Negotiated Accounting Measurement Rules in Debt Contracts*

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Abstract

We propose an incomplete debt contracting model featuring accounting-based covenants between a manager and a lender to study how accounting measurement rules can be designed to enhance the role of accounting-based covenants. Extant accounting rules may exhibit false alarm and undue optimism errors and the manager may choose to exert costly effort to privately find out the suitable accounting methods that correct those errors (with some probability). The manager may also choose whether to disclose such findings. In the case of non-disclosure, costly renegotiation may occur when eventually both parties find out the suitability of the extant accounting rules. We find that 1) the manager has a tendency to disclose the existence of false alarm but not undue optimism errors; and 2) the manager has a tendency to exert socially wasteful effort in finding out the suitability of the extant accounting rules. Our results provide empirical and policy implications.

JEL classification: M40, M41, G32

Key Words: debt contracting; accounting rules; renegotiation; incomplete contracting; accounting-based covenant

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1 Introduction

Accounting information is widely used in debt contracting (see, e.g., Smith Jr and Warner (1979), Leftwich (1983), Smith Jr (1993)) and understanding the role that accounting information plays in debt contracting is, according to Christensen et al. (2016), “at the heart of the positive accounting theory developed by Watts (1977), Watts and Zimmerman (1978), Holthausen (1981), Leftwich (1983), Holthausen and Leftwich (1983), and Watts and Zimmerman (1986)”. Contracts based on accounting numbers, however, are also known to be incomplete and frequently renegotiated (e.g., Dichev and Skinner (2002), Roberts and Sufi (2009)), resulting in both Armstrong et al. (2010) and Christensen et al. (2016) calling for more research using the “incomplete contracting theory” approach to better understand “the mechanisms through which accounting information enhances debt contract efficiency”.

In this paper we respond to such calls and propose a model to study how accounting measurement rules can be designed to enhance the role of accounting-based covenants, one of the open questions proposed in Christensen et al. (2016). We focus on one particular mechanism: how errors of extant accounting rules (referred to as the GAAP rules thereafter) and the subsequent renegotiations to correct those errors affect debt contract efficiency through manager’s tendency to search for and disclose such errors. Our model exhibits two novel features: first, the contract is incomplete because whether the GAAP rules are suitable or not cannot be contracted upon ex ante; and second, managers exert effort trying to find out the suitability of the GAAP rules. Those features are realistic as accounting numbers used in debt contracts are frequently adjusted from extant GAAP numbers (e.g., EBITD or EBITDA)\(^1\) while other times the GAAP numbers are preserved (Leftwich (1983), Li (2010), Li (2016)). We offer an explanation regarding 1) why and how extant GAAP numbers exhibit different properties from the adjusted numbers and; 2) why extant GAAP numbers are sometimes preserved in debt contracts.

In our model, a manager signs a debt contract to obtain financing of a project from a lender. The debt contracting is incomplete, following Aghion and Bolton (1992). The payoff

\(^1\)See https://www.sec.gov/Archives/edgar/data/1747079/000114420419025412/tv521256_ex4-1.htm for an example of a contract that defines EBITDA and uses that metric elsewhere. We thank Judson Caskey for this example.
of the project depends on both an underlying state and on an intermediate action upon receiving accounting information, to be discussed in more detail below. The underlying state can be either good or bad and the action taken can be either continuing or restructuring the project. There is a conflict of interest between the manager and the lender in that the lender would always prefer restructuring whereas the manager would always prefer continuing the project, due to the private benefit from continuation. Socially optimal decision, on the other hand, dictates restructuring when the state is bad and continuation when the state is good. The state is not contractible, even though it is observable ex-post, as in Aghion and Bolton (1992). What is contractible is an earnings signal generated from accounting rules. We assume that it is more likely to see good (bad) earnings when the state is good (bad). Therefore, a key insight from Aghion and Bolton (1992) is that the earnings-based covenant gives decision rights to the manager (lender) when the earnings signal is good (bad).

Extending Aghion and Bolton (1992), the contract is also incomplete in the sense that whether the GAAP rules are the correct measurement rules of the underlying economic transactions is not contractible. The GAAP rules may not be the correct measurement rule, in the sense that the GAAP rules may generate false alarm (i.e., report bad earnings in the good state) and/or undue optimism (i.e., report good earnings in the bad state) errors. The manager can choose to ex-ante exert costly effort to find out whether the GAAP rules generate such errors. The more effort the manager exerts, the more likely the manager will find out whether there is any error. Once finding out, the manager can then choose whether to disclose immediately the finding, i.e., whether the GAAP rules exhibit any error. The manager’s disclosure choice is strategic following Dye (1985) and Jung and Kwon (1988), i.e., the manager can remain silent but has to truthfully reveal the information if disclosing. If the manager does not find out whether the GAAP rules generate errors, the manager has to remain silent, i.e., there is no credible way of verifying that the manager is uninformed.

If the manager discloses that the GAAP rules exhibit errors, the contract is based on earnings from an alternative set of accounting rules that are suitable and correct such errors. Otherwise, the contract is based on default GAAP earnings. In a later date, both parties will eventually find out whether the GAAP rules are suitable and may have to renegotiate
if GAAP rules are not suitable. Renegotiation results in the socially optimal decision but incurs a deadweight cost.\textsuperscript{2} During renegotiation, whoever has the control rights, which is determined by the covenant, is in a better position as renegotiation has to make the party who has the control rights at least as well off as without renegotiation, since the party can threaten to not renegotiate and choose the party’s preferred action.

We establish two results. First, the manager has a tendency to disclose false alarm errors and hide the non-existence of such errors. In contrast, the manager has a tendency to hide undue optimism errors but disclose the non-existence of such errors. Second, the propensity for the manager to strategically disclose or withhold certain information results in the manager exerting (socially) excessive effort to find out whether the GAAP rules are suitable.

The intuition for the first result is as follows. The manager’s disclosure of the private findings removes any information asymmetry between the manager and the lender. The interest rate the lender charges is thus the information symmetry rate. With false alarm errors, low accounting signals become more likely. This results in the lender more likely to possess the control rights, which reduces the interest rate the lender is asking for when the manager does not disclose such errors, relative to the information symmetry rate (i.e., “lender opportunism” reduces the interest rate the lender asks for). However, due to the information asymmetry between the manager and the lender, the interest rate upon non-disclosure is not as low as when the lender knows for sure that a false alarm error occurs. The manager who observes the existence of false alarm errors therefore has an incentive to disclose such errors, as the disclosure reduces the interest rate. However, the manager who observes the non-existence of false alarm errors has no incentive to disclose as disclosing will result in the higher information symmetry rate.

On the other hand, with undue optimism errors, high accounting signals become more likely. This results in the lender less likely to possess the control rights. So the lender charges a higher interest rate than the information symmetry rate upon non-disclosure (i.e., the lender “price protects”). Similarly, due to information asymmetry, the interest rate is not as high

\textsuperscript{2}This cost is needed as otherwise Coarse Theorem will apply. Please see detailed discussions in footnote 8.
as when the lender knows for sure an undue optimism error occurs. The manager who knows that an undue optimism error exists therefore has an incentive to withhold disclosing and taking advantage of the (relatively) lower interest rate, when we do not consider renegotiation cost. Since renegotiation incurs social cost and thus will reduce the manager’s payoff (as the lender breaks even), the manager faces a trade-off and may choose to disclose (at least with some probability) when such social cost is sufficiently high. The manager who knows that an undue optimism error does not exist, on the other hand, always has an incentive to disclose and lower the interest rate.

The intuition for the second result comes from the first result. Since the manager can strategically choose to disclose or withhold when finding out whether the GAAP rules are correct, the manager has an excessive incentive to search for such information, relative to the socially optimal case, which is equivalent to the manager always having to disclose the finding, i.e., the manager cannot strategically disclose.

Our results have empirical and policy implications. Our first result explains why accounting rules that generate false alarm errors are preferred to those that generate undue optimism errors as false alarm errors are more likely to be found out and disclosed by the manager upfront whereas the undue optimism errors are more likely to be withheld. The results also imply that we are more likely to see correction of false alarm errors when adjusting GAAP numbers for debt contracting, consistent with the usual practice to exclude certain expense items (i.e., EBIT or EBITDA). Our second result explains why it is sometimes better to commit to using the GAAP rules even if such rules are prone to measurement errors, as such commitment prevents managers’ excessive effort of finding out whether the GAAP rules are suitable. In other words, our result provides a justification to why debt contracts sometimes exclude accounting changes, as empirically documented in Beatty et al. (2002).

Our paper contributes and is related to several streams of literature. First, it is related to the broad empirical literature documenting the differences between accounting rules used in the actual debt contract and standard rules such as U.S. GAAP. We are, to the best of our knowledge, the first to provide an analytical explanation of why such difference exists and what type of difference is preferred. Leftwich (1983) is probably among the first to
empirically document such difference and he provides evidence consistent with the explanation that adjustments from U.S. GAAP is meant to prevent managers from engaging in behaviors that exploit debtholders. His explanation is also consistent with our finding that GAAP precludes undue optimism errors to prevent managers from withholding them. Leftwich (1983) also argues that debtholders rely on “price protection” and so may be indifferent to the terms of the debt issue but such “price protection” will affect shareholders’ preferences. In our model, it is indeed the case. Debtholders rely on such “price protection” to break even whereas such “price protection” affects the manager’s disclosure decisions and thus the manager’s preference to different accounting rules.

More recently, Li (2010) studies the adjustments of net income and net worth in private debt contracts. He finds that transitory earnings are usually excluded. To the extent that transitory earnings merely add noise and is equally likely to generate false alarm errors and undue optimism errors, it is not directly related to our predictions. Nevertheless, Li’s finding that there is not much conservative adjustment in the debt contract is generally consistent with our results as we predict that the GAAP rules should generate more false alarm errors relative to undue optimism errors so there is not much additional conservative adjustment left.

Our paper is also related to the literature on the role of accounting information in debt contracting. Armstrong et al. (2010) provides an excellent review on the role of information and financial reporting in debt contracting. They emphasize the endogenous nature of debt contracts with respect to information asymmetries between contracting parties, a key feature of our model. They also call for more research to “investigate the financial reporting attributes that debt holders value by examining the modifications to GAAP that are made in the calculation of compliance with covenants”. In this paper we find that debtholders prefer financial reporting rules that generate false alarm errors as opposed to undue optimism errors. The reason is that managers are more likely to disclose false alarm errors, allowing for modifications to the GAAP rules to improve efficiency.

Christensen et al. (2016) provides another excellent review on the role of accounting information in financial contracting. In particular, they expect the incomplete contracting
theory approach to be useful in “advancing our knowledge of how accounting information affects contract efficiency”, an approach we adopt in our paper. In particular, they discuss lender opportunism and future renegotiation. Those two features play a crucial role in our model, by affecting the value of the debt and thus managers’ differential disclosure incentives. We also respond to their call for more research on the role of accounting standards in debt contracting. In our setting, accounting standards generate false alarm errors ex-ante in anticipation that such false alarm errors are more likely to be disclosed and corrected ex-post whereas undue optimism errors may not be.

Analytically, numerous papers have studied the optimal properties of earnings in various debt contracting settings, with a particular focus on conservatism. Those papers mostly focus on how the information properties of the accounting system affects debt contracts, and the underlying assumption is that the earnings numbers used in the debt contracts are identical to those generated from the accounting system. However, there are empirical findings (e.g., Dyreng et al. (2017)) documenting that accounting numbers used in debt contracts are adjusted from the numbers prepared under extant accounting rules. Therefore, there is a lack of explanation regarding why extant accounting numbers exhibit different properties from the adjusted numbers in this literature. We offer an explanation: managers may find that extant accounting numbers generate measurement errors that need to be corrected. However, managers may not have the right incentive to disclose those errors. Therefore, the design of extant accounting rules and subsequent correction of errors need to take such managerial incentives into account.

Finally, our paper is related to the economics literature on endogenously incomplete contracts. Allen and Gale (1992) finds that it may be optimal to exclude contingencies from optimal contracts because 1) contingencies may be based on information subject to manipulation; and 2) the choice of certain contingencies may induce inefficiencies due to signalling of contracting parties’ private information. Glaeser and Shleifer (2001) find that excluding contingencies may be optimal because it is less costly to enforce. More recently,

3Similar point has also been raised in Armstrong et al. (2010).
Tirole (2009) finds that it may be optimal to exclude certain contingencies to prevent socially costly cognitive effort in seeing through the implications of such contingencies. We apply the insights of Tirole (2009) in a debt contracting setting and study how properties of the GAAP rules affect the manager’s effort and subsequent disclosure decisions.

The paper is organized as follows. Section 2 presents the model. Section 3 provides some preliminary analysis. Section 4 and Section 5 solve for the equilibrium and obtains our main results when GAAP rules may exhibit false alarm errors and undue optimism errors, respectively. Section 6 discusses empirical and policy implications and Section 7 concludes. All proofs are in the appendix.

2 The model

We augment a basic incomplete contracting setting à la Aghion and Bolton (1992) with ex-ante negotiation of accounting measurement rules. A penniless borrower-manager seeks funding for the set-up costs \( K \) of his new project at date 0. At this stage, the manager is facing identical lenders and thus has all the bargaining power. The manager makes a take-it-or-leave-it offer to a lender who accepts the offer that makes the lender break-even. This defines the lender’s individual rationality constraint. Due to the lender’s price protection, the manager’s expected payoffs at date 0 is equal to the expected social surplus or the (ex-ante) firm value. We thus use these three terms interchangeably. We sometimes refer to the manager as “he” and the lender as “she” for convenience.

If the project is funded, the state \( \theta \) is publicly realized and observed at date 1. State \( \theta \) can be interpreted as the project’s underlying economic profitability. It is either good or bad, i.e., \( \theta \in \{G, B\} \), with a common prior \( \Pr(\theta = G) = q \). After observing the state \( \theta \), the project can be either continued (kept at status quo) or restructured at date 2. The decision to continue is denoted as \( a = 1 \) and to restructure as \( a = 0 \), i.e., \( a \in \{0, 1\} \). The project’s stochastic payoffs, realized at date 3, consist of both cash flows and non-pledgeable private benefit to the manager. Both components of the payoffs are jointly determined by the state \( \theta \) and the action \( a \). Specifically, if it is continued in state \( \theta \), the project pays out cash flow
Y with probability \( \gamma_\theta \) and 0 otherwise. If it is restructured in state \( \theta \), the project pays out cash flow \( Y \) with probability \( \gamma_\theta \) and cash flow \( y < Y \) with probability \( 1 - \gamma_\theta \). We assume that \( 1 > \gamma_G > \gamma_B \geq 0 \) so that the good and bad states are properly defined and that \( Y \) is sufficiently large so that the project is always funded. In contrast, the manager receives a private benefit \( X \) if and only if the project is continued, regardless of the state. In other words, the private benefit is not “comonotonic” with the total payoffs, an interesting case studied in Aghion and Bolton (1992). In essence, the restructuring improves the project’s cash flow at the expense of sacrificing his private benefit. The expected joint surplus, defined as the sum of the cash flows and the private benefit, is thus

\[
w(\theta, a) = \gamma_\theta Y + aX + (1 - a)(1 - \gamma_\theta)y.
\] (1)

The central friction in this incomplete contracting setting is that state \( \theta \) is ex post observable but ex ante not contractible.\(^5\) Instead, at date 1, there is a contractible signal \( s \in \{g, b\} \) that measures state \( \theta \). Naturally, we interpret this contractible signal \( s \) as an accounting measurement of the state. In other words, the underlying economic profitability is not contractible, but its accounting measurement is contractible.

To deal with the contractual incompleteness, the debt contract designed at date 0 includes a face value \( D \) and an accounting-based covenant, i.e., covenant based on \( s \). In exchange for the initial investment \( K \), the manager promises to pay back an amount up to \( D \) at date 3 and to concede the control rights at date 2 if and only if the accounting measurement is \( s = b \). After the initial assignment of the control rights at date 2, the manager and the lender may renegotiate the contract.\(^6\) Since they are locked into the bilateral relation at this stage, the manager and the lender split the bargaining power with \( \tau \in [0, 1] \) and \( 1 - \tau \), respectively. Renegotiation is costly and consumes \( \kappa \in (0, 1) \) fraction of the joint surplus.\(^7\) For simplicity,

\(^5\)As Aghion and Bolton (1992) have discussed, the assumption that \( \theta \) is publicly observed ex-post is “mostly for convenience since it allows us to abstract away from issues of bargaining under asymmetric information.” (page 477)

\(^6\)As an example of such renegotiation, consider a firm that increases its estimate of valuation allowance from prior tax loss carryforward, due to expectation of increasing future sales. This results in low earnings, but everybody observes that sales are getting better so they renegotiate, even though this expectation of increasing future sales is observable but not verifiable. We thank Judson Caskey for this example.

\(^7\)In practice, renegotiation of a debt contract is not costless. In addition to direct costs such as legal fees,
we assume that the renegotiation cost is paid by the manager. After possible renegotiation, the action is taken and the project’s payouts are divided between the lender and the manager according to the (possibly renegotiated) contract.

Now we introduce our main departure from Aghion and Bolton (1992). We define the degree of contractual incompleteness as the distance between state $\theta$ and its measurement $s$, which is treated as exogenous (Aghion and Bolton (1992, p.477)). Instead, we assume that the contracting parties in our model can expand resources to improve the quality of the measurement rules.

Specifically, the default measurement rule, denoted as $R$ and referred to as the GAAP rule, has one of the two possibilities, represented by

$$\omega \in \{\omega_G, \omega_B\}.$$ 

The first possibility is that the measurement rule has no undue optimism error, but it may or may not have a false alarm error.

Denote $\omega_G \in \{0, 1\}$ to represent whether the measurement rule exhibits a false alarm error ($\omega_G = 1$) or not ($\omega_G = 0$). When $\omega_G = 0$, the measurement rule satisfies

$$\Pr(r = b|G) = \Pr(r = g|B) = 0,$$

i.e., the default measurement rule exhibits no error.

When $\omega_G = 1$,

$$\Pr(r = b|G) = 1/2 \text{ and } \Pr(r = g|B) = 0,$$

i.e., there is maximum amount of false alarm error as $r = b$ is completely uninformative. We

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renegotiation is also costly in the form of time and effort both the lender and borrower spend in understanding the proposed transactions and implications for both parties. The cost is also increasing in the dispersion of lenders. It is more costly to renegotiate a public bond contract than a syndicated loan contract, which in turn is more costly to renegotiate than a single-bank loan contract. We treat the renegotiation cost as exogenous to focus on its comparative statics. Tirole (2006) provides multiple ways to micro-found the indirect cost of renegotiation. Aghion and Bolton (1992) endogenize the cost of renegotiation from the manager’s limited wealth. When the manager doesn’t have enough wealth to pay the lender for the control rights ex-post, renegotiation fails and the project is inefficiently liquidated.
assume that

\[ \Pr(\omega_G = 1) = \beta_G, \]

in other words, the measurement rule has false alarm error with probability \( \beta_G \).

By exerting effort \( e_G \in [0, 1] \) with cost \( \frac{1}{2}e_G^2 \), the manager can acquire a perfect signal \( f_G = \omega_G \) with probability \( e_G \).

The second possibility is that it has no false alarm error, but it may or may not have an undue optimism error.

Denote

\[ \omega_B \in \{0, 1\} \]

to represent whether the measurement rule exhibits an undue optimism error \( (\omega_B = 1) \) or not \( (\omega_B = 0) \).

When \( \omega_B = 0 \), the measurement rule satisfies

\[ \Pr(r = b|G) = \Pr(r = g|B) = 0, \]

i.e., there is no undue optimism error and the default measurement rule is perfect.

When \( \omega_B = 1 \), the measurement rule satisfies

\[ \Pr(r = b|G) = 0 \text{ and } \Pr(r = g|B) = 1/2, \]

i.e., there is maximum amount of undue optimism error as \( r = g \) is completely uninformative. We assume that

\[ \Pr(\omega_B = 1) = \beta_B, \]

in other words, the measurement rule has undue optimism error with probability \( \beta_B \).

Similarly, by exerting effort \( e_B \in [0, 1] \) with cost \( \frac{1}{2}e_B^2 \), the manager can acquire a perfect signal \( f_B = \omega_B \) with probability \( e_B \).

Note that this way of modelling measurement error incorporates the possibility that the
default measurement rule contains both types of errors (i.e., $\omega_G = \omega_B = 1$). In our subsequent analysis, however, we will focus on the separate case when only one type of error is present to simplify the algebra. Our results are robust when both types of errors may exist so long as the cost of the efforts to investigate the two types of errors are separable, e.g., the total cost is $\frac{1}{2}c e^2_G + \frac{1}{2}c e^2_B$. The reason is that separation of the costs implies the separation of the choice of effort levels into two problems: choice of effort in finding out the false alarm error and in finding out the undue optimism error, corresponding to the $e^*_G$ and $e^*_B$ solution in the main model.

In case $R$ incurs measurement error, an alternative rule, denoted as $R'$, exists that measures the states perfectly. We call $R'$ a deviation (from the default measurement rule) or, equivalently, a negotiated measurement rule that will be used in debt contracting. For simplicity, we assume that the negotiated measurement rule generates a perfect measurement of the states. If the manager observes and discloses $f_i = 1$ for $i = G, B$, $R'$ will be applied, resulting in no measurement error. However, if either the manager does not observe $f_i$, or the manager observes $f_i$ but chooses not to disclose, then the default $R$ will be used in contracting, resulting in measurement errors when $f_i = 1$. We use $F_i \in \{0, 1, \phi\}$ to denote the manager’s disclosure decision.

Finally, we make two assumptions so as to assure that the control rights allocation rule is non-trivial.

Assumption 1 : $$(1 - \gamma_B)y > X > (1 - \gamma_G)y$$

Assumption 2 : $$K > y$$

We will explain below how Assumption 1 and Assumption 2 create a demand for state-contingent allocation of control rights, which we discuss below. Figure 1 summarizes the timeline of the model.
Manager exerts effort $e_i$, The debt contract is signed Possible renegotiation occurs All uncertainties (possibly) finds about $f_i$ based on either $R$ or $R'$, based on $r$ and makes disclosure depending on $F_i$, and manager’s disclosure $F_i$ decision $F_i$

Figure 1: The Time-line

3 Preliminary analysis

Before we proceed, we prepare preliminary analysis for solving the model. We first explain how Assumption 1 and Assumption 2 create a demand for state-contingent allocation of control rights.

First, the project’s total expected payoff $w(\theta, a)$ is defined in equation (1). The restructuring ($a = 0$) essentially converts the manager’s private benefit $X$ to stochastic cash flow ($y$). Assumption 1 requires that this conversion is socially optimal (maximizes the joint surplus) if and only if the state is bad, i.e., when $\theta = B$. To see this, consider the difference of the total expected payoff under restructuring versus under continuation when $\theta = B$:

$$L_B \equiv w(B, 0) - w(B, 1) = (1 - \gamma_B) y - X > 0.$$

$L_B > 0$ due to the first part of Assumption 1, which implies that it is socially optimal to restructure the project in the bad state. $L_B$ thus measures the efficiency loss in the bad state when the action deviates from the first-best.

Similarly, when $\theta = G$, the difference of the total expected payoff under continuation versus under restructuring is

$$L_G \equiv w(G, 1) - w(G, 0) = X - (1 - \gamma_G) y > 0.$$

$L_G > 0$ due to the second part of Assumption 1, which implies that it is socially optimal to
continue the project in the good state. \( L_G \) measures the efficiency loss in the good state when the action deviates from the first-best. Therefore, under Assumption 1, the socially optimal action is state contingent: \( a_{FB}^G = 1 \) and \( a_{FB}^B = 0 \). When first-best actions are applied, the firm value, which is the project’s date-0 expected payoffs net of cost \( K \), is

\[
V^{FB} = E_\theta[w(\theta, a_{FB}^\theta)] - K = \gamma Y + (1 - q) X + q(1 - \gamma_B)y - K,
\]

where \( \gamma \equiv (1 - q) \gamma_G + p \gamma_B \).

The project’s total expected payoff is divided between the manager \( w^M(\theta, a) \) and the lender under the debt contract \( D \). For a given face value, the lender’s share of the project’s expected payoff in state \( \theta \) with action \( a \), denoted as \( w^L(\theta, a) \), is

\[
w^L(\theta, a) = \gamma_D + (1 - a)(1 - \gamma_B)\min\{D, y\}.
\]

Even though the value \( D \) will be determined in equilibrium, we know that \( D \geq K \). Otherwise, the lender cannot recoup the principal \( K \). Assumption 2 then implies that \( D > y \). As a result, \( w^L(\theta, a) \) is simplified as

\[
w^L(\theta, a) = \gamma_D + (1 - a)(1 - \gamma_B)y.
\]

It can be verified that \( w^L(\theta, 0) - w^L(\theta, 1) = (1 - \gamma_B)y > 0 \). Thus, under Assumption 2, the lender always prefers restructuring, regardless of the state.

Similarly, the manager’s share of the project’s expected payoff in state \( \theta \) with action \( a \), denoted as \( w^M(\theta, a) \), is

\[
w^M(\theta, a) = w(\theta, a) - w^L(\theta, a) = \gamma_D(Y - D) + aX.
\]

It is straightforward that \( w^M(\theta, 1) - w^M(\theta, 0) = X > 0 \). Thus, under Assumption 2, the manager always prefers continuation, regardless of the state.

Collecting these results, we have the following lemma, with its proof already laid out
Lemma 1 Under Assumptions 1 and 2, the socially optimal action is to continue the project if and only if the state is good. However, regardless of the state, the manager prefers continuation while the lender prefers restructuring. That is, $a^M_G = 1$ and $a^M_B = 0$, $a^L_G = a^L_B = 1$, and $a^L_B = 0$.

The first-best state-contingent action is obtained in the absence of the fundamental friction of contractual incompleteness. If the state is contractible, then the first-best action can be contracted upon in the debt contract. Renegotiation and accounting-based allocation rule are two instruments to deal with the contractual incompleteness. In one extreme, if renegotiation ex-post is costless, then Coase Theorem (Coase (1937)) states that renegotiation leads to the first-best action without additional cost.\footnote{When renegotiation is costless, first-best can be achieved by not referring to accounting information and simply giving decision rights to either the manager or the creditor. However, if the decisions rights is (inefficiently) based on accounting information, Coarse Theorem does not apply as there is the friction that the manager may exert effort to establish an informational advantage about the suitability of accounting rules. Our results remain largely unchanged if we always use accounting-based covenants, even if renegotiation is costless.} In the other extreme, if accounting information perfectly reveals the state, that is, $\Pr(r = b | \theta = G) = \Pr(r = g | \theta = B) = 0$, then the accounting-based allocation of control rights, which gives control to the manager if and only if $r = g$, also leads to the first-best action without renegotiation. Therefore, the interaction between renegotiation and accounting-based control rights allocation arises only when renegotiation is costly and when accounting information is imperfect, the interesting case we will focus on. To simplify the algebra from now on we will assume that $\gamma_B = 0$ but our results hold for general $\gamma_B > 0$.

4 Solving for optimal $e_G$

4.1 First-best benchmark

In the first-best benchmark, any information that the manager obtains regarding $\omega_G$ is observed by the social planner to correct for possible measurement errors and thus avoid any
Table 1: Payoff for the manager and the creditor at \( t=2 \) with only false alarm errors

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( r )</th>
<th>( u_\theta^r(F_G) )</th>
<th>( v_\theta^r(F_G) )</th>
<th>( w_\theta^r(F_G) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G )</td>
<td>( g )</td>
<td>( \gamma_G(Y - D_{FG}) + X )</td>
<td>( \gamma_GD_{FG} )</td>
<td>( \gamma_GY + X )</td>
</tr>
<tr>
<td>( G )</td>
<td>( b )</td>
<td>( \gamma_G(Y - D_{FG}) + \tau(1 - \kappa)L_G )</td>
<td>( \gamma_GD_{FG} + (1 - \gamma_G)y + (1 - \tau)(1 - \kappa)L_G )</td>
<td>( \gamma_GY + X - \kappa L_G )</td>
</tr>
<tr>
<td>( B )</td>
<td>( b )</td>
<td>0</td>
<td>( y )</td>
<td>( y )</td>
</tr>
</tbody>
</table>

possible social loss from renegotiation. As shown in the appendix, \( e^F_B \) satisfies the following first-order condition:

\[
\frac{1}{2} q \beta_G \kappa L_G = c e^F_B
\]  

Intuitively, increasing effort increases the probability that the social planner is informed thus avoiding the social loss \( (\kappa L_G) \), which occurs with probability \( \frac{1}{2} q \beta_G \) (i.e., when \( \theta = G \), \( \omega_G = 1 \), and \( r = b \)). This implies that the marginal benefit is \( \frac{1}{2} q \beta_G \kappa L_G \) and that the marginal cost is \( ce_G \). The first-best effort balances the marginal benefit with the marginal cost, resulting in the following Proposition.

**Proposition 1** The first-best effort \( e^F_B \) is the unique solutions to equation (3).

### 4.2 General case

We use backward induction to solve the general case. We first solve for the payoff after possible renegotiation decision at \( t = 2 \), followed by the manager’s optimal disclosure decision at \( t = 1 \), and finally the manager’s optimal effort decision at \( t = 0 \).

#### 4.2.1 \( t=2 \): payoff after possible renegotiation

After renegotiation, we can calculate each party’s payoff, which is of course a function of manager’s disclosure \( F_G \in \{0, 1, \phi\} \). Denote the manager’s and the lender’s payoff when the state is \( \theta \), the accounting signal is \( r \) and the disclosure is \( F_G \) as \( u_\theta^r(F_G) \) and \( v_\theta^r(F_G) \), respectively, and denote \( w_\theta^r(F_G) \) as the social payoff, which is the sum of \( u_\theta^r(F_G) \) and \( v_\theta^r(F_G) \). Table 1 lists the payoff for the manager and the lender at \( t = 2 \), which we now explain.

Since there is no undue optimism error, we only have three possible combinations of \( \theta \) and \( r \) (i.e., \( \theta = B \) and \( r = g \) is impossible). When \( \theta = G \) and \( r = g \), the manager has the control rights and continuation is the socially efficient decision, resulting in no renegotiation. Since
$DF_G \leq Y$, the manager gets the expected cash flow, $\gamma_G(Y - DF_G)$, plus the private benefit $X$, whereas the lender gets the expected cash flow, $\gamma_GD_{FG}$. Similarly, when $\theta = B$ and $r = b$, the lender has the control rights and restructuring is the socially efficient decision, again resulting in no renegotiation, the lender getting $y$ and the manager getting 0 as $y < D_{FG}$.

When $\theta = G$ and $r = b$, the lender has the control rights but continuation is the socially optimal decision. Therefore there is renegotiation and the project will be continued. The manager gets the expected cash flow from restructuring, $\gamma_G(Y - DF_G)$, plus his share of the surplus from renegotiation, $\tau(1 - \kappa)L_G$. Similarly, the lender gets the expected cash flow from restructuring, $\gamma_GD_{FG} + (1 - \gamma_G)y$, plus her share of the surplus from renegotiation, $(1 - \tau)(1 - \kappa)L_G$.

Comparing the payoffs in table 1 generates the following intuitive results:

\[
\begin{align*}
  u^g_G(F_G) - u^b_G(F_G) & = X - \tau(1 - \kappa)L_G > 0, \\
  v^g_G(F_G) - v^b_G(F_G) & = -[(1 - \gamma_G)y + (1 - \tau)(1 - \kappa)L_G] < 0, \\
  w^g_G(F_G) - w^b_G(F_G) & = \kappa L_G > 0.
\end{align*}
\]

Regardless of the manager’s disclosure, the manager prefers the signal to be $g$, the lender prefers the signal to be $b$, and it is socially optimal to continue when the signal is $g$.

### 4.2.2 $t=1$: the manager’s disclosure strategy

The manager’s disclosure $F_G$ depends on his private information $f_G$ and his disclosure strategy $d_G$, i.e., $d_G$ is a mapping from $f_G \in \{0, 1, \phi\}$ to $F_G \in \{0, 1, \phi\}$. Clearly $d_G(\phi) = \phi$ and $d_G(f_G) \in \{\phi, f_G\}$. Denote the manager’s disclosure strategy as $d_G(f_G)$ and the lender’s conjecture as $\hat{\theta}_G(f_G)$, i.e., the manager chooses (lender conjectures that the manager chooses) $F_G = f_G$ with probability $d_G(f_G)$ ($\hat{\theta}_G(f_G)$) and $F_G = \phi$ with probability $1 - d_G(f_G)$ ($1 - \hat{\theta}_G(f_G)$). In equilibrium, the conjecture has to be the same as the actual choice, i.e., $\hat{\theta}_G(f_G) = d_G(f_G) \forall f_G$.

To solve for the manager’s optimal disclosure strategy, we need to solve for the manager’s payoff as a function of his disclosure strategies, which requires solving for the face value of the
debt as a function of his disclosure strategies. Conditional on \( f_G = \phi \) and conjecture of the manager’s effort \( \hat{e}_G \) and of the manager’s disclosure strategy \( \hat{d}_G(f_G) \), the lender’s expected payoff can be calculated as

\[
V_{\hat{e}_G}(\phi) = E_{\theta, s}[v_G^s(\phi, \hat{e}_G, \hat{d}_G)] = q\rho_G v^G_G(\phi) + q(1 - \rho_G)v_G^b(\phi) + (1 - q)v^b_B(\phi)
\)

where

\[
\rho_G = \Pr(g|G, \phi, \hat{e}_G, \hat{d}_G) > \frac{1}{2},
\]

which is intuitive as \( d_G = \phi \) indicates that either the manager is uninformed or knows that \( f_G = 0 \) (which results in \( \Pr(g|G) > \frac{1}{2} \)), or knows that \( f_G = 1 \) (which results in \( \Pr(g|G) = \frac{1}{2} \)).

To understand equation (4), note that in the absence of disclosure, the lender knows that with probability \( 1 - q \), \( \theta = B \) and \( r = b \) as there is no undue optimism error (and thus \( \Pr(r = b, \theta = B|\phi) = \Pr(r = b|\theta = B, \phi) \Pr(\theta = B|\phi) = 1 \times (1 - q) = 1 - q \)); with probability \( q\rho_G \), \( \theta = G \) and \( r = g \) (as \( \Pr(r = g, \theta = G|\phi) = \Pr(r = g|\theta = G, \phi) \Pr(\theta = G|\phi) = \rho_G q \)); and with probability \( q(1 - \rho_G) \), \( \theta = G \) and \( r = b \) (as \( \Pr(r = b, \theta = G|\phi) = \Pr(r = b|\theta = G|\phi) = (1 - q)q \)).

In equilibrium the lender breaks even, resulting in

\[
q\gamma_G D_\phi + (1 - q) y + q(1 - \rho_G)(v_G^b(\phi) - v^a_G(\phi)) = K,
\]

or, equivalently,

\[
D_\phi = \frac{K - (1 - q)y - q(1 - \rho_G)(v_G^b(\phi) - v^a_G(\phi))}{q\gamma_G}
\)

When the firm makes disclosures, i.e., \( d_G = 1 \) or 0, then either there is no error or the false alarm error is corrected, implying that the accounting report is now perfect, i.e., with probability \( q \) we have \( \theta = G \) and \( r = g \) and with probability \( 1 - q \) we have \( \theta = B \) and \( r = b \).
Therefore,

\[
V_{\tilde{c}_G}(i) = E_{\theta,s}[v^s_G|i, \tilde{c}_G, \tilde{d}_G] \\
= q v^G_G(i) + (1 - q) v^B_B(i) \\
= K,
\]

resulting in

\[
D_i = \frac{K - (1 - q)y}{q \gamma G} > y
\]

and

\[
D_i > D_\phi
\]

for \( i = 0, 1 \). Intuitively, not disclosing implies that it is possible that the accounting system may generate false alarm errors, i.e., \( \theta = G \) and \( r = b \). Since the lender has the control rights, she can extract surplus from renegotiation and get a higher payoff relative to the case when she does not, with the surplus being the product of the probability that \( \theta = G \) and \( r = b \), \( q (1 - \rho_G) \), and the lender’s extra payoff, \( v^B_G(\phi) - v^B_B(\phi) \). This results in the lender willing to accept a lower interest rate upfront, i.e., \( D_i > D_\phi \).

Given the lender’s choice of \( D \), we now look at the manager’s disclosure strategy when observing \( f_G \). We write the manager’s payoff as \( U_f(F) \), where \( f \) is his signal and \( F \) is his disclosure.

If the manager observes \( f_G = 1 \) and discloses it, his payoff will be

\[
U_1(1) = qu^G_G(1) + (1 - q) u^B_B(1) \\
= q(\gamma_G Y + X) + (1 - q)y - K.
\]

This expression is intuitive, as when the manager discloses, the error is corrected, and the manager’s payoff is the first-best social payoff minus the lender’s expected payoff, i.e., \( K \), as the lender breaks even.
If the manager does not disclose, his payoff will be

\[
U_1(\phi) = q\left(\frac{1}{2} u_G^g(\phi) + \frac{1}{2} u_B^g(\phi)\right) + (1 - q) u_B^b(\phi) \\
= U_1(1) - q(1 - \rho_G)(w_G^g(\phi) - w_G^b(\phi)) - q(\rho_G - \frac{1}{2})(u_G^g - u_G^b) < 0,
\]

as it can be shown that \(\rho_G > \frac{1}{2}\). Therefore, \(U_1(\phi) < U_1(1)\) so it is a dominant strategy for the manager to disclose \(f_G = 1\) for any conjecture \(\hat{d}_G(1)\), resulting in \(d^*_G(1) = 1\). Intuitively, if the manager does not disclose \(f_G = 1\), he suffers two types of losses: first, with probability \(q(1 - \rho_G)\), not disclosing results in \(\theta = G\) but \(r = b\), generating a social loss from renegotiation of \(w_G^g(\phi) - w_G^b(\phi)\) and the burden falls on the manager as the lender breaks even; second, not disclosing results in only \(\frac{q}{2}\) probability that \(\theta = G\) and \(r = g\) (relative to the \(q\rho_G\) probability upon disclosing), generating a smaller payoff for the manager as \(u_G^g > u_G^b\).

If the manager observes \(f_G = 0\) and discloses it, his payoff will again be

\[
U_0(0) = qu_G^g(0) + (1 - q)u_B^b(0) \\
= q(\gamma_G Y + X) + (1 - q)y - K \\
= U_1(1),
\]

as there is no error and the manager’s payoff is the first-best social payoff minus the investment.

If the manager chooses not to disclose, his payoff can be shown to be

\[
U_0(\phi) = qu_G^g(\phi) + (1 - q)u_B^b(\phi) \\
= U_0(0) + q (1 - \rho_G) (v_G^b(\phi) - v_G^g(\phi)) \\
> U_0(0)
\]

Therefore, \(U_0(\phi) > U_0(0)\) so it is a dominant strategy for the manager to not disclose \(f_G = 0\) for any conjecture \(\hat{d}_G(0)\), resulting in \(d^*_G(0) = 0\). Intuitively, since \(D_0 > D_\phi\), the manager has the incentive to withhold disclosure, reduce the interest rate and thus obtain a
higher payoff, as the manager knows that the event of $\theta = G$ and $r = b$ will not occur. As can be seen from equation (6), the increased payoff is due to the expected lower payment to the lender, i.e., $q_G(D_0 - D_\phi)$.

We summarize the results in the following Proposition.

**Proposition 2** The manager always discloses $f_G = 1$ and withholds $f_G = 0$, i.e., $d^*_G(1) = 1$ and $d^*_G(0) = 0$.

### 4.2.3 $t=0$: the manager’s effort decision

Given that $d^*_G(1) = 1$ and $d^*_G(0) = 0$, we can simplify the expression of $\rho_G$ to

$$\rho_G(e_G) = 1 - \frac{1}{2} \beta_G(1 - e_G) \frac{1}{1 - \hat{\beta}_G e_G}.$$  

The manager’s payoff from exerting effort $e_G$ and following the optimal disclosure strategy is then

$$\beta_G[e_G U_1(1) + (1 - e_G)U_\phi(\phi)] + (1 - \beta_G)[e_G U_0(\phi) + (1 - e_G)U_\phi(\phi)]$$

$$- \frac{1}{2} \hat{e}_G^2,$$

i.e., with probability $\beta_G e_G$, the manager discovers and discloses $f_G = 1$, resulting in a payoff of $U_1(1)$; with probability $\beta_G(1 - e_G)$, the manager does not discover $f_G$ and thus does not disclose, resulting in a payoff of $U_\phi(\phi)$; with probability $(1 - \beta_G)e_G$, the manager discovers but withholds $f_G = 0$, resulting in a payoff of $U_0(\phi)$; with probability $(1 - \beta_G)(1 - e_G)$, the manager does not discover $f_G$ and thus does not disclose, resulting in a payoff of $U_\phi(\phi)$.

Note that $U_1(1)$, $U_0(\phi)$ and $U_\phi(\phi)$ are independent of actual effort choice, resulting in the first order condition being

$$\beta_G(U_1(1) - U_\phi(\phi)) + (1 - \beta_G)[U_0(\phi) - U_\phi(\phi)] = ce_G^*.$$  

Intuitively, exerting effort both increases the payoff when $\omega_G = 1$ (as the manager will disclose) and the payoff when $\omega_G = 0$ (as the manager will withhold).
Inserting the expressions of $U_1(1)$, $U_0(\phi)$ and $U_\phi(\phi)$ and after some tedious algebra we have
\[ ce^*_G = ce^*_G + q\beta_G(\rho_G - \frac{1}{2})(v_G^b(\phi) - v_G^b(\phi)) > ce^*_G, \tag{7} \]

implying that $e^*_G > e^*_{FB}$. Intuitively, exerting effort has a higher marginal benefit than that in the first-best case. The reason is that finding $\omega_G = 0$ and not disclosing has no social value but allows the manager to exploit the low face value from the lender upon no disclosure. This low face value is due to the lender expecting no disclosure to be an indication of possibly extracting value from the manager, when false alarm error occurs and they have the control rights (i.e., $\omega_G = 1$, $\theta = G$ and $r = b$). Ex-ante the manager expects the face value to be lower by $q\beta_G(\rho_G - \frac{1}{2})(v_G^b(\phi) - v_G^b(\phi))$, i.e., the marginal benefit of exerting effort is higher than the first-best by $q\beta_G(\rho_G - \frac{1}{2})(v_G^b(\phi) - v_G^b(\phi))$, which is the payoff the lender expects to gain in the absence of any disclosure. To see the intuition of this expression, note that conditional on no disclosure, the lender expects that false alarm error occurs with probability $q\beta_G(\rho_G - \frac{1}{2})$. On the other hand, without any strategic considerations, the lender expects that false alarm error occurs with probability $q\beta_G(\rho_G - \frac{1}{2})$ conditional on no disclosure. Therefore, the lender’s gain is $q\beta_G(\rho_G - \frac{1}{2})(v_G^b(\phi) - v_G^b(\phi))$.

We summarize the results in the following Proposition.

**Proposition 3** The manager chooses $e^*_G$ as the solution to equation (7). It is always true that $e^*_G > e^*_{FB}$.

### 4.3 Comparative statics

We now explore the comparative statics of both $e^*_G$ and the difference between $e^*_G$ and $e^*_{FB}$.

The comparative statics of $e^*_G$ is summarized in the following corollary.

**Corollary 1** $e^*_G$ is ambiguous in $\gamma_G$, $Y$, $X$, $y$ and $\beta_G$ but increases in $\beta_G$ when $\beta_G$ is very small; increases in $q$ and $\kappa$ and decreases in $\tau$.

The ambiguity of the comparative statics of $e^*_G$ with respect to $\gamma_G$ and $X$ comes from the ambiguity of the comparative statics of $e^*_G$ with respect to $L_G = X - (1 - \gamma_G)y$, i.e.,
the social loss from restructuring relative to continuation when the state is good. Intuitively, increasing $L_G$ has two opposing effects on effort: first, it increases the social loss and thus motivating more effort. This effect increases with $\kappa$. Second, it reduces the lender’s expected surplus from renegotiation, which in turn does not reduce the face value very much, resulting in less effort for the manager to find $\omega_G = 0$ and not disclose it (and thus enjoy the lower face value). This effect increases with $1 - \kappa$ as $1 - \kappa$ represents the potential expected surplus from renegotiation. When $\kappa$ is small, the second effect dominates whereas when $\kappa$ is large, the first effect dominates. Therefore the overall effect is ambiguous.

Similarly, the comparative statics of $e^*_G$, with respect to $y$ is ambiguous, as increasing $y$ reduces the social loss from renegotiation.

High likelihood of state being $G$ increases $e^*_G$ as 1) it is more likely that a false alarm error occurs and thus increases social losses and increases the effort level; and 2) the manager is motivated to increase effort, find $\omega_G = 0$ and not disclose it (and thus enjoy the lower face value).

Increasing $\beta_G$ also has two opposing effects on $e^*_G$: first, it increases social loss thus motivating more effort; second, it has an ambiguous effect on the expected amount of information asymmetry and thus has an ambiguous effect on the manager finding $\omega_G = 0$ and not disclosing it (and thus enjoying the lower face value). When $\beta_G$ is very small, increasing $\beta_G$ increases information asymmetry and thus motivates the manager to exert more effort, i.e., the two effects work in the same direction. When $\beta_G$ is very large, increasing $\beta_G$ reduces information asymmetry (i.e., without information both parties know that it is very likely the GAAP rule is not accurate) and thus the manager has less motivation to exert effort.

Higher $\kappa$ resulting in higher $e^*_G$ can be explained as follows. Higher $\kappa$, while increasing social loss thus motivating effort, also reduces the expected surplus that lender can extract from renegotiation and dampens the reduction of face value that the manager enjoys. In this case, the first effect always dominates because both are linear functions of $\kappa$ and social loss has a higher sensitivity to $\kappa$ as the change in social loss is larger than the change in the lender’s extraction from renegotiation.

Higher $\tau$ decreases $e^*_G$ as if the manager extracts more surplus, then the lender extracts
less from renegotiation, therefore the reduction of face value is less, which reduces the value from the manager to increase effort, find $\omega_G = 0$ and not disclose it (and thus enjoy the lower face value).

We now look at the comparative statics of $e^*_G - e^{FB}_G$, which can be summarized in the following corollary.

**Corollary 2** $e^*_G - e^{FB}_G$ is ambiguous in $\gamma_G, Y, X, y, \kappa$ and $\beta_G$ but increases in $\beta_G$ when $\beta_G$ is very small; increases in $q$ and decreases in $\tau$.

The comparative statics of $e^*_G - e^{FB}_G$ is similar to that of $e^*_G$, except for that of $\kappa$. The intuition for the comparative statics of $e^*_G - e^{FB}_G$ is also similar to that of $e^*_G$, except for two aspects.

First, the motivation of increasing effort to avoid social loss is not in the difference as this motivation also drives the first-best effort; second, if some variable $x$ causes effort to increase (decrease), this will amplify (diminish) the effort difference between $e^*_G$ and $e^{FB}_G$. The reason is that the additional benefit from being able to find $\omega_G = 0$ and not disclose it (and thus enjoy the lower face value) increases in $e^*_G$ as higher $e^*_G$ is more likely to increase such benefit. Therefore, from the chain rule

$$
\frac{d(e^*_G - e^{FB}_G)}{dx} = \frac{\partial(e^*_G - e^{FB}_G)}{\partial x} + \frac{\partial(e^*_G - e^{FB}_G)}{\partial e^*_G} \frac{de^*_G}{dx}.
$$

Since

$$
\frac{\partial(e^*_G - e^{FB}_G)}{\partial e^*_G} > 0,
$$

$$
\text{sgn}\left(\frac{\partial(e^*_G - e^{FB}_G)}{\partial e^*_G} \frac{de^*_G}{dx}\right) = \text{sgn}\left(\frac{de^*_G}{dx}\right),
$$

which explains the second difference.

The two differences explain the ambiguity of $e^*_G - e^{FB}_G$ with respect to $\kappa$. Higher $\kappa$ has two effects on the difference: 1) everything else being the same, it reduces the private benefit from being able to find $\omega_G = 0$ and not disclose it as the reduction in face value is dampened by the increase in $\kappa$; second, everything else is not the same as higher $\kappa$ increases $e^*_G$ and thus
increases the chances of being able to find $\omega_G = 0$ and not disclose it and therefore increase the private benefit, when keeping everything else equal.

5 Solving for optimal $e_B$

5.1 First-best

As shown in the appendix, in the first-best case, $e_{FB}^B$ satisfies the following first-order condition:

$$\frac{1}{2}(1-q)\beta_B^L \kappa_L B = ce_{FB}^B$$

(8)

Similar to the first-best case of exerting effort to find the false alarm error, increasing effort has the marginal benefit of increasing the probability that the social planner is informed thus avoiding the social loss ($\kappa_L B$), which occurs with probability $\frac{1}{2}(1-q)\beta_B$ (i.e., when $\omega_G = 1, \theta = B$ and $r = g$) but incurs a marginal cost of $ce_{FB}^B$. The first-best effort balances the marginal benefit with the marginal cost.

We summarize the results in the following Proposition.

**Proposition 4** The first-best effort $e_{FB}^B$ is the unique solutions to equation (8).

5.2 General case

We again solve the main model through backward induction. We first solve for the payoff after possible renegotiation decision at $t = 2$, followed by the manager’s optimal disclosure decision at $t = 1$, and finally the manager’s optimal effort decision at $t = 0$.

5.2.1 $t=2$: payoff after possible renegotiation

Table 2 lists the payoff for the manager and the lender at $t = 2$, which we explain now.

Since there is no false alarm error, we again have three possible combinations of $\theta$ and $r$ (i.e., $\theta = G$ and $r = b$ is impossible). When $\theta = G$ and $r = g$, the manager has the control rights and continuation is the socially efficient decision, resulting in no renegotiation. Since $D_{FB} \leq Y$, the manager gets the expected cash flow, $\gamma_G(Y - D_{FB})$, plus the private benefit
Table 2: Payoff for the manager and the creditor at t=2 with only undue optimism errors

<table>
<thead>
<tr>
<th>θ</th>
<th>r</th>
<th>$u^r_\theta(F_B)$</th>
<th>$v^r_\theta(F_B)$</th>
<th>$w^r_\theta(F_B)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>g</td>
<td>$\gamma_G(Y - D_{FB}) + X$</td>
<td>$\gamma_G D_{FB}$</td>
<td>$\gamma_G Y + X$</td>
</tr>
<tr>
<td>B</td>
<td>g</td>
<td>$X + \tau(1 - \kappa)L_B$</td>
<td>$(1 - \tau)(1 - \kappa)L_B$</td>
<td>$y - \kappa L_B$</td>
</tr>
<tr>
<td>B</td>
<td>b</td>
<td>0</td>
<td>$y$</td>
<td>$y$</td>
</tr>
</tbody>
</table>

Comparing the payoffs in table 2 generates the intuitive results that regardless of the manager’s disclosure, the manager prefers the signal to be $g$, the lender prefers the signal to be $b$, and it is socially optimal to continue when the signal is $g$:

$$u^g_B(F_B) - u^b_B(F_B) = X + \tau(1 - \kappa)L_B > 0,$$

$$v^g_B(F_B) - v^b_B(F_B) = [-y - (1 - \tau)(1 - \kappa)L_B] < 0,$$

$$w^g_C(F_B) - w^b_C(F_B) = \kappa L_B > 0.$$

5.2.2 t=1: the manager’s disclosure strategy

Similar to the false alarm error case, the manager’s disclosure $F_B$ depends on his private information $f_B$ and his disclosure strategy $d_B$, i.e., $d_B$ is a mapping from $f_B \in \{0, 1, \phi\}$ to $F_B \in \{0, 1, \phi\}$. Clearly $d_B(\phi) = \phi$ and $d_B(f_B) \in \{\phi, f_B\}$. Denote the manager’s disclosure strategy as $d_B(f_B)$ and the lender’s conjecture as $\hat{d}_B(f_B)$. In equilibrium again we require $\hat{d}_B(f_B) = d_B(f_B) \forall f_B$.

Conditional on $f_B = \phi$ and conjecture of the manager’s effort $\hat{e}_B$ and the manager’s
disclosure strategy $\hat{d}_B(f_B)$, the lender’s expected payoff can be similarly calculated as

$$V_{\hat{c}_B}(\phi) = E_{\theta,s}[v_{B}^\phi|\phi,\hat{c}_B,\hat{d}_B]$$

$$= qv_{G}^\phi(\phi) + (1-q)v_{B}^b(\phi) - (1-q)(1-\rho_B)(v_{B}^b(\phi) - v_{B}^b(\phi))$$

$$= K,$$

where

$$\rho_B \equiv \Pr(b|B,\phi,\hat{c}_B,\hat{d}_B) > \frac{1}{2}.$$  

In equilibrium, the lender breaks even, resulting in

$$V_{\hat{c}_B}(\phi) = K,$$

and thus

$$D_{\phi} = \frac{K - (1-q)y + (1-q)(1-\rho_B)(v_{G}^b(\phi) - v_{G}^b(\phi))}{q\gamma_G} > y \quad (10)$$

When the manager discloses, then either there is no error or the undue optimism error is corrected and we again have

$$V_{\hat{c}_B}(i) = E_{\theta,s}[v_{B}^\phi|i,\hat{c}_B,\hat{d}_B]$$

$$= qv_{G}^\phi(i) + (1-q)v_{B}^b(i)$$

$$= K,$$

resulting in

$$D_{\phi} > D_i = \frac{K - (1-q)y}{q\gamma_G} > y$$

for $i = 0, 1$.  

Note that relative to the case with false alarm errors, $D_{\phi} > D_i$. The reason is that the manager has the control rights and can extract a higher payoff when undue optimism error occurs (i.e., $\theta = B$ and $r = g$) relative to when undue optimism error does not occur (i.e.,
\( \theta = B \) and \( r = b \). As a result,

\[
V_{EB}(\phi) < qu_{G}(\phi) + (1 - q)v_{B}(\phi).
\]

Since the lender needs to break even, the lender “price-protects” and endogenously increases the interest rate to compensate for this loss. Therefore \( D_{\phi} > D_{i} \).

We now look at the manager’s disclosure strategy when observing \( f_{B} \). Again we write the manager’s payoff as \( U_{f}(F) \), where \( f \) is his signal and \( F \) is his disclosure.

If the manager observes \( f_{B} = 0 \) and discloses it, the GAAP rule has no error and his payoff will be

\[
U_{0}(0) = qu_{G}(0) + (1 - q)v_{B}(0)
\]

\[
= q(\gamma_{G}Y + X) + (1 - q)y - K,
\]

i.e., the first-best expected total payoff from the project minus the lender’s expected payoff, \( K \).

If the manager does not disclose, his payoff can be calculated to be

\[
U_{0}(\phi) = qu_{G}(\phi) + (1 - q)v_{B}(\phi)
\]

\[
= U_{0}(0) - (1 - q)(1 - \rho_{B})(v_{B}(\phi) - v_{B}(\phi))
\]

\[
< U_{0}(0).
\]

Therefore, \( U_{0}(\phi) < U_{0}(0) \) so it is dominant for the manager to disclose \( f_{B} = 0 \) for any conjecture \( \hat{d}_{B}(0) \), resulting in \( d_{B}(0) = 1 \). Intuitively, when not disclosing, the lender will demand a higher interest rate. Knowing that there is no chance that he will extract extra payoffs (as the manager knows that \( \theta = B \) and \( r = g \) is impossible), the manager chooses to disclose \( f_{B} = 0 \) to lower the interest rate.

If the manager observes \( f_{B} = 1 \) and discloses it, the undue optimism error is corrected.
His payoff will be

\[
U_1(1) = qu^g_G(1) + (1 - q)u^b_B(1) \\
= q(\gamma Y + X) + (1 - q)y - K \\
= U_1(0),
\]

i.e., the same payoff as no error.

If the manager does not disclose, his payoff can be shown to be

\[
U_1(\phi) = qu^g_G(\phi) + (1 - q)[\frac{1}{2}u^g_B(\phi) + \frac{1}{2}u^b_B(\phi)] \\
\equiv U_1(1) + (1 - q)\Delta(\hat{d}_B(1)),
\]

where

\[
\Delta(\hat{d}_B(1)) \\
= \frac{1}{2}(u^g_B(\phi) - u^b_B(\phi)) - \frac{1}{2}\left(1 - \rho_B(\hat{d}_B(1))\right)(v^b_B(\phi) - v^g_B(\phi)).
\]

By not disclosing \( f_B = 0 \) and thus not correcting errors, the manager incurs a benefit and a cost. The benefit is that the manager knows that \( \theta = B \) and \( r = g \) will occur with probability \( \frac{1}{2} \), resulting in a benefit of \( \frac{1}{2}(u^g_B(\phi) - u^b_B(\phi)) \), i.e., the first term of \( \Delta(\hat{d}_B(1)) \). The cost is that \( D_\phi \) is larger than \( D_{-1} \) as the lender price protects herself, resulting in an expected cost of \( \frac{1}{2}\left(1 - \rho_B(\hat{d}_B(1))\right)(v^b_B(\phi) - v^g_B(\phi)) \), i.e., the second term of \( \Delta(\hat{d}_B(1)) \).

Since the sign of \( \Delta(\hat{d}_B(1)) \) is ambiguous, all three possibilities (i.e., \( d^*_B(1) = 0 \), \( d^*_B(1) = 1 \) and \( d^*_B(1) \in [0, 1] \)) can occur. We summarize all three cases in the next Proposition, while leaving all the technical details in the appendix.

**Proposition 5** The manager always disclose \( f_B = 0 \), i.e., \( d^*_B(0) = 1 \). The manager withholding \( f_B = 1 \), i.e., \( d^*_B(1) = 0 \) is the unique equilibrium if equation (21) holds. The manager disclosing \( f_B = 1 \), i.e., \( d^*_B(1) = 1 \) is the unique equilibrium if equation (20) holds. Otherwise, i.e., when equation (22) is satisfied, there is a unique mixed strategy equilibrium \( d^*_B(1) \in (0, 1) \).
that satisfies equation (23). All referred equations are in the appendix.

Proposition 5 states that all three possibilities of $d_B(1)$ can occur. Specifically, it can be shown that $\Delta(\hat{d}_B(1))$ is increasing in $\hat{d}_B(1)$, i.e., if the lender believes that the manager is more likely to disclose $f_B = 1$, the lender will ask for less price protection, resulting in lower $D_\phi$, which in turn makes not disclosing more attractive.

Therefore, when the benefit from non-disclosure is higher than the cost even when $\hat{d}_B(1) = 0$, $d_B(1) = 0$ is the unique equilibrium. When the benefit from non-disclosure is lower than the cost even when $\hat{d}_B(1) = 1$, $d_B(1) = 1$ is the unique equilibrium. In between, neither $d_B(1) = 0$ nor $d_B(1) = 1$ is the unique equilibrium, resulting in a unique mixed strategy equilibrium $d_B(1) \in (0, 1)$.

5.2.3 $t=0$: the Manager’s effort decision

We already know that $d_B(0) = 1$. Proposition 5 identifies the conditions for $d_B(1) = 0$, $d_B(1) = 1$ and $d_B(1) \in (0, 1)$. We now solve $e_B^*$ in three steps. In the first step, we solve for the optimal $e_B^*$ given the three equilibrium possibilities of $d_B(1)$. In the second step, we identify conditions on exogenous parameters that satisfy the equations for the three equilibrium possibilities of $d_B(1)$, as identified in Proposition 5.

Step 1: find the optimal $e_B^*$ given various possibilities of $d_B(1)$ When $d_B(1) = 1$, first order condition for $e_B^*$, which we denote as $e_B^{*1}$, results in

$$ce_B^{*1} = \beta_B[U_1(1) - U_\phi(\phi)] + (1 - \beta_B)[U_0(0) - U_\phi(\phi)]$$

$$= \frac{(1 - q)\beta_B}{2} KL_B$$

$$= ce_B^{FB}. \quad (11)$$

Intuitively, when $d_B(0) = d_B(1) = 1$, there is no strategic withholding of information and we are back to the first-best world.

When $d_B(1) = 0$, the first order condition for $e_B^*$, which we denote as $e_B^{*2}$, results in
\[
\begin{align*}
ce_B^* &= ce_B^{FB} + \frac{1}{2}(1-q)\beta_B \{X + \tau(1-\kappa)L_B - \frac{\beta_B}{1-e_B^*(1-\beta_B)}[y - (1-\tau)(1-\kappa)L_B]\} \\
&\quad + \frac{(1-q)\beta_B}{2} \frac{(1-\beta_B)e_B^*[1-d_B^*(1)]}{1-e_B^*(1-\beta_B)}(v_B^b(\phi) - v_B^g(\phi)) \\
&= ce_B^{FB} + \frac{1}{2}(1-q)\beta_B \{X + \tau(1-\kappa)L_B - \frac{\beta_B}{1-e_B^*(1-\beta_B)}[y - (1-\tau)(1-\kappa)L_B]\} \\
&\quad + \frac{(1-q)\beta_B}{2} \frac{(1-\beta_B)e_B^*[1-d_B^*(1)]}{1-e_B^*(1-\beta_B)}(v_B^b(\phi) - v_B^g(\phi)) \\
&= ce_B^{FB} + \frac{1}{2}(1-q)\beta_B \{X + \tau(1-\kappa)L_B - \frac{\beta_B}{1-e_B^*(1-\beta_B)}[y - (1-\tau)(1-\kappa)L_B]\} \\
&\quad + \frac{(1-q)\beta_B}{2} \frac{(1-\beta_B)e_B^*[1-d_B^*(1)]}{1-e_B^*(1-\beta_B)}(v_B^b(\phi) - v_B^g(\phi)) \\
&> ce_B^{FB}
\end{align*}
\]

Note that by assuming \(d_B^*(1) = 0\) holds, we know that equation (21) holds at \(e_B^*\), implying that
\[
X + \tau(1-\kappa)L_B \geq \frac{\beta_B}{1-e_B^*(1-\beta_B)}[y - (1-\tau)(1-\kappa)L_B]
\]

Therefore, \(e_B^* > e_B^{FB}\). Intuitively, when \(d_B^*(1) = 0\), the benefit from not disclosing is so high that the manager always withholds information when \(f_B = 1\). While the lender anticipates this and increases \(D\phi\) to price-protect, the protection is incomplete because of the information asymmetry between the lender and the manager. This results in the manager exerting a higher than socially optimal effort to try to find and withhold the information that \(f_B = 1\).

When \(d_B^*(1) \in (0, 1)\), the first order condition for \(e_B^*\), which we denote as \(e_B^*\), results in
\[
\begin{align*}
\ce_B^* &= ce_B^{FB} + \frac{1}{2}(1-q)\beta_B \{X + \tau(1-\kappa)L_B - \frac{\beta_B}{1-e_B^*(1-\beta_B)}[y - (1-\tau)(1-\kappa)L_B]\} \\
&\quad + \frac{(1-q)\beta_B}{2} \frac{(1-\beta_B)e_B^*[1-d_B^*(1)]}{1-e_B^*(1-\beta_B)}(v_B^b(\phi) - v_B^g(\phi)) \\
&> ce_B^{FB}
\end{align*}
\]

Comparing equations (12) and (13) also results in \(e_B^* < e_B^*\), which we prove formally in the appendix. Intuitively, given that the manager sometimes withholds \(f_B = 1\), the manager still has an incentive to exert excessive effort to find it out. However, given that the benefit is smaller than that when the manager always withholds, such incentive is also smaller.

We summarize the results in the following Proposition.

**Proposition 6** When equation (20) holds, the optimal solution \(e_B^* = e_B^{FB}\) satisfies equation (11); when equation (21) holds, the optimal solution \(e_B^* > e_B^{FB}\) satisfies equation (12); when equation (23) holds, the optimal solution \(e_B^* \in (e_B^{FB}, e_B^*)\) satisfies equation (13).
Step 2: Identify conditions for various possibilities of \(d_B^*(1)\) and complete the characterization of \(e^*_B\). We now need to verify whether the solutions \(e^*_B1\), \(e^*_B2\) and \(e^*_B3\) are indeed consistent with the conditions that determine the optimal disclosure decision. We leave the technical details in the appendix and summarize the results below.

**Proposition 7** The equilibrium can be characterized as follows.

First, when \(X \geq \overline{X}\), \(e^*_B = e^*_B2\):

Second, when \(\beta_By \leq X < \overline{X}\), \(e^*_B = \begin{cases} e^*_B2 \text{ if } \kappa \leq \kappa_0 \\ e^*_B3 \text{ if } \kappa > \kappa_0 \end{cases}\)

Third, when \(X < \beta_By\), \(e^*_B = \begin{cases} e^*_B2 \text{ if } \kappa \leq \kappa_0 \\ e^*_B3 \text{ if } \kappa_0 < \kappa < \kappa_1 \\ e^*_B1 \text{ if } \kappa \geq \kappa_1 \end{cases}\)

We can also combine all three cases as \(e^*_B = \begin{cases} e^*_B2 \text{ if } \kappa \leq \min(\kappa_0, 1) \\ e^*_B3 \text{ if } \kappa \in (\kappa_0, \min(\kappa_1, 1)) \end{cases}\), where \(\overline{X}\) is defined in the appendix.

Proposition 7 states that the degree of excessive effort depends on two parameters, \(X\) and \(\kappa\). Recall that the degree of excessive effort depends on the manager’s disclosure strategies when \(f_B = 1\), which in turn depends on the net benefit of withholding. The net benefit comes from the manager’s ability to extract surplus when \(\theta = B\) and \(r = g\), given that the manager has control and would like to continue. This benefit is increasing in the private benefit from continuation, \(X\) and decreasing in the efficiency loss from renegotiation, \(\kappa\). Thus, when \(X\) is sufficiently large, the manager always withholds, resulting in \(e^*_B = e^*_B2\). When \(X\) is not large enough, then always withholding is still optimal if \(\kappa\) is sufficiently small. When \(X\) further decreases and \(\kappa\) further increases, we would have partial withholding and eventually no withholding to be optimal.

Figure 1 below provides an illustration of the optimal effort when \(X < \beta_By\) so that \(e^*_B\) has three segments. Note that this graph is not inconsistent with Proposition 6 as \(e^*_B2 > e^*_B3 > e^*_BFB\) holds when holding all other exogenous parameters (e.g., \(\kappa\)) fixed. In the figure, \(e^*_B\) achieves \(e^*_B2\), \(e^*_B3\) and \(e^*_BFB\) on different levels of \(\kappa\).
5.2.4 Comparative statics

We now explore the comparative statics of $e^*_B$, $e^*_B$, and their difference from first-best, i.e., $e^*_B - e^*_B$ and $e^*_B - e^*_B$.

We first look at the comparative statics of $e^*_B$, which is summarized in the following corollary.

**Corollary 3** $e^*_B$ is ambiguous in $B$ but increases in $B$ when $B$ is very small; increases in $X, y, k$ and $\tau$ and decreases in $q$.

Intuitively, increasing $L_B = y - X$ has two ambiguous effects on effort: first, it increases the social loss and thus motivating more effort. Second, it reduces the manager’s expected surplus from renegotiation, which in turn does not increase the face value from non-disclosure very much, resulting in less effort for the manager to find $\omega_B = 1$ and not disclose it (and thus enjoy the surplus from renegotiation). Overall the second effect dominates. Therefore $e^*_B$ decreases in $L_B$ and thus increases in $X$.

Intuitively, increasing $y$ has two effects on effort: first, it increases the social loss, thus motivating more effort. Second, it increases the manager’s expected surplus from renegotiation, which in turn increase the face value from non-disclosure, resulting in more effort for the manager to find $\omega_B = 1$ and not disclose it (and thus enjoy the surplus from renegotiation).
The two effects move in the same direction, resulting in $e_{B2}^*$ increasing in $y$.

The intuition for $e_{B2}^*$ to be decreasing in $q$ is that high likelihood of state being $B$ increases effort as 1) it is more likely that an undue optimism error occurs and thus increases social losses; and 2) the manager is motivated to increase effort, find $\omega_B = 0$ and not disclose it (and thus enjoy the surplus from renegotiation) as the probability that $\omega_B = 0$ increases.

Increasing $\beta_B$ has two opposing effects on $e_{B2}^*$: first, it increases social loss thus motivating more effort; second, it has an ambiguous effect on the expected amount of information asymmetry and thus has an ambiguous effect on finding $\omega_B = 0$ and not disclose it (and thus enjoy the surplus from renegotiation). When $\beta_B$ is very small, increasing $\beta_B$ increases information asymmetry and thus motivates the manager to exert more effort, i.e., the two effects work in the same direction. When $\beta_B$ is very large, increasing $\beta_B$ reduces information asymmetry (i.e., without information both parties know that it is very likely the GAAP rule is not accurate) and thus the manager has less motivation to exert effort.

Higher $\kappa$, while increasing social loss thus motivating effort, also reduces the expected surplus that the manager can extract from renegotiation and thus dampens the reduction of face value that the manager enjoys. The first effect always dominates when the manager cannot extract the whole surplus (i.e., $\tau < 1$), resulting in $e_{B2}^*$ increasing in $\kappa$.

The reason that $e_{B2}^*$ increases with $\tau$ is that the more surplus the manager can extract from renegotiation, the higher face value the lender asks for non-disclosure, and the more incentive for the manager to exert effort, find $\omega = 1$ and disclose it.

We next look at the comparative statics of $e_{B2}^* - e_{FB}^*$, which is summarized in the following corollary.

**Corollary 4** $e_{B2}^* - e_{FB}^*$ is ambiguous in $y, \kappa$ and $\beta_B$ but increases in $\beta_B$ when $\beta_B$ is very small; increases in $\tau$ and $X$ and decreases in $q$.

Compared with the comparative statics of $e_{B2}^*$, $e_{B2}^* - e_{FB}^*$ is now ambiguous on $y$ and $\kappa$. The reason is that $e_{FB}^*$ is also increasing in $y$ and $\kappa$ as both higher $y$ and higher $\kappa$ increases the social loss from renegotiation, therefore increasing the marginal benefit of exerting effort. Since $e_{FB}^*$ is not affected by $\tau$, the comparative statics of $e_{B2}^* - e_{FB}^*$ with respect to $\tau$ stays
the same. While $e^*_B$ is also decreasing in $q$ as lower probability that $\theta = B$ reduces the marginal benefit of exerting effort, $e^*_{B2}$ decreases even faster than $e^*_B$, as the manager’s excessive incentive of finding $\omega_B$ reduces when the probability that $\theta = B$ reduces. Similarly, while $e^*_B$ is also increasing in $X$ as higher social loss increases the marginal benefit of exerting effort, this increase is less than the private benefit (as the additional benefit accrues disproportionately more to the manager), resulting in $e^*_B - e^*_B$ increasing in $X$.

We now look at the comparative statics of $e^*_B$ and $e^*_B - e^*_B$.

In the appendix, we can solve for $e^*_B$ as

$$e^*_B = \frac{(1 - q)(1 - \beta_B)[X + \tau(1 - \kappa)L_B]}{2c}$$

It is straightforward to take the derivatives of $e^*_B$ with respect to all exogenous variables. The comparative statics of $e^*_B$ is summarized in the next corollary.

**Corollary 5** $e^*_B$ increases in $X, y, \tau$ and decreases in $q, \beta_B$ and $\kappa$.

The results are all very intuitive. For example, an increase in $X$ increases the lender’s concern of manager extracting surplus from negotiation, resulting in an increase in face value and a higher incentive to find $\omega_B = 0$ and disclose it so as to obtain a lower face value from the lender. Similarly, an increase in $\beta_B$ reduces the likelihood of finding $\omega_B = 0$ and disclosing, thus reducing the marginal benefit of investing in effort.

The comparative statics of $e^*_B - e^*_B$ is summarized in the following corollary.

**Corollary 6** $e^*_B - e^*_B$ increases in $\tau$ and $X$, decreases in $q, \beta_B$ and $\kappa$ and is ambiguous in $y$.

Compared with the comparative statics of $e^*_B$, $e^*_B - e^*_B$ is now ambiguous on $y$, as $e^*_B$ is also increasing in $y$. Similar to the case of $e^*_B$, $e^*_B - e^*_B$ is also increasing in $\tau$ and $X$ and decreasing in $q$. Different from the case of $e^*_B$, $e^*_B - e^*_B$ is now decreasing in $\beta_B$ and $\kappa$. The reason is that the manager withholds less in this case, which reduces the excessive private benefit of exerting effort. Therefore, the increase in $e^*_B$ due to higher $\beta_B$ and $\kappa$ dominates the increase in $e^*_B$, resulting in a decrease in $e^*_B - e^*_B$.
6 Implications

Our results provide the following implications.

First, there are more corrections for false alarm errors than corrections of undue optimism errors, implying subsequently there will be more renegotiations due to undue optimism errors. Our results provide a justification for accounting rules that generate more false alarm errors relative to undue optimism errors when errors are unavoidable: the managers have strong incentives to find out and subsequently disclose false alarm errors relative to undue optimism errors.

Second, our comparative statics results show that withholding of undue optimism errors would be more severe if the firm is less profitable, renegotiation is more costly and the conflict of interest between debtholders and shareholders is sufficiently large. We thus predict that the GAAP rules would be more likely to exhibit false alarm errors when the number of lenders in a syndicated loan is larger or for public versus private debt (as renegotiation cost will be larger); and for debt contract with more earnings-based covenants (as the conflict of interest will be higher). To the extent that more false alarm errors imply more conservative accounting, the latter prediction is consistent with Nikolaev (2010).

Thirdly, the correction of accounting errors in the debt contracts is more likely to be correcting for false alarm errors (e.g., excluding certain expense items), which is consistent with anecdotal evidence that EBIT or EBITDA are often used to replace GAAP earnings in debt contracts. To the extent that more false alarm errors imply more conservative accounting, our results can be viewed as providing a reconciliation between why GAAP earnings are conservative and adjusted earnings numbers usually undo such conservatism, as documented in Dyreng et al. (2017). Conservative accounting, by reducing undue optimism error, reduces the chances that managers withhold such error. Managers would more often disclose false alarm errors and then adjust the accounting numbers correspondingly to correct for such errors.

Finally, we provide an explanation on why debt contract may exclude accounting method changes. Empirically, Beatty et al. (2002) find that in some debt contracts, lenders exclude
both voluntary and mandatory accounting changes in calculation of covenant compliance and in return charge a lower interest rate. They interpret this as the value of borrower’s “accounting flexibility”. In our model, such “accounting flexibility” stems from the borrower’s excess incentive in searching for information related to accounting changes. Since such search can be socially wasteful, the lender may find it optimal to exclude accounting changes to discourage the search.

7 Conclusion

We analytically study how accounting measurement rules can be designed to enhance the role of accounting-based covenants. We model a manager that obtains debt financing from a lender and the debt contract contains a covenant based on accounting information. The accounting information can be based on the extant GAAP rules, which may contain false alarm and undue optimism errors, or can be based on alternative rules that correct those errors. The manager can privately exert costly effort trying to find out whether GAAP rules are suitable but such information cannot be contracted upon ex ante. The manager can choose to credibly disclose such information (and correct the errors) or remain silent, as in Dye (1985) and Jung and Kwon (1988). We find that undue optimism error and false alarm errors have different effects on manager’s disclosure decisions. Specifically, the manager is more willing to disclose false alarm errors than undue optimism errors. Nevertheless, the option of strategic withholding results in the manager exerting excessive effort to exploit such option. Our results provide an explanation of why the GAAP numbers are more likely to be corrected for false alarm errors in debt contracting, and also why sometimes contracts exclude all changes to the GAAP rules. Our results also provide additional empirical and policy implications. We hope future research can build on our framework to provide further insights in how properties of accounting numbers used for financial reporting purposes differ from the properties of accounting numbers used in debt contracting and how those properties are related to various managerial incentives, e.g., manager’s incentives to manipulate earnings.
8 Appendix: Proofs

Proof of Proposition 1:

**Proof.** In the first-best, $e_G$ is chosen to maximize the objective function

$$
\beta_G[e_Gw(0) + (1 - e_G)w^F(1)] + (1 - \beta_G)w(0) - \frac{1}{2}ce_G^2,
$$

where

$$
w(0) = q(\gamma_G Y + X) + (1 - q)y,
$$

and

$$
w^F(1) = q(\gamma_G Y + X) + (1 - q)y - \frac{1}{2}q\kappa L_G.
$$

First order condition with respect to $e_G$ results in

$$
\beta_G[w(0) - w^F(1)] = ce_G^F,
$$

or, equivalently,

$$
\frac{1}{2}q\beta_G\kappa L_G = ce_G^F.
$$

Proof of Proposition 2:

**Proof.** What remains to be shown is the algebraic details for $\rho_G$, $U_1(\phi) < U_1(1)$ and $U_0(\phi) > U_0(0)$. Conditional on $f_G = \phi$ and conjecture of the manager’s effort $\hat{e}_G$ and the manager’s disclosure strategy $\hat{d}_G(f_G)$, the lender’s expected payoff is

$$
V_{\hat{e}_G}(\phi) = E_{\theta,s}[v^g_b|\phi, \hat{e}_G; \hat{d}_G]
$$

$$
= q \Pr(g|G, \phi, \hat{e}_G, \hat{d}_G)v^g_G(\phi) + q \Pr(b|G, \phi, \hat{e}_G, \hat{d}_G)v^b_G(\phi) + (1 - q)v^b_B(\phi)
$$

$$
= q\rho_G v^g_G(\phi) + q(1 - \rho_G)v^b_G(\phi) + (1 - q)v^b_B(\phi)
$$

$$
= qv^g_G(\phi) + (1 - q)v^b_B(\phi) + q(1 - \rho_G)(v^b_G(\phi) - v^g_G(\phi))
$$

$$
= K,
$$

(14)
where the last equality represents the lender’s break-even constraint.

Note that in arriving at the third equality of equation (14), we use

\[
\rho_G \\
\equiv \Pr(g|G, \phi, \hat{e}_G, \hat{d}_G) \\
= \Pr(g|G, \omega_G = 0, \phi, \hat{e}_G, \hat{d}_G) \times \Pr(\omega_G = 0|G, \phi, \hat{e}_G, \hat{d}_G) \\
+ \Pr(g|G, \omega_G = 1, \phi, \hat{e}_G, \hat{d}_G) \times \Pr(\omega_G = 1|G, \phi, \hat{e}_G, \hat{d}_G).
\]

Note that

\[
\Pr(g|G, \omega_G = 0, \phi, \hat{e}_G, \hat{d}_G) = \Pr(g|G, \omega_G = 0) = 1,
\]

and

\[
\Pr(g|G, \omega_G = 1, \phi, \hat{e}_G, \hat{d}_G) = \Pr(g|G, \omega_G = 1) = \frac{1}{2}.
\]

Therefore

\[
\rho_G = 1 \times \Pr(\omega_G = 0|G, \phi, \hat{e}_G, \hat{d}_G) + \frac{1}{2} \times \Pr(\omega_G = 1|G, \phi, \hat{e}_G, \hat{d}_G) \\
= 1 - \frac{1}{2} \times \Pr(\omega_G = 1|G, \phi, \hat{e}_G, \hat{d}_G) \\
= 1 - \frac{1}{2} \times \Pr(\omega_G = 1|\phi, \hat{e}_G, \hat{d}_G) \\
= 1 - \frac{1}{2} \times \Pr(\omega_G = 1|\phi, \hat{e}_G, \hat{d}_G) \\
= 1 - \frac{1}{2} \times \frac{\Pr(\phi|\omega_G = 1, \hat{e}_G, \hat{d}_G) \times \Pr(\omega_G = 1|\hat{e}_G, \hat{d}_G)}{\sum_{i=0}^{1} \Pr(\phi|\omega_G = i, \hat{e}_G, \hat{d}_G) \times \Pr(\omega_G = i|\hat{e}_G, \hat{d}_G)} \\
= 1 - \frac{1}{2} \times \frac{\Pr(\phi|\omega_G = 1, \hat{e}_G, \hat{d}_G) \times \Pr(\omega_G = 1|\hat{e}_G, \hat{d}_G)}{\sum_{i=0}^{1} \Pr(\phi|\omega_G = i, \hat{e}_G, \hat{d}_G) \times \Pr(\omega_G = i)} \\
= 1 - \frac{1}{2} \times \frac{\beta_G[1 - \hat{e}_G \hat{d}_G(1)]}{\beta_G(1 - \hat{e}_G \hat{d}_G(1)) + (1 - \beta_G)[1 - \hat{e}_G \hat{d}_G(0)]} > \frac{1}{2}.
\]
If the manager observes \( f_G = 1 \) and discloses it, his payoff will be

\[
U_1(1) = q u_G^g(1) + (1 - q) u_B^b(1) \\
= q(\gamma_G Y + X - v_G^g(1)) + (1 - q)(y - v_B^b(1)) \\
= q(\gamma_G Y + X) + (1 - q)y - K,
\]

whereas if he does not disclose, his payoff will be

\[
U_1(g) = q u_G^g(1) + \frac{1}{2} u_G^b(1) + (1 - q) u_B^b(1) \\
= q(\gamma_G Y + X + (1 - q)y - K + q(1 - \rho_G)(v_G^g(1) - v_G^b(1))) - \frac{1}{2}(u_G^g(1) - u_G^b(1)) \\
= U_1(1) - q(1 - \rho_G)(u_G^g(1) - u_G^b(1)) - q(1 - \rho_G)(u_G^b(1)) \\
= U_1(1) - q(1 - \rho_G)(u_G^g(1) - u_G^b(1)) - q(1 - \rho_G)(u_G^b(1)) < U_1(1).
\]

as \( \rho_G \in (\frac{1}{2}, 1) \). Therefore, \( U_1(0) < U_1(1) \).

If the manager observes \( f_G = 0 \) and discloses it, his payoff will be

\[
U_0(0) = q u_G^g(0) + (1 - q) u_B^b(0) \\
= q(\gamma_G Y + X - v_G^g(0)) + (1 - q)(y - v_B^b(0)) \\
= q(\gamma_G Y + X) + (1 - q)y - K,
\]

whereas if he does not disclose, his payoff will be

\[
U_0(\phi) = q u_G^g(\phi) + (1 - q) u_B^b(\phi) \\
= q(\gamma_G Y + X - v_G^g(\phi)) + (1 - q)(y - v_B^b(\phi)) \\
= q(\gamma_G Y + X) + (1 - q)y - K + q(1 - \rho_G)(v_G^b(\phi) - v_G^g(\phi)) \\
> U_0(0).
\]
Therefore, $U_0(\phi) > U_0(0)$. ■

Proof of Proposition 3:

**Proof.** Given $d^*_G(1) = 1$ and $d^*_G(0) = 0$, we can simplify the expression of $\rho_G$:

$$
\rho_G(\bar{e}_G) = 1 - \frac{1}{2} \frac{\beta_G(1 - \bar{e}_G)}{1 - \beta_G \bar{e}_G}.
$$

The manager’s payoff from exerting effort $e_G$ and following the optimal disclosure strategy will be

$$
\beta_G[e_G U_1(1) + (1 - e_G)U_\phi(\phi)] + (1 - \beta_G)[e_G U_0(\phi) + (1 - e_G)U_\phi(\phi)]
$$

$$
\quad - \frac{1}{2} c e_G^2.
$$

First order condition results in

$$
\beta_G(U_1(1) - U_\phi(\phi)) + (1 - \beta_G)[U_0(\phi) - U_\phi(\phi)] = c e_G^*.
$$

Note that

$$
U_\phi(\phi)
$$

$$
= q[(1 - \frac{1}{2} \beta_G)u_G^q(\phi) + \frac{1}{2} \beta_G u_G^b(\phi)] + (1 - q)u_B^b(\phi)
$$

$$
= q u_G^q(\phi) + (1 - q)u_B^b(\phi) - \frac{q \beta_G}{2} [u_G^q(\phi) - u_G^b(\phi)]
$$

$$
= q(\gamma_G Y + X - v_G^b(\phi)) + (1 - q)(y - v_B^b(\phi)) - \frac{q \beta_G}{2} [u_G^q(\phi) - u_G^b(\phi)]
$$

$$
= q(\gamma_G Y + X) + (1 - q)y - K
$$

$$
+ q(1 - \rho_G)(v_G^b(\phi) - v_G^q(\phi)) - \frac{q \beta_G}{2} [u_G^q(\phi) - u_G^b(\phi)],
$$

Therefore,

$$
U_1(1) - U_\phi(\phi)
$$

$$
= \frac{q \beta_G}{2} [u_G^q(\phi) - u_G^b(\phi)] - q(1 - \rho_G)(v_G^b(\phi) - v_G^q(\phi)),
$$

41
and

\[ U_0(\phi) - U_\phi(\phi) = \frac{q\beta G}{2} [u_G^b(\phi) - v_G(\phi)]. \]

We therefore have

\begin{align*}
ce^*_G &= \beta G[U_1(1) - U_\phi(\phi)] + (1 - \beta G)[U_0(\phi) - U_\phi(\phi)] \\
&= \frac{q\beta G}{2} [u_G^b(\phi) - u_G^b(\phi)] - q\beta G(1 - \rho_G^b)(v_G^b(\phi) - v_G^b(\phi)) \\
&= \frac{q\beta G}{2} [w_G^b(\phi) - w_G^b(\phi)] + q\beta G(1 - \beta G - \frac{1}{2}) (v_G^b(\phi) - v_G^b(\phi)) \\
&= ce^{FB} + \frac{q\beta G(1 - \beta G)}{2(1 - \beta Ge_G^*)} (v_G^b(\phi) - v_G^b(\phi)) \\
&= ce^{FB} + \frac{q\beta G(1 - \beta G)}{2(1 - \beta Ge_G^*)} [(1 - \gamma_G)y + (1 - \tau) (1 - \kappa)L_G] = ce^{FB}. \quad (15)
\end{align*}

\textbf{Proof of Corollary 1:}

\textbf{Proof.} Recall that \(e^*_G\) is defined by the following equation, which is simplified to

\[ q\beta G \kappa L_G + \frac{q\beta G (1 - \beta G)}{(1 - \beta Ge_G^*)} [(1 - \gamma_G)y + (1 - \tau) (1 - \kappa)L_G] - ce^*_G = 0. \quad (16) \]

Denote the left hand side of equation (16) as \(M^G\). Note that the second order condition will be satisfied if \(c\) is sufficiently large, i.e., \(M^G\) will be decreasing in \(e^*_G\). Therefore, by implicit function theorem,

\[ sgn(\frac{\partial e_G^*}{\partial \phi}) = sgn\left(\frac{\partial M^G}{\partial x}\right) = sgn\left(\frac{\partial M^G}{\partial \phi}\right), \]

for any exogenous variable \(x\) on the left hand side of equation (16).

We therefore have

\[ \frac{\partial e_G^*}{\partial L_G} \propto q\beta G [\kappa - \frac{(1 - \beta G)(1 - \tau)(1 - \kappa)}{(1 - \beta Ge_G^*)}], \]
and thus is ambiguous.

Similarly, we have

$$\frac{\partial e^*_G}{\partial y} \propto -q\beta_G[\kappa - \frac{(1 - \beta_G)}{(1 - \beta_G e^*_G)} + (1 - \tau)(1 - \kappa)],$$

which again is ambiguous;

$$\frac{\partial e^*_G}{\partial\beta_G} \propto q\beta_G\kappa L_G + \frac{\beta_G(1 - \beta_G)}{(1 - \beta_G e^*_G)}[(1 - \gamma_G)y + (1 - \tau)(1 - \kappa)L_G] > 0;$$

$$\frac{\partial e^*_G}{\partial\kappa} \propto q\beta_G L_G + \frac{q[1 - \beta_G(2 - \beta_G e^*_G)]}{(1 - \beta_G e^*_G)^2}[(1 - \gamma_G)y + (1 - \tau)(1 - \kappa)L_G],$$

which is ambiguous;

$$\frac{\partial e^*_G}{\partial\kappa} \propto q\beta_G L_G[1 - \frac{q\beta_G(1 - \beta_G)}{(1 - \beta_G e^*_G)}(1 - \tau)] > 0,$$

and

$$\frac{\partial e^*_G}{\partial\tau} \propto -q\beta_G(1 - \beta_G)(1 - \kappa)L_G < 0.$$

Proof of Corollary 2:

**Proof.** Recall that this difference is defined by

$$\begin{align*}
c(e^*_G - e^{FB}_G) & \propto \frac{q\beta_G(1 - \beta_G)}{(1 - \beta_G e^*_G)}[(1 - \gamma_G)y + (1 - \tau)(1 - \kappa)L_G] \\
& \equiv T^G.
\end{align*}$$

Therefore

$$\frac{\partial(e^*_G - e^{FB}_G)}{\partial x} \propto \frac{\partial T^G}{\partial x} + \frac{\partial T^G}{\partial e^*_G}\frac{de^*_G}{dx},$$

and

$$\frac{\partial T^G}{\partial e^*_G} \propto \frac{1}{(1 - \beta_G e^*_G)^2} > 0.$$

We therefore have \(\frac{\partial(e^*_G - e^{FB}_G)}{\partial L_G}\), \(\frac{\partial(e^*_G - e^{FB}_G)}{\partial y}\) and \(\frac{\partial(e^*_G - e^{FB}_G)}{\partial\beta_G}\) being ambiguous as \(\frac{de^*_G}{dx}\) is ambigu-
ous for \( x = L_G, y \) and \( \beta_G \). Nevertheless, when \( \beta_G \) is very small, \( \frac{d\epsilon^*_G}{d\beta_G} > 0 \) and

\[
\frac{\partial T^G}{\partial \beta_G} \propto \frac{q[1 - \beta_G (2 - \beta_G e^*_G)]}{(1 - \beta_G e^*_G)^2} > 0,
\]

when \( \beta_G \) is very small. Therefore \( \frac{\partial (\epsilon^*_G - \epsilon^*_{FB})}{\partial \beta_G} > 0 \) when \( \beta_G \) is very small.

We also have \( \frac{\partial (\epsilon^*_G - \epsilon^*_{FB})}{\partial q} > 0 \) as \( \frac{\partial T^G}{\partial q} > 0 \) and \( \frac{d\epsilon^*_G}{dq} > 0 \).

We also have \( \frac{\partial (\epsilon^*_G - \epsilon^*_{FB})}{\partial \tau} < 0 \) as \( \frac{\partial T^G}{\partial \tau} < 0 \) and \( \frac{d\epsilon^*_G}{d\tau} < 0 \).

The sign of \( \frac{\partial (\epsilon^*_G - \epsilon^*_{FB})}{\partial \kappa} \) is ambiguous as \( \frac{\partial T^G}{\partial \kappa} < 0 \) but \( \frac{d\epsilon^*_G}{d\kappa} > 0 \).

Proof of Proposition 4:

**Proof.** Again denote \( w(0) \) as the social payoff where there is no error (or error has been detected) and \( w^{UO}(1) \) as the social payoff where there is undetected undue optimism error.

In the first-best, \( e_B \) is chosen to maximize the objective function

\[
\beta_B [e_B w(0) + (1 - e_B) w^{UO}(1)] + (1 - \beta_B) w(0) - \frac{1}{2} c e_B^2
\]

where

\[
w(0) = q(\gamma_G Y + X) + (1 - q) y
\]

and

\[
w^{UO}(1) = q(\gamma_G Y + X) + (1 - q) y - \frac{1}{2}(1 - q) \kappa_L B
\]

First order condition with respect to \( e_B \) results in

\[
\beta_B [w(0) - w^{UO}(1)] = c e^*_B,
\]

or, equivalently,

\[
\frac{1}{2}(1 - q) \beta_B L_B = c e^*_B \quad (17)
\]

Proof of Proposition 5:

**Proof.** What remains to be shown is the expression of \( \rho_B, U'_0(\phi) < U_0(0) \) and the conditions
for $d_B^*(1)$ to be 0, 1, or between 0 and 1.

Conditional on $f_B = \phi$ and conjecture of the manager’s effort $\hat{e}_B$ and the manager’s disclosure strategy $\hat{d}_B(f_B)$, the lender’s expected payoff is

\[
V_{\hat{e}_B}(\phi) = E_{\theta,t}[v^*_B|\phi, \hat{e}_B, \hat{d}_B]
\]

\[
= q \nu_G^B(\phi) + (1 - q) \Pr(g|B, \phi, \hat{e}_B, \hat{d}_B)\nu_B^B(\phi) + (1 - q) \Pr(b|B, \phi, \hat{e}_B, \hat{d}_B)\nu_B^b(\phi)
\]

\[
= q \nu_G^B(\phi) + (1 - q)(1 - \rho_B)\nu_B^B(\phi) + (1 - q)\rho_B\nu_B^b(\phi)
\]

\[
= q \nu_G^B(\phi) + (1 - q)\nu_B^b(\phi) - (1 - q)(1 - \rho_B)(\nu_B^b(\phi) - \nu_B^g(\phi))
\]

\[
= K
\]

where the last equality represents the lender’s break-even constraint.

Note that in arriving at the third equality of equation (18), we use

\[
\rho_B = \Pr(b|B, \phi, \hat{e}_B, \hat{d}_B)
\]

\[
= \Pr(b|B, \omega_G = 0, \phi, \hat{e}_B, \hat{d}_B) \times \Pr(\omega_B = 0|B, \phi, \hat{e}_G, \hat{d}_G)
\]

\[
+ \Pr(b|B, \omega_G = 1, \phi, \hat{e}_B, \hat{d}_B) \times \Pr(\omega_B = 1|B, \phi, \hat{e}_B, \hat{d}_B)
\]

Note that

\[
\Pr(b|B, \omega_B = 0, \phi, \hat{e}_B, \hat{d}_B) = \Pr(b|B, \omega_B = 0) = 1
\]

and

\[
\Pr(b|B, \omega_B = 1, \phi, \hat{e}_B, \hat{d}_B) = \Pr(b|B, \omega_B = 1) = \frac{1}{2}
\]
Therefore

\[
\rho_B = 1 \times \Pr(\omega_B = 0|B, \phi, \hat{c}_G, \hat{d}_G) + \frac{1}{2} \times \Pr(\omega_B = 1|B, \phi, \hat{c}_B, \hat{d}_B)
\]

\[
= 1 - \frac{1}{2} \times \Pr(\omega_B = 1|B, \phi, \hat{c}_B, \hat{d}_B)
\]

\[
= 1 - \frac{1}{2} \times \Pr(\omega_B = 1|B, \phi, \hat{c}_B, \hat{d}_B)
\]

\[
= 1 - \frac{1}{2} \times \Pr(\omega_B = 1|B, \phi, \hat{c}_B, \hat{d}_B)
\]

\[
= 1 - \frac{1}{2} \times \Pr(\omega_B = 1|B, \phi, \hat{c}_B, \hat{d}_B)
\]

\[
= 1 - \frac{1}{2} \times \frac{\Pr(\phi|\omega_B = 1, \hat{c}_B, \hat{d}_B) \times \Pr(\omega_B = 1|\hat{c}_B, \hat{d}_B)}{\sum_{i=0}^{1} \Pr(\phi|\omega_B = i, \hat{c}_B, \hat{d}_B) \times \Pr(\omega_B = i|\hat{c}_B, \hat{d}_B)}
\]

\[
= 1 - \frac{1}{2} \times \frac{\beta_B [1 - \hat{c}_B \hat{d}_B(1)]}{\beta_B [1 - \hat{c}_B \hat{d}_B(1)] + (1 - \beta_B)[1 - \hat{c}_B \hat{d}_B(0)]} > \frac{1}{2}
\]

We now look at the manager’s disclosure strategy when observing \(f_B\). Again we write the manager’s payoff as \(U_f(F)\), where \(f\) is his signal and \(F\) is his disclosure.

If the manager observes \(f_B = 0\) and discloses it, his payoff will be

\[
U_0(0) = q\mu^G_B(0) + (1 - q)\mu^*_B(0)
\]

\[
= q(\gamma G X - \nu^G_B(0)) + (1 - q)(y - \nu^*_B(0))
\]

\[
= q(\gamma G Y + X) + (1 - q)y - K,
\]

whereas if he does not disclose, his payoff will be

\[
U_0(\phi) = q\mu^G_B(\phi) + (1 - q)\mu^*_B(\phi)
\]

\[
= q(\gamma G Y + X - \nu^G_B(\phi)) + (1 - q)(y - \nu^*_B(\phi))
\]

\[
= q(\gamma G Y + X) + (1 - q)y - K - (1 - q)(1 - \rho_B)(\nu^*_B(\phi) - \nu^*_B(\phi))
\]

\[
< U_0(0).
\]

Therefore, \(U_0(\phi) < U_0(0)\).
If the manager observes \( f_B = 1 \) and discloses it, his payoff will be

\[
U_1(1) = qu_G(1) + (1 - q)u_B^b(1)
\]
\[
= q(\gamma_GY + X - u_G^g(1)) + (1 - q)(y - v_B^b(1))
\]
\[
= q(\gamma_GY + X) + (1 - q)y - K,
\]

whereas if he does not disclose, his payoff will be

\[
U_1(\phi) = qu_G^g(\phi) + (1 - q)[\frac{1}{2}u_B^g(\phi) + \frac{1}{2}u_B^b(\phi)]
\]
\[
= qu_G^g(\phi) + (1 - q)u_B^b(\phi) + (1 - q)\frac{1}{2}(u_B^g(\phi) - u_B^b(\phi))
\]
\[
= q\gamma_GY + X + (1 - q)y - K - (1 - q)(1 - \rho_B)(v_B^b(\phi) - v_B^g(\phi))
\]
\[
\quad + (1 - q)\frac{1}{2}(u_B^g(\phi) - u_B^b(\phi))
\]
\[
= U_1(1) + (1 - q)[\frac{1}{2}(u_B^g(\phi) - u_B^b(\phi)) - (1 - \rho_B)(v_B^b(\phi) - v_B^g(\phi))]
\]
\[
\equiv U_1(1) + (1 - q)\Delta(\hat{d}_B(1)),
\]

where

\[
\Delta(\hat{d}_B(1)) = \frac{1}{2}[(u_B^g(\phi) - u_B^b(\phi)) - (1 - \rho_B(\hat{d}_B(1)))(v_B^b(\phi) - v_B^g(\phi))].
\]

Note that \( d_B(1) = 0 \) if \( \Delta(\hat{d}_B(1)) > 0 \), \( d_B(1) = 1 \) if \( \Delta(\hat{d}_B(1)) < 0 \) and \( d_B(1) \in [0, 1] \) if \( \Delta(\hat{d}_B(1)) = 0 \). In equilibrium, we need \( \hat{d}_B(1) = d_B(1) = d_B^*(1) \).

Note that since \( d_B^*(0) = 1 \),

\[
\rho_B = 1 - 2 \frac{\beta_B[1 - \hat{c}_B\hat{d}_B(1)]}{\beta_B[1 - \hat{c}_B\hat{d}_B(1)] + (1 - \beta_B)(1 - \hat{c}_B)}
\]

(19)
and is increasing in $\hat{d}_B(1)$ as
\[
\frac{1}{2(1 - \rho_B)} = \frac{\beta_B[1 - \hat{\epsilon}_Bd_B(1)] + (1 - \beta_B)(1 - \hat{\epsilon}_B)}{\beta_B[1 - \hat{\epsilon}_Bd_B(1)]} = 1 + \frac{(1 - \beta_B)(1 - \hat{\epsilon}_B)}{\beta_B[1 - \hat{\epsilon}_Bd_B(1)]}
\]

is increasing in $\hat{d}_B(1)$. Therefore $\Delta(\hat{d}_B(1))$ is increasing in $\hat{d}_B(1)$ and there are three cases:

First, if $\Delta(1) \leq 0$, then $\Delta(\hat{d}_B(1)) \leq 0$ always holds and the equality only holds when $\hat{d}_B(1) = 1$. In this case, $\hat{d}_B(1) = d_B(1) = d_B^*(1) = 1$ is the unique equilibrium. Note that $\Delta(1) < 0$ is equivalent to
\[
X + \tau(1 - \kappa)L_B \leq \beta_B[y - (1 - \tau)(1 - \kappa)L_B]
\]  
(20)

Second, if $\Delta(0) \geq 0$, then $\Delta(\hat{d}_B(1)) \geq 0$ always holds and the equality only holds when $\hat{d}_B(1) = 0$. In this case $\hat{d}_B(1) = d_B(1) = d_B^*(1) = 0$ is the unique equilibrium. Note that $\Delta(0) \geq 0$ is equivalent to
\[
X + \tau(1 - \kappa)L_B \geq \beta_B \frac{\beta_B}{1 - \hat{\epsilon}_B(1 - \beta_B)}[y - (1 - \tau)(1 - \kappa)L_B].
\]  
(21)

Third, if $\Delta(0) < 0 < \Delta(1)$, corresponding to
\[
\beta_B[y - (1 - \tau)(1 - \kappa)L_B] < X + \tau(1 - \kappa)L_B < \frac{\beta_B}{1 - \hat{\epsilon}_B(1 - \beta_B)}[y - (1 - \tau)(1 - \kappa)L_B].
\]  
(22)

Then since $\Delta(1) > 0$, $d_B^*(1) = 1$ is clearly not an equilibrium. Second, since $\Delta(0) < 0$, $d_B^*(1) = 0$ is clearly not an equilibrium. There is a unique mixed strategy equilibrium with $d_B^*(1) \in (0, 1)$ such that
\[
\Delta(d_B^*(1)) = 0,
\]
which is equivalent to

\[ X + \tau(1 - \kappa)L_B = \frac{\beta_B(1 - \hat{e}_B d_B^*(1))}{1 - \hat{e}_B d_B^*(1) - (1 - \beta_B)\hat{e}_B(1 - d_B^*(1))}[y - (1 - \tau)(1 - \kappa)L_B] \] (23)

\[ \square \]

Proof of Proposition 6:

**Proof.** We divide the derivation into three cases, corresponding to \( d_B^*(1) = 1 \), \( d_B^*(1) = 0 \) and \( d_B^*(1) \in (0, 1) \).  

Case 1: \( d_B^*(1) = 1 \);

Given \( d_B^*(1) = 1 \), we can simplify the expression of \( \rho_B \) from equation (19) as

\[ \rho_B(\hat{e}_B) = 1 - \frac{1}{2}\beta_B \]

The manager’s payoff from exerting effort \( e_B \) and following the optimal disclosure strategy will be

\[ \beta_B[e_B U_1(1) + (1 - e_B)U_\phi(\phi)] + (1 - \beta_B)[e_B U_0(0) + (1 - e_B)U_\phi(\phi)] - \frac{1}{2}\epsilon e_B^2 \]

Note that \( U_1(1), U_0(\phi) \) and \( U_\phi(\phi) \) are independent of actual effort choice, resulting in

\[ \beta_B(U_1(1) - U_\phi(\phi)) + (1 - \beta_B)[U_0(0) - U_\phi(\phi)] = ce_{B1}^* \]

Intuitively, exerting effort both increases the payoff when \( \omega_B = 1 \) and the payoff when \( \omega_B = 0 \).
Note that

\[ U_\phi(\phi) \]
\[ = qu^0_G(\phi) + (1 - q)[(1 - \frac{1}{2}\beta_B)u^0_B(\phi) + \frac{1}{2}\beta_Bu^0_B(\phi)] \]
\[ = qu^0_G(\phi) + (1 - q)u^0_B(\phi) + \frac{(1 - q)\beta_B}{2}[u^0_B(\phi) - u^0_B(\phi)] \]
\[ = q(\gamma_GY + X - v^0_G(\phi)) + (1 - q)(y - v^0_B(\phi)) + \frac{(1 - q)\beta_B}{2}[u^0_B(\phi) - u^0_B(\phi)] \]
\[ = q(\gamma_GY + X) + (1 - q)y - K \]
\[ -(1 - q)(1 - \rho_B)(v^b_B(\phi) - v^0_B(\phi)) + \frac{(1 - q)\beta_B}{2}[u^0_B(\phi) - u^0_B(\phi)] \]
\[ = q(\gamma_GY + X) + (1 - q)y - K - \frac{(1 - q)\beta_B}{2}(w^b_B(\phi) - w^0_B(\phi)), \tag{24} \]

and

\[ U_1(1) = U_0(0) = q(\gamma_GY + X) + (1 - q)y - K. \]

Therefore, we have

\[ U_1(1) - U_0(\phi) = U_0(0) - U_0(\phi) \]
\[ = \frac{(1 - q)\beta_B}{2}(w^b_B(\phi) - w^0_B(\phi)) \]
\[ = \frac{(1 - q)\beta_B}{2}\kappa L_B, \]

and the first order condition becomes

\[ ce_{B1}^{*} = \beta_B[U_1(1) - U_0(\phi)] + (1 - \beta_B)[U_0(0) - U_0(\phi)] \]
\[ = \frac{(1 - q)\beta_B}{2}\kappa L_B \]
\[ = ce_{FB}^{F}. \tag{25} \]

Case 2: \( d_B^*(1) = 0; \)

Given \( d_B^*(1) = 0, \) we can simplify the expression of \( \rho_B \) from equation (19) as

\[ \rho_B(\bar{c}_B) = 1 - \frac{1}{2} \frac{\beta_B}{1 - \bar{c}_B(1 - \beta_B)} \]
The manager’s payoff from exerting effort $e_B$ and following the optimal disclosure strategy will be

$$\beta_B[e_B U_1(\phi) + (1 - e_B) U_\phi(\phi)] + (1 - \beta_B)[e_B U_0(0) + (1 - e_B) U_\phi(\phi)] - \frac{1}{2}ce_B^2.$$  

Note that $U_1(1)$, $U_0(\phi)$ and $U_\phi(\phi)$ are independent of actual effort choice, resulting in

$$\beta_B(U_1(\phi) - U_\phi(\phi)) + (1 - \beta_B)[U_0(0) - U_\phi(\phi)] = ce_B^*.$$  

Intuitively, exerting effort both increases the payoff when $\omega_B = 0$ (as the manager will disclose) and the payoff when $\omega_B = 1$ (as the manager will withhold).

Note that we can similarly calculate that

$$U_\phi(\phi) = q(\gamma_G Y + X) + (1 - q)y - K - (1 - q)(1 - \rho_B)(v_B^b(\phi) - v_B^g(\phi)) + \frac{(1 - q)\beta_B}{2}[u_B^g(\phi) - u_B^b(\phi)].$$

In addition,  

$$U_0(0) = q(\gamma_G Y + X) + (1 - q)y - K,$$

and

$$U_1(\phi)$$

$$= q u_B^g(\phi) + (1 - q)\frac{1}{2}u_B^b(\phi) + \frac{1}{2}u_B^g(\phi)]$$

$$= q u_B^g(\phi) + (1 - q)u_B^b(\phi) + \frac{1 - q}{2}[u_B^g(\phi) - u_B^b(\phi)]$$

$$= q(\gamma_G Y + X) + (1 - q)y - K - (1 - q)(1 - \rho_B)(v_B^b(\phi) - v_B^g(\phi)) + \frac{1 - q}{2}[u_B^g(\phi) - u_B^b(\phi)].$$
Therefore, we have

\[
U_1(\phi) - U_\phi(\phi) = \frac{(1-q)(1-\beta_B)}{2}[u_B^g(\phi) - u_B^b(\phi)],
\]

and

\[
U_0(0) - U_\phi(\phi) = (1-q)(1-\rho_B)(v_B^b(\phi) - v_B^g(\phi)) - \frac{(1-q)\beta_B}{2}[u_B^g(\phi) - u_B^b(\phi)],
\]

and the first order condition becomes

\[
ce_{B2}^* = (1-\beta_B)(1-q)(1-\rho_B)(v_B^b(\phi) - v_B^g(\phi))
\]

\[
= ce_{B2}^F + (1-\beta_B)(1-q)(1-\rho_B)(v_B^b(\phi) - v_B^g(\phi))
- \frac{1}{2}(1-q)\beta_B(u_B^b(\phi) - u_B^g(\phi)) - \frac{1}{2}(1-q)\beta_B(v_B^b(\phi) - v_B^g(\phi))
\]

\[
= ce_{B2}^F + (1-q)\beta_B\left\{\frac{1}{2}(u_B^g(\phi) - u_B^b(\phi)) - (1-\rho_B)(v_B^b(\phi) - v_B^g(\phi))\right\}
+ (1-q)(1-\rho_B - \frac{1}{2}\beta_B)(v_B^b(\phi) - v_B^g(\phi))
\]

\[
= ce_{B2}^F + \frac{1}{2}(1-q)\beta_B\left\{[X + \tau(1-\kappa)L_B - \frac{\beta_B}{1 - e_{B2}^*(1-\beta_B)}y - (1-\tau)(1-\kappa)L_B]\right\}
+ \frac{(1-q)\beta_B}{2}(1-\beta_B)e_{B2}^*(v_B^b(\phi) - v_B^g(\phi)).
\]

(26)

(27)

Note that by assuming \(d_B^*(1) = 0\) holds, we know that equation (21) holds at \(e_{B2}^*\), implying that

\[X + \tau(1-\kappa)L_B \geq \frac{\beta_B}{1 - e_{B2}^*(1-\beta_B)}y - (1-\tau)(1-\kappa)L_B\]

Therefore, \(e_{B2}^* > e_{B2}^{FB}\).

Case 3: \(d_B^*(1) \in (0,1)\):

Given \(d_B^*(1) \in (0,1)\), we can simplify the expression of \(\rho_B\) from equation (19) as

\[
\rho_B(\hat{e}_B) = 1 - \frac{1}{2} \times \frac{\beta_B[1 - \hat{e}_Bd_B^*(1)]}{\beta_B[1 - \hat{e}_B^*d_B(1)] + (1-\beta_B)(1-\hat{e}_B)}
\]

52
such that
\[
\frac{1}{2}(u_B^g(\phi) - u_B^b(\phi)) = (1 - \rho_B(\phi)) (v_B^b(\phi) - v_B^g(\phi))
\]

The manager’s payoff from exerting effort \(e_B\) and following the optimal disclosure strategy will be

\[
\begin{align*}
\beta_B[e_Bd_B^*(1)U_1(1) + e_B(1 - d_B^*(1))U_1(\phi) + (1 - e_B)U_\phi(\phi)] \\
+ (1 - \beta_B)[e_BU_0(0) + (1 - e_B)U_\phi(\phi)] - \frac{1}{2}ce_B^2.
\end{align*}
\]

Note that \(U_1(1), U_0(\phi)\) and \(U_\phi(\phi)\) are independent of actual effort choice, resulting in

\[
\begin{align*}
\beta_Bd_B^*(1)(U_1(1) - U_\phi(\phi)) \\
+ \beta_B(1 - d_B^*(1)(U_1(\phi) - U_\phi(\phi)) \\
+ (1 - \beta_B)[U_0(0) - U_\phi(\phi)] \\
= ce_B^*.
\end{align*}
\]

Note that we can similarly calculate that

\[
U_\phi(\phi) = q(\gamma G Y + X) + (1 - q)y - K
\]

\[
-(1 - q)(1 - \rho_B)(v_B^b(\phi) - v_B^g(\phi)) + \frac{(1 - q)\beta_B}{2}[u_B^g(\phi) - u_B^b(\phi)].
\]

In addition,

\[
U_1(\phi) = U_1(1) = U_0(0) = q(\gamma G Y + X) + (1 - q)y - K,
\]

where the first equality is because of indifference that is necessary for mixed strategies.
Therefore, we have

\[ U_1(\phi) - U_\phi(\phi) \]
\[ = U_1(1) - U_\phi(\phi) \]
\[ = U_0(0) - U_\phi(\phi) \]
\[ = (1 - q)(1 - \rho_B) (v_B^b(\phi) - v_B^g(\phi)) - \frac{(1 - q)\beta_B}{2} [u_B^q(\phi) - u_B^b(\phi)], \]

and the first order condition becomes

\[ c e_{B3}^* = (1 - q)(1 - \rho_B) (v_B^b(\phi) - v_B^g(\phi)) - \frac{(1 - q)\beta_B}{2} [u_B^q(\phi) - u_B^b(\phi)] \]
\[ = (1 - q)(1 - \rho_B) (v_B^b(\phi) - v_B^g(\phi)) \]
\[ = c e_{EB}^F + \frac{1}{2} (1 - q)\beta_B [X + \tau(1 - \kappa) L_B \]
\[ - \frac{\beta_B [1 - e_{B3}^* d_B^*(1)]}{\beta_B [1 - e_{B3}^* d_B^*(1)] + (1 - \beta_B)(1 - e_{B3}^*)} (v_B^b(\phi) - v_B^g(\phi)) \]
\[ + \frac{(1 - q)\beta_B}{2} \frac{1 - e_{B3}^* d_B^*(1)}{1 - e_{B3}^*} (v_B^b(\phi) - v_B^g(\phi)),(28) \]

where the second and fourth equality of equation (28) comes from

\[ \frac{1}{2} (u_B^q(\phi) - u_B^b(\phi)) = (1 - \rho_B(e_{B3}^*)) (v_B^b(\phi) - v_B^g(\phi)). \]

Therefore, \( e_{B2}^* > e_{B3}^* > e_{EB}^F. \)

**Proof of Proposition 7:**

**Proof.** We now need to verify whether the solutions \( e_{B1}^* \), \( e_{B2}^* \) and \( e_{B3}^* \) are indeed consistent with the conditions that determine the optimal disclosure decision.

Recall that \( d_B^*(1) = 1 \) if

\[ X + \tau(1 - \kappa) L_B \leq \beta_B [y - (1 - \tau)(1 - \kappa) L_B], \]

(29)
\[ d^*_B(1) \in (0, 1) \text{ if} \]
\[ \beta_B[y - (1 - \tau)(1 - \kappa)L_B] < X + \tau(1 - \kappa)L_B < \frac{\beta_B}{1 - e^*_B(1 - \beta_B)}[y - (1 - \tau)(1 - \kappa)L_B], \quad (30) \]

and \( d^*_B(1) = 0 \) if
\[ X + \tau(1 - \kappa)L_B \geq \frac{\beta_B}{1 - e^*_B(1 - \beta_B)}[y - (1 - \tau)(1 - \kappa)L_B]. \quad (31) \]

Essentially those are three constraints of the optimization problem.

We divide our discussion into cases.

First, when
\[ X + \tau(1 - \kappa)L_B \leq \beta_B[y - (1 - \tau)(1 - \kappa)L_B], \quad (32) \]
then equation (29) holds, implying that \( d^*_B(1) = 1 \) and that \( e^*_B = e^*_B = e^*_FB \).

Note that the left hand of equation (32) is decreasing in \( \kappa \) whereas the right hand side is increasing in \( \kappa \). Thus, equation (32) will be automatically satisfied if it holds when \( \kappa = 0 \), or, equivalently,
\[ X + \tau L_B \leq \beta_B[y - (1 - \tau)L_B]. \quad (33) \]

Note, however, that equation (33) cannot be satisfied as
\[ X + \tau L_B + \beta_B(1 - \tau)L_B > \beta_BX + \beta_B\tau L_B + \beta_B(1 - \tau)L_B \]
\[ = \beta_B(X + L_B) = \beta_By. \]

Note that equation (32) will not be satisfied if it does not hold when \( \kappa = 1 \), or, equivalently,
\[ X > \beta_By, \quad (34) \]
which is equivalent to
\[ X > \bar{X} \equiv \beta_By. \quad (35) \]
When $X \leq \bar{X}$, equation (33) will hold if

$$\kappa \geq \kappa_1,$$

where $\kappa_1 \in (0, 1]$ is the unique solution of

$$X + \tau(1 - \kappa_1)L_B = \beta_B[y - (1 - \tau)(1 - \kappa_1)L_B]. \quad (36)$$

Therefore, $e_B^* = e_{B1}^* = e_{FB}^*$ is the unique equilibrium when $\beta_B \geq \beta_B$ and $\kappa \geq \kappa_1$.

Second, note that $\frac{\beta_B}{1 - \epsilon_B(1 - \beta_B)}[y - (1 - \tau)(1 - \kappa)L_B]$ is increasing in $e_B^*$. Thus, when

$$X + \tau(1 - \kappa)L_B \geq \frac{\beta_B}{1 - \epsilon_B^2(1 - \beta_B)}[y - (1 - \tau)(1 - \kappa)L_B] \quad (37)$$

$$> \frac{\beta_B}{1 - \epsilon_B^3(1 - \beta_B)}[y - (1 - \tau)(1 - \kappa)L_B], \quad (38)$$

then $e_B^* = e_{B2}^* > e_{FB}^*$ is the unique equilibrium.

Third, when

$$\beta_B[y - (1 - \tau)(1 - \kappa)L_B] < X + \tau(1 - \kappa)L_B < \frac{\beta_B}{1 - \epsilon_B^2(1 - \beta_B)}[y - (1 - \tau)(1 - \kappa)L_B], \quad (39)$$

then $e_{B2}^*$ cannot be optimal. Note that since $\beta_B[y - (1 - \tau)(1 - \kappa)L_B] < X + \tau(1 - \kappa)L_B$, $d_B^*(1) = 0$ cannot be optimal.

Note that $d_B^*(1) \in (0, 1)$ and $e_{B3}^*$ is optimal since

$$X + \tau(1 - \kappa)L_B$$

$$= \frac{\beta_B[1 - e_{B3}^*d_B^*(1)]}{\beta_B[1 - e_{B3}^*d_B^*(1)] + (1 - \beta_B)(1 - e_{B3}^*)}[y - (1 - \tau)(1 - \kappa)L_B]$$

$$< \frac{\beta_B}{1 - e_{B3}^*(1 - \beta_B)}[y - (1 - \tau)(1 - \kappa)L_B], \quad (40)$$

as

$$\frac{\beta_B[1 - e_{B3}^*d_B^*(1)]}{\beta_B[1 - e_{B3}^*d_B^*(1)] + (1 - \beta_B)(1 - e_{B3}^*)}$$
is decreasing in $d_B^*(1)$ and reaches its maximum of $\frac{\beta_B}{1-e_{B3}^*(1-\beta_B)}$ when $d_B^*(1) = 0$.

This results in $e_{B}^* = e_{B3}^*$.

We now find conditions on exogenous parameters $X$ and $\kappa$ that satisfy equation (37), i.e.,

$$X + \tau(1-\kappa)L_B \geq \frac{\beta_B}{1-e_{B2}^*(1-\beta_B)}[y - (1-\tau)(1-\kappa)L_B]. \quad (41)$$

If this condition is satisfied, then $e_{B2}^*$ is the solution. Otherwise $e_{B3}^*$ is the solution.

As will be shown below, $e_{B2}^*$ is increasing in $\kappa$ so the left hand side of equation (41) is decreasing in $\kappa$ whereas the right hand side is increasing in $\kappa$. We now write $e_{B2}^*$ explicitly as a function of $\kappa$, i.e., $e_{B2}^*(\kappa)$.

$$X + \tau(1-\kappa)L_B - \frac{\beta_B}{1-e_{B2}^*(1-\beta_B)}[y - (1-\tau)(1-\kappa)L_B] \geq 0 \quad (42)$$

Therefore the left hand side of equation (42) is decreasing in $\kappa$. When $\kappa = 0$, this condition clearly holds as

$$X + \tau L_B - \frac{\beta_B}{1-e_{B2}^*(1-\beta_B)}[y - (1-\tau)L_B]$$

$$= [\tau y + (1-\tau)X][1 - \frac{\beta_B}{1-e_{B2}^*(1-\beta_B)}] > 0,$$

as $\frac{\beta_B}{1-e_{B2}^*(1-\beta_B)}$ is increasing in $e_{B2}^*$ and thus

$$\frac{\beta_B}{1-e_{B2}^*(1-\beta_B)} < \frac{\beta_B}{\beta_B} = 1$$

First, when $X > \bar{X} = \beta_B y$, we need to check whether equation (42) holds when $\kappa = 1$ as if it holds when $\kappa = 1$ then $e_{B}^* = e_{B2}^*$. If it does not hold, then $e_{B}^* = e_{B2}^*$ if and only if $\kappa$ is smaller than some threshold. When $\kappa = 1$, the left hand side of equation (42) becomes

$$X - \frac{\beta_B y}{1-e_{B2}^*(1)(1-\beta_B)}.$$

Clearly $e_{B2}^*(1)$ does not depend on $X$. When $X \to \bar{X} = \beta_B y$, this equation clearly cannot
satisfy as $1-e_{B2}^*(1-\beta_B) > 1$. When $X \to y$, this equation is clearly satisfied as $1-e_{B2}^*(1-\beta_B) < 1$. Therefore, there exists a unique $\bar{X} \in (\hat{X}, y)$ such that $e_{B}^* = e_{B2}^*$ when $X \geq \bar{X}$ where $\bar{X}$ satisfies

$$\bar{X} = \frac{\beta_B y}{1-e_{B2}^*(1-\beta_B)}. \quad (43)$$

When $X < \bar{X}$, then there exists a unique $\kappa_0 \in (0, 1)$ such that $e_{B}^* = e_{B2}^*(e_{B3}^*)$ is the equilibrium effort if and only if $\kappa \leq \kappa_0(\kappa > \kappa_0)$ where $\kappa_0$ satisfies

$$X + \tau(1-\kappa_0)L_B - \frac{\beta_B}{1-e_{B2}^*(\kappa_0)(1-\beta_B)}[y - (1-\tau)(1-\kappa_0)L_B] = 0. \quad (44)$$

Second, when $X \leq \hat{X}$ and $\kappa = \kappa_1$, clearly equation (42) is not satisfied as when $\kappa = \kappa_1$,

$$X + \tau(1-\kappa_1)L_B = \beta_B[y - (1-\tau)(1-\kappa_1)L_B] < \frac{\beta_B}{1-e_{B2}^*(1-\beta_B)}[y - (1-\tau)L_B].$$

Therefore, there exists a unique $\kappa_0 \in (0, \kappa_1)$ such that $e_{B}^* = e_{B2}^*(e_{B3}^*)$ is the equilibrium effort if and only if $\kappa \leq \kappa_0(\kappa_0 < \kappa < \kappa_1)$ where $\kappa_0$ satisfies equation (44). ■

Proof of Corollary 3:

**Proof.** Recall that $e_{B2}^*$ is defined by

$$\frac{(1-\beta_B)(1-q)(1-\rho_B)(v_B^b(\phi) - v_B^a(\phi)) - ce_{B2}^*}{1-e_{B2}^*(1-\beta_B)} - \frac{1}{2} \frac{(1-\beta_B)(1-q)\beta_B}{1-e_{B2}^*(1-\beta_B)} [y - (1-\tau)(1-\kappa)L_B] - ce_{B2}^* = 0. \quad (45)$$

Denote the left hand side of equation (45) as $M_{B2}$. Note that the second order condition will be satisfied if $c$ is sufficiently large, i.e., $M_{B2}$ will be decreasing in $e_{B2}^*$. Therefore, by implicit function theorem,

$$\text{sgn}(\frac{\partial e_{B2}^*}{\partial x}) = \text{sgn}(-\frac{\partial M_{B2}}{\partial x}(\frac{\partial M_{B2}}{\partial e_{B2}^*})) = \text{sgn}(\frac{\partial M_{B2}}{\partial x})$$

58
for any exogenous variable $x$ on the left hand side of equation (45).

We therefore have
\[
\frac{\partial e_{B2}^*}{\partial L_B} \propto (1 - \tau)(1 - \kappa) < 0.
\]

Similarly, we have
\[
\frac{\partial e_{B2}^*}{\partial y} \propto [1 - (1 - \tau)(1 - \kappa)] = [\kappa + \tau(1 - \kappa)] > 0;
\]
\[
\frac{\partial e_{B2}^*}{\partial q} \propto -\beta_B(1 - \beta_B) < 0;
\]
\[
\frac{\partial e_{B2}^*}{\partial \beta_B} \propto \frac{(1 - q)[1 - 2\beta_B - (1 - \beta_B)^2e_{B2}^*]}{[1 - (1 - \beta_B)e_{B2}^*]^2};
\]
\[
\frac{\partial e_{B2}^*}{\partial \kappa} \propto (1 - \tau)L_B > 0;
\]
and
\[
\frac{\partial e_{B2}^*}{\partial \tau} \propto (1 - \kappa)L_B > 0.
\]

**Proof of Corollary 4:**

**Proof.** We now explore the comparative statics of $e_{B2}^* - e_{B}^{FB}$. Note that
\[
\alpha e_{B}^{FB} = \frac{1}{2}(1 - q)\beta_B \kappa L_B.
\]

Therefore
\[
c(e_{B2}^* - e_{B}^{FB}) \propto (1 - q)\left\{\frac{(1 - \beta_B)\beta_B}{1 - e_{B2}^*(1 - \beta_B)}[y - (1 - \tau)(1 - \kappa)L_B] - \beta_B \kappa L_B\right\}
\equiv T^{B2}.
\]
and we already know that $T^{B2} > 0$, i.e.,
\[
\frac{(1 - \beta_B)\beta_B}{1 - e_{B2}^*(1 - \beta_B)}[y - (1 - \tau)(1 - \kappa)L_B] - \beta_B \kappa L_B > 0.
\]
Therefore
\[
\frac{\partial (e_B^* - e_B^F)}{\partial x} \propto \frac{\partial T^B_2}{\partial x} + \frac{\partial T^B_2}{\partial e_B^*} \frac{de_B^*}{dx},
\]
and
\[
\frac{\partial T^B_2}{\partial e_B^*} \propto \frac{1}{[1 - (1 - \beta_B)e_B^*]^2} > 0.
\]

We therefore have the sign of \( \frac{\partial (e_B^* - e_B^F)}{\partial \beta_B} \) being ambiguous as \( \frac{de_B^*}{d\beta_G} \) is ambiguous. Nevertheless, since when \( \beta_B \) is very small, \( \frac{de_B^*}{d\beta_G} > 0 \) and
\[
\frac{\partial T^B_2}{\partial e_B^*} \propto \frac{(1 - q)[1 - 2\beta_B - (1 - \beta_B)^2e_B^*]}{[1 - (1 - \beta_B)e_B^*]^2} > 0
\]
when \( \beta_B \) is very small, \( \frac{\partial (e_B^* - e_B^F)}{\partial \beta_B} > 0 \) when \( \beta_B \) is very small.

We also have \( \frac{\partial (e_B^* - e_B^F)}{\partial L_B} < 0 \)
as
\[
\frac{\partial T^B_2}{\partial L_B} \propto - \frac{(1 - \beta_B)\beta_B}{1 - e_B^*(1 - \beta_B)} (1 - \tau) (1 - \kappa) - \beta_B\kappa < 0,
\]
and
\[
\frac{de_B^*}{dL_B} < 0.
\]
This implies that \( e_B^* - e_B^F \) is increasing in \( X \).

We also have \( \frac{\partial (e_B^* - e_B^F)}{\partial \tau} < 0 \) as \( \frac{\partial T^B_2}{\partial \tau} < 0 \) and \( \frac{de_B^*}{d\tau} < 0 \).
We also have \( \frac{\partial (e_B^* - e_B^F)}{\partial \kappa} > 0 \) as \( \frac{\partial T^B_2}{\partial \kappa} > 0 \) and \( \frac{de_B^*}{d\kappa} > 0 \).

The signs of \( \frac{\partial (e_B^* - e_B^F)}{\partial \beta} \) and \( \frac{\partial (e_B^* - e_B^F)}{\partial \kappa} \) are ambiguous as
\[
\frac{\partial T^B_2}{\partial \beta} \propto \frac{(1 - \beta_B)\beta_B}{1 - e_B^*(1 - \beta_B)} [1 - (1 - \tau) (1 - \kappa)] - \beta_B\kappa
\]
is ambiguous and
\[
\frac{\partial T^B_2}{\partial \kappa} \propto \frac{(1 - \beta_B)\beta_B}{1 - e_B^*(1 - \beta_B)} (1 - \tau) - \beta_B
\]

60
is also ambiguous. ■

Proof of Corollary 5:

**Proof.** Recall that $e_{B3}^*$ is defined by

$$
(1 - q)(1 - \beta_B)(1 - \beta_B)(v_B^b(\phi) - v_B^g(\phi)) - ce_{B3}^*
$$

$$
= \frac{1}{2}(1 - q)(1 - \beta_B)(u_B^q(\phi) - u_B^b(\phi)) - ce_{B3}^*
$$

$$
= \frac{1}{2}(1 - q)(1 - \beta_B)[X + \tau(1 - \kappa)L_B] - ce_{B3}^*
$$

$$
= 0
$$

where the first equality comes from equation (19) evaluated at $e_{B3}^*$. Intuitively, the firm chooses $e_{B3}^*$ such that its marginal benefit, which comes from finding $\omega_B = 0$ (which happens with probability $1 - \beta_B$) and disclosing it so as to obtain a lower face value from the lender (in the amount of $u_B^q(\phi) - u_B^b(\phi)$) when the state is bad (which happens with probability $1 - q$). The disclosure decisions at $\omega_B = 1$ does not matter as the manager is indifferent from disclosing $\omega_B = 1$, i.e., he gets the same off whether disclose or not, i.e., he cannot extract any surplus in this case. We can solve for $e_{B3}^*$ as

$$
e_{B3}^* = \frac{(1 - q)(1 - \beta_B)[X + \tau(1 - \kappa)L_B]}{2c}
$$

$$
= \frac{(1 - q)(1 - \beta_B)[X + \tau(1 - \kappa)(y - X)]}{2c}
$$

Therefore,

$$
\frac{\partial e_{B3}^*}{\partial X} \propto \frac{1 - \tau(1 - \kappa)}{2c} > 0;
$$

$$
\frac{\partial e_{B3}^*}{\partial y} \propto \frac{\tau(1 - \kappa)}{2c} > 0;
$$

$$
\frac{\partial e_{B3}^*}{\partial \tau} \propto \frac{(1 - \kappa)(y - X)}{2c} > 0;
$$

$$
\frac{\partial e_{B3}^*}{\partial q} \propto -\frac{(1 - \beta_B)}{2c} < 0;
$$

$$
\frac{\partial e_{B3}^*}{\partial \beta_B} \propto -\frac{1 - q}{2c} < 0;
$$

61
and
\[
\frac{\partial e_{B3}^*}{\partial \kappa} \propto -\frac{(1-q)(1-\beta_B)}{2c} < 0.
\]

\begin{proof}
Proof of Corollary 6:

\textbf{Proof.} We now compare \(e_{B3}^*\) and \(e_{FB}^*\). Note that
\[
e_{FB}^* = \frac{(1-q)\beta_B \kappa L_B}{2c}.
\]

Therefore
\[
e_{B3}^* - e_{FB}^* = \frac{(1-q)\{X + \tau(1-\kappa)L_B - \beta_B \kappa L_B\}}{2c}
= \frac{(1-q)\{X + \tau(1-\kappa)(y - X) - \beta_B \kappa (y - X)\}}{2c}.
\]

Note that \(e_{B3}^* - e_{FB}^* > 0\) implies that
\[
(1-\beta_B)[X + \tau(1-\kappa)L_B] - \beta_B \kappa L_B > 0.
\]

Therefore,
\[
\frac{\partial(e_{B3}^* - e_{FB}^*)}{\partial X} \propto (1-\beta_B)[1-\tau(1-\kappa)] + \beta_B \kappa > 0;
\]
\[
\frac{\partial(e_{B3}^* - e_{FB}^*)}{\partial \tau} \propto \frac{1-\beta_B}{2c} > 0;
\]
\[
\frac{\partial(e_{B3}^* - e_{FB}^*)}{\partial q} \propto \frac{1}{2c} < 0;
\]
\[
\frac{\partial(e_{B3}^* - e_{FB}^*)}{\partial \beta_B} \propto \frac{1-q}{2c} < 0;
\]
\[
\frac{\partial(e_{B3}^* - e_{FB}^*)}{\partial \kappa} \propto \frac{1-q}{2c} < 0.
\]

Finally,
\[
\frac{\partial(e_{B3}^* - e_{FB}^*)}{\partial y} \propto (1-\beta_B)\tau(1-\kappa) - \beta_B \kappa
\]
and thus is ambiguous. \hfill \blacksquare
\end{proof}
References


