DISCLOSURE PATERNALISM*

Jeremy Bertomeu

March 5, 2022

Abstract

Investors lacking good judgment may miscalculate the strategic motives causing withholding of material information. The resulting inadequate professional skepticism encourages excessively optimistic expectations after a non-disclosure and breaks the economic forces inducing forthcoming voluntary disclosure. A regulator may intervene to correct the problem by mandating disclosure over events that would otherwise be withheld; however, such paternalistic interventions come with a drawback: over-protection prevents investors from learning to be skeptical through repeated experiences of non-disclosure losses. While an unregulated market will converge over time toward greater transparency, paternalism may lead to cycles characterized by high levels of compliance followed by excessive optimism. The model further predicts negative market reactions to regulation, an association between positive price drift and transparency, and when regulators prefer to shut down entire markets.

Keywords: accounting, standards, regulation, disclosure, unravelling.

JEL codes: D5, D6, G1, G2, G3, G4, M4.

*Jeremy Bertomeu is from the Olin School of Business at Washington University, Campus Box 1156, One Brookings Drive St. Louis, MO 63130-4899. Many thanks to J. Barrios, E. Cheynel, R. Dye, P. Dybvig, R. Frankel, P. Liang, P. Gao, I. Marinovic, X. Martin and other seminar participants at SWUFE and Washington University in St Louis for valuable suggestions.
M. Seguin was at a loss for words. Yet another one of his cherished goats was going to be devoured by the wolf. He put some thought into the love he felt for his dear Blanquette and said “Good, now I know and I am determined to save you, despite that terrible force that’s pulling you to the mountain. I know you’ll try and chew your chord, so I’m closing you up into a pen, so you will stay with me forever!”

from Monsieur Seguin’s Goat (1866), Alphonse Daudet

In his children tale, Alphonse Daudet describes the impulse of a young goat to wander despite the certitude of being ultimately eaten by a wolf. Its owner, convinced that no rational argument will keep her safe, chooses to restrain her but, ultimately, his efforts are in vain and the goat escapes toward her fate. The tale resonates with many forms of regulation that are commonly recognized, yet rarely openly discussed by economists. Paternalism is a theory in which a benevolent government corrects individual errors in decision-making, presupposing a role for intervention other than the traditional viewpoint of addressing social externalities. Most modern societies feature some explicit acceptance of paternalistic regulations such as laws preventing voluntary servitude, prohibitions on victimless crimes or mandatory contributions to retirement and healthcare.

There are, however, sound arguments against an unrestricted application of this principle given that most theories of behavior do not view individuals as devoid of self-interest even absent supervision. The law and economics literature recognizes that there is a potential benefit of paternalism, provided the theory is organized around a behavioral model that, in particular, recognizes individuals’ information and choices and what they learn from the environment, see, e.g., Thaler and Sunstein (2003), O’Donoghue and Rabin (2003), and Ambuehl et al. (2021). Despite continuing interest in this area, there are continuing debates about the desirable scope of paternalism and which decision problems are best addressed with intervention.

I adopt in this study the narrower context of verifiable communication. This setting remains highly consequential in the development of modern capital markets: models of disclosure present a natural solution to the classic lemon’s problem of Akerlof (1970) with early unravelling theorems showing that sellers would voluntarily give all of their information prior to a sale and resolve the lemons problem (Milgrom 1981). Observed failures of the theorem present an unresolved puzzle and contribute to recurrent market breakdowns.

The model is built around a principal behavioral premise: individuals learn adaptively over time, an-
oring their expectations on realized past experience. They lack appreciation for the strategic component of disclosure choices and, instead, dynamically adapt using realized outcomes to set their present beliefs. A regulator can implement paternalistic disclosure regulations, which can mandate the disclosure of news that would not be correctly assessed by behavioral individuals. However, such regulations lead to a price path in which individuals insulated from bad news no longer learn and, therefore, convergence toward high levels of voluntary disclosure is prevented. In the long-run, paternalistic regulations may lead to overly optimistic beliefs and reduce transparency.

Contexts in which regulations mandate disclosures that could have been made voluntarily are common in practice. Restaurants must issue a health grade and are not allowed to operate without such disclosure. Grades are verifiable, nearly costless to post voluntarily and known to the restaurant owner; hence, that many establishments did not post their grades voluntarily and that better health outcomes followed the disclosure mandate (Jin and Leslie 2003) is in contradiction to the unravelling principle. Similar findings are documented in other contexts. When required to publicly report workforce accidents in their financial statements, mining companies were found to reduce their productivity and accident rates (Christensen et al. 2017). Presumably, these companies would know their accident rates and reporting entails no direct cost (as they were reporting in other filings). The limited information dissemination prior to the mandate suggests that investors had incorrect expectations about mining risks.

Many other examples exist in the financial industry: firms cannot redact their financial statements without approval by the Securities and Exchange Commission (Heinle et al. 2020), omit an auditor report or material information, even though the unravelling principle would suggest that no such mandates are necessary. Most financial securities and money management instruments require disclosures, such as past performance or portfolio composition, that cannot be voluntarily withheld. In the U.S. and most countries with active capital markets, a publicly-owned corporation is required to file financial statements according to generally-accepted accounting practices (GAAP). The corporation does not have the option to customize its own accounting standards and substitute its own pro-forma financial statements, implying that regulators do not fully trust markets to discipline firms to make forthcoming disclosures.¹

An extensive empirical evidence rejects that individuals immediately regard with the greatest skepticism any non-disclosure, even if they know that the non-disclosing firm was informed and omitted this informa-

¹Regulators have been historically distrustful of firms’ motives when choosing their own measurements. Regulation G requires all firms issuing non-GAAP financial statements to provide a reconciliation detailing itemized differences between non-GAAP and GAAP.
tion with strategic intent (Dickhaut et al. 2003; Jin et al. 2015; Zhou and Zhou 2020; Bourveau et al. 2020). This incomplete strategic understanding may lead an investor to view the information too optimistically. As the experience repeats, investors will adapt their expectation to become more skeptical, unravelling in the long run. Such convergence, however, may take many repetitions and a regulator may wish to intervene to protect the investor against error. By doing so, however, the regulator necessarily insulates the investor from the costs of mispricing. Under protection of the law, the individual can ignore the costs of strategic behavior but will never fully learn.\footnote{The study by Barrios et al. (2021) develops a related theory: entrepreneurs exposed to pioneer areas, where learning occurred with little or no social protection, transmitted values encouraging entrepreneurship across generations. Their theory suggests that exposure to experimentation may have led to better outcomes in the long run.}

The study is organized in five sections. Section 1 presents the main model, and includes a definition of the dynamics of disclosure and prices and the learning process. Section 2 contrasts the dynamics of investor beliefs, and the associated mispricing, by comparing laissez-faire to regulated environments. Section 4 shows that periods of higher (lower) contemporaneous disclosure are usually predictive of positive (negative) price drift. Section 5 explores various extensions of the baseline model that can affect learning dynamics and the desirability of regulation.

\section{The Model}

A short non-technical overview of the model follows. Firms may make verifiable voluntary disclosures each period to capital markets. Investors are miscalibrated and do not understand a non-disclosure event strategically; instead, they price the firm according to adaptive expectations, using recent realized cash flows to anchor their current non-disclosure expectation. They are more optimistic in the current period if undisclosed news was favorable, and more pessimistic if it was unfavorable. Firms strategically withhold information, implying that behavioral investors using an adaptive (stale) expectation may systematically overpay conditional on non-disclosure. Regulators can step in and mandate disclosure of events that would be strategically withheld in order to reduce investor losses. However, with some probability, a financial innovation allows firms to bypass the regulation, leaving investors with no other protection but their own skepticism toward non-disclosure. The more protection investors receive when regulations are effective, the less skeptical they become and the more miscalibrated they are to the financial innovation. Understanding the consequences of this trade-off is the main objective of the study.
Time is denoted $t = 0, 1, \ldots, \infty$ and firms are indexed by $i \in [0, 1]$. Each period, with probability $q \in (0, 1)$, firms privately observe an i.i.d. cash flow $\tilde{x}_{i,t}$ drawn from a distribution with support over $[x, \pi]$, c.d.f. $F(.)$, p.d.f. $f(.)$, and finite mean $\mu$. This event is objectively verifiable and can be disclosed at no cost. With probability $1 - q$, the firm does not observe any information, or equivalently, the information is soft and unverifiable (Dye 1985; Jung and Kwon 1988). This assumption is meant to capture the inherent incompleteness of the mapping from economic events to transactions: later on, I also discuss the special case in which $q$ is exactly zero, when the receipt of information is common-knowledge.

As the critical channel for the model is a feedback from learning each period, $\tilde{x}_{i,t}$ is assumed to be observed at the end of the period. It is unimportant for the model if $\tilde{x}_{i,t}$ is observed with noise and, therefore, one period in the model should be interpreted the time horizon to receive information about undisclosed events. As an example, a firm may make a forecast of earnings in the next year (or withhold that forecast) and earnings realize in the following earnings announcement - allowing investors at this point to compute average earnings as a function of whether the firm did or did not forecast.

At the start of the period, firms issue a report $d_t(\tilde{x}_{i,t}) \in \{\tilde{x}_{i,t}, ND\}$. The uninformed firm must issue $d_t(\tilde{x}_{i,t}) = ND$, hereafter no-disclosure. In comparison, the informed firm is subject to a regulation $k_t \in \{0, \hat{k}\}$, which indicates a probability that the firm is required to disclose when informed. Hereafter, $k_t = \hat{k} \in (0, 1]$ indicates a disclosure mandate while $k_t = 0$ indicates laissez-faire. If $\hat{k} = 1$, the disclosure mandate is potentially perfect while $\hat{k} < 1$ reflects situations in which the law cannot describe all possible transactions.

Certain periods feature breakdowns in reporting and enforcement. There is an event each period such that a new financial innovation is developed, denoted by an indicator variable $\tilde{\theta}_t \in \{0, 1\}$, where $\tilde{\theta}_t$ may be serially correlated with $Pr(\tilde{\theta}_t = 1|\tilde{\theta}_{t-1} = 1) = p_1 \geq .5 \geq Pr(\tilde{\theta}_t = 1|\tilde{\theta}_{t-1} = 0) = p_0$. Put differently, there is an arms race between financial experts, accounting innovations and regulators, such that regulators intervene with delay (Glode et al. 2011; Dye et al. 2015). For example, empirically, Hail et al. (2018) and Bourveau et al. (2021) provide evidence consistent with the arms race theory, with regulations typically

---

3 The proofs are readily adapted to the case of unbounded support, if $x = -\infty$ or $\pi = \infty$, so I use generically $[x, \pi]$ to refer to an arbitrary support.

4 To reduce notation, the disclosure $d_t(\tilde{x}_{i,t})$ is defined implicitly as a function of whether the firm receives the information and the current disclosure mandate. Formally, one can explicitly write $d_t(\tilde{x}_{i,t}; \tilde{\nu}^1_t, \tilde{\nu}^2_t)$, where $\tilde{\nu}^1_t$ is an indicator variable equal to one when the firm is informed and $\tilde{\nu}^2_t$ is an indicator variable equal to one when the mandate is effective. Then, (i) $d_t(\cdot; 0, \cdot) = ND$, i.e., an uninformed firm does not disclose, (ii) $d_t(x; 1, 1) = x$, i.e., an informed firm subject to the mandate discloses and $d_t(x; 1, 0) \in \{x, ND\}$ describes the strategic firm. With some minor adjustments, all the results can be shown more generally if $k_t$ varies on $[0, 1]$ or, alternatively, if $k_t = \hat{k}$ under laissez-faire.
lagging firms’ innovation in fraudulent or misleading practices.⁵

When the disclosure mandate is in place and there is no innovation (\( \tilde{\theta}_t = 0 \)), informed firm are required to disclose.⁶ With the financial innovation (\( \tilde{\theta}_t = 1 \)), there is an alternative recording of the event that can avoid the disclosure requirement, so that firms are no longer subject to mandatory disclosure. This event can have multiple interpretations, such as innovations in financial securities or means to evade a threshold classification for certain types of news (Dye et al. 2015). Informed firms that are not subject to mandatory disclosure, either because there is no disclosure mandate or they are able to sidestep the regulation, choose \( d_t(x) \in \{x, ND\} \) to maximize their market price \( P_t(x) = x \) forms conditional on disclosure and \( P_t(ND) \) conditional on non-disclosure. Hence, \( d_t(x) \) is chosen to satisfy

\[
P_t(d_t(x)) \geq \max(x, P_t(ND))
\]

for any \( x \).⁷ As is common in this type of model, a firm discloses voluntarily if and only if the information received is sufficiently favorable \( x \geq P_t(ND) \).

The next part of the model departs from the standard rational expectations approach by assuming that investors adapt dynamically to observed non-disclosing firms after each period to update their non-disclosure beliefs. Specifically, investors are learning to play the equilibrium by iterating best responses to non-disclosure (Jin et al. 2015; Zhou and Zhou 2020; Bourveau et al. 2020). Following research on miscalibrated expectations, see, e.g., Evans and Ramey (2006) or Orphanides and Williams (2007), assume below that expectations are adaptive as a function of realized cash flows in the prior period conditional on non-disclosure.⁸

---

⁵The context of derivatives, which became widespread across the 80s and 90s across non-financial firms may illustrate these properties of the model (Géczy et al. 1997). Derivatives can be complex instruments that are difficult to comprehensively codify and can have speculative or hedging rationales (sometimes both). Innovations in the derivative markets led to regulators trailing financial innovations and, repeatedly, bear unexpected losses via unreported off-balance sheet exposures. A complete overhaul of accounting for derivatives was completed in FAS 133 (effective 2000), and, subsequently, standard-setters imposed many updates directly related to options, such as reporting obligations over off-balance sheet financing (Sarbanes-Oxley Act of 2004), revenue recognition with purchase options (ASC 606, effective 2017) and leases (ASC 842, effective 2019 but delayed for certain issuers). Continuing debates about these transactions suggest that settings with a high rate of financial innovation present a particular challenges to regulation.

⁶In additional analyses in Section 4.3, I consider other types of asymmetric regulation in which the regulation applies to bad or good news. As will become clear by solving the baseline model, these alternative formulations do not affect the main intuition.

⁷Without loss of generality, one may interpret the manager as selling the firm to a new generation of investors, where the value of the firm is an increasing continuous function of the posterior expectation \( P_t \). Later, assume that the regulator may care about more information in the form of a convex payoff \( \phi(P_t) \): a special case of this objective function may occur under the assumption that \( \phi(P_t) \) is the price paid by the firm and the regulator maximizes expected prices.

⁸There is a growing literature in accounting exploring behavioral deviations from rational expectations anchored on fundamental cash flows. While it would be too extensive to review fully, a few notable contributions from this work are given below. Fischer
Let the initial non-disclosure belief $P_0(ND) = \mu_0 > \bar{x}$ and suppose that, as time progresses, beliefs are updated according to the law of motion

$$
P_{t+1}(ND) = \mathbb{E}(\tilde{x}_{i,t} | d_t(\tilde{x}_{i,t}) = ND, k_t, \tilde{\theta}_t = \theta_t).
$$

The conditioning event “$d_t(\tilde{x}_{i,t}) = ND$” does not follow rational expectations: investors incorrectly set the non-disclosure price $P_{t+1}(ND)$ today to the average non-disclosure realizations observed at the end of the prior period, the right-hand side of (2), rather than the correct (rational) conditioning event “$d_{t+1}(\tilde{x}_{i,t}) = ND$.”

Developing the conditional expectation in (2),

$$
P_{t+1}(ND) = \frac{q(1 - k_t(1 - \theta_t))F(P_t(ND))\mathbb{E}(\tilde{x}_{i,t} \leq P_t(ND)) + (1 - q)\mu}{q(1 - k_t(1 - \theta_t))F(P_t(ND)) + 1 - q} \equiv \zeta(P_t(ND); k_t(1 - \theta_t)),
$$

since, with probability $q(1 - k_t(1 - \theta_t))F(P_t(ND))$, a firm withholds strategically and realizes an expected cash flow $\mathbb{E}(\tilde{x}_{i,t} \leq P_t(ND))$ and, with probability $1 - q$, the firm makes a non-disclosure due to lack of information, which implies an expected cash flow $\mathbb{E}(\tilde{x}_{i,t}) = \mu$. The function $\zeta(\cdot)$, captures how investors update their belief after period $t$. The equation is similar to Jung and Kwon (1988) except that the adaptive belief $P_t(ND)$ is used instead of a rational expectation.

In what follows, to simplify notation, $\zeta_1(\cdot) = \zeta(\cdot; \hat{k})$ refers to the update when the regulation is effective and $\zeta_0(\cdot) = \zeta(\cdot; 0)$ when there is no regulation or there is an unregulated financial innovation. When there is no ambiguity or a statement applies to both cases, $\zeta(\cdot)$ refers in short-hand to the updating function. Figure 1 summarizes the sequence of events and notation.

Note that if the end-of-period realization is $\tilde{y}_{it} = \tilde{x}_{it} + \epsilon_{it}$, where $\epsilon_{it}$ is white noise with mean zero, the belief $P_t(ND) = \mathbb{E}(\tilde{y}_{it} | d(\tilde{x}_{i,t}) = ND)$ is equal to the realizations averaged around all non-disclosing firms. Simplifying $\mathbb{E}(\epsilon_{it} | d(\tilde{x}_{i,t}) = ND) = 0$ yields equation (2) and only requires a noisy firm-level observation of $\tilde{x}_{i,t}$ conditional on non-disclosure. This result can be strengthened to $\epsilon_{it}$ correlated across firms, as long as disclosed $\tilde{x}_{i,t}$ and $\tilde{y}_{i,t}$ are sufficient to extract common factors.

and Verrecchia (1999) examine decision rules in the form of approximate heuristics. Hirshleifer and Teoh (2003) develop a model in which investors may not use all the available public information. In Bloomfield and Fischer (2011), investors may have different posterior beliefs conditional on the same public information. Fischer et al. (2016) examine the dynamic rational pricing of a bubble component. To my knowledge, this work does not focus on adaptive expectations; this type of behavioral process captures how investor adapt their expectations as a function of past errors and is central to the question asked in this study.
Informed firms observe $\bar{x}_{i,t}$ with prob. $q$ and, if uninformed, must report $d_t(\bar{x}_{i,t}) = ND$.

Informed firms report $d_t(\bar{x}_{i,t}) = \bar{x}_{i,t}$ if constrained by regulation; otherwise, $d_t(\bar{x}_{i,t}) = ND$ iff $\bar{x}_{i,t} \leq P_t(ND)$.

New investor belief $P_{t+1}(ND)$ forms from eq. (2).

The next lemma, shown in prior literature, is a key step to many results in this type of model.\(^\text{10}\)

**Lemma 1.1** For $z \in \{0, 1\}$, $\zeta_h(\cdot)$ is $U$-shaped with a unique minimum $\tau^*_h$; this minimum also satisfies the rational equilibrium condition $\zeta_h(\tau^*_h) = \tau^*_h$.

To summarize, the fixed point $\tau^*_h$ of $\zeta_h(\cdot)$ corresponds to the standard Nash equilibrium solution because it solves for $P_{t+1}(ND) = \zeta_h(P_{t+1}(ND))$ such that investors price a non-disclosure event with rational expectations. The solution satisfies a minimum principle property, achieving the lowest possible non-disclosure price $\zeta_h(\cdot)$ given any conjectured disclosure choice. In other words, relative to any other belief, the rational expectations solution $\tau^*_h$ is the one with lowest non-disclosure price and, therefore, the highest likelihood of disclosure.

**Discussion of the Assumptions.** The model is stated with several stylized assumptions to make the argument as transparent as possible by minimizing clutter; the discussion below addresses interpretations and possible variations.

**Reporting frictions.** The model incorporates two frictions: a friction to information endowment (or verifiability) and, to avoid a trivial solution where disclosure mandates can implement maximal disclosure, a friction to the effectiveness of regulations. Suppose, instead, that firms are always informed and, absent regulation, disclose with probability one, i.e., $k = q = 1$. In this corner of the model, beliefs $P_t(ND)$ must decrease conditional on ineffective regulations and, if the regulation is effective, the equilibrium will feature full-disclosure. Hence, investors cannot condition their belief on the current non-disclosures and may,

---

\(^{10}\)For a proof that the minimum of $\zeta$ is unique, see Lemma 1 in Cheynel (2009) and Proposition 1 in Acharya et al. (2011).
realistically, consider the most recent period with non-disclosure $P_{t-k}(ND)$ to form their expectations.\textsuperscript{11}

Therefore, in this model, beliefs cannot decrease and, while regulations will slow the learning process, the probability of non-disclosure will always converge, in the long run, to full-disclosure. However, this property is fragile and only occurs in the knife-edge corner with full-disclosure under effective regulation; in fact, as will be shown later on, if the friction is small but non-zero $q \approx 1$, a period with near full disclosure would imply that non-disclosure will be “almost” certainly due to lack of information and therefore make investors least skeptical $P_1(ND) \approx \mu$.

\textit{Learning Process.} The learning process is kept to its simplest form where investors naively form their expectation as the past non-disclosure. My conjecture is that similar intuitions will hold over a more general class of models in which past non-disclosures affect pricing errors, even without equating current prices and a prior non-disclosure - that is, if one writes more generally the update $P_{t+1}(ND)$ in (3) as a fixed point incorporating jointly the behavioral and rational beliefs such as, for example, $P_{t+1}(ND) = \rho \zeta \theta(P_t(ND)) + (1 - \rho) \zeta \theta(P_{t+1}(ND))$. Relatedly, Aghamolla and Smith (2021) propose a price function that is an average of the beliefs of sophisticated and unsophisticated investors. Similar dynamics can be obtained using level-k beliefs similar to Bourveau et al. (2020), see section 4.4. Another motivation is that the expectation may reflect entry by new unsophisticated investors with little history to form their beliefs and using recent realizations rather than a complete understanding of the game.

\textit{Value of information.} As is common in disclosure models, there is no purpose here for regulation within the scope of the modelled parties because the expected cash flow is exogenously given. At this point, no normative judgment is made about the desirability or undesirability of regulation - observations are made only about consequences on investor behavior. As will be shown in Section 4.2, the model can be completed with a specific decision affected by the disclosure with minor changes to the analysis, if firms prefer to send more favorable information. Such a model involves putting more structure on the regulator’s preferences, so implications about the preferences of regulators (or why regulations exist) are deferred until later sections.

\textsuperscript{11}I thank the reviewer for this suggestion. Note that one could similarly use a weighted average of prior periods with a similar insight. More generally, predictions relative to cycles require a regulation period where non-disclosures are excessively skeptical (relative to the regulation in place). Along the long-run convergence to the rational threshold, all results derived in the main model apply as a special case to this setting; of course, the “long run” may not be a practical criterion and the process may be constantly renewed if new firms constantly enter and exit, or, within the firm, new generations of investors or new cash flow generating activities come to be.
Transaction types and decay. Implied by the serial correlation in \((\theta_t)\), the model features an innovation that is persistent but may be randomly addressed by regulators given time. One can express the same economic trade-off with the random occurrence of a transaction allowing a decaying proportion \(\rho_t\) of firms to avoid the regulation, with \(t'\) representing the time since the transaction appeared. Similarly, it is unimportant for the main results if the transaction is only feasible for a subset of all the firms.

Asymmetric disclosure. The disclosure mandate is unconditional and requires firms to disclose irrespective of the income implication: a large number of accounting rules are based on economic characteristics of the transaction such as reporting a hedging position, a purchase commitment or stock option expenses. Other reporting rules are asymmetric and favor reporting over potentially adverse outcomes (Basu 1997; Watts 2003). Asymmetries in regulations cannot remove the negative effect of regulation on learning but they can affect the speed of investor learning, especially if investors are insulated from large losses. In Section 4.3, I formally extend the analysis to asymmetric reporting requirements (Goex and Wagenhofer 2009; Guay and Verrecchia 2018).

2 Dynamics of Investor Learning

2.1 Laissez-Faire

This section examines the special case of this model in which the regulation is set to \(k_t = 0\) so that no firm is required to disclose information, hereafter, the laissez-faire (or unregulated) economy. To examine the dynamic properties of the model, one needs to first characterize the convergence process when investors adapt their beliefs by responding to a sequence of pricing errors.

The pricing errors are obtained from (3) when evaluating at \(k_t(1 - \theta_t) = 0\). The following benchmark demonstrates that this economy always converges to a rational equilibrium with the highest possible level of transparency.

**Proposition 2.1** Let \(k_t = 0\) for any \(t\), i.e., there is no disclosure regulation. Then, \(P_t(ND)\) (and, hence, the probability of non-disclosure) is decreasing for any \(t \geq 1\) and converges to the belief \(\tau^*_0\) under rational expectations. In particular, if firms are very likely to be informed, \(q \to 0\), the equilibrium unravels to full-disclosure.
When firms are very likely to be informed, the argument behind Proposition 2.1 is in line with standard unravelling theory. Standard unravelling implies that investors instantly realize an infinite sequence of higher-order beliefs, by updating their non-disclosure beliefs against a suspected amount of withholding and such that withholding best responds to their pricing choices. Here, the process does not occur immediately. Investors correct their pricing errors over time, becoming increasingly skeptical which, in turn, disciplines firms to be forthcoming. The same intuition holds of course if $q > 0$ in which case beliefs unravel to the equilibrium $P_t(ND) \rightarrow \tau_0^*$ in Jung and Kwon (1988). This belief is know to satisfy the minimum principle: it achieves the lowest possible non-disclosure price (Acharya et al. 2011; Guttman et al. 2014). In the long-run, therefore, investors always settle toward the greatest possible skepticism and the probability of disclosure increases.

Figure 2 below illustrates convergence in the special case of Normally-distributed uncertainty $\tilde{x}_{it} \sim N(0, 1)$. The update in (3) can be rewritten in terms of a modified inverse Mills ratio:

$$P_{t+1}(ND) = \frac{-qf(P_t(ND))}{qF(P_t(ND)) + 1 - q}. \tag{4}$$

The belief process converges to the fixed point $\tau^*$. In the limit case of $q \approx 1$, which updates according to the lower curve, $P_{t+1}(ND) = f(P_t(ND))/F(P_t(ND))$ is exactly Mill’s ratio, which is increasing and
lower than \( P_t(ND) \). Because Mill’s ratio \( \frac{f(y)}{F(y)} \) is asymptotically equal to \( y \) in the lower tail, the belief error \( |P_{t+1}(ND) - P_t(ND)| \) also becomes smaller over time. In summary, the adaptive model makes a more nuanced prediction than standard equilibrium theory: firms initially fail to disclose in a forthcoming manner, exploiting errors in beliefs, but the probability of disclosure increases over time. This prediction is consistent with the observed time trends toward more comprehensive disclosure as well as evidence pertaining to unregulated disclosure environments (Bourveau et al. 2020).

**Corollary 2.1** For any logconcave distribution with \( q \) sufficiently small or if \( \tilde{x}_{it} \) is normally-distributed, the belief error \( |P_{t+1}(ND) - P_t(ND)| \) decreases for any \( t \geq 1 \).\(^{12} \) Further, the pricing error \( \mathbb{E}((\tilde{x}_{it} - d_t(\tilde{x}_{it}) = ND)) \) is decreasing over time.

As time progresses, investors under laissez-faire learn the rational equilibrium and, along this convergence process, strategic firms dynamically re-adapt their withholding choices to the lower non-disclosure price. Price efficiency increases over time, that is, the pricing error \( \mathbb{E}((\tilde{x}_{it} - d_t(\tilde{x}_{it}))^2) \) is reduced. This property can be intuitively obtained by joining together two heuristic arguments. First, conditional on any \( \tilde{x}_{it} \), the probability of disclosure increases over time because the disclosure threshold \( P_t(ND) \) decreases toward \( \tau^* \). So every event is more likely to have no pricing error. Second, for events in the non-disclosure region, the error is less than the more precise expectation \( P_{t+1}(ND) < P_t(ND) \) since \( P_{t+1}(ND) \) is closer to the “correct” conditional expectation.

Note that the variance conditional on non-disclosure need not decrease as investors learn, if non-disclosures are increasingly dominated by tail events that feature more uncertainty. To see this, continuing on the example of \( \tilde{x}_{it} \sim N(0, 1) \), for any belief \( P_t(ND) \in (\tau^*, 0) \), one can rewrite the conditional variance

\[
Var(\tilde{x}_{it}|d_t(\tilde{x}_{it}) = ND) = 1 + \zeta_0(P_t(ND))(P_t(ND) - \zeta_0(P_t(ND))) < 0.
\]

(5)

As noted in prior work by Dye and Hughes (2017), the right-hand side of (5) is equal to one at the rational equilibrium \( \tau^* = \zeta_\theta(\tau^*) \), at which case the uncertainty conditional on non-disclosure is exactly equal to the unconditional variance. Before this point is reached, the conditional variance is always lower, \( Var(\tilde{x}_{it}|d_t(\tilde{x}_{it}) = ND) < 1 \). Hence, as investor learn to become more skeptical, the uncertainty conditional on non-disclosure must eventually increase.

\(^{12}\)A distribution \( F(.) \) is logconcave if \( \ln(F(x)) \) is concave. Logconcavity of the distribution is satisfied by most common distribution functions, see Bagnoli and Bergstrom (2005) Tables 1 and 3.
Corollary 2.2. Let there be two distributions for $\tilde{x}_{i,t}$, with c.d.f. indexed with upper scripts $j = 1, 2$, and starting from the same initial non-disclosure price $\mu_0$. If $F_2$ reverse hazard rate dominates $F_1$, the non-disclosure price $P^1_t(ND)$ is lower than $P^2_t(ND)$ for any $t \geq 1$.\(^{13}\)

Corollary 2.2 shows that certain factors accelerate convergence toward unravelling, in the sense of implying more skepticism over the entire dynamic path of prices. If a distribution is more unfavorable, featuring lower values in the sense of the reverse hazard rate, outcomes are less favorable under non-disclosure and unravelling occurs faster. The hazard rate condition, which is stronger than first-order stochastic dominance (but weaker a monotone likelihood ratio property), guarantees that truncating the distribution preserves the comparison between the two distributions.

Of note, the Corollary applies to comparative statics on prices and cutoffs rather than probabilities of disclosure since these are also affected by changes in distributions. There is no general result on these probabilities but the example of the Normal distribution can offer telling intuition.

Corollary 2.3. Suppose $\tilde{x}_{i,t} \sim N(m, \sigma^2)$. The probability of disclosure $1 - F(P_t(ND))$ is increasing in $m$ and $\sigma$ for a given $\mu_0$. If $\mu_0$ is a function of $(m, \sigma)$ such that $(\mu_0 - m)/\sigma$ is held constant, the probability of disclosure does not depend on $m$ or $\sigma$.

The second part of Corollary 2.3, expressed by holding the normalized starting belief constant, is the most intuitive and yields a key intuition for the rest of the Corollary. Consistent with observations in Acharya et al. (2011), a change in the mean or variance of the random cash flows, leaving all else equal, is equivalent to a Von Neumann-Morgenstern linear transformation of payoffs and, while it rescales price, has no effect on the probability of observed disclosure choices. In particular, a change in mean and variance does not affect choices as long as the prior belief is proportionately changed. In comparison, if the initial belief $\mu_0$ is kept fixed, a change in the mean or variance of the distribution may slow down or accelerate the convergence. An increase in the mean $m$ leads to more favorable beliefs along the entire price path and increase disclosure as firms intend to report that their information is more favorable than the current belief - see also Acharya et al. (2011) for an application of this argument in a dynamic disclosure setting. An increase in the uncertainty $\sigma^2$ leads to more dispersed beliefs and, given that low outcomes become commensurately more likely conditional on no-disclosure, implies lower prices and facilitates convergence to unravelling.\(^{14}\)

\(^{13}\) $F_2$ reverse hazard rate $F_1$ dominates if $f_2(x)/F_2(x) > f_1(x)/F_1(x)$ for any $x$.

\(^{14}\) In the proof, it is show that the observation holds more generally for any class of homothetic distributions such that $\tilde{x}_{i,t} = \sigma^2 \tilde{x}_{i,t}^0 + m$ can be written as a linear function of a random variable $\tilde{x}_{i,t}^0$ that does not depend on $m$ or $\sigma$. 

13
2.2 Regulated Economy

Consider the dynamics of investor learning, and its associated disclosure decisions, when a regulator may affect strategic withholding by setting a mandatory disclosure regulation. A disclosure mandate \(k_t = \hat{k}\) is imposed on reporting firms in all periods. Conditional on no financial innovation, the non-disclosure price in (3) can be evaluated to

\[
P_{t+1}(ND) = \zeta_1(P_t(ND)) = \frac{q(1-\hat{k})F(P_t(ND))\mathbb{E}(\tilde{x}_{it}|\tilde{x}_{it} \leq P_t(ND)) + (1-q)\mu}{q(1-\hat{k})F(P_t(ND)) + 1-q}. \tag{6}
\]

In (6), regulation reduces the probability of a strategic disclosure by \(\hat{k}\) but it also makes investors less skeptical against no-disclosure - placing more weight on the unconditional mean \(\mu\) after current cash flows realize. If the regulation can be made arbitrarily effective with \(\hat{k}\) close to one, for example, investors will almost entirely ignore self-selection in the next period. Hence, the effect of regulation on investor skepticism depends on the ratio of regulation effectiveness to reporting frictions.

A process for \(P_t(ND)\) is illustrated in Figure 3, starting with two initial periods with a regulation set at \(\hat{k}\) and no financial innovation. The process converges toward point (a), featuring high levels of compliance and a relatively high non-disclosure belief. The beliefs errors are small and the regulation is successful at mitigating adverse selection into non-disclosure. Then, after a financial innovation, \(\theta_t = 1\) and the updating function \(\zeta_1(P_t(ND))\) switches to the lower curve. Investors make large errors and the updating function adapts to more severe adverse selection. As the innovation is unlikely to be addressed immediately, the process continues along this non-compliance stage for several periods toward the lower point (b), learning in the process more skepticism and reducing over time the probability of strategic withholding.

Up to this point, investors have only become more skeptical throughout the process, in line with laissez-faire, until the financial transaction is ultimately addressed and the process reverts back to \(\theta_t = 0\). At this regulation stage, the belief changes to the upper updating curve because strategic withholding is reduced by the mandate. The non-disclosure price is then more skeptical than actual realized cash flows which resets expectations toward lower skepticism. These dynamics are summarized in the next proposition.

**Proposition 2.2** For any \(t \geq 0\), denoting by convention \(\mu_0 \equiv P_0(ND) \leq \mu\), the non-disclosure price evolves according to the following four stages:\(^{15}\)

\(^{15}\)It is unimportant for the analysis if \(\mu_0 > \mu\) but this would only occur for the first period \(t = 0\) so, for expositional purposes, we remove this special (transient) starting point.
Figure 3: Investor Learning with Regulation

(1) [compliance stage] If the regulation is effective, $\theta_{t-1} = \theta_t = 0$, $P_{t+1}(ND) < P_t(ND)$ updates to $P_{t+1}(ND) = \zeta_1(P_t(ND)) < P_t(ND)$ and the process converges toward $\tau^*_1$;

(2) [innovation stage] when the financial innovation emerges, $\theta_{t-1} = 0 < \theta_t = 1$, $P_{t+1}(ND) < P_t(ND)$ updates to $P_{t+1}(ND) = \zeta_0(P_t(ND)) < P_t(ND)$ with greater investor losses and more skeptical belief than in (1);

(3) [non-compliance stage] If the regulation has been ineffective, $\theta_{t-1} = \theta_t = 1$, $P_{t+1}(ND) < P_t(ND)$ updates to $P_{t+1}(ND) = \zeta_0(P_t(ND))$ and the process converges toward $\tau^*_0$;

(4) [regulation stage] If the regulation becomes effective, $\theta_{t-1} = 1 > \theta_t = 0$, the process updates to $P_{t+1}(ND) = \zeta_1(P_t(ND))$ and satisfies $P_{t+1}(ND) > P_t(ND)$ if and only if $P_t(ND) < \tau^*_1$.

We summarize below the key take-aways of Proposition 2.2. As in laissez-faire, the regulation does not fully eliminate skepticism. Investors make errors throughout the compliance stage (1) because they learn over time to converge to the equilibrium skepticism in $\tau^*_1$; however, the errors are smaller than what they would have been absent the regulation. By contrast, in the innovation and non-compliance stages (2) and (3), the regulation facilitates excessively optimistic beliefs and will imply greater errors than under laissez-faire.

The regulation stage (4) is the critical step that resets expectations toward less skepticism because adapt-
tive beliefs learn to trust a low level of adverse selection. Indeed, if \( \hat{k} \) is almost one so that effective regulations eliminate all adverse selection, investors always reset their expectations toward the minimal level of skepticism \( P_{t+1}(ND) \approx \mu \). At this stage, the effective regulation has fully undone the skepticism formed by investors along the pricing errors in stages (1)-(3).

Additional intuition can be gained by asking whether expectations overshoot the rational level of skepticism when resetting: are investors overly confident in regulations when moving from the regulator stage (4) to compliance (1)? To answer this question, note that the “rational” belief \( \tau^*_1 \) satisfies the minimum principle property and therefore, any investor update following \( \zeta_1(.) \) in (4) must fall above \( \tau^*_1 \) - strictly so except in the knife-edge \( P_t(ND) = P_{t+1}(ND) = \tau^*_1 \). Hence, for any resetting of investor expectations in (4) such that the investor becomes less skeptical \( P_{t+1}(ND) > P_t(ND) \), the investor must always overshoot \( \tau^*_1 \). As illustrated in periods \( t = 3 \) onward in Figure 3, excess skepticism is short-lived (one period in the model) and is always followed by excess optimism regardless of the presence of regulation. These observations are summarized in the next Corollary.

**Corollary 2.4** In any dynamic path, a period with excess skepticism \( P_{t+1}(ND) > P_t(ND) \) can only occur after an existing transaction is regulated (4), and then must necessarily be followed by one or more periods with insufficient skepticism \( P_{t+2}(ND) < P_{t+1}(ND) \). In particular, for any \( t \in [0, T] \), excess skepticism is less frequent than excess optimism.

Naturally, the resetting of expectations at the regulation stage needs not always occur if investors have not yet become sufficiently skeptical. If investors have optimistic beliefs at \( P_t(ND) > \tau^*_1 \) above the long-term rational level under compliance, regulation does not cause beliefs to increase. Hence, counter-intuitively, skepticism in the presence of regulation is self-defeating: the loss of skepticism in stage (4) must be preceded by a long enough spell compliance and non-compliance, sufficient to erode investor confidence to the point they would excessively penalize a non-disclosure. In this respect, greater correlation in \( \theta_t \), interpreted as the transaction being more persistent and/or longer to address, is more likely to give rise to a sharper decline in beliefs prior to the regulatory stage and thus lead to a cycle of excess optimism.

To further illustrate the comparative statics, consider next a simplified economy with a single transaction event, occurring at a date \( t_0 \) and a single regulation date \( t_1 \geq t_0 \). This is a reasonable perspective when considering individual transactions rather than overall financial statements: to set ideas, we may consider the accounting for employee compensation, the innovation of using derivatives such as stock options (which
became widespread during the 90s), i.e., \( t_0 \), and largely bypassed recognition in the income statement, followed by disclosure mandates, i.e., \( t_1 \). Many other financial reporting innovations have followed a similar pattern, with the examples of regulations increasing recognition over financial instruments and derivatives, leases and, more generally, many off balance sheet obligations.

The resulting beliefs \( (P_t(ND)) \) depend on the realized dates \((t_0, t_1)\) with (1) compliance from \( t = 0 \) to \( t_0 \) and \( t_1 + 1 \) onwards, (2) innovation at \( t_0 \), (3) non-compliance between \( t_0 + 1 \) and \( t_1 - 1 \) and (4) regulation at \( t_1 \). Corollary 2.5, below, examines how the length of each of these stages affects investor beliefs post regulation.

**Corollary 2.5** *In the simplified economy with a single financial innovation at \( t_0 \) and regulation at \( t_1 \),*

(i) For any \( t \leq t_1 \), \( P_{t_1}(ND) \) is increasing in \( t_1 - t_0 \), that is, longer compliance and shorter non-compliance spells initially decrease skepticism;

(ii) For any \( t > t_1 \), (ii.a) \( P_t(ND) \) decreases in \( t_1 - t_0 \) if \( P_{t_1}(ND) < \tau_1^* \) but, otherwise, (ii.b) \( P_t(ND) \) increases in \( t_1 - t_0 \), implying then that a shorter compliance and longer non-compliance spells increase optimism, implying larger pricing errors in the long run.

Prior to \( t_1 \), more non-compliance tends to generate more skepticism \( P_{t_1}(ND) \) as investors experience greater losses and the belief process converges toward \( \tau_0^* < \tau_1^* \). Hence, \( P_{t_1}(ND) \), the belief prior to regulation, is increasing in compliance. Next, post regulation \( t \geq t_1 + 1 \), the investor belief must be greater or equal than the long-term equilibrium belief \( \tau_1^* \), due to the minimum principle property used earlier. Investors’ pricing errors are lower when \( P_{t_1+1}(ND) \) is lower because, then, beliefs are closer to the long-term equilibrium \( \tau_1^* \).

Having tied \( P_{t_1}(ND) \) to the length of the compliance spell and investor errors to \( P_{t_1+1}(ND) \), the two beliefs need to be linked together by asking whether more skeptical pre-regulation beliefs increase or decrease post-regulation beliefs. This relation is ambiguous. If beliefs become sufficiently skeptical, which will occur if the non-compliance spell is long, the low resulting non-disclosure price will deter a large number of strategic withholders: when the regulation is put in place, prices will feature an abnormally low level of strategic withholding. This causes a positive cash flow surprise but also generates more optimism post regulation.

In addition, the greater the severity of the non-compliance, i.e., the lower \( \zeta_0(.) \) is, the greater the re-
bound in expectation in (4) in response to excessive skepticism. Investors make greater errors even after the regulation is in place (in stage (1)) after a more innovation: regulations convert skepticism into excessive optimism.

3 Implications for Price Drift and Sentiment

3.1 Cash Flows and Valuation

This section examines the empirical implications of the model in terms of dynamics of prices, and their interaction with disclosure and earnings surprises. One needs to make an adjustment to the model so that the baseline model can be mapped to the value of a long-lived asset with noisy forecasts. The firm is characterized by a stream of (potentially correlated) cash flows \( \tilde{y}_{it} = \eta \tilde{y}_{it-1} + (1 - \eta) \mu + \tilde{\epsilon}_{it} \) distributed at the end of the period and discounted at rate \( r > 0 \) and such that \( \eta \in [0, 1] \) is the correlation in cash flows and \( \tilde{\epsilon}_{it} \) is centered white noise.\(^{16}\) Using only backward looking information, the discounted value of future cash flows conditional on \( \tilde{y}_{it} = y \) is

\[
P(y) \equiv \mathbb{E} \left( \sum_{t' = t}^{\infty} \frac{\tilde{y}_{it'}}{(1 + r)^{t' - t}} | \tilde{y}_{it} = y \right) = \sum_{t' = t-1}^{\infty} \frac{\eta^{t' - t} y + (1 - \eta^{t' - t}) \mu}{(1 + r)^{t' - t}}
\]

\[
= \frac{1 + r}{1 + r - \eta} (y - \mu) + \mu (1 + \frac{1}{r})
\]

\[
\equiv \beta
\]

decomposed in terms of a \( \beta > 1 \) multiplier on the surprise \( y - \mu \) and the value of a perpetuity. This payoff structure is common-knowledge and, containing no strategic component, well understood by investors.

In what follows, it is convenient to interpret the timing in terms of an earnings announcement containing both a disclosure \( d_{it} \) and a realized cash flow \( y_{it-1} \) from the prior period.\(^ {17}\) For expositional purposes, the cash flow and the matching disclosure are set in the same period, so that a period \( t \) starts after the date \( t - 1 \) earnings announcement (after payment of \( y_{it-1} \)) and ends with the date \( t \) earnings announcement (after

\(^{16}\)As in this literature, I assume for expositional purposes that the cash flows are distributed (Beyer et al. 2019; Bertomeu et al. 2022) but one can also interpret the cash flow as being held by the firm with a rate of return \( r \), in which case the value of the firm is simply the value of future cash flows described above plus the accumulated value of its (undistributed) prior cash flows growing at the rate of return. The results do not depend on the distribution policy provided the accumulated cash is netted out of the firm’s financial position (i.e., adjusting for net debt and income from invested non-operating assets).

\(^{17}\)In practice, forecasts \( d_{it+1} \) and/or actuals \( y_{it} \) may become known to investors before an annual earnings date (Beyer et al. 2010), for example, during quarterly announcements, unbundled forecasts or other disclosures (such as an 8K). This has no consequence on the model and implies that, empirically, a window for measuring the return could be defined between actual earnings announcements.
payment of \( y_{it} \).\(^{18}\) Unless explicitly specified otherwise, prices and expectations are taken at the start of each period, and cash flows are discounted between periods.

The firm does not observe \( y_{it} \) at the start of the period but observes \( x_{it} \) where, without loss of generality, \( x_{it} \) is defined to represent the posterior expectation about \( y_{it} \).\(^{19}\) The price is then given by

\[
P_{it} \equiv P(\hat{E}_{t}(y_{it})) = 1_{d_{it} \neq ND}P(d_{it}) + 1_{d_{it} = ND}P(z_{t} + \eta y_{it-1} + (1 - \eta)\mu),
\]

where, hereafter, \( \hat{E}_{t}(.\) is the investors’ subjective expectation after the date \( t \) earnings announcement and the sequence of subjective expectations \( z_{t} \equiv \hat{E}_{t}(\hat{\epsilon}_{it}|d_{it} = ND) \) is updated as in the baseline model with \( z_{t+1} = \zeta(z_{t}; k_{t}(1 - \theta_{t})) \).

### 3.2 Price Drift

Below, the return \( R_{it} \) is defined as the buy-and-hold return from buying the firm after the earnings announcement in period \( t \) and selling the firm at the end of the period:

\[
R_{it} \equiv \Delta_{it} = \frac{P_{t+1}(1 + r) + y_{it} - P_{it}}{P_{it}}.
\]

In the next benchmark, suppose that investor expectations are perfectly calibrated at \( t \), with \( E_{t}(.) = \hat{E}_{t}(.) \). Then, it must hold from the law of iterated expectations and the linearity of the pricing function in (7) that the expected return is zero.

**Proposition 3.1** Under Bayesian pricing, i.e., setting \( E_{t}(.) = \hat{E}_{t}(.) \), the expected return \( E_{t}(R_{it}) \) is zero.

With adaptive expectations, the equality \( E_{t}(.) = \hat{E}_{t}(.) \) needs not hold because the objective expectation used to obtain \( E_{t}(R_{it}) \) is not identical to the expectation used to form \( P_{it+1} = P(\hat{E}_{t}(y_{it+1})) \). The incorrect calibration of expectations yields a simple mechanism to make specific predictions about mispricing and its (predictable) direction depends on the firm’s level of transparency in date \( t \). Specifically, the resulting drift

---

\(^{18}\)This timing is purely expositional and one could equivalently assume that \( y_{it} \) is distributed in the next period after suitably adjusting for discounting so that a forecast \( x \) would map to a discounted cash flow \( x/(1 + r) \). Similarly, when defining \( R_{it} \) in (9), the return is defined within a period to avoid straightforward considerations over the rate of return \( r \).

\(^{19}\)I write the signal structure in posterior expectation space to reduce notation but this representation is without loss of generality.

If the signal was defined as a noisy garbling of fundamentals, with (for example) \( \tilde{x}_{it} = \tilde{y}_{it} + \epsilon_{it} \) (Verrecchia 1983; Dye 1985), one can simply redefine \( \tilde{x}_{it} = \hat{E}_{t}(\tilde{y}_{it+1}) \) as the implied posterior expectation.

\(^{20}\)To avoid consideration of discounting, the return is defined before the start of the next period, i.e., before discounting. Hence, the selling price is \( P_{it}/(1 + r) \) given that, by definition, the firm is sold after discounting (in the next period) for \( P_{it} \).
is the result of two forces. First, the pricing error in \( t - 1 \) may generate a predictable surprise \( \tilde{x}_{t-1} - P_{t-1} \) after a non-disclosure. Second, new prices form in period \( t + 1 \) conditional on the disclosure \( d_{t+1}(x_{t+1}) \), which may be mispriced relative to true fundamental cash flows.

**Proposition 3.2** Conditional on a disclosure at date \( t \), the price drift is (a) positive if the disclosure mandate is unchanged or the innovation exists, and (b) negative conditional on more effective mandatory disclosure and a sufficiently low non-disclosure belief. Formally, denoting \( A_t = \{(1 - \theta_t)k_t < (1 - \theta_{t+1})k_{t+1}, \hat{E}_t(\tilde{y}_{it}|ND) < \tau_1^* \} \) and \( A^c_t \) the complementary event,

\[
\mathbb{E}_t(R_{it}|d_{it} \neq ND, A^c_t) < 0 < \mathbb{E}_t(R_{it}|d_{it} \neq ND, A_t).
\]

In Proposition 3.2, when the firm discloses at date \( t \), the drift is entirely driven by the mispricing in \( t + 1 \). On average, the firm features a higher price when the regulation is less effective and there is a greater degree of overvaluation. Vice-versa, an increase in the effectiveness of regulation will tend to lead to decrease in prices, as date \( t \) owners do not benefit from selling at inflated prices. This intuition implies that, absent changes in regulation, firms that disclose relatively more than their peers in the current period tend to feature positive drift; in other words, disclosing firms predictably achieve a positive drift (post disclosure), especially when expectations are optimistic since this tends to facilitate overvaluation in the next period.

**Corollary 3.1** A firm that tends to disclose relatively more than its peers tends to feature positive price drift (i) under laissez-faire, (ii) in a regulated economy if the innovation is unlikely to occur or the regulation is not very effective, or (iii) when expectations are optimistic.

Consider next the average price drift, by taking expectations over the possible regulations in \( t + 1 \). The next Corollary derives conditions under which the price drift is positive.

**Corollary 3.2** In the regulated economy, expected price drift is positive after a disclosure if and only beliefs are sufficiently optimistic, that is, \( z_{t+1} > z^*_t \) where \( z^*_t \) is defined as the unique solution to \( z^*_t = \zeta(z^*_t, (1 - p_{\theta_t})\hat{k}) \).

The drift that follows non-disclosure is slightly more complex because there is a predictable cash flow surprise in the current period; hence, the investor buying at date \( t \) may bear a negative cash flow surprise
followed by selling at overvalued prices at $t + 1$, or the opposite, as a function of the disclosure environment. The next statement in Proposition 3.3 extends some of the previous insights to this environment.

**Corollary 3.3** *Conditional on a non-disclosure at date $t$,*

(i) Suppose that $z_t < \tau^*_\theta_t$ (in which case $z_{t+1} > z_t$ and the date $t$ cash flow surprise is positive), the price drift is positive; 

(ii) otherwise (in which case $z_{t+1} < z_t$ and the cash flow surprise is negative), the price drift is negative if $r$ is sufficiently large, conditional on sequential non-disclosures or if the regulation becomes more demanding $k_{t+1}(1 - \theta_{t+1}) > k_t(1 - \theta_t)$.

To summarize, current investors earn positive predictable returns when buying non-disclosing firms given an increase in regulation and pessimistic beliefs, since this allows them to buy at deflated prices. By contrast, buying with a status-quo or declining disclosure mandates earns negative returns, especially if a period with higher regulation follows. Indeed, in (i) and (ii) of Corollary 3.3, current investors are always better-off with the innovation, thus demonstrating how the existence of the innovation can be supported by current investors. The next Corollary follows immediately.

**Corollary 3.4** *Price drift is non-decreasing in $\theta_{t+1} - \theta_t$, and, in expectation, increases in $p_{\theta_t}$.*

Corollary 3.4 is in sharp contrast with the prediction in a rational model. In principle, rational investors can reverse engineer the overall degree of transparency by inverting the fraction of non-disclosing firms

$q + (1 - q)(1 - k_t(1 - \theta_t))(1 - F(\tau^*_\theta_t))$ which will reveal whether the transaction $\theta_t$ is available. With adaptive expectations, investors make no such calculation and, using their realized non-disclosure prices, will be surprised by a change in the disclosure environments. As a result, the financial innovation tends to increase price drift.

### 3.3 Market Sentiment and Trust

The concept of trust is essential to the functioning of financial markets, given that investors make risky investments subject to asymmetric information (Guiso et al. 2008); however, unwarranted trust can also lead to excessively optimistic expectations (Marinovic 2013; Shiller 2015). In the context of trust following non-transparency, the model captures the dynamics of sentiment and their interaction with price. These
predictions can be further tested using proxies for trust in corporate disclosure, for example, the Financial Trust index in Guiso et al. (2008) captures individuals’ trust in market institutions.\textsuperscript{21} In what follows, I use interchangeably the notion of sentiment and trust for $z_t$ since, by reflecting more favorable beliefs, they are the same concept in the confines of the model.\textsuperscript{22}

**Implication 3.1** Market sentiment increases conditional on stricter effective regulations and sufficiently pessimistic sentiment, but decreases otherwise.

The implication is driven by the fact that market sentiment tends to decrease in the model in a status-quo disclosure environment, because strategic disclosers strategically withhold when their information is below the non-disclosure price (which tends to cause negative surprises). This is, naturally, all the more so if regulations become less effective because of a new financial innovation since it leads to the greatest decrease in sentiment. However, if disclosure mandates becomes more effective in the context of pessimistic beliefs, the non-disclosure belief will be excessively pessimistic given the nature of the disclosure environment: in this case, in response to the regulation, cash flow surprises after a non-disclosure are positive in expectation and sentiment increases.

**Implication 3.2** Market sentiment is mean-reverting, and is more likely to increase (decrease) when sufficiently pessimistic (optimistic).

Given optimistic beliefs, market sentiment can only decrease (in this stylized model) regardless of the regulatory effectiveness. By contrast, once market sentiment is excessively pessimistic relative to the optimal level of skepticism with regulation, market sentiment increases if $(1 - \theta_t)k_t$ increases. In fact, as shown earlier, the increase in sentiment is greatest when sentiment is lowest before a new regulation.

**Implication 3.3** When market sentiment is high (low), investors tend to make positive (negative) pricing errors. Given a change in reporting regime, mispricing is larger for extreme market sentiment.

Sentiment affects the errors made by investors, with high sentiment leading to excessively optimistic beliefs and positive errors (overvaluation), and low sentiment leading to pessimistic beliefs and negative errors.

\textsuperscript{21}See http://www.financialtrustindex.org/.

\textsuperscript{22}In general, trust and market sentiment can refer to distinct objects. For example, market sentiment is a broad concept that may reflect investors’ beliefs about long-term fundamentals, even absent any strategic motives. One should therefore cautiously restrict interpretations of this model to sentiment about the existence of adverse selection, which only affects beliefs about fundamentals via a strategic channel.
(undervaluation). Extreme sentiments are more likely to be mismatched given a change in the disclosure environment and can lead to larger errors (for example, larger surprises, larger analysts errors, etc.).

4 Additional Implications

4.1 Asset Pricing and Uncertainty

Below, I examine additional asset pricing implications along the dynamics of the baseline model. As is common in this literature, such implications require additional structure on cash flow innovations so suppose (to set intuition) that \( F(.) \) is uniform on \([-1, 1]\). Let \( h_t = 1_{k_t(1-\theta_t)} \) denote an effective disclosure mandate. For a given non-disclosure belief \( z \leq \mu \), the updating function simplifies to

\[
\zeta(z; h) = \frac{q'(z^2 - 1)}{4 + 2q'(z - 1)},
\]

where \( q' = 1 - (1 - q)/(1 - qhk) \) is the relative ratio of (unconstrained) strategic to uninformed firms. Solving for \( \zeta(z; h) = z \), the rational equilibrium features a non-disclosure belief (and disclosure threshold) given by

\[
\tau^* = 1 - 2 \frac{1 - \sqrt{1 - q'}}{q'}.
\]

**Proposition 4.1** Suppose that \( \tilde{x}_{it} \) is uniform. Then, for any belief \( z_t \in (\tau^*_h, 0) \), given no change to the reporting environment \( h_t \), the variance of cash flows conditional on non-disclosure \( \text{Var}(\tilde{x}_{it}|ND) \) first decreases in \( z \) and then increases as beliefs converge toward \( \tau^*_h \), remaining lower than \( \text{Var}(\tilde{x}_{it}|ND) \).

A non-disclosure is a statistical message that leads to a revision in the posterior distribution of cash flows; hence, there is information in the message that could, in principle, decrease volatility as it better identifies strategic withholders (e.g., close to unravelling, a non-disclosure would fully reveal the lowest cash flows). Consistent with this intuition, a non-disclosure tends to reduce uncertainty relative to the unconditional variance of the cash flow.

However, a secondary counter-effect also applies: more skeptical beliefs induce firms with lower cash flows to withhold. These firms are substantially different from the genuinely uninformed firms, causing an increase in volatility after a non-disclosure. This effect implies that the greater the adverse selection - which occurs more strongly when strategic firms with better cash flows are deterred from withholding by
pessimistic beliefs - the more volatility is created. Indeed, as long as there is sufficient pessimism, volatility increases over time.

A change in the effectiveness of regulation supplements to this trade-off because (i) investor optimism increases in response to a more effective regulation if beliefs are sufficiently pessimistic (Proposition 2.2 (4)) , (ii) there is an adjustment in the relative ratio of strategic firms $q'_t$. The first effect implies that the comparative statics in Proposition (4.1) acts in the opposite direction for regulations that increase optimism. The second effect is more complex because a change in $q'_t$ changes the mix of strategic and non-strategic firms. The next Corollary below shows that more effective regulations leads to more effective regulations increasing volatility.

**Corollary 4.1** Holding the belief $z_t$ constant, volatility decreases (increases) with a less (more) effective disclosure mandate.

A heuristic intuition behind Corollary 4.1 follows. The result is not as surprising as it may a-priori seem, because an increase in disclosure requirements need not make non-disclosing firms more similar. The greater mandate removes information from the non-disclosure message by increasing the relative fraction of uninformed non-disclosing firms. In the limit, a near-perfect disclosure mandate implies that the non-disclosure contains no information, which yields the maximal volatility $Var(\tilde{x}_{it})$. The net effect of a regulatory change can nevertheless be ambiguous, especially when the effect of the change in beliefs in Proposition 4.1 contrasts with the adjustment in regulatory effectiveness in Corollary 4.1. Table 1 summarizes the main implications when the two effects act in the same direction, as a function of whether beliefs are optimistic or pessimistic, and whether there is a regime change.

McNichols (1988) and Smith (2020) suggest that, because capital market communication often leads to incentives to withhold bad news, environments with strategic withholding feature abnormal skewness. Skewness is not solely an asset pricing fact and has been shown to have other testable implications, for example increasing analyst forecast bias (Gu and Wu 2003) or altering the convexity of contracts (Hemmer et al. 1999). Consistent with this argument, in this model - given no change to the reporting environment - skewness increases over time: more extreme firms withhold strategically as investors become more pessimistic, lowering the distribution of low cash flow (strategic) relative to more favorable cash flows firms.

**Corollary 4.2** Given no change to the disclosure regime $h_{t+1} = h_t$, skewness is increasing as beliefs
Same regime  
\( h_t = h_{t+1} \)  
-  
- if \( z_t > \hat{z}_{ht} \), + otherwise

Less effective regulation  
\( h_t > h_{t+1} \)  
-  
- if \( z_t > \hat{z}_{ht} \), +/- if \( z_t > \hat{z}'_{ht} \)

More effective regulation  
\( h_t < h_{t+1} \)  
- if \( z_t > \tau^*_1 \)  
+/- if \( z_t > \hat{z}_{ht} \), +/- if \( z_t > \hat{z}'_{ht} \)

+/ otherwise

+/- otherwise

Table 1: Comparative Statics Summary (\( \hat{z}_{ht}, \hat{z}'_{ht} \in [-1, 0] \))

become more skeptical. For a given belief \( z_t \), skewness is increasing in \( q' \) for \( z_t \) sufficiently low and decreases in \( q' \) otherwise.

As in the case of volatility, the effect on skewness of a change in the regulatory environment is usually ambiguous and depends in investor optimism. To see why, consider a less effective regulation which leads to more strategic firms withholding. Given that the withholding strategic firms with unfavorable events are more similar and less dispersed, skewness tends to increase. However, at the same time, the less effective regulation has relatively fewer uninformed firms, reducing dispersion on the upper tail and reducing skewness. The first effect dominates when withholding firms tend to be more similar, which occurs in particular when investor beliefs are more pessimistic and only attract firms with sufficiently low cash flows. Naturally, the same intuition applies in reverse given more effective regulations.

I examine next whether these intuitions may be unique to a functional form; however, some inspection of the Normal distribution suggests that the economic intuitions apply in more general models. In the context of bell-shaped cash flows \( \tilde{x}_{it} \sim N(\mu, \sigma^2) \), the volatility conditional on non-disclosure is equal to the unconditional volatility when evaluated at the equilibrium threshold \( \tau^2_{ht} \), and decreases for \( z_t \in (\tau^2_{ht}, \mu) \) (Dye and Hughes 2018). This implies a similar property as the prior analysis: under no change to the disclosure regime, volatility conditional on non-disclosure decreases over time. The behavior of skewness is, unfortunately, not analytically tractable; however, Figure 4 suggests that, given no change to the disclosure regime, skewness increases over time. In most cases, a more effective regulation increases volatility and
reduces skewness. Further, skewness increases in $q'$ for $z_t$ sufficiently pessimistic and decreasing otherwise, confirming prior intuitions.

4.2 Regulatory choice and Trade Restrictions

For purposes of interpreting the regulatory choices of a regulator, consider a simplified two-period version of the model $t = 0, 1$ where agents maximize market expectations (Brandenburger and Polak 1996). Suppose that $\bar{x}$ $\geq$ 0 and the following decision problem is solved by the regulator:

(i) conditional on a disclosure $x_{it}$, firms make a decision $I_{it}$ to maximize an output given by

$$\pi(I_{it}, x_{it}) = I_{it}x_{it} - \xi(I_{it}),$$

where $\xi(.)$ is a convex differentiable function with $\xi'(\bar{x}) = 0$, implying a solution $I^*_{it}$ given by the unique solution to $\xi'(I^*_{it}) = x_{it}$.

(ii) conditional on non-disclosure, firms make the decision $I$ that maximizes the market value of the output

$$\pi(I_{it}, P_t(ND)) = E(I_{it}\bar{x}_{it} - \xi(I_{it})|P_t(ND)) = I_{it}P_t(ND) - \xi(I_{it}),$$

23See also Marinovic (2013), Beyer and Dye (2012), and Aghamolla et al. (2021) for recent studies in which managers make a disclosure to maximize price.

24To be rigorous, this periodic production function should be introduced earlier in the model, so that the firm maximizes perceptions about the expected production rather than perception about $\bar{x}_{it}$. However, it is readily seen that these two objectives are the same, because the perceived output is monotonic in expectations about $\bar{x}_{it}$. Given that this production decision does not affect the disclosure decision, I delay its specification solely for expositional purposes.

25This model can be made slightly more general to $\pi(x, y) = \xi_0(x)\xi_1(y) - \xi_2(y)$, as long as $x$ and $y$ are multiplicatively separable. In this more general formulation, one can map to the original problem by redefining $\bar{x}' = \xi_0(\bar{x})$ and $y' = \xi_1(y)$, which implies $\pi(x, y) = x'y' - \xi_2 \circ \xi_1^{-1}(y')$. 

Figure 4: Volatility and Skewness conditional on non-disclosure ($\bar{x}_{it} \sim N(0, 1)$)
which implies a maximum attained at $\xi'(I_{it}^*) = P_t(ND)$.

Decisions are a function of posterior expectations (Ganuza and Penalva 2010), which implies that the process of investor expectations $P_t(ND)$ is a sufficient statistic in the regulator’s preference.

The regulator has no direct control over optimal actions $I_{it}^*$ and knows only that actions are taken given the investors’ information set. This creates two sources of inefficiency: first, investors take incorrect actions in the withholding region and, second, these average actions are based on miscalibrated expectations when $P_{t+1}(ND)$, the correct non-disclosure expectation in (2), is different from investor beliefs $P_t(ND)$.

Specifically, the regulator calculates the current period surplus based on correct expectations

$$V_t \equiv \mathbb{E}_t(\tilde{x}_{it} \tilde{I}_{it} - \xi(\tilde{I}_{it}^*)),$$

where $\mathbb{E}_t(.)$ indicates the expectation according to all information known at date $t$.

**Proposition 4.2** The current period surplus $V_t$ increases in the effectiveness of the regulation $k_t(1 - \theta_t)$ and, as long as either $k_t(1 - \theta_t) = 0$ or $P_t(ND) \geq \tau_1^*$, decreases in current optimism $P_t(ND)$.

As is entirely intuitive, a more effective regulation strictly improves communication in the present by reducing strategic withholding. By contrast, optimism affects efficiency via two channels. A higher non-disclosure expectation favors more withholding, which reduces efficiency by distorting decisions over a larger non-disclosure set. This channel unambiguously implies that an increase in optimism decreases surplus. However, investor beliefs in the model are also distorted conditional on non-disclosure away from the correct expectation $\zeta(P_t(ND))$, causing an additional inefficiency. If the firm is overpriced, pessimism offsets the distortion by moving expectations toward better calibrated expectations and more efficient investments. By contrast, if a more effective regulation passes jointly with pessimistic expectations, more optimism may better match the non-disclosure (correct) expectation $\zeta(P_t(ND))$.

In what follows, suppose that the regulator chooses $k_1$ to maximize $V_1$ at date 1, and, at date 0, chooses $k_0$ to maximize a weighted average of both periods $S = \mathbb{E}((1 - \delta)V_0 + \delta V_1)$, where $\delta \in [0, 1]$ jointly represents the regulator patience as well as the expected growth in the market over the periods (e.g., $\delta$ is the product of the discount factor and the growth rate). The next Corollary characterizes the effect of $\delta$ on the regulatory choice. As this plays no role for this result, assume that the financial innovation does not exist at date 0 and can only occur at $t = 1$. 

27
Corollary 4.3 The regulator chooses $k_1^* = \hat{k}$ and $k_0^*$ is decreasing in $\delta$ if the financial innovation is sufficiently likely or initial beliefs are sufficiently optimistic with $\zeta(P_0(ND), 0) > \tau_1^*$. Vice-versa, $k_0^*$ is increasing in $\delta$ if the transaction is sufficiently unlikely and initial beliefs are sufficiently pessimistic. The regulation $k_0^*$ is decreasing in the probability of the financial innovation.

The regulator balances the benefit of more regulation today, which reduces mispricing, with reduced investor learning in future periods. In the last period $t = 1$, the regulator should only focus on mispricing and implements the highest level of regulation. In the starting period $t = 0$, by contrast, the trade-off is affected by the preferences of the regulator and the presence of the innovation: a more impatient regulator prefers to focus on mispricing today, while a more patient regulator willing to discipline investor optimism will favor laissez-faire.

An interesting application of this model helps draw further intuition about how the regulator manages investor optimism. Suppose that $\delta$ is equal to one (to focus only on investor learning) but $k_0$ may now be set in the entire interval $[0, \hat{k}]$ (with $\hat{k} < 1$) allowing for different levels of regulatory effectiveness. Any regulation such that $\zeta(P_0(ND)) \geq \hat{k}$ triggers inefficient optimism in period 1 and is dominated by a less effective regulation $\zeta(P_0(ND)) = \tau_1^*$, implying some degree of laissez-faire.

Which degree regulation is optimal depends on the current investor optimism. First, if the initial belief is optimistic and satisfies $P_0(ND) > \tau_1^*$ (tends to feature overpricing relative the date $t = 1$ regulation), the regulator will prefer the less effective regulation $k_0^* < \hat{k}$ that implements $\zeta(P_0(ND)) = \tau_1^*$ to offset excess optimism. If the initial belief is pessimistic and satisfies $P_0(ND) < \tau_1^*$, choosing $k_0 = \hat{k}$ would increase the next period belief above $\tau_1^*$ so, as for the previous case, there is a less effective $k_0^* < k_1$ such that $\zeta(P_0(ND)) = \tau_1^*$ that dominates choosing $k_0 = \hat{k}$. In summary, in both cases, the optimal regulation $k_0^*$ is strictly less than the maximal possible regulation.

I consider next another common manner in which regulators address reporting issues through partial or complete prohibitions of trade. For example, individuals cannot freely trade substances with known risks (controlled substances) and trading over certain goods or services occasionally requires a license. In the U.S., new issuers of financial securities face a number of regulatory constraints beyond mandatory requirements, including requirements on liquidity, governance and minimum trading prices. In China, regulators have made some requirements explicit, for example barring trade for firms with three or more consecutive periods of losses.
It is of course surprising that such restrictions should exist in a model with perfectly rational expectations. If investors decide rationally to invest based on correct priors, revealed preferences imply that they are weakly better-off trading than they would be when exogenously restricted not to trade. Risk-neutral price-protected investors will make zero profit and any restriction on trade will prevent issuers from earning positive surplus from selling. But this is not the case with adaptive expectations: investors always trail expectations and may make losses at each period of trade. While these losses are redistributive (earned by the seller), the incorrect expectation is reflected via lower investment efficiency $V_t$. Assume, for the result below, that the surplus is fully dissipated if there is a trade prohibition, i.e., the firm requires external financing from a market to operate.

**Corollary 4.4** Suppose $\int \pi(x, \psi(x)) f(x) dx < 0$. Then, there exists $\hat{\delta} > 0$ such that, for any $\delta < \hat{\delta}$, a prohibition is optimal for sufficiently optimistic beliefs and if the financial innovation is sufficiently likely.

Interestingly, the model provides a rationale for entirely shutting down a market when market expectations are too optimistic and would cause excessive inefficiencies. This context occurs specifically when the regulator is impatient, since it reduces the future benefits of learning, and when innovations are likely, because this weakens the effectiveness of regulations. Transparency, under these circumstances, is an imperfect substitute for a trading prohibition.

### 4.3 Asymmetric Reporting

In the baseline model, the regulator imposes a disclosure requirement in the form of mandatory disclosure when information is received. Importantly, the mandate is not a function of the implication of the news for valuation. However, in practice, some disclosure requirements asymmetrically apply to potentially negative news (Basu 1997; Watts 2003). This section revisits the model in the context of (first) asymmetric disclosure requirements for bad news and (second) potential implications of the model for regulating good news.

The regulation $k_t$ is redefined in terms of a mandatory threshold such that, absent the financial innovation, informed firms with $x_{it} \leq k_t$ must disclose. Uninformed firms do not disclose, and firms are not subject to $k_t$ when the innovation is present. The updating rule (3) is unchanged when the financial innovation is present but, otherwise, must be adapted to a lower-tail disclosure mandate:

$$P_{t+1}(ND) = \zeta(P_t(ND); k_t) = \frac{q(F(P_t(ND)) - F(k_t)) \mathbb{E}(\tilde{x}_{it}|k_t \leq \tilde{x}_{it} \leq P_t(ND)) + (1 - q)\mu}{q(F(P_t(ND)) - F(k_t)) + 1 - q}. \tag{16}$$
In Figure 5, the dynamics of this model are similar to those under an unconditional disclosure mandate, except that the beliefs post regulation are now more favorable as regulators target withholding of bad news. The more favorable non-disclosure, in turn, leads to more optimism in later periods. If one or more periods without regulation move pessimism below the regulatory threshold, $P_t(ND) \leq k_t$, then all firms will be better-off disclosing voluntarily when the regulation is effective - therefore, leading to the maximally optimistic belief in the next period $P_{t+1}(ND) = \mu$. This situation does not occur in the baseline model unless the regulation is perfectly effective ($\hat{k} = 1$).

From this intuition, the model suggests a downside of lower-tail asymmetric reporting on investor learning: while it effectively reveals bad news in the current period, bad news is a special form of paternalism that protects investors against negative surprises. I revisit this restriction here, noting that strategic withholding carries two externalities that depend on the information being withheld. The first externality (discussed earlier) is the effect of withholding on current decisions and is largest when the withheld news is furthest from expectation. The second externality is that more favorable withheld information most increases investor optimism $P_{t+1}(ND)$ and negatively affects future decisions. When this second asymmetry is strongest, mandatory disclosure over favorable news is desirable because it most decreases future optimism.

The trade-off between the two externalities is ambiguous, depending on the social costs of making the incorrect decisions. However, mandatory disclosure over good news yields benefits over laissez-faire. To
illustrate this point, consider an infinite horizon version of the regulator problem in Section 4.2:

\[ S_t = \mathbb{E}(\sum_{t'=t}^{\infty} \delta^{t'-t} V_{t'}). \tag{17} \]

For the next result, suppose that \( P_0(ND) > \tau_0^* \) since any other belief is transient (given that \( \zeta(z) \geq \tau_0^* \)) or implies that laissez-faire has converged to its long-term equilibrium and features no learning.

**Proposition 4.3** Suppose that \( P_0(ND) \geq \tau_0^* \). Consider regulations in which all information above \( k_t \) must be disclosed (but firms have discretion for news below \( k_t \)). Then, there exists a positive sequence \( (u_t) \) such that \( k_t = P_t(ND) - u_t \) such that \( k_t \) implies higher \( V_t \) all periods over laissez-faire.

Certain regulations can be shown to always dominate laissez-faire, even if the regulator is patient. However, these regulations do not seek to protect investors; on the contrary: “anti” paternalistic regulations increase the losses borne investors by requiring disclosure of better news. The regulation better internalizes the negative externality of optimism and increases the speed of convergence toward unravelling. This is achieved by imposing mandatory disclosure over some news \( \tilde{x}_{it} \in (P_t(ND), P_{t+1}(ND)) \) that tend to make investors more optimistic in the next period.

### 4.4 Investor Sophistication

Prior research finds that investor have varying degrees of sophistication, which in turn affects market pricing (Bourveau et al. 2020). To capture the effect of investor sophistication, the following extension of the model develops a market with heterogenous expectations. There is an exogenous supply of \( N \) shares and \( M = \sum_{j \leq J} n_j \) investors, of which \( n_j \) are level-\( j \) rational investors to be defined shortly.\(^{26}\) Since the main intuition does not require it, suppose that the financial innovation does not occur so that \( \hat{k} \) is enforced every period and investor beliefs remain greater than \( \tau_1^* \).

Level 0 investors, as in the baseline, do not account for strategic considerations and set their beliefs to \( P^0_t(ND) \equiv \zeta(P_{t-1}(ND)) \). Then, define the level \( j \) non-disclosure price \( P^j_t(ND) \) recursively by

\[ P^{j+1}_t(ND) \equiv \zeta(P^j_t(ND)). \tag{18} \]

\(^{26}\)The presence of the innovation presents similar forces but requires more structure on how level \( j \) investor consider the presence of the innovation, given that fully rational investors would be able to condition on the innovation after observing the frequency of non-disclosure.
Each investor can buy at most one share and cannot short sell. The equilibrium price $P_t(ND)$ is then defined as the price such that investors demand all $N$ units such that an investor with level $j$ buys when $P_t(ND) \geq P_j^*(ND)$. By convention, if there are multiple market-clearing price, I set the highest market-clearing price $P_t(ND)$.

**Proposition 4.4** There exists $j^*$ given by $\sum_{j \leq j^* - 1} n_j < N \leq \sum_{j \leq j^*} n_j$ such that $P_t(ND) = P_j^*(ND)$. As $N$ increases or $n_j$ decreases, the non-disclosure price and pricing errors decrease.

Type $j^*$ is the pivotal level of rationality that determines prices in this model. Investors who are less rational (lower $j$) are less skeptical after a non-disclosure and tend to be willing to buy more. In turn, their miscalibrated expectation supports higher prices and the less rational the investor base, the higher the price. By contrast, greater investor rationality will lead to a decline in price. In the limit, as the market converges toward greater rationality, beliefs must converge to the rational price $P_j^*(ND) \to \tau^*_1$ with the highest level of disclosure.

This logic can be extended to environments with the financial innovation; assume below that the presence of an effective regulation can be inferred by rational investors. Proposition 4.4 is unchanged if the current realized non-disclosure payoff is $P_0^*(ND) \geq \tau_1$. However, consider the case in which $P_0^*(ND) \lt \tau_1^*$ which would lead to $P_{t+1}(ND) \gt P_t(ND)$ in the baseline model with a positive price surprise in the next period. With a higher level of rationality, the price increases in the current period as long as the pivotal trader $j^*$ is greater than zero.

**Corollary 4.5** The price is given by $P_t(ND) = P_0^*(ND)$ if $N > M - n_0$. Otherwise, $P_t(ND) = P_j^*(ND)$ where $j^*$ given by $\sum_{0 \lt j \leq j^* - 1} n_j < N \leq \sum_{0 \lt j \leq j^*}$. In particular, as $N$ increases, $P_t(ND)$ first increases from $P_0^*(ND)$ to $P_1^*(ND)$ and then decreases in $N$ and, if $N > M - n_0$, investors always overprice current cash flows.

When beliefs are sufficiently pessimistic (uafter one or more periods with the transaction) and there is a regulatory change, one level of investor rationality will increase price, by correcting the excess pessimism. As long as the pivotal trader is not the level 0 investors, all level 0 investors stay out of the market and the price adjusts immediately to the presence of the new regulation. This implies, in particular, that for a

---

27 This rule is unimportant for the results (all properties can be stated on the price correspondence) and only occurs for knife-edge values of $N$.

28 Absent this assumption, prices have properties similar to those under Proposition 4.4, taking expectations over whether an effective regulation exists.
sufficiently rational investor base, prices always overprice the current cash flow as $\zeta(P_t(ND)) < P_t(ND)$. In other words, more investor rationality reduces the potential for positive surprises but does not eliminate negative surprises.

5 Concluding Remarks

Economic agents will err in judgment, engaging in a continuous learning process to avoid past mistakes. I apply this argument in the context of verifiable communication to demonstrate the following principle: paternalism that insulates investors from excessive optimism will cause them to make more mistakes in the long run. In the model, convergence toward high levels of disclosure will occur as a result of market forces, even if investors do not understand strategic motives, as long as investors respond to errors by becoming more skeptical. By contrast, an impatient regulator will implement regulations that prevent unravelling and may create recurring cycles of excessive optimism followed by negative drifts in prices. Put more starkly, overly-protective regulations create the very problem that they intend to solve.

The analysis suggests that a trade-off between solving current problems and maintaining a degree of long-term investor learning. Less paternalism is thus desirable when there is more innovation and when current risks are smaller relative to future hazards. Paternalism is counter-cyclical relative to disclosure cycles: it is most desirable after expectations are the most optimistic following strings of high transparency. However, regulations tend to correct excess optimism and affect prices negatively.

While the working assumption is that of a well-calibrated regulator, actual regulators need not know more than market participants and may be subject to the same errors of judgment, learning themselves from realized adverse events after a non-disclosure. For example, across 25 countries spanning two centuries of financial scandals and regulations, Hail et al. (2018) document that most interventions do not occur proactively during periods of optimism but after a scandal. This evidence suggests a positive framework for regulatory action where the learning process jointly drives the expectation of investors and regulators. While prior work overwhelmingly focuses on rational expectations as a guiding principle, I argue that concerns for learnability and how the learning interacts with reporting may offer a rich framework to understand observed reporting choices and their economic consequences.
Appendix

Proof of Proposition 2.1: Consider a sequence of non-disclosure prices \((P_{it}(ND))\). In (4), \(\zeta(P_{it}(ND)) < \mu\) so that all beliefs for any \(t \geq 1\) must be less than \(\mu\). Bertomeu et al. (2019) (Proposition 1) show that the belief updating function satisfies \(\zeta(y) \in [\tau^*, y]\) for any \(y \in [\tau^*, \bar{y}]\). Hence, the process process \(P_{it}(ND)\) is decreasing and bounded by \(\tau^*\), so that it must converge on \([\tau^*, \zeta(\mu_0)]\). The only possible convergence is the unique fixed point \(\zeta(\tau^*) = \tau^*\). As \(q \to 0\), \(\tau^*\) converges to \(x\) (Jung and Kwon 1988). \(\square\)

Proof of Corollary 2.1: Recall from Proposition 2.1 that \(P_{it}(ND)\) is decreasing for \(t \geq 1\); hence, the Corollary follows from showing that the belief error \(\Delta(y) \equiv |\zeta(y) - y| = y - \zeta(y)\) is decreasing.

(A) Suppose that \(F(.)\) is logconcave. By continuity, if \(q\) is close to one, the belief error converges to \(\Delta_0(y) \equiv y - \mathbb{E}(|\tilde{x}| \tilde{x} \leq y)\). It is well-known that this term is increasing in \(y\), establishing that the first part of the Corollary.\(^{29}\)

(B) Since the analysis is adapted to any normal distribution by rescaling, assume that \(\tilde{x}_{it} \sim N(0, 1)\). The conditional expectation \(\mathbb{E}(\tilde{x}_{it}|\tilde{x}_{it} \leq y) = -f(y)/F(y)\) is equal to the (inverse) Mills ratio. The belief error and its derivative simplify to

\[
\Delta(y) = y + \frac{qf(y)}{qF(y) + 1 - q},
\]

\[
\Delta'(y) = 1 - qf(y)\frac{qf(y) + y(1 - q(1 - F(y)))}{(1 - q(1 - F(y)))^2}.
\]

To determine the sign of \(\Delta'(y)\), consider its variation as a function of \(q \in [0, 1]\),

\[
\frac{\partial \Delta'(y)}{\partial q} = -f(y)\frac{A_1}{(1 - q(1 - F(y)))^3},
\]

where the numerator \(A_1 \equiv 2f(y)q + y(1 - (1 - F(y))q)\) can be readily verified to be increasing in \(q\) so \(\Delta'(y)\) is decreasing.

\(^{29}\)I provide a short proof of this property below:

\[
\frac{1}{\Delta_0(y)} = \left(y - \frac{\int_{-\infty}^{y} zf(z)dz}{F(y)}\right)^{-1} = \left(y - \frac{yF(y) - \int_{-\infty}^{y} F(z)dz}{F(y)}\right)^{-1} = \frac{F(y)}{\int_{-\infty}^{y} F(z)dz} = (\ln(\int_{-\infty}^{y} F(z)dz))',
\]

where the second step follows by integration by parts. Bagnoli and Bergstrom (2005) show that any function whose derivative is logconcave is also logconcave (Theorem 1), which implies that \(\int_{-\infty}^{y} F(z)dz\) is logconcave and therefore \((\ln(\int_{-\infty}^{y} F(z)dz))'\) is decreasing.
that $\Delta'(y)$ is either monotonic or hump-shaped in $q$, i.e., its minimum is at $q = 0$ or $q = 1$. At $q = 0$, $\Delta'(y) = 1$ is positive; at $q = 1$, $\Delta(y) = \Delta_0(y)$ and, given that the normal distribution is logconcave, it follows immediately from part (A) that $\Delta_0(y)$ is increasing.

Consider next the pricing error

$$\Lambda_t \equiv \mathbb{E}((\tilde{x}_{it} - P_t(d_t(\tilde{x}_{it})))^2) = \mathbb{E}(1_{d_t(\tilde{x}_{it})=ND}(\tilde{x}_{it} - P_t(ND))^2).$$

Developing this expectation,

$$\Lambda_t = q \int_{\mathbb{R}} P_{t+1}(ND) (x - P_t(ND))^2 dx + (1 - q) \int_{\mathbb{R}} (x - P_t(ND))^2 dx$$

$$= q \int_{\mathbb{R}} P_{t+1}(ND) (x - P_t(ND))^2 dx + (1 - q) \int_{\mathbb{R}} (x - P_t(ND))^2 dx$$

$$> \Lambda_{t+1},$$

where the first bound comes from the fact that the mean $P_{t+1}(ND)$ minimizes $\mathbb{E}((\tilde{x}_{it} - m)^2|d_{t+1}(\tilde{x}_{it}) = ND)). \Box$

**Proof of Proposition 2.1:** Conjecture that $P_t(ND) > \bar{x}$ is finite at time $t$. Then, all firms with $\tilde{x}_{i,t} \leq P_t(ND)$ strategically withhold implying that the conditional expectation given by

$$P_{t+1}(ND) = \mathbb{E}(\tilde{x}_{i,t} | \tilde{x}_{i,t} \leq P_t(ND)) < P_t(ND).$$

(19)

The conjecture that $P_t(ND)$ remains finite can then be verified a simple recursive argument.

By contradiction, if $P_t(ND)$ converges to a finite $P^*$, continuity of (19) implies that $P^*$ must satisfy $P^* = \mathbb{E}(\tilde{x}_{i,t} | \tilde{x}_{i,t} \leq P^*)$, which implies that $P^*$ is the infimum of the support of $\tilde{x}_{i,t}$ and must be consistent with $Pr(d(\tilde{x}_{i,t}) = ND) \rightarrow 0. \Box$

**Proof of Corollary 2.2:** In what follows, I use upperscripts to index all symbols that depend on each distribution $j = 1, 2$. Suppose that $h^1(x) \equiv f^1(x)/F^1(x) < h^2(x) = f^2(x)/F^2(x)$ for any $x$, i.e., $F^2$

---

30A minor extension of the mathematical argument applies to distributions with point mass and/or finite support. If, for example, $\tilde{x}_{i,t}$ has a point mass at the lowest point of its support, the equilibrium may feature non-disclosure at this point but would nevertheless remain fully-revealing given the existence of a one-to-one mapping between report and private information.
reverse hazard rate dominates $F^1$. At $t = 0$,

$$
E^1(\tilde{x}_{i,t} | \tilde{x}_{i,t} \leq \mu_0) = \mu_0 - \frac{\int_{\mu_0}^{\mu_0} F^1(x) dx}{F^1(\mu_0)} = \mu_0 - \int_{\mu_0}^{\mu_0} \exp \left( - \int_{x}^{\mu_0} h^1(y) dy \right) dx
\leq \mu_0 - \int_{\mu_0}^{\mu_0} \exp \left( - \int_{x}^{\mu_0} h^2(y) dy \right) dx = E^2(\tilde{x}_{i,t} | \tilde{x}_{i,t} \leq \mu_0),
$$

(20)

where the first equality follows by integration by parts, the second equality is due to $F(x) = \exp(-\int_{x}^{\infty} h(x) dx)$ when $h(.)$ is the reverse hazard rate corresponding to $F(.)$, and the inequality follows from the assumption that $F^2$ reverse hazard rate dominates $F^1$. Then,

$$
P^1(ND) = \zeta^1(\mu_0) = \frac{qF^1(\mu_0)E^1(\tilde{x}_{i,t} | \tilde{x}_{i,t} \leq \mu_0) + (1 - q)E^1(\tilde{x}_{i,t})}{qF^1(\mu_0) + 1 - q} < \frac{qF^2(\mu_0)E^2(\tilde{x}_{i,t} | \tilde{x}_{i,t} \leq \mu_0) + (1 - q)E^2(\tilde{x}_{i,t})}{qF^2(\mu_0) + 1 - q} = \zeta^2(\mu_0) = P^2(ND)
$$

Since the truncation $\mu_0$ is arbitrary, the above argument demonstrates that $\zeta^1(x) < \zeta^2(x)$ for any $x$.

The rest of the proof follows by recursion. Suppose that $P^1_{it}(ND) < P^2_{it}(ND)$ for some $t \geq 1$. Then, the recursion is shown to be true from the following sequence of inequalities

$$
P^1_{it+1}(ND) = \zeta^1(P^1_{it}(ND)) \leq \zeta^1(P^2_{it}(ND)) \leq \zeta^2(P^2_{it}(ND)) = P^2_{it+1}(ND),
$$

where (A) is proven below and (B) follows immediately from $\zeta^1 < \zeta^2$.

(A) Denoting $\tau^*_1$ as the Jung and Kwon (1988) equilibrium threshold, it has been shown in prior literature that $\zeta^1(\tau^*_1) = \tau^*_1$ and $\zeta^1(y)$ is increasing for $y > \tau^*_1$, see footnote 8. From Proposition 2.1, $P^1_{it}(ND)$ is always greater than $\tau^*_1$, which readily establishes inequality (A).□

**Proof of Corollary 2.3:** The claim is proved in the general case of homothetic distributions such that

$$
F(x) = \Phi(\frac{x - \mu}{\sigma}),
$$

such that $\Phi(.)$ is a c.d.f. that does not depend on $m$ or $\sigma$ and with mean zero (for example, $\Phi(.)$ can be set to a standard normal). Then:

36
\[ P_{it+1}(ND) = \zeta(P_{it}(ND)) = \frac{q F(P_{it}(ND)) \mathbb{E}(\tilde{x}_{it} | \tilde{x}_{it} \leq P_{it}(ND)) + (1 - q) \mu}{q F(P_{it}(ND)) + 1 - q} \]

\[ P'_{it+1}(ND) = \frac{q \Phi(P'_{it}(ND)) \mathbb{E}(\tilde{x}'_{it} | \tilde{x}'_{it} \leq P'_{it}(ND))}{q \Phi(P'_{it}(ND)) + 1 - q} , \]

where any prime \( g' = (g - m) / \sigma \) refers to a standardized variable, which in turn implies that \( \{P'_{it}\} \) does not depend on \( m \) or \( \sigma \). Further, given that \( 1 - \Phi(P'_{it}(ND)) \) is the probability of disclosure, this probability of disclosure is decreasing in

\[ P'_{0} \equiv \frac{\mu_0 - \mu}{\sigma} . \]

If \( P'_{0} \) is held constant, \( P'_{it}(ND) \) does not depend on \( \mu \) or \( \sigma \). If \( \mu_0 \) is held constant, \( P'_{0} \) is decreasing in \( m \) and \( \sigma \), which implies that the opposite comparative static on the probability of disclosure. \( \square \)

**Proof of Proposition 2.2:** The law of motion from evaluating the adaptive update at \( \tilde{\theta}_t = 1 \). The law of motions in case (ii) are given in text for the case with positive probability of non-disclosure or with full-disclosure. What remains to be shown is that, in case (ii.b), \( P_{it+1}(ND) < P_{it}(ND) \) if and only if \( P_{it}(ND) < \hat{x} \).

To show this, consider the properties of the updating functional \( P_{it+1}(ND) = \zeta(P_{it}(ND)) \). Differentiating \( \zeta(y) \) twice in \( y \), it can be readily verified that this function is decreasing and then increasing, with a global minimum \( \hat{x} \) which further satisfies \( \hat{x} = \phi(\hat{x}) \). This latter equation implies that the minimum satisfies the equilibrium condition in Jung and Kwon (1988). \( \square \)

**Proof of Proposition 3.1:** Recall that \( \Delta_{it} = \mathbb{E}_t(\frac{P_{it+1}}{1 + r} + y_{it} - P_{it}) \) is the numerator of the return in (9) and let \( \hat{y} = \mathbb{E}_t(\hat{y}_{it}) \). Then:

\[
\Delta_{it} = \frac{P(\mathbb{E}_t(\hat{y}_{it+1}))}{1 + r} + \hat{y} - P(\hat{y})
\]

\[
= \frac{P(\eta \hat{y} + (1 - \eta) \mu)}{1 + r} + \hat{y} - P(\hat{y})
\]

\[
= \frac{\eta \beta(\hat{y} - \mu) + \mu(1 + \frac{1}{r})}{1 + r} + \hat{y} - \beta(\hat{y} - \mu) - \mu(1 + \frac{1}{r})
\]

\[
= \left( \frac{1}{1 + r - \eta} + \frac{1 + r}{1 + r - \eta} \right) \hat{y} + \left( \frac{1 + r}{1 + r - \eta} - \eta \right) \frac{1}{1 + r - \eta} - 1 \mu ,
\]

37
where the first equality follows from linearity of $P(\cdot)$, the second from the auto-regressive process for $y_{it}$, the third and fourth from developing the price in (7) and substituting $\beta$. □

**Proof of Proposition 3.2:** Let $d_{it} \neq ND$ denote the correct (disclosed) expected cash flows.

$$E_t(\Delta_{it}|d_{it}) = \frac{P(E_t(\hat{E}_{t+1}(\hat{y}_{it+1})))}{1 + r} + d_{it} - P(d_{it}). \quad (21)$$

From Proposition 2.2, the following inequalities hold:

$$E_{t+1}(\hat{y}_{it+1}|d_{t+1} = ND, A_t) > \hat{E}_{t+1}(\hat{y}_{it,t+1}|d_{t+1} = ND, A_t)$$

which implies, after taking expectations and reinjecting in the pricing equation (21),

$$E_t(\Delta_{it}|d_{it} \neq ND, A_t) < \frac{P(E_t(\hat{y}_{it,t+1})))}{1 + r} + d_{it} - P(d_{it}) \leq E_t(\Delta_{it}|d_{it} \neq ND, A_t^c),$$

where the fact that the middle term is zero is shown in Proposition 3.1. □

**Proof of Corollary 3.2:** Suppose that the firm discloses $d_{it} = x_{it}$ at period $t$,

$$\Delta_{it} = \frac{1}{1 + r - \eta} \left( \hat{E}_{t+1}(\hat{y}_{it+1}) - \mu \right) + \frac{\mu}{r} + y_{it} - \frac{1 + r}{1 + r - \eta} \left( x_{it} - \mu \right) - \mu (1 + \frac{1}{r})$$

$$\frac{\Delta_{it}}{\beta} = \frac{1}{1 + r} \left( \hat{E}_{t+1}(\hat{y}_{it+1}) - \eta y_{it} - (1 - \eta) \mu \right) + y_{it} - x_{it}$$

$$\frac{(1 + r)E_t(\Delta_{it})}{\beta} = E_t(\hat{E}_{t+1}(\hat{y}_{it+1})) - \eta y_{it} - (1 - \eta) \mu$$

$$= qa_t E_t(\max(\hat{y}_{it+1}, z_{t+1})) + (1 - p_{\theta_t}) \hat{k} E_t(\hat{y}_{it+1}) + (1 - q) z_{t+1}$$

$$= (1 - (1 - p_{\theta_t}) \hat{k}) q a_t E_t(\max(\hat{y}_{it+1}, z_{t+1}) + (1 - q) z_{t+1})$$

where $a_t = p_{\theta_t} + (1 - p_{\theta_t})(1 - \hat{k})$ of not being subject to the mandate.

The term $B$ is the expected belief in the disclosure game of Jung and Kwon (1988) conditional on an exogenously set non-disclosure $z_{t+1}$ and a probability of information endowment $qa/(1 - (1 - p_{\theta_t}) \hat{k})$.

Hence, if $z_{t+1}$ is Bayesian and set at $z_{t+1}^* = \zeta(z_{t+1}, (1 - p_{\theta_t}) \hat{k})$, $B$ must equate the unconditional expectation
\( E_t(\tilde{y}_{t+1}) \) which, by Proposition 3.1, implies no price drift. By monotonicity in \( z_{t+1} \), it follows that the price drift is positive if \( z_{t+1} > z^*_t \) and negative otherwise. \( \square \)

**Proof of Corollary 3.3:** This follows from the same argument as in Proposition 3.2, but adding the effect of the current cash flow surprise \( z_{t+1} - z_t \). It has been shown from the minimum principle in Proposition 2.2 that \( z_{t+1} - z_t > 0 \) if and only if \( z_t < \tau^*_t \). In case (i), the current cash flow surprise is positive, which can only occur if \( k_t(1 - \theta_t) = 1 > k_{t-1}(1 - \theta_{t-1}) = 0 \). Hence, at \( t+1 \), from Proposition 3.2 (a), the belief must be greater than the objective value, implying positive drift. In case (ii), a sufficiently large discount rate \( r \) implies that the negative cash flow surprise dominates the effect of future price and two sequential non-disclosure imply a negative drift due to \( z_{t+1} < z_t \). For the last sub case, if the regulation becomes more demanding in \( t+1 \), Proposition 3.2 (b) implies a negative price drift. \( \square \)

**Proof of Proposition 4.1:** To prove this statement, consider the variance of cash flows (hereafter, volatility) as a function of an exogenously set threshold \( z \in (\tau^*_h, 0) \),

\[
E((\tilde{x}_{it} - \zeta(z, h))^2|ND) = \frac{q' \int_{-1}^{z} f(x)(x - \zeta(z, h))^2 dx + (1 - q') \int f(x)(x - \zeta(z, h))^2 dx}{q' F(z) + 1 - q'}
\]

\[
= \frac{16 + (q')^2(1 - z)^4 - 8q'(2 - z^3 - z)}{12(2 - q'(1 - z))^2} < 1/3 = Var(\tilde{x}_{it})
\]

\[
E((\tilde{x}_{it} - \zeta(\tau^*_{1-h}, h))^2|ND, \tau^*_{1-h}) = \frac{4\sqrt{1 - q'}(2 - q') - 8(1 - q')}{3(q')^2}
\]

\[
\frac{\partial E((\tilde{x}_{it} - \zeta(z, h))^2|ND)}{\partial z} = \frac{q' ((q')^2(z - 1)^4 + 8q'(z^3 - 3z^2 + z + 1) - 8)}{6(2 - q'(1 - z))^3}
\]

\[
\sim D(q', z) \equiv (q')^2(z - 1)^4 + 8q'(z^3 - 3z^2 + z + 1) - 8.
\]

\( D(q', z) \) convex in \( z \), so the volatility is (a) increasing, (b) decreasing and then (c) increasing. Evaluating at this expression,

\[
D(q', -1) = 16(1 - (q'))^2 > 0
\]

\[
D(q', \tau^*_h) = \frac{32(1 - q')(2(1 - q') - (1 - q')\sqrt{1 - q'})}{(q')^2} < 0
\]

\[
D(q', 0) = q'(8 + q') - 8,
\]

\[
39
\]
which implies that (a) is non-empty for \( z \) sufficiently small, (b) is non-empty and (c) occurs for \( z \) close enough to zero if and only if \( q' > 2(\sqrt{6} - 2) \approx .9 \). When \( h_{t+1} = h_t \), given \( q' \) remains constant and \( z_{t+1} < z_t \), the claim follows immediately. □

**Proof of Corollary 4.1:** Differentiating the volatility in \( q' \)

\[
\frac{\partial \mathbb{E}((\tilde{x}_{it} - \zeta(z, h))^2|ND)}{\partial q'} \sim -4z + q'(z - 1)(4 + z),
\]

implies that the volatility is hump-shaped in \( q' \) with maximum at \( \hat{q}(z) \equiv 4z/(3 + 2z + z^2) \). Evaluating this expression at the lower bound \( z = \tau^*_h, \hat{q}(z) < q' \) which implies that this term is in the decreasing part of the curve, implying the claim in the Corollary. □

**Proof of Corollary 4.2:** As in Proposition 4.1, consider the skewness of cash flows as a function of an exogenously set threshold \( z \in (\tau^*_h, 0) \),

\[
\mathbb{E}((\tilde{x}_{it} - \zeta(z, h))^3|ND) = \frac{q' \int_{-\infty}^{x} f(x) (x - \zeta(z, h))^3 dx + (1 - q') \int f(x) (x - \zeta(z, h))^3 dx}{q' F(z) + 1 - q'}
\]

\[
= \frac{q'(1 - q')(1 - z^2)^2}{(2 - q'(1 - z))^3} > 0 = \text{Skew}(\tilde{x}_{it})
\]

\[
\mathbb{E}((\tilde{x}_{it} - \zeta(\tau^*_h, h))^3|ND, \tau^*_h) = \frac{4\sqrt{1 - q'(2 - q') - 8(1 - q')}}{3(q')^2}
\]

\[
\frac{\partial \mathbb{E}((\tilde{x}_{it} - \zeta(z, h))^3|ND)}{\partial z} \sim D_2(q', z) \equiv -8z - q'(3 - z)(1 - z),
\]

where \( D_2 \) is readily verified to be decreasing and satisfies:

\[
D_2(q', 0) = -3q' < D_2(q', \tau^*_h) = \frac{8(1 - q') - \sqrt{1 - q'(8 - 4q')}}{q'} < 0 < D_2(q', -1) = 8(1 - q').
\]

To obtain the threshold in Table 1, differentiate the skewness in \( q \) and note that it is decreasing with a unique root at \( (2q' - 4 + \sqrt{16 + q'(q' - 16)})/q' \). □
Proof of Proposition 4.2: From (13), the current surplus is written as a function of the actual shock $x_{it}$ and the belief $P_{it}$,
\[
\pi(x_{it}, \psi(P_{it})) = x_{it}\psi(P_{it}) - \xi \circ \psi(P_{it}).
\]
(23)

where $P_{it} \equiv x_{it}$ conditional on disclosure, $P_{it} \equiv P_{it}(ND)$ conditional on non-disclosure, and $\psi(P) \equiv (\zeta')^{-1}(P)$. In short-hand, omitting time indices, denote $\pi^*(x) \equiv \pi(x, \psi(x))$, $p_{nd} \equiv P_{t}(ND)$, $k' \equiv k(1 - \theta_t)$ and

\[
V = (q(1 - k')F(p_{nd}) + 1 - q)\pi(\zeta(p_{nd}), \psi(p_{nd})) + q(1 - k')\int_{\mathbb{R}} \pi^*(x) f(x) dx + qk' \int_{\mathbb{R}} \pi^*(x) f(x) dx.
\]

Differentiating this expression:
\[
\frac{1}{q} \frac{\partial V}{\partial k'} = -F(p_{nd})\pi(\zeta(p_{nd}), \psi(p_{nd})) + \int_{\mathbb{R}} \pi^*(x) f(x) dx - \int_{p_{nd}}^{\mathbb{R}} \pi^*(x) f(x) dx \]

\[
> - \int_{\mathbb{R}} \pi^*(x) f(x) dx + \int_{\mathbb{R}} \pi^*(x) f(x) dx - \int_{p_{nd}}^{\mathbb{R}} \pi^*(x) f(x) dx = 0,
\]

where the bound follows immediately from Jensen’s inequality.

\[
V = q(1 - k') \int_{\mathbb{R}} f(x) \pi(x, \psi(p_{nd})) dx + (1 - q) \int_{\mathbb{R}} f(x) \pi(x, \psi(p_{nd})) dx
\]

Holding $k'$ constant, let $V$ (resp., $V'$) denote the surplus at $p_{nd}$ (resp., $p'_{nd}$), with $p_{nd} < p'_{nd}$.

\[
V - V' = q(1 - k') \int_{\mathbb{R}} \delta(x) f(x) dx + (1 - q) \int_{\mathbb{R}} \delta(x) f(x) dx
\]

\[
+ q(1 - k') \int_{p_{nd}}^{p'_{nd}} (\pi^*(x) - \pi(x, \psi(p'_{nd}))) f(x) dx,
\]

with $\delta(x) = \pi(x, \psi(p_{nd})) - \pi(x, \psi(p'_{nd}))$. The positivity of the second row is immediate. For the first row, let $A$ refer to the event in which either the firm is uninformed or decides to withhold information. Due to the convexity of the production function in (13), the expected surplus conditional on $\mathbb{E}(\pi(\bar{x}, I|A))$ is hump-shape with its maximum at $I^* = \psi(\mathbb{E}(\bar{x}|A)) = \psi(\zeta(p_{nd}))$. Positivity of the first row follows from $I^* < p_{nd} < p'_{nd}$. □
Proof of Corollary 4.3: The Corollary follows immediately from Proposition 4.2: if initial beliefs are such that $\zeta(P_0(ND),0) > \tau_1^*$ or if the financial innovation occurs, then beliefs in the next period are in the region where optimism reduces efficiency. For the last claim, note that, if the innovation occurs, more pessimism is always desirable. □

Proof of Corollary 4.4: If beliefs are sufficiently optimistic, all strategic firms withhold conditional on the financial innovation, and the current investment is $\psi(\bar{x})$, causing $V_0 < 0$. □

Proof of Proposition 4.3: In this proof, define $k_t$ such that $\bar{x}_t$ above $k_t$ must be disclosed. Define the sequence of prices $P_t^{LF}(ND)$ under laissez-faire, which is decreasing and converges to $\tau_0^*$ (Proposition 2.1). To prove the claim, it suffices to show that there exists a regulation with $k_0 < \mu_0$ but $k_t = \bar{x}$ for $t > 0$ such that the regulator is better-off than laissez-faire (of course, this construction can be repeated for $k_t$ with $t \geq 1$ to increase surplus even further). Denote $P_t(ND)$ as the associated price sequence.

I make two important preliminary observation. Under laissez-faire after $t \geq 1$, $S_1$ is decreasing in $P_t(ND)$. This follows immediately from the fact that $P_{t+1}(ND)$ is decreasing in $P_t(ND)$, and, from Proposition 4.2, the regulator achieves more current surplus $V_t$ with more pessimistic beliefs. Hence, given that any $k_0$ reduces withholding, one needs only find a $k_0$ such that $P_1(ND) < P_1^{LF}(ND)$, where $P_1^{LF}(ND)$ refers to the non-disclosure price under laissez faire (with $k_0 = \bar{x}$). Now, set instead $k_0 \in (\zeta(P_t(ND)), P_t(ND));$ then, this policy enforces more disclosure in the present than laissez-faire and and a lower next period belief $P_{t+1}(ND)$. It follows that, for any given $\mu_0$, $k_0$ yields higher $V_0$ and more pessimistic beliefs in $t + 1$ than one period of laissez-faire, concluding the proof. □

Proof of Proposition 4.4: Assume that there is an exogenous supply of $N$ shares being put of sale, and a set of investors $M > 1$. Note that $P_t^j(ND)$ is decreasing in $j$. Conditional on non-disclosure and a price $P$, the inverse demand correspondence is given by

$$D_t(P) = \sum_{P_t^j(ND) \geq P} n_j$$

for any $P \neq P_t^j(ND)$ and $D_t(P) \in [\sum_{j' \leq j-1} n_{j'}, \sum_{j' \leq j} n_{j'}]$ for $P = P_t^j(ND)$. The market-clearing solution to $D_t(P_t(ND)) = N$ with highest price must be given by $P_t(ND) = P_t^{\star}(ND)$ where $\sum_{j \leq j_{\star}-1} n_j <$
\[ N \leq \sum_{j \leq j^*} n_j. \] The comparative statics then follow from the fact that \( D_t(P) \) is increasing in \( n_j \), implying that prices must increase when \( n_j \) increases, and that a shift in the supply \( N \) must decrease prices. \( \Box \)

**Proof of Corollary 4.5:** The proof is identical to Proposition 4.4 except that \( P^0_t(ND) < \tau_1^* < P^1_t(ND) < \ldots < \zeta(P^2_t(ND)) < \zeta(P^1_t(ND)). \) \( \Box \)

**Bibliography**


Aghamolla, Cyrus, and Kevin Smith (2021) ‘Strategic complexity in disclosure.’ Available at SSRN 3936805


Bertomeu, Jeremy, Edwige Cheynel, and Davide Cianciaruso (2019) ‘Strategic withholding and imprecision in asset measurement.’ *Journal of Accounting Research*


Bourveau, Thomas, Matthias Breuer, and Robert C. Stoumbos (2020) ‘Learning to disclose: Disclosure dynamics in the 1890s streetcar industry.’ *Available at SSRN 3757679*

Bourveau, Thomas, Matthias Breuer, Jeroen Koenraadt, and Robert C Stoumbos (2021) ‘Public company auditing around the securities exchange act.’ *Available at SSRN 3837593*

Brandenburger, Adam, and Ben Polak (1996) ‘When managers cover their posteriors: Making the decisions the market wants to see.’ *The Rand Journal of Economics* pp. 523–541


Ganuza, Juan-Jose, and Jose S. Penalva (2010) ‘Signal orderings based on dispersion and the supply of private information in auctions.’ *Econometrica* 78(3), 10071030


Smith, Kevin (2020) ‘An option-based approach to measuring disclosure asymmetry.’ *Available at SSRN 3068855*


