Fail-Safe Federalism

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Abstract

We explore the consequences for social welfare and the national political conflict of several key institutional features of federalism in the United States: supermajoritarian national institutions and permeable boundaries in the provision of by national and state governments, where the actions by the former can crowd out the latter. States with high demand for public good provision are better positioned to adjust state-level policies to accommodate local demand in the presence of low national provision than corresponding states with low demand in the presence of high national provision. This asymmetry implies that the level of federal provision preferred by moderate-demanders may be socially inefficient, and can exacerbate political polarization when national provision is gridlocked at a high level. Symmetric cross-state negative externalities can reduce conflict at the national level by generating consensus for national action; whereas positive externalities, or asymmetric negative externalities, can increase it. We also explore how, in a dynamic setting, exogenous shocks to demand can create inefficiencies while expanding the “gridlock interval” of national policies; and the limits of Coaseian bargaining over national public goods.
1 Introduction

A characteristic feature of many democracies that operate over an extensive territory is federalism: the constitutional apportionment of sovereignty between central and constituent governments. Federalism, along with the separation of powers at the national level, is a core opponent of the United States’ constitutional design. Indeed, an enormous portion of the political conflict of the United States has centered on the proper roles, responsibilities, and limitations of different levels of government – from the Virginia and Kentucky Resolutions of 1798 (and the vociferous responses thereto) to the Supreme Court’s recent jurisprudence regarding gun control, violence against women, and gay marriage. How does shared responsibility for governance across multiple levels, in a world where decisions at one level might affect decisions at the other, constrain or exacerbate political conflict?

To address this question, we describe a model of federalism that incorporates several critical features of the U.S. political system: (1) overlapping provision of public goods by the federal and state governments with “crowding out” of state provision; and (2) gridlock at the national level, brought about by supermajoritarian legislative procedures. Our primary focus is the consequences of these features and their interactions for social welfare and political polarization at the national level.

In the model, the magnitude of the national government’s presence affects the ability of the federation’s constituent states to raise revenue. The anticipated crowding out effect that results from this distortion affects the preferences of state actors over national provision. As long as the magnitude of the distortion is not too large, and in the absence of cross-state externalities, all states prefer a mix of federal and state provision. While states have heterogeneous demand, the system functions smoothly: when the differences in demand for public good are too great to move federal provision from the status quo level, states adjust their state-level provision up and/or down, respectively. In effect, when, from the state’s perspective, there is a failure at the federal level, the state “fails safely” into the state-level action. We refer to this system as fail-safe federalism to evoke the design principle that potentially dangerous devices should contain features that automatically correct for the failure of a component part by reverting to a harmless state rather than a hazardous one.

One of the key aspects of the functioning of the federal system we analyze is the limits of its “safe” performance. A sufficiently large level of national provision will eventually crowd out states with low demand for public goods, making it impossible for those states to adjust their state-level provision to
meed local demand in response to what they would perceive as overprovision at the federal level. For these states, further increases in the level of national provision hurt more than they help states with high demand. This asymmetry has a number of implications for political conflict at the national level: first, it leads to a normative bias for a small national government. At first glance, this would seem to justify the supermajoritarian constraints in the original U.S. constitution. Indeed, when the status quo level of national public goods provision is low (as it was at the moment of ratification), these supermajoritarian constraints can be calibrated to yield the social welfare optimum (holding the demand for public goods constant). However, in a dynamic environment where shocks to the demand for public good provision may occur, the political system may find itself in a position where the status quo national provision is too generous. The baseline model also yields novel results concerning the extent of ideal-point polarization in the political system.

We then consider three extensions to the baseline model. First, we introduce negative and positive cross-state externalities of state policy making. Counterintuitively, an increase in the magnitude of symmetric negative externalities actually tends to decrease polarization at the national level, because it will endogenously generate consensus regarding the need for national solutions. Asymmetric externalities can either increase or decrease national polarization: it can increase polarization when the negative externality is imposed by the low-demand state on the high-demand; and decrease polarization if the high-demand is imposing the costs. By contrast, an increase in the magnitude of positive externalities will tend to increase polarization, because states with low demand for public goods provision will find there demand satiated by the activities of other states at a faster rate than similarly-situated high-demand states.

Second, we consider an environment in which transfers across states can generate the socially efficient policy. Per the Coase Theorem, there is a set of cross-state transfers that, in conjunction with the (non-centrist) social welfare optimal policy, is Pareto efficient. However, if states are constrained in their ability to make such transfers, an inefficient policy can remain gridlocked. We interpret this result in the context of the “Lacy paradox” (Lacy 2009): the observation that politically conservative states receive a disproportionate per capita share of government expenditures. If low-demand states tend to be poorer than high-demand states, our result concerning bargaining is consistent with an account of the expansion of national policy-making in which increases were achieved via the willingness of rich states to exchange a higher overall level of expenditures for a smaller share of those
expenditures; but in which poor states lack the resources to forgo their high share in exchange for a reduction in the overall level.

Finally, we embed our baseline model in a dynamic two-period framework to analyze how exogenous shocks to the national preference profile affect the national-level political conflict resulting from heterogeneous state-level demands. We show that the expectation of the future preference shock increases the present-period gridlock by pushing both boundaries of the gridlock interval out in the direction of extreme demands.

2 Background

2.1 Related Research

The current research relates to the literature on fiscal federalism dating to Oates (1972), whose “decentralization theorem” posits that in the absence of externalities and scale economies, social welfare will be at least as high if public goods are provided at Pareto efficient levels locally than if they are provided via a uniform national policy. With spillovers, free-rider problems will lead to a suboptimal level of local provision, and so whether a centralized or decentralized system is to be preferred on social welfare grounds will depend on the severity of the spillovers and the degree of preference heterogeneity in the polity. Rose-Ackerman (1981) considers the effects of spillovers and status quo policies at the state level on demand for, and feasibility of, national policies. Besley and Coate (2003) relax the assumption of a uniform national policy under different assumptions about behavior in a national legislature, demonstrating that centralization can still be inefficient owing to misallocation and uncertainty (in a Baron-Ferejohn [1989] style bargaining environment) or because voters face incentives to elect high-demanders to the legislature, leading to inefficient overprovision (see also Inman and Rubinfeld 1996; and Lockwood 2002).

Recent work on political economy of federations has focused on the strategic analysis of the implications of the inter-state spillovers in the provision of public goods. Thus, Cremer and Palfrey (2000, 2006) analyze the federal systems in which federal policy comes in the form of public goods provision floors (mandates) that must be met by the state-level public good provision with positive spillovers. Bolton and Roland (1997) consider the effects of interregional income heterogeneity, factor mobility, and efficiency gains from centralization on the incentives of nations to unify or go their separate ways. Closer to the present model, Alesina et al. (2005) and Hafer and Landa (2006) analyze “dual provision” models of federalism, in which
public goods provision with spillovers across states takes place both at state
and federal levels. Hafer and Landa induce differences in state demand
dependently from the interaction between the differences in state incomes
(wealth), the costs of public good provision, and the relative efficiency of pro-
viding the public good at the federal as opposed to the state level. Unlike
the current paper, the focus of that work is on the the effect of redistribu-
tive tensions and externalities on coalition formation at the national level.
As in the current paper, Alesina et al. posit primitive differences in states' in demand for the public good. Whereas the focus of that work is on the the decisions of (potential) member states to join or enlarge an international union to take advantage of the positive cross-state spillovers of its members, our model takes the union as a given and is chiefly concerned with the consequences of the interaction of the supermajoritarian procedures of the central government with the fiscal consequences of shared policy making on political conflict at the national level.

A critical feature of our analysis that distinguishes it from the above research is “crowding out” of state expenditures by federal expenditures. Crowding out was initially hypothesized to result as the straightforward consequence of exogenous nonmatching federal grants from to states and localities (Bradford and Oates 1971); however, empirical research starting with Courant, Granlich, and Rubinfeld (1979) has documented a “flypaper effect” wherein state and local expenditures appear to increase in response to intergovernmental aid (but see Knight 2002). The crowding out we consider below (in reduced form) is more akin to an economic distortion induced by the overall size of the national government, an approach similar to, but distinct from, the one taken in Bolton and Roland (1997).\footnote{In that paper, the authors model a deadweight economic cost of taxation. We model the effect of increases in taxation at the national level on the cost of raising revenue at the state level.}

Our model also explores the consequences of gridlock-inducing procedures (e.g., supermajority requirements; gatekeeping opportunities); in the context of U.S. lawmaking, canonical work on the subject includes Krehbiel (1996, 1998), who focuses on the filibuster and veto override pivots; Cox and McCubbins (2005); who examine majority party gatekeeping.

2.2 The Object of Study

The model endeavors to capture three aspects of a federal political system. The first, which is of course not unique to federalism, is the existence of heterogeneity in the demand for public good provision. In the presence of
significant heterogeneity, federalism may be a particularly useful way to per-
mit government to tailor government provision to variation in local demand,
rather than subject each subordinate community to the same national, uni-
form rule (or, alternatively, delegate policy making to the executive to tailor
its application to local circumstances.)

The second is the existence of antimajoritarian institutions for lawmak-
ing at the national level. The federal constitution contains a number of
important antimajoritarian features. These include, for example, institu-
tions that grant some degree of insulation for elected officials, e.g., six year
terms for senators; formal constraints on the powers of national government
(e.g., the enumerated powers and restrictions on Congress listed in Article
I, sections 8 and 9 and the Bill of Rights); and institutions such as bicam-
eralism, the presidential veto, the filibuster, and any legislative rules that
limit proposal rights to a restricted group of public officials.

The third critical aspect of the federal system we wish to capture in our
model is the presence of de facto shared sovereignty between the national
and state governments with permeable boundaries (Rose-Ackerman 1981).
To be sure, spheres of autonomy between the state and national governments
are necessary for a coherent federal structure to exist; and the enumerated
and reserved powers of the national and state governments listed in the
constitution and interpreted of two centuries of judicial precedent to a large
extent define these boundaries. However, we depart from the principal of
“dual federalism” (Corwin 1950) by arguing that these boundaries do not
represent a clean partition; and moreover, that they are defined much more
by practical politics and the exigencies of the day than normative theories
about the appropriate division of authorities between levels of government.
Moreover, the framers anticipated that these boundaries would be fuzzy.
This is not simply a point about the ambiguity of language. The existence
of the supremacy clause, for example, implies the potential for federal and
state laws to come into conflict (which would be impossible if the spheres
were truly separate). And Madison, writing in the National Gazette in
February, 1792, pointed to relative ease of distinguishing executive, judicial,
and legislative power, noting of the national and state governments, “the
powers being of a more kindred nature, their boundaries are more obscure
and run more into each other.” Related to the permeable boundaries feature
is the fact crowding out. As noted above, in our model we assume that
increases in the size of the national government have a direct effect on the

\footnote{Madison goes on to admonish, “if the task be difficult, however, it must by no means
be abandoned.}
ability of the states to raise revenue.

In focusing on these three features of federalism, we necessarily abstract away from several others. The first of these is the relationship between income and demand for public good provision. Second, to focus on across-state preference heterogeneity, we abstract away from within-state heterogeneity. The interaction between these two different forms of diversity is clearly important, but beyond the scope of the current inquiry. Having abstracted away from within-state heterogeneity, we also do not consider representation failure.

3 The Baseline Model

3.1 Primitives and Equilibrium Concept

There is a continuum of states with measure one. Each state \(i\) is characterized by a preference parameter whose support is a compact, convex subset of the positive real line, \(\alpha_i \in [\alpha, \bar{\alpha}]\), with probability density function \(p(\alpha)\). As a shorthand, we will refer to state \(i\)'s preference parameter as its demand for public good provision. Each states has endowment, \(y_i\), and each pays a lump sum for a given level of state government activity, \(S_i \in \mathbb{R}_+\), and a lump sum for federal activity, \(F \in \mathbb{R}_+\). The costs of these respective activities are quadratic in the level of service provided. State \(i\)'s utility function is given by

\[
  u_i(F, S_i; \alpha_i, y_i, \gamma, \delta) = y_i + \alpha_i(F + S_i) - \frac{\delta F^2}{2} - \frac{S_i^2}{2} - \gamma FS_i,
\]

where the parameter \(\delta\) represents the relative inefficiency of raising revenue at the federal level. Note that a high \(\delta\) may stem from diseconomies of scale, or it may capture constitutional restrictions on the federal government involvement in that particular domain of public policy. The multiplicative term, \(\gamma FS_i\), represents, in reduced form, the distortionary effect of the size of the federal government on state revenue collection. The parameter \(\gamma\) scales the magnitude of the distortion.

The game unfolds as follows:

1. The Federation decides on a level of federal provision, \(F\), given status quo level \(\tilde{F}\);
2. Each state decides on its own level of state provision, \(S_i\);
3. Payoffs are realized
Let \( \mathcal{U}(p(\cdot|Q,\bar{\pi})) \) be the preference profile within the federation given the distribution of the \( \alpha_i \)'s, \( p(\cdot|Q,\bar{\pi}) \), and \( \mathcal{U} \) be the set of all preference profiles. Generally, we can think of the federal bargaining protocol \( B \) as a mapping from the status quo policy \( \tilde{F} \in \mathbb{R}_+ \) and the preference profile \( \mathcal{U}(\cdot) \) into a policy \( F \in \mathbb{R}_+ \). Formally, \( B := \mathcal{U} \times \mathbb{R}_+ \rightarrow \mathbb{R}_+ \). Rather than commit ourselves to a particular bargaining protocol, we simply restrict our attention to protocols that, given single-peaked preferences, generate a gridlock interval, that is, a compact and convex set of policies that cannot be beaten by another policy under the protocol. Such protocols include q-rules (Austen-Smith and Banks 1999, Banks and Duggan 2006), which are of particular relevance to the supermajoritarian federalist decision-making that is our motivation in this paper. The equilibrium concept is subgame perfect Nash equilibrium.

### 3.2 Equilibrium

We solve by backward induction. To summarize, the states will choose optimal levels of taxation given the size of the federal budget; federal policy will be set in anticipation of optimal play in the state policy-making subgame.

**State-level policy making.** We begin by considering the state policy-making subgame. In the last stage of the game, federal policy \( F \) is set, and the states condition their choices on \( F \). Cursory inspection of equation (1) reveals that it is globally concave in \( S_i \). Solving state \( i \)'s first-order condition yields the optimal state policy

\[
S_i^*(F) \equiv \max\{0, \alpha_i - \gamma F\}.
\]

This expression immediately gives rise to the following remark:

**Remark 1 (Crowding Out.)** A state’s level of provision is weakly decreasing in the level of federal provision.

That the state’s provision should be decreasing in the level of federal provision is natural: imagine a counterfactual world in which the federal government eliminated a major policy like social security or medicaid. In such circumstances, we might reasonably expect at least some states to compensate by substituting state-level policies in the absence of the federal policy.

Another natural feature of state-level provision we wish to highlight appears in the next remark:
Remark 2 (Monotonicity in State Provision.) For a given level of federal provision $F$, state $i$’s equilibrium level of state provision $S_i^*$ is weakly higher than state $j$’s equilibrium level of state provision if and only if $\alpha_i > \alpha_j$. If $S_i^* > 0$, this relationship is strict.

Moving backward in the game, we next consider federal policy making. Anticipating the policy it will set in the second stage, each state seeks to maximize

$$E[u_i(F|\alpha_i; \delta\gamma)] = y_i + \alpha_i(F + S_i^*(F)) - \frac{\delta F^2}{2} - \frac{(S_i^*(F))^2}{2} - \gamma FS_i^*(F)$$ (3)

The substantive focus of our paper is on federalism as a mixed provision of public goods by both the national and state governments; thus, we wish to focus on conditions under which such a mix is feasible. The following lemma establishes those conditions.

Lemma 1 (Preference for Mixed Provision) The following statements are true if and only if $\gamma < \min\{1, \delta\}$:

1. all states have single-peaked preferences over federal provision with ideal point $\hat{F}(\alpha_i; \gamma, \delta) = \left(\frac{1 - \gamma}{\delta - \gamma^2}\right) \alpha_i > 0$;

2. a state’s ideal federal policy induces a strictly positive level of state provision in that state.

Proof. Substituting the expression for $S_i^*$ from equation (2) into equation (3) and simplifying yields

$$E[u_i(F|\alpha_i; \delta\gamma)] = \begin{cases} y_i - \frac{\delta - \gamma^2}{2} F^2 + (1 - \gamma)\alpha_i F + \frac{\alpha_i^2}{2} & \text{if } F < \frac{\alpha_i}{\gamma} \\ y_i - \frac{\delta - \gamma^2}{2} F^2 + \alpha_i F & \text{otherwise.} \end{cases}$$ (4)

This first line of (4) is globally concave if and only if $\delta > \gamma^2$, or $\gamma < \sqrt{\delta}$. Solving the state’s first order condition under the supposition $F < \frac{\alpha_i}{\gamma}$ gives the expression for $\hat{F}(\alpha_i; \gamma, \delta)$ in part 1 of the lemma. Given $\delta > \gamma^2$, the expression for $\hat{F}$ is strictly positive if and only if $\gamma < 1$. To establish single-peakedness with strictly positive state provision at the federal ideal point, it is sufficient to demonstrate that (a) for all $F \geq \frac{\alpha_i}{\gamma}$, $\frac{\partial E[u_i(F)]}{\partial F} < 0$; (b) $\hat{F}(\alpha_i; \gamma, \delta) < \frac{\alpha_i}{\gamma}$; and (c) no discontinuity exists at in $E[u_i(F)]$ at $F = \frac{\alpha_i}{\gamma}$. Differentiating the second line of (4) with respect to $F$ yields $\alpha_i - F\delta$, which
is strictly negative if and only if $F > \frac{\delta}{\gamma}$. $F \geq \frac{\alpha}{\gamma}$ implies $F > \frac{\alpha}{\gamma}$ if and only if $\delta > \gamma$. Substituting the expression for the federal ideal point in part 1 of the lemma, $\hat{F}(\alpha; \gamma, \delta) < \frac{\alpha}{\gamma}$ if and only if $\delta > \gamma$. Finally, to show that there are no discontinuities in $i$’s induced utility, set the first and second lines of (4) to equal each other. Simple algebra reveals a solution at $F = \frac{\alpha}{\gamma}$. Because $\delta < \sqrt{\delta}$ if and only if $\delta < 1$, the three conditions derived above ($\gamma < \sqrt{\delta}$, $\gamma < \delta$, and $\gamma < 1$) are jointly equivalent to $\gamma < \min\{\delta, 1\}$. ■

This result establishes conditions under which the preferences over federal policy are “well-behaved.” If federal provision is “too efficient” ($\delta$ too low), or if state provision is crowded out too quickly, then all states will prefer a completely centralized policy with all authority for policy making surrendered to the national government. In such a circumstance, any federal policy that does not fully crowd out state governance will be strictly Pareto dominated by one that does. Likewise, if the distortion parameter $\gamma$ is sufficiently large, all states will prefer zero federal provision. In the remainder of the paper, we will focus on situations in which the restriction on parameters given in the Lemma is met.

Having established single-peakedness over federal policy making, we note that the bargaining protocol $B$ will induce a nonempty gridlock interval (Krehbiel 1996, 1998). Let $\alpha_L$ represent the demand parameter of the pivotal actor at the extreme low-end of the gridlock region, and $\hat{F}(\alpha_L)$ that actor’s ideal federal policy; likewise, let $\alpha_H$ represent the demand parameter of the actor at the extreme right, and $\hat{F}(\alpha_H)$ its associated ideal point. In the context of Krehbiel’s pivotal politics model, these actors might represent the filibuster and veto override pivots; in the context of Cox and McCubbins, it might represent the floor median and its reflection about the majority caucus median. The point is that any status quo federal policy between $\hat{F}(\alpha_L)$ and $\hat{F}(\alpha_H)$ will be gridlocked, whereas any status quo policy outside of $[\hat{F}(\alpha_L), \hat{F}(\alpha_H)]$ will be, given the federal bargaining protocol, amended to a point in that interval. (Rewrite to say that we are restricting our attention to bargaining protocols that yield this.)

The following proposition encapsulates the discussion above into a full characterization of equilibrium behavior:

**Proposition 1** In equilibrium:

1. The Federation chooses a federal policy $F^* \in [\hat{F}(\alpha_L), \hat{F}(\alpha_H)]$ via bargaining protocol $B$;

2. Each state $i$ chooses a state-level policy $\hat{S}_i^* = \max\{0, \alpha_i - \gamma F\}$. 


Lemma 1 establishes conditions under which, at a state’s optimal level of federal provision, that state will continue to provide locally. Additionally, for each state, there exists a level of federal provision higher than that state’s federal ideal point $F_0(\alpha_i) = \frac{\alpha_i}{\gamma} > \bar{F}(\alpha_i)$ such that for all $F > F_0(\alpha_i)$, state provision is fully crowded out. For federal provision between these values, $F \in (\bar{F}(\alpha_i), F_0(\alpha_i))$, an upward departure from the state’s ideal policy imposes a cost on the state, but one that it can mitigate through a compensating reduction in the level of state provision. However, when the state provision is already fully crowded out, such mitigation is not possible, thus increasing the burden on the state. We formalize this result in terms of the shadow cost of the non-negativity constraint on the state provision:

Lemma 2 (Shadow Cost of Crowding Out) The following conditions hold if and only if $\gamma < \min\{1, \delta\}$ and $F > \frac{\alpha_i}{\gamma}$:

1. the state’s utility is strictly lower than, and decreasing in $F$ at a faster rate, than it would be if the non-negativity constraint were relaxed; and

2. the shadow cost of the non-negativity constraint is increasing in the rate of crowding out $\gamma$ and decreasing in $i$’s overall demand for public goods $\alpha_i$.

Proof. 1. Comparing the first and second lines of (4) gives

$$y_i - \frac{\delta - \gamma^2}{2}F^2 + (1 - \gamma)\alpha_iF + \frac{\alpha^2}{2} > y_i - \frac{\delta}{2}F^2 + \alpha_iF.$$  

Simplifying and solving for $F$ yields $F > \frac{\alpha_i}{\gamma}$. From Lemma 2, given $\gamma < \min\{1, \delta\}$ and $F > \frac{\alpha_i}{\gamma}$, both lines of (4) are decreasing in $F$; the second line is decreasing faster if and only if

$$-(\delta - \gamma)^2F + (1 - \gamma)\alpha > -\delta F + \alpha,$$

which is true if and only if $F > \frac{\alpha_i}{\gamma}$. 2. The shadow cost of the constraint is given by

$$\lambda = \frac{\gamma^2}{2}F^2 - \alpha \gamma F + \frac{\alpha^2}{2}.$$  

Then if and only if $F > \frac{\alpha_i}{\gamma}$, $\frac{\partial \lambda}{\partial \gamma} = \gamma F^2 - \alpha F > 0$ and $\frac{\partial \lambda}{\partial \alpha} = \alpha_i - \gamma F < 0$.

The first part of Lemma 2 implies a kink in a state’s utility function strictly to the right of the state’s ideal point. At the kink, the rate of decline in the state’s welfare associated with additional increases in federal provision experiences a discontinuous jump. This implies that although, given the
restrictions on parameters described above, a state’s utility function is single-peaked, it is not symmetric. This asymmetry will play a key role in the substantive results that follow. The second part establishes the conditions under which the constraint bites more strongly. In particular, the more quickly a state is crowded out, the greater the cost, for a given level of federal provision, associated with having been crowded out.

3.3 Welfare and Polarization

Having characterized the equilibrium to the baseline model, we now move to the analysis of welfare and polarization. We will assume throughout this section that \( p(\alpha) \), the distribution of overall demand for public goods provision, is symmetric. The motivation for this assumption is not strict verisimilitude; rather, we adopt it to clarify how the strategic incentives of the states yield important asymmetries that deviate from canonical models, even in the absence of distributional asymmetries.

Our first result in this regard concerns the aggregate welfare of the polity:

**Proposition 2 (Aggregate Welfare and Federal Policymaking)** Suppose \( p(\alpha) \) is symmetric. Then the socially optimal level of federal provision is strictly less than the median ideal policy if and only if at least some states are fully crowded out at the median’s ideal policy. Otherwise, the socially optimal level of provision is the median ideal policy.

**Proof.** (Sketch.) Let \( \alpha_m \) be the demand of the median state. Given symmetric \( p(\alpha) \), social welfare is given by

\[
V = \int_{\alpha}^{\alpha_m} (E[u_i(F|\alpha; \delta, \gamma)] + E[u_i(F|2\alpha_m - \alpha; \delta, \gamma)]) p(\alpha) d\alpha.
\]

First, note that if all states are crowded out (\( F > \overline{\alpha}/\gamma \)), a decrease in \( F \) is strictly Pareto improving. Next, suppose \( F < \overline{\alpha}/\gamma \), so no state is crowded out at policy \( F \). Substituting from (4) and simplifying yields

\[
V = -\frac{(\delta - \gamma)^2}{2} F^2 + (1 - \gamma)\alpha_m F + K,
\]

where \( K = \int_{\alpha}^{\alpha_m} (\alpha^2 + 2\alpha_m^2 - 2\alpha_m \alpha) p(\alpha) d\alpha \). Given the assumption \( \gamma < \min\{1, \delta\} \), \( V \) is maximized at \( \hat{F}(\alpha_m) \). Consider two states, \( l \) and \( h \), with \( \alpha_l < \alpha_h \) and \( \frac{\alpha_l + \alpha_h}{2} = \alpha_m \). If \( l \) is crowded out at \( \hat{F}(\alpha_m) \), their joint utility is given by

\[
E[u_l(F|\cdot)] + E[u_l(F|\cdot)] = -\frac{2\delta - \gamma^2}{2} F^2 + (\alpha_l + (1 - \gamma)\alpha_h) F + \frac{\alpha_h^2}{2}.
\]
This expression is globally concave, and the joint welfare maximizing policy is given by

\[ F^\text{jwm} = \frac{\alpha_l + (1 - \gamma)\alpha_h}{2\delta - \gamma^2}. \]

Comparing \( F^\text{jwm} \) with \( \hat{F}(\alpha_m) \), the latter exceeds the former if and only if

\[ \alpha_l < \frac{\gamma(1 - \gamma)}{\gamma - 2\delta + \gamma^2}\alpha_h. \]  

(5)

Note that if \( l \) is crowded out at \( \hat{F}(\alpha_m) \), then it must be the case that

\[ \frac{\alpha_l}{\gamma} < \left( \frac{1 - \gamma}{\delta - \gamma^2} \right) \left( \frac{\alpha_l + \alpha_h}{2} \right). \]

Simplifying gives a condition identical to inequality (5). The joint welfare for the entire polity extends this logic by integrating over \( \alpha_l \) and the associated \( \alpha_h = 2\alpha_m - \alpha_l \), relying on the symmetry of \( p(\alpha) \).

To understand the intuition behind this result, suppose the federal policy is greater than or equal to \( \hat{F}_m \), and imagine two states: a “low-demander” state whose demand \( \alpha' \) is less than \( \alpha_m \), and a corresponding “high-demander” state whose demand \( \alpha'' \) is higher than, but equidistant to, \( \alpha_m \) (i.e., \( \alpha'' = 2\alpha_m - \alpha' \)). If the low-demander state is fully crowded out at \( \hat{F}_m \), it will be crowded out for any federal policy greater than \( \hat{F}_m \). Similarly, by Lemma 1, the median state will not be fully crowded out at its own ideal point, and so neither will the high-demander state. By part (a) of Lemma 2, the marginal benefit of a reduction in \( F \) for the low-demander state will be larger than the marginal cost of the high-demander state for that reduction. The sum of the welfare of these two states will be maximized when the marginal benefit to the low-demander of a further reduction equals the marginal cost to the high-demander, which must occur at a point lower than \( \hat{F}_m \). If, by contrast, neither the low-demander nor the high-demander is crowded out at \( \hat{F}_m \), the joint welfare is maximized at the midpoint of their ideal points, which is \( \hat{F}_m \).

The proposition simply extends this two-state intuition to the continuum of states, exploiting the symmetry of the distribution.

A corollary to this proposition concerns institutional design, and in particular the extent to which democratic political institutions yield normatively appealing results. Given single-peaked preferences with an open agenda on the single-dimensional policy space, the ideal point of the median voter must be unbeatable in pairwise competition between alternatives. Thus, in our model, pure majority rule with an open agenda yields the median state’s ideal point as an equilibrium policy. However, whereas
in canonical spatial models, this result has an appealing normative utilitarian implication, it does not have that implication in our federalist setting. Specifically, if utility functions are Euclidean (i.e., a function of the absolute distance between an actor’s ideal policy and the enacted policy), then the welfare maximizing policy corresponds to a central tendency of the distribution of ideal points: for example, the median (in the case of absolute-value preferences) or mean (in the case of quadratic preferences). If the distribution is symmetric, as we have assumed here, these central tendencies correspond, and so majority rule with an open agenda and Euclidean preferences would yield the socially optimal level of federal provision. The asymmetry induced by the crowding out effect of federalism, however, undermines this normative implication, because the induced preferences are no longer strictly Euclidean. The consequence here is that the median voter’s preferred policy is not necessarily the social welfare-maximizing (and so the institutional configuration that results in it may not be justified on utilitarian grounds). More precisely:

Corollary 1 Pure majority rule with an open agenda yields a socially suboptimal level of provision if and only if at least some states are fully crowded out at the median’s ideal policy.

Our next result relates aggregate welfare to the degree of preference heterogeneity across the states. In particular, suppose heterogeneity in the demand for federal provision increased, while the mean and median of the distribution remained unchanged. If preferences were Euclidean, as it is standard to assume, the social welfare maximizing policy would remain at the ideal point of the median/mean, $\hat{F}_m$. Proposition 2 suggests that the welfare maximizing policy is strictly less than $\hat{F}_m$. The next proposition goes further, however, by documenting a relationship between the extent of heterogeneity and that policy.

Proposition 3 (Aggregate Welfare and Preference Heterogeneity)
Suppose $p(\alpha)$ is symmetric, and let $p'(\alpha)$ be a mean-preserving spread of $p(\alpha)$. Then the socially optimal level of federal provision is strictly lower with $p'(\alpha)$ than with $p(\alpha)$ if and only if at least some states are fully crowded out at the median’s ideal policy with $p'(\alpha)$.

Proof. (Intuition) Social welfare is the average of the joint welfare of pairs of states symmetrically distributed about $\alpha_m$. If some low-demander states are crowded out under $p(\alpha)$, then under $p'(\alpha)$ there will be proportionately more crowded out states, exerting a stronger pull on the social welfare maximizing policy away from the median’s ideal point. 

13
The logic underlying Proposition 3 is similar to that underlying Proposition 2. The spread described in the proposition results a strictly larger contingent of states that are fully crowded out. Moreover, the concavity of utility functions implies that the lowest demanders among them are particularly disadvantaged relative to the high demanders. In the aggregate, this effect increases the aggregate benefit associated with a reduction in the federal policy, relative to the cost to the set of high-demander states.

**Polarization.** Next, we consider how the shadow cost of crowding out affects political polarization. To begin, we distinguish between two kinds of polarization. The first, which is standard in the literature, is ideal point (IP-) polarization (e.g., McCarty, Poole, and Rosenthal 2006; Poole and Rosenthal 1984). The IP-polarization between states $i$ and $j$ is simply the absolute difference in their ideal points: $\Pi_{ij}^{IP} \equiv |\hat{F}(\alpha_i) - \hat{F}(\alpha_j)|$. A limitation of IP-polarization is that by definition, it is invariant to the location of the status quo. As an alternative, we propose policy-contingent (PC-) polarization: the absolute utility differential between $i$ and $j$ for a given status quo level of federal provision $F$: $\Pi_{ij}^{PC} \equiv |E[u_i(F|\alpha_i; \delta, \gamma)] - E[u_j(F|\alpha_j; \delta, \gamma)]|

The next result describes the effect of changes in the relative efficiency of federal provision and the magnitude of fiscal distortion on IP-polarization.

**Proposition 4 (Ideal Point Polarization in the Baseline Model)** Suppose $\gamma < \min\{1, \delta\}$. Then IP-polarization between any two states is (1) strictly increasing with the efficiency of federal provision; (2) strictly decreasing in the magnitude of the fiscal distortion when federal provision is less efficient than, or just slightly more efficient than, state provision, and otherwise increasing in the magnitude of the fiscal distortion; and (3) independent from the location of the equilibrium federal provision.

**Proof.**

1. $\frac{\partial^2 \hat{F}(\alpha_i)}{\partial \alpha_i \partial \delta} = -\frac{1-\gamma}{(\delta-\gamma)^2} < 0$, which implies that the distance between any two ideal points is decreasing in $\delta$ (i.e., increasing in efficiency).

2. $\frac{\partial^2 \hat{F}(\alpha_i)}{\partial \alpha_i \partial \delta} = \frac{-\delta+\gamma(2-\gamma)}{(\delta-\gamma^2)^2}$. This quantity is strictly negative if and only if $\delta > \gamma(2-\gamma)$. The right side of this inequality is strictly less than one for all $\gamma < \min\{1, \delta\}$, so the inequality holds for any $\delta > 1$. For $\delta \leq 1$, note that the right side of the inequality is strictly increasing in $\gamma$ for $\gamma < 1$, globally concave, and maximized at $\gamma = 1$. Simple algebra reveals that the inequality is reversed at $\gamma = 1 - \sqrt{1 - \delta}$, which is strictly less than $\delta$ for all $\delta < 1$, thus satisfying the condition $\gamma < \min\{1, \delta\}$.
3. Immediate.

The first part of the proposition can be thought of as a price effect: as the cost or providing at the national level decreases, all states want proportionately more national provision; the effect is to increase the distance between the ideal levels of provision of different states. This may seem a mathematical artifact, but we point it out because it reveals a potential limit of the IP-polarization measure: to the extent that a measure of polarization is intended to capture animosity between parties or individuals, it is hard to imagine a situation in which a technological change that benefits all players would increase, rather than decrease acrimony.

To understand the intuition behind the second part of the proposition, note that when federal provision is relatively inefficient, any increase in the fiscal distortion compounds the desire of all parties not to rely on it – thus producing a compression of ideal points. When federal provision is efficient relative to that of the states (perhaps because of scale economies), a second effect comes into play: the willingness of all states to tolerate being largely crowded out to take advantage of the more efficient federal provision. This effect dominates for very high levels of federal efficiency (low \( \delta \)). Because the states differ in the rate at which they wish to substitute from state to federal provision, however, their ideal points will pull diverge as \( \gamma \) increases.

The third part of the proposition is immediate: obviously, one’s ideal level of provision cannot depend on the level of provision. We insert this as a point of comparison with the next proposition, which concerns policy-contingent (PC-) polarization.

**Proposition 5 (Policy-Contingent Polarization in the Baseline Model)**

PC-polarization between two states is (1) strictly increasing in the magnitude of the federal policy; (2) strictly decreasing in the magnitude of the fiscal distortion if neither or one state is crowded out, and unresponsive otherwise; and (3) unresponsive to changes in the relative efficiency of federal provision.

**Proof.** Consider two states with demand \( \alpha_l \) and \( \alpha_h \), with \( \alpha_l < \alpha_h \). Noting that from the above, it can never be the case that state \( h \) is crowded out while state \( l \) is not, there are three cases to consider.

1. Neither state crowded out. Then

\[
E[u_h(F|\cdot)] - E[u_l(F|\cdot)] = (\alpha_h - \alpha_l)(1 - \gamma)F + \frac{\alpha_h^2 - \alpha_l^2}{2}.
\]
This quantity is strictly positive and increasing in $F$. Further, this quantity is strictly decreasing in $\gamma$, and unresponsive to changes in $\delta$.

2. Both states crowded out. Then

$$E[u_h(F|\cdot)] - E[u_l(F|\cdot)] = (\alpha_h - \alpha_l)F,$$

which is strictly positive, increasing in $F$, and unresponsive to $\gamma$ and $\delta$.

3. State $l$ crowded out. If $l$ is crowded out and $h$ is not, then $F$ must lie between the ideal points of the two states. By Lemma 2, state $l$’s utility is falling at a faster rate than it would be if the nonnegativity constraint were not binding. Thus given that the expected utility differential in case 1 is strictly positive, it must be positive in this case as well, and given that its derivative with respect to $F$ is strictly positive in case 1, it must be positive in this case as well. Further, the derivative of $E[u_h(F|\cdot)] - E[u_l(F|\cdot)]$ with respect to $\gamma$ is $\gamma F^2 - F\alpha_h$, which is negative if and only if $F < \frac{\alpha_h}{\gamma}$. But this must be true given the initial supposition that $h$ is not crowded out.

### 4 Extensions

#### 4.1 Externalities

In the next extension, we vary the baseline model in a way that highlights the effects of cross-state spillovers on preferences for federal provision. In particular, suppose that state’s $i$’s utility incorporates an externality term, as follows:

$$u_i(F, S_i; \alpha_i, \beta_i, y, \gamma, \delta) = y_i + \alpha_i (F + S_i) - \frac{\delta F^2}{2} - \frac{S_i^2}{2} - \gamma F S_i + \beta_i \int S(\alpha) dP(\alpha),$$

where $\beta_i \in \mathbb{R}$ is a (state-specific) parameter scaling the magnitude of the externality. Given this parameterization, it is immediate that the state’s own level of provision, $S_i$, will be unaffected by the externality; as above, a state $i$ will be crowded out for all $F > \frac{\alpha_i}{\gamma}$, or rearranging terms, $\alpha_i < \gamma F$. Given the expectation that states not crowded out will implement $S^*(\alpha) = \alpha - \gamma F$.
in equilibrium, the last term in equation (6) is equal to $\beta \int_{\alpha}^{\gamma F} (\alpha - \gamma F) p(\alpha) d\alpha$ if $\gamma F < \alpha$ and 0 otherwise.

In the results that follow in this section, we will assume for analytic tractability that $p(\alpha)$ is $U[0, 1]$. First, we consider how the introduction of cross-state externalities affects induced preferences over federal provision.

**Lemma 3** Suppose all states have common externality parameter $\beta$, and $\alpha \sim U[0, 1]$. If and only if $\beta < \frac{\delta - \gamma^2}{\gamma^2}$, all states have single-peaked preferences over national policy. Further,

1. If $0 < \beta < \frac{\delta - \gamma^2}{\gamma^2}$, all states have a strictly positive level of state provision at their respective ideal national levels of provision. If $\delta - \gamma^2 - \beta \gamma^2 > 0$, then all states have ideal national provision of zero.

2. If $\beta < 0$, then all states have strictly positive ideal national policies, but a proper subset of states ($\alpha \in (0, \frac{\beta \gamma^2}{\gamma^2 - (\delta - \gamma^2)})$) have zero state provision at their respective ideal national level of provision.

**Proof.** Given $\alpha \sim U[0, 1]$, the externality term in state $i$’s utility is given by $\frac{\beta (1 - \gamma) F^2}{2}$. Substituting into the state’s induced utility over federal provision gives

$$E[u_i(F; \alpha_i, \beta, \alpha, \gamma)] = \begin{cases} 
-\frac{\delta - \gamma^2 - \beta \gamma^2}{\gamma^2} F^2 + (\alpha_i - \beta \gamma) F + \frac{\alpha_i^2 + \beta}{2} & \text{if } \alpha_i - \gamma F > 0 \\
-\frac{\delta - \beta \gamma^2}{\gamma^2} F^2 + (\alpha_i - \beta \gamma) + \frac{\beta}{2} & \text{otherwise}.
\end{cases}$$

(7)

The first line of (7) is globally concave if and only if $\delta - \gamma^2 - \beta \gamma^2 > 0$, or $\beta < \frac{\delta - \gamma^2}{\gamma^2}$. This condition implies global concavity of the second line, because $\frac{\delta - \gamma^2}{\gamma^2} < \frac{\delta}{\gamma^2}$. To establish single-peakedness it is sufficient to demonstrate that (a) there is no discontinuity at $F = \frac{\alpha_i}{\gamma}$ and (b) the derivative of the first and second lines of (7) share the same sign. Substituting $F = \frac{\alpha_i}{\gamma}$ into both expressions yields

$$\frac{\alpha_i^2 \beta \gamma^2 - \alpha_i^2 \delta + 2 \alpha_i^2 \gamma - 2 \alpha_i \beta \gamma^2 + \beta \gamma^2}{2 \gamma^2}.$$.

Differentiating both lines of (7) with respect to $F$ and evaluating at $F = \frac{\alpha_i}{\gamma}$ gives

$$\frac{\alpha_i \beta \gamma - \alpha_i \delta + \alpha \gamma - \beta \gamma^2}{\gamma^2}$$.

1. $\beta > 0$. From the foregoing, state $i$’s induced utility is maximized at $F = \frac{\alpha_i (1 - \gamma)}{\beta \gamma}$. From the condition given in the proposition, the denominator of this expression is strictly positive. The numerator
is strictly positive if and only if $\alpha_i > \frac{\beta \gamma}{1 - \gamma}$. For $\alpha_i \in [0, \frac{\beta \gamma}{1 - \gamma})$, the expression is weakly negative, and so the state’s (constrained) ideal level of federal provision is zero. If $\beta > 1$, $\frac{\beta \gamma}{1 - \gamma} > 1$, and so given $\alpha_i \leq 1$ by assumption, all states have an ideal federal provision of zero. At $F = 0$, $S^*_i = \alpha_i > 0$.

2. $\beta < 0$. From above, $i$’s induced utility is maximized at $F = \frac{\alpha_i (1 - \gamma) - \beta \gamma}{\delta - (1 - \beta) \gamma^2}$ if it is not fully crowded out at its own ideal point, and $F = \frac{\alpha_i - \beta \gamma}{\delta - \beta \gamma^2}$ if it is. The first of these candidate ideal points is positive if and only if $\beta < \frac{1 - \gamma}{\gamma} \alpha_i$. The right side of this condition is strictly positive for any $\alpha_i > 0$, and thus holds for all $\beta < 0$. The second candidate ideal point is positive if and only if $\beta < \frac{\alpha_i}{\gamma}$, which holds for all $\beta < 0$. Noting that state $i$ will be fully crowded out for all $F \geq \frac{\alpha_i}{\gamma}$, and substituting the expression for $i$’s ideal federal provision and solving for $\alpha_i$ gives

$$\alpha_i < \frac{\beta \gamma^2}{\beta \gamma^2 - (\delta - \gamma)}.$$  
(8)

This quantity is positive and bounded between zero and one.

Part 1 of the Lemma describes the effect of positive, symmetric externalities. When spillovers are positive but small, there is no qualitative difference from the baseline model. For intermediate levels of positive externalities, a set of states with low demand for public good provision will have that demand met by the provision of other states; consequently, they will be driven to prefer that all provision be local. Finally, for large values of positive externalities, all states will belong to that set, and purely decentralized provision will be unanimously preferred.

Part 2 describes what happens when the externalities are negative and symmetric. In that case, there will always be a set of states with low demand that prefer a level of federal provision sufficient to fully crowd themselves (and other states) out. As the magnitude of the negative externalities increases, the fraction of states in that set increases, but there will always be a subset of states that continues to prefer mixed provision.

The next result is a corollary to Lemma 3.

**Corollary 2** Suppose $\beta < \frac{\delta - \gamma^2}{\gamma}$, and that a state with demand $\alpha'$ would be fully crowded out at its own ideal level of national provision. Then there exists a neighborhood of states with demand strictly greater than $\alpha'$ that would also be fully crowded out at that level of national provision.
Proof. From the Proof of Lemma 3, the first state’s ideal level of provision is given by $F^* = \frac{\alpha' - \beta \gamma}{\beta \gamma^2 - \delta}$. It is sufficient to demonstrate that $F^* > \frac{\alpha''}{\gamma}$ for some $\alpha'' > \alpha'$. Simplifying gives

$$\alpha'' < \frac{\alpha' \gamma - \beta \gamma^2}{\delta - \beta \gamma^2}.$$  

The right side of this inequality is strictly larger than $\alpha'$ if and only if $\alpha' < \frac{\beta \gamma^2}{\beta \gamma^2 - (\delta - \gamma)}$, which is the condition delineating which states prefer themselves crowded out given in inequality (8). □

This corollary simply points out that any state that most prefers a level of federal provision sufficient to put itself out of the business of public goods provision (and, by extension, all states with lower demand) also wants to put higher-demander states out of business as well.

The next result relates the magnitude of a state’s externality parameter with its ideal level of federal provision:

Proposition 6 (Externalities and Ideal Federal Provision) Suppose $0 < \beta < \frac{\delta - \gamma^2}{\delta - \gamma}$. If $\gamma < \min\{1, \delta\}$ and a state prefers provision at the national level, then its ideal level of federal provision is decreasing in the magnitude of externalities if externalities are positive, and increasing if they are negative.

Proof. If the state is not fully crowded out at its ideal point, then

$$\frac{\partial F}{\partial \beta} = -\frac{\gamma(\delta - \gamma^2 - \alpha \gamma + \alpha \gamma^2)}{(\beta \gamma^2 - (\delta - \gamma))^2}.$$  

This quantity is negative if and only if $\delta - \gamma^2 - \alpha \gamma (1 - \gamma) > 0$. Rearranging terms yields $\alpha < \frac{\delta - \gamma^2}{\gamma(1 - \gamma)}$. The right side of this inequality exceeds one for all $\gamma < \min\{1, \delta\}$. If the state is fully crowded out at its own ideal point, then $\frac{\partial F}{\partial \beta} = -\frac{\gamma(\delta - \alpha \gamma)}{(\delta - \beta \gamma)^2}$. This quantity is negative if and only if $\alpha_i < \frac{\delta}{\gamma}$. The right side of this inequality exceeds one for all $\gamma < \delta$. □

This result stems naturally from the logic of the preceding Lemma. A reduction in federal provision implies a concomitant increase in state provision. When externalities are positive, this is a good thing; when they are negative, a bad thing.

Next, we turn to polarization. The first result relating externalities and polarization concerns the specific case of IP-polarization in the presence of a common externality $\beta$:

Proposition 7 (Ideal Point Polarization with Common Externalities) Suppose all states have common externality parameter $\beta$, and single-peaked preferences over federal policy. Then the following statements are true:
1. **IP-polarization between two states that prefer mixed provision is increasing in the magnitude of the externality if the externality is positive, and decreasing if it is negative;**

2. **IP-polarization between a state that prefers mixed provision and a state that prefers only state provision is increasing in the magnitude of a positive externality;**

3. **IP-polarization between two states that prefer only federal provision is decreasing in the magnitude of a negative externality; and**

4. **IP-polarization between a state that prefers mixed provision and a state that prefers only federal provision is either increasing or decreasing in the magnitude of a negative externality.**

**Proof.** The proof proceeds by taking derivatives of \( \Pi_{ij}^{IP} \) for the different cases.

**Proposition 8 (Policy-Contingent Polarization with Common Externalities)**
Suppose all states have common externality parameter \( \beta \) and single-peaked preferences over federal policy. Then PC-contingent polarization between two states is independent of \( \beta \).

**Proof.** The proof immediately follows by inspection of the expression for \( \Pi_{ij}^{PC} \).

### 4.2 Transfers Payments and Efficiency

Suppose each state can make voluntary positive utility transfers to any subset of other states. Let \( T \) be the set of realized inter-state transfers. Let \( F_{sum} \) be the social welfare maximizing level of federal provision.

**Proposition 9 (Coaseian Bargaining)** There exists a pair \((T, F_{sum})\) that is jointly (Pareto) efficient if the endowment constraints do not bind.

**Proof.** To be added.

This result says, in essence, that in the world with positive utility transfers and no endowment constraints, we should expect the status quo location not to affect social welfare. This conclusion is reminiscent of the Coase theorem’s assertion of efficient outcomes regardless of the allocation of property rights between the potential bargainers. The requirement that state endowments do not bind is, however, important here. Intuitively, if the endowment
size is correlated with demand, as we would expect to be the case in the U.S. – low-demand states are poorer than the high-demand states – the result breaks down. We state it as a conjecture with the proof to be added:

**Conjecture 1** Gridlock will persist at a socially suboptimal level of provision if $F^{sym}$ lies closer to resource-constrained states than the status quo level of federal provision.

### 4.3 Dynamics

In this section, we consider a two-period extension of our model in which at the beginning of the second period, all agents’ parameter $\alpha$ is shocked by some $\sigma$ symmetrically distributed around 0, with pdf $p(\sigma)$. We will show that the expectation of the second period leads to the increase in the size of the gridlock interval in the first period. In other words, the weight of the future will increase the present-day disagreement. In fact, we show that it does so by pushing toward their respective extremes both the left and the right bounds of the gridlock interval.

Recall that $\alpha_L$ and $\alpha_H$ are types defining lower and upper bounds of the gridlock interval, respectively. If the status quo is low (below $\hat{F}(\alpha_L)$), suppose that the new policy is then $\hat{F}(\alpha_L)$. If the status quo is high, then the new policy is $\hat{F}(\alpha_H)$. Let $F^2(\alpha^2)$ be optimal choice in $t = 2$ given preference $\alpha^2$ and optimal $S_i$. We are going to assume that $F^1 \in [F^2(\alpha_L + \sigma), F^2(\alpha_H + \sigma)]$ – that is, if the status quo is below the induced optimal preference of the state defining the left bound of the second period’s gridlock interval, then the policy will be pulled up to that lower bound, and if it is above the induced optimal preference of the state defining the right bound of the second period’s gridlock interval, then the policy will be pulled down to that upper bound. Formally, we have the following proposition:

**Proposition 10** There exists $\hat{\alpha}_i \in (\alpha_L, \alpha_H)$, such that for all $\alpha_i < \hat{\alpha}_i$, $F^1(\alpha_i) < \hat{F}(\alpha_i)$ and for all $\alpha_i > \hat{\alpha}_i$, $F^1(\alpha_i) > \hat{F}(\alpha_i)$.

**Proof.** Note first that state levels of provision $S_i(F, \alpha)$ are not sticky, and are chosen optimally given $\alpha$ and $F$. It is clear that $F^2(\alpha^2) = \hat{F}(\alpha^2)$ from the one-period model. Because $F^2(\alpha) = \hat{F}(\alpha)$ is monotone, its inverse is well-defined. Let $A(F)$ be inverse of $F^2(\alpha)$. Then $A(F^1) \in [\alpha_L + \sigma, \alpha_H + \sigma]$ and so for policy in $t = 1$ (which is status-quo in $t = 2$) to be in the gridlock interval in $t = 2$, it must be that the shock to preferences $\sigma$ is such that $\sigma \in [A(F^1) - \alpha_H, A(F^1) - \alpha_L]$. 21
The expected utility from choice \( F^1 \) is

\[
\begin{align*}
&= u(F^1, \alpha_i, \cdot) + \lambda \int_{A(F^1) - \alpha_L}^{A(F^1) - \alpha_H} p(\sigma) u(F^1, \alpha_i + \sigma, \cdot) d\sigma \\
&+ \lambda \int_{-\infty}^{\alpha_H} p(\sigma) u(F^2(\alpha_H + \sigma), \alpha_i + \sigma, \cdot) d\sigma \\
&+ \lambda \int_{\alpha_L}^{\infty} p(\sigma) u(F^2(\alpha_L + \sigma), \alpha_i + \sigma, \cdot) d\sigma,
\end{align*}
\]

where \( u(F^1, \alpha_i, \cdot) \) is the indirect utility given that \( S_i = S_i^*(F^1, \cdot) \), the first integral is the expected utility from federal provision when the shock lends the system in the gridlock interval, and the second and third integrals are when the shock moves the system to the right of the right bound and the left of the left bound of the gridlock interval.

Taking the first-order condition, we get

\[
\frac{\partial u(F^1, \alpha_i, \cdot)}{\partial F^1} + \lambda p(A(F^1) - \alpha_L) u(F^1, \alpha_i + A(F^1) - \alpha_L, \cdot) \frac{\partial A(F^1)}{\partial F^1}
\]

\[
+ \lambda p(A(F^1) - \alpha_H) u(F^1, \alpha_i + A(F^1) - \alpha_H, \cdot) \frac{\partial A(F^1)}{\partial F^1}
\]

\[
+ \int_{A(F^1) - \alpha_L}^{A(F^1) - \alpha_H} p(\sigma) \frac{\partial u(F^1, \alpha_i + \sigma)}{\partial F^1} d\sigma
\]

\[
+ \lambda p(A(F^1) - \alpha_H) u(F^2(\alpha_H + A(F^1) - \alpha_H), \alpha_i + A(F^1) - \alpha_H, \cdot) \frac{\partial A(F^1)}{\partial F^1}
\]

\[
+ \lambda p(A(F^1) - \alpha_L) u(F^2(\alpha_L + A(F^1) - \alpha_L), \alpha_i + A(F^1) - \alpha_L, \cdot) \frac{\partial A(F^1)}{\partial F^1}
\]

\[
= 0
\]

Noting that \( F^2(A(F^1)) = F^1 \) and canceling terms, we obtain an equivalent condition

\[
\frac{\partial u(F^1, \alpha_i, \cdot)}{\partial F^1} + \lambda \int_{A(F^1) - \alpha_H}^{A(F^1) - \alpha_L} p(\sigma) \frac{\partial u(F^1, \alpha_i + \sigma, \cdot)}{\partial F^1} d\sigma = 0 \quad (9)
\]
If \( \lambda \int_{A(F^1) - \alpha_H}^{A(F^1) - \alpha_L} p(\sigma) \frac{\partial u(F^1, \alpha_i + \sigma)}{\partial F^1} d\sigma \) less (greater) than 0 then \( F^1 \) is less (greater) than in the one-shot model.

\( A(F^1) \) is the state for which \( \frac{\partial u(F^1, \alpha_i + \sigma)}{\partial F^1} \) is equal to zero. If \( \alpha_i + \sigma < A(F^1) \), then the integrand in the equation (9) is less than 0 and so for \( \sigma < A(F^1) - \alpha_i \), \( \frac{\partial u(F^1, \alpha_i + \sigma)}{\partial F^1} < 0 \). So, \( A(F^1) - \alpha_L \leq A(F^1) - \alpha_1 \), the integrand is less or equal to zero, and thus the value of the integral is less than 0, and thus \( F^1(\alpha_i) < F^2(\alpha_i) = \hat{F}(\alpha_i) \).

If \( \alpha_i + \sigma > A(F^1) \), then \( \frac{\partial u(F^1, \alpha_i + \sigma)}{\partial F^1} > 0 \). Thus, if \( \sigma > A(F^1) - \alpha_i \), \( \frac{\partial u(F^1, \alpha_i + \sigma)}{\partial F^1} > 0 \). So, if \( A(F^1) - \alpha_H \geq A(F^1) - \alpha_1 \), then the value of the integral is greater than 0, and thus \( \alpha_i \geq \alpha_H \) and so \( F^1(\alpha_i) > F^2(\alpha_i) = \hat{F}(\alpha_i) \).

It follows that \( F^1(\alpha_L) < \hat{F}(\alpha_L) \) and \( F^1(\alpha_H) > \hat{F}(\alpha_H) \). Note next that the integral is monotonic (increasing) in \( \alpha_i \). Thus, there is a unique \( \hat{\alpha}_i \in (\alpha_L, \alpha_H) \) such that the value of integral is 0, and so \( F^1(\hat{\alpha}_i) = \hat{F}(\hat{\alpha}_i) \). Given monotonicity, then, the result follows.

References


