Asset measurement in imperfect credit markets

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Abstract

How should a firm measure a productive asset used as collateral in a credit agreement? To answer this question, we develop a model in which firms borrow funds subject to collateral constraints. In this environment, we characterize the qualities of optimal asset measurements and analyze their interactions with financing needs, collateral constraints and interest rates. We demonstrate that greater financing needs or tighter credit market conditions may, counter-intuitively, lead to more opaque measurements and increased investment, generally in the form of inefficient continuations. The optimal measurement adapts to credit market conditions.

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This paper develops an analysis of the optimal measurement of productive assets held as collateral, when firms are subject to a financing need. In practice, most bank loans are secured by some type of collateral. For example, in their sample of private debt from 1988 to 2007, Erel, Julio, Kim, and Weisbach (2012) report that 79% of debt contracts were secured. While the pricing of collateralized loan has been the object of extensive prior research (Stulz and Johnson 1985), our focus is on the optimal measurement of the underlying productive asset used as collateral. Collateral is a first-order consideration in the design and valuation of debt issues.

Our theory demonstrates that the qualities of the optimal measurement are a function of financing needs, the interest rate, and collateral constraints. In general, full disclosure is inefficient because it prevents firms whose collateral has low resale value from raising capital. When financing needs are large or credit market conditions are tight, the optimal measurement prescribes more precise disclosures over low asset values. Otherwise, the optimal measurement prescribes more precise disclosures over high asset values. In this latter case, we show that, counter-intuitively, firms respond by increasing investment and reducing transparency in response to increases in financing needs. In summary, the nature of the optimal measurement and its consequences on investment are a function of credit market conditions.

In our model, firms are subject to a financing need and must raise capital from an outside source in order to continue operating. Each firm has a single operating asset that can be used either as an input of production or liquidated in a competitive capital market. When the firm is liquidated, the productive asset is sold for cash and the proceeds are competitively reinvested at the risk-free rate. Key to our approach, there is incomplete information about the collateral value of the asset when it is sold externally or seized and liquidated by other parties; for example, some assets may be more efficiently used by a competitor while a firm-specific productive asset may have low value if used by other firms. The firm commits ex-ante to release information about collateral values to
maximize its ex-ante surplus.

If the financing need is small to moderate, a measurement prescribing more precise information about high asset values, hereafter, an *upper measurement*, is optimal. For example, this may be interpreted as a write-up over certain asset classes or, alternatively, as a higher degree of verification for good news. The advantage of this measurement is to identify assets whose outside resale value is attractive relative to the cash flows if they were operated. By contrast, firms whose collateral has low outside resale value are better-off operating the asset internally. If they were to disclose their collateral value, some of these firms would fail to meet the minimal collateral constraint, causing an inefficient liquidation. Hence, the upper measurement prescribes to withhold information about low asset values.

As the financing need increases, the minimum collateral constraint becomes more difficult to meet. Then, upper measurements are problematic because they indirectly deplete expected collateral when the firm withholds. The optimal measurement first responds to the depletion of collateral by reducing upper disclosures, which increases the expected collateral of withholding firms at the expense of over-investment by firms that should have disclosed and sold their asset. In this case, an increase in the financing need requires more firms with high collateral values to inefficiently continue, reduces the precision of the measurement and *increases* the probability of investment.

As the financing need increases even further, all firms with high collateral withhold and are inefficiently continued; yet, this will no longer suffice to meet the collateral constraint. At this point, the optimal measurement must prescribe some disclosures for low collateral values, hereafter a *lower measurement*. For example, a lower measurement may correspond in practice to an asset impairment in which a decline in asset value is reported. Lower measurements reveal a firm that cannot meet the collateral requirement and lead to inefficient liquidations.

**Related Literature.** Our model is part of the broader literature on the real effects of
disclosure, defined as the strategic consequences of information on the actions of market participants, i.e., “how accountants measure and disclose a firm’s economic transactions changes those transactions” (Kanodia and Sapra 2015). Our model specializes this idea to transactions that involve financing with collateral, by considering how the measurement may change market perceptions about collateral and, in doing so, affect the ability of a firm to raise capital.

There is an extensive literature in the area of real effects, and, to settle ideas further, we discuss a few related studies below. Kanodia, Singh, and Spero (2005) consider a model in which the investment choice can signal a firm’s inherent characteristics. An excessively precise disclosure of investment might cause an over-investment distortion as the observed investment acts as a signal of quality. Suijs (2008) examines whether asymmetric disclosures can affect the allocation of the risk of the firm’s investments between generations and, like us, argues that the degree of asymmetry is a function of the production technology. Focusing on voluntary disclosures, Beyer and Guttman (2010) and Hughes and Pae (2013) examine the interaction between incentives to release information, adverse selection, and their effects on productive decisions.1 A recent literature examines when changes in the public information environment can shift expectations across multiple equilibria (Morris and Shin (1998)). Applying this theory in the context of mark-to-market accounting, Plantin, Sapra, and Shin (2008) find that measurement rules based on market prices tend to increase asset sales during a downturn, draining liquidity and magnifying the adverse consequences of the downturn.2

Our model also extends the literature on credit rationing under asymmetric information. In this area, the paper most closely related to ours is Holmström and Tirole (1997),

1For other studies on the real effects of disclosure, see also Kanodia (1980), Sapra (2002), Caskey and Hughes (2012), Beyer (2012), Gigler, Kanodia, Sapra, and Venugopalan (2012) and Corona and Nan (2013).

2A recent study by Corona, Nan, and Zhang (2014) analyzes the effect of loan measurement on banks, although its focus is slightly different from ours in that these studies focus on the banks’ accounting of its own assets while we focus on the measurement by the debt issuer.
who link investment to the firm’s available collateral in a model of financial intermediation. There are two key differences between their model and ours. First, we specifically focus on a setting in which collateral values are not fully observable and determine the optimal measurement. Second, while their focus is on pure financial assets (which, admittedly, form a very small portion of the type of collateral used in practice), we focus on productive assets used in the firm’s operations.

Within this area, a study closely related to ours is Goex and Wagenhofer (2009), who examine a commitment to an information system in which the value of the collateral can be measured. In their model, lower measurements are always preferred to any other measurements. Their baseline setting is different from ours in that they do not analyze productive collateral so that, in their model, there can be no inefficient continuation. They also assume that a liquidation of the asset after cash flows are observed is costly, implying that firms with higher asset values endure higher liquidation costs.

Several prior studies have examined whether pre-decision information can be useful for an organization, and our study fits within this literature. Baiman and Evans (1983), Penno (1984), and Baiman and Sivaramakrishnan (1991) examine this question in the context of a control problem and analyze when giving more information to an agent can reduce agency costs. Our model presents a slightly different environment because, in the context of an end-of-period sale by the manager, pre-decision information can only have an impact if it is publicly revealed to both the manager and outside investors. More recently, Demski, Lin, and Sappington (2008) also focus on asymmetric asset reevaluations, but their primary focus is on solving a lemon’s problem at the time of sale rather than the shortage of collateral considered here. In a model where disclosures are entirely voluntary and information is produced by analysts, Langberg and Sivaramakrishnan (2010) argue that some unfavorable disclosures are made to improve production efficiency. Teoh (1997) considers the social value of disclosure in the problem of the commons, and shows that the consequences of disclosure depend on the nature of the production function and
that with decreasing returns disclosure can increase free-riding behaviors.

1 The model

The model builds on Holmström and Tirole (1997), hereafter HT, to which we add the measurement of real collateral, defined as assets that are essential parts of firms’ productive activities. We first lay out the assumptions required for our analysis verbally, with emphasis in italics to critical aspects of our analysis.

**Assumption 1.** Firms must raise financing from a capital market, where funds can be borrowed at the competitive interest rate equal to the return on alternative investments.

**Assumption 2.** Firms hold a productive asset with an uncertain outside value when used by another party. This asset is necessary to operate the firm.

**Assumption 3.** Firms can make an ex-ante commitment to the measurement of the outside value.

**Assumption 4.** The measurement is costless and can reveal any information about the outside value.

**Assumption 5.** Lenders require a minimum asset market value, defined as the resale value of the asset conditional on the measurement, to be held as collateral.

We operationalize next these assumptions in greater detail. Funds can be borrowed competitively at a risk-free rate \( r \geq 0 \). Firms have a risky project that delivers a non-contractible expected cash flow \( H \) but requires an outside capital infusion \( I \in (0, H/(1 + r)) \). To keep our discussion focused, we will interpret \( I \) as a financing need in the analysis (Assumption 1). For example, we might think about \( I \) as a monetary outlay required to finance a single project or investment opportunity.

Each firm is endowed with a productive asset that can be transferred to a lender and sold. Whenever sold, the benefit of the asset to an outside party is uncertain, and denoted \( \tilde{A} \), drawn from a distribution with p.d.f. \( f(.) > 0 \), c.d.f. \( F(.) \), mean \( m \geq 0 \), and full
support over $\mathbb{R}^+$. For example, an outside party may acquire a productive asset, such as an inventory of materials, a building, a brand name or a patent, and then deploy it within its own business. Put differently, the realization of $\tilde{A}$ is the net present value that can be achieved from the best alternative use of the asset (Assumption 2).

Following Kamenica and Gentzkow (2011), we model the asset measurement as an ex-ante commitment to report information about the asset. A measurement is a set $D \subseteq \mathbb{R}^+$ indicating the asset realizations that are disclosed and, then, become publicly known.\(^3\) Otherwise, no disclosure is made. Investors in the market are risk-neutral and, applying Bayes rule, the asset’s (exit) value $P_D(A)$ is the resale price of the asset

$$P_D(A) = \begin{cases} A & \text{if } A \in D, \\ P_{ND} = \mathbb{E}(\tilde{A} | \tilde{A} \notin D) & \text{if } A \notin D. \end{cases}$$

As in other models of Bayesian persuasion, we abstract away from frictions that may restrict the set of feasible measurements, such as for example measurement costs or imperfect commitment (Assumption 3).

If the firm continues, it borrows for an expected cost $(1 + r)I$ in a competitive market, but must retain and use its productive asset. The productive asset is, then, sold after production has taken place.\(^4\) A continuing firm is subject to an agency problem and must have enough collateral value to raise capital. That is, in order to continue, a firm must meet a minimal collateral requirement $P_D(A) \geq A(I, r)$. Our results will hold for different agency problems so we do not fully specify the functional form of this minimum collateral. For our results to hold, we only need the function $A(I, r)$ to be continuous and

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\(^3\)In the model of Kamenica and Gentzkow (2011), an optimal information system takes the form of the sender choosing a distribution of posterior beliefs. Because in our model the role of the measurement is to induce continuation, the optimal information system can be implemented with an intuitive signal structure inducing a single (imprecise) withholding region. This withholding region can be equivalently thought of as a posterior expectation $\mathbb{E}(\tilde{A} | \tilde{A} \in ND)$ induced with probability $Pr(\tilde{A} \in ND)$.

\(^4\)The assumption can be contrasted with HT. In their model, the asset is a financial asset that is always invested and yields the risk-free interest rate (e.g., a cash balance or a security); to pay this competitive interest, the market must know the value of the collateral so the issue of the measurement is moot.
increasing in \( I \) and \( r \) with \( \lim_{I,r \to \infty} A(I, r) = \infty \). These assumptions are in line with reality because when the financing need \( I \) or the risk-free rate \( r \) increases, lenders ask for more collateral to compensate for the the funding they grant and the higher opportunity cost represented by \( r \). This requirement is satisfied, for example, by the cash-on-hand constraint of Kiyotaki and Moore (1997) (i.e., \( \bar{A}(I, r) = (1 + r)I \)). In section 3, we expand on this collateral constraint and derive the collateral constraint and debt security endogenously, as a function of explicitly stated agency frictions. We shall show then that the quoted interest rate on the debt security (i.e., the debt repayment net of principal absent default) is negatively associated to collateral. All results presented in our baseline line model carry over to the setting with endogenous collateral constraints.

If the firm is unable or unwilling to continue, it optimally liquidates and sells its productive asset for \( P_D(A) \), reinvesting the proceeds for an end-of-period payoff \((1 + r)P_D(A)\). Hence, even if the minimum collateral requirement is met, a firm prefers to liquidate if

\[
\underbrace{(1 + r)P_D(A)}_{\text{liquidation}} \geq \underbrace{H - (1 + r)I + P_D(A)}_{\text{continuation}}.
\]

This inequality can be rewritten as

\[
P_D(A) \geq \overline{A}(I, r) = \frac{H - (1 + r)I}{r}
\]

so that the liquidation threshold is decreasing in \( I \) and \( r \). In what follows, we lighten the notation by writing \( A \) and \( \overline{A} \), and drop the explicit dependence on \( I \) and \( r \). We further focus on \((I, r)\) such that, given \( r \), \( \underline{A}(I, r) < \overline{A}(I, r) \) for some non-empty set of \( I \). When this inequality does not hold, all firms liquidate regardless of the measurement.

To characterize the optimal measurement, it is convenient to define \( \theta_D(A) \in \{0, 1\} \) as a policy function equal to one when a firm continues and zero if a firm liquidates. As noted earlier, a feasible policy prescribes continuation, or \( \theta_D(A) = 1 \), if and only if the value of
the collateral $P_D(A)$ is in the region $[\underline{A}, \overline{A}]$. Then, we define an optimal measurement as a measurement that maximizes total surplus

$$D^* \in \arg\max_D \mathbb{E}(\theta_D(\tilde{A})(H - (1 + r)I - r\tilde{A})) + (1 + r)m,$$

where $H - (1 - r) - r\tilde{A}$ the social benefit of continuing the project, net of the financing cost $(1 + r)I$ and the opportunity cost of holding the productive asset $r\tilde{A}$.

**First-best benchmark.** To evaluate the incremental effect of informational frictions, we define the *first-best* as a benchmark problem by lifting the collateral requirement (Assumption 4) and therefore imposing a continuation function $\theta_D^{fb}(A) = 1$ if and only if $P_D(A) \leq \underline{A}$.

The net surplus of a continuing project is $H - (1 + r)I - r\tilde{A}$, positive if and only if $A \leq \underline{A}$. Therefore, the first-best policy is to continue any firm with $A \leq \underline{A}$, but liquidate any firm for which the productive asset $A > \underline{A}$ has a better alternative use. This feature is a critical tension in our analysis and motivates the role of disclosure in efficiently allocating the asset. For example, an unsold inventory may be sold to a discounter (say, better equipped to cater to a price-discriminating clientele), patents may be sold to a firm better equipped to manufacture and sell product, a plant from a firm facing low demand may be acquired by a competitor, or a growing store may acquire a central real estate location from another firm.\(^5\)

In first-best, full disclosure is an optimal measurement in that it implements the efficient continuation policy for all firms with $A \leq \underline{A}$. In what follows, we will assume that the collateral constraints bind and this first-best benchmark can no longer be attained.

In the Appendix, we formally show that the first-best surplus is infeasible if and only if the financing need $I$ is greater than a lower bound $I_{fb}$. We show that this condi-

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\(^5\)By contrast, HT only consider financial collateral that is always invested at the risk-free rate and Goex and Wagenhofer (2009) do not focus on the opportunity cost of capital and thus normalize the risk-free rate to zero.
tion is always met if \( E(\tilde{A} | \tilde{A} \leq \frac{H}{r}) = A(0, r) \) and, otherwise, \( I_{fb} \) is uniquely given by \( E(\tilde{A} | \tilde{A} \leq \bar{A}(I_{fb}, r)) = \bar{A}(I_{fb}, r) \). This represents the point at which there is no longer enough aggregate collateral to finance all firms that should continue in first-best while, simultaneously, efficiently liquidating all firms with \( A \geq \bar{A}(I_{fb}, r) \).

2 The optimal measurement

2.1 Full disclosure

We first examine full disclosure as a candidate optimal measurement. Full disclosure of all material events is a general principle in financial reporting and standard-setters have traditionally insisted in providing as much information as possible to investors (see, e.g., *Conceptual Framework for Financial Reporting*, FASB 2006). Recently, the expansion of fair-value measurement may have led to more comprehensive disclosure from a standard-setting perspective since, relative to pure historical cost, it requires new and updated information in the balance sheet.

Full disclosure maximizes investment efficiency in a single-person decision problem and, in our model, is an optimal measurement in the first-best benchmark. We demonstrate next, however, that full disclosure is always suboptimal in an imperfect credit market.

**Proposition 1** Full disclosure, i.e., \( D = \mathbb{R}^+ \), is not an optimal measurement.

Proposition 1 is an example of a real effect caused by excessive provision of information. Disclosure alters market expectations about collateral, revealing to the market that certain firms have assets with little resale value \( P_D(A) = A < \bar{A} \), forcing these firms to liquidate at low prices. A less precise measurement can help address this real effect by raising the market perception of collateral. As shown in Figure 1, nondisclosure pools together undisclosed asset values into a collateral value \( P_{ND} \): as long as \( P_{ND} \) remains
greater than the minimum required collateral, some firms with $A < \bar{A}$ can borrow funds provided they withhold.

In summary, imprecise disclosures help sustain a cross-subsidization of collateral among non-disclosing firms, as markets perceive an average level of collateral. The financing benefits of high levels collateral (for which the incentive constraint is slack) are redistributed to other non-disclosing with low levels of actual collateral and which require more collateral to obtain outside financing.

### 2.2 Asymmetric measurements

We are interested next in the characteristics of an optimal measurement and, to solve this problem, we borrow the following general principle from persuasion theory. When the first-best surplus is unattainable, the optimal measurement implements the minimum collateral $P_{ND} = \bar{A}$, because it makes the capital provider indifferent between their two possible actions of financing versus not financing the firm (Kamenica and Gentzkow (2011), proposition 5 p. 2605).

The economic intuition for this result in our model is straightforward. If $P_{ND}$ is below
the minimum required collateral, all withholding firms are liquidated, leading to a surplus that is below a full-disclosure measurement. If \( P_{ND} \) is above the minimum required collateral, the extra collateral does not affect investment in the withholding region. While increasing perceived collateral beyond \( A \) provides no benefit, it is socially costly to do so, because it requires a measurement such that (a) some firms with low assets \( A < A \) disclose and inefficiently liquidate, or (b) some firms with high assets \( A > A \) do not disclose and inefficiently continue. To avoid any extra inefficiency, the optimal measurement always implements the minimum required collateral to continue the firm.

**Proposition 2**  *In an optimal measurement*,

(i) the collateral of a withholding firm is \( P_{ND} = \mathbb{E}(\tilde{A}|\tilde{A} \notin D) = A \):

(ii) only withholding firms continue and any \( A \) in the region \((A, \overline{A})\) is withheld.

As noted in part (ii), intermediate asset realizations \( A \) that would be efficiently continued even if they were disclosed, should be withheld in the optimal measurement. By definition, these realizations of \( A \) are greater than the minimum required collateral and, therefore, pooling them in the withholding region helps raise the continuation collateral \( P_{ND} \).

We are now equipped to derive the optimal measurement. Proposition 2 establishes that \( \theta_D(A) \) can be equivalently thought as continuation or withholding, since the two actions always coincide. Hence, the optimal measurement sets the minimum required collateral conditional on nondisclosure \( P_{ND} = \mathbb{E}(\tilde{A}|\tilde{A} \notin D) = A \), which simplifies to

\[
\int (A - A) \theta_D(A) f(A) dA = 0, \tag{C}\]

and which states that the nondisclosure collateral must be above \( A \).
The optimal measurement solves the following program

$$\theta^*_D(.) \in \arg\max_{\theta_D(.) \in \{0,1\}} \int \theta_D(A)(H - (1 + r)I - rA)f(A)dA$$

subject to (C) and $\theta_D(A) = 1$ if $A \in (A, \overline{A})$.

To solve this problem with standard calculus, it is convenient to solve a relaxed program, searching across policies $\hat{\theta}_D(A) \in [0, 1]$ in which the disclosure policy is a continuous variable (e.g., the probability to withhold). We shall prove that a solution of this relaxed problem involves $\hat{\theta}_D^*(A) \in \{0, 1\}$, and thus it is also a solution of the original program.

Denoting the lagrangian $L$ of the relaxed program and $\mu \geq 0$ the multiplier associated to (C), for any $A /\in (A, \overline{A})$,

$$\frac{1}{f(A)} \frac{\partial L}{\partial \hat{\theta}_D(A)} = H - (1 + r)I - rA + \mu(A - A).$$

This first-order condition represents the benefit of not disclosing a particular realization of $A$. Part (i) corresponds to the net continuation payoff of a withholding firm and part (ii) corresponds to the contribution of the firm to the continuation collateral. When the first-order condition is positive, the optimal measurement implies $\hat{\theta}_D(A) = 1$, that is, the firm withholds and continues; otherwise, when this equation is negative, the optimal measurement implies $\hat{\theta}_D(A) = 0$, that is, the firm discloses and is liquidated.

For some asset realizations, the marginal effect in part (i) dominates part (ii). For example, evaluating equation (1) at the minimum required collateral $A = A$, the first-order condition simplifies to $H - (1 + r)I - rA > 0$. That is, a firm that falls slightly short of the minimum required collateral if its collateral is disclosed will never disclose and will always be financed. This is intuitive as such a firm has almost no effect on the equilibrium continuation collateral $P_{ND} = A$ but generates some value when it is
operated.

In summary, the optimal measurement solves a trade-off between the opportunity cost of continuing with valuable assets $A$, because any continuing firm forfeits the financial return $rA$, and the collateral value of nondisclosure. We resolve this trade-off in the next proposition.

**Proposition 3** The optimal measurement takes the following form:

(i) if $m > A(I, r)$, that is, aggregate collateral is large, firms implement an upper measurement in which collateral values $A \in D = (A_{up}, \infty)$ are disclosed, where $\mathbb{E}(\tilde{A} | \tilde{A} \leq A_{up}) = A(I, r)$;

(ii) if $m = A(I, r)$, $D = \emptyset$, i.e., the optimal measurement is no-information;

(iii) if $m < A(I, r)$, that is, aggregate collateral is small, firms implement a lower measurement in which collateral values $A \in D = (0, A_{low})$ are disclosed, where $\mathbb{E}(\tilde{A} | \tilde{A} \geq A_{low}) = A(I, r)$.

We illustrate this proposition in Figure 2. Given a low liquidity need $I$, the opportunity cost of collateral is dominant; then, the optimal measurement focuses on disclosing firms with high asset realizations that yield the greatest financial return (part (i)). Note that withholding firms raise capital and, hence, capital providers can, in equilibrium, understand that high collateral values $A \geq A_{up}$ have been liquidated. We do not mean that withholding firms factually report to capital providers that their collateral is below $A_{up}$ as this information is conveyed via equilibrium expectations.

Given a high financing need $I$, the financing constraint is dominant; then, the optimal measurement focuses on disclosing firms with low asset realizations that most decrease the nondisclosure collateral (part (ii)). Similarly, withholding firms indirectly convey to capital providers, via equilibrium expectations, that their collateral is greater than $A_{low}$ since they have not been subject to a disclosure.
Note that upper and lower measurements yield different levels of investment efficiency. Upper measurements exhibit some inefficient continuations, for firms with high $A \in (\bar{A}, A_{up})$ that withhold and inefficiently continue. Lower measurements also feature these inefficient continuations, since all firms that with $A > \bar{A}$ withhold and continue; in addition, these measurements feature a second sort of investment inefficiency since all firms with $A < A_{low}$ disclose and inefficiently liquidate. Hence, upper measurements are always preferred to lower measurements if they can meet the collateral constraints; if $I$ is large, however, only lower measurements might be feasible.

![Disclosure region](image)

**Figure 2**: financing need and the optimal measurement

### 2.3 Comparative statics

We describe next how the optimal measurement responds to changes in the characteristics of the project and the financing need.

**Corollary 1** *The following comparative statics hold:*
(i) In an upper measurement, the probability of disclosure and the probability of liquidation decrease \((A_{up} \uparrow)\) in the interest rate \(r\) and the financing need \(I\).

(ii) In a lower measurement, the probability of disclosure and the probability of liquidation increase \((A_{low} \uparrow)\) in the interest rate \(r\) and the financing need \(I\).

Under both upper and lower measurements, the collateral value of a withholding firm is set equal to the minimum bound \(A\) required for financing. Conditional on an upper measurement, this bound is implemented by inefficiently continuing certain firms with high asset realizations: the greater the financing need or the interest rate, the greater the need for inefficient continuations and the more opaque the measurement. Conditional on a lower measurement, the collateral requirement is implemented by disclosing and inefficiently liquidating certain firms with low asset realizations: the greater the interest rate or the required collateral, the greater the level of inefficient liquidations.

In our model, the probability of a disclosure is also the probability of a liquidation, since disclosure is used as a means to identify when to liquidate. This implies that greater financing needs increase inefficient continuations in an upper measurement. The behavior of investment is illustrated in Figure 2. More firms with high collateral must continue in response to a greater financing need, because their continuation serves to increase the withholding collateral. Then, in the range of low to moderate financing needs, the model predicts a growth investment boom given greater financing needs. This effect caused by the collateral constraint is in contrast to the first-best investment policy or the lower measurement, where investment will decrease given greater financing need. Indeed, when the financing need is close to setting the aggregate collateral equal to the minimum collateral requirement, all firms are continued.
3 Endogenous collateral

In this section, we elaborate on the origins of the collateral constraint $\Delta(I, r)$ by deriving it as a function of a simple model of agency frictions. Then, we determine the debt security used for financing, and examine its face contractual interest rate (i.e., the interest rate that must be paid to lenders absent default).\(^6\)

Suppose, next, that the realized payoff of a continuing project is random, and either $\pi = S$ (success) with probability $p \in (0, 1)$ or $\pi = 0$ with probability $1 - p$ (failure). Note that $H = pS$, assumed greater than $(1 + r)I$, maps to the expected payoff in the baseline model. The firm’s owner may now take a bad unobserved action which yields a private benefit $B > 0$ but reduces the probability of $S$ by $\Delta p \in (0, p)$.\(^7\) To rule out uninteresting settings where the unobserved action is elicited and the firm might be financed with no collateral requirement, we set $(p - \Delta p)S + B - (1 + r)I < 0$. This means that the firm would have negative value if the unobserved action were elicited.

To raise capital, withholding firms issue a security with repayment schedule $W = (w_\pi)_{\pi \in \{S, 0\}}$. This security cannot pay more than the firm’s end-of-period cash flow, that is, $w_\pi \leq \pi + P_D(A)$ (LL). Furthermore, the firm must issue a security that credibly convinces the capital providers that the bad action will not be chosen, which can be written as $\Delta p(S - w_S + w_0) \geq B$ (IC). Lastly, this security maximizes the residual cash flow of the firm if it binds the participation of the capital provider $pw_S + (1 - p)w_0 \geq (1 + r)I$ (PC). A security $W$ is optimal if it satisfies (LL), (IC) and (PC).\(^8\)

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\(^6\)By construction, the (debt) security used for financing will yield an expected return $(1 + r)I$. Here, we map this expected return to the face interest rate that would be placed in the legal description of the debt contract; that is, the debt contract will prescribe the actual interest to be paid absent a default, which will typically be higher $(1 + r)I$ as a function of the probability of default and the transfer to debt holders conditional on default.

\(^7\)This action is a short-hand to represent many possible actions that could be detrimental to the value of the firm, such as shirking on the job, empire building, picking unproductive pet projects or diverting assets for a personal use.

\(^8\)Once (PC) is imposed, the firm’s owners will achieve an expected payoff $p(S + P_{ND} - w_S) + (1 - p)(P_{ND} - w_0) = pS - (1 + r)I + P_{ND}$ equal to the expected value of the project and the collateral minus the present value of the required financing.
Lemma 1  An optimal security exists if and only if $P_D(A) \geq (1+r)I - pS + \frac{pB}{\Delta p} \equiv A(I, r)$.

Lemma 1 closely follows the collateral constraint in HT and states that, to be financed, a firm must have a collateral valued by the market for at least $A(I, r)$. The greater the private benefit $B$ or the information friction $p/\Delta p$, the greater the required collateral.

We characterize next the nature of the optimal security. As is common in these problems, the optimal security is not unique if $P_D(A) > A(I, r)$ (as is the case in HT). In our model, however, Proposition 2 implies that only withholding firms continue and, then, use the minimum possible collateral, so that $P_D(A) = P_{ND} = A(I, r)$.

Proposition 4  All continuing firms use a collateral $P_{ND} = A(I, r)$. The optimal contract is unique and is such that the firm must repay $\min(P_{ND}, (1+r')I)$ where the face interest rate $r'$ is given by

$$r' = r + (1-p)\frac{(1+r)I - P_{ND}}{pI} = r + (1-p)\frac{S - \frac{B}{\Delta p}}{I}.$$  

The face interest rate $r'$ is the contractual interest to be paid in the event that the firm does not default, as directly observed in a debt contract. Unsurprisingly, this interest rate is greater than the expected return demanded by lenders $r$, because it incorporates the probability that the project fails and, in a default situation, repays a lower amount $P_{ND}$. This implies, as is intuitive, that the face interest rate is increasing in the collateral $P_{ND}$ since more collateral offers more protection to the lenders.

Substituting in the (endogenous) minimum collateral $P_{ND} = A(I, r)$, a new fact becomes apparent. As informational frictions become greater, i.e., $B$ or $1/\Delta p$ increase, the debt security requires a greater collateral $A(I, r)$. While this makes it more difficult to obtain external finance, equation (2) reveals that such more highly collateralized loans require a lower face interest rate. Hence, we show that the face interest rate (the most directly observed empirical property of loans) is negatively associated to the underlying agency frictions.
We conclude with an important implication of this section for empirical analysis. Many empirical studies assume that lower face interest rates are associated to lower cost of debt, better financing conditions and lower agency frictions (see the survey by Armstrong, Guay, and Weber (2010)). Furthermore, this interpretation is widely used in the context of bank debt contracts which tend to be heavily collateralized.

Within our model, this interpretation should be considered with caution. The “cost of debt”, defined as the expected payment to capital providers, is by construction equal to the expected return \((1 + r)I\) and is not equal to the face interest rate. As a result, an observed change in the face interest rate as a situation more attractive to borrowers, unless the value of the associated collateral is empirically measured.\(^9\) In fact, it is difficult to think about a reason why investors would demand a lower expected return on their securities - the notion ex-ante relevant to borrowers - because of characteristics of measurements or agency frictions. In addition, a low face interest rate is a companion to loans that are more heavily collateralized and, hence, does indicate a situation where many firms cannot obtain financing. This indicates an opportunity cost to firms that do not raise capital and, in this respect, low face interest rates might indicate greater investment inefficiencies.

4 **Caveats to the analysis**

We discuss, below, some variations on the assumptions of the baseline model and how these variations would affect our analysis.

**Private information.** In our baseline, we have assumed that firm owners commit to an information system which reveals information to outsiders. Our analysis holds if the manager is privately informed about \(\hat{A}\) but (a) the asset is sold competitively to *uninformed* investors, and (b) private information cannot be costlessly and truthfully disclosed. If (a)

\(^9\)To address this, many studies control for the risk of default; however, note that, in our model, the probability of default \(1 - p\) will not vary conditional on a change to the agency friction.
does not hold such as (for example) the manager consumes the realized value of the asset or the realized \( A \) becomes publicly known for exogenous reasons, the firm owners will divert the asset as a function of the privately-observed \( A \) and, within this scenario, more disclosure is always desirable. If (b) does not hold, firm owners with high asset realizations obtain better financing terms and, from a standard unravelling argument, ex-post frictionless voluntary disclosure would imply that all information would be revealed. A variation on (b) would be to assume that some voluntary disclosure are possible, albeit with a friction. This alternative setting implies very similar results to our baseline setting, except that some firms would voluntary disclose high realized collateral values.

**Variable investment.** Our baseline setting is one in which the firm has access to a single investment opportunity or, equivalently, needs an infusion of capital to continue operating. A possible extension of this setting may involve a variable scale of investment, in which the firm chooses not just whether to continue but, also, how many resources to put into its project. This alternative setting causes an additional inefficiency when withholding, because it prevents the firm from choosing its preferred scale of investment. In extreme cases where this inefficiency is very large, this can cause the firm to revert to full disclosure. For example, if the investment technology is perfectly linear, so that the expected cash flow is \( I_H \), the required collateral is \( I_A \) and the investment can be increased unboundedly, it can be shown that the value of a continued project is linear in the available collateral \( P_D(A) \). Then, the expected surplus of a continuing firm is independent of the measurement of the collateral.

**Timeless rules.** As our study has tried to derive the preferred measurement, we have solved for the measurement given knowledge of the financing needed \( I \). Currently, however, standard-setting bodies do not have the institutional design to quickly adapt to financial shocks; for example, standard-setting takes the form of written rules rather than
flexible policies, and the due process implies that any change in standards is relatively slow. One may ask, then, what a measurement would look like if it were chosen for a long horizon, that is, without conditioning on the knowledge of a realized financing need. Although both upper and lower measurements may be optimal in this problem, as a function of the distribution of financing needs, upper measurements are somewhat problematic because, if the financing need is greater than a certain threshold, a fixed upper measurement will occasionally cause a complete breakdown in all financing. Lower measurements also run this risk, but the required financing need to cause this behavior is greater.

5 Conclusion

In this study, we challenge a conventional view that asset measurements should be designed with an emphasis on full disclosure regardless of credit market conditions and, more generally, prescribe fixed measurement rules. In contrast to this view, we describe a simple setting in which the optimal measurement is a function of several economic determinants, including current financing needs, interest rates (or cost of funds), collateral requirements, and other frictions in the credit market. Credit market conditions affect both the optimal level of disclosure as well as what such disclosures should focus on (for example, whether providing more informative disclosures about high or low asset values).

We illustrate the economic trade-offs in a simple economic model involving the measurement of the collateral value of a firm’s productive assets. Excessive measurements trigger inefficient liquidations of productive assets whose collateral value is low, while insufficient measurements dampen the market’s confidence in the collateral value of assets whose value has not been assessed. The resolution of these fundamentals depends on credit market conditions and involves flexible measurements with varying degrees of information being released or a changing focus on measurements of high collateral values versus low collateral values.
The research on disclosure and financing needs is still nascent and we observe that our
discussion is, due to its theoretical nature, limited in its scope and, hence, its predictions
should be considered illustrative of basic trade-offs rather than prescriptive. It is clear that
accounting does not serve only the purpose of valuing collateral prior to lending agree-
ments and we have taken aside, for example, other important functions of measurements,
such as reducing adverse selection in capital markets or deciding whether to liquidate an
asset at some interim stage. Having noted this, we hope that our analysis can help shed
some light into an important aspect of asset measurements in debt contracts.

Appendix

**Conditions on $I$ to achieve first-best:** A measurement that implements the first-best
surplus induces $\theta_D(A) = 0$ for $A > \overline{A}$ and $\theta_D(A) = 0$ for $A < \underline{A}$. Note that firms
with $A < \underline{A}$ would liquidate if they were to disclose, so that it must be that all $A \in
[0, \underline{A})$ are withheld and withholding induces continuation. Then, all $A > \overline{A}$ should be
liquidated, implying that all $A > \overline{A}$ should be disclosed. Whether the measurement
prescribes disclosure or withholding for $A \in [\underline{A}, \overline{A}]$ is irrelevant since the firm continues
in both cases (its efficient choice).

The only constraint is that the measurement must satisfy $\mathbb{E}(\tilde{A}|\tilde{A} \notin D) \geq \underline{A}$, that is,
withholding firms must meet the collateral constraint on average. This constraint is easiest
to meet with a measurement such that all $A \leq \overline{A}$ are withheld. In other words, the first-
best investment policy can be implemented if and only if (making the dependence explicit)

$$\mathbb{E}(\tilde{A}|\tilde{A} \leq H - (1 + r)I) \geq \underline{A}(I, r).$$

The left-hand side of this equation is decreasing in $I$ while the right-hand side is increas-
ing in $I$; therefore, this equation must be satisfied on a set with the form $\{I : 0 \leq I \leq I_{fb}\}$.
where either (a) \( \mathbb{E}(\tilde{A} | \tilde{A} \leq H) < A(0, r) \), in which case \( I_{fb} < 0 \) and this set is empty, 
or (b) \( \mathbb{E}(\tilde{A} | \tilde{A} \leq H) \geq A(0, r) \), in which case \( I_{fb} \) is given by the relationship 
\[ \mathbb{E}(\tilde{A} | \tilde{A} \leq H - (1 + r)I_{fb}) = A(I_{fb}, r) \] (*). Since we focus on \( A(I, r) < \bar{A}(I, r) \), non-empty, letting 
\( I_{\text{max}} \) denoting \( A(I_{\text{max}}, r) = A(I_{\text{max}}, r) \), it is readily verified that a unique \( I_{fb} \) solution to 
(*') exists in \([0, I_{\text{max}}]\).

**Proof of Proposition 1:** With full disclosure, firms inefficiently liquidate if their asset value is below \( A \). Any measurement rule that prescribes \( \theta_D(A) = 1 \) for \( A \in [A, \bar{A}] \), 
\( \theta_D(A) = 0 \) for \( A > \bar{A} \) and \( \theta_D(A) = 1 \) for some \( A < A \) provided that \( \mathbb{E}(\tilde{A}|\tilde{A} = 0) \geq A \) improves the ex-ante surplus relatively to full disclosure.

**Proof of Proposition 2:** We first show that non-disclosing firms continue. We make a reasoning by contradiction. We assume that when a firm does not disclose, it liquidates. Under this assumption, the optimal measurement system maximizes:

\[
\max_{\hat{\theta}_D(A) \in [0, 1]} \int_{A}^{\bar{A}} (1 + r)A(1 - \hat{\theta}_D(A))f(A)dA + \int_{\bar{A}}^{\infty} (1 + r)A(1 - \hat{\theta}_D(A))f(A)dA + \int_{A}^{\bar{A}} (H - (1 + r)I + A)(1 - \hat{\theta}_D(A))f(A)dA + \int_{0}^{+\infty} (1 + r)A\hat{\theta}_D(A))f(A)dA
\]

Taking the first order condition (F.O.C) yields:

\[-f(A)(H - (1 + r)I - rA) < 0 \text{ if } A \in (A, \bar{A}), \text{ otherwise } 0.\]

The solution is \( \hat{\theta}_D(A) = 0 \) for \( A \in (A, \bar{A}) \). Otherwise any \( \hat{\theta}_D(A) \) can be set. As a result, all firms with \( A \in (A, \bar{A}) \) disclose and continue, while the other firms liquidate.

This measurement rule displays the same investment allocation as full disclosure, which is never optimal. Therefore, non-disclosing firms continue the project and \( P_{ND} \in [A, \bar{A}] \).

We assume by contradiction that \( P_{ND} > A \). This measurement rule is not optimal because we can improve welfare by measuring more firms’ collateral \( \tilde{A} < A \). Thus, \( P_{ND} = A \).

**Proof of proposition 3:** We assume that \( I > I_{fb} \). Taking the F.O.C on the Lagrangian
yields:

$$\frac{1}{f(A)} \frac{\partial C}{\partial \theta_D(A)} = H - (1 + r)I - rA + \mu(A - A)$$

For $A \in [A, \bar{A}]$, $\frac{1}{f(A)} \frac{\partial C}{\partial \theta_D(A)} \geq 0$, otherwise if $\mu \neq 0$, the sign of the FOC is ambiguous. To determine it, we study the monotonicity of $\frac{1}{f(A)} \frac{\partial C}{\partial \theta_D(A)}$. Depending on $\mu$, we analyze three cases: (a) for $\mu > r$, $\frac{1}{f(A)} \frac{\partial C}{\partial \theta_D(A)}$ is increasing in $A$, (b) for $\mu = r$, $\frac{1}{f(A)} \frac{\partial C}{\partial \theta_D(A)}$ is flat in $A$, (c) for $\mu < r$, $\frac{1}{f(A)} \frac{\partial C}{\partial \theta_D(A)}$ is decreasing in $A$.

(a) For $\mu > r$, there exists a unique $A_{low} < A$ such that:

(i) For $A < A_{low}$, $\frac{1}{f(A)} \frac{\partial C}{\partial \theta_D(A)} < 0$ and $\hat{\theta}_D(A) = 0$.

(ii) For $A \geq A_{low}$, $\frac{1}{f(A)} \frac{\partial C}{\partial \theta_D(A)} \geq 0$ and $\hat{\theta}_D(A) = 1$.

where $A_{low}$ is defined by $E(\bar{A}|\bar{A} \geq A_{low}) = A$.

Let us define $\forall Y \leq A, \Phi_{low}(Y) = (1 - F(Y))E(\bar{A}|\bar{A} \geq Y) - (1 - F(Y))A$.

Note that $\Phi_{low}(0) = E(\bar{A}) - A$ and $\Phi_{imp}(A) = (1 - F(A))E(\bar{A}|\bar{A} \geq A) - (1 - F(A))A > 0$. Further, $\frac{\partial \Phi_{low}(Y)}{\partial Y} = f(Y)(A - Y) > 0$. Thus, if $E(\bar{A}) < A$, $A_{low}$ is unique.

(b) For $\mu = r$ and for $A \not\in (A, \bar{A})$, $\frac{1}{f(A)} \frac{\partial C}{\partial \theta_D(A)} = r(\bar{A} - A) \geq 0$ and $\forall A, \hat{\theta}_D(A) = 1$. This case prescribes no measurement, which is optimal if and only if $I = I_{nd}$ defined by $E(A) = A(I_{nd})$.

(c) For $\mu < r$, there exists a unique $A_{up} > \bar{A}$ such that

(i) For $A < A_{up}$, $\frac{1}{f(A)} \frac{\partial C}{\partial \theta_D(A)} > 0$ and $\hat{\theta}_D(A) = 1$.

(ii) For $A \geq A_{up}$, $\frac{1}{f(A)} \frac{\partial C}{\partial \theta_D(A)} \leq 0$ and $\hat{\theta}_D(A) = 0$.

where $A_{up}$ is defined by $E(\bar{A}|\bar{A} \leq A_{up}) = A$. Define $\forall Y > A, \Phi_{up}(Y) = F(Y)E(\bar{A}|\bar{A} \leq Y) - F(Y)A$.

$\Phi_{up}(A) = F(A)E(\bar{A}|\bar{A} \leq A) - F(A)A < 0$ and $\lim_{Y \to \infty} \Phi_{up}(Y) = E(\bar{A}) - A$. Further, $\frac{\partial \Phi_{up}(Y)}{\partial Y} = f(Y)(Y - A) \geq 0$. Thus, $A_{up}$ is unique.
if $E(A) > A$.

**Proof of corollary 1**: Applying the implicit function theorem,

$$\frac{\partial A_{up}}{\partial q} = -\frac{\partial \Phi_{up}}{\partial q} / \frac{\partial \Phi_{up}}{\partial A_{up}}$$

where $q = r$ or $I$. From proposition 3, $\frac{\partial \Phi_{up}}{\partial A_{up}} > 0$. Further,

$$\frac{\partial \Phi_{up}}{\partial q} = -F(Y) \frac{\partial A}{\partial q}.$$

We conclude that $\frac{\partial A_{up}}{\partial q}$ is the same sign as $\frac{\partial A}{\partial q}$. Similarly applying the implicit function theorem,

$$\frac{\partial A_{low}}{\partial q} = -\frac{\partial \Phi_{low}}{\partial q} / \frac{\partial \Phi_{low}}{\partial A_{low}}.$$

From proposition 3, $\frac{\partial \Phi_{low}}{\partial A_{low}} > 0$. Further,

$$\frac{\partial \Phi_{low}}{\partial q} = -(1 - F(Y)) \frac{\partial A}{\partial q}.$$

We conclude that $\frac{\partial A_{low}}{\partial q}$ is the same sign as $\frac{\partial A}{\partial q}$.

**Proof of Lemma 1**: The manager selects a contract that elicits the value-enhancing action and maximizes his utility,

$$(w^*_S, w^*_0) \in \arg\max p(S + P_D(\hat{A}) - w_S) + (1 - p)(P_D(A) - w_0)$$
subject to:

\[ \Delta p(S - w_S + w_0) \geq B, \]  

\[ pw_S + (1 - p)w_0 \geq (1 + r)I, \]  

and \( w_S \leq S + P_D(A), \quad w_0 \leq P_D(A). \)

Rearranging the (IC) constraint,

\[ S - w_S + w_0 \geq \frac{B}{\Delta p}. \]

The lenders’ participation (PC) must be satisfied at equality, and rewriting it in terms of \( w_S, w_S = \frac{1+r}{p}I - \frac{1-p}{p}w_0. \) Substituting this equation in the above inequality,

\[ w_0 \geq (1 + r)I - pS + p \frac{B}{\Delta p} \equiv A(I, r). \]

Combining this inequality with the (LL) constraint, we conclude that

\[ P_D(A) \geq (1 + r)I - pS + p \frac{B}{\Delta p} \equiv A(I, r). \]

**Proof of Proposition 4:** If \( P_D(A) > (1 + r)I, \) one can set \( w_0 = (1 + r)I \) and thus, \( w_S = (1 + r)I. \) Otherwise, if \( P_D(A) \leq (1 + r)I, \) one can set \( w_0 = P_D(A) \) and \( w_S = \frac{(1+r)}{p}I - \frac{1-p}{p}P_D(A). \)

Further, if \( P_D(A) = A(I, r) \) then \( w_0 = A(I, r) \) and thus, the only efficient contract is the risky debt contract.

Further, from Proposition 2, we know that \( P_D(A) = P_{ND}. \) and from Proposition 3, we substitute \( A(I, r) \) by expression (3) to conclude that under the optimal measurement, the face value of the risky debt contract is equal to \( \frac{(1+r)}{p}A(I, r) = (1+r)I + (1-p)(S - \frac{B}{\Delta p}). \)
Thus this risky debt contract charges a face interest rate $r'$ given by

$$\frac{(1 + r)I + (1 - p)(S - \frac{B}{\Delta p})}{1 + r'} = I.$$ 

Rearranging the above expression,

$$r' = r + (1 - p)\frac{(S - \frac{B}{\Delta p})}{I}.$$ 

**Bibliography**


