>
</tr>
</tbody>
</table>
Figure 1: Portfolio allocations with and without lotteries

Panel A
Private equity allocation with and without lotteries

Optimal allocations to private equity with and without convexification of the objective function using gambling/lotteries.

Panel A: $\gamma = 5, \eta = 10^4, \zeta = 4$
Panel B: $\gamma = 5, \eta = 10^4, \zeta = 40$
Figure 2: **Portfolio allocations with multiple common assets**

Panel A

Allocation to public asset 1

Panel B

Allocation to private asset

Optimal policies for the one-period model with 2 common assets and an individual assets; $\gamma = 5$, $\eta = 10^4$, $\zeta = 40$. Panel A plots the fraction of total allocation to the two common assets invested in $R^{a1}$. Panel B depicts the fraction of total wealth invested in the individual-specific asset.