Quota Dynamics and the Intertemporal Allocation of Sales-Force Effort

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April 2009‡
WORK-IN-PROGRESS

Abstract

We empirically investigate the effect of sales-force compensation schemes on the intertemporal allocation of sales-agents’ effort. Real-world sales force compensation schemes are typically concave and non-linear, involving quotas and bonuses that depend on outputs. Such non-linearities may introduce intertemporal inefficiencies, arising from the potential for strategic timing on the part of sales-agents. We develop a structural model of sales agent behavior to empirically measure the extent of such dynamic moral hazard. The non-linear aspects of compensation plans induce dynamic considerations into sales-agents’ actions. Quantifying the empirical relevance of these dynamic considerations requires modeling the forward-looking behavior by which sales-agents allocate effort over time. We model agents as maximizing intertemporal utility, conditional on the current compensation scheme, and their expectations about how quotas would be updated based on their chosen actions. Our empirical approach is to estimate, in a first stage, the structural parameters involving the sales person’s utility function, and to then simulate in a second stage, his behavior given a changed compensation profile. We utilize a rich dataset that involves complete details of sales, and compensation plans for a set of 90 sales-people for a period of 4 years at a large consumer-product company in the US. Our estimates from the data suggest significant incentives for strategic timing arising from the structure of the compensation scheme.

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‡We thank Dan Ackerberg, Lanier Benkard, Paul Ellickson, Liran Einav, Wes Hartmann, Gunter Hitsch, Phil Haile, Eddie Lazear, Philip Leslie, Katherine Shaw, Seenu Srinivasan, John Van Reenan, and seminar participants at Berkeley, Kellogg, NYU, Rochester, Stanford, UC Davis, as well as the 2008 Marketing Science, SICS, and UTD FORMS conferences, for their feedback.
1 Introduction

Salesforce compensation schemes in the real world often involve sales targets or quotas which specify discontinuous, non-linear compensation policies for agents when their output crosses pre-specified thresholds (Joseph and Kalwani 1992). It is well known (c.f. Holmstrom 1979; Lazear 1986) that output-based contracts, in general, have the beneficial effect of inducing agents to exert effort, even when effort is unobservable by the firm. However, surprisingly little is known about the role of quotas in motivating agent effort. In the salesforce context there is a large literature that investigates the design and implementation of compensation plans that induce optimal levels of salesforce effort, and examines the role of various factors on the nature and curvature of the optimal contract (see for e.g. Basu et al. 1985; Lal and Srinivasan 1993; Rao 1990). Most of this literature, however, has little to say about quotas (Coughlan 1993). The limited theory that does exist on quota based plans (c.f. Raju and Srinivasan 1996; Oyer 2000), while rationalizing the existence of quotas as approximations to optimal non-linear compensation schemes, has noted that quotas may introduce significant dynamic inefficiencies arising from the potential for strategic timing and manipulation of effort by agents. Again, given their static nature, the current theory has even less to say about the intertemporal inefficiencies in effort allocation caused by the presence of dynamic quotas. Investigating whether, how and to what extent these inefficiencies manifest themselves is the key aim of this paper.

To illustrate the nature of such dynamic inefficiencies consider the following example: a given firm’s compensation scheme involves a guaranteed salary plus a bonus if sales are higher than a pre-specified quota. Under this scheme sales-agents who achieve the level of sales required for the bonus in the current compensation cycle may have a perverse incentive to postpone additional effort to the future. This enables the agent to use the sales generated from the postponed effort to attain the quota in the next compensation cycle. Indeed, in some settings, it is possible that such intertemporal reallocation of effort may well negate the effort-inducing benefits from utilizing output-based contracts. The extent of such dynamic moral hazard indexes the inefficiency caused by the firm’s chosen compensation scheme. Our approach to

\[1\] An alternative motivation of output-based contracts is that it may help attract and retain the best sales-people (Lazear 1986; Godes 2003; Zenger and Lazarini 2004). This paper abstracts away from these issues since our data does not exhibit any significant turnover in the sales-force.

\[2\] Similar distortions can take place with a commission based plan (as in our application) when coupled with compensation plans that involve discontinuities based on quota ceilings and floors.
quantifying these inefficiencies relies on the development and implementation of a novel, fully dynamic model of agent effort allocation in response the compensation profile faced.

The construct of inefficiency is not absolute and has to be measured relative to a counterfactual compensation profile in which the agent faces a different set of incentives. An obvious theoretical benchmark is a comparison to the first-best contract in which effort is observable by the firm and compensation is a direct function of such effort. This contract does not generate any dynamic inefficiency; but, it is not practical from a real world, implementation standpoint since effort is never truly observable to the firm. Hence, we might wish to measure how much better or worse off the firm may be under other more feasible contracts. One possibility is for the firm to move to a linear contract, which specifies a simple linear commission on sales. Under the specific LEN assumptions (“linear exponential utility and normal errors”), Holmstrom and Milgrom (1987) showed that this scheme can achieve the best possible outcomes for the firm. A different possibility for the firm is to use subjective quotas that are updated according to the agent’s currently observed performance. Such a policy, sometimes referred to as “ratcheting”, is a common feature of several real world compensation schemes (e.g. Weitzman 1980; Leone, Misra and Zimmerman 2004). While ratcheting can help the firm fine-tune effort, it may also potentially accentuate dynamic inefficiencies if forward-looking agents manipulate current effort so as to obtain favorable quotas in the future. Yet another option would be for the firm to increase the length of the quota-cycle under which the generated sales count for a commission (for example, moving from a quarterly quota to an annual quota). However, this policy may generate undesirable shirking in the early part of the quota-cycle by agents, which increases the volatility of the firm’s revenue stream. It may well be that when taking these aspects of alternative contracts into account, the conclusions regarding the desirability of the firm’s current policies may be reversed. Thus, constructing a reasonable answer to the question “Are quotas good or bad for the firm?” rests on a credible evaluation of these alternative, counterfactual profiles.

Evaluating such policies requires a framework that incorporates the features of real world compensation schemes, and the dynamic incentives generated by them. The

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3Normatively, we can interpret the results from this counterfactual exercise as measuring the maximum willingness-to-pay of the firm for investments in additional technology that may help monitor agent effort better (for example, Hubbard 2003 documents the effects of better monitoring on worker productivity in the trucking industry).
extant models in the theoretical literature are inadequate for analyzing the counterfac-tuals because actual compensation schemes contain several aspects - kinked profiles, discrete bonuses, finite horizons, heterogeneity across agents and significant demand uncertainty - that are hard to accommodate into a single all-encompassing theoretical framework, but nevertheless, significantly impact on agent behavior. Developing randomized experiments in which alternative compensation policies are implemented is not feasible either. Our approach is to develop a structural model of agent behavior that incorporates the forward-looking behavior in which agents allocate effort over time by maximizing a discounted stream of payoffs. We outline methods to estimate the parameters indexing the model and show how the model can be solved using numerical dynamic programming techniques. The flexibility of the model formulation enables us to accommodate several important aspects of real world salesforce compensation policies. The structural derivation of the model facilitates the counterfactual analysis we are interested in.

Estimation of the model is complicated by the fact that effort is unobserved. We introduce a methodology that exploits the richness of our data, an informative structure, and recent advances in estimation methods to facilitate the identification of this latent construct. In particular, we describe how intertemporal linkages helps identify effort from sales data in salesforce compensation settings. We model agents as maximizing intertemporal utility, conditional on the current compensation scheme, and their expectations about the process by which quotas would be updated based on their chosen actions. Our empirical approach is to estimate, in a first stage, the structural parameters involving the sales person’s utility function. We then simulate, in a second stage, his behavior given a changed compensation profile. The estimator for the 1st stage of our empirical strategy is based on the recent literature on 2-step estimation of dynamic decisions (Hotz and Miller 1993; Bajari, Benkard and Levin 2007). Our approach is to non-parametrically estimate agent-specific policy functions, and use these, along with the conditions for the optimality of the observed actions, to estimate the structural parameters. We discuss how an individual rationality constraint as well as the assumption of agent optimality identifies sharp bounds on agent preferences. We use these bounds to generate the empirical distribution of agent preferences, which we use to simulate the behavior of the agent-pool under counterfactual compensation profiles.

A practical concern with the use of two-step estimators has been the presence of
unobserved serially-correlated state variables which prevent consistent non-parametric estimation of first-stage policy functions and transitions. In particular, this ruled out models with unobserved heterogeneity (though see Arcidiacono & Miller 2008 for a recent approach that handles discrete unobserved heterogeneity). We are able to address this problem due to the availability of panel data of relatively long cross-section and duration for each agent, which facilitates estimation agent-by-agent. This enables a non-parametric accommodation of unobserved heterogeneity. Given the estimates from the first stage, we evaluate agent behavior and sales under the counterfactual by solving the agents’ dynamic programming problem numerically. We believe we are the first in the empirical literature to model the intertemporal problem facing sales-agents and to measure the dynamic effect of quotas and ratcheting in a real world setting. More generally, our model can extended in a straightforward manner to analyze dynamics in other settings in which agents take actions to earn non-linear payoffs within pre-specifed deadlines.

Our model-free analysis of the data reveals significant evidence for strategic timing considerations by sales-agents. In particular, we find evidence that high effort levels are extended as the agent strives to “make quota” within the quarter, but that effort is adjusted downward once the agent is “in the money”. Our results indicate that the current compensation scheme is highly leveraged - the equivalent, linear contract that achieves the same quarterly sales on average as the current policy, requires a large, 9%, commission on sales. We find that the first-best can achieve about 98% higher revenues than the current policy, indicating significant costs associated with asymmetric information in this compensation scheme.

Our paper adds into a small empirical literature that has explored the effect of salesforce compensation schemes. Despite the preponderance of these schemes in practice, the empirical literature analyzing the effect of quotas on salesforce effort has remained sparse. Part of the reason for the paucity of work has been the lack of availability of agent-level compensation and output data. The limited empirical work has primarily sought to provide descriptive evidence of the effect of compensation schemes on outcomes (e.g. Healy 1985, in the context of executive compensation, and Asch 1990, in the context of army-recruiters paid via non-linear incentives). Oyer (1998) was the first to empirically document the timing effects of quotas, by providing evidence of jumps in firms’ revenues at the end of quota-cycles, that are unrelated to demand-side factors.
A related literature also seeks to empirically describe the effect of incentives, more broadly, on output (e.g. Chevalier and Ellison 1999; Lazear 2000a; Bandiera, Baransky and Rasul 2005). We complement this literature by detecting and measuring the dynamic inefficiencies associated with compensation schemes. The descriptive evidence on quotas are mixed. Using data from a different context, and a different compensation scheme, Steenburgh (forthcoming) reports descriptive evidence that agents facing quotas in a durable-goods company do not tend to reduce effort in response to lump-sum bonuses. In contrast, Larkin (2006) uses reduced form methods to document the effect of compensation schemes on the timing and pricing of transactions in technology-markets. Our paper is also related to the work of Ferrall and Shearer (1999), and Paarsch and Shearer (2000), who estimate static, structural models of worker behavior, while modeling the optimal contract choice by the firm. Unlike our context, the compensation contracts in their papers are linear, and do not generate dynamic incentives for the agent. The closest paper to ours in spirit is Copeland and Monnett (2007) who estimate a dynamic model to analyze the effects of non-linear incentives on agents’ productivity in sorting checks. Our institutional context, personal selling by salesforce agents, adds several aspects that warrant a different model, analysis, and empirical strategy from Copeland and Monnet’s context. Unlike their industry, demand uncertainty plays a key role in our setting; further, ratcheting, an important dynamic affecting agent effort in our setting, is not a feature of their compensation scheme.

Finally, this paper also adds to the theoretical literature on salesforce compensation by offering a new framework in which to examine more realistic comparative statics that involve arbitrarily complex and dynamic compensation plans and effort policies of agents that respond to these dynamics.

The rest of this paper is structured as follows: We begin with a description of our data and some stylized facts. We then introduce our model followed by the estimation methodology. Finally we present some counterfactuals and conclude with a discussion.

2 Patterns in the Data and Stylized Facts

In this section, we start by presenting some stylized facts of our empirical application, and also provide model-free evidence for the existence of strategic timing by sales-
agents in our data. We use the reduced form evidence and the stylized facts presented here to motivate our subsequent model formulation and empirical strategy.

2.1 Data and Compensation Scheme

Our data come from the direct selling arm of the salesforce division of a large consumer-product manufacturer in the US with significant market-share in the focal category (we cannot reveal the name of the manufacturer, or the name of the category due to confidentiality reasons). The category of interest involves a non-pharmaceutical product available via prescriptions to consumers from certified physicians. Importantly, industry observers and casual empiricism suggests that there is little or no seasonality in the underlying demand for the product. The manufacturer employs 87 sales-agents in the U.S. to advertise and sell its product directly to each physician (also referred to as a “client”), who is the source of demand origination. Agents are assigned their own, non-overlapping, geographic territories, and are paid according to a non-linear period-dependent compensation schedule. We note in passing that prices play an insignificant role since the salesperson has no control over the pricing decision and because price levels remained fairly stable during the period for which we have data.\footnote{In other industries, agents may have control over prices (e.g. Bharadwaj 2002). In such situations, the compensation scheme may also provide incentives to agents to distort prices to “make quota”. See Larkin (2006), for empirical evidence from the enterprise resource software category.}

The compensation schedule involves a fixed salary that is paid irrespective of the sales achieved by the agent, as well as a commission on any sales generated above a quota, and below a ceiling. The sales on which the output-based compensation is earned are reset every quarter. Additionally, the quota may be updated at end of every quarter depending on the agent’s performance (“ratcheting”).

Our data includes the complete history of compensation profiles and payments for every sales-agent for a period of 4 years, and monthly sales at the client-level for each of these sales-agents for 2 of these years.

Quarterly, kinked compensation profiles of the sort in our data are typical of most real world compensation schemes (e.g. Joseph and Kalwani 1998 report that 95% of compensation schemes they survey had some combination of quotas and commissions, or both), and have been justified in the theory literature as a trade-off between the optimal provision of incentives versus the cost of implementing more complicated schemes (Raju and Srinivasan 1996), or as optimal under specific assumptions on
agent preferences and the distribution of demand (Oyer 2000). Consistent with the literature, our conversations with the management at the firm revealed that the primary motivation for quotas and commissions is to provide “high-powered” incentives to the salesforce for exerting effort in the absence of perfect monitoring. We also learned that the motivation for maintaining a “ceiling” on the compensation scheme stemmed from a desire to hedge against large “windfall” payouts to agents, which may result from large changes in demand due to reasons unrelated to agent effort. The latter observation suggests that unanticipated shocks to demand are important in driving sales.

### 2.2 The Timing of Effort

Our primary interest is in the extent to which nonlinearities in the compensation schemes provide incentives to salespeople to manipulate the timing of transactions. We start by looking in the data to see whether there exists patterns consistent with such strategic timing. As Oyer (1998) pointed out, when incentives exist for agents to manipulate timing, output (i.e. sales) should look lumpy over the course of the sales-cycle. In particular, we expect to see spikes in output when agents are close to the end of the quarter (and most likely to be close to “making quota”). Figure 1 plots the sales (normalized by total sales across all months in the data) achieved in each month by a set of sales-agents for a one year window. Figure 1 reveals significant spikes at the end of quarters suggesting that agents tend to increase effort as they reach closer to quota. In Figure 2, we present analogous plots that suggest that agents also tend to reduce effort within the quarter. The shaded regions in Figure 2 highlights quarters in which sales fell in the last month of the quarter, perhaps because the agent realized a very large negative shock to demand early in the quarter and reduced effort, or because he “made quota” early enough, and hence decided to postpone effort to the next sales-cycle.\(^5\)

We now explore how these sales-patterns are related to how far the agent is from his quarterly quota. Figure 3 shows nonparametric estimates of the relationship between sales (y-axis) and the distance to quota (x-axis), computed across all the

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\(^5\)One alternative explanation for these patterns is demand side seasonality, which we believe can be reasonably ruled out due to the nature of the category. Another explanation is that the spikes reflect promotions or price changes offered by the firm. Our extensive interactions with the management at the firm revealed that prices were held fixed during the time-period of the data (in fact, prices are rarely changed), and no additional promotions were offered during this period.
Figure 1: Sales are high at end of quarters

Figure 2: Agents reduce effort within quarters
sales-people for the first two months of each quarter in the data. We define the distance to quota as $\frac{\text{Cumulative Sales at beginning of month} - \text{quota}}{\text{quota}}$. From Figure 3, we see that the distance to quota has a significant influence on the sales profile. Sales (proportional to effort) tend to increase as agents get closer to quota, suggesting increasing effort allocation, but fall once the agent reaches about 40% of the quota in the first 2 months, suggesting the agent anticipates he would “make the quota” by the end of the quarter. The decline in sales as the agent approaches quota is also consistent with the ratcheting incentive, whereby the agent reduces effort anticipating his quota may be increased in the next cycle, if he exceeds the ceiling this quarter. To further explore the effect of quotas, we present in Figure 4, non-parametric plots of the % quota attained by the end of month three versus the % quota attained by the end of month two, across all agents and quarters. Figure 4 suggests patterns that are consistent with intertemporal effort allocation due to quotas. In particular, when far away from quota in month 2 ($x \approx 0.2, 0.4$), the profile is convex, suggesting a ramping up of effort. When the agent is close to quota in month 2 ($x \approx 0.5, 0.8$), the profile is concave suggesting a reduction in the rate of effort allocation. Finally, 4 also shows that most agents do not achieve sales more than $1.4 \times \text{quota}$, which is consistent with the effect of the ceiling (which was set to be $1.33 \times \text{quota}$ by the firm during the time-
period of the data). Figure 5 presents the analogous relationship, with plots for each

agent in the data. Figure 5 shows that the concavity that we uncover is robust, and is not driven by pooling across agents. Taken together, these results point to the existence of significant effects of the compensation scheme on agent’s intertemporal effort allocations in these data, and motivates the dynamics incorporated into the model of agent effort.

Our above discussion highlights three facts regarding salesperson effort: (i) Salespeople are forward looking in that they allocate current effort in anticipation of future rewards; (ii) they act in response to their current quarter compensation environment by increasing and reducing effort relative to their quarter goals; and, (iii) salespeople take into account the impact of their current actions on subsequent changes in future firm compensation policies. These facts will play key roles in the development of our formal model of dynamic effort allocation. We discuss this next.

3 A Model of Dynamic Effort Allocation

We consider the intertemporal effort allocation of an agent facing a period-dependent, non-linear compensation scheme. The compensation scheme involves a salary, $\alpha_t$, 

![Figure 4: Concavity in Quota Attainment within Each Quarter](image)
Figure 5: Concavity in Quota Attainment within Each Quarter by Agent

paid in month \( t \), as well as a commission on sales, \( \beta_t \). The compensation scheme is period-dependent in the sense that it specifies that sales on which the commission is accrued is reset every \( N \) months. The compensation scheme is non-linear in the sense that the commission \( \beta_t \) may depend discontinuously on the extent to which his total sales over the sales-cycle, \( Q_t \), exceeds a quota, \( a_t \), or falls below a ceiling \( b_t \). The extent to which the ceiling is higher than the quota determine the range of sales over which the agent is paid the marginal compensation. While our framework is general enough to accommodate compensation schemes where \( \{\alpha_t, \beta_t, a_t, b_t\} \) change over time, our empirical application has the feature that the salary, \( \alpha \) and the commission-rate, \( \beta \) are time-invariant, and that the ceiling \( b_t \) is a known deterministic function of the quota \( a_t \). We develop the model in the context of this simpler compensation plan. The choice of the structure of the incentive scheme by the firm is determined by reasons outside of our model. Our approach will be to solve for the agent’s effort policy taking the firm’s compensation policy as given, and to use the model to simulate agent-effort for counterfactual compensation profiles. Let \( I_t \) denote the months since the beginning of the sales-cycle, and let \( q_t \) denote the agent’s sales in month \( t \). The total sales, \( Q_t \), the current quota, \( a_t \), and the months since the beginning of the
cycle $I_t$ are the state variables for the agent’s problem. We collect these in a vector $s_t = \{Q_t, a_t, I_t\}$, and collect the observed parameters of his compensation scheme in a vector $\Psi = \{\alpha, \beta\}$.

### 3.1 Actions

At the beginning of each period, the agent observes his state, and chooses to exert effort $e_t$. Based on his effort, sales $q_t$ are realized at the end of the period. We assume that the sales production function satisfies three conditions.

1. Current sales is a strictly increasing function of current effort.

2. Current sales are affected by the state variables only through their effect on the agent’s effort.

3. Unobservable (to the agent) shocks to sales are additively separable from the effect of effort.

Condition 1 is a fairly innocuous restriction that more effort result in more sales. Monotonicity of the sales function in effort enables inversion of the effort policy function from observed sales data. Condition 2 implies that the quota, cumulative sales or months of the quarter do not have a direct effect on sales, over and above their effect on the agent’s effort. As is discussed in more detail below, this “exclusion” restriction is a required condition for nonparametric identification of effort from sales data. Condition 2 rules out reputation effects for the agent (the fact that an agent has achieved high sales in the quarter does not make him more likely to achieve higher sales today); and also rules out direct end-of-the-quarter effects on sales (we find support for these restrictions in our data). Condition 3 is a standard econometric assumption. Based on the above, we consider sales-functions of the form,

$$q_t = g(e_t; z, \mu) + \varepsilon_t$$

where, $g(.)$ is the sales production function, such that $\frac{\partial g(e_t; \mu)}{\partial e} > 0$, $\mu$ is a vector of parameters indexing $g(.)$; $z$ is a vector of observed factors (such as the number and type of clients in an agent’s sales-territory) that affects his demand; and $\varepsilon_t$ is a mean-zero agent and month specific shock to demand that is realized at the end of the period, which is unobserved by the agent at the time of making his effort decision. We assume that $\varepsilon_t$ is distributed IID over agents and time-periods. $\varepsilon_t$ serves
as the econometric error term in our empirical model (we present our econometric assumptions in detail in §4.1). In our empirical work, we will consider specifications in which the production function \( g(.) \) is heterogeneous across agents. For now, we suppress the subscript “i” for agent for expositional clarity.

### 3.2 Per-period utility

The agents’ utility is derived from his compensation, which is determined by the incentive scheme. We write the agent’s monthly wealth from the firm as, \( w_t = w(s_t, e_t, \varepsilon_t; \mu, \Psi) \). We model his utility each month as derived from the wealth from the firm minus the cost of exerting effort. We denote the cost function as \( C(e_t; d) \), where \( d \) is a set of parameters. We assume that agents are risk-averse, and that their per-period utility function is,

\[
  u_t = E[w_t] - r \var{w_t} - C(e_t; d) \tag{2}
\]

where \( r \) is the agent’s coefficient of constant absolute risk aversion, and the expectation and variance of wealth is taken with respect to the demand shocks, \( \varepsilon_t \). The specification in equation (2) is attractive since it can be regarded as a second order approximation to an arbitrary utility function. We now discuss the transition of the state variables that generate the dynamics in the agent’s effort allocation problem.

### 3.3 State Transitions

There are two sources of dynamics in the model. The non-linearity in the compensation scheme generates a dynamic into the agent’s problem because reducing current effort increases the chance to cross, say, the quota threshold tomorrow. A second dynamic is introduced since the agent’s current effort also affects the probability of his compensation structure being updated for the future. Hence, in allocating his effort each period, the agent also needs to take into account his expectations regarding his future compensation structure. These aspects are embedded in the transitions of the state variables in the model. In the remainder of this section, we discuss these transitions. Subsequently, we present the value functions that encapsulate the optimal intertemporal decisions of the agent.

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6In case of the standard linear compensation plan, exponential CARA utilities and normal errors this specification corresponds to an exact representation of the agent’s certainty equivalent utility.
The first state variable, total sales, is augmented by the realized sales each month, except at the end of the quarter, when the agent begins with a fresh sales schedule, i.e.,

\[ Q_{t+1} = \begin{cases} Q_t + q_t & \text{if } I_t < N \\ 0 & \text{if } I_t = N \end{cases} \] (3)

We assume that the agent has rational expectations about the transition of his quota, \( a_t \). We use the observed empirical data on the evolution the agent’s quotas to obtain the transition density of quotas over time. We estimate the following transition function that relates the updated quota to the current quota, as well as the performance of the agent relative to that quota in the current quarter,

\[ a_{t+1} = \begin{cases} a_t & \text{if } I_t < N \\ \sum_{k=1}^{K} \theta_k \Gamma (a_t, Q_t + q_t) + v_{t+1} & \text{if } I_t = N \end{cases} \] (4)

In equation (4) above, we allow the new quota to depend flexibly on \( a_t \) and \( Q_t + q_t \), via a \( K \)-order orthogonal polynomial basis indexed by parameters, \( \theta_k \).\(^7\) The term \( v_{t+1} \) is an IID normal variate with standard deviation \( \sigma_v \), which is unobserved by the agent in month \( t \). Allowing for \( v_{t+1} \) in the transition equation enables us to introduce uncertainty into the agent’s problem.

The transition of the third state variable, months since the beginning of the quarter, is deterministic,

\[ I_{t+1} = \begin{cases} I_t + 1 & \text{if } I_t < N \\ 1 & \text{if } I_t = N \end{cases} \] (5)

### 3.4 Optimal Effort

Given the above state-transitions, we can write the agent’s problem as choosing effort to maximize the present-discounted value of utility each period, where future utilities are discounted by the factor, \( \rho \). We collect all the parameters describing the agent’s preferences and transitions in a vector \( \Omega = \{ \mu, d, r, \sigma_v, \sigma_v, \theta_{k, k=1, \ldots, K} \} \). In month \( I_t < N \), the agent’s present-discounted utility under the optimal effort policy can be represented by a value function that satisfies the following Bellman equation,

\[ V(Q_t, a_t, I_t; \Omega, \Psi) = \]

\(^7\)We use this flexible polynomial to capture in a reduced-form way, the manager’s policy for updating agents’ quotas.
\[
\max_{e > 0} \left\{ u(Q_t, a_t, I_t, e; \Omega, \Psi) + \rho \int_{\varepsilon} V(Q_{t+1} = Q_t, q(\varepsilon_1, e), a_{t+1} = a_t, I_{t+1}; \Omega, \Psi) f(\varepsilon_t) d\varepsilon_t \right\} \tag{6}
\]

The value in period \( I_t + 1 \) is stochastic from period \( I_t \)'s perspective because the effort in period \( I_t \) is decided prior to the realization of \( \varepsilon_t \), which introduces uncertainty into the cumulative sales attainable next period. Hence, the Bellman equation involves an expectation of the \((I_t + 1)-period\) value function against the distribution of \( \varepsilon_t \), evaluated at the states tomorrow. Similarly, the Bellman equation determining effort in the last period of the sales-cycle is,

\[
V(Q_t, a_t, N; \Omega, \Psi) = \\
\max_{e > 0} \left\{ u(Q_t, a_t, N, e; \Omega, \Psi) + \rho \int_{\varepsilon} \int_{v} V(Q_{t+1} = 0, a_{t+1} = a(Q_t, q(\varepsilon_t, e), a_t, v_{t+1}), 1; \Omega, \Psi) f(\varepsilon_t) \phi(v_{t+1}) d\varepsilon_t dv_{t+1} \right\} \tag{7}
\]

At the end of the sales-cycle, the cumulative sales is reset and the quota is updated. The value in the beginning of the next cycle is again stochastic from the current perspective on account of the uncertainty introduced into the ratcheted future quota by the demand shock, \( \varepsilon_t \), and the quota-shock, \( v_{t+1} \). Hence, the Bellman equation in (7) involves an expectation of the \(1\text{-st}\) period value function against the distribution of both \( \varepsilon_t \) and \( v_{t+1} \).

The optimal effort in period \( t \), \( e_t = e(s_t; \Omega, \Psi) \) maximizes the value function,

\[
e(s_t; \Omega, \Psi) = \arg\max_{e > 0} \{ V(s_t; \Omega, \Psi) \} \tag{8}
\]

Given the structure of the agent’s payoffs and transitions, it is not possible to solve for the value function analytically. We solve for the optimal effort policy numerically via modified policy iteration. The state-space for the problem is discrete-continuous, of dimension \( \mathbb{R}^2 \times N \). The two continuous dimensions (\( Q_t \) and \( a_t \)) are discretized, and the value function is approximated over this grid for each discrete value of \( N \). One iteration of the solution took 120 seconds on a standard Pentium PC. Further computational details of the algorithm are provided in Appendix A. We now present the technique for the estimation of the model parameters.
4 Empirical Strategy and Estimation

Our empirical strategy comprises two steps. In step 1, we use the observed data on sales and compensation plans across agents to estimate the parameters of the agents’ preferences, as well as the functions linking sales to effort. In step 2, we use these parameters, along with our dynamic programming (henceforth DP) solution to simulate the agent’s actions under counterfactual compensation profiles. In the remainder of this section, we first discuss our econometric assumptions, and then present details on the specific compensation scheme in our data. Subsequently, we describe the procedure for estimation of the parameters of the model.

4.1 Econometric Assumptions

The econometric assumptions on the model are motivated by the nature of the data, as well as the intended procedure for estimation. The observed variation to be explained by the model are sales across agents and months, which are a function of the agents’ effort. The computational challenge in estimation derives from the fact that the model implies that each agent’s effort, and consequently, their sales, are solutions to a dynamic problem that cannot be solved analytically.

One approach to estimation would be to nest the numerical solution of the associated DP into the estimation procedure. This would be significantly numerically intensive since the DP has to be repeatedly solved for each guess of the parameter vector. Instead, our estimation method builds on recently developed methods for two-stage estimation of dynamic models (e.g. Hotz and Miller 1993; Bajari, Benkard and Levin 2007), which obviates the need to solve the DP repeatedly. Under this approach, agents’ policy functions - i.e., his optimal actions expressed as a function of his state - as well as the transition densities of the state variables are estimated non-parametrically in a first-stage; and subsequently, the parameters of the underlying model are estimated from the conditions for optimality of the chosen actions in the data. We face two difficulties in adapting this approach to our context. First, the relevant action - effort - is unobserved to the econometrician, and has to be inferred from the observed sales. This implies that we need a way to translate the sales policy function to an “effort policy function”. Second, unobserved agent heterogeneity is likely to be significant in this context, since we expect agents to vary significantly in their productivity. The dependence of sales on quotas induced by the compensation
scheme generates a form of state dependence in sales over time, which in the absence of adequate controls for agent heterogeneity generates well-known biases in the estimates of the effort policy. However, practical two-stage methods have not been developed to date which can handle unobserved heterogeneity.

We address both issues in our proposed method. To handle the first issue, we make a parametric assumption about the sales-production function. We discuss below why a non-parametric solution is not possible. We are able to handle the second issue due to the availability of sales-information at the agent-level of unusually large cross-section and duration, which enables us to estimate agent-specific policy functions, and to accommodate non-parametrically the heterogeneity across agents. We discuss the specific assumptions in more detail below.

4.1.1 Preliminaries

The model of agent optimization presented in §3 implies that the optimal effort each period is a function of only the current state $s_t$. To implement a two-step method, we thus need to estimate non-parametrically in a first-stage, the effort policy function, $e_t = \hat{e}(s_t)$. The effort policy function is obtained parametrically from the sales-policy function. To see the need for a parametric assumption, recall from §3 that we consider sales-production functions of the form,

$$q_t = g(e_t(s_t), z) + \varepsilon_t$$

For clarity, we suppress the variable $z$, as the argument below holds for each value of $z$. Let $f(s_t) \equiv g(e_t(s_t))$.

**Remark 1** If at least two observations on $q$ are available for a given value of $s$, the density of $f(s)$ and $\varepsilon$ are separately non-parametrically identified (Li and Vuong 1998).

**Remark 2** Given the density of $f(s)$, only either $g(s)$ or $e(s)$ can be estimated non-parametrically.

Remark 2 underscores the need for a parametric assumption on the relationship between sales and effort. One option to relax this would be to obtain direct observations on agent’s effort, via say, survey data, or monitoring. This of course, changes the character of the principal-agent problem between the agent and the firm.
Unobservability of agent effort is the crux of the moral hazard problem in designing compensation schemes. Hence, we view this parameterization as unavoidable in empirical models of salesforce compensation.

We now discuss how we use this assumption, along with the sales data to estimate the sale-production function. For each agent in the data, we observe sales at each of \( J \) clients, for a period of \( T \) months. In our empirical application \( T \) is 38 (i.e., about 4 years), and \( J \) is of the order of 60-300 for each agent. The client data adds cross-sectional variation to agent-level sales which aids estimation. To reflect this aspect of the data, we add the subscript \( j \) for client from this point onward. In light of remark 2 we assume that the production function at each client \( j \) is linear in effort,

\[
q_{jt} = h_j + e_t + \varepsilon_{jt} \\
= h_j (z_j) + e(s_t) + \varepsilon_{jt}
\]

(9) (10)

The linear specification is easy to interpret: \( h_j \) can be interpreted as the agent’s time-invariant intrinsic “ability” to sell to client \( j \), which is shifted by client characteristics \( z_j \). We now let \( h_j = \delta' z_j \), and let \( \hat{e}(s_t) = \lambda' \theta(s_t) \), where \( \gamma \) is a \( R \times 1 \) vector of parameters indexing a flexible polynomial basis approximation to the monthly effort policy function. Then, the effort policy function satisfies,

\[
q_{jt} = \delta' z_j + \lambda' \theta(s_t) + \varepsilon_{jt}
\]

(11)

We assume that \( \varepsilon_{jt} \) is distributed IID across clients. We can then obtain the demand parameters and the effort policy function parameters from the following minimization routine,

\[
\min_{\delta, \lambda} \| q_{jt} - (\delta' z_j + \lambda' \theta(s_t) D_{jt}) \|
\]

(12)

As a by product, we also obtain the effort policy function for the month \( t \) as,

\[
\hat{e}_t = \hat{\lambda}' \theta(s_t)
\]

(13)

and the time-specific error distribution,

\[
\hat{\varepsilon}_t = \sum_j \left( q_{jt} - \left( \delta' z_j + \hat{\lambda}' \theta(s_t) D_{jt} \right) \right)
\]

(14)

which is then used to estimate the empirical distribution of \( \varepsilon_t \).\(^8\) Recall that the distribution of \( \varepsilon_t \) is a required input into the computation of the dynamic programming

\(^8\)Alternatively, one could assume a parametric density for \( \varepsilon \) and use maximum likelihood methods. The advantage of our nonparametric approach is that we avoid the possibility of extreme draws inherent in parametric densities and the pitfalls that go along with such draws.
problem. To estimate the error distribution parameters, we fit a log-normal distribution to the recovered residuals, \( \tilde{\varepsilon}_t \), from equation (14). In practice, any distribution that restricts the support of \( \varepsilon_t \) to \((0, \infty)\), such that sales are always positive, would be appropriate. We choose the log-normal since it provided the best fit to our residuals data.\(^9\) These two entities are inputs into the next stage of the estimation procedure.

Finally, at the end of this step, we can recover the predicted overall sales for the agent which determines the agent’s overall compensation. Summing equation (11) across clients, the overall sales in a given month is simply,

\[
q_t = \sum_j q_{jt} = h + J\varepsilon_t + \varepsilon_t
\]

where, \( h = \sum_{j=1}^J \delta' z_j \), and \( \varepsilon_t = \sum_{j}^J \varepsilon_{jt} \) is distributed \( \pi_{LN}(\varepsilon_t; \mu_{\varepsilon}, \sigma_{\varepsilon}) \), where \( \pi_{LN}(\cdot) \) is the PDF of a log-normal distribution.

Intuitively, we can think of identification of effort above, by casting the estimator in equation (11) in two steps,

- **Step 1:** Estimate time-period fixed effects \( \varpi_t \) as, \( q_{jt} = \delta' z_j + \varpi_t + \varepsilon_{jt} \)
- **Step 2:** Project \( \varpi_t \) on a flexible function of the state variables as \( \varpi_t = \lambda' \vartheta(s_t) \)

The client-level data facilitates the estimation of time-period specific fixed effects in Step 1. Equation (11) combines steps 1 & 2 into one procedure. We discuss the identification of the model in further detail below.

We now discuss the specifics of the compensation scheme in our dataset, and derive the expression for the monthly expected wealth for the agent given the above econometric assumptions.

### 4.2 Compensation scheme

The incentive scheme in our empirical application has two noteworthy features. First, the agent’s payout is determined based on his quarter-specific performance. Thus, \( N = 3 \), and cumulative sales, which affect the payout, are reset at the end of each quarter. Second, the commission scheme is non-linear, involving a salary, a quota and a ceiling. The monthly salary \( \alpha \) is paid out to the agent irrespective of his sales.

\(^9\) Note, while we restrict the support of \( \varepsilon_t \) in the computation of the DP, we let \( \varepsilon_{jt} \) be unrestricted in our estimation procedure above. The estimated \( \tilde{\varepsilon}_t \) values were always positive.
If his current cumulative sales are above quota, the agent receives a percentage of a fixed amount $\beta$ as commission. The percentage is determined as the proportion of sales above $a_t$, and below a maximum ceiling of $b_t$, that the agent achieves in the quarter. Beyond $b_t$, the agent receives no commission. For the firm in our empirical application, $\beta = $ 20,000, and the ceiling was always set 33% above the quota, i.e., $b_t = \frac{4}{3}a_t$. Figure 6 depicts the compensation scheme. We can write the agent’s wealth, $w(s_t, e_t, \varepsilon_t; \mu, \Psi)$ in equation (2) as,

$$w(s_t, e_t, \varepsilon_t; \mu, \Psi) = \alpha + \beta \left[ \frac{Q_t + q_t - a_t}{b_t - a_t} \mathbb{I}(a_t \leq Q_t + q_t \leq b_t) + \mathbb{I}(Q_t + q_t > b_t) \right] \mathbb{I}(I_t = N)$$

$$= \alpha + \beta \left[ 3 \frac{Q_t + q_t - a_t}{a_t} \mathbb{I}(a_t \leq Q_t + q_t \leq b_t) + \mathbb{I}(Q_t + q_t > b_t) \right] \mathbb{I}(I_t = N)$$

$$= \alpha + \beta \min \left\{ \frac{3(Q_t + q_t - a_t)}{a_t}, 1 \right\} \mathbb{I}(Q_t + q_t > a_t) \mathbb{I}(I_t = N)$$

Thus, at the end of each sales-cycle, $I_t = N$, the agent receives the salary $\alpha$, as well as a incentive component, $\beta \times \min \left\{ \frac{3(Q_t + q_t - a_t)}{a_t}, 1 \right\}$, on any sales in excess of quota. If it is not the end of the quarter, $\mathbb{I}(I_t = N) = 0$, and only the salary is received.

Finally, assume that the cost function in (2), $C(e)$, is quadratic in effort, i.e. $C(e_t) = \frac{de^2}{2}$, where $d$ is a parameter to be estimated.

4.3 Estimation procedure

We now present the steps for estimation of the model parameters. The estimation consists of 4 steps, which are discussed in sequence below.

Step 0: Nonparametric estimation of policy function and state transitions

The effort policy function is related to observed sales via equation (11). We estimate equation (11) agent-by-agent. For each, we pool the data across all his clients to estimate the parameters $(\delta, \lambda, \mu_{\varepsilon}, \sigma_{\varepsilon})$. This gives us an estimate of the optimal effort policy function for each agent in the data. An advantage of this approach is that we are able to handle agent heterogeneity non-parametrically.

The next step is to estimate the parameters $(\theta_k, \sigma_{\varepsilon})$ describing the transition of the agent’s quotas in equation (4). From equation (4), the quota process has a normal likelihood on account of the assumption of the normality of $v_{t+1}$. We estimate the
Figure 6: Compensation Scheme
parameters by maximum likelihood. Since quotas vary only at the quarter-level, we do not estimate the transitions agent-by-agent. Instead, we pool the data across agents to estimate the quota transition function allowing for fixed-effects across agents.

The law of motion of the other state variables (month of the quarter) does not have to be estimated since it does not involve any unknown parameters. This concludes step 0. The only parameters remaining to be estimated are the agent’s risk aversion, \( r \), and the cost parameter, \( d \). The following 4-step procedure delivers estimates of \( r \) and \( d \).

**Step 1: Simulate optimal policy paths** For each agent, we start by making a guess of the remaining parameters to be estimated, \( (d, r) \). Given the guess, the estimate of the agent’s effort policy function, and the estimate of the quota transition equation, we forward simulate the agent’s value function as,

\[
\hat{V}^e (s_t; \Omega, \Psi) = \sum_{\tau=t}^{\infty} \rho^{\tau-t} u (s_t, \hat{e} (s_t); \Omega, \Psi) \tag{17}
\]

**Step 2: Simulate sub-optimal policy paths** We now perturb the estimated effort policy to simulate \( NR \) sub-optimal effort policies, \( \tilde{e}_\tau (s_t) \). For each of these, we obtain the corresponding value functions,

\[
\hat{V}^q (s_t; \Omega, \Psi) = \sum_{\tau=t}^{\infty} \rho^{\tau-t} u (s_t, \tilde{e}_\tau (s_t); \Omega, \Psi) \tag{18}
\]

**Step 3: Minimize violations of optimality** We then obtain the following objective function which takes a weighted sum of the squared violations of the optimality of the chosen action,

\[
\left( \hat{d}, \hat{r} \right) = \arg \min_{(d, r)} \left[ \left( \hat{V}^q (s; \Omega, \Psi) - \hat{V}^e (s; \Omega, \Psi) \right)^+ \right] \Psi \left[ \left( \hat{V}^q (s; \Omega, \Psi) - \hat{V}^e (s; \Omega, \Psi) \right)^+ \right] \tag{19}
\]

We then implement a non-linear search to find the parameters that minimize this objective function. The resulting estimator has an asymptotic normal distribution, as shown by Bajari, Benkard and Levin (2007).

**4.4 A Simple Bounds Estimator**

While the BBL estimator outlined above is estimable it is computationally quite expensive and requires many draws of alternate (perturbed) policy functions. Our
application exacerbates the problem since we have to implement the estimation algorithm separately for each of the salespeople in our sample (i.e. about 90 times). In what follows we present a simple bounds estimator that will allow us to recover the empirical distribution of the cost of effort parameter with much lower computational burden.

Begin by defining the following quantities,

\[
Z(s_0; e^*) = \begin{bmatrix} \mathbb{E}(W) & \nabla(W) & \mathbb{C}(e) \end{bmatrix}
\]

\[
\theta = \begin{bmatrix} 1 & r & d \end{bmatrix}
\]

(20)

where \(\theta\) are parameters of interest, \(s_0\) is an initial state, \(e^*\) is the estimated optimal effort policy function and \(Z(s_0; e^*)\) has components,

\[
\mathbb{E}(W) = E_{e^*|s_0} \sum_{t=0}^{\infty} \beta^t E_{\epsilon} [W(s, e^*(s))]
\]

\[
\nabla(W) = E_{e^*|s_0} \sum_{t=0}^{\infty} \beta^t E_{\epsilon} \left[ W(s, e^*(s))^2 - E_{\epsilon} [W(s, e^*(s))]^2 \right]
\]

\[
\mathbb{C}(e) = E_{e^*|s_0} \frac{1}{2} \sum_{t=0}^{\infty} \beta^t e^*(s)^2
\]

(21)

The value function based on the optimal effort policy can then be expressed as

\[
V(s_0; e^*, \theta) = Z(s_0; e^*)' \theta.
\]

(22)

Similarly, for any alternative policy function \((e' \neq e^*)\), the perturbed value function is

\[
V(s_0; e', \theta) = Z(s_0; e')' \theta.
\]

(23)

Clearly, at the true parameter vector \((\theta = \theta^*)\), we must have

\[
V(s_0; e^*, \theta^*) \geq 0
\]

\[
V(s_0; e^*, \theta^*) \geq V(s_0; e', \theta^*)
\]

(24)

Together, these inequalities reflect the individual rationality (IR) and incentive compatibility (IC) constraints.
4.4.1 Constructing Bounds for $d$

In what follows we will assume that the risk aversion parameter $r$ is known\(^\text{10}\) and aim to recover bounds for the cost of effort parameter $d$. The estimators described below assume that the quantities $Z(s_0; e^*)$ and $Z(s_0; e')$ are available to us from forward simulations described earlier.

Our first estimator relies solely on the individual rationality (IR) constraints which are enough to characterize the cumulative distribution function of $d$. To see this note that the IR constraints imply $V(s_0; e^*, \theta^*) \geq 0$, which using the definitions above gives

$$E(W) - rV(W) - dC(e) \geq 0.$$ 

Since $r$ is assumed known we have

$$d \leq \bar{d} = \frac{E(W) - rV(W)}{C(e)}.$$ \hspace{1cm} (25)

Once the vector of bounds $\{\bar{d}\}$ are known the CDF for $d$ can then be constructed as

$$\mathcal{F}(d) = \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}(\bar{d}_i < d).$$ \hspace{1cm} (26)

For the purposes of constructing counterfactuals, $\mathcal{F}(d)$ is the only relevant entity since we care about changes in compensation that impact the entire salesforce. Thus, even though the individual bounds are fairly loose the identification of the CDF allows us to construct counterfactuals. Further, this simple estimator is computationally inexpensive as it require no alternate policy function forward simulations (i.e. $Z(s_0; e')$ may not be constructed.)

While the above estimator is simple it can be improved upon by incorporating the incentive compatibility constraints. From the definitions described earlier, $V(s_0; e^*, \theta^*) \geq V(s_0; e', \theta^*)$ implies

$$E(W) - rV(W) - dC(e) \geq E(W') - rV(W') - dC(e')$$ \hspace{1cm} (27)

Thus, when $C(e) > C(e')$, we can write

$$d \leq \bar{d} = \frac{(E(W) - rV(W)) - (E(W') - rV(W'))}{(C(e) - C(e'))}.$$ \hspace{1cm} (28)

\(^{10}\)In the identification section we point out that $r$ is only weakly identified and we therefore calibrate its value based on prior studies.
Similarly, when $C(e) < C(e')$ we have

$$d \geq d = \frac{(E(W') - rV(W')) - (E(W) - rV(W))}{(C(e') - C(e))}$$ (29)

Together these give us lower and upper bounds for each agent’s $d$ and, as earlier, can be used to construct $F(d)$.

4.5 Discussion: Identification

We now provide a more detailed discussion of identification in our model. In particular, we discuss how intertemporal linkages in observed sales identifies an agent’s unobserved effort allocation over time. The first concern is that effort has to be inferred from sales. In particular, looking at equation (9), we see that sales is explained by two unobservables, the first, effort, and the second, client-specific demand shocks. How can the data sort between the effects of either? The key identifying assumptions are,

1. Effort is a deterministic function of only the state variables.

2. Effort is not client specific - i.e., the agent allocates the same effort to each client in a given month.

We believe the first assumption is valid since we believe we have captured the key relevant state variables generating the intertemporal variation in agent effort. Further, after including a rich-enough polynomial in the state variables in equation (9), we can reject serial correlation in the residuals, $\varepsilon_{jt}$ (i.e. the remaining variation in sales is only white noise). Assumption 1 is also consistent with our dynamic programming model which generates a deterministic policy by construction. We believe the second assumption is reasonable. In separate analysis (not reported), we use limited data on the number of sales calls made by agents to each of the clients to check the validity of this assumption. Our analysis of these data finds that the allocation of calls across clients is not significantly related to the quotas and past performance, suggesting that effort more broadly, is not being tailored to each individual client.

Given these two assumptions, effort is identified by the joint distribution over time of the agent’s current sales, and the extent to which cumulative sales are below or above the quota and the ceiling. To see this, recall that the optimal policy implies
that the agent expends high effort when he is close to the quota, irrespective of month. The agent expends low effort when he has either crossed the ceiling in a given quarter, or when he is very far away from the quota in an early month. Under the former situation, the marginal benefit of an additional unit of effort is higher when expended in the next quarter; the same is true under the latter, since he has very little chance of reaching the quota in the current quarter. The model assumes that sales are strictly increasing in effort. Hence, if we see an agent achieve high sales across clients when he is close to the quota we conclude that effort is high. If we see low sales early on in the quarter, and when the quarter’s sales have crossed the ceiling, we conclude that effort is low. Our identification argument is based essentially on the fact that variation in effort over time is related to variation in the distance to quota over time, and is similar to the identification of productivity shocks in the production economics literature (see e.g. Olley and Pakes 1996; Ackerberg, Caves and Frazer 2006). In addition to the dynamic patterns observed in sales the identification of effort is also aided by the manner in which it enters the sales function. Note that, in our model, effort is implemented as the ‘effectiveness’ of detailing. Since the marginal effect of detailing (given adequate data) can be non-parameterically identified across time and individuals, a flexible projection of these effects on state variables will give us a non-parametric (and consistent) estimate of the effort policy function.

A related concern is how the effect of ratcheting is identified separately from the intertemporal substitution induced by the quota structure. The data are able to sort between these two separate dynamics in the following way. The extent of decline in the agent’s observed sales after he crossed the ceiling in any quarter informs the model about the extent of intertemporal effort allocation induced by the quota structure. However, note that in the absence of ratcheting, effort, and hence, sales, should be strictly increasing between the quota and the ceiling. Hence, the extent of decline in the agent’s observed sales after he crosses the quota, and before he attains the ceiling informs the model about the extent to which ratcheting plays a role. Figure 7 depicts the identification argument pictorially.

The two other key parameters that are estimated in step 3 above are the cost \((d)\) and risk aversion parameter \((r)\). The cost of effort parameter is identified from the fact that sales are above the intercept in the first two months of the quarter. That is, if effort were costless, it would be optimal to exert no effort in the first two months and meet any target in the third month alone. The fact that effort is
Figure 7: Identification of intertemporal inefficiencies from sales profile
costly induces a capacity constraint on how much sales can be generated in any given month. This, along with the structure of the sales response function, acts as the primary identification mechanism for the cost of effort parameter. Finally, the risk aversion parameter is in principle, weakly identified by the degree to which effort (sales) changes due to changes in the variance of wealth. In our empirical work, we found that there was not enough variation to identify this parameter; hence, we fix this to calibrated values reported in the literature, and report sensitivity to these values.

5 Data and Estimation Results

Table 8 presents summary statistics from our data. The salesforce has 87 salespeople who are about 43 years of age on average, and have been with the firm for approximately 9 years. The firm did not significantly hire, nor have significant employee turnover in the sales-department during the time-period of the data.\(^\text{11}\) The average salesperson in the salesforce earns $67,632 per annum via a fixed salary component. The annual salary ranges across the salesforce from around $50,000 to about $90,000. The firm’s output-based compensation is calibrated such that, on a net basis, it pays out a maximum of $20,000 per agent per quarter, if the agent achieves 133% of their quarterly quota. On an average this implies that the agent has a 77%-23% split between fixed salary and incentive components if they achieve all targets. The agents differ in terms of the number of clients they have, but are balanced in terms of the type of clients and the total detailing calls they are required to make. For example, a particular salesperson may have a small number of clients but may be required to call on them more frequently, while another may have many more clients but may be asked by the firm to call on each of them relatively infrequently. The firm attempts to ensure that the total number of detailing calls is balanced across agents.\(^\text{12}\)

The mean quota for the salesforce is about $397,020 per quarter. The mean attained sales is only slightly less, at $374,755, suggesting that the firm does a fairly good job of calibrating quotas and effort levels. This is further evidenced by the fact that the range and dispersion parameters of the cumulative sales at the end of

\(^\text{11}\)So as to avoid concerns about learning-on-the-job, and its interactions with quotas, 5 sales-agents, who had been with the firm for less than 2 years were dropped from the data.

\(^\text{12}\)For a more involved discussion of calls and the role it plays, along with empirical support of agents’ adherence to management policy regarding calls, see Appendix A.
the quarter and the quota levels are also fairly close. From Table (8), it appears on average that the firm adopts an asymmetric ratcheting approach to quota setting. When salespeople beat quotas the average increase in subsequent quarter quotas is about 10%, but on the flip side, falling short of quotas only reduces the next quarter quota by about 5.5%. This is consistent with some other earlier studies (e.g. Leone, Misra and Zimmerman 2005) that document such behavior at other firms, and is also consistent with our conversations with the firm management. Finally, the table documents that monthly sales average about $138,149, a fairly significant sum.

5.1 Results from estimation

The estimation results include, a) effort policy function, b) quota transition process, and c) cost function estimates. We discuss these in sequence below.

5.1.1 Effort policy

The effort policy function was estimated separately for each agent using a flexible bivariate spline polynomial basis approximation. On average, we are able to explain about 53.6% ($R^2 = .535; R^2 \in [.361,.728]$) of the variation in monthly sales. The joint test for all basis functions coefficients (30 in all) being zero was categorically rejected for all salespeople ($p = 0.0005; p \in [0,0.0026]$).

Rather then present estimates of the parameters of the basis functions approximating the effort policy (30 × 87 in total), we present the estimates in graphical form. Figure 8 and Figure 9 represent the the average estimated policy across salespeople using a perspective and its contour plot. Looking at Figure 8 and Figure 9, we see that the data shows a clear pattern whereby effort tends to increase in quotas, which supports the “effort inducement” motivation for quotas noted by the theory starting with Holmstron 1978. The variation of effort with cumulative sales is also intuitive. When cumulative sales are less than quota (areas to the left of the diagonal), the agent tends to increase effort. When cumulative sales are much greater than quota (areas to the right of the diagonal line), there is little incentive for the agent to exert further effort, and sales decline.

We now present contours of the effort policy estimated for the individual agent. Figure 10 shows the contours for nine of the sales-people. We find that there is considerable heterogeneity across the salespeople, which is consistent with wide variation in agent productivity. At the same time, we find that the basic pattern described
Figure 8: Estimated Effort Policy Function

Figure 9: Contours of the Estimated Effort Policy
above remain true. Similar to the average contour plot discussed below, we see sales increase with quota but fall after cumulative sales have exceeded quota.

Figure 10: Examples of estimated effort policy functions across salespeople.

5.1.2 Quota transition process

For now, we work with a simple specification for the quota transition process that is linear in the past quota and the past cumulative sales, and plan to update this in future versions. We estimate this specification by pooling the data across all agents and quarters. The estimates from this specification are given below (standard errors
We find that the fit of this naive model is reasonably good, and that we are not significantly under or over predicting the quotas in any systematic manner. We also implemented a more flexible model that uses a flexible (bivariate spline) function for \((a_t, Q_t)\) that also incorporates individual level random effects. This specification improves the fit (approximate \(R^2 = .55\)) and is better able to capture nuances of the firm’s quota policy. The contours from this model are presented in Figure (11).

Figure 11: Contour plot from quota policy function estimation. (Lighter shades indicate higher current quotas)

The flexible quota policy function shows a clear asymmetric pattern of quota ratcheting. That is, quotas are increased at a disproportionately larger rate when past quarter performance (cumulative sales) is high (relative to past quota). While we find that there is some heterogeneity, for most part the quota policy seems remarkably similar across agents. This suggests that the policy itself is applied fairly uniformly across salespeople. For now, we choose to use equation (30) as the estimated quota policy function and are working on incorporating the more flexible model into our model. We are also examining other approaches to modeling the quota policy that would significantly improve fit.
5.2 Cost of Effort Estimates

Discussion to be added.

6 Results from the Dynamic Model

We now discuss the results from simulations of the optimal policy. The optimal policy is an output from the DP, taking the parameter estimates as given.\textsuperscript{13} We discuss the main qualitative features of the solution, and present simulations comparing output under the observed compensation scheme to the first-best, as well as to the linear plan suggested by Holmstrom and Milgrom (1987).

6.1 Optimal Policy

The optimal policy evaluated for a representative agent is presented in Figure 12. The optimal policy predicts the agent exerts positive effort when cumulative sales are close to quota. When the agent is past quota, the incentive to put in additional effort is zero, and effort is set to zero. When the cumulative sales are far less than quota, the chance of making quota in the current quarter is low, and the agent again sets effort to zero, preferring to postpone effort to the next quarter.

\textsuperscript{13}The simulation below use a risk aversion of 0, and set the cost function parameter $d = 5000$. 

Figure 12: Effort Policy
The value function for the agent is plotted in Figure 13. Consistent with the pattern in the policy function, we see that the agent generates value from expending effort only in a range around quota. At the current level of effort, the agent’s values are all positive; the agent is thus making positive utility on a present discounted basis, which we interpret as some indication that the agent’s participation constraint is not binding.

We now compare the predictions from the dynamic model of effort to the sales patterns in the data. Recall that the model takes the demand-side parameters as given, and produces an effort policy for these estimates. Since we intend to use our model to generate counterfactual predictions of agent effort, we wish to verify that the output from our fairly complex dynamic model, computed for the observed compensation policy, is able to reproduce reasonably the patterns we see in the data (for similar arguments, see for example, Dube, Hitsch and Chintagunta 2005; Nair 2007). In figure 14, we plot the observed sales in the data versus the predicted sales from the model for the 3 months of the quarter. We simulate the predicted sales by a three steps procedure. In step 1, we make 1000 draws of a 3×1 vector of demand shocks (ε_t), from the empirical distribution of the demand shocks estimated from
the data (see §4.1.1). In step 2, we used the predicted effort policy from the model to simulate agent effort, as well as the associated realization of sales, for the three months of the quarter, for each vector of the demand shocks. For now, we use the average quota in the data as the value of the quota state variable. Finally, in step 3 we average across the 1000 predicted sales histories to generate a monthly sales prediction from the model. In Figure 14, the prediction is plotted as the solid line, and the observed sales (averaged across all agents and quarters) is plotted as the dotted line. We see that the policy does a remarkably good job of replicating the patterns, as well as the level of sales, in the data. The predicted policy underpredicts sales slightly in the initial months of the quarter, and overpredicts in the last month; nevertheless, the basic pattern - that effort is ramped up in the last month of the quarter - is captured well by the model.

\footnote{We plan to update this by running the simulation agent-by-agent, and also by incorporating ratcheting.}
6.1.1 Evaluating the compensation scheme

We now return to the main research question posed in the paper, i.e., evaluating the value of the current quota scheme. As noted in the introduction, this evaluation is relative and requires comparison to a relevant counterfactual. Our benchmark simulation is a comparison to a counterfactual compensation profile in which the firm can observe effort, and achieve the theoretical, first best outcome. Observability of effort solves the moral hazard problem for the firm. However, while theoretically relevant, by construction, this first-best scheme is not fully realistic. Hence, we also compute the output under a situation in which the firm implements the linear contract suggested by Holmstrom and Milgrom (1987), who show that a linear contract will achieve the best possible outcomes for the firm, under the LEN assumptions (“linear exponential utility and normal errors”). As a first cut, we first solve for a linear contract that achieves the same level of overall sales over a quarter as the observed contract. This enables us to assess intuitively, the degree to which the current contract provides incentives to the agents. We implement the counterfactual as follows. We first solve for the optimal effort and compensation under the first-best, and under the linear contract. As before, we make 1000 draws of a $3 \times 1$ vector of demand shocks from the empirical distribution of the shocks, and use these to compute the predicted sales corresponding to both these scenarios, for each of the 1000 draws. These are then averaged across the 1000 draws to obtain a prediction. Figure 15 presents the predicted sales pattern under the two counterfactuals and under the current policy.

Both the first-best and the linear contracts do not generate any intertemporal switching, and result in the agent extending the same level of effort in every month. As a result, the sales profile over the quarter is flat. Under the first-best, the agent achieves about $810,000 (\$2.7K \times 3)$ worth of sales over the average quarter, which is approximately 98% higher than the sales achieved under the observed compensation scheme. The profits associated with this increased revenue represents the costs of asymmetric information at this firm. This also suggests that investments in better monitoring technology is valuable to the company, since the upside potential of better observability of effort is of the order of double the current revenues. Finally, the sales under the linear contract is plotted in green. The linear scheme that achieves the same quarterly sales as the observed compensation policy requires a monthly commission rate of 9%. This is a fairly high incentive scheme compared to common sales-force
policies in business settings. We are currently exploring how sales will change under the optimal linear contract.

### 6.2 Effect of Quota Ratcheting

Finally, we briefly discuss the impact of quota ratcheting on the agent’s incentives to allocate effort. Our approach is to simulate the agent’s present the effort policy under the no ratcheting scenario. Figure 16 presents the policy function. At the currently estimated parameter values, and under the observed ratcheting policy, we find that the agent’s effort policy is not significantly altered by the addition of a ratcheting incentive. Although not reported, we also found in other simulations that the agent’s effort is impacted significantly if the ratcheting policy is modified so has to have a larger impact on new quotas than policy currently implemented at the firm. As a normative take away, we believe this suggests a promising avenue for improvement in the firm’s compensation policies.
7 Conclusions

This paper considers the dynamics induced by non-linear output based incentive schemes for compensating sales-force agents. Nonlinearities and kinks in the sales-force contract generates dynamic moral hazard that creates incentives for agents to postpone effort when they are “in-the-money” or have already “made quota”, or when agents shade effort to influence expected revisions in their quotas. Such intertemporal reallocation of effort can have adverse consequences for the firm. This paper seeks to provide evidence for, and measures of, the extent of such dynamic inefficiencies. The model free evidence suggest that quotas has a significant effect on the allocation of effort. Preliminary estimates from the dynamic model suggest that the extent of inefficiency is large, and suggest that re-optimizing the current compensation scheme will have large payoffs to the firm.
8 References (Incomplete)


Table 1: Descriptive Statistics of Data

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>SD</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Agent Demographics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Salary</td>
<td>$67,632.28</td>
<td>$8,585.13</td>
<td>$51,001.14</td>
<td>$88,149.78</td>
</tr>
<tr>
<td>Incentive Proportion at Quota</td>
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<td>0.02</td>
<td>0.8</td>
<td>0.28</td>
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<td>Age</td>
<td>43.23</td>
<td>10.03</td>
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<td>Tenure</td>
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<td>8.42</td>
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<tr>
<td>Number of Clients</td>
<td>162.20</td>
<td>19.09</td>
<td>63</td>
<td>314</td>
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<tr>
<td><strong>Quarter Level Variables</strong></td>
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<td></td>
<td></td>
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<tr>
<td>Quota</td>
<td>$397,020.5</td>
<td>$95,680.74</td>
<td>$197,898.81</td>
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<tr>
<td>Cumulative Sales (end of quarter)</td>
<td>$374,755.5</td>
<td>$89,947.66</td>
<td>$171,009.11</td>
<td>$767,040.98</td>
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<tr>
<td>Percent Change in Quota (when positive)</td>
<td>10.01%</td>
<td>12.48%</td>
<td>0.00%</td>
<td>92.51%</td>
</tr>
<tr>
<td>Percent Change in Quota (when negative)</td>
<td>-5.53%</td>
<td>10.15%</td>
<td>-53.81%</td>
<td>-0.00%</td>
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<td><strong>Monthly Level Variables</strong></td>
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<tr>
<td>Monthly Sales</td>
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<td>Cumulative Sales (beginning of month)</td>
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<td>$985,94.65</td>
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<td>$65,2474.25</td>
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<td>Distance to Quota (beginning of month)</td>
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<td>$121,594.2</td>
<td>$20,245.52</td>
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<tr>
<td><strong>Number of Salespeople</strong></td>
<td>87</td>
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</table>
A  Appendix A: Computational Details

This appendix provides computational details of solving for the optimal policy function in equation (8). The optimal effort policy was solved using modified policy iteration (see, for e.g., Rust 1996 for a discussion of the algorithm). The policy was approximated over the two continuous states using 10 points in each state dimension, and separately computed for each of the \( N \) discrete states. The expectation over the distribution of the demand shocks \( \varepsilon_t \) was implemented using Monte Carlo integration using 1000 draws, and the expectation over the ratcheting error, \( v_{t+1} \), was implemented using Gauss Hermite quadrature using 8 nodes. The maximization involved in computing the optimal policy was implemented using the highly efficient SNOPT solver, using a policy tolerance of 1E-5.