What Quantity of Reserves Is Sufficient? *

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Abstract

What quantity of reserves should the Fed supply to support effective monetary policy implementation and an efficient interbank payment system? To answer this question, I construct a model linking interbank intraday payment timing with monetary policy implementation. A low supply of reserves causes banks to delay payments to each other and strategically hoard reserves, which in turn disincentsivizes banks from providing liquidity to short-term funding markets, driving up the spreads between overnight risk-free market rates and the central bank deposit rate. As reserve balances get sufficiently low, even small reductions in reserves can have large impacts on these spreads, as in September 2019. My fitted model captures the funding rate spikes of September 16-18, 2019 as an out-of-sample event.

This is a preliminary draft.

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Our goal is to provide an ample supply of reserves to ensure that control of the federal funds rate and other short-term interest rates is exercised primarily by setting our administered rates and not through frequent market interventions [...] it is clear that without a sufficient quantity of reserves in the banking system, even routine increases in funding pressures can lead to outsized movements in money market interest rates. This volatility can impede the effective implementation of monetary policy, and we are addressing it.

—Jerome Powell, “Data-Dependent Monetary Policy in an Evolving Economy,” October 08, 2019

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1 Introduction

What quantity of central bank deposits (“reserves”) is sufficient to support effective monetary policy implementation and an efficient interbank payment system? I show that the sufficiency of reserves depends on (1) strategic behavior in intraday interbank payment timing and (2) demands in wholesale funding markets. A low supply of reserves causes banks to delay payments to each other, reinforced in equilibrium by strategic complementarity in interbank intraday payment timing, resulting in strategic hoarding of reserves. This in turn disincentivizes banks from providing liquidity in wholesale funding markets, driving up the spreads between wholesale funding market rates and the central bank’s deposit rate, called “IOER.” I estimate my model with a method of moments procedure and provide the first structural model estimate of the minimum level of reserves necessary to support effective monetary policy implementation, efficient interbank payment system, and liquid wholesale funding markets.

The aggregate quantity of reserves supplied by the Federal Reserve (Fed) is an important policy concern: A sufficient level of reserve balances supports an efficient interbank payment system as well as effective monetary policy implementation (pass-through of the Fed-administered rate into risk-free overnight market rates). An efficient interbank payment system relies on banks making large volumes of timely payments. Because those payments are predominately settled with reserves, a sufficient quantity of reserves “lubricates” the payment system (Atalay, Martin and McAndrews, 2010). Sufficient reserve balances also support monetary policy implementation by allowing banks to close the spreads between market rates and IOER (Ihrig, Senyuz and Weinbach, 2020): When wholesale market risk-free rates, such as Treasury repo rates, are higher than IOER, banks active in wholesale funding markets (“dealer banks”) with extra reserves can in principle lend reserves in wholesale markets, making an arbitrage profit, which forces the spreads between wholesale funding rates and IOER close to

1Jerome Powell is the chair of the Federal Reserve. See his full speech here.
zero. When reserves are insufficient, however, banks may hoard reserves and not enforce this arbitrage, impeding monetary policy implementation.

In 2019, as total reserve balances decreased\(^2\), several risk-free funding rates, including Treasury repo rates, experienced sporadic spikes, as shown in Fig. 1. Notably, on September 16-18, 2019, overnight repo rates spiked more than 300 basis points, attracting considerable attention from global policymakers, academics, and market participants.\(^3\) These spikes surprised the Fed,\(^4\) and necessitated emergency interventions. Other overnight risk-free funding markets also suffered from liquidity shortages (e.g., the overnight synthetic dollar interest rate\(^5\) as documented by Correa, Du and Liao 2020. See also Fig. 12). These disruptions rippled through various related securities markets (see Figs. 13 to 18). Low reserve balances also reduced the efficiency of the intraday interbank payment system: Copeland, Duffie and Yang (2020) document that the payment timing stress\(^6\) on September 17, 2019, reached its highest level since 2015. The observed link between payment timing stress and funding market spikes is consistent with the mechanism of my paper, which I explain as follows.

My model links two critical empirical facts. First, large dealer banks are the marginal lenders in short-term funding markets, such as repo markets and FX swap markets (Avalos, Ehlers and Eren, 2019; Correa, Du and Liao, 2020). Thus, the equilibrium funding rates are closely related to large dealer banks’ marginal value of reserves.\(^7\) Second, in modern interbank payment system, large banks rely heavily on incoming payments from other banks before being able to make the bulk of their own outgoing payments. This reliance gives rise to strategic

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\(^2\)Total reserve balances reached a peak of about $2.8 trillion in October 2014. In 2017, the Federal Open Market Committee began implementing balance sheet normalization and planned to reduce its assets and liabilities, including reserves, to the greatest extent consistent with “efficient and effective monetary policy.” System-wide reserve balances gradually declined to a low of about $1.4 trillion in early September 2019. See Board of Governors of the Federal Reserve System (2019) for an overview of the Fed’s balance sheet normalization policies.


\(^4\)Lorie Logan, manager of the System Open Market Account (SOMA) for the Federal Open Market Committee (FOMC), stated on September 20, 2019, “The expectation had been that as repo rates rose, banks would withdraw excess cash held at the Fed and lend it into the repo market... Instead the New York Fed had to step in to provide that cash as banks remained on the sidelines.”

\(^5\)The synthetic dollar interest rate is the implied dollar interest rate in the foreign exchange (FX) swap markets (borrowing dollars by first borrowing in foreign currency and swapping this foreign funding for dollars).

\(^6\)Payment timing stress is measured by the payment time net of its sample mean when 50% of the day’s total incoming value has been received by the 10 largest dealer banks over Fedwire. The later dealer banks receive their incoming payments, the higher the payment timing stress.

\(^7\)Most transactions in wholesale funding markets are settled using reserve balances at the central bank, so lending in the funding markets causes an outflow of banks’ excess reserves (Bech, Martin and McAndrews, 2012; Correa et al., 2020).
complementarity in banks’ payment timing decisions (Bech and Garratt, 2003; McAndrews and Rajan, 2000): For any large bank—say, JPMorgan—incoming payments from other banks provide the balances needed to cover its own outgoing payments, so when JPMorgan believes other banks will send it payments early in the day, it will likewise tend to pay others early. On the other hand, when JPMorgan believes other banks are paying it late—for example, because of low reserve balances—JPMorgan will also pay late.

At the center of my mechanism is the aforementioned strategic complementarity in inter-bank intraday payment timing. A sufficiently low supply of reserves causes banks to suddenly hoard reserves, reinforced by a feedback effect stemming from the strategic complementarity of intraday payment timing, and leads to intraday payment timing stress. This results in high marginal values of reserves, and disincentivizes banks from efficiently allocating liquidity into wholesale funding markets. As such, when reserves are close to being insufficient, even small reductions in reserve balances can have strong nonlinear or discontinuous impacts on short-term wholesale funding rates, mirroring observed market events such as the wholesale funding rate spikes of September 2019.
Figure 2: Sufficient reserves for large dealer banks to keep the expected GCF-IOER spread below 13 bps as suggested by my model (green line), conditional on other macroeconomic variables (e.g., Treasury issuance, repo lending quantity and other large banks’ reserves). GCF is an index of overnight Treasury general collateral interdealer repo rates. IOER, the interest rate paid on reserves, is the Fed’s target policy rate. Large U.S. dealer banks are the total reserve balances of 10 large and repo-active banks identified by Copeland, Duffie and Yang (2020). Other large banks are the largest 100 banks except for the 10 large dealer banks. Data: Fedwire Funds Service, FRBNY.

To gauge the quantitative implications of this strategic complementarity, I estimate the parameters of my model using a method of moments procedure with data from 2019. My sample comprises days leading up to but not including the repo spike event of September 16-18, 2019. My estimated model is able to treat the funding rate spikes of September 16-18, 2019, as an out-of-sample event. This supports the validity of the mechanism of my model and underscores the importance of incorporating the effect of strategic complementarity of payment timing in shaping central-bank policy. I use my estimated model to calculate the first counterfactual estimate of the minimum level of reserves of large U.S. dealer banks that would have mitigated reserve hoarding and have kept the expected spread between GCF repo rates and IOER below 13 basis points (see Fig. 2).
2 Background

The notion of reserve sufficiency has changed substantially since the GFC. Before the 2007-2009 crisis, the aggregate reserves were typically under $50 billion. Relatively small reserve balances were sufficient both for efficient functioning of wholesale funding markets and for banks to manage their intraday liquidity demands. During this period, the Fed achieved its policy rate by actively managing the supply of reserves in the banking system. By contrast, today’s post-GFC liquidity requirements incentivize large banks to hold substantial reserve balances at the Fed throughout each day (see Appendix B.3 for more details), and the level of reserves necessary to maintain efficient funding markets and the interbank intraday payment system has been an open question. The level of aggregate reserves also increased substantially with the Fed’s crisis facilities and post-crisis quantitative easing programs. Thus, in the post-GFC regulatory and macroeconomic environment, the Fed implements monetary policy by adjusting the overnight interest rate on excess reserves (IOER)\(^8\) on balances held at the Fed.

Although the Fed does not directly target risk-free short-term wholesale funding rates (e.g., repo rates), a volatile funding market with rate spikes outside the Fed’s target range raises concerns about the Fed reacted quickly to the funding rate spikes on September 16-18, 2019, by announcing a series of repo operations and changed the course of its balance sheet normalization process. Additional supply of large amounts of reserves by the Fed eventually drove the overnight repo rates back to moderately low levels. However, disruptions in the term wholesale funding markets lasted longer (Figs. 13 to 15).

The funding rate spikes of September 16-18, 2019 were surprising to policymakers and market participants. From 2011 to 2019, the total excess reserves exceeded $1.4 trillion, far above the pre-GFC level. In addition, according to Senior Financial Officer Surveys conducted by the Fed in September 2018 and February 2019 regarding the “lowest comfortable level of reserves,” there should still have been ample reserve balances in early September 2019 (Andros, Beall, Martinez, Rodrigues, Styczynski and Thorp, 2019), more than enough for banks to conduct the repo-IOER arbitrage, lending excess reserves in the Treasury repo market and keeping overnight Treasury repo rates near IOER.

Regarding the important factors determining reserve sufficiency, a large body of academic work and many finance industry commentaries highlight the roles of supervisory and regulatory requirements, especially Basel III liquidity regulations,\(^9\) which encourage large U.S.

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\(^8\)Starting July 29, 2021, the Fed also started to refer to its policy rate as the “interest rate on reserve balances (IORB)”.  
\(^9\)See more details in Appendix B.3.
banks to hold substantial reserve balances at the Fed throughout each day. Those regulations may have weakened monetary policy implementation by preventing banks from lending those reserve balances in the short-term funding markets (see Anbil, Anderson and Senyuz 2020b, d’Avernas and Vandeweyer 2020, and Nelson and Covas 2019 among others). However, given that banks’ excess reserves exceeded the regulatory required level as suggested by actual excess reserve levels and the aforementioned Senior Financial Officer Surveys conducted by the Fed, one might wonder how these regulations actually constrained the banks from lending in the repo market. In fact, in their NY Fed staff report, Afonso, Cipriani, Copeland, Kovner, La Spada and Martin (2020b) point out that “it seems unlikely that regulation itself may have been a key contributing factor to the money market stress of mid-September. Banks typically hold considerable buffers above their regulatory minima, which means that the regulatory constraints were, in all likelihood, not binding.”

My paper addresses the above puzzle by demonstrating that the reserves can suddenly become insufficient at a level well above the level required by regulation. Whereas banks’ liquidity positions and stress testing information are available to the policymaker on a daily basis (Bush, Kirk, Martin, Weed and Zobel, 2019), my model implies that this information alone is inadequate for gauging the minimum aggregate reserve supply required for a well-functioning money market and effective monetary policy implementation. Hence, a holistic understanding of the way banks use reserves to make payments is necessary to guide the Fed’s policy on the quantity of reserve supply.

3 Related literature

This paper is closely related to the empirical literature studying financial intermediaries and the mechanism of wholesale funding markets such as the repo market. Large financial intermediaries, especially large U.S. banks, play increasingly important roles in wholesale funding markets. Post-crisis regulations thus have profound implications for wholesale funding markets by influencing intermediaries’ balance sheets and liquidity management decisions (Duffie, 2018; Adrian and Shin, 2011; Ranaldo, Schaffner and Vasios, 2020; Egelhof, Martin and Zinsmeister, 2017). Correa, Du and Liao (2020) examine the daily balance sheet information of the large U.S. dealer banks and find they substantially increased the liquidity provision in the FX swap markets and repo markets from 2016 to 2020. In particular, Correa, Du and Liao (2020) point out that post-GFC, key Basel III regulatory ratios such as SLR and GSIB capital surcharge scoring.

Fig. 11 shows the net liquidity provision (lending minus borrowing) of the large U.S. dealer banks to the repo markets, in comparison with the liquidity provision by the money market funds (MMF).
have significantly increased banks’ balance sheet costs, so banks heavily rely on draining down their own reserves for liquidity provision. Avalos, Ehlers and Eren (2019) show that large U.S. banks have become important net lenders in the repo market since 2011.

Concurrent empirical work by Adam Copland, Darrell Duffie, and myself explores the empirical relationship between the supply of total reserves and repo rates. One of the key findings is illustrated in Fig. 23 and Table 2, which both suggest that lower aggregate reserves among large dealer banks are associated with elevated repo rates. Copeland, Duffie and Yang (2020) also find a strong relationship between interbank intraday payment timing delays and repo rate spikes, as shown in Fig. 24. Additionally, in line with Correa, Du and Liao (2020), Copeland, Duffie and Yang (2020) find evidence supporting the role of demand factors such as Treasury issuance on repo rates. The repo spike of mid-September 2019 is a good example of how supply and demand factors come into play, which is also described in detail by Afonso, Cipriani, Copeland, Kovner, La Spada and Martin (2020b), Anbil, Anderson and Senyuz (2020a), Anbil, Anderson and Senyuz (2020b), Ihrig, Senyuz and Weinbach (2020), Avalos, Ehlers and Eren (2019), and Martin, James, Palida and Skeie (2020), among others.

While consistent with the above-mentioned empirical research showing a negative relationship between aggregate measures of reserves and wholesale funding rates in normal times, my work is one of the first to show that small reductions in the probability distribution of discretionary reserve balances can cause large discontinuous increases in the wholesale funding rates. In a contemporaneous work, d’Avernas and Vandeweyer (2020) focus on the repo rate spike event of September 2019; they build a model that explores the implications of binding regulations such as internal Liquidity Stress Tests (LST) on repo rate spikes. Their work points to the theoretical possibility that anticipation of future funding market disruption might have played a role in the unexpected rise in Treasury spreads in March 2020. Relative to their paper, my paper analyzes the central role of strategic complementarity in banks’ intraday payment timing on wholesale funding rate spikes and focuses on understanding reserve sufficiency from the angle of an efficient interbank payment system and effective monetary policy implementation.

My paper is related to the literature studying global games and their applications in finance. In my paper, the key mechanism is the strategic complementarity among banks’ payment-timing decisions. The technique of global games with strategic complementarity, in various forms, is extensively applied to study currency attacks, bank runs, debt crises, safe asset determination, and so on (Carlsson and Van Damme, 1993; Morris and Shin, 1998, 2001; Goldstein and Pauzner, 2005; He and Xiong, 2012; Heider, Hoerova and Holthausen, 2015; He, Krishnamurthy and Milbradt, 2019). My paper differs from the standard global games models in one important
dimension: In my model, the agents’ beliefs about the payoff-relevant states are endogenously determined in equilibrium. In that regard, my model is closely related to Angeletos, Hellwig and Pavan (2006) and Angeletos and Werning (2006), who show that endogenously generated information ensures equilibria multiplicity when uncertainty in the economy is small. In my model, the equilibrium is unique under only mild technical conditions. My model also features an interaction between strategic substitutability and complementarity (see discussion under Lemma 1), which presents several new complications. My setting does not allow for standard techniques in global games to be applied directly; I also develop a new solution method to characterize the equilibrium.

A strand of literature considers banks’ liquidity hoarding in the overnight-funding markets that arises from adverse selection and counterparty risk (Heider, Hoerova and Holthausen, 2015; Gorton and Metrick, 2012; Acharya and Merrouche, 2013). By contrast, in my model, precautionary hoarding of reserves in interbank intraday payment systems is caused by strategic complementarity.

Copeland, Duffie and Yang (2020) and this paper are also the first to point out the intimate connection between the intraday interbank payment mechanism and monetary policy implementation in the post-GFC regulatory environment. Due to financial-stability concerns, central banks and academic scholars around the world have extensively studied the intraday payment mechanism among banks (see Armantier, McAndrews and Arnold (2008); McAndrews and Rajan (2000); Bech (2008); Bech and Garratt (2003); Schoenmaker (1995); Zhou (2000); McAndrews and Potter (2002); McAndrews and Rajan (2000) among others). There is also a rich literature on monetary implementation with an ample supply of reserves (Logan, 2019; Afonso, Kim, Martin, Nosal, Potter and Schulhofer-Wohl, 2020a; Piazzesi, Rogers and Schneider, 2019; Lenel, Piazzesi and Schneider, 2019), but intraday payment mechanisms were not thought to be related to monetary policy implementation in the current macroeconomic conditions. My paper points out that after the introduction of post-crisis liquidity regulations, banks are now reluctant to use the Fed’s intraday overdraft facility, and that their own reserves have become the most important source of liquidity. Naturally, this means the effects of strategic complementarity become significantly larger post-GFC, creating a tight link between the intraday payment system and monetary policy implementation.

The rest of the paper proceeds as follows. Section 4 describes the baseline model with two representative banks and two representative wholesale funding borrowers. In the baseline model, banks and borrowers are competitive. (Appendix C.1 considers a model extension that incorporates market power in wholesale funding markets.) Section 5 describes the data.
Bank $i$ learns $R_i$. Borrower $i$ learns $D_i$.

Early payment: Bank $i$ learns total payment $N_i$ to other banks and makes early payment $a_i$.

Funding market opens. Bank $i$ trades with borrower $i$.

Late payment: Bank $i$ makes the remaining payment $N_i - a_i$. Game ends.

Figure 3: The timeline of the model.

Section 6 estimates the baseline model and calculates the counterfactual sufficient level of reserves to support effective monetary policy implementation and an efficient interbank payment system. Section 7 concludes.

4 Baseline model

There are two types of agents: $n$ dealer banks active in liquidity provision in the overnight USD wholesale funding markets\(^\text{11}\) and $n$ overnight wholesale funding borrowers. The timing of the model is shown in Fig. 3. Each business day is divided into four time periods: 0, 1, 2, and 3. Initially, at time 0, bank $i$ observes only $R_i$, its beginning reserve balance, and borrower $i$ observes only $D_i$, its financing target to borrow. The day opens with trading in the wholesale funding markets at time 1. In this market, bank $i$ and borrower $i$ play the trading game. For simplicity, I assume that whenever bank $i$ lends an amount, say, $S_i$, to borrower $i$ in the overnight wholesale funding markets, bank $i$ will transfer $S_i$ quantity of reserves to the clearing bank of borrower $i$.

After making an overnight loan $S_i$ in the funding markets, at time $t = 2$, bank $i$ observes the total payment obligation $N_i$ that bank $i$ must send to other banks by the end of the business day ($t = 3$), and that the banks play a payment game. In this subgame, bank $i$ can choose to pay any smaller amount $a_i$ early to the other banks at $t=2$, and delivers the reminder $N_i - a_i$ late at

\(^{11}\)One prominent feature of the USD wholesale funding markets is the outsized importance of a few large U.S. banks. Copeland, Duffie and Yang (2020) focus on 10 large repo-active dealer banks and show the total reserve balances of large financial institutions outside these 10 were much less influential with respect to repo rates. Similarly, Correa, Du and Liao (2020) study six global systemically important banks (GSIBs)—Bank of America, Citi, Goldman Sachs, JP Morgan, Morgan Stanley, and Wells Fargo—and show they are major liquidity providers in both the repo markets and the FX swap markets.
The timeline is broadly consistent with the operational details of the USD wholesale funding markets and the interbank payment system (i.e., the Fedwire). For example, one of the most important wholesale funding markets, the repo market, operates throughout the business day, but 90% of the trading volume is completed before 9 a.m. (EST), as documented in Copeland, Duffie and Yang (2020), whereas 50% of interbank intraday payments are typically not completed before 1 p.m (EST), as shown by Copeland, Molloy and Tarascina (2019). (To fix ideas, think of $t = 1$ as 6 a.m. to 9 a.m., $t = 2$ as 9 a.m. to 2 p.m., and $t = 3$ as 2 p.m. to 7 p.m.)

To clarify the main ideas, I assume there are only two banks (bank 1 and bank 2) and 2 borrowers (borrower 1 and borrower 2) in the baseline model. I assume the random variables $N_1$ and $N_2$ are identically and independently distributed according to a cumulative distribution function (cdf) $F_N(\cdot)$ with finite mean and support $[N_{\text{min}}, \infty)$ for some $N_{\text{min}} > 0$ and $F_N(\cdot)$ is atomless and strictly increases on $(N_{\text{min}}, \infty)$. In addition, assume $R_1$ and $R_2$ are identically and independently distributed with cdf $F_R(\cdot)$. Similarly, assume $D_1$ and $D_2$ are identically and independently distributed with cdf $F_D(\cdot)$. To simplify notations, I assume the random variables are independent, but my results extend to arbitrary joint distribution over $(R_1, R_2, D_1, D_2)$. The distributions of all random variables $R_1, R_2, D_1, D_2, N_1$ and $N_2$ are common knowledge, but their realizations are private information. Let $i \in \{1, 2\}$. I describe my model from the perspective of bank $i$, and the other bank is denoted bank $j$. The setups and results are symmetric for bank $i$ and bank $j$.

### 4.1 Payment subgame from $t = 2$

In modern interbank payment system, settlement of time-critical interbank payments relies on reserves (McAndrews and Kroeger, 2016; Soramäki, Bech, Arnold, Glass and Beyeler, 2007). The volume of payment requests far exceeds the amount of total reserves in the banking system, as shown in Fig. 19. Large banks have to rely heavily on incoming payments from other banks before being able to make the bulk of their own outgoing payments, and therefore face a serious liquidity management problem when payment requests outbalance incoming payment flows. (See Appendix B.2 for more details.) From the lens of my model, this means that $N_i$ is likely to be much larger than $R_i$.

As documented by Bech, Martin and McAndrews (2012), once bank $i$ transfers the overnight loan $S_i$ to the clearing bank of borrower $i$, that amount of reserves becomes unavailable for the

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12See Appendix C.2 for a general model with $n > 2$ banks and borrowers.
interbank payment system. After observing $N_i$, bank $i$ can pay any positive amount $a_i \leq N_i$ at time 2 and postpone the rest of the payment to time 3. The amount $a_i$ paid by bank $i$ must be measurable with respect to the information of bank $i$ after trading in the funding market. In the current setup, this information is captured entirely by the reserve balances $L_i$ of bank $i$ post trade, net of the minimum required reserve balances of bank $i$ before time 2. That is, $L_i = R_i - S_i - Q$, where $Q > 0$ is an exogenous constant minimum level of reserves required by liquidity regulations. It is worth mentioning that in the pre-crisis era, in the absence of those liquidity regulations, banks had often kept minimum reserve balances and relied heavily on borrowing from the Fed’s intraday overdraft facility to meet ongoing payment demands (Fig. 20). Borrowing from the Fed means banks would have negative intraday reserve balances. By contrast, the post-GFC new liquidity requirements provided incentives for large U.S. dealer banks to maintain substantial balances at the Fed and strongly discouraged them from incurring daylight overdrafts on their reserve accounts at the Fed.\textsuperscript{13} This explains why the mechanism of my paper is important in the post-GFC regulatory environment but did not play a large role in monetary policy implementation pre-GFC.\textsuperscript{14}

After payments at time 1, $L_i - a_i + a_j$ is the residual reserve balance of bank $i$. Bank $i$ bears a cost if this residual balance is below 0, modeled as a per-unit “regulatory cost” of $\psi > 0$. In reality, banks are especially worried about the cost of a failure to satisfy strict new supervision of liquidity sufficiency (Correa, Du and Liao, 2020; d’Avernas and Vandeweyer, 2020), and the stigma in the eyes of their supervisors associated with borrowing from the Fed’s intraday overdraft facility. This implies that $\psi$ is very large. Bank $i$ also suffers a linear cost $c \cdot (N_i - a_i)^+$ caused by paying bank $j$ late, for some late payment cost coefficient $c > 0$. (I call $c$ the “marginal cost of delay.”) Costs associated with late payments are discussed extensively in the literature studying banks’ intraday liquidity management, including work by Ashcraft, McAndrews and Skeie (2011), Afonso and Shin (2011), Bech and Garratt (2003), and Bech (2008). The final cost to bank $i$ associated with payment timing is thus

$$\psi \cdot (L_i - a_i + a_j)^- + c \cdot (N_i - a_i)^+.$$

\textsuperscript{13}This is supported by conversations with relevant senior managers at several GSIBs, as documented by Copeland, Duffie and Yang (2020).

\textsuperscript{14}Large U.S. banks can in principle borrow additional reserves from other financial institutions, for example, from money market funds in the Tri-party repo market, to make outgoing payments. However, borrowing reserves increases the supplementary leverage ratio (SLR) of the large U.S. banks, and the settlement of the Tri-party market is late (typically from 3 p.m. to 6 p.m. EST). Thus, borrowing reserves is much less useful for the purpose of making interbank payments for large banks. For simplicity, I assume banks cannot borrow reserves at time 2.
Here, I adopt the convention that

$$(L_i - a_i + a_j)^- = \begin{cases} 0 & \text{if } L_i - a_i + a_j \geq 0, \\ |L_i - a_i + a_j| & \text{if } L_i - a_i + a_j < 0. \end{cases}$$

The cost function to bank $i$ captures the force of strategic complementarity in banks’ payment timing decisions (Afonso and Shin, 2011). That is, the higher the early payment strategy $a_j$ of bank $j$ conjectured by bank $i$, the higher the best-response early payment $a_i$ of bank $i$. Given the payment strategy $a_j$ of bank $j$, bank $i$ chooses $a_i$ to optimize the conditional expected payoff,

$$U(L_i, N_i) = \mathbb{E}[-\psi (L_i - a_i + a_j)^- - c (N_i - a_i)^+ \mid N_i, L_i]. \quad (1)$$

**Lemma 1.** Suppose $L_i$ and $L_j$ are arbitrarily distributed such that $\mathbb{P}(L_i > 0) > 0$ and $\mathbb{P}(L_j > 0) > 0$. Then there is a Perfect Bayes payment game equilibrium of the form $a_i^* = \min((L_i + \alpha_i)^+, N_i)$, $a_j^* = \min((L_i + \alpha_j)^+, N_j)$ for some constants $\alpha_i, \alpha_j$ such that

$$\alpha_i = \inf \left\{ \vartheta \geq 0 : \mathbb{P}(\min((L_j + \alpha_j)^+, N_j) \leq \vartheta) \geq \frac{c}{\psi} \right\}$$

$$\alpha_j = \inf \left\{ \vartheta \geq 0 : \mathbb{P}(\min((L_i + \alpha_i)^+, N_i) \leq \vartheta) \geq \frac{c}{\psi} \right\}.$$

In particular, when $L_i$ and $L_j$ have the same distribution\footnotemark, then $\alpha_i = \alpha_j = \alpha$. If $\mathbb{P}(L_j \leq 0) > c/\psi,$

\footnotetext{It turns out that on the equilibrium path, $L_i$ and $L_j$ will always have the same distribution, which will be proved by Theorems 1 and 2.}
then $\alpha = 0$. If $\Pr(L_j \leq 0) \leq c/\psi$, then $\alpha \geq N_{\min}$ and

$$\alpha = \inf \left\{ \vartheta \geq 0 : \Pr(N_j \leq \vartheta) \geq \frac{\psi - \Pr(L_j \leq 0)}{1 - \Pr(L_j \leq 0)} \right\}. \quad (2)$$

The equilibrium is unique, except for the knife-edge case where $\Pr(L_j \leq 0) = \Pr(L_i \leq 0) = c/\psi$.

Proofs of all results, including Lemma 1, are in Appendix E. Intuitively, when $\Pr(L_j \leq 0)$ is high, bank $i$ thinks that the bank $j$ will not have enough liquidity to pay early. Thus, bank $i$ is conservative about paying bank $j$ early. Bank $j$ thinks the same, so $a_j$ is also likely to be lower. Due to strategic complementarity, when bank $i$ expects $a_j$ to be lower, bank $i$ wants to make an even lower $a_i$, and so on. This starts a “vicious circle”. Eventually, when $\Pr(L_j \leq 0)$ is above some threshold, both banks start to hoard reserves, making $\alpha = 0$. This intuition is illustrated in Fig. 4.

The force that makes the equilibrium unique is similar to the force in the standard global games literature (Carlsson and Van Damme, 1993; Morris and Shin, 1998, 2001; Goldstein and Pauzner, 2005). Here, the strategic complementarity in banks’ payment strategies and the uncertainty of the other bank’s liquidity situation $L_j$ lead bank $i$ to make the risk dominant payment action: $a_i = \min((L_i + \alpha_i)^+, N_i)$.\textsuperscript{16} However, it is worthwhile highlighting the difference between my model and the standard global games models. In those models, agents are assumed to receive exogenous private signals of the payoff-relevant states. In my model, the payoff-relevant states $L_i, L_j$ are endogenously determined by the trading outcome in the funding market: $L_i = R_i - Q - S_i$. Hence, even though there is a unique equilibrium for the payment subgame, it does not necessarily follow that the entire game has a unique equilibrium (Angeletos, Hellwig and Pavan, 2006; Angeletos and Werning, 2006).

To derive the banks’ optimal lending decisions in the funding market, I first need to characterize banks’ marginal value of reserves for the payment subgame. The continuation value of bank $i$ for reserve balances at the beginning of the payment game, before observing its payment obligation $N_i$, is

$$V(L_i) = \mathbb{E}[U(L_i, N_i) \mid L_i]. \quad (3)$$

When well defined, the left-hand derivative of the value function (3) at a given reserve balance

\textsuperscript{16}See Morris and Shin (2001) for a discussion of the risk dominant strategy.
\[ y \] is
\[ V'(y) = \lim_{x \uparrow y} \frac{V(x) - V(y)}{x - y}. \]

**Lemma 2.** Suppose that in the payment game, bank \( j \) makes payment \( a_j = \min((L_j + \alpha_j)^+, N_i) \) and bank \( i \) makes payment \( a_i = \min((L_i + \alpha_i)^+, N_i) \), for some constants \( \alpha_j, \alpha_i \). When \( L_i + \alpha_i > 0 \), then for bank \( i \),
\[
V'(L_i) = \int_{n \in (L_i^+, L_i + \alpha_i)} \psi P(a_j \leq n - L_i) \, dF_N(n) + \int_{n \in [(L_i + \alpha_i), \infty)} cF_N(n),
\]

When \( L_i + \alpha_i \leq 0 \), for bank \( i \),
\[
V'(L_i) = \psi P(a_j \leq -L_i).
\]

Also, \( V'(\cdot) \) weakly decreases.

**Lemma 2** guarantees the existence of the left-hand derivative of the value function, although \( V(\cdot) \) may not be differentiable. Note \( V'(\cdot) \) for bank \( i \) depends on the payment strategy \( a_i \) and \( a_j \). To make this dependence relationship explicit, I define the marginal value of liquidity function \( \Gamma_i = V'_i(L_i) \) for bank \( i \) as follows:

**Definition 1.** Fix any payment strategy \( a_i = \min((L_i + \alpha_i)^+, N_i) \) and the probability distribution of \( L_i \) for all \( i \in \{1, 2\} \). Let \( j \in \{1, 2\} \backslash \{i\} \). Define the marginal value of liquidity function for bank \( i \) to be the function \( \Gamma_i : \mathbb{R} \times [0, \infty) \rightarrow \mathbb{R}^+ \) such that
\[
\Gamma_i(y, \alpha_i, \alpha_j) = \begin{cases} 
\int_{n \in (y^+, (y + \alpha_i))} \psi P((L_j + \alpha_j)^+ \leq n - y) \, dF_N(n) + \int_{n \in [(y + \alpha_i), \infty)} cF_N(n), & \forall y > -\alpha_i \\
\psi P((L_j + \alpha_j)^+ \leq -y), & \forall y \leq -\alpha_i.
\end{cases}
\]

\( \Gamma(\cdot) \) naturally arises when large dealer banks optimize the quantity of reserves to lend out in the funding market.

### 4.2 Trading game at \( t = 1 \)

I focus on the overnight wholesale funding markets in this paper, but my model can be applied to other short-term funding markets. I model the overnight wholesale funding markets as
over-the-counter markets in which borrower $i$ is matched with bank $i$ and they trade bilaterally. The net cost of borrower $i$ associated with financing some amount $q$ at some funding rate $r$ (endogenously determined in equilibrium) is\(^{17}\)

\[
qr + \frac{\xi}{2}((D_i - q)^+)^2,
\]

where the marginal value of financing, $\xi$, is a positive coefficient determining the sensitivity of the cost to borrower $i$ of unmet financing. I assume $D_i > D_{\text{min}}$ almost surely for a constant $D_{\text{min}}$, and $\mathbb{P}(R_i - Q - D_{\text{min}} > 0) > 0$.\(^{18}\)

The exact form of the equilibrium funding rate $r$ largely depends on the market microstructure of the funding markets, for example, the bargaining power between borrowers and lenders. However, the qualitative predictions of my main results are robust to any market microstructure setup under which the funding rate is an increasing function of $\Gamma_i(y, \alpha_i, \alpha_j)$. This is not surprising, because the driving force is the aforementioned strategic complementarity in the payment subgame. To fix ideas, I discuss results for one concrete example of competitive pricing in the baseline model, where bank $i$ and borrower $i$ act as price takers (Section 4.2.1). In Appendix C.1, I include results under another funding market structure—the case of monopolistic pricing, where bank $i$ acts as a local monopolist by offering a supply schedule $g : \mathbb{R} \to \mathbb{R}$ to screen the borrower’s demand and maximize profit.

### 4.2.1 Competitive pricing setup

In this section, I assume bank $i$ and borrower $i$ are fully competitive. Bank $i$ takes the overnight funding rate $r_i$ as given and lends the quantity $s$, solving

\[
\sup_s sr_i + V(R_i - s),
\]

\(^{17}\)For simplicity, I normalize the Fed’s policy rate benchmark, IOER, to be zero. When IOER is not zero, $r$ should be understood as the spread between funding rates $r^{\text{gross}}$ and IOER, and the cost for borrower $i$ should be adjusted to $qr^{\text{gross}} + (D_i - q)^+ \cdot \text{IOER} + \frac{\xi}{2}((D_i - q)^+)^2 = qr + \frac{\xi}{2}((D_i - q)^+)^2 + D_i \cdot \text{IOER}$ (in equilibrium $q < D_i$). The rest of my analysis does not change.

\(^{18}\)Empirically, banks almost never have negative reserve balances before they start making interbank intraday payments.
where \( sr_i \) is the interest paid by borrower \( i \) to bank \( i \) on the next business day. Borrower \( i \) minimizes net cost by solving

\[
\inf_s sr_i + \frac{\xi}{2}((D_i - s)^+)^2.
\]

The first order conditions for bank \( i \) and borrower \( i \) imply the equilibrium funding rate \( r_i^* \) and quantity \( S_i^* \) satisfy

\[
S_i^* = \inf \left\{ s : \Gamma_i(R_i - Q - s, \alpha_i, \alpha_j) \geq \xi(D_i - s) \right\}
\]

\[
r_i^* = \xi(D_i - S_i^*),
\]

where the marginal value of liquidity \( \Gamma_i \) is defined in Definition 1. Clearly, \( \xi \) governs the demand elasticity.

Throughout the paper, the equilibrium concept is a perfect Bayesian equilibrium. By definition, the distribution of \( L_i = R_i - Q - S_i \) and \( L_j = R_j - Q - S_j \) is endogenously determined by the equilibrium outcome of the trading game. From Eq. (4), the equilibrium outcome of the trading game depends on \( \Gamma_i \) and \( \Gamma_j \), which in turn depend on the distribution of \( L_i = R_i - Q - S_i \) and \( L_j = R_j - Q - S_j \) (Lemma 1). Thus, characterizing the full equilibrium will necessarily involve solving a complicated fixed-point problem. In particular, there is no a priori reason why the model has a unique equilibrium.

It is worthwhile mentioning that there is also strategic substitutability in banks’ lending decisions. The combination of strategic complementarity and strategic substitutability is rarely studied and presents several new complications. For example, when bank \( i \) expects bank \( j \) to lend a smaller loan \( S_j \) in the funding market, bank \( i \) knows bank \( j \) may have more reserves \( L_j = R_j - S_j - Q \) to make a larger payment \( a_j \). Therefore, bank \( i \) has a stronger incentive to lend a larger amount \( S_i \) in the funding market. As a result, even if \( R_i \) and \( R_j \) have the same probability distribution, \( L_i \) and \( L_j \) may not be symmetrically distributed. At first glance, the equilibrium of the entire game seems to inevitably depend on an infinite hierarchy of beliefs between the two banks, because bank \( i \)'s decision to lend \( S_i \) depends on its belief about bank \( j \)'s decision of \( \alpha_j \), which, in turn, depends on bank \( j \)'s belief about bank \( i \)'s strategy, and so on. Theorem 1 and Theorem 3 address this challenge.
4.2.2 Liquidity stress index and equilibrium

Any change in the macroeconomic conditions, such as a reduction of reserve balances, an increase in borrowing demand in the funding markets, or an increase in the regulatory requirement, may change the distributions of $R_i$, $D_i$ and the value of $Q$. Those changes will eventually change banks’ beliefs, their strategies in the payment subgame, and the equilibrium funding rates. I show the complicated macroeconomic conditions can be summarized by one index—the **liquidity stress index**.

**Definition 2.** The **liquidity stress index** is

$$m \equiv \mathbb{P}\left(R_j - D_j - Q \leq -\frac{c}{\xi}\right) - \frac{c}{\psi} = E \left[F_R\left(D_j + Q - \frac{c}{\xi}\right)\right] - \frac{c}{\psi},$$

where $F_R$ is the cdf of $R_j$.

The **liquidity hoarding condition** is when

$$m = E \left[F_R\left(D_j + Q - \frac{c}{\xi}\right)\right] - \frac{c}{\psi} > 0. \quad (5)$$

The **no hoarding condition** is when

$$m = E \left[F_R\left(D_j + Q - \frac{c}{\xi}\right)\right] - \frac{c}{\psi} < 0. \quad (6)$$

The liquidity hoarding condition applies whenever the probability distribution $F_R$ of initial reserve balances is sufficiently low in the sense of first order stochastic dominance, for given parameters $Q, c, \xi, \psi, D_{\min}$. Let $F_{RD}(y) = \mathbb{P}(R_j - D_j - Q \leq y)$. The next two key theorems state how the liquidity stress index determines the level of reserve hoarding in equilibrium. Fig. 5 illustrates the intuition for the mechanism of the following results.

**Theorem 1.** Under the liquidity hoarding condition, there is a unique equilibrium. In this equilibrium, bank $i$ hoards reserves and pays $a^*_i = \min(L^+_i, N_i)$ in the payment subgame. The marginal value of liquidity functions are the same for both banks $\Gamma_i(y, 0, 0) = \Gamma_j(y, 0, 0) = \Gamma(y, 0, 0)$ such that

1. When $y > 0$, $\Gamma(y, 0, 0) = c(1 - F_N(y))$;

2. When $y \leq 0$, $\Gamma(y, 0, 0) = \psi \left(F_N(-y) + (1 - F_N(-y))F_{RD}(-y - \frac{F_N(y)}{\xi})\right)$.

**Theorem 2.** Under no liquidity hoarding, there always exists at least one equilibrium. Any equilibrium must be symmetric (in the sense that $\alpha_i = \alpha_j = \alpha$) with pure payment strategy $a^*_i = \min(N_i, (L_i + \alpha)^+)$. 
Total reserves are insufficient: trigger liquidity hoarding

Bank $i$ believes that $R_j - S_j - Q$ is low

Marginal value of reserves $\Gamma(R_i - S_i - Q)$ suddenly becomes larger

Inefficient payment system

Bank $j$ lowers $a_j$

Lower incoming $a_j$ for bank $i$

Bank $i$ lowers $a_i$

Lower incoming $a_i$ for bank $j$

Equilibrium funding rate $r^*$ jumps up

Ineffective monetary policy implementation

Figure 5: Low supply of reserves causes ineffective monetary policy implementation and an inefficient interbank payment system.

for some $\alpha > N_{\text{min}}$. The marginal value of liquidity functions are the same for both banks $\Gamma_i(y, \alpha, \alpha) = \Gamma_j(y, \alpha, \alpha) = \Gamma(y, \alpha, \alpha)$. In addition, $\alpha$ and $\Gamma$ solve a system of integral equations:

$$
\begin{align*}
\mathbb{P} \left( R_i - D_i - Q \leq -\frac{\Gamma(0, \alpha, \alpha)}{\xi} \right) + \mathbb{P}(N_i \leq \alpha) \left( 1 - \mathbb{P} \left( R_i - D_i - Q \leq -\frac{\Gamma(0, \alpha, \alpha)}{\xi} \right) \right) &= \frac{c}{\psi}; \\
\Gamma(y, \alpha, \alpha) &= \left\{ \begin{array}{ll}
\psi \int_{n \in (y^+, y+\alpha)} F_N(n-y) + (1-F_N(n-y)) F_{RD}(n-y-\alpha - \frac{\Gamma(n-y-\alpha, \alpha, \alpha)}{\xi}) \, dF_N(n) \\
+ \int_{n \in [y+\alpha, \infty)} c \, dF_N(n), \quad & \forall y > -\alpha; \\
\psi \left( F_N(-y) + (1-F_N(-y)) F_{RD}(-y-\alpha - \frac{\Gamma(-y-\alpha, \alpha, \alpha)}{\xi}) \right) \quad & \forall y \leq -\alpha.
\end{array} \right.
\end{align*}
$$

From Eq. (4) and Theorems 1 and 2, the equilibrium funding rate depends on (1) initial reserve balances $R_i$ of bank $i$, (2) equilibrium trading quantity $S_i$\footnote{It can be easily shown that conditional on $R_i = \zeta$, there is a monotone relationship between $D_i$ and the equilibrium trading quantity $S_i$.} and (3) liquidity stress index $m$.

Whereas Theorem 2 does not show uniqueness under the general distributional assumptions for the exogenous state variables, it does characterize all possible equilibria by a system of non-standard integral equations. In particular, all equilibria share the same prediction: the funding rate spikes when the liquidity stress index turns from negative to positive (see Theorem 4, Theorem 5, and Fig. 6). The next theorem proves the equilibrium is unique under mild technical conditions.

**Theorem 3.** Assume $N_i - N_{\text{min}}$ ($i = 1, 2$) is exponentially distributed with parameter $\lambda_N$ and $F_{RD}$ is differentiable with density function $f_{RD}$. Let $f_{RD}^m = \sup\{f_{RD}(t) : t \leq 0\}$. If $\sqrt{2 e \xi / \psi} > f_{RD}^m$, the
equilibrium is unique under the no hoarding condition.

The assumptions for Theorem 3 are likely to hold. Realistically, the marginal value of financing $\xi$ is much larger than the marginal cost of delay $c$ (as confirmed by quantitative estimates from my sample). Recall that under the no hoarding condition, $F_{RD}(\xi) < \frac{c}{\psi} < \sqrt{\frac{2\xi}{\psi}}$, so $\sqrt{\frac{2\xi}{\psi}} > f_{RD}^m$ is generally satisfied for most common families of probability density functions. Moreover, if bank $i$ has better information about the other bank’s reserve condition, that is, uncertainty about $R_j - D_j - Q$ is smaller, $f_{RD}^m$ is smaller. Therefore, the equilibrium is unique, especially when banks have more precise information about each other.

While it is not surprising that a large change in the macroeconomic conditions may have a significant impact on banks’ equilibrium payment timing decisions and wholesale funding markets, I show that small or moderate changes in macroeconomic conditions are enough to have strong nonlinear or discontinuous impacts on wholesale funding rates. I formalize this idea by first defining when two macroeconomic conditions are close to each other.

**Definition 3.** A set of macroeconomic condition, $\mathcal{M}_C$, is the set of constants and cdf of the exogenous random variables in this economy, that is, $\mathcal{M}_C = \{F_R(\cdot), F_N(\cdot), Q, \lambda, D_{\text{min}}, c, \psi, \xi\}$. Two sets of macroeconomic conditions $\mathcal{M}_1^{m_1} = \{F_R^1(\cdot), F_N^1(\cdot), Q^1, \lambda^1, D_{\text{min}}^1, c^1, \psi^1, \xi^1\}$ and $\mathcal{M}_2^{m_2} = \{F_R^2(\cdot), F_N^2(\cdot), Q^2, \lambda^2, D_{\text{min}}^2, c^2, \psi^2, \xi^2\}$ are close with respect to the liquidity stress index if

$$\sup\left\{ |c^1 - c^2|, |\psi^1 - \psi^2|, |\xi^1 - \xi^2|, |\lambda^1 - \lambda^2|, |D_{\text{min}}^1 - D_{\text{min}}^2|, \|F_{RD}^1 - F_{RD}^2\|_\infty \right\} < m^1 - m^2, \quad \|F_N^1 - F_N^2\|_\infty = 0, \quad (7)$$

where $m^1$ and $m^2$ are the liquidity stress indexes under $\mathcal{M}_1^{m_1}$ and $\mathcal{M}_2^{m_2}$, respectively.

By Definition 3, if $\mathcal{M}_1^{m_1}$ and $\mathcal{M}_2^{m_2}$ are close with respect to the liquidity stress index, and their associated liquidity stress indexes are the same, $m_1 = m_2$, $\mathcal{M}_C^{m_1} = \mathcal{M}_C^{m_2}$; when $m_1$ and $m_2$ are close, the distances between any elements from $\mathcal{M}_C^{m_1}$ and $\mathcal{M}_C^{m_2}$ are also close to zero.

Aggregate reserves steadily but slowly declined from March 2017 to September 2019 under the Fed’s balance sheet normalization. The day-to-day variations in the macroeconomic conditions were small. The macroeconomic conditions in the period Friday September 13–Monday September 16, 2019, were close, as formalized by Definition 3. However, that small difference could be enough to trigger the liquidity hoarding condition and cause a big impact on the equilibrium wholesale funding rates, as demonstrated by Theorems 4 and 5. This explains why the repo rate spiked on September 16.

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20 As usual, $\| \cdot \|_\infty$ is the sup-norm: $\| f \|_\infty = \sup \{ |f(x)| \}$
Figure 6: When the liquidity stress index turns positive, bank \( i \) requests a much higher funding rate \( r^* \) to be willing to lend a fixed quantity \( S^*_i \) of $16 billion in the wholesale funding market. The realization of the opening reserve balance \( R_i - Q \) is fixed at $40 billion. Parameters: \( \xi = 38.47\% \); \( c = 1.60\% \); \( \psi = $820.86 \) (per dollar); \( D_i - 10 \) ($bn) is exponentially distributed with mean $2.3 billion; \( N_i - 33 \) ($bn) is exponentially distributed with mean $1.1 billion.

**Theorem 4.** Fix some realization \( \zeta \) of beginning reserve balances \( R_i \) and a quantity \( S^* \) traded in the funding market such that \( \zeta - S^* \neq Q \). The equilibrium funding rate \( r^* \) jumps up as a function of the liquidity stress \( m \) at the threshold \( m = 0 \) that triggers liquidity hoarding. More specifically, there exists some \( \delta(\zeta, S^*) > 0 \) such that

\[
\lim_{\epsilon \downarrow 0} r^*(\zeta, S^*, \epsilon^m) - r^*(\zeta, S^*, -\epsilon^m) > \delta(\zeta, S^*),
\]

provided that the sets of macroeconomic conditions \( \mathcal{M}_e^m \) are mutually close with respect to the liquidity stress index to each other.\(^{21}\)

**Theorem 4** shows how the equilibrium funding rate for a fixed amount of overnight loan changes with the liquidity stress index. It implies the equilibrium funding rate can jump up discontinuously with only (a) slight reductions in the amount of reserves in the banking system,

\(^{21}\)The equilibrium funding rate function in this theorem is \( r^* : \mathbb{R}^3 \rightarrow \mathbb{R}^+ \), which determines the equilibrium supply curve: given some beginning reserve balances \( R_i = \zeta \) and the liquidity stress index \( m \) that depends on the macroeconomic conditions, \( r^*(\zeta, s, m) \) is the funding rate that bank \( i \) will charge borrower \( i \) for borrowing quantity \( S_i = s \) in equilibrium.
(b) slight increases in the demand for wholesale financing, or (c) slight increases in the per-unit liquidity cost $\psi$, and so on. This is illustrated in Fig. 6.

**Theorem 5.** Fix some realization $\zeta$ of beginning reserve balances $R_i$ and wholesale borrowing demand $\mathcal{D}$. The equilibrium trading quantity $S^*(\zeta, \mathcal{D}, m)$ decreases and the funding rate $r^*$ jumps up as a function of the liquidity stress $m$ at the threshold $m = 0$ that triggers liquidity hoarding. More specifically, there exists some $\delta(\zeta, \mathcal{D}) > 0$ such that

$\lim_{\epsilon m \downarrow 0} S^*(\zeta, \mathcal{D}, -\epsilon m) - S^*(\zeta, \mathcal{D}, \epsilon m) > \delta(\zeta, \mathcal{D})$

$\lim_{\epsilon m \downarrow 0} r^*(\zeta, \mathcal{D}, \epsilon m) - r^*(\zeta, \mathcal{D}, -\epsilon m) > \xi \delta(\zeta, \mathcal{D}),$

provided the sets of macroeconomic conditions $\mathcal{M}_c^{\epsilon m}$ are mutually close to each other with respect to the liquidity stress index, and $\zeta - S^*(\zeta, \mathcal{D}, 0) - Q \neq 0$.

Importantly, Theorems 4 and 5 state that funding rate spikes can happen without large drops in the opening reserve balances of bank $i$. Under the liquidity hoarding condition, even if bank $i$ has a large amount of initial reserves $R_i = \zeta >> 0$, it will still be conservative about lending out reserves and will charge a higher funding rate, because it worries the early payment $a_{ij}$ is likely to be low. When total reserves balances are lower, banks not only expect lower incoming early payments, but also actually have smaller opening reserve balances to start with. Therefore, the equilibrium funding rate nonlinearly increases even under the no hoarding condition and jumps up once the liquidity hoarding condition is triggered. This is illustrated in Fig. 7.

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$^{22}$The equilibrium funding rate function in this theorem is $r^* : \mathbb{R}^3 \rightarrow \mathbb{R}^+$, which determines the equilibrium supply curve: given some beginning reserve balances $R_i = \zeta$ and the liquidity stress index $m$, which depends on the macroeconomic conditions, $r^*(\zeta, \mathcal{D}, m)$ is the equilibrium funding rate at which bank $i$ lends to borrower $i$, who has a total financing need $D_i = \mathcal{D}$. Similar definitions apply to the equilibrium trading function $S^* : \mathbb{R}^3 \rightarrow \mathbb{R}^+$. 

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Figure 7: Lower total reserve balances cause the overnight funding rate to increase nonlinearly under the no hoarding condition and jump up discontinuously when the liquidity hoarding condition is triggered. Parameters: $\xi = 38.47\%$; $c = 1.60\%$; $\psi = $820.86 (per dollar); $D_i - 10$ ($\text{bn}$) is exponentially distributed with mean $2.3$ billion; $N_i - 33$ ($\text{bn}$) is exponentially distributed with mean $1.1$ billion.

5 Data

To quantitatively understand the effect of strategic complementarity and calculate the counterfactual sufficient reserves to support effective monetary policy implementation, I estimate my model in the context of the GCF Treasury repo market. This section describes the data I use for this exercise.

I use the GCF repo rate as the proxy for large U.S. dealer banks’ lending rate in wholesale funding markets. GCF repo rates data are published by FICC. I obtain two daily trading volume data from the New York Fed: The volumes for calculating the Secured Overnight Financing Rate (SOFR) index and the Triparty General Collateral Rate (TGCR) index. I obtain the quarterly average Treasury repo lending quantity from U.S. GSIB form 10-Q for a set of large U.S. banks including JP Morgan, Bank of America, Goldman Sachs, Morgan Stanley, Citi, and State Street. I combine those two data to estimate the daily repo lending quantity of large dealer banks.

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23The GCF repo market is an interdealer market where large dealers lend to smaller dealers. As a result, the GCF-IOER spread measures the compensation that large dealers require when they provide liquidity by draining reserves.
I use two types of information about reserve balances held at the Federal Reserve Banks: daily opening balances and the timing of cash transfers between accounts within each day. Both types of data are provided by Copeland, Duffie and Yang (2020), sourced from the Fedwire Funds Service. Specifically, I observe the total opening reserve balances of the largest 100 accounts managed by depository institutions, and the total opening reserve balances of 10 dealer banks, as identified by Copeland, Duffie and Yang (2020). The total opening reserve balances of 10 dealer banks are held by depository institutions owned by bank holding companies that have a large presence in U.S. repo markets. The total opening balances of these 10 dealer banks is about 40% of total opening balances of the accounts of the 100 largest banks over 2018-19, and the total balances of the 100 accounts are about 85% of total reserves held at Federal Reserve Banks over 2018-19. I call the banks of those 100 accounts other than the 10 dealer banks “other large banks.” I do not observe the identities of any banks in my sample.

In addition to daily opening-balance information, Copeland, Duffie and Yang (2020) provide statistics regarding the timing of payments sent over Fedwire within the day. In particular, I observe the timestamp when 50% of the total value of transfers to the 10 dealer banks’ accounts has been received by the 10 dealer banks in the day. For example, on September 3, 2019, 50% of the total transfers to the 10 dealer banks had been received by 2:06 pm. I subtract this time stamp from 12:09 pm, the average of this statistic between January 2, 2014, and October 9, 2020, to calculate a measure of payment timing stress, which I call “median time of receives.”

I obtain Treasury issuance and redemption data from the Treasury Department. I also obtain total Treasuries outstanding from the U.S. Treasury Fiscal Data and the Treasuries held by the Fed from the Federal Reserve Bank of St. Louis’s FRED database. Finally, I obtain total payment volume data from Fedwire. Summary statistics of the key variables are provided in Table 1.

6 Model estimation and counterfactual

I focus on a sample from January 3, 2019, to September 18, 2019. I divide the sample into two subsamples: one is the period from January 3, 2019, to September 15, 2019, and the other is the period from September 16, 2019, to September 18, 2019. I call the first subsample the “training

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24This measure is based on standard payment timing metrics used in previous research on intraday payments, such as Armantier, McAndrews and Arnold (2008), McAndrews and Kroeger (2016), and Copeland, Molloy and Tarascina (2019).
dataset” and the second one the “test dataset.” I estimate the parameters of my model using the method of moments on the training dataset. I do not include data from days in previous years for my training dataset, because large dealer banks need to change the assumptions of their regulation YY stress testing and resolution planning on an annual basis (i.e., $Q$ in my baseline model may differ every year). In addition, before 2019, the total reserve balances were abundant, and the effects of strategic complementarity were relatively small. I also exclude observations for the period January 1-3, 2019, to eliminate the impact from year-end capital requirements (Correa, Du and Liao, 2020). I do not include dates after September 18, 2019, for my test dataset, because the Fed reacted to the September repo spikes by directly offering liquidity every day until June 2020. The Fed’s reaction distorted the GCF-IOER spread in a way that is outside of my model.

My theory allows me to estimate the $n$-bank model in principle, but numerically computing a fixed point of the integral-equation system characterizing the $n$-bank equilibrium is time consuming. Therefore, in this current version, I estimate my two-bank baseline model. (As discussed in Appendix C.2, the theoretical predictions for the $n$-bank case are similar to the 2-bank case.) Specifically, I assume the total dealer reserve balances are evenly distributed among the aforementioned 10 largest dealer banks. I divide those 10 banks into five pairs such that each bank only needs to consider the payment strategy of one other bank. This assumption reduces the computation complexity while preserving the effects of strategic complementarity. I assume all pairs of the dealer banks are identical. In other words, I estimate my model for a representative bank pair \{i, j\}.

From my sample, I observe the following inputs to my model on every business day:

- total opening reserve balances $R_D$ of 10 dealer banks.
- total opening reserve balances $R_O$ of other large banks.
- net Treasury issuance $T_I$.
- total Treasuries outstanding $T_D$.
- total repo lending quantity $S$ of the dealer banks.\(^{27}\)
- median time of receives $D_P$ of the large dealer banks.

I make several additional assumptions before taking the model to the data. First, motivated by empirical findings by Copeland, Duffie and Yang (2020), I assume each dealer bank $i$ has a

\(^{25}\)There are usually acute funding constraints on year ends when GSIBs adjust their balance sheets, due to the scoring that takes place predominantly at year-end to determine the capital surcharge for these institutions.

\(^{26}\)The Fed announced it would lend cash to borrowers after most repo trading occurred on September 17. Market participants were uncertain whether the New York Fed would continue its intervention on the following days, so the GCF repo rates remained elevated until the morning of September 18.

\(^{27}\)For details on how I calculate $S$, see Appendix D.
usable beginning reserve balance given by

\[ R_i = \frac{1}{10}(R_D - E_I T_I + E_O R_O) - Q, \]

where \( E_I, E_O \) are parameters to be estimated. Treasury issuance settlements result in cash transfers from dealer banks’ accounts at the Fed to the TGA account, and these transfers must occur near the beginning of the day (Copeland, Duffie and Yang, 2020). The term \( E_I T_I \) captures this effect. In addition, the opening-of-day reserve balances of the other large banks are highly related to the early payments received by the large dealer banks. This relationship seems linear (Copeland, Duffie and Yang 2020, see also Fig. 25). Therefore, large dealer banks can treat the incoming payments from other large banks as part of their usable beginning reserve balances. The term \( E_O R_O \) captures this effect.

Second, I assume bank \( i \) believes \( R_j \sim \mathcal{N}(R_i, 0.001) \). Therefore, bank \( i \) knows the correct mean of bank \( j \)'s beginning reserve balance with a small uncertainty of the realization of \( R_j \).\(^{28}\) I also assume repo borrowing demand for borrower \( j \) follows \( D_j - E_D T_D \sim \exp(\lambda) \).\(^{29}\) This assumes the minimum repo borrowing demand linearly increases with total Treasuries outstanding. Increased repo borrowing demand drives up the equilibrium repo trading quantity, consistent with reduced form empirical evidence (see Table 3). I further assume total payment volume follows \( N_j - N_{min} \sim \exp(\lambda_N) \), where \( N_{min} \) and \( \lambda_N \) are estimated to match the mean and variance of the payment volume data from 2019.

Third, I use the median time of receives to approximate early payment strategy \( a^*_i \). I do not observe equilibrium early payment strategy \( a^*_i \) directly, but the co-movement among \( R_i, a^*_i, \) and \( r \) is important in identifying the level of strategic complementarity. Intuitively, the median time of receives measures the payment delay that large dealer banks face each business day. Therefore, it should be highly related to the early payments of large banks; that is, when bank \( j \) pays bank \( i \) a larger \( a_j \) on day \( t \), the time by which bank \( i \) receives 50\% of its total incoming payments over Fedwire on that day should be earlier. I assume and estimate the following linear relationship: on day \( t \),

\[ D_{it}^P - \mathbb{E}[D_{it}^P] = \beta_1^e (a^*_it - \mathbb{E}[a^*_it]) + \beta_2^e (R_{it}^O - \mathbb{E}[R_{it}^O]) + \epsilon_{it}^D, \]

Eq. (8) implies \( \mathbb{E}[\epsilon_{it}^D] = 0 \) by definition. Because I assume \( R_i \) and \( D_i \) have atomless distributions,\(^{29}\) I find that making this uncertainty smaller leaves the estimates and the model fit virtually unchanged.\(^{28}\) I normalized \( T_D \) and \( E_O \) by subtracting their sample minimums.

\(^{28}\)I find that making this uncertainty smaller leaves the estimates and the model fit virtually unchanged.

\(^{29}\)I normalized \( T_D \) and \( E_O \) by subtracting their sample minimums.
the equilibrium funding rates can be simplified from Eq. (4) to

$$\Gamma_i(R_i - Q - S^*_i, \alpha_i, \alpha_j) = r^*_i.$$  

Recall that in the baseline model, I normalized the funding rates by the Fed’s policy rate, IOER. Thus, the above equation implies the following empirical equation on day $t$:

$$(GCF - IOER)^t = \Gamma_i(R_i - Q - S^*_i, \alpha_i^*, \alpha_j^*) + \vartheta + \epsilon^t.$$  

where $S^*_t = \frac{1}{10} S^t$ and $\vartheta$ is another parameter that I estimate to capture the combined effect from other factors that influence GCF repo rates.

There are a total of 10 parameters, listed in Table 4. Given a set of candidate model parameters, observable inputs $R^t_D, R^t_O, T^t_I, T^t_D$ from day $t$ allow me to calculate the equilibrium early payment $a^*_i, a^*_j$ and $\Gamma_i, \Gamma_j$ on that day based on Theorems 1 and 2. With two additional observable inputs, $S^*_t, D^*_P$, I can calculate the empirical equation Eqs. (8) and (9) as well as $\epsilon^t, \epsilon^t_D$. The identification assumption is that $\epsilon^t, \epsilon^t_D$ are orthogonal to the information observable at the beginning of each day $t$:

$$\mathbb{E} \begin{bmatrix} \epsilon^t_r \\ \epsilon^t_D R^t_D \\ \epsilon^t_D R^t_O \\ \epsilon^t_D T^t_I \\ \epsilon^t_D T^t_D \end{bmatrix} = 0; \quad \mathbb{E} \begin{bmatrix} \epsilon^t_D R^t_D \\ \epsilon^t_D R^t_O \\ \epsilon^t_D T^t_I \\ \epsilon^t_D T^t_D \\ \epsilon^t_D a^*_j \end{bmatrix} = 0.$$

I estimate the model based on the test data set of my sample (business days from January 3, 2019, to September 15, 2019). The point estimates are recorded in Table 4. Admittedly, the GCF repo market is a complicated OTC market, featuring relationship trading (Paddrik, Ramirez, McCormick et al., 2021), search frictions (Afonso and Lagos, 2015), and market segmentation (Han, 2020; Avalos, Ehlers and Eren, 2019; Duffie and Krishnamurthy, 2016). GCF-IOER spread is usually elevated at month end and quarter end due to regulatory capital requirements on foreign bank holding companies that cause them to reduce their provision of liquidity to interdealer markets. My model abstracts away from those frictions to focus only on the relationship between the quantity of reserve supply and GCF-IOER spread. This could bias the estimate of $\vartheta$ by a few basis points. However, the goal of my quantity exercise is not to
provide the most accurate quantitative model to describe the GCF repo market. Rather, my focus is on exploring the impact of strategic complementarity on overnight wholesale funding rate spreads. Nevertheless, my model fits the in-sample variations of GCF-IOER spread and median payment timing simultaneously reasonably well on my test data set.

Table 5 compares the goodness of fit of my model with three other types of models: (1) one linear model for the GCF-IOER spread and one linear model for the median time of receives, (2) a machine learning model of random forest trained using cross-validation, and (3) an otherwise identical model of mine shutting down the effect of strategic complementarity (see details in Appendix C.3). The measures of goodness of fit include mean squared error, correlation between the model-predicted and actual GCF-IOER spread, and correlation between the model-predicted and actual median time of receives. For linear models, I calculate R². My model is the only one that is able to simultaneously fit both the GCF-IOER spread and the median time of receives, while achieving similar performance relative to other models in each dimension.

Although I excluded the September repo spike events in my sample when estimating my model, my quantitative model correctly predicts the repo spikes on September 16-19, 2019, as an out-of-sample event. Figs. 8 and 9 compare the performance across all four models. In these two plots, the days to the left of the dashed vertical line are my training dataset, and the days to the right are my test dataset. To study the out-of-sample performance of all aforementioned models, I use observable \{R_{pd}, R_{po}, T_{id}, T_{od}, S_{ai}, D_{dp}\} from my test dataset as inputs into my model and three other models with parameters estimated from my training dataset. The predicted GCF-IOER spread generated by all four models tracks the variation of the actual GCF-IOER spread closely in sample, but only my model can predict the sharp GCF-IOER spread spikes on September 16-19, 2019.

The performance of my quantitative model on the test dataset validates the mechanism of my theory. Without prior knowledge of how strategic complementarity in interbank intraday payment timing works, simple statistical models or even machine learning models are unlikely to capture the nonlinear or discontinuous reactions of short-term wholesale funding rates when reserves balances are close to being insufficient. Thus, those models tend to underestimate the level of sufficient reserves. Because the Fed has access to granular data of intraday interbank payment timing, real-time changes of every bank’s reserve account balance, and activities in the short-term funding markets, a good understanding of strategic complementarity in interbank intraday payment timing could have allowed the Fed to develop a full-fledged quantitative

30For completeness, I also calculate Pseudo-R² defined by Schabenberger and Pierce (2001) for nonlinear models, although Pseudo-R² is not a great measure for comparing different nonlinear models.
model to shape its policy of reserve supply. An effective quantitative model of this nature that helps predict funding rate distortions out of sample is a useful tool for central bankers because it allows precautionary interventions.

6.1 Counterfactual: What quantity of reserves would have been sufficient?

Copeland, Duffie and Yang (2020) note the reserve balances of the top 10 repo active dealer banks are more important in directly determining the spreads of various repo rates over IOER than reserve balances of other non-dealer large banks. My quantitative model allows me to back out the minimum reserve level for the large dealer banks to keep short-term funding rates close to the Fed’s policy target and support an efficient payment system. This level may vary daily due to changes in the macroeconomic conditions, such as borrowing demands in the wholesale funding markets, Treasury issuance, and reserve balances of other non-dealer large banks.

Fig. 10 shows the sufficient amount of reserves of large dealer banks required to keep the expected GCF-IOER spread below 13 basis points, 26 basis points, and 52 basis points, respectively, as predicted by my model.31 My estimated model suggests that since starting

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31Due to noise that is not captured by my theoretical model, the actual GCF-IOER spread may fluctuate above
in September, 2019, the historical level of reserve balances of the large dealer banks became insufficient to keep the expected GCF-IOER spread below 13 basis points, as confirmed by the data. All three lines increase progressively in this plot, because up to September 18, 2019, reserve balances of other large banks declined substantially, yet repo borrowing demand steadily increased throughout 2019. This plot also shows that Treasury issuance temporarily elevates the level of necessary reserves, as demonstrated by the vertical dashed lines. In addition, the three lines get closer and eventually converge in September, because increased repo borrowing demand makes the liquidity stress index more sensitive to changes in reserve balances. As a result, my model predicts that monetary policy implementation became highly unstable in September.

Note that on July 1, 2019, dealer banks’ total reserve balances were lower than those of September 17, 2019, but the GCF-IOER spread on July 1 was much lower than the spread on September 17. This observation would be a puzzle through the lenses of models that study the repo market in isolation. My model provides an explanation for this observation: The reserve balances of other large banks were larger on July 1, so large dealer banks expected they could rely on early incoming payments from other large banks to make their outgoing payments. Therefore, large dealer banks did not hoard reserves in paying other banks on July and below the expected GCF-IOER spread.
1, and the GCF-IOER spread did not spike with a large magnitude (though with a smaller amount of opening reserve balances, the dealer banks demanded higher repo rates). On September 16, 2019, however, a combined effect of low reserve balances of all large banks and increased repo borrowing demand triggered the liquidity hoarding condition, according to my model’s prediction, thus causing the large repo spike on that day. By integrating the interbank payment market and wholesale funding markets, my model captures both events with comparable magnitudes (Fig. 8). Fig. 10 shows the sufficient level of reserves for large dealer banks is higher on September 16-18 than on July 1.

7 Conclusion

The post-GFC liquidity rules and supervision significantly increase the incentives of large U.S. dealer banks to maintain substantial intraday reserve buffers. As a result, instead of using the Fed’s intraday overdraft facility, large banks rely heavily on incoming payments from other banks to cover their own outgoing payments. A smaller quantity of reserves relative to wholesale borrowing demands means less liquidity is available for the intraday interbank
payment system. I show that a sufficiently low supply of reserves causes banks to suddenly hoard reserves, reinforced by a feedback effect stemming from the strategic complementarity of intraday payment timing, and leads to intraday payment timing stress. My paper is the first to study the impact of this strategic complementarity on monetary policy implementation. My mechanism explains how small increases in funding pressures can lead to outsized movements in wholesale funding markets without sufficient reserves: Monetary policy implementation becomes highly unstable when reserve balances are low, as observed in September 2019.

My main results suggest that to avoid reserve hoarding and wholesale funding rate spikes, the Fed would want to ensure banks have enough reserves to meet (1) intraday interbank payment needs and (2) borrowing demand in wholesale funding markets. I show that factors determining reserve sufficiency can be summarized by one liquidity stress index (Definition 2). To reduce frictions in monetary policy implementation and the interbank payment system, the Fed may also relax post-crisis liquidity regulations to encourage the use of the Fed’s intraday overdraft facility and reduce large banks’ dependence on incoming payments in sending their outgoing payments.

I bring data to my model through a method of moments procedure and provide the first structural model estimates of the levels of sufficient reserves to keep the wholesale funding rates close to the Fed’s target policy rate within three particular bandwidths (i.e., 13 basis points, 26 basis points, and 52 basis points). As such, my paper contributes a quantitative framework for the Fed to manage the supply of reserves with the objectives of ensuring effective monetary policy implementation, liquid wholesale funding markets, and an efficient interbank payment system.
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## Appendix: Additional tables and figures

### Table 1: Summary statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Min</th>
<th>Q(0.25)</th>
<th>Q(0.75)</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>dealer opening balances ($ billions)</td>
<td>1,465</td>
<td>686.150</td>
<td>362.150</td>
<td>619.150</td>
<td>759.150</td>
<td>1,224.150</td>
<td></td>
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<tr>
<td>other large bank balances ($ billions)</td>
<td>1,465</td>
<td>1,153.231</td>
<td>652.231</td>
<td>960.231</td>
<td>1,335.231</td>
<td>1,632.231</td>
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<tr>
<td>Tbills outstanding ($ billions)</td>
<td>1,522</td>
<td>2,137.841</td>
<td>1,233.841</td>
<td>1,557.841</td>
<td>2,312.841</td>
<td>4,802.841</td>
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<tr>
<td>Bill issuance ($ billions)</td>
<td>1,463</td>
<td>33.568</td>
<td>-0.017</td>
<td>0</td>
<td>75.273</td>
<td>273.273</td>
<td></td>
</tr>
<tr>
<td>Coupon issuance ($ billions)</td>
<td>1,463</td>
<td>10.231</td>
<td>-0.011</td>
<td>0</td>
<td>0</td>
<td>218.218</td>
<td></td>
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<tr>
<td>Treasuries redemption ($ billions)</td>
<td>1,463</td>
<td>37.956</td>
<td>0</td>
<td>0</td>
<td>84.7259</td>
<td>259.259</td>
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<td>median time of receives (minutes)</td>
<td>1,467</td>
<td>-0.375</td>
<td>58.172</td>
<td>-49.4</td>
<td>49.6</td>
<td>155.155</td>
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<tr>
<td>SOFR - IOER (basis points)</td>
<td>1,455</td>
<td>-7.843</td>
<td>13.829</td>
<td>-29</td>
<td>-16</td>
<td>-1316</td>
<td></td>
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<tr>
<td>GCF Repo - IOER (basis points)</td>
<td>1,421</td>
<td>-0.724</td>
<td>17.430</td>
<td>-30.2</td>
<td>-9</td>
<td>5.3</td>
<td></td>
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<tr>
<td>Treasuries issuance ($ billions)</td>
<td>1,463</td>
<td>43.264</td>
<td>64.301</td>
<td>-0.017</td>
<td>87.365</td>
<td>391.391</td>
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<tr>
<td>quarter-end fixed effect</td>
<td>1,524</td>
<td>0.0164</td>
<td>0.127</td>
<td>0</td>
<td>0</td>
<td>1</td>
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<tr>
<td>Note issuance ($ billions)</td>
<td>1,463</td>
<td>9.27</td>
<td>29.5</td>
<td>-0.011</td>
<td>0</td>
<td>199.199</td>
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Note: This table includes days from January 1, 2015, to October 30, 2020.
Table 2: Basic regression models for interdealer Treasury repo (GCF Repo) as a spread to IOER. The units of the explanatory variables are trillions of dollars and minutes.

<table>
<thead>
<tr>
<th></th>
<th>GCF - IOER</th>
<th></th>
<th></th>
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<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
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<tr>
<td>Dealer opening balances</td>
<td>$-38.6^{**}$</td>
<td>$-38.4^{**}$</td>
<td>$-18.3^{***}$</td>
<td>$-50.5^{***}$</td>
<td>$-31.7^{***}$</td>
<td>$-30.2^{***}$</td>
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<tr>
<td></td>
<td>(4.60)</td>
<td>(4.51)</td>
<td>(2.87)</td>
<td>(4.66)</td>
<td>(4.76)</td>
<td>(4.40)</td>
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<tr>
<td>Median time of receives</td>
<td>$0.137^{***}$</td>
<td>$0.117^{***}$</td>
<td>$0.0731^{***}$</td>
<td>$0.0728^{***}$</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>(0.0132)</td>
<td>(0.0116)</td>
<td>(0.0216)</td>
<td>(0.0246)</td>
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<td></td>
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<tr>
<td>Quarter-end fixed effect</td>
<td>$31.0^{**}$</td>
<td>$30.8^{**}$</td>
<td>$30.7^{**}$</td>
<td>$32.5^{**}$</td>
<td>$32.4^{**}$</td>
<td>$26.1^{**}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(12.5)</td>
<td>(12.8)</td>
<td>(12.6)</td>
<td>(12.9)</td>
<td>(13.0)</td>
<td>(12.8)</td>
<td></td>
</tr>
<tr>
<td>T-bills outstanding</td>
<td>$8.26^{***}$</td>
<td>$4.06^{***}$</td>
<td>$2.69^{***}$</td>
<td></td>
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<tr>
<td></td>
<td>(0.514)</td>
<td>(1.10)</td>
<td>(0.926)</td>
<td></td>
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<td>Net Treasuries inventory</td>
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<td>Treasuries redemption</td>
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<td>Bill issuance</td>
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<td></td>
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</tr>
<tr>
<td>Observations</td>
<td>1,419</td>
<td>1,419</td>
<td>1,420</td>
<td>1,418</td>
<td>1,417</td>
<td>1,416</td>
<td>1,413</td>
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<tr>
<td>$R^2$</td>
<td>0.105</td>
<td>0.151</td>
<td>0.251</td>
<td>0.270</td>
<td>0.266</td>
<td>0.283</td>
<td>0.298</td>
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<tr>
<td>Adjusted $R^2$</td>
<td>0.104</td>
<td>0.150</td>
<td>0.250</td>
<td>0.268</td>
<td>0.264</td>
<td>0.281</td>
<td>0.294</td>
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<tr>
<td>Residual Std. Error</td>
<td>16.5</td>
<td>16.0</td>
<td>15.1</td>
<td>14.9</td>
<td>14.9</td>
<td>14.8</td>
<td>14.6</td>
</tr>
</tbody>
</table>

Notes: Standard errors are adjusted for heteroskedasticity. *$p < 0.1$; **$p < 0.05$; ***$p < 0.01$. A constant was included for each specification.
Table 3: Basic regression models for repo trading volume.

<table>
<thead>
<tr>
<th></th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td>(0.00633)</td>
</tr>
<tr>
<td>Treasuries Outstanding</td>
<td>0.0799***</td>
</tr>
<tr>
<td>Large Banks’ Reserve Balances</td>
<td>−0.186***</td>
</tr>
<tr>
<td>Treasury Issuance</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Treasury Redemptions</td>
<td>−0.00915</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>153</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.571</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.568</td>
</tr>
<tr>
<td>Residual Std. Error</td>
<td>62.8</td>
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</table>

Note: Standard errors are adjusted for heteroskedasticity. *$p < 0.1$; **$p < 0.05$; ***$p < 0.01$.
Constant included for each specification. Sample: biweekly 09-28-2016 to 09-18-2019.
Table 4: Estimated parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Meaning</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>late payment cost</td>
<td>1.60%</td>
</tr>
<tr>
<td>$\xi$</td>
<td>governs demand elasticity</td>
<td>38.47%</td>
</tr>
<tr>
<td>$\psi$</td>
<td>regulatory cost</td>
<td>$820.86</td>
</tr>
<tr>
<td>$Q$</td>
<td>regulatory minimum</td>
<td>11.1 ($bn)</td>
</tr>
<tr>
<td>$1/\lambda$</td>
<td>$\mathbb{E}[D_i - D_{min}]$</td>
<td>2.3 ($bn$)</td>
</tr>
<tr>
<td>$E_I$</td>
<td>Treasury issuance effect</td>
<td>0.42</td>
</tr>
<tr>
<td>$E_O$</td>
<td>early payment from other banks is $E_O R_O$</td>
<td>0.32</td>
</tr>
<tr>
<td>$E_D$</td>
<td>$D_{min} = E_D T_D$</td>
<td>0.54</td>
</tr>
<tr>
<td>$\beta_1^e$</td>
<td>coefficient</td>
<td>-15.40</td>
</tr>
<tr>
<td>$\beta_2^e$</td>
<td>coefficient</td>
<td>-85.30</td>
</tr>
<tr>
<td>$\vartheta$</td>
<td>other factors</td>
<td>0.95 (bp)</td>
</tr>
<tr>
<td>$N_{min}$</td>
<td>minimum total payment volume</td>
<td>33 ($bn$)</td>
</tr>
<tr>
<td>$1/\lambda_N$</td>
<td>$\mathbb{E}[N_i - N_{min}]$</td>
<td>1.1 ($bn$)</td>
</tr>
</tbody>
</table>

Note: $\psi$ is the dollar penalty per dollar of overdraft.
Table 5: In-sample model fit for four types of models: linear models, my model with strategic complementarity, a model without strategic complementarity, and a random forest machine learning model.

<table>
<thead>
<tr>
<th>Model Type</th>
<th>MSE for fitted GCF-IOER</th>
<th>R² for fitted GCF-IOER</th>
<th>Correlation between fitted and actual GCF-IOER</th>
<th>MSE for fitted median time of receives</th>
<th>R² for fitted median time of receives</th>
<th>Correlation between fitted and actual median time of receives</th>
</tr>
</thead>
<tbody>
<tr>
<td>model with strategic complementarity</td>
<td>26.71</td>
<td>0.40</td>
<td>0.67</td>
<td>0.56</td>
<td>0.40</td>
<td>0.63</td>
</tr>
<tr>
<td>model without strategic complementarity</td>
<td>22.65</td>
<td>0.49</td>
<td>0.70</td>
<td></td>
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<td></td>
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<tr>
<td>linear model for GCF-IOER</td>
<td>22.09</td>
<td>0.52</td>
<td>0.72</td>
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<tr>
<td>linear model for median time of receives</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>random forest</td>
<td>23.5</td>
<td>0.47</td>
<td>0.96</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: R² for nonlinear models is the Pseudo-R² defined by Schabenberger and Pierce (2002). MSE stands for mean squared error. For random forest, MSE and R² are based on the out-of-bag prediction error. The linear models used here correspond to the third columns from Table 6 and Table 7, respectively.
### Table 6: Linear model on GCF-IOER.

<table>
<thead>
<tr>
<th>GCF-IOER</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>dealer opening balances</td>
<td>−117.71***</td>
<td>−118.50***</td>
<td>−118.59***</td>
<td>−117.37***</td>
</tr>
<tr>
<td></td>
<td>(27.20)</td>
<td>(24.96)</td>
<td>(25.10)</td>
<td>(25.26)</td>
</tr>
<tr>
<td>other large bank balances</td>
<td>−26.86***</td>
<td>−53.58***</td>
<td>−52.38***</td>
<td>−47.42***</td>
</tr>
<tr>
<td></td>
<td>(5.44)</td>
<td>(6.48)</td>
<td>(7.15)</td>
<td>(8.39)</td>
</tr>
<tr>
<td>net Treasury issuance</td>
<td>155.71***</td>
<td>191.62***</td>
<td>187.72***</td>
<td>194.17***</td>
</tr>
<tr>
<td></td>
<td>(34.79)</td>
<td>(33.96)</td>
<td>(35.12)</td>
<td>(35.18)</td>
</tr>
<tr>
<td>Treasuries outstanding</td>
<td>−13.67***</td>
<td>−13.16***</td>
<td>−13.58***</td>
<td>−13.58***</td>
</tr>
<tr>
<td></td>
<td>(2.73)</td>
<td>(2.81)</td>
<td>(2.76)</td>
<td>(2.76)</td>
</tr>
<tr>
<td>repo lending quantity</td>
<td>20.49</td>
<td>16.61</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(33.34)</td>
<td>(32.44)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>median time of receives</td>
<td></td>
<td></td>
<td>0.64</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.42)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>175</td>
<td>175</td>
<td>175</td>
<td>175</td>
</tr>
<tr>
<td>R²</td>
<td>0.45</td>
<td>0.52</td>
<td>0.52</td>
<td>0.52</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.44</td>
<td>0.51</td>
<td>0.51</td>
<td>0.51</td>
</tr>
<tr>
<td>Residual Std. Error</td>
<td>5.00</td>
<td>4.69</td>
<td>4.70</td>
<td>4.69</td>
</tr>
</tbody>
</table>

Note: Standard errors are adjusted for heteroskedasticity. *p < 0.1; **p < 0.05; ***p < 0.01. Constant included for each specification.
Table 7: Linear model on median time of receives.

<table>
<thead>
<tr>
<th></th>
<th>median time of receives</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>dealer opening balances</td>
<td>−1.98</td>
</tr>
<tr>
<td></td>
<td>(2.07)</td>
</tr>
<tr>
<td>other large bank balances</td>
<td>−9.42***</td>
</tr>
<tr>
<td></td>
<td>(1.05)</td>
</tr>
<tr>
<td>net Treasury issuance</td>
<td>−7.90*</td>
</tr>
<tr>
<td></td>
<td>(4.06)</td>
</tr>
<tr>
<td>Treasuries outstanding</td>
<td>0.52</td>
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<tr>
<td>repo lending quantity</td>
<td>6.28</td>
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<td>GCF-IOER</td>
<td>0.02</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>Observations</th>
<th>R²</th>
<th>Adjusted R²</th>
<th>Residual Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>175</td>
<td>0.41</td>
<td>0.40</td>
<td>0.77</td>
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<td></td>
<td>175</td>
<td>0.42</td>
<td>0.42</td>
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<tr>
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<td>0.42</td>
<td>0.42</td>
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<tr>
<td></td>
<td>175</td>
<td>0.43</td>
<td>0.41</td>
<td>0.77</td>
</tr>
</tbody>
</table>

Note: Standard errors are adjusted for heteroskedasticity. *p < 0.1; **p < 0.05; ***p < 0.01. Constant included for each specification.
Figure 11: Liquidity provided by MMF is quarterly average of the money market mutual fund (MMF) investments in the overnight Treasury repo less MMF RRP facility usage. Liquidity provided by large U.S. banks is the quarterly average of net lending of all repo products and all tenors (reverse repos + Fed Funds lent - repos - Fed Funds borrowed). There is no exact data on the net lending of large U.S. banks in the overnight Treasury repo, but based on information from large banks’ 10-Q, a lower bound is about 49% of the total net lending of all repo products. The set of large U.S. banks includes JPMorgan Chase, Bank of America, Goldman Sachs, Morgan Stanley, Citibank, Wells Fargo, and State Street. Data: FFIEC Call Reports, OFR, 10-Q.
Figure 12: Reserve balances and the spread of the overnight synthetic dollar interest rates over IOER (FX-IOER spread). Synthetic dollar interest rate is the implied dollar interest rate in the foreign exchange (FX) swap (borrowing dollars by first borrowing in foreign currency and swapping this foreign funding for dollars, and entering into an FX forward contract to hedge the exchange-rate risk). IOER is the interest rate paid on reserves. The reserve balances of the large repo-active banks are shown in blue (right axis). The spread of the overnight synthetic dollar funding rate by swapping the ECB deposit rate over the Fed IOER (EUR) is shown in green (left axis). Source: Fedwire Funds Service, FRBNY, Correa, Du and Liao (2020).
Figure 13: Reserve balances and the spread of the one-week synthetic dollar interest rates over the Overnight Index Swap (OIS) rates (left axis). For each currency, the synthetic dollar rates are calculated as the forward premium minus maturity-matched foreign currency OIS rate as in Wallen (2020). The reserve balances of the large repo-active banks are shown in blue (right axis). Source: Fedwire Funds Service, FRBNY, Bloomberg.

Note: Unlike Treasury repo loans, lending dollars in the FX markets incurs large balance sheet costs for dealer banks, especially the global systemically important banks. Due to the balance sheet constraints of international large dealer banks, the spreads of synthetic dollar interest rates over OIS (FX-OIS spread) usually spike with large magnitudes near quarter-ends and are generally more volatile (Du, Tepper and Verdelhan, 2018; Ivashina, Scharfstein and Stein, 2015; Wallen, 2020). Nevertheless, when reserve balances were low, borrowing in the FX markets became more costly.
Figure 14: Reserve balances and the spread of the two-week synthetic dollar interest rates over the Overnight Index Swap (OIS) rates (left axis). For each currency, the synthetic dollar rates are calculated as the forward premium minus maturity-matched foreign currency OIS rate as in Wallen (2020). The reserve balances of the large repo-active banks are shown in blue (right axis). Source: Fedwire Funds Service, FRBNY, Bloomberg.
Figure 15: Reserve balances and the spread of the three-week synthetic dollar interest rates over the Overnight Index Swap (OIS) rates (left axis). For each currency, the synthetic dollar rates are calculated as the forward premium minus maturity-matched foreign currency OIS rate as in Wallen (2020). The reserve balances of the large repo-active banks are shown in blue (right axis). Source: Fedwire Funds Service, FRBNY, Bloomberg.
Figure 16: Reserve balances and the spreads of overnight commercial paper rates over IOER (left axis). The reserve balances of the large repo-active banks are shown in blue (right axis). Data: Federal Reserve Board.
Figure 17: Reserve balances and the spreads of seven-day commercial paper rates over IOER (left axis). The reserve balances of the large repo-active banks are shown in blue (right axis). Data: Federal Reserve Board.
Figure 18: Reserve balances and the spreads of 15-day commercial paper rates over IOER (left axis). The reserve balances of the large repo-active banks are shown in blue (right axis). Data: Federal Reserve Board.
Figure 19: Quarterly average daily payments: the average daily payment value calculated every quarter by the Fedwire Funds Service. The reserve balances of the large repo-active dealer banks are shown in blue. Large repo-active dealer banks are the total reserve balances of the 10 large and repo-active account holders. Other large banks are the total reserve balances of the other large account holders of the largest 100 reserve accounts. Most of the payment activities are concentrated among the large banks (Soramäki et al., 2007). Data: Fedwire Funds Service, FRBNY, Copeland, Duffie and Yang (2020).
Figure 20: Peak intraday overdrafts are calculated over two-week periods and published by the Federal Reserve. The peak daylight overdraft for a given day is the greatest value reached by the sum of the daylight overdrafts for all institutions at the end of each operating minute of the day. Sources: Federal Reserve and Fedwire Funds Service.
The big four US banks turned into key lenders in the repo market

Graph A1

Reserves and Treasury holdings of banks in the US

Net lending through repos and federal funds by size group

Share of Treasury holdings in fungible liquid assets by size group

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1 All banks filing US Call Reports, including foreign banking operations in the US, but excluding credit unions. Excludes broker-dealer affiliates.

2 Size = total assets. Aggregated across all bank entities of the same holding company.

3 Net lending = reverse repos (assets) − repos (liabilities) + fed funds (assets) − fed funds (liabilities).

4 Fungible liquid assets are defined as cash + fed funds + reserves + Treasury securities.

Sources: Federal Financial Institutions Examination Council, Call Reports 031, 041 and 002; BIS calculations.

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Figure 21: Source: Avalos, Ehlers and Eren (2019)
Figure 22: Quarterly average daily payments is the average daily payment value calculated every quarter by the Fedwire Funds Service. Total reserve balances average daily level of reserve balances of all depository institutions calculated every week by the Fed. Sources: Federal Reserve and Fedwire Funds Service.
Figure 23: This plot shows the local-linear regression of two repo rate indices, SOFR and VWATR. SOFR is the Secured Overnight Financing Rate. VWATR is the volume-weighted average repo rate calculated from Tradition transaction data. Source: Copeland, Duffie and Yang (2020). Data: Fedwire Funds Service, FRBNY, Tradition.
Figure 24: Payment time net of sample mean when 50% of the day’s total incoming value has been received by dealer banks over Fedwire against the repo rate spread (VWATR). VWATR is the value-weighted average of the Treasury general collateral repo rate calculated from Tradition transaction data. Because of the log scale, I drop the observations for which this rate spread is negative. The upper-right dot corresponds to September 17, 2019, when repo rates had a huge spike. Clearly, payment timing had been significantly delayed on this day. Source: Copeland, Duffie and Yang (2020). Data: FRBNY and Tradition.
"Other large bank balances" for a given day is the total of the opening-of-day reserve balances of all accounts in our sample, except for the ten dealer banks. The payment timing measure is the half-received time of payments to the dealer banks. The date corresponding to the red dot in the upper-left corner is September 17, 2019, on which GCF–IOER spiked to its sample-record high and the total opening balances of the other large banks reached its sample-record low. Data source: Fedwire Funds Service.
B Appendix: Institutional background

B.1 The repo market

Repo transactions are economically similar to collateralized loans. Unlike traditional collateral, repo collateral is not pledged but rather sold and then repurchased at maturity, which gives the lender greater control over the collateral. A general collateral (GC) repo is a transaction whereby the cash investor agrees to accept any security within an asset class, such as U.S. Treasuries. This paper focuses on overnight GC repo transactions collateralized by Treasuries, which constitute the largest segment of the repo market. The GC repo rates with Treasury collateral are typically free of counterparty risk and repo specialness.\(^{(32)}\)

The repo market is among the most important global money markets. Financial institutions participating in the repo market include securities dealers, primary dealers, domestic and international banks, insurance companies, asset managers, money market funds, mutual funds, pension funds and hedge funds. The repo market redistributes liquidity among these financial institutions, and in doing so allows other financial markets to function more efficiently. Disruptions in the repo market may undermine the efficiency and stability of the financial system.

The initial leg of an overnight repo market has “\(T + 0\)” settlement, meaning that settlement of the exchange of collateral and reserves occurs on the day the transaction is negotiated. Importantly, banks and borrowers are unable to sell other assets to provide same-day liquidity because they are unable to obtain cash settlement for asset sales on the same day in most cases. The “\(T + 0\)” settlement makes the repo market essential for intraday funding needs.

The repo market is critical to the implementation of monetary policy. The Federal Reserve makes heavy use of repos to manage its balance sheet and to target short term rates, including its official target rate (i.e., the federal funds rate). After the Global Financial Crisis (GFC) of 2007-2009, the repo market supplanted the federal funds market by becoming the dominant market in which U.S. banks and dealers borrow and lend reserves with each other. Currently, more than $5 trillion of repo products of various tenors and collateral types are traded every day.\(^{(33)}\) One component of the market, the overnight funding market collateralized by Treasury

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\(^{(32)}\)On occasion, one cash lender may seek a specific security as collateral in the repo market. In this case, the cash lender is willing to earn a below-market rate on the loan because the securities posted as collateral are “special,” meaning that they have an intrinsic value that the cash lender will attempt to monetize. This adjustment of repo rates is known as the repo specialness premium (Duffie, 1996).

\(^{(33)}\)See Baklanova, Copeland and McCaughrin (2015) and US Repo Market Fact Sheet for more details.
securities and covered by SOFR, had a daily average trading volume of $1.08 trillion between January 1, 2019 and July 10, 2020. In contrast, the concurrent daily average trading volume for the federal funds market is only $0.071 trillion. As a result, the Treasury-collateralized repo rate has become the most important indicator of U.S. short term money market conditions. In addition, the Secured Overnight Financing Rate (SOFR) is replacing LIBOR as the main benchmark interest rate in U.S. money markets. Therefore, an understanding of the factors that determine repo rates, especially the supply of reserves provided by the Fed, is critical to the conduct of U.S. monetary policy in the post-GFC regulatory environment.

Fig. 26 provides a stylized overview of the U.S. GC Treasury repo market. Large U.S. banks are central intermediaries in this market. On the one hand, large U.S. banks channel liquidity from ultimate cash lenders (such as MMFs, government-sponsored enterprises, and exchange-traded funds) to ultimate cash borrowers, including hedge funds, smaller banks, and foreign institutions. In this mechanism, every dollar lent by large U.S. banks is financed by a corresponding one dollar increase in liabilities such as repo borrowing. I follow Correa, Du and Liao (2020) and call this mechanism the “matched-book intermediation” of large U.S. banks. On

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34 These are estimates based on daily volume data from NYFed and from Fred.
35 More details of the transition from LIBOR can be found on the Alternative Reference Rates Committee (ARRC)'s webpage.
36 Foreign banks also function as dealers between lenders and borrowers, but they are primarily net cash borrowers in the GC Treasury repo market (Kahn and Olson, 2021).
the other hand, large U.S. banks run down their reserve balances to provide additional liquidity in the U.S. GC Treasury repo market—so-called “reserve-draining intermediation” (Correa, Du and Liao, 2020). As demonstrated by Fig. 11, reserve-draining intermediation has played an increasingly important role in liquidity provision in short-term wholesale funding markets.

The term “repo” is usually associated with the activity of borrowing liquidity. Liquidity provision by large U.S. banks increase their reverse repo position. The market segment where large, high-quality, dealer banks borrow from U.S. money market funds is called the “triparty repo market.” The market where large dealer banks lend to smaller dealers is called the “GCF repo market.” Therefore, in a fully competitive market, the GCF-IOER spread represents the marginal value of liquidity for large dealer banks.

The New York Fed (NY Fed) publishes SOFR as one important broad measure of the cost of borrowing cash overnight collateralized by Treasury securities. The SOFR is a volume-weighted median of transaction-level triparty repo data (collected from the Bank of New York Mellon) as well as GCF repo transaction data and data on bilateral Treasury repo transactions cleared through FICC’s DVP service, which is filtered to remove a portion of transactions considered “specials.” Due to its complicated composition, SOFR is not a good measure of the marginal value of liquidity for large U.S. banks, but it is highly related, and typically co-moves with GCF repo rates.

B.2 The interbank payment system details

The clearing and settlement system for U.S.-dollar denominated wholesale transactions is the largest in the world. It is highly complex and consists of a multitude of platforms which form an intricate network, connecting multiple financial institutions. The center of the network is the interbank payment system—the system that commercial banks use to send large-value or time-critical payments to each other across the accounts of the Federal Reserve, which is called the “Federal Reserve’s Fedwire Funds Service” (Fedwire Funds). Fedwire Funds is a real-time gross settlement (RTGS) system and processes payments individually, immediately, unconditionally, and with finality during 22 hours of any given business day.\footnote{A more detailed description can be found here} Transactions on all other platforms in the wholesale clearing and settlement system almost always involve a payment from one bank to another in the Fedwire Funds system (Bech, Martin and McAndrews, 2012). Therefore, banks face real-time demand for payment services by their clients, who wish to send money to their business counterparts who may hold accounts at other banks. Often, clients have
urgent payment requests (e.g., settling foreign exchange transactions) and desire settlement by banks of potentially very large payments with minimal delay. In such a case, it is costly for banks to postpone making those payments as clients might either demand compensation for late settlement or take their business elsewhere in the future, imposing costs on the delaying banks. In general, a bank has little control over the arrival of its customers’ outgoing payment requests and the flow of its incoming funds transfers that depend on other banks’ timing decisions of payment initiation. However, banks can strategically delay sending those payments (albeit delaying is costly) in order to smooth non-synchronized payment flows and to economize on their use of reserves throughout the day. This is because under post-crisis liquidity regulations and supervision, large U.S. banks appeared to have become extremely averse to allowing their intraday reserve balances to drop below a certain desired level. (See Appendix B.3 for more details.) Throughout each business day, large banks face both sizable incoming payments flows and outgoing payments requests (see Figs. 19 and 22). Therefore, they have to rely heavily on incoming payments from other banks to meet their own payment requests, and they face a serious liquidity management problem when payment requests outbalance incoming payment flows.

B.3 Post-GFC liquidity regulations

I summarize some relevant liquidity rules and supervision that constrain the large dealer banks as follows:

- The Federal Reserve created the Large Institution Supervision Coordinating Committee (LISCC) supervisory program in 2010, which supervises the intraday liquidity risk of large banks. The Federal Reserve Board stated: “In 2019, LISCC liquidity supervision is focusing on the adequacy of a firm’s cash-flow forecasting capabilities, practices for establishing liquidity risk limits, and measurement of intraday liquidity risk” (May, 2019 Report on Supervisory Developments).

- The Federal Reserve Board’s Regulation YY, Enhanced Prudential Standards, includes rules covering intraday liquidity exposures, which state: “If the bank holding company is a global systemically important BHC, Category II bank holding company, or a Category III bank holding company, these procedures must address how the management of the bank holding company will: (i) Monitor and measure expected daily gross liquidity inflows and outflows; (ii) Manage and transfer collateral to obtain intraday credit; (iii) Identify and prioritize time-specific obligations so that the bank holding company can meet these
obligations as expected and settle less critical obligations as soon as possible; (iv) Manage the issuance of credit to customers where necessary; and (v) Consider the amounts of collateral and liquidity needed to meet payment systems obligations when assessing the bank holding company’s overall liquidity needs.”

- Resolution Liquidity Adequacy and Positioning (RLAP) under the Dodd-Frank Act includes the intraday “resolution” liquidity requirement. The associated FDIC and Federal Reserve Board guidance states that banks must “ensure that liquidity is readily available to meet any deficits.” Additionally, the RLAP methodology should take into account (A) the daily contractual mismatches between inflows and outflows; (B) the daily flows from movement of cash and collateral for all inter-affiliate transactions; and (C) the daily stressed liquidity flows and trapped liquidity as a result of actions taken by clients, counterparties, key FMUs, and foreign supervisors, among others.”

Liquidity Coverage Ratio (LCR) is another frequently mentioned regulatory constraint that may have prevented banks from lending their excess reserves to take advantage of higher repo rates, but it is unlikely that LCR presents a hurdle. The LCR requires banks to hold high-quality liquid assets (HQLA) equal to a projected 30-day net cash outflow under stress. Excess reserves and the Treasury securities received in a reverse repo as collateral count equally as HQLA by LCR, so trading one for the other leaves a bank’s HQLA unchanged. Moreover, the reverse repo is assumed to roll over 100% for 30 days, so there is no implication for net cash outflows. Consequently, any bank’s LCR is unchanged regardless of the amount of reserves it lends in the repo market.

Numerous industry reports and academic work have documented how the set of liquidity regulations constrained the large banks (Pozsar, 2019a,b; Younger, John and Aggarwal, 2020; Nicolae, 2020). Jamie Dimon, the Chairman and CEO of J.P. Morgan, commented on the September 2019 repo market disruption during J.P. Morgan’s third-quarter 2019 earnings call, by saying

“. . . we have a checking account at the Fed with a certain amount of cash in it. Last year [2018] we had more cash than we needed for regulatory requirements. So when repo rates went up, we went from the checking account, which was paying IOR into repo. Obviously makes sense, you make more money. But

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38According to the Federal Reserve Board’s August 2019 Senior Financial Officer Survey, “satisfying internal liquidity stress metrics, meeting routine intraday payment flows, and meeting potential deposit outflows were important or very important determinants” of banks’ holdings of excess reserves. In a related BIP survey, over three-quarters of the banks to which the Regulation YY liquidity buffer is applicable indicated this to be an “important” or “very important” consideration.

39An FMU is a designated financial market utility, such as a designated payment system or a settlement system.
now the cash in the account, which is still huge. It’s $120 billion in the morning and goes down to $60 billion during the course of the day and back to $120 billion at the end of the day. That cash, we believe, is required under resolution and recovery and liquidity stress testing. And therefore, we could not redeploy it into repo market, which we would have been happy to do. And I think it’s up to the regulators to decide they want to recalibrate the kind of liquidity they expect us to keep in that account. Again, I look at this as technical; a lot of reasons why those balances dropped to where they were. I think a lot of banks were in the same position, by the way. But I think the real issue, when you think about it, is what does that mean if we ever have bad markets? Because that’s kind of hitting the red line in the Fed checking account, you’re also going to hit a red line in LCR, like HQLA, which cannot redeployed either. So, to me, that will be the issue when the time comes. And it’s not about JPMorgan. JPMorgan will be fine in any event. It’s about how the regulators want to manage the system and who they want to intermediate when the time comes.”
C Appendix: Model extensions

C.1 Monopolistic pricing

Many wholesale funding markets feature relationship trading and central-peripheral network structures. Dealer banks usually have market power over the borrowers. In this section, I extend my baseline model and assume dealer banks are local monopoly to the borrowers.

As in the baseline model, in the funding market, borrower $i$ is matched with bank $i$. The net cost of borrower $i$ associated with financing amount $q$ at funding rate $r$ (endogenously determined in equilibrium) is

$$qr + \frac{\xi}{2}((D_i - q)^+)^2,$$

where $((D_i - q)^+)$ is the cost of reduced financing.

For simplicity, assume that the quantities $D_i$ to be financed by borrowers have a density $f_D(x) = \lambda e^{-\lambda(x-D_{min})}$ on $[D_{min}, \infty)$.

Bank $i$ and borrower $i$ bilaterally negotiate the quantity-rate pair $(S_i, r_i)$, where $S_i$ is the quantity of reserves that bank $i$ provides to borrower $i$, and $r_i$ is the funding rate. The bilateral negotiation is modeled as a monopolistic screening model (Mussa and Rosen, 1978). Bank $i$ acts as the local monopolist by offering a supply schedule $g_i: \mathbb{R} \to \mathbb{R}$, which may depend on the initial balance $R_i$ of bank $i$. That is, for some measurable $G: \mathbb{R} \times \mathbb{R}_+ \to \mathbb{R}$, bank $i$ is willing to charge, at any quantity $s$ chosen by the borrower, the funding rate of $g(s) = G(s, R_i)$. After observing $D_i$, given the supply schedule $g$ announced by bank $i$, borrower $i$ picks its desired quantity $S_i$ by solving

$$\inf_s \left(\frac{\xi}{2}((D_i - s)^+)^2 + g_i(s)s\right),$$

or leaves the market without trading. To define the problem of bank $i$, I temporarily assume that there is a unique measurable solution $\rho(D_i, g_i)$ to (10), and that borrower $i$ prefers obtaining $\rho(D_i, g_i)$ in funding at rate $g_i(\rho(D_i, g_i))$ to the alternative of leaving the market without trading. I show in Lemma 3 that these assumptions are satisfied in equilibrium. Having observed $R_i$, bank $i$ thus chooses the supply schedule $g_i$ to solve

$$\sup_g \mathbb{E}\left[V(R_i - Q - \rho(D_i, g)) + g(\rho(D_i, g))\rho(D_i, g) \mid R_i\right].$$

\(^{40}\)As usual, $\mathbb{R} = \mathbb{R} \cup \{\infty, -\infty\}$. 

In summary, an equilibrium of the trading game consists of contingent supply schedule $G$ and quantity of funding $S_i$ such that given $G(\cdot, R_i)$, $S_i$ solves the problem (10) of borrower $i$, and the funding schedule $G(\cdot, R_i)$ solves problem (11) of bank $i$. The equilibrium funding rate is $r_i = G(S_i, R_i)$.

**Lemma 3.** Fix any strategy $a_i = \min((L_i + \alpha_i)^+, N_i)$ and $a_j = \min((L_j + \alpha_j)^+, N_j)$ for the payment subgame. There is a unique equilibrium of the trading game which is determined by the equations

\[ S_i = \mathcal{S}(D_i) \overset{\text{def}}{=} \inf \left\{ s : \Gamma_i(R_i - Q - s, \alpha_i, \alpha_j) \geq \xi(D_i - s - \frac{1 - F_D(D_i)}{f_D(D_i)}) \right\} \]

\[ T_i = \mathcal{T}(D_i) \overset{\text{def}}{=} -\frac{\xi}{2} \left( D_i - \mathcal{S}(D_i) \right)^2 + \frac{\xi}{2} D_{\min}^2 + \int_{D_{\min}}^{D_i} \xi(x - \mathcal{S}(x)) \, dx \]

\[ r_i = \frac{T_i}{S_i}, \tag{12} \]

where $F_D$ is the cumulative distribution function of $D_i$.

This formulation reflects consistent beliefs and rational expectations by bank $i$ about the equilibrium in the subsequent payment subgame. The term $\frac{1 - F_D(D_i)}{f_D(D_i)} = \lambda^{-1}$ in Eq. (12) is usually called the borrower’s “information rent” in the mechanism design literature.

**Lemma 4.** Fix the payment subgame equilibrium strategy profile $\{a_i^* = \min((L_i + \alpha_i)^+, N_i)\}$. For any $o \in \mathbb{R}$, and reserve balances after trading game $L_j = R_j - Q - S_j$ possible in equilibrium,

\[ L_j < o \iff R_j - Q - D_j + \frac{\Gamma_j(o, \alpha_j, \alpha_i)}{\xi} + \lambda^{-1} < o \]

\[ L_j \geq o \iff R_j - Q - D_j + \frac{\Gamma_j(o, \alpha_j, \alpha_i)}{\xi} + \lambda^{-1} \geq o. \]

Let $\Gamma_j^+(o, \alpha_j, \alpha_i) = \lim_{x \uparrow o} \Gamma_j(x, \alpha_j, \alpha_i)$, then

\[ \mathbb{P}(L_j \leq o) = \mathbb{P}(R_j - Q - D_j + \frac{\Gamma_j^+(o, \alpha_j, \alpha_i)}{\xi} + \lambda^{-1} \leq o) = \mathbb{E} \left[ F_R \left( D_j + Q - \frac{\Gamma_j^+(o, \alpha_j, \alpha_i)}{\xi} - \lambda^{-1} \right) \right]. \]

The lemma is important since it almost bridges the endogenous state variable $L_j$ for the payment subgame at $t = 1$ and the exogenous state variable $R_j$ and $D_j$ at $t = 0$ under monoplistic pricing, under which, I need to revise the definition of liquidity stress index to incorporate the information rent of borrower $j$:  

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Definition 4. The liquidity stress index is

\[ m = \mathbb{P}\left( R_j - D_j - Q + \lambda^{-1} \leq -\frac{c}{\xi}\right) - \frac{c}{\psi} = \mathbb{E}\left[ F_R\left( D_j + Q - \frac{c}{\xi} - \lambda^{-1}\right)\right] - \frac{c}{\psi}, \]

where \( F_R \) is the cumulative distribution function of \( R_i \).

The liquidity hoarding condition is when

\[ m = \mathbb{E}\left[ F_R\left( D_j + Q - \frac{c}{\xi} - \lambda^{-1}\right)\right] - \frac{c}{\psi} > 0. \] (13)

The no hoarding condition is when

\[ m = \mathbb{E}\left[ F_R\left( D_j + Q - \frac{c}{\xi} - \lambda^{-1}\right)\right] - \frac{c}{\psi} < 0. \] (14)

For a given probability distribution of reserves satisfying a non-degeneracy condition, liquidity hoarding occurs whenever \( \lambda \) is sufficiently low, \( Q \) is sufficiently high, \( c \) is sufficiently low, \( \xi \) is sufficiently high, or \( \psi \) is sufficiently high.

Theorem 6. Under liquidity hoarding condition, there is a unique equilibrium. In this equilibrium, bank \( i \) hoards liquidity and pays \( a^*_i = \min(L^+_i, N_i) \) in the payment subgame. The marginal value of liquidity functions are the same for both banks \( \Gamma_i(y, 0, 0) = \Gamma_j(y, 0, 0) = \Gamma(y, 0, 0) \) such that

1. When \( y > 0 \), \( \Gamma(y, 0, 0) = c(1 - F_N(y)) \);
2. When \( y \leq 0 \), \( \Gamma(y, 0, 0) = \psi \left( F_N(-y) + (1 - F_N(-y))F_{RD}(-\lambda^{-1} - y - \frac{\Gamma(-y, 0, 0)}{\xi})\right) \).

Theorem 7. Under no liquidity hoarding, there always exists at least one equilibrium. Any equilibrium must be symmetric (in the sense that \( \alpha_i = \alpha_j = \alpha \)) with pure payment strategy \( \alpha^*_i = \min(N_i, (L_i + \alpha)^+) \) for some \( \alpha > N_{\min} \). The marginal value of liquidity functions are the same for both banks \( \Gamma_i(y, \alpha, \alpha) = \Gamma_j(y, \alpha, \alpha) = \Gamma(y, \alpha, \alpha) \). In addition, \( \alpha \) and \( \Gamma \) solve a system of integral equations:

\[ \mathbb{P}\left( R_i - D_i - Q \leq -\frac{\Gamma(0, \alpha, \alpha)}{\xi}\right) + \mathbb{P}(N_i \leq \alpha) \left( 1 - \mathbb{P}\left( R_i - D_i - Q \leq -\frac{\Gamma(0, \alpha, \alpha)}{\xi}\right)\right) = \frac{c}{\psi}; \]

\[ \Gamma(y, \alpha, \alpha) = \begin{cases} \psi \int_{n \in (y^+, (y+\alpha))} F_N(n - y) + (1 - F_N(n - y))F_{RD}(-\lambda^{-1} - n - y - \alpha - \frac{\Gamma(n - y, \alpha, \alpha)}{\xi}) \, dF_N(n), & \forall y > -\alpha; \\
\psi \left( F_N(-y) + (1 - F_N(-y))F_{RD}(-\lambda^{-1} - y - \alpha - \frac{\Gamma(-y, \alpha, \alpha)}{\xi})\right) & \forall y \leq -\alpha. \end{cases} \]

It turns that the equilibrium is unique under the same technical conditions as in Section 4.2.1.
**Theorem 8.** Assume $N_i - N_{\min}$ ($i = 1, 2$) is exponentially distributed with parameter $\lambda_N$ and $F_{RD}$ is differentiable with density function $f_{RD}$. Let $f_{RD}^{m} = \sup\{f_{RD}(t) : t \leq 0\}$. If $\sqrt{\frac{2\epsilon}{\psi}} > f_{RD}^{m}$, then the equilibrium is unique under the no hoarding condition.

Theorem 6 and Theorem 8 implies that the equilibrium is unique under both the liquidity hoarding condition and the no hoarding condition. Once the payment subgame and the marginal value for liquidity is determined, the equilibrium funding rate can be solved directly:

**Theorem 9.** Given realizations of $R_i$ and $D_i$, then equilibrium trading quantity $S_i = \mathcal{S}(D_i)$ is determined by Eq. (12) in Lemma 3. The equilibrium funding rate $r_i$ is

$$r_i = \frac{R_i - Q - \mathcal{S}(D_{\min})}{\mathcal{S}(D_i)} \int_{R_i - Q - \mathcal{S}(D_i)}^{R_i - Q - \mathcal{S}(D_{\min})} \Gamma(y, \alpha, \alpha) \, dy + (\xi \lambda)^{-1} \mathcal{S}(D_{\min}) + (\xi \lambda)^{-1},$$

where the marginal value $\Gamma(y, \alpha, \alpha)$ of reserves is solved explicitly in Theorem 6 and Theorem 7, case by case.

Theorem 7 and Theorem 9 state that when banks expect other banks to have abundant opening reserves, they have a better incentive to send more payments early in the day and to lend more liberally in the funding market. Theorem 6 and Theorem 9 predict that when market conditions (including the probability distribution of reserve levels) change slightly, yet enough to trigger the hoarding condition Eq. (13), the marginal value of liquidity $\Gamma$ can jump up from $\Gamma(R_i - Q - S_i, \alpha, \alpha)$ to $\Gamma(R_i - Q - S_i, 0, 0)$, causing short-term funding rates to spike:

**Theorem 10.** Fix some outcome $\zeta$ of beginning reserve balances $R_i$ and a quantity $S^*$ traded in the funding market. The equilibrium funding rate $r^*$ jumps up as a function of the liquidity stress $m$ at the threshold $m = 0$ that triggers liquidity hoarding. More specifically, there exists some $\delta(\zeta, S^*) > 0$ such that

$$\lim_{\epsilon_m \downarrow 0} r^*(\zeta, S^*, \epsilon_m) - r^*(\zeta, S^*, -\epsilon_m) > \delta(\zeta, S^*),$$

provided that the sets of macroeconomic conditions $\mathcal{M}_C^{\epsilon_m}$ are mutually close with respect to liquidity stress index to each other.
C.2 General case: $n$ dealer banks

In this section I present and solve the model for $n > 2$ dealer banks. For simplicity, assume that $n$ dealer banks are symmetric. Assume that the random variables of total payment needs for each bank $N_1, N_2, \cdots, N_n$ are identically and independently distributed according to a probability distribution function (pdf) $f_N(\cdot)$ on the support $[N_{min}, \infty)$ for some $N_{min} > 0$. In addition, assume the initial reserve balances for each bank $R_1, R_2, \cdots, R_n$ are identically and independently distributed with pdf $f_R(\cdot)$. Similarly, assume the borrowing demands from each short-term borrower $D_1, D_2, \cdots$ and $D_n$ are identically and independently distributed with pdf $f_D(\cdot)$. Let $F_{RD}(x) = \mathbb{P}(R_i - D_i - Q \leq x)$. Under stated conditions, $F_{RD}$ is differentiable with pdf $f_{RD}$.

Suppose bank $i$ makes an early payment $a_{i,j}$ to bank $j$, and gets incoming early payment $a_{j,i}$ from bank $j$. Then in the payment subgame the cost to bank $i$ associated with payment timing is

$$\psi(L_i - \sum_{j \neq i} a_{i,j} + \sum_{j \neq i} a_{j,i})^- + c(N_i - \sum_{j \neq i} a_{i,j})^+ = \psi(L_i - a_i + a_{-i})^- + c(N_i - a_i)^+. \tag{15}$$

where

$$a_i \overset{\text{def}}{=} \sum_{j \neq i} a_{i,j}, \quad \text{and} \quad a_{-i} \overset{\text{def}}{=} \sum_{j \neq i} a_{j,i},$$

and $L_i$ is the reserve balances of bank $i$ after the lending in the funding market. Clearly, bank $i$ is indifferent in how to split its total outgoing payment to other banks as long as the total payment $a_i$ is the same. To simplify the analysis, I assume that each of the large dealer banks have the same business relationships with all other dealer banks. (Soramäki, Bech, Arnold, Glass and Beyeler (2007) show that 25 large banks form a densely connected sub-graph, or clique, in the payment network of the Fedwire system.) Therefore, $a_{i,j} = a_{i,k} = a_{i,n-1}$ for any $j, k \neq i$.

Given the payment strategy $a_{-i}$ of all other banks, bank $i$ chooses $a_i$ to optimize the conditional expected payoff in the payment subgame

$$U(L_i, N_i) = \mathbb{E}[-\psi(L_i - a_i + a_{-i})^- - c(N_i - a_i)^+ \mid N_i, L_i]. \tag{15}$$

Lemma 5. Suppose that either $\prod_{j \neq i} \mathbb{P}(L_j \leq 0) > \frac{c}{\psi}$ for all $i$, or $\prod_{j \neq i} \mathbb{P}(L_j \leq 0) < \frac{c}{\psi}$ for all $i$. Then

---

$41$ Although it is possible to solve the model with asymmetric banks numerically, there is a lack of granular data for us to examine the implications of asymmetric banks empirically.
there is a unique Perfect Bayes payment game equilibrium. In this equilibrium, each bank $i$ chooses the payment $a_i^* = \min((L_i + \alpha_i)^+, N_i)$, where $(\alpha_i)$ solves

$$
\alpha_i = \inf \left\{ \vartheta \geq 0 : \mathbb{P}\left( \sum_{j \neq i} \min((L_j + \alpha_j)^+, N_j) \leq \vartheta \right) \geq \frac{c}{\psi} \right\}
$$

(16)

In particular, if $L_i$ are i.i.d. distributed, then there is a Perfect Bayes payment game equilibrium of the form $a_i^* = \min((L_i + \alpha)^+, N_i)$, for some constants $\alpha$ such that

$$
\alpha = \inf \left\{ \vartheta \geq 0 : \mathbb{P}\left( \sum_{j \neq i} \min((L_j + \vartheta)^+, \vartheta, N_j - \vartheta) \leq 0 \right) \geq \frac{c}{\psi} \right\}.
$$

The equilibrium is unique, except for the knife-edge case where $\mathbb{P}(L_j \leq 0) = \frac{n-1}{p}$. The funding market is modeled as an OTC market where bank $i$ and borrower $i$ is matched and they are price takers, just as in Section 4.2.1: The cost to borrower $i$ of borrowing $S_i$ at funding rate $r_i$ is $S_i r_i + (D_i - S_i)^+ r^{IOER} + \frac{\xi}{2}((D_i - S_i)^+)^2$. Bank $i$'s payoff for lending $S_i$ at funding rate $r_i$ is $S_i (r_i - r^{IOER}) + V(R_i - S_i)$. Since I assume that the distribution functions of $R_i$ and $D_i$ are atomless, the equilibrium funding rate $r^*$ and trading quantity $S^*_i$ satisfy

$$
V'(R_i - S^*_i) = r^*_i - r^{IOER} = \xi (D_i - S^*_i),
$$

where the marginal value of liquidity $\Gamma = V'$ depends on the strategy profile of all other banks in the payment subgame and can be calculated directly:

**Lemma 6.** Given the payment subgame strategy profile $\{ a_i = \min((L_i + \alpha_i)^+, N_i) \}$ and some joint probability distribution for $\{L_i\}$, let $\alpha_{-i} = (\alpha_j)_{j \neq i}$. Then the marginal value of liquidity function for bank $i$ to be the function $\Gamma_i : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^+$ such that

$$
\Gamma_i(y, \alpha, \alpha_{-i}) = \begin{cases} 
\int \psi\mathbb{P}\left( \sum_{i \neq j} \min((L_j + \alpha_j)^+, N_j) \leq \eta - y \right) dF_N(\eta) + \int_{\eta \in [y + \alpha_i, \infty]} cdF_N(\eta), & \forall y > -\alpha_i \\
\psi\mathbb{P}\left( \sum_{i \neq j} \min((L_j + \alpha_j)^+, N_j) \leq -y \right), & \forall y \leq -\alpha_i.
\end{cases}
$$

With identical proof as in the two-banks case, we have

**Lemma 7.** Fix the payment subgame equilibrium strategy profile $\{ a_i^* = \min((L_i + \alpha_i)^+, N_i) \}$. For any
Let $\in [R, \text{and reserve balances after trade } L_j = R_j - Q - S_j \text{ possible in equilibrium,}$

\[ L_j < o \iff R_j - Q - D_j + \frac{\Gamma_j(o, \alpha_j, \alpha_{-j})}{\xi} < o \]
\[ L_j \geq o \iff R_j - Q - D_j + \frac{\Gamma_j(o, \alpha_j, \alpha_{-j})}{\xi} \geq o. \]

Let $\Gamma^+_j(o, \alpha_j, \alpha_{-j}) = \lim_{x \downarrow o} \Gamma^+_j(x, \alpha_j, \alpha_{-j}),$ then $P(L_j \leq o) = P(R_j - Q - D_j + \frac{\Gamma^+_j(o, \alpha_j, \alpha_{-j})}{\xi} \leq o).$

Recall that by assumption, $\{R_i\}$ and $\{D_i\}$ are i.i.d. As in the two-banks case, Lemma 7 motivates the following definitions:

**Definition 5.** The liquidity hoarding condition for $n$ banks is

\[ P(R_i - D_i - Q \leq -\frac{c}{\xi}) > \left(\frac{c}{\psi}\right)^{1/(n-1)}. \] (17)

The no hoarding condition for $n$ banks is

\[ P(R_i - D_i - Q \leq -\frac{c}{\xi}) < \left(\frac{c}{\psi}\right)^{1/(n-1)}. \] (18)

Note that the liquidity hoarding condition for $n$ banks is less stringent than the liquidity hoarding condition for two banks. This is due to the diversification benefits. Each bank receives incoming payments from $n - 1 > 1$ other banks, so it is not as concerned about the liquidity condition of any particular bank. When $n \to \infty,$ bank $i$ knows that it will receive a positive amount of incoming payments: $a_{-i} = \sum_{j \neq i} \min((L_j + \alpha_j)^+, N_j) > 0$ almost surely if $P(R_j - D_j - Q \leq -\frac{c}{\xi}) < 1.$

In this case, bank $i$ will never hoard liquidity in the payment game. In other words, liquidity hoarding condition for $n$ banks will never hold as $n \to \infty$ when $P(R_j - D_j - Q \leq -\frac{c}{\xi}) < 1.$

Apparently, it is useful for bank $i$ to consider the probability distribution of $a_{-i}.$ Fix some payment subgame equilibrium strategy profile $\{a^*_i = \min((L_i + \alpha_i)^+, N_i)\}.$ By assumption, the joint probability distribution of $\{a^*_i\}$ is common knowledge for all banks. Let

\[ F_{a_j}(x) \overset{\text{def}}{=} P(\min((L_j + \alpha_j)^+, N_j) \leq x). \]

Lemma 7 implies that for any $x \geq 0,$

\[ F_{a_j}(x) = F_N(x) + (1 - F_N(x))P(R_j - D_j - Q \leq x - \alpha_j - \frac{\Gamma_j(\xi, \alpha_j, \alpha_{-j})}{\xi}). \]

Under the stated conditions, when $x > 0,$ $F_{a_j}(x)$ is differentiable. Let $f_{a_j}(\cdot)$ be the pdf for $a_j.$
Direct calculation gives

**Lemma 8.** Given some payment subgame strategy profile \( \{a_i = \min((L_i + \alpha_i)^+, N_i)\} \) and some joint probability distribution for \( \{L_i\} \), then for any \( x \geq N_{\text{min}} \),

\[
fa_j(x) = \frac{dFa_j(x)}{dx} = (1 - FN(x))fRD(x - \alpha_j - \frac{\Gamma_j(x - \alpha_j)}{\xi})(1 - \frac{\Gamma_j'(x - \alpha_j)}{\xi}) + FN(x)(1 - FRD(x - \alpha_j - \frac{\Gamma_j(x - \alpha_j)}{\xi})),
\]

and for any \( 0 < x < N_{\text{min}} \),

\[
fa_j(x) = fRD(x - \alpha_j - \frac{\Gamma_j(x - \alpha_j)}{\xi})(1 - \frac{\Gamma_j'(x - \alpha_j)}{\xi}).
\]

For any \( x \geq 0 \),

\[
F_{a_{-i}}(x) \overset{\text{def}}{=} \mathbb{P}(\sum_{j \neq i} \min((L_j + \alpha_j)^+, N_j) \leq x) = \prod_{j \neq i} Fa_j(0) + \int_0^{(n-1)x} f\sum_{j \neq i} a_j(t) dt,
\]

where \( f\sum_{j \neq i} a_j(\cdot) \) is the pdf for the random variable \( \sum_{j \neq i} a_j \) that can be calculated using the convolution operation on \( \{fa_j\}_{j \neq i} \).

With **Lemma 8**, the equilibrium can be characterized by the following theorems:

**Theorem 11.** Under liquidity hoarding condition for \( n \) banks, there is a unique equilibrium, in which bank \( i \) pays \( a_i^* = \min(N_i, L_i^+) \) in the payment subgame. The marginal value of liquidity functions are the same for all banks \( \Gamma_i(y, 0, 0) = \Gamma(y, 0, 0) \). In addition,

1. When \( y > 0 \), \( \Gamma(y, 0, 0) = c(1 - FN(y)) \);
2. When \( y \leq 0 \), \( \Gamma(y, 0, 0) = \psi Fa_{-i}(-y) \).

**Theorem 12.** Under no liquidity hoarding, there always exists at least one equilibrium. Any equilibrium must be symmetric with pure payment strategy \( a_i^* = \min(N_i, (L_i + \alpha)^+) \) for some \( \alpha > N_{\text{min}} \). The marginal value of liquidity functions are the same for both banks \( \Gamma_i(y, \alpha, \alpha) = \Gamma_j(y, \alpha, \alpha) = \Gamma(y, \alpha, \alpha) \). In addition, \( \alpha \) and \( \Gamma \) solve the following system of integral equations:

\[
F_{a_{-i}}(\alpha) = \frac{c}{\psi},
\]

\[
\Gamma(y, \alpha, \alpha) = \begin{cases} 
\int_{n\in(y^+, (y+\alpha))} \psi Fa_{-i}(n - y) dF_N(n) + \int_{n\in[y(y+\alpha), \infty)} c dF_N(n), & \forall y > -\alpha \\
\psi Fa_{-i}(-y), & \forall y \leq -\alpha.
\end{cases}
\]
Finally, Theorems 11 and 12 imply that the Theorems 4 and 5 hold for the \( n \)–banks case. The proofs are almost identical to the two-banks case.

C.3 A quantitative model without strategic complementarity

To fully explore the important role of strategic complementarity, I consider an otherwise-identical model without the component of strategic complementarity. Following Section 4.2.1, assume that bank \( i \) is still competitive in the funding markets. After having loaned out \( S_i \) in the funding market, the reserve balance left for the payment subgame of bank \( i \) is \( R_i - S_i - Q \). To shut down strategic complementarity, I assume that in the payment subgame, instead of rationally incorporating the payment strategy of bank \( j \) as in Section 4.1, bank \( i \) simply believes that the early payment from bank \( j \) is a constant fraction of bank \( j \)'s expected total reserve balances net of expected bank \( j \)'s loan in the funding markets, i.e. bank \( i \) believes that \( a_j^* = E_R(E^i[R_j - S_j]) \). Bank \( i \) non-strategically believes that \( E^i[E_R S_j] = E_D T_D \), where \( T_D \) is the total Treasuries outstanding.

Bank \( i \) can pay any positive amount \( a_i \leq N_i \) at time 1 and postpones the rest of the payment to time 2. Given bank \( i \)'s belief about the payment strategy \( a_j^* \) of bank \( j \), bank \( i \) chooses \( a_i \) to optimize the conditional expected payoff

\[
U(L_i, N_i) = \mathbb{E}[-\psi(R_i - S_i - a_i + a_j^*) - c(N_i - a_i)^+ | N_i, R_i - S_i] = \mathbb{E}[-\psi(R_i - S_i - a_i + E_R(E^i[R_j] - E_D T_D)^+) - c(N_i - a_i)^+ | N_i, R_i - S_i].
\]

It is straightforward to see that the optimal strategy for bank \( i \) is then \( a_i^* = \min((R_i - S_i - a_i + E_R(E^i[R_j] - E_D T_D)^+), N_i) \). The continuation value of bank \( i \) for reserve balances at the beginning of the payment game, before observing its payment obligation \( N_i \), is

\[
V(R_i - S_i - Q) = \mathbb{E}[-\psi(R_i - S_i - a_i^* + E_R(E^i[R_j])) - c(N_i - a_i)^+ | R_i - S_i - Q].
\]

(20)

Therefore, direct calculation yields

\[
\Gamma(y) \equiv V'_i(y) \equiv \lim_{x \uparrow y} \frac{V(x) - V(y)}{x - y} = c \mathbb{P}(N_i \geq R_i - S_i + E_R(E^i[R_j] - E_D T_D - Q)),
\]

when \( R_i - S_i + E_R(E^i[R_j] - E_D T_D - Q \geq 0 \) and

\[
\Gamma(y) = \psi
\]

when \( R_i - S_i + E_R(E^i[R_j] - E_D T_D - Q < 0 \). Throughout the year of 2019, there were no reports
of any large bank which had violated liquidity regulation, which implies that $R_i - S_i - Q > 0$ (Afonso, Cipriani, Copeland, Kovner, La Spada and Martin, 2020b). In addition, understanding that bank $j$ is unlikely to lend any $S_j > R_j$, bank $i$ believes that $E_R[E_i^i[R_j] - E_D T_D] > 0$. Therefore, I focus on parameters such that $R_i - S_i + E_R[E_i^i[R_j] - E_D T_D - Q > 0$ in the sample. (This means that $\psi$ cannot be identified. I will discuss this point in more detail later.) In addition, I assume that $E_i[R_j] = R_i$. Similar to Section 6, the impact of Treasury issuance on large dealer banks’ reserve balances is assumed to be linear, and other large banks’ early payment is also linear in their total reserve balances. Thus, from observed dealer balance $R_D$,

$$R_i = \frac{1}{10}(R_D - E_i T_I + E_O R_O).$$

I assume that $N_i - N_{min} \sim \exp(\lambda_N)$ where $\lambda_N$ and $N_{min}$ are estimated from payment data. This implies that

$$\Gamma(y) = ce^{-\lambda_N\left(\max\left(\frac{1+E_R}{10}(R_D+E_O R_O)-E_i T_I-S_i-E_D T_D-Q-N_{min}, 0\right)\right)}.$$

Note that it is not possible to jointly identify $Q$ and $c$ in my sample, so I take $Q = $11.1 billion from my estimates as in Section 6.

As in Section 6, I estimate this model in the context of GCF Treasury repo market. Eq. (21) implies the empirical relation under this particular model without strategic complementarity on day $t$ is:

$$(GCF - IOER)^t = ce^{-\lambda_N\left(\max\left(\frac{1+E_R}{10}(R_D+E_O R_O)-E_i T_I-S_i-E_D T_D-Q-N_{min}, 0\right)\right)} + \bar{\theta} + \epsilon_r.$$  

From payment volume data, I recover $N_{min} = $33 billion and $\lambda_N = 109.4$ as in Section 6. The unknown parameters to be estimated are $\{E_R, E_O, E_I, E_D, c, \bar{\theta}\}$. The moments that I use to identify parameters are

$$E_\epsilon = \begin{bmatrix} \epsilon_r \\ \epsilon_r R_D \\ \epsilon_r R_O \\ \epsilon_r T_I \\ \epsilon_r T_D \\ \epsilon_r S_i \end{bmatrix} = 0.$$
Table 8: Estimated parameters for the model without strategic complementarity

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Meaning</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>late payment cost</td>
<td>0.67%</td>
</tr>
<tr>
<td>$E_R$</td>
<td>early payment from bank $j$ is $E_R R_i - E_D T_D$</td>
<td>0.73</td>
</tr>
<tr>
<td>$Q$</td>
<td>regulatory minimum</td>
<td>11.1 ($bn$)</td>
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<tr>
<td>$E_I$</td>
<td>Treasury issuance effect</td>
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<td>$E_O$</td>
<td>early payment from other banks is $E_O R_O$</td>
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</tr>
<tr>
<td>$E_D$</td>
<td>early payment from bank $j$ is $E_R R_i - E_D T_D$</td>
<td>0</td>
</tr>
<tr>
<td>$\vartheta$</td>
<td>other factors</td>
<td>5.52 (bp)</td>
</tr>
<tr>
<td>$N_{min}$</td>
<td>minimum total payment volume</td>
<td>33 ($bn$)</td>
</tr>
<tr>
<td>$1/\lambda_N$</td>
<td>$\mathbb{E}[N_i - N_{min}]$</td>
<td>1.1 ($bn$)</td>
</tr>
</tbody>
</table>

Without strategic complementarity, the information from payment delay is not used. In addition, the parameters governing bank $i$’s belief about bank $j$’s repo lending can be represented by one single parameter, $E_D$. However, this model captures the dynamic interactions of all other observable variables similar to the quantitative model described in Section 6. Table 8 presents the estimated parameters.

Finally, we do not identify $\psi$ from my training data set. Is it possible that an unobserved large $\psi$ explains the repo spike events on September 16, 2019? No. If this were the case, then $R_i - S_i + E_R \mathbb{E}^t[R_j] - E_D T_D - Q \geq 0$ for every day before September 16, 2019, and $R_i - S_i + E_R \mathbb{E}^t[R_j] - E_D T_D - Q < 0$ on September 16-18, 2019. However, this is unlikely: In fact, there were several days in my training data set such that the value of $R_i - S_i + E_R \mathbb{E}^t[R_j] - E_D T_D - Q$ is smaller on those days than the value on any days from September 16 to September 18. For example, the level of total reserve balances of large dealer banks reached its lowest level on July 1, 2019, in my entire sample. Thus, this model without strategic complementarity would always predict that the repo spike on July 1 should be no smaller than any of the large repo spikes on September 16-18. Clearly this is rejected by the data.
D Appendix: Estimating repo lending quantity of large dealer banks

To estimate the daily repo lending quantity of the large U.S. dealer banks, I obtain daily transaction volume underlying the calculation of Secured Overnight Financing Rate (SOFR) and Tri-Party General Collateral Rate (TGCR) from the New York Fed. Let $Volume_t$ be the difference between the underlying volume of SOFR and the underlying volume of TGCR. By construction, $Volume_t$ contains the total reserves lent by large U.S. dealer banks (i.e., the total reserve-draining intermediation in Fig. 26) on day $t$. However, because dealers are borrowing and lending to each other, $Volume_t$ double counts the total repo lending quantity of the large dealer banks; $Volume_t$ also contains liquidity provided by other lenders in the repo market. Therefore, $Volume_t$ overstates the reserves lent by large U.S. dealer banks on day $t$. To estimate the fraction of $Volume_t$ that comes from reserves lent by large dealer banks, I use U.S. GSIB form 10-Q and call reports.\footnote{Large U.S. GSIBs include JPMorgan Chase, Bank of America, Goldman Sachs, Morgan Stanley, Citibank, Wells Fargo, State Street, and Bank of New York Mellon. I exclude Bank of New York Mellon in my analysis since it primarily functions as a clearinghouse in the repo markets.} I extract the daily-average net overnight repo lending quantity of each U.S. GSIB in quarter $q$ from those reports. I then take the sum of these quantities over all U.S. GSIBs in quarter $q$ to get $S_{GSIB}^q$. I assume $S_{GSIB}^q$ approximates the daily average of total repo lending quantity of all large dealer banks in quarter $q$. Let $Volume_q$ be the daily average of $Volume_t$ in quarter $q$. I assume that the ratio $\varphi_q = S_{GSIB}^q / Volume_q$ is constant everyday in quarter $q$. Thus, on day $t$ in quarter $q$, total repo lending quantity can be approximately calculated as $S_t = \varphi_q Volume_t$.\footnote{Large U.S. GSIBs include JPMorgan Chase, Bank of America, Goldman Sachs, Morgan Stanley, Citibank, Wells Fargo, State Street, and Bank of New York Mellon. I exclude Bank of New York Mellon in my analysis since it primarily functions as a clearinghouse in the repo markets.}
E Appendix: Proofs

This appendix contains proofs.

E.1 Proof of Lemma 1

Consider bank $i$’s decision problem. Conditional on the realizations of $N_i$ and $L_i$, when $N_i \leq L_i$, the optimal strategy is $a_i = N_i$. Thus, for the remainder, I only consider the case in which $N_i > L_i$. The equilibrium strategy $a_i$ of bank $i$ is permitted to be a mixed strategy conditional on $N_i$ and $L_i$. Any such mixed strategy $a_i$ can be represented in the form $A(N_i, L_i, \epsilon_i)$ for some measurable, $A: [N_{min}, \infty) \times \mathbb{R} \times [0, 1] \to \mathbb{R}$ and some uniform random variable $\epsilon_i$ independent of $\{N_i, L_i, N_j, L_j, a_j\}$. Letting $A$ denote the space of mixed payment strategies of this form, and given the payment strategy $a_j$ of bank $j$, bank $i$ solves

$$
\text{ess inf}_{A \in A} \mathbb{E}[\psi(A(N_i, L_i, \epsilon_i) - L_i - a_j)^+ + c(N_i - A(N_i, L_i, \epsilon_i))^+ \mid N_i, L_i].
$$

subject to

$$
A(N_i, L_i, \epsilon_i) \geq 0 \quad \text{almost surely}
$$

$$
A(N_i, L_i, \epsilon_i) \leq N_i \quad \text{almost surely}.
$$

An optimal $A$ must satisfy $A(N_i, L_i, \epsilon) \geq L_i$ almost surely for all $N_i$, since $a_j \geq 0$ almost surely. In other words, any strategy $A$ of bank $i$ satisfying $A(N_i, L_i, \epsilon_i) < L_i$ with positive probability is dominated. Since bank $j$ faces the same problem, bank $i$ can correctly infer that $a_j \geq L_j$ by eliminating the dominated strategies of bank $j$.

Let $z = A(N_i, L_i, \epsilon_i) - L_i$. We have shown that $z \geq 0$ almost surely. The problem of bank $i$ can be expressed as

$$
\text{ess inf}_{z} \mathbb{E}\left[\mathbb{E}\left[\psi(z - a_j)^+ \mid z\right] + c(N_i - (\mathbb{E}[z] + L_i))\right]
$$

subject to

$$
z \geq (-L_i)^+ \quad \text{almost surely} \quad (23)
$$

$$
z + L_i \leq N_i \quad \text{almost surely,} \quad (24)
$$

First we observe that when constraint (23) binds, $z = (-L_i)^+$ and $a_i = L_i^+$. Likewise, when constraint (23) binds, $z = N_i - L_i$ and $a_i = N_i$. 

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Let us consider the case when neither constraint (23) nor constraint (24) binds. Since \((z - a_j)^+\) is increasing and convex in \(z\) for all realizations of \(a_j\), the mapping from real \(x\) to \(\mathbb{E}[\psi(x - a_j)^+]\) is also convex. Thus, by Jensen’s inequality, replacing \(z\) by \(\mathbb{E}[z]\) weakly decreases the objective function (22). This implies that conditional on \(L_i, N_i\), if bank \(i\) optimally chooses some \(z\) with support \(I_z \subset [−L_i, N_i − L_i]\), then for any \(v_1\) and \(v_2\) in \(I_z\) and any \(v' \in [−L_i, N_i − L_i]\),

\[
\mathbb{E}[\psi(v_1 - a_j)^+] + c(N_i - (v_1 + L_i)) = \mathbb{E}[\psi(v_2 - a_j)^+] + c(N_i - (v_2 + L_i)) \tag{25}
\]

\[
\mathbb{E}[\psi(v_1 - a_j)^+] + c(N_i - (v_1 + L_i)) \geq \mathbb{E}[\psi(v' - a_j)^+] + c(N_i - (v' + L_i)). \tag{26}
\]

The first order condition for optimality in problem (10) implies that for any \(v \in I_z\),

\[
\mathbb{P}(a_j \leq v) \geq \frac{c}{\psi}. \tag{27}
\]

If there does not exist \(v\) such that \(\mathbb{P}(a_j \leq v) = c/\psi\), then \(I_z\) must be a singleton, and any optimal \(z\) is a constant function. We summarize the above arguments in the following lemma.

**Lemma 9.** Suppose bank \(j\) choose any strategy \(a_j\), the best response actions of bank \(i\) are of the form

\[
a_i = \min((L_i + v_i)^+, N_i),
\]

where \(v_i\) is some non negative random variable with support \(I_z\). For any \(v \in I_z\),

\[
\mathbb{P}(a_j \leq v) \geq \frac{c}{\psi}. \tag{28}
\]

Moreover, if there does not exist \(v^*\) such that \(\mathbb{P}(a_j \leq v^*) = c/\psi\), then \(I_z = \{v^*\}\) is a singleton and \(v^* = \inf\{\vartheta \geq 0, \mathbb{P}(a_j \leq \vartheta) > \frac{c}{\psi}\}\). If there is an \(v^*\) such that \(\mathbb{P}(a_j \leq v^*) = c/\psi\), then for any \(v \in I_z\), \(\mathbb{P}(a_j \leq v) = c/\psi\).

We first the characterize the class of pure strategy equilibrium.

**Lemma 10.** There exists an equilibrium in the class of equilibria in which \(I_z\) is a singleton. In this equilibrium, bank \(i\) chooses the payment \(a_i^* = \min((L_i + \alpha_i)^+, N_i)\), and bank \(j\) chooses payment \(a_j^* = \min((L_j + \alpha_j)^+, N_j)\), where \(\alpha_i\) and \(\alpha_j\) solve

\[
\alpha_i = \inf \left\{ \vartheta \geq 0 : \mathbb{P}(\min((L_j + \alpha_j)^+, N_j) \leq \vartheta) \geq \frac{c}{\psi} \right\},
\]

\[
\alpha_j = \inf \left\{ \vartheta \geq 0 : \mathbb{P}(\min((L_i + \alpha_i)^+, N_i) \leq \vartheta) \geq \frac{c}{\psi} \right\}. \tag{29}
\]
In particular, when $L_i$ and $L_j$ have the same distribution, then $\alpha_i = \alpha_j = \alpha$ where

$$\alpha = \inf \left\{ \vartheta \geq 0 : \mathbb{P}(N_j \leq \vartheta) \geq \frac{c \psi - \mathbb{P}(L_j \leq 0)}{1 - \mathbb{P}(L_j \leq 0)} \right\}.$$

**Proof.** If $I_z = \{\alpha_i\}$ is a singleton, taking constraints (23) and (24) into account, $a^*_i = \min((L_i + \alpha_i)^+, N_i)$. Given $a^*_i$, a similar analysis shows that $a^*_j = \min((L_j + \alpha_j)^+, N_j)$. Under the conditions of Lemma 1, $N_i$ and $N_j$ have the same distribution. When $L_i$ and $L_j$ also have the same distribution, then if $0 \leq \alpha_i < \alpha_j$,

$$\frac{c}{\psi} \leq \mathbb{P}(\min((L_j + \alpha_j)^+, N_j) \leq \alpha_i) \leq \mathbb{P}(\min((L_i + \alpha_i)^+, N_i) \leq \alpha_i)$$

$$< \mathbb{P}(\min((L_i + \alpha_i)^+, N_i) \leq \alpha_j) = \frac{c}{\psi}.$$

This is a contradiction. The last inequality follows from that $F_N$ strictly increases on the interior of its support. Hence $\alpha_i \geq \alpha_j$. A symmetric argument shows that it must be that $\alpha_i = \alpha_j$. Denote this common value by $\alpha$. By Lemma 9,

$$\alpha = \inf \left\{ \vartheta \geq 0 : \mathbb{P}(\min((L_j + \alpha)^+, N_j) \leq \vartheta) \geq \frac{c}{\psi} \right\}.$$

When $\mathbb{P}(L_j \leq 0) \neq \frac{c}{\psi}$, the solution is unique. Since $N_i, N_j \in \mathbb{R}_{++}$,

$$\mathbb{P}(\min((L_j + \alpha)^+, N_j) \leq \alpha) = \mathbb{P}(\min(L_j + \alpha, N_j) \leq \alpha)$$

$$= \mathbb{P}(L_j + \alpha \leq \alpha) + \mathbb{P}(N_j \leq \alpha)(1 - \mathbb{P}(L_j + \alpha \leq \alpha)).$$

This shows that $\alpha$ is determined by

$$\alpha = \inf \left\{ \vartheta \geq 0 : \mathbb{P}(N_j \leq \vartheta) \geq \frac{c}{\psi} - \frac{\mathbb{P}(L_j \leq 0)}{1 - \mathbb{P}(L_j \leq 0)} \right\}.$$

By Lemma 9, when $L_i$ and $L_j$ have different distributions, $\alpha_i$ and $\alpha_j$ solve

$$\alpha_i = \inf \left\{ \vartheta \geq 0 : \mathbb{P}(\min((L_j + \alpha_j)^+, N_j) \leq \vartheta) \geq \frac{c}{\psi} \right\}$$

$$\alpha_j = \inf \left\{ \vartheta \geq 0 : \mathbb{P}(\min((L_i + \alpha_i)^+, N_i) \leq \vartheta) \geq \frac{c}{\psi} \right\}.$$  \hspace{1cm} (27)

Clearly any possible solutions $\alpha_i, \alpha_j$ is less than $F_N^{-1}(\frac{c}{\psi})$. Consider a map $\mathcal{T}^\alpha : [0, F_N^{-1}(\frac{c}{\psi})]^2 \to$
where $T^\alpha(x, y) = (a_i, a_j)$ such that

$$a_i = \inf \left\{ \vartheta \geq 0 : \mathbb{P}(\min((L_j + y)^+, N_j) \leq \vartheta) \geq \frac{c}{\psi} \right\},$$

$$a_j = \inf \left\{ \vartheta \geq 0 : \mathbb{P}(\min((L_i + x)^+, N_i) \leq \vartheta) \geq \frac{c}{\psi} \right\}.$$

It is easy to check that $T^\alpha$ is continuous. By Schauder fixed-point theorem there is at least one fixed point of $T^\alpha$. \qed

Next, I rule out the possibility of mixed equilibria. Suppose that there is a mixed strategy equilibrium such that $a_i = \min((L_i + z_i)^+, N_i)$ and $a_j = \min((L_j + z_j)^+, N_j)$. Let $I_z^i$ denote the support of $z_i$. Since $I_z^i$ and $I_z^j$ are bounded, let

$$v_i^i \overset{\text{def}}{=} \inf I_z^i, \quad v_j^i \overset{\text{def}}{=} \inf I_z^j,$$

$$\bar{v}_i \overset{\text{def}}{=} \sup I_z^i, \quad \bar{v}_j \overset{\text{def}}{=} \sup I_z^j.$$

At least one of $I_z^i$ and $I_z^j$ must have more than one element, for otherwise it is a pure strategy equilibrium. Say $I_z^i$ has at least two elements, then $v_i < v_i^i$ and for any $v \in [v_i, v_i^i)$,

$$\mathbb{P}(a_j \leq v) = \frac{c}{\psi}. \quad (28)$$

**Lemma 11.** There is no mixed strategy equilibrium.

**Proof.** Suppose that there is a mixed strategy equilibrium. If $v_i \geq N_{min}$ Pick any $N_{min} \leq v_i' < v_i''$ in $I_z^i$. Since $F_N$ strictly increases,

$$\mathbb{P}(N_j \leq v_i'') > \mathbb{P}(N_j \leq v_i'),$$

so if it is the case that

$$\mathbb{P}(a_j \leq v_i'') = \mathbb{P}(\min((L_j + z_j)^+, N_j) \leq v_i'') = \mathbb{P}(\min((L_j + z_j)^+, N_j) \leq v_i') = \mathbb{P}(a_j \leq v_i'),$$

then it must be that

$$\mathbb{P}((L_j + z_j)^+ \leq v_i' \mid N_j \in (v_i', v_i'')] = 1.$$

Thus, $v_j \leq \max(v_i, N_{min}) - \max(L_j) < \max(v_i, N_{min})$ as $\mathbb{P}(L_j > 0) > 0$. If $v_j = \bar{v}_j$ (i.e. $z_j$ is degenerate), then $\mathbb{P}(\min((L_i + z_i)^+, N_i) \leq z_j) = 1 > \frac{c}{\psi}$. Thus, $v_j < \bar{v}_j$. By a similar argument,
\[ \nu_i < \max(\nu_j, N_{\text{min}}) \]. It follows that \( \nu_i < N_{\text{min}} \) and \( \nu_j < N_{\text{min}} \). Now if \( \nu_i \leq \nu_j \),

\[ P(a_j \leq \nu_i) = P(L_j + z_j \leq \nu_i) \leq P(L_j \leq 0) < \frac{c}{\psi}. \]

This contradicts Eq. (28), so \( \nu_i > \nu_j \), but a symmetric argument shows that \( \nu_i < \nu_j \), contradiction. Thus, \( I_i^j \) and \( I_j^i \) must be singletons.

It is easy to check that when \( P(L_j \leq 0) = c/\psi \), there are infinitely many equilibrium strategy profiles of the form \( a_i = \min((L_i + \alpha)^+, N_i) \) for any \( \alpha \in [0, N_{\text{min}}) \).

**Lemma 12.** Suppose \( L_i \) and \( L_j \) are arbitrarily distributed. There is a unique equilibrium to the payment game when

1. \( P(L_i \leq 0) > c/\psi \) and \( P(L_j \leq 0) > c/\psi \), in which case \( \alpha_i = \alpha_j = 0 \);
2. \( P(L_i \leq 0) < c/\psi \) and \( P(L_j \leq 0) < c/\psi \), in which case \( \alpha_i > N_{\text{min}} \) and \( \alpha_j > N_{\text{min}} \).

**Proof.** Suppose that \( P(L_i \leq 0) > c/\psi \) and \( P(L_j \leq 0) > c/\psi \). If \( \alpha_i \geq \alpha_j \) and \( \alpha_i > 0 \), then

\[ P(L_j + \alpha_j \leq \alpha_i) \geq P(L_j \leq 0) > \frac{c}{\psi}. \]

Bank \( i \) can be better off to deviate by making payment \( \min(L_i^+, N_i) \). Thus, in this case both \( \alpha_i = \alpha_j = 0 \). Suppose that \( P(L_i \leq 0) < c/\psi \) and \( P(L_j \leq 0) < c/\psi \). Without loss of generality, assume \( \alpha_i \geq \alpha_j \). If \( \alpha_j < N_{\text{min}} \), then

\[ P(\min((L_i + \alpha_i)^+, N_i) \leq \alpha_j) = P(L_i \leq \alpha_j - \alpha_i) < \frac{c}{\psi}, \]

a contradiction to Lemma 9. Thus, \( \alpha_j > N_{\text{min}} \) and \( \alpha_i > N_{\text{min}} \). It can be easily checked that this case there can be only one solution to Eq. (27). \( \square \)
E.2 Proof of Lemma 2

Let $F_{a_j}(o) = \mathbb{P}(a_j^* \leq o)$ where $a_j^*$ is bank $j$’s equilibrium strategy, then

$$V(L) = \int_{n \in (L^+, (L + \alpha)^+]} \int_{o \in [0, n-L]} -\psi(n - L - o)dF_{a_j}(o)dF_N(n) + \int_{n \in ((L + \alpha)^+, \infty)} \int_{o \in [0, (L + \alpha)^+ - L]} -\psi((L + \alpha)^+ - L - o) \, dF_{a_j}(o)dF_N(n) + \int_{n \in ((L + \alpha)^+, \infty)} -c(n - (L + \alpha)^+) \, dF_N(n)$$

Let $\epsilon$ be a small positive real number.

$$V(L - \epsilon) = \int_{n \in ((L - \epsilon)^+, (L - \epsilon + \alpha)^+]} \int_{o \in [0, n-(L-\epsilon)]} -\psi(n - (L - \epsilon) - o)dF_{a_j}(o)dF_N(n) + \int_{n \in ((L - \epsilon + \alpha)^+, \infty)} \int_{o \in [0, (L - \epsilon + \alpha)^+ -(L-\epsilon)]} -\psi((L - \epsilon + \alpha)^+ - (L - \epsilon) - o) \, dF_{a_j}(o)dF_N(n) + \int_{n \in ((L - \epsilon + \alpha)^+, \infty)} -c(n - (L - \epsilon + \alpha)^+) \, dF_N(n)$$

Direct calculation yields

$$\Gamma(L) = \lim_{\epsilon \to 0} \frac{V(L) - V(L - \epsilon)}{\epsilon}$$

$$= \lim_{\epsilon \to 0} \int_{n \in (L^+, (L - \epsilon + \alpha)^+]} \int_{o \in [0, n-L]} \psi \, dF_{a_j}(o)dF_N(n)$$

$$+ \mathbb{P}(a_j \leq (L + \alpha)^+ - L) \mathbb{P}(N = (L + \alpha)^+) \lim_{\epsilon \to 0} \frac{(L - \epsilon + \alpha)^+ - (L + \alpha)^+ + \epsilon}{\epsilon}$$

$$+ \lim_{\epsilon \to 0} \mathbb{P}(N = (L + \alpha)^+)((L + \alpha)^+ - (L - \epsilon + \alpha)^+)$$

$$+ \lim_{\epsilon \to 0} \int_{n \in ((L + \alpha)^+, \infty)} \frac{\psi((L - \epsilon + \alpha)^+ - (L + \alpha)^+ + \epsilon) F_{a_j}((L + \alpha)^+ - L)}{\epsilon}$$

Which can be further simplified to the desired results.
E.3 Proofs of results in Section 4.2.1

I first state an important lemma that links the endogenous distribution of \( L_i \) and \( L_j \) with the exogenous state variables, given the payment subgame strategy profile.

**Lemma 13.** Fix the payment subgame equilibrium strategy profile \( \{a^*_i = \min((L_i + \alpha_i)^+, N_i)\} \). For any \( o \in \mathbb{R} \), and reserve balances after trade \( L_j = R_j - Q - S_j \) possible in equilibrium,

\[
L_j < o \iff R_j - Q - D_j + \frac{\Gamma_j(o, \alpha_j, \alpha_i)}{\xi} < o \\
L_j \geq o \iff R_j - Q - D_j + \frac{\Gamma_j(o, \alpha_j, \alpha_i)}{\xi} \geq o.
\]

Let \( \Gamma_j^+(o, \alpha_j, \alpha_i) = \lim_{x \downarrow o} \Gamma_j(x, \alpha_j, \alpha_i) \), then

\[
\mathbb{P}(L_j \leq o) = \mathbb{P}(R_j - Q - D_j + \frac{\Gamma_j^+(o, \alpha_j, \alpha_i)}{\xi} \leq o) = \mathbb{E} \left[ F_R \left( D_j + Q - \frac{\Gamma_j^+(o, \alpha_j, \alpha_i)}{\xi} \right) \right].
\]

**Proof.** The proof is identical to the proof of Lemma 4 after we set \( \lambda^{-1} = 0 \) in the proof of Lemma 4.

Comparing Lemma 13 and Lemma 4, we see same results except that in the competitive case, we omit the term \( \lambda^{-1} \). The information rent \( \lambda^{-1} \) for borrower is zero under competitive pricing, but is positive under monopolistic screening, as discussed in Appendix C.1. In fact: the case of competitive pricing where bank \( i \) and borrower \( i \) acts as price takers in Section 4.2.1 is a simpler version of case of monopolistic screening.

**Proof of Theorems 1 to 3.** Rewrite \( \lambda^{-1} \) as zero in Appendices E.6 to E.8. The rest of the proofs are exactly the same.

For the proofs of Theorems 4 and 5, I abuse notations and write \( \Gamma(y, \alpha, \alpha) \) as \( \Gamma(y ; m) \) to denote the marginal value of liquidity at \( y \) when the liquidity stress index is \( m \) under some macroeconomic condition \( \mathcal{M}_C^m \). Under macroeconomic condition \( \mathcal{M}_C^0 \), there could be multiple equilibria. For the following analysis, we select the equilibrium with the \( \alpha = N_{\text{min}} \), because this equilibrium is the limiting equilibrium of a sequence of economies under no hoarding condition such that \( m \uparrow 0 \).

**Proof of Theorem 4.** It suffices to show that for all small \( \epsilon^m > 0 \), there is some \( \delta_0(\zeta, s) \geq 0 \) and
constant $O^1 \geq 0$ such that

$$\Gamma(\zeta - Q - s ; \epsilon^m) - \Gamma(\zeta - Q - s ; 0) > \delta_0(\zeta, s) - O^1 \epsilon^m.$$  

In addition, whenever $\zeta - Q - s \neq 0$, $\delta_0(\zeta, s) > 0$. Recall that under no hoarding condition, $a_j^* = \min((L_j + \alpha)^+, N_j)$ for some $\alpha \geq N_{\min}$. Under no hoarding condition, $a_j^* = \min(L_j^+, N_j)$. Note that $\Gamma(y ; 0) \geq \Gamma(y ; \epsilon)$ for any $\epsilon$. We discuss $\Gamma(y ; \epsilon) - \Gamma(y ; 0)$ case by case. First, when $y \geq 0$, by directly calculation the assumption that two macroeconomic conditions are close with respect to liquidity stress index, there is some constant $O^3 > 0$ such that

$$\Gamma(y ; \epsilon^m) - \Gamma(y ; 0) \geq \int_{y}^{y+\alpha} c - \psi \mathbb{P}(\min\{L_j + \alpha, N_j\} \leq n - y) dF_N(n) - O^3 \epsilon^m.$$  

When $y \geq N_{\min}$,

$$\int_{y}^{y+\alpha} c - \psi \mathbb{P}(\min\{L_j + \alpha, N_j\} \leq n - y) dF_N(n)$$

$$= \int_{y}^{y+\frac{N_{\min}}{2}} c - \psi \mathbb{P}(\min\{L_j + \alpha, N_j\} \leq n - y) dF_N(n) + \int_{y+\frac{N_{\min}}{2}}^{y+\alpha} c - \psi \mathbb{P}(\min\{L_j + \alpha, N_j\} \leq n - y) dF_N(n)$$

$$\geq \int_{y}^{y+\frac{N_{\min}}{2}} c - \psi \mathbb{P}(L_j + \alpha \leq \frac{N_{\min}}{2}) dF_N(n) + \int_{y+\frac{N_{\min}}{2}}^{y+\alpha} c - \psi \mathbb{P}(\min\{L_j + \alpha, N_j\} \leq n - y) dF_N(n).$$

Since $c - \psi \mathbb{P}(\min\{L_j + \alpha, N_j\} \leq \alpha) = 0$ and $\mathbb{P}(\min\{L_j + \alpha, N_j\} \leq \theta)$ strictly increases in $\theta$, there is some $\delta_1(\zeta, y)$ such that

$$\int_{y}^{y+\frac{N_{\min}}{2}} c - \psi \mathbb{P}(L_j + \alpha \leq \frac{N_{\min}}{2}) dF_N(n) > \delta_1(\zeta, y) > 0,$$

and

$$\int_{y+\frac{N_{\min}}{2}}^{y+\alpha} c - \psi \mathbb{P}(\min\{L_j + \alpha, N_j\} \leq n - y) dF_N(n) \geq 0.$$
Thus,
\[
\int_{y}^{y+\alpha} c - \psi \mathbb{P}(\min\{(L_j + \alpha)^+, N_j\} \leq n - y) \, d F_N(n) > \delta_1(\zeta, y) > 0,
\]

Hence, when \( y \geq N_{\min} \),
\[
\Gamma(y ; e^m) - \Gamma(y ; 0) > \delta_1(\zeta, y) - \mathcal{O}^3 e^m.
\]

When \( N_{\min} > y > 0 \),
\[
\Gamma(y ; e^m) - \Gamma(y ; 0) \geq \int_{N_{\min}}^{y+\alpha} c - \psi \mathbb{P}(\min\{(L_j + \alpha)^+, N_j\} \leq n - y) \, d F_N(n) - \mathcal{O}^3 e^m
\]
\[
> \int_{N_{\min}}^{y+\alpha} c - \psi \mathbb{P}(\min\{(L_j + \alpha)^+, N_j\} \leq n - N_{\min}) \, d F_N(n) - \mathcal{O}^3 e^m.
\]

When \( y = 0 \), clearly \( \Gamma(y ; e^m) - \Gamma(y ; 0) \geq -\mathcal{O}^3 e^m \). When \( 0 > y \geq -\alpha \), \( \exists \mathcal{O}^4 > 0 \) such that
\[
\Gamma(y ; e^m) - \Gamma(y ; 0)
\]
\[
\geq \psi \mathbb{P}^m(\min(L_j^+, N_j) \leq -y) - \psi \int_{0}^{y+\alpha} \mathbb{P}(a_j^* \leq n - y) \, d F_N(n) - c \mathbb{P}(N_i > y + \alpha) - \mathcal{O}^4 e^m
\]
\[
> \delta_2(\zeta, y) + c - \psi \int_{0}^{y+\alpha} \mathbb{P}(a_j^* \leq n - y ; T + \epsilon) \, d F_N(n) - c \mathbb{P}(N_i > y + \alpha) - \mathcal{O}^4 e^m > \delta_2(\zeta, y) - \mathcal{O}^4 e^m.
\]

where the second inequality derives from the fact that under liquidity hoarding condition, \( \psi \mathbb{P}^m(L_j \leq 0) > c \). Finally, when \( y < -\alpha \), then
\[
\Gamma(y ; e^m) - \Gamma(y ; 0) = \psi \mathbb{P}^m(\min(L_j^+, N_j) \leq -y) - \psi \mathbb{P}(\min((L_j + \alpha)^+, N_j) \leq -y)
\]
\[
> \psi \mathbb{P}^m(\min(L_j^+, N_j) \leq -y) - \psi \mathbb{P}(\min(L_j^+, N_j) \leq -y)
\]
\[
+ \psi \left( \mathbb{P}(\min((L_j + \alpha)^+, N_j) \leq -y) - \psi \mathbb{P}(\min(L_j^+, N_j) \leq -y) \right)
\]
\[
> \psi \left( \mathbb{P}(\min((L_j + \alpha)^+, N_j) \leq -y) - \psi \mathbb{P}(\min((L_j)^+, N_j) \leq -y) \right) > \delta_3(\zeta, y)
\]

for some \( \delta_3(\zeta, y) > 0 \) regardless of \( \epsilon \). Let \( \delta_0(\zeta, \zeta - Q - y) \) be the corresponding \( \delta_i(\zeta, y) \) in each case corresponding to different \( y \). This finishes the proof. \( \Box \)
Proof of Theorem 5. Based on the proof above, for all small \( \epsilon^m \), there is some \( \delta_0(\zeta, S^*(\zeta, \mathcal{D}, 0)) \geq 0 \) and constant \( \mathcal{O}^1 \geq 0 \) such that \( \Gamma(\zeta - Q - S^*(\zeta, \mathcal{D}, 0) ; \epsilon^m) - \Gamma(\zeta - Q - S^*(\zeta, \mathcal{D}, 0) ; 0) > \delta_0(\zeta, S^*(\zeta, \mathcal{D}, \epsilon^m)) - \mathcal{O}^1 \epsilon^m \) and \( \delta_0(\zeta, S^*(\zeta, \mathcal{D}, 0)) > 0 \). For \( \epsilon^m \) small enough, \( \exists \delta_1(\zeta, \mathcal{D}) > 0 \) such that

\[
\Gamma_i(\zeta - Q - S^*(\zeta, \mathcal{D}, 0) ; \epsilon^m) + \xi S^*(\zeta, \mathcal{D}, 0) > \Gamma_i(\zeta - Q - S^*(\zeta, \mathcal{D}, 0) ; 0) + \xi S^*(\zeta, \mathcal{D}, 0) + \delta_1(\zeta, \mathcal{D}).
\]

From Eq. (4), this implies that

\[
S^*(\zeta, \mathcal{D}, 0) - S^*(\zeta, \mathcal{D}, \epsilon^m) > \delta(\zeta, \mathcal{D})
\]

for some \( \delta(\zeta, \mathcal{D}) > 0 \). Since \( r^*(\zeta, \mathcal{D}, \epsilon^m) = \xi(\mathcal{D} - S^*(\zeta, \mathcal{D}, \epsilon^m)) \), the result follows. \( \Box \)

E.4 Proof of Lemma 3

First consider the following relaxed version of the problem for bank \( i \). Suppose that bank \( i \) designs a mechanism for selecting the transaction quantity \( S_i \) and repo rates \( r_i \). By the revelation principle (Myerson, 1986), we can focus on direct mechanisms without loss of generality. I will characterize the optimal direct mechanism and show that the optimal direct mechanism can be implemented by a supply schedule.

The type space of a repo borrower is \([D_{min}, \infty)\). A direct mechanism consists of functions \( Q : [D_{min}, \infty) \rightarrow \mathbb{R} \) and \( T : [D_{min}, \infty) \rightarrow \mathbb{R} \). The direct mechanism design problem of bank \( i \) is

\[
\sup_{Q, T} \mathbb{E}[V(R_i - Q - Q(D)) + T(D)],
\]

subject to the incentive-compatibility (IC) constraint

\[
-\frac{\xi}{2} ((D - Q(D))^+)^2 - T(D) \geq -\frac{\xi}{2} ((D - Q(\theta))^+)^2 - T(\theta), \quad (D, \theta) \in [D_{min}, \infty)^2,
\]

and the individually rational (IR) constraint

\[
-\frac{\xi}{2} ((D - Q(D))^+)^2 - T(D) \geq -\frac{\xi}{2} D^2.
\]

Lemma 14. Under any optimal contract, \( Q(D) \leq D \) for all \( D \) in \([D_{min}, \infty)\).

Proof. When \( Q(D) > D \) for some \( D \), bank \( i \) can set \( Q(D) = D \) without changing the cost of
the repo borrower, thus respecting the IC constraints of all types. However, this would weakly increase the payoff of bank $i$. □

By Lemma 14, we can focus without loss of generality on a mechanism $(Q, T)$ satisfying $Q(D) \leq D$. In the following lemma, I characterize the set of allocations and payment strategies that satisfy the IC constraint. This simplifies the subsequent analysis.

**Lemma 15.** For any mechanism $(Q, T)$, any incentive compatible allocation rule $Q$ is weakly increasing with $D$.

**Proof.** Consider two borrower types $D$ and $D'$ with $D > D'$. Incentive compatibility requires that

$$-rac{\xi}{2} ((D - Q(D))^+)^2 - T(D) \geq -\frac{\xi}{2} ((D - Q(D'))^+)^2 - T(D')$$

and that

$$-\frac{\xi}{2} ((D' - Q(D'))^+)^2 - T(D') \geq -\frac{\xi}{2} ((D' - Q(D))^+)^2 - T(D).$$

Adding these inequalities, we see that

$$-\frac{\xi}{2} ((D - Q(D))^+)^2 - \frac{\xi}{2} ((D' - Q(D'))^+)^2 \geq -\frac{\xi}{2} ((D' - Q(D))^+)^2 - \frac{\xi}{2} ((D - Q(D'))^+)^2.$$ 

Rearranging and invoking Lemma 14, we get

$$\left( (D' - Q(D))^+ + (D - Q(D)) \right) \left( D - Q(D) - (D' - Q(D))^+ \right) \leq (D' + D - 2Q(D')) (D - D'), \tag{29}$$

If $Q(D) < Q(D')$, then $Q(D) < D'$ and

$$\left( (D' - Q(D))^+ + (D - Q(D)) \right) \left( D - Q(D) - (D' - Q(D))^+ \right) = \left( D' - Q(D) + D - Q(D) \right) (D - D') > (D' + D - 2Q(D')) (D - D'),$$

contradicting inequality (29). Thus, it is necessary that $Q(D) \geq Q(D')$. □

Let $u : \mathbb{R} \rightarrow \mathbb{R}$ be the value function of a repo borrower under a given mechanism $(Q, T)$,
in that

\[ u(D) \overset{\text{def}}{=} \sup_{\theta \in [D_{\text{min}}, \infty)} -\frac{\xi}{2} \left( (D - Q(\theta))^+ \right)^2 - T(\theta). \]

**Lemma 16.** Any incentive compatible truthful mechanism \((Q, T)\) must satisfy

\[ u(D) = -\frac{\xi}{2} \left( (D - Q(D))^+ \right)^2 - T(D) = u(D_{\text{min}}) - \int_{D_{\text{min}}}^{D} \xi(x - Q(x)) \, dx. \]

**Proof.** By Lemma 14, we can focus on an IC mechanism \((Q, T)\) with \(Q(D) \leq D\). We first show that \(u\) is absolutely continuous on any bounded interval. Let \(M > 0\). We claim that \(u(\cdot)\) is absolutely continuous on \((-M, M)\). Indeed, a truthful incentive compatible mechanism must satisfy

\[ u(D) = -\frac{\xi}{2} \left( (D - Q(D))^+ \right)^2 - T(D) \geq -\frac{\xi}{2} \left( (D - Q(\theta))^+ \right)^2 - T(\theta) \]

\[ u(\theta) = -\frac{\xi}{2} \left( (\theta - Q(\theta))^+ \right)^2 - T(\theta) \geq -\frac{\xi}{2} \left( (\theta - Q(D))^+ \right)^2 - T(D), \]

whenever \(-M < \theta < D < M\). Thus,

\[ u(D) - u(\theta) \leq -\frac{\xi}{2} \left( (D - Q(D))^+ \right)^2 + \frac{\xi}{2} \left( (\theta - Q(D))^+ \right)^2 = \frac{\xi}{2} \left( (\theta - Q(D))^+ - (D - Q(D)) \right) \left( (\theta - Q(D))^+ + (D - Q(D)) \right) \leq 0 \]

and

\[ u(D) - u(\theta) \geq -\frac{\xi}{2} \left( (D - Q(\theta))^+ \right)^2 + \frac{\xi}{2} \left( (\theta - Q(\theta))^+ \right)^2 = \frac{\xi}{2} \left( \theta - D \right) \left( \theta + D - 2Q(D) \right). \]

Thus,

\[ \left| \frac{u(D) - u(\theta)}{D - \theta} \right| \leq \frac{\xi}{2} \left( \theta + D - 2Q(D) \right) \leq \frac{\xi}{2} \left( 2M - 2Q(-M) \right), \]

where the last inequality holds due to the monotonicity of \(Q\) shown in Lemma 15. The above inequality holds true whenever \(-M < \theta < D < M\).

Finally, since the above argument holds for arbitrary \(M > 0\), the envelope theorem (Milgrom and Segal, 2002) implies that for all \(D\), \(u'(D) = -\xi(D - Q(D))\). By the fundamental theorem of
calculus,
\[-\frac{\xi}{2} \left( (D - Q(D))^+ \right)^2 - T(D) = u(D) = u(D_{\text{min}}) - \int_{D_{\text{min}}}^{D} \xi(x - Q(x)) \, dx.\]

**Lemma 17.** A direct mechanism \((Q, T)\) satisfying \(Q(D) \leq D\) for all \(D\) is incentive-compatible if and only if

1. \(Q(D)\) is weakly increasing in \(D\).
2. \(T(D) = -\frac{\xi}{2} \left( (D - Q(D))^+ \right)^2 - u(D_{\text{min}}) + \int_{D_{\text{min}}}^{D} \xi(x - S(x)) \, dx.\)

**Proof.** We just need to show the “if” part. We need to show the IC condition
\[u(D) \geq -\frac{\xi}{2} \left( (D - Q(D))^+ \right)^2 - T(\theta).\]

Substituting \(T(D)\), the IC condition holds if and only if
\[-\int_{\theta}^{D} \xi(x - Q(x)) \, dx \geq -\frac{\xi}{2} \left( (D - Q(\theta))^+ \right)^2 + \frac{\xi}{2} \left( (\theta - Q(\theta))^+ \right)^2,
\]
which holds since \(Q(D)\) is weakly increasing. \(\square\)

We now turn to a bank’s problem. The bank will take \(D\) as random. Since \(T\) is determined by \(Q\), we substitute \(T(D)\) from Lemma 3 to get
\[\sup_{Q} \mathbb{E} \left[ V(R_0 - Q - Q(D)) - \frac{\xi}{2} \left( (D - Q(D))^+ \right)^2 - u(D_{\text{min}}) + \int_{D_{\text{min}}}^{D} \xi(x - Q(x)) \, dx \right],\]subject to the condition that \(Q(D)\) is weakly increasing.
Using standard trick of integration by part as in Börgers (2015), we get

\[
\begin{align*}
\mathbb{E} \left[ V(R_0 - Q - Q(D)) - \frac{\xi}{2} (D - Q(D))^2 - u(D_{\min}) + \int_{D_{\min}}^{D} \xi(x - Q(x)) \, dx \right] \\
= \mathbb{E} \left[ V(R_0 - Q - Q(D)) - \frac{\xi}{2} (D - Q(D))^2 - u(D_{\min}) \right] + \int_{D_{\min}}^{D} \xi(x - Q(x)) \, dx f_D(D) \, dD \\
= \mathbb{E} \left[ V(R_0 - Q - Q(D)) - \frac{\xi}{2} (D - Q(D))^2 - u(D_{\min}) \right] + \int_{D_{\min}}^{\infty} \int_{x}^{D} \xi(x - Q(x)) f_D(D) \, dD \, dx \\
= \mathbb{E} \left[ V(R_0 - Q - Q(D)) - \frac{\xi}{2} (D - Q(D))^2 - u(D_{\min}) \right] + \int_{D_{\min}}^{\infty} \xi(x - Q(x)) \frac{1 - F_D(x)}{f_D(x)} f_D(x) \, dx \\
= \mathbb{E} \left[ V(R_0 - Q - Q(D)) - \frac{\xi}{2} (D - Q(D))^2 - u(D_{\min}) + \xi(D - Q(D)) \frac{1 - F_D(D)}{f_D(D)} \right].
\end{align*}
\]

Thus, the solution to (30) must satisfy

\[
\begin{align*}
Q(D_i) &= \inf \left\{ s : \Gamma(R_i - Q - s) \geq \xi D_i - s - \frac{1 - F_D(D_i)}{f_D(D_i)} \right\} \\
T(D_i) &= -\frac{\xi}{2} (D - Q(D))^2 + \frac{\xi}{2} D_{\min}^2 + \int_{D_{\min}}^{D_i} \xi(x - Q(x)) \, dx,
\end{align*}
\]

where \( F_D \) is the cumulative distribution function of \( D \). The mapping from \( D \) to \( D - \frac{1 - F_D(D)}{f_D(D)} \) is strictly increasing by the assumed form of the density \( f(\cdot) \). Thus, \( Q(\cdot) \) is weakly increasing. Finally, the IR condition is satisfied because

\[
u(D) = u(D_{\min}) - \int_{D_{\min}}^{D} \xi(x - Q(x)) \, dx = -\frac{\xi}{2} D_{\min}^2 - \int_{D_{\min}}^{D} \xi(x - Q(x)) \, dx \\
\geq -\frac{\xi}{2} D_{\min}^2 - \int_{D_{\min}}^{D} \xi x \, dx = -\frac{\xi}{2} D^2.
\]

Lemma 18. The repo game has a unique equilibrium. This unique equilibrium is fully separating. The outcome \((r_i, S_i)\) is the same as that implied by the direct mechanism \((Q, T)\) defined by Eq. (31).
Proof. The direct mechanism can be implemented by posting a schedule. This is implied by the taxation principle (Mussa and Rosen, 1978). Indeed, by the IC constraint, if \( Q(D) = Q(D') \), then \( T(D) = T(D') \). The mapping \( D \mapsto (Q(D), T(D)) \) generates a graph on \( \mathbb{R}^2 \) whose trace defines an associated supply schedule \( g \). More specifically, let \( Q^{-1} : Q([D_{\text{min}}, \infty)) \rightarrow \mathbb{R} \) be the inverse function of quantity defined by \( Q^{-1}(t) = \inf \{ \vartheta \in [D_{\text{min}}, \infty) : Q(\vartheta) = t \} \).

Then the supply schedule \( g \) is defined on \( Q([D_{\text{min}}, \infty)) \) by

\[
g(s) = \frac{T(Q^{-1}(s))}{s}.
\]

When \( s \notin Q([D_{\text{min}}, \infty)) \), define \( g(s) = \infty \). Since the direct mechanism \( (Q, T) \) maximizes the payoff of bank \( i \), the associated supply schedule \( g \) also maximizes the payoff of bank \( i \). This direct mechanism is dominant implementable. Thus, the repo game has a unique equilibrium. This unique equilibrium is separating. \( \square \)

### E.5 Proof of Lemma 4

Fix any \( o \in \mathbb{R} \). By Lemma 2, \( \Gamma_j(y, \alpha_j, \alpha_i) \) is weakly decreasing in \( y \). By Lemma 3, \( \Gamma_j(R_j - Q - S_j, \alpha_j, \alpha_i) \geq \xi(D_j - S_j - \lambda^{-1}) \) in equilibrium. Hence, when \( L_j = R_j - Q - S_j \geq o \),

\[
\Gamma_j(R_j - Q - S_j, \alpha_j, \alpha_i) \leq \Gamma_j(o, \alpha_j, \alpha_i) \Rightarrow D_j - S_j - \lambda^{-1} \leq \frac{\Gamma_j(o, \alpha_j, \alpha_i)}{\xi} \Rightarrow \\
R_j - Q - S_j \leq \frac{\Gamma_j(o, \alpha_j, \alpha_i)}{\xi} - D_j + R_j - Q + \lambda^{-1} \Rightarrow \\
o \leq L_i \leq \frac{\Gamma_j(o, \alpha_j, \alpha_i)}{\xi} - D_j + R_j - Q + \lambda^{-1}.
\]

Claim 1. Suppose \( S_j \) is the equilibrium trading quantity. Then for any \( S' < S_j \), \( \Gamma_j(R_j - Q - S', \alpha_j, \alpha_i) \leq \xi(D_j - S_j - \lambda^{-1}) \).

Suppose \( o \in \mathbb{R} \) such that \( L_j = R_j - Q - S_j < o \). Let \( S_o = R_j - Q - o < S_j = R_j - Q - L_j \). Claim 1 implies that \( \Gamma_j(o, \alpha_j, \alpha_i) = \Gamma_j(R_j - Q - S_o, \alpha_j, \alpha_i) \leq \xi(D_j - S_j - \lambda^{-1}) \). Thus,

\[
R_j - Q - D_j + \frac{\Gamma_j(o, \alpha_j, \alpha_i)}{\xi} + \lambda^{-1} \leq R_j - Q - S_j = L_j < o.
\]
Thus,
\[
L_j = R_j - Q - S_j \geq o \iff R_j - Q - D_j + \frac{\Gamma_j(a, \alpha_j, \alpha_i)}{\xi} + \lambda^{-1} \geq o
\]
\[
L_j = R_j - Q - S_j < o \iff R_j - Q - D_j + \frac{\Gamma_j(a, \alpha_j, \alpha_i)}{\xi} + \lambda^{-1} < o.
\]

Finally, since probability measure is continuous from above, \(\mathbb{P}(L_j \leq o) = \mathbb{P}(R_j - Q - D_j + \frac{\Gamma^+(a, \alpha_j, \alpha_i)}{\xi} + \lambda^{-1} \leq o)\).

**Proof of Claim 1.** By definition, \(S_j = \inf \left\{ s : \Gamma_j(R_j - Q - s, \alpha_j, \alpha_i) \geq \xi(D_j - s - \lambda^{-1}) \right\}\). Thus, \(\Gamma_j(R_j - Q - S_j, \alpha_j, \alpha_i) + \xi S_j \geq \xi(D_j - \lambda^{-1})\). In addition, for any \(S'' < S_j\),
\[
\Gamma_j(R_j - Q - S'', \alpha_j, \alpha_i) + \xi S'' < \xi(D_j - \lambda^{-1}) \Rightarrow \lim_{s \uparrow S_j} (\Gamma_j(R_j - Q - s, \alpha_j, \alpha_i) + \xi s) \leq \xi(D_j - \lambda^{-1}) \Rightarrow
\]
\[
\lim_{s \uparrow S_j} \Gamma_j(R_j - Q - s, \alpha_j, \alpha_i) + \xi S_j \leq \xi(D_j - \lambda^{-1}).
\]

Since \(\Gamma_j(y, \alpha_j, \alpha_i)\) is weakly decreasing in \(y\), for any \(S' < S_j\), \(\Gamma_j(R_j - Q - S', \alpha_j, \alpha_i) + \xi S_j \leq \xi(D_j - \lambda^{-1})\).

\[\Box\]

**E.6 Proof of Theorem 6**

Suppose that liquidity hoarding condition Eq. (13) holds for \(j \in \{1, 2\}\). We prove by contradiction that banks hoard liquidity in equilibrium at time 1. Assume that bank \(j\) does not hoard liquidity, i.e. bank \(j\) pays \(\min(L_j^+ + z_j, N_j)\) at time 1 for some non-degenerate random variable \(z_j \geq 0\). Let \(i \in \{1, 2\} \setminus \{j\}\). Let \(\mathcal{I}_j\) be the support of \(z_j\). Trivially, \(\mathcal{I}_j\) is bounded above. By Lemma 9, for any \(v_j \in \mathcal{I}_j \cap (0, \infty)\), either \(\mathbb{P}(a_i \leq v_j) = \frac{c}{\psi}\) or \(v_j = \inf \{\vartheta \geq 0, \mathbb{P}(a_i \leq \vartheta) > \frac{c}{\psi}\}\). Let \(V^{\prime}_{j,-}(y) = \mathbb{E}[\Gamma_j^+(y, z_j, z_i)]\) be the marginal value of liquidity for bank \(j\), where the expectation is taken over the random realizations of the mixed strategy profile. By Lemma 2, \(V^{\prime}_{j,-}(0) \leq c\).

Then by Lemma 4 and Eq. (13),
\[
\mathbb{P}(L_j \leq 0) = \mathbb{P}\left(\frac{R_j - D_j - Q + \lambda^{-1} \leq -\frac{V^{\prime}_{j,-}(0)}{\xi}}{\xi}\right) > \mathbb{P}\left(\frac{R_j - D_j - Q + \lambda^{-1} \leq -\frac{c}{\xi}}{\xi}\right) = \frac{c}{\psi}.
\]

If bank \(i\) does not hoard liquidity either, then \(\mathbb{P}(L_i \leq 0) > \frac{c}{\psi}\) as well. However, as shown in Lemma 12, when \(\mathbb{P}(L_i \leq 0) > \frac{c}{\psi}\) and \(\mathbb{P}(L_j \leq 0) > \frac{c}{\psi}\), banks must both hoard liquidity, a contradiction. If bank \(i\) hoards liquidity but bank \(j\) does not hoard liquidity, then \(\mathbb{P}(L_j \leq 0) > \frac{c}{\psi}\). Since bank \(j\) is best responding, it must still hold that for any \(v_j \in \mathcal{I}_j \cap (0, \infty)\), either
\( \mathbb{P}(a_i \leq v_j) = \frac{c}{\psi} \), or \( \mathbb{P}(a_i \leq v_j) > \frac{c}{\psi} \) and \( v_j = \inf\{\vartheta \geq 0, \mathbb{P}(a_j \leq \vartheta) > \frac{c}{\psi}\} \). Note that

\[
\mathbb{P}(a_i \leq v_j) = \mathbb{P}(\min((L_i)^+, N_i) \leq v_j) \geq \mathbb{P}(L_i \leq v_j) = \mathbb{P}
\left( R_i - D_i - Q + \lambda^{-1} \leq v_j - \frac{V'_{i-}(v_j)}{\xi} \right).
\]

From Lemma 2, \( V'_{i-}(v_j) \leq c \) when \( v_j > 0 \). Thus,

\[
\mathbb{P}(L_i \leq v_j) = \mathbb{P}
\left( R_i - D_i - Q + \lambda^{-1} \leq v_j - \frac{V'_{i-}(v_j)}{\xi} \right) \geq \mathbb{P}
\left( R_i - D_i - Q + \lambda^{-1} \leq -\frac{c}{\xi} \right) > \frac{c}{\psi}.
\]

If \( \mathbb{P}(a_i \leq v_j) = \frac{c}{\psi} \), then \( \frac{c}{\psi} > \frac{c}{\psi} \), a contradiction. If \( \mathbb{P}(a_i \leq v_j) > \frac{c}{\psi} \) and \( v_j = \inf\{\vartheta \geq 0, \mathbb{P}(a_j \leq \vartheta) > \frac{c}{\psi}\} \), then pick a very small \( \epsilon > 0 \). Since \( F_N(\cdot) \) is strictly increasing, \( \mathbb{P}(a_i \leq v_j) \geq \mathbb{P}(a_i \leq v_j - \epsilon) > \frac{c}{\psi} \), a contradiction.

### E.7 Proof of Theorem 7

Suppose that the no hoarding condition Eq. (14) holds for \( j \in \{1, 2\} \). By Lemma 4, the assumption \( \mathbb{P}(R_i - Q - D_{min} > 0) > 0 \) and Lemma 1, no banks will not play mixed strategy in the payment subgame in equilibrium. Furthermore, bank \( i \) pays \( a_i = \min((L_i + \alpha_i)^+, N_i) \) and bank \( j \) pays \( a_j = \min((L_j + \alpha_j)^+, N_j) \) and

\[
\alpha_i = \inf\{\vartheta \geq 0, \mathbb{P}(a_j \leq \vartheta) \geq \frac{c}{\psi}\}, \quad \alpha_j = \inf\{\vartheta \geq 0, \mathbb{P}(a_i \leq \vartheta) \geq \frac{c}{\psi}\}.
\]

Our first goal is to show that the real numbers \( \alpha_i, \alpha_j \geq N_{min} \). First, by Lemma 2 and Definition 1,

\[
\Gamma_j(\alpha_i - \alpha_j, \alpha_j, \alpha_i) = \left\{ \begin{array}{ll}
\int_{n \in ((\alpha_i - \alpha_j)^+, \alpha_i) \cap [\alpha_i, \infty)} \psi \mathbb{P}(\min((L_i + \alpha_i)^+, N_i) \leq n + \alpha_j - \alpha_i) dF_N(n) + \int_{n \in [\alpha_i, \infty)} cdF_N(n), & \text{if } \alpha_i > 0; \\
\psi \mathbb{P}(\min((L_i + \alpha_i)^+, N_i) \leq \alpha_j - \alpha_i), & \text{if } \alpha_i \leq 0.
\end{array} \right.
\]

Thus, if \( \alpha_i < N_{min}, \Gamma_j(\alpha_i - \alpha_j, \alpha_j, \alpha_i) \geq c. \) If \( \alpha_i \leq \alpha_j \), then Eq. (14) implies that

\[
\mathbb{P}(R_j - Q - D_j + \frac{\Gamma_j(\alpha_i - \alpha_j, \alpha_j, \alpha_i)}{\xi} + \lambda^{-1} \leq \alpha_i - \alpha_j) < \mathbb{P}(R_j - Q - D_j + \frac{c}{\xi} + \lambda^{-1} \leq 0) < \frac{c}{\psi}.
\]

Recall that bank \( i \) is optimizing in the payment subgame: by Lemma 9 and Lemma 4,

\[
\mathbb{P}(a_j \leq \alpha_i) = \mathbb{P}(L_j + \alpha_j \leq \alpha_i) = \mathbb{P}(R_j - Q - D_j + \frac{\Gamma_j(\alpha_i - \alpha_j, \alpha_j, \alpha_i)}{\xi} + \lambda^{-1} \leq \alpha_i - \alpha_j) \geq \frac{c}{\psi}.
\]

This is a contradiction. Thus, \( \alpha_i < N_{min} \Rightarrow \alpha_i > \alpha_j. \) It further implies that when \( \alpha_i < N_{min}, \alpha_j < N_{min}. \) However, a symmetric argument shows that when \( \alpha_j < N_{min}, \alpha_j > \alpha_i, \) a contradiction.
Thus, \( \alpha_i \geq N_{\text{min}} \) and \( \alpha_j \geq N_{\text{min}} \).

Our next goal is to show \( \alpha_i = \alpha_j \). By Lemma 9 and Lemma 4,

\[
\frac{c}{\psi} \leq \mathbb{P}(a_i \leq \alpha_j) = \mathbb{P}(\min((L_i + \alpha_i)^+, N_i) \leq \alpha_j)
= \mathbb{P}(L_i \leq \alpha_j - \alpha_i) + \mathbb{P}(N_i \leq \alpha_j)(1 - \mathbb{P}(L_i \leq \alpha_j - \alpha_i))
= \mathbb{P}\left( R_i - D_i - Q + \lambda^{-1} \leq \alpha_j - \alpha_i - \frac{\Gamma_i(\alpha_j - \alpha_i, \alpha_i)}{\xi} \right) + \mathbb{P}(N_i \leq \alpha_j)\left(1 - \mathbb{P}\left( R_i - D_i - Q + \lambda^{-1} \leq \alpha_j - \alpha_i - \frac{\Gamma_i(\alpha_j - \alpha_i, \alpha_i)}{\xi} \right)\right)
\]

and

\[
\frac{c}{\psi} \leq \mathbb{P}(a_j \leq \alpha_i) = \mathbb{P}\left( R_j - D_j - Q + \lambda^{-1} \leq \alpha_i - \alpha_j - \frac{\Gamma_j(\alpha_i - \alpha_j, \alpha_j)}{\xi} \right) + \mathbb{P}(N_i \leq \alpha_i)\left(1 - \mathbb{P}\left( R_j - D_j - Q + \lambda^{-1} \leq \alpha_i - \alpha_j - \frac{\Gamma_j(\alpha_i - \alpha_j, \alpha_j)}{\xi} \right)\right).
\]

By Lemma 2 and Definition 1,

\[
\Gamma_j(\alpha_i - \alpha_j, \alpha_j, \alpha_i) = \int_{n \in ((\alpha_i - \alpha_j)^+, \alpha_i]} \psi \mathbb{P}(a_i \leq n - (\alpha_i - \alpha_j)) dF_N(n) + \int_{n \in [\alpha_i, \infty)} cdF_N(n),
\]

\[
\Gamma_i(\alpha_j - \alpha_i, \alpha_i, \alpha_j) = \int_{n \in ((\alpha_j - \alpha_i)^+, \alpha_j]} \psi \mathbb{P}(a_j \leq n - (\alpha_j - \alpha_i)) dF_N(n) + \int_{n \in [\alpha_j, \infty)} cdF_N(n).
\]

**Lemma 19.** Suppose that \( (R_i, D_i, N_i) \) and \( (R_j, D_j, N_j) \) have the same distribution. If in equilibrium bank \( i \) pays \( a_i = \min((L_i + \alpha_i)^+, N_i) \) and bank \( j \) pays \( a_j = \min((L_j + \alpha_j)^+, N_j) \) in the payment subgame and \( \alpha_j > \alpha_i \), then

\[
\mathbb{P}(a_j \leq n) \leq \mathbb{P}(a_i \leq n).
\]

for all \( n \in [N_{\text{min}}, \alpha_i) \).

The proof of this lemma is very technical and is postponed in Appendix F.1. In nontechnical terms, when \( \alpha_j > \alpha_i \) bank \( j \) pays more than bank \( i \). Therefore, \( \Gamma_j(y, \alpha_j, \alpha_i) \) is larger than \( \Gamma_i(y, \alpha_i, \alpha_j) \) for \( y \) in some relevant range. Thus, bank \( j \) quotes higher funding rates and lends out less liquidity in the funding market, so \( L_j \) is higher than \( L_i \). This confirms that \( a_j = \min((L_j + \alpha_j)^+, N_j) \) is larger than \( a_i = \min((L_i + \alpha_i)^+, N_i) \), so \( \mathbb{P}(a_j \leq n) \leq \mathbb{P}(a_i \leq n) \).

With Lemma 19 we are ready to show that \( \alpha_j - \alpha_i = 0 \). Suppose that \( \alpha_j - \alpha_i > 0 \). By
Lemma 19,

\[ \Gamma_i(0, \alpha_i, \alpha_j) = \int_{n \in [N_{\min}, \alpha_i]} \psi P(a_j \leq n) \, dF_N(n) + \int_{n \in [\alpha_i, \infty)} c \, dF_N(n) \]

\[ \leq \int_{n \in [N_{\min}, \alpha_i]} \psi P(a_i \leq n) \, dF_N(n) + \int_{n \in [\alpha_i, \infty)} c \, dF_N(n) \]

\[ \leq \int_{n \in [N_{\min}, \alpha_i]} \psi P(a_i \leq n + (\alpha_j - \alpha_i)) \, dF_N(n) + \int_{n \in [\alpha_i, \infty)} c \, dF_N(n) \]

\[ = \Gamma_j(\alpha_i - \alpha_j, \alpha_j, \alpha_i) \]

Therefore,

\[ \mathbb{P} \left( R_i - D_i - Q + \lambda^{-1} \leq -\frac{\Gamma_i(0, \alpha_i, \alpha_j)}{\xi} \right) \geq \mathbb{P} \left( R_j - D_j - Q + \lambda^{-1} \leq \alpha_i - \alpha_j - \frac{\Gamma_j(\alpha_i - \alpha_j, \alpha_j, \alpha_i)}{\xi} \right). \]

This implies that bank \( j \) can deviates to choose to pay \( a_j' = \min((L_j + \alpha_i)^+, N_j) \) and still satisfies

\[ \mathbb{P}(a_i \leq \alpha_i) = \mathbb{P}(\min((L_i + \alpha_i)^+, N_i) \leq \alpha_i) \]

\[ = \mathbb{P} \left( R_i - D_i - Q + \lambda^{-1} \leq -\frac{\Gamma_i(0, \alpha_i, \alpha_j)}{\xi} \right) \]

\[ + \mathbb{P}(N_i \leq \alpha_i) \left( 1 - \mathbb{P} \left( R_i - D_i - Q + \lambda^{-1} \leq -\frac{\Gamma_i(0, \alpha_i, \alpha_j)}{\xi} \right) \right) \]

\[ \geq \mathbb{P} \left( R_j - D_j - Q + \lambda^{-1} \leq \alpha_i - \alpha_j - \frac{\Gamma_j(\alpha_i - \alpha_j, \alpha_j, \alpha_i)}{\xi} \right) \]

\[ + \mathbb{P}(N_i \leq \alpha_i) \left( 1 - \mathbb{P} \left( R_j - D_j - Q + \lambda^{-1} \leq \alpha_i - \alpha_j - \frac{\Gamma_j(\alpha_i - \alpha_j, \alpha_j, \alpha_i)}{\xi} \right) \right) \]

\[ = \mathbb{P}(a_j' \leq \alpha_i) \geq \frac{c}{\psi}. \]

In addition, \( a_i < a_j \Rightarrow \mathbb{P}(a_i \leq \alpha_i) < \mathbb{P}(a_i \leq a_j) \). This contradicts with Lemma 9.

Similarly, it cannot be the case that \( \alpha_i > \alpha_j \). Thus, \( \alpha_i = \alpha_j \). Let \( \alpha_i = \alpha_j = \alpha \). Lemma 19 also implies that \( \Gamma_i = \Gamma_j = \Gamma \) for some function \( \Gamma \). To sum up, the value of \( \alpha \) and the function \( \Gamma \) are jointly determined in the following way:
1. Fix the function $\Gamma$ in equilibrium. Then $\alpha$ satisfies

$$\alpha = \inf \left\{ \theta \geq N_{\min}, \mathbb{P}\left(R_i - D_i - Q + \lambda^{-1} \leq -\frac{\Gamma(0, \alpha, \alpha)}{\xi}\right) + \mathbb{P}(N_i \leq \theta) \right\} \geq \frac{c}{\psi}.$$

When $F_N$ is atomless, then

$$\mathbb{P}(N_i \leq \alpha) = \frac{\frac{c}{\psi} - \mathbb{P}\left(R_i - D_i - Q + \lambda^{-1} \leq -\frac{\Gamma(0, \alpha, \alpha)}{\xi}\right)}{1 - \mathbb{P}\left(R_i - D_i - Q + \lambda^{-1} \leq -\frac{\Gamma(0, \alpha, \alpha)}{\xi}\right)}.$$

(32)

2. Fix the value of $\alpha$. Define functional

$$F_{a_{-i}}(y, \alpha, \Gamma) = F_N(y) + (1 - F_N(y))\mathbb{P}(R_j - D_j - Q + \lambda^{-1} \leq y - \alpha - \frac{\Gamma(y - \alpha, \alpha, \alpha)}{\xi}).$$

In equilibrium, when $y + \alpha > 0$,

$$\Gamma(y, \alpha, \alpha) = \int_{n \in (y^+, (y+\alpha))} \psi F_{a_{-i}}(n - y, \alpha, \Gamma) \, dF_N(n) + \int_{n \in [(y+\alpha), \infty)} c \, dF_N(n). \quad (33)$$

When $y + \alpha \leq 0$,

$$\Gamma(y, \alpha, \alpha) = \psi F_{a_{-i}}(-y, \alpha, \Gamma). \quad (34)$$

The equilibrium $\alpha$ and $\Gamma$ is the fixed point of the above system of integral equations.

**Lemma 20.** There is at least one pair of $\alpha$ and $\Gamma$ that satisfies the above system.

(Proof see Appendix F.2.)

Lemma 20 implies that there is at least one equilibrium under the no hoarding condition.
E.8 Proof of Theorem 8

First, we explore properties for any equilibrium \((\alpha, \Gamma)\) under the given conditions. When \(N_i - N_{\text{min}}\) is exponentially distributed, from Eq. (32),

\[
\frac{\zeta}{\psi} - P\left( \frac{R_i - D_i - Q + \lambda^{-1} \leq - \frac{\Gamma(0,0,\alpha)}{\xi}}{1 - P\left( \frac{R_i - D_i - Q + \lambda^{-1} \leq - \frac{\Gamma(0,0,\alpha)}{\xi}}{1 - \zeta/\psi} \right)} \right) = P(N_i \leq \alpha) = 1 - e^{-\lambda N_i(\alpha - N_{\text{min}})}.
\]

Thus,

\[
e^{\lambda N_i(\alpha - N_{\text{min}})} = \frac{1 - P\left( \frac{R_i - D_i - Q + \lambda^{-1} \leq - \frac{\Gamma(0,0,\alpha)}{\xi}}{1 - \zeta/\psi} \right)}{1 - \zeta/\psi} \tag{35}
\]

From Eq. (33) and Eq. (34), when \(L \leq -\alpha\)

\[
\Gamma(L, \alpha, \alpha) = \psi F_{a_{-1}}(-L, \alpha, \Gamma) = \psi \left( e^{\lambda N(L+N_{\text{min}})} P\left( R_i - D_i - Q + \lambda^{-1} \leq -L - \alpha - \frac{\Gamma(-L-\alpha,\alpha,\alpha)}{\xi} \right) + 1 - e^{\lambda N(L+N_{\text{min}})} \right); \tag{36}
\]

when \(-\alpha < L \leq N_{\text{min}} - \alpha\),

\[
\Gamma(L, \alpha, \alpha) = \int_{n \in (L^+, (L+\alpha))} \psi F_{a_{-1}}(n - L) dF_N(n) + \int_{n \in ([L+\alpha), \infty)} c dF_N(n) = c;
\]

when \(N_{\text{min}} - \alpha < L \leq N_{\text{min}}\),

\[
\Gamma(L, \alpha, \alpha) = \int_{n \in (N_{\text{min}}, (L+\alpha))} \psi F_{a_{-1}}(n - L) dF_N(n) + \int_{n \in ([L+\alpha), \infty)} c dF_N(n) < c;
\]

when \(N_{\text{min}} < L\)

\[
\Gamma(L, \alpha, \alpha) = \int_{n \in (L, (L+\alpha))} \psi F_{a_{-1}}(n - L) dF_N(n) + \int_{n \in ([L+\alpha), \infty)} c dF_N(n).
\]

Therefore, when \(-\alpha < L \leq N_{\text{min}} - \alpha\),

\[
\Gamma'(L, \alpha, \alpha) = 0.
\]
Fix $\alpha$ and consider changing $L$ for $\Gamma(L, \alpha, \alpha)$ and $F_{a_{-l}}(L, \alpha, \Gamma)$. To simplify notations, write $\Gamma(L, \alpha, \alpha)$ as $\Gamma(L)$ and $F_{a_{-l}}(L, \alpha, \Gamma)$ as $F_{a_j}(L)$ when there is no confusion. Our next step is to transfer the integral equation into a non-standard ODE when $L > N_{\min} - \alpha$. Note by assumption $F_{a_j}(L)$ is differentiable. Let $f_{a_j}(L) = F'_{a_j}(L)$.

When $N_{\min} - \alpha < L \leq N_{\min}$,

$$
\Gamma'(L) = \int_{n \in [N_{\min}, (L+\alpha))] -f_{a_j}^2(n - L) dF_N(n) + f_{a_j}(L + \alpha) f_N(L + \alpha) - c f_N(L + \alpha) \lambda_N e^{-\lambda_N(n-N_{\min})} dn.
$$

Since

$$
\frac{d}{dn} f_{a_j}^2(n - L) \lambda_N e^{-\lambda_N(n-N_{\min})} = -f_{a_j}(n - L) \lambda_N^2 e^{-\lambda_N(n-N_{\min})} + f_{a_j}(n - L) \lambda_N e^{-\lambda_N(n-N_{\min})},
$$

we have

$$
\Gamma'(L) = -\int_{n \in [N_{\min}, (L+\alpha))] \frac{d}{dn} f_{a_j}^2(n - L) \lambda_N e^{-\lambda_N(n-N_{\min})} dn - \int_{n \in [N_{\min}, (L+\alpha))] f_{a_j}^2(n - L) \lambda_N e^{-\lambda_N(n-N_{\min})} dn
$$

$$
= \psi_{a_j}(N_{\min} - L) \lambda_N - \lambda_N \Gamma(L).
$$

When $0 < L \leq N_{\min}, N_{\min} - L < N_{\min}$ and $\Gamma(N_{\min} - L - \alpha) = c$, so

$$
\Gamma'(L) = -\lambda_N \Gamma(L) + \psi \lambda_N P(R_j - D_j - Q + \lambda^{-1} \leq N_{\min} - L - \alpha - \frac{\Gamma(N_{\min} - L - \alpha)}{\xi})
$$

$$
= -\lambda_N \Gamma(L) + \psi \lambda_N P(R_j - D_j - Q + \lambda^{-1} \leq N_{\min} - L - \alpha - \frac{c}{\xi}).
$$

When $N_{\min} - \alpha < L \leq 0, N_{\min} - L \geq N_{\min}$, so

$$
\Gamma'(L) = -\lambda_N \Gamma(L) + \psi \lambda_N F_{a_j}(N_{\min} - L)
$$

$$
= -\lambda_N \Gamma(L) + \psi \lambda_N ((1 - e^{\lambda_N L} + e^{\lambda_N L} P(R_j - D_j - Q + \lambda^{-1} \leq N_{\min} - L - \alpha - \frac{\Gamma(N_{\min} - L - \alpha)}{\xi})).
$$
Define \( H(k) = e^{\lambda_N k} \Gamma(k) \). Then
\[
H'(k) = e^{\lambda_N k} \Gamma'(k) + \lambda_N e^{\lambda_N k} \Gamma(k) = e^{\lambda_N k} \psi \lambda_N F_{a_j}(N_{\text{min}} - k)
\]

\[
= e^{\lambda_N k} \psi \lambda_N \left( (1 - e^{\lambda_N k}) + e^{\lambda_N k} P(R_j - D_j - Q + \lambda^{-1} \leq N_{\text{min}} - k - \alpha - \frac{\Gamma(N_{\text{min}} - k - \alpha)}{\xi} \right)
\]

(37)

so
\[
H(L) = e^{\lambda_N L} \Gamma(L) = \Gamma(0) + \psi \lambda_N \int_0^L e^{\lambda_N k} F_{a_j}(N_{\text{min}} - k) \, dk.
\]

Thus \( \Gamma(L) \) is uniquely determined by \( \Gamma(0) \) when \( 0 < L \leq N_{\text{min}} \). Next, when \( N_{\text{min}} < L \),
\[
\Gamma'(L) = \int_{n \in [L, (L+\alpha)])} -\psi f_{a_j}(n - L) \lambda_N e^{-\lambda_N(n - N_{\text{min}})} \, dn - \psi F_{a_j}(0) f_N(L)
\]

\[
= - \int_{n \in [L, (L+\alpha)])} \frac{d}{dn} \psi F_{a_j}(n - L) \lambda_N e^{-\lambda_N(n - N_{\text{min}})} \, dn
\]

\[
- \int_{n \in [L, (L+\alpha)])} \psi F_{a_j}(n - L) \lambda_N^2 e^{-\lambda_N(n - N_{\text{min}})} \, dn - \psi F_{a_j}(0) f_N(L)
\]

\[
= \psi F_{a_j}(L - L) \lambda_N e^{-\lambda_N(L - N_{\text{min}})} - \psi F_{a_j}(\alpha) \lambda_N e^{-\lambda_N(L + \alpha - N_{\text{min}})} - \psi F_{a_j}(0) f_N(L)
\]

\[
- \lambda_N \int_{n \in [N_{\text{min}}, (L+\alpha)])} \psi F_{a_j}(n - L) \lambda_N e^{-\lambda_N(n - N_{\text{min}})} \, dn
\]

\[
= - c \lambda_N e^{-\lambda_N(L + \alpha - N_{\text{min}})} - \lambda_N \Gamma(L) + \lambda_N c e^{-\lambda_N(L + \alpha - N_{\text{min}})}
\]

\[
= - \lambda_N \Gamma(L).
\]

Thus, \( \Gamma(L) \) is uniquely determined by \( \Gamma(N_{\text{min}}) \) when \( L > N_{\text{min}} \). Therefore, \( \Gamma(0) \) uniquely determines \( \Gamma(L) \) for all \( L \geq 0 \). By Eq. (36), \( \Gamma(L) \) for \( L \geq 0 \) in turn uniquely determines the value of \( \Gamma(L) \) for \( L \leq -\alpha \). Thus it suffices to show that the system of integral equations has a unique solution \( \Gamma(L) \) for \( L \in [N_{\text{min}} - \alpha, 0] \).

Suppose there are two equilibria, characterized by \( (\alpha_1, \Gamma_1) \) and \( (\alpha_2, \Gamma_2) \). Assume that \( \Gamma_1(0, \alpha_1, \alpha_1) > \Gamma_2(0, \alpha_2, \alpha_2) \), then it follows that \( \alpha_1 > \alpha_2 \). We want to establish a contradiction. Define \( H_1(L) = e^{\lambda_N L} \Gamma_1(L) \) and \( H_2(L) = e^{\lambda_N L} \Gamma_2(L) \). Then \( H_1(0) > H_2(0) \) and
\[
H_1(N_{\text{min}} - \alpha_1) = e^{\lambda_N (N_{\text{min}} - \alpha_1)} c \leq e^{\lambda_N (N_{\text{min}} - \alpha_2)} c = H_2(N_{\text{min}} - \alpha_2),
\]
\[
H_1(N_{\text{min}} - \alpha_2) = e^{\lambda_N (N_{\text{min}} - \alpha_2)} \Gamma_1(N_{\text{min}} - \alpha_2) \leq e^{\lambda_N (N_{\text{min}} - \alpha_2)} c = H_2(N_{\text{min}} - \alpha_2).
\]

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Similarly, we can show \( H'_1(N_{\min} - \alpha_2) < H'_2(N_{\min} - \alpha_2) \) and \( H'_1(0) < H'_2(0) \). Note that \( H'_1 \) and \( H'_2 \) are continuous.

Since \( H_1(N_{\min} - \alpha_2) \leq H_2(N_{\min} - \alpha_2) \) and \( H_1(0) > H_2(0) \), there must be \( x \in (N_{\min} - \alpha_2, 0) \) such that \( H_1'(x) > H_2'(x) \). Then there exists \( t_1 \) such that \( H_1'(N_{\min} - \alpha_2 + x) < H_2'(N_{\min} - \alpha_2 + x), \forall 0 \leq x < t_1 \) and \( H_1'(N_{\min} - \alpha_2 + t_1) = H_2'(N_{\min} - \alpha_2 + t_1) \). Since \( H_1'(0) < H_2'(0) \), there also exists \( t_0 \) such that \( H_1'(x) < H_2'(x), \forall 0 \geq x > -t_0 \) and \( H_1'(-t_0) = H_2'(-t_0) \).

First, \(-t_0 > \frac{N_{\min} - \alpha_2}{2} \). If not, then \( \Gamma_1(x) > \Gamma_1(x), \forall 0 \geq x > \frac{N_{\min} - \alpha_2}{2} \). This implies \( H_1'(x) < H_2'(x), \forall N_{\min} - \alpha_2 \leq x \leq \frac{N_{\min} - \alpha_2}{2} \), so \( H_1(N_{\min} - \alpha_2) > H_2(N_{\min} - \alpha_2) \), a contradiction. Second, by \( H_1'(N_{\min} - \alpha_2 + t_1) = H_2'(N_{\min} - \alpha_2 + t_1) \) and Eq. (37),

\[
\Gamma_1(-t_1 - (\alpha_1 - \alpha_2)) + \xi(\alpha_1 - \alpha_2) = \Gamma_2(-t_1).
\]

Also \( \Gamma_1(-x - (\alpha_1 - \alpha_2)) + \xi(\alpha_1 - \alpha_2) > \Gamma_2(-x), \forall 0 \leq x < t_1 \). Similarly,

\[
\Gamma_1(N_{\min} - \alpha_2 + t_0 - (\alpha_1 - \alpha_2)) + \xi(\alpha_1 - \alpha_2) = \Gamma_2(N_{\min} - \alpha_2 + t_0)
\]

and \( \Gamma_1(N_{\min} - \alpha_2 + x - (\alpha_1 - \alpha_2)) + \xi(\alpha_1 - \alpha_2) < \Gamma_2(N_{\min} - \alpha_2 + x), \forall 0 \leq x < t_0 \). Thus \(-t_1 < -t_0 \).

Since \( H_1(0) > H_2(0) \) and \( H_1'(x) < H_2'(x), \forall 0 \geq x > -t_0 \), \( H_1(0) > H_2(-t_0) \) and \( \Gamma_1(-t_0) - \Gamma_2(-t_0) > 0 \). Let \( J_1 = \sup\{\Gamma_1(x) - \Gamma_2(x) \mid x \in [-t_1, -t_2]\} \). By continuity, \( \exists t_2 \in [t_0, t_1] \) such that \( \Gamma_1(-t_2) = \Gamma_2(-t_2) = J_1 \) and \( H_1'(-t_2) = H_2'(-t_2) \). Since \( \Gamma_1(-t_1 - (\alpha_1 - \alpha_2)) + \xi(\alpha_1 - \alpha_2) = \Gamma_2(-t_1) \),

\[
H_2(-t_1) - H_1(-t_1) = e^{-\lambda N t_1}\left(\Gamma_2(-t_1) - \Gamma_1(-t_1)\right)
\geq e^{-\lambda N t_1}\left(\Gamma_2(-t_1) - \Gamma_1(-t_1 - (\alpha_1 - \alpha_2))\right) = e^{-\lambda N t_1}\xi(\alpha_1 - \alpha_2).
\]

Then by mean value theorem, \( \exists t_3 \in (t_2, t_1) \) such that

\[
(t_1 - t_2)(H_1'(-t_3) - H_2'(-t_3)) = (H_1(-t_2) - H_2(-t_2)) - (H_1(-t_1) - H_2(-t_1))
\geq e^{-\lambda N t_1}\xi(\alpha_1 - \alpha_2) + e^{-\lambda N t_2}J_1.
\]

By Eq. (37),

\[
H_1'(-t_3) - H_2'(-t_3) = \psi Ne^{-2\lambda N t_3}\left(F_{RD}(\lambda^{-1} + N_{\min} - \alpha_1 + t_3 - \frac{\Gamma_1(N_{\min} - \alpha_1 + t_3)}{\xi})
- F_{RD}(\lambda^{-1} + N_{\min} - \alpha_2 + t_3 - \frac{\Gamma_2(N_{\min} - \alpha_2 + t_3)}{\xi})\right),
\]

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Then by mean value theorem the previous two equations imply that

\[
(-\alpha_1 - \frac{\Gamma_1(N_{\min} - \alpha_1 + t_3)}{\xi} + \alpha_2 + \frac{\Gamma_2(N_{\min} - \alpha_2 + t_3)}{\xi})f_{RD}(N_{\min} - \lambda^{-1} - \alpha_2 - \frac{\Gamma_2(N_{\min} - \alpha_2 + t_3)}{\xi} + \bar{s}) \geq \frac{e^{\lambda(N(2t_3-t_1))}(\alpha_1 - \alpha_2) + e^{\lambda N(2t_3-t_2)}J_1}{\psi \lambda N(t_1 - t_2)}
\]

for some \(\bar{s} \in [0, (-\alpha_1 - \frac{\Gamma_1(N_{\min} - \alpha_1 + t_3)}{\xi} - (-\alpha_2 - \frac{\Gamma_2(N_{\min} - \alpha_2 + t_3)}{\xi}))].\) Since \(f_{RD}(t) \leq f_{RD}^{m}\) when \(t \in (-\infty, 0],\)

\[
\frac{\Gamma_2(N_{\min} - \alpha_2 + t_3) - \Gamma_1(N_{\min} - \alpha_1 + t_3)}{\xi} \geq \alpha_1 - \alpha_2 + \frac{e^{\lambda(N(2t_3-t_1))}(\alpha_1 - \alpha_2) + e^{\lambda N(2t_3-t_2)}J_1}{f_{RD}^{m} \psi \lambda N(t_1 - t_2)}.
\]

Again by mean value theorem, \(\exists t_4 \in (t_2, t_3)\) such that

\[
\Gamma_2(N_{\min} - \alpha_2 + t_3) - \Gamma_1(N_{\min} - \alpha_1 + t_3) = \Gamma_2(N_{\min} - \alpha_2 + t_2) - \Gamma_1(N_{\min} - \alpha_1 + t_2) + (t_3 - t_2)(\Gamma_2(N_{\min} - \alpha_2 + t_4) - \Gamma_1(N_{\min} - \alpha_1 + t_4)) = \xi(\alpha_1 - \alpha_2) + (t_3 - t_2)(\Gamma_2(N_{\min} - \alpha_2 + t_4) - \Gamma_1(N_{\min} - \alpha_1 + t_4)).
\]

Thus,

\[
\psi \lambda N\left(e^{\lambda(N_{\min} - \alpha_1 + t_4)} - e^{\lambda N(N_{\min} - \alpha_2 + t_4)} + e^{\lambda N(N_{\min} - \alpha_2 + t_4)}F_{RD}(-t_4 - \frac{\Gamma_2(-t_4)}{\xi} - \lambda^{-1}) - e^{\lambda N(N_{\min} - \alpha_1 + t_4)}F_{RD}(-t_4 - \frac{\Gamma_1(-t_4)}{\xi} - \lambda^{-1})\right) \geq \frac{e^{\lambda(N(2t_3-t_1))}(\alpha_1 - \alpha_2) + \xi e^{\lambda N(2t_3-t_2)}J_1}{f_{RD}^{m} (t_3 - t_2) \psi \lambda N(t_1 - t_2)}.
\]

In other words,

\[
e^{\lambda N(N_{\min} - \alpha_1 + t_4)}(F_{RD}(-t_4 - \frac{\Gamma_2(-t_4)}{\xi} - \lambda^{-1}) - F_{RD}(-t_4 - \frac{\Gamma_1(-t_4)}{\xi} - \lambda^{-1})) \geq
t\left(e^{\lambda(N_{\min} - \alpha_2 + t_4)} - e^{\lambda N(N_{\min} - \alpha_1 + t_4)}(1 - F_{RD}(-t_4 - \frac{\Gamma_2(-t_4)}{\xi} - \lambda^{-1})) + \frac{e^{\lambda N(2t_3-t_1))}(\alpha_1 - \alpha_2) + \xi e^{\lambda N(2t_3-t_2)}J_1}{f_{RD}^{m} (t_3 - t_2) \psi \lambda N(t_1 - t_2)}\right)
\]

Thus,

\[
\left(\frac{\Gamma_1(-t_4)}{\xi} - \frac{\Gamma_2(-t_4)}{\xi}\right) f_{RD}^{m} > \frac{e^{\lambda N(2t_3-t_1-t_4+\alpha_1-N_{\min})}(\alpha_1 - \alpha_2) + \xi e^{\lambda N(2t_3-t_2+t_4+\alpha_1-N_{\min})}J_1}{f_{RD}^{m} (t_3 - t_2) \psi \lambda N(t_1 - t_2)}.
\]
Note that
\[
e^{\lambda_N (t_3 - t_2 - t_4 + \alpha_1 - N_{\min})} \left( \frac{t_3 - t_2}{(t_1 - t_2)^2} \right) \frac{e^{\lambda_N (t_3 - t_2)}}{(t_3 - t_2) \lambda_N (t_1 - t_2) \lambda_N} > 2e^2.
\]
Thus,
\[
\Gamma_1 (-t_2) - \Gamma_2 (-t_2) \geq \Gamma_1 (-t_4) - \Gamma_2 (-t_4) > \left( \frac{1}{f_{\text{RD}}^m} \right) 2e^2 t_2 \frac{J_1}{\psi^2} > J_1.
\]
The last line follows from the assumption of Theorem 8. Then we have \(J_1 = \Gamma_1 (-t_2) - \Gamma_2 (-t_2) > J_1\), a contradiction. Thus, there cannot be two \((\alpha_1, \Gamma_1)\) and \((\alpha_2, \Gamma_2)\) that solves the system of integral equations defined in Theorem 7. In other words, the equilibrium is unique.

E.9 Proof of Theorem 9

For simplicity, I suppress notation and write \(\Gamma(y)\) as \(\Gamma(y, \alpha, \alpha)\). Repo trading is characterized by Eq. (12). Immediately, we have the following lemma.

**Lemma 21.** \(\mathcal{S}(D)\) is almost everywhere differentiable, and
\[
\mathcal{S}(D) = \mathcal{S}(D_{\min}) + \int_{D_{\min}}^{D} \mathcal{S}'(\theta) \, d\theta.
\]

**Proof.** Consider \(\theta_1 < \theta_2\) in the support of \(D_j\). By Lemma 3, \(\mathcal{S}(\theta_1) < \mathcal{S}(\theta_2)\) and
\[
\mathcal{S}(\theta_1) = \inf \left\{ s : \Gamma(R_i - Q - s) + \xi s \geq \xi (\theta_1 - \lambda^{-1}) \right\},
\]
\[
\mathcal{S}(\theta_2) = \inf \left\{ s : \Gamma(R_i - Q - s) + \xi s \geq \xi (\theta_2 - \lambda^{-1}) \right\}.
\]
Since \(\Gamma(R_i - Q - s)\) is right continuous,
\[
\Gamma(R_i - Q - \mathcal{S}(\theta_1)) + \xi \mathcal{S}(\theta_1) \geq \xi (\theta_1 - \lambda^{-1})
\]
\[
\Gamma(R_i - Q - \mathcal{S}(\theta_2)) + \xi \mathcal{S}(\theta_2) \geq \xi (\theta_2 - \lambda^{-1}).
\]
We notice that
\[
\Gamma(R_i - Q - (\mathcal{S}(\theta_1) + \theta_2 - \theta_1)) + \xi (\mathcal{S}(\theta_1) + \theta_2 - \theta_1) \geq \xi (\theta_1 + \theta_2 - \theta_1 - \lambda^{-1}) = \xi (\theta_2 - \lambda^{-1}).
\]
Since $\mathcal{I}(\theta_2) = \inf \left\{ s : \Gamma(R_i - Q - s) + \xi s \geq \xi(\theta_2 - \lambda^{-1}) \right\}$,

$$\mathcal{I}(\theta_1) \leq \mathcal{I}(\theta_2) \leq \mathcal{I}(\theta_1) + \theta_2 - \theta_1.$$ 

Thus,

$$|\mathcal{I}(\theta_2) - \mathcal{I}(\theta_1)| \leq \theta_2 - \theta_1.$$

Hence $\mathcal{I}(\theta)$ is absolute continuous. By Lebesgue differentiation theorem, $\mathcal{I}(D)$ is differentiable almost everywhere, and

$$\mathcal{I}(D) = \mathcal{I}(D_{\text{min}}) + \int_{D_{\text{min}}}^{D} \mathcal{I}'(\theta) \, d\theta.$$ 

By Eq. (12),

$$\Gamma(R_i - Q - \mathcal{I}(D)) \geq \xi(D - \mathcal{I}(D) - \lambda^{-1})$$

$$\mathcal{T}(D) = -\frac{\xi}{2} \left( D - \mathcal{I}(D) \right)^2 + \frac{\xi}{2} D_{\text{min}}^2 + \int_{D_{\text{min}}}^{D} \xi(x - \mathcal{I}(x)) \, dx$$

$$r_i = \frac{\mathcal{T}(D)}{\mathcal{I}(D)}.$$ 

By Lemma 21, $\mathcal{I}(D)$ is differentiable for all $D \in [D_{\text{min}}, \infty)$, so $\mathcal{T}(D)$ is differentiable almost everywhere (a.e):

$$\mathcal{T}'(D) = \xi(D - \mathcal{I})\mathcal{I}'(D) \quad \text{a.e.}$$

**Lemma 22.** Suppose that for some $\theta_1$, $\Gamma(R_i - Q - \mathcal{I}(\theta_1)) > \xi(\theta_1 - \mathcal{I}(\theta_1) - \lambda^{-1})$, then there is an $\theta_2 > \theta_1$ such that

$$\Gamma(R_i - Q - \mathcal{I}(\theta_2)) = \xi(\theta_2 - \mathcal{I}(\theta_2) - \lambda^{-1})$$

$$\mathcal{I}(\theta_2) = \mathcal{I}(\theta_1)$$

$$\mathcal{I}'(x) = 0, \forall x \in (\theta_1, \theta_2).$$

Moreover, there are at most countably many $\theta$ such that $\Gamma(R_i - Q - \mathcal{I}(\theta_1)) > \xi(\theta_1 - \mathcal{I}(\theta_1) - \lambda^{-1})$.  

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Proof. Suppose that for some $\theta_1$, $\Gamma(R_i - Q - \mathcal{I}(\theta_1)) > \xi(\theta_1 - \mathcal{I}(\theta_1) - \lambda^{-1})$, then

$$\Gamma(R_i - Q - \mathcal{I}(\theta_1)) + \xi \mathcal{I}(\theta_1) > \xi(\theta_1 - \lambda^{-1}).$$

Since $\xi(x - \lambda^{-1})$ is strictly increasing in $x$, there is a $\theta_2 > \theta_1$ such that

$$\Gamma(R_i - Q - \mathcal{I}(\theta_1)) + \xi \mathcal{I}(\theta_1) = \xi(\theta_1 - \lambda^{-1}).$$

For any $x \in [\theta_1, \theta_2]$,

$$\Gamma(R_i - Q - \mathcal{I}(\theta_1)) + \xi \mathcal{I}(\theta_1) \geq \xi(x - \lambda^{-1})$$

and for any $s < \mathcal{I}(\theta_1)$

$$\Gamma(R_i - Q - s) + \xi s < \xi(\theta_1 - \lambda^{-1}) < \xi(x - \lambda^{-1}).$$

Thus,

$$\mathcal{I}(x) = \mathcal{I}(\theta_1), \quad \forall x \in [\theta_1, \theta_2].$$

It follows that $\mathcal{I}'(x) = 0, \forall x \in (\theta_1, \theta_2)$.

Finally, the function $x \mapsto \Gamma(R_i - Q - \mathcal{I}(x)) + \xi \mathcal{I}(x)$ is strictly increasing in $x$, so it has at most countably many discontinuity.

Following Lemma 22, $\xi(x - \mathcal{I}(x)) \mathcal{I}'(x) = \left(\Gamma(R_i - Q - \mathcal{I}(x)) + \xi \lambda^{-1}\right) \mathcal{I}'(x)$ almost everywhere. Thus,

$$\mathcal{T}(D) = \mathcal{T}(D_{\text{min}}) + \int_{D_{\text{min}}}^{D} \xi(x - \mathcal{I}(x)) \mathcal{I}'(x) \, dx$$

$$= \mathcal{T}(D_{\text{min}}) + \int_{D_{\text{min}}}^{D} \left(V'(R_i - Q - \mathcal{I}(x)) + \xi \lambda^{-1}\right) \mathcal{I}'(x) \, dx$$

$$= \mathcal{T}(D_{\text{min}}) + \int_{D_{\text{min}}}^{\mathcal{I}(D)} \Gamma(R_i - Q - s) \, ds + \xi \lambda^{-1} (\mathcal{I}(D) - \mathcal{I}(D_{\text{min}})).$$

Plug the calculation of $\mathcal{T}(D)$ into Eq. (12) we get the desired result.
E.10 Proof of Theorem 10

By Lemma 3, Theorem 6 and Theorem 7, the equilibrium outcome variables $S_i$, $T_i$ and $r_i$ of the trading game depends on the realizations of $R_i$, $D_i$ and $m$. Let $S(\zeta, \mathcal{D}(S^*, m), m)$ and $\mathcal{T}(\zeta, \mathcal{D}(S^*, m), m)$ be the equilibrium amount of financing and total transfer respectively, in the trading game in the state of the world such that the realization of $R_i$ is $\zeta$ and the realization of $D_i$ is $\mathcal{D}(S^*, m)$. Obviously, $\mathcal{D}(S^*, m)$ has to satisfy $S(\zeta, \mathcal{D}(S^*, m), m) = S^*$. Abuse notations and write $\Gamma(y, \alpha, \alpha)$ as $\Gamma(y; m)$ to denote the marginal value of liquidity at $y$ when the liquidity stress index is $m$. Let $\ell = \zeta - Q - S^*$. By Eq. (38), under the macroeconomic conditions indexed by $m$,

$$\mathcal{T}(\zeta, \mathcal{D}(S^*, m), m) = \mathcal{T}(\zeta, D_{\text{min}}, m) + \int_{S(\zeta, D_{\text{min}}, m)}^{S(\zeta, \mathcal{D}(S^*, m), m)} \Gamma(\zeta - Q - s, m) \, ds \tag{39}$$

By Lemma 3,

$$\mathcal{T}(\zeta, D_{\text{min}}, m) = -\frac{\xi^m}{2} \left(D_{\text{min}} - S(\zeta, D_{\text{min}}, m)\right)^2 + \frac{\xi^m}{2} D_{\text{min}}^2$$

$$= \xi^m D_{\text{min}} S(\zeta, D_{\text{min}}, m) - \frac{\xi^m}{2} S(\zeta, D_{\text{min}}, m)^2.$$

Thus,

$$\mathcal{T}(\zeta, D_{\text{min}}, m) - \frac{\xi^m}{\lambda^m} S(\zeta, D_{\text{min}}, m)$$

$$= (\xi^m D_{\text{min}} - \frac{\xi^m}{\lambda^m}) S(\zeta, D_{\text{min}}, m) - \frac{\xi^m}{2} S(\zeta, D_{\text{min}}, m)^2. \tag{40}$$

By definition,

$$r^*(\zeta, S^*, m) = \frac{\mathcal{T}(\zeta, \mathcal{D}(S^*, m), m)}{S^*}.$$  

Combining Eqs. (39) and (40), we can simplify the equilibrium funding rate as

$$r^*(\zeta, S^*, m) = \frac{1}{S^*} \left(\left(\xi^m D_{\text{min}} - \frac{\xi^m}{\lambda^m}\right) S(\zeta, D_{\text{min}}, m) - \frac{\xi^m}{2} S(\zeta, D_{\text{min}}, m)^2 \right)$$

$$+ \int_{S(\zeta, D_{\text{min}}, m)}^{\zeta - Q - \ell} \Gamma(\zeta - Q - s; m) \, ds + \frac{\xi^m}{\lambda^m}.$$
To simplify notations, let \( \{ F_R(\cdot), F_N(\cdot), Q, \lambda, D_{\text{min}}, c, \psi, \xi \} = \{ F^0_R(\cdot), F^0_N(\cdot), Q^0, \lambda^0, D^0_{\text{min}}, c^0, \psi^0, \xi^0 \} \). When \( m = 0 \), there could be multiple equilibria. For the following analysis, we select the equilibrium with the max \( \alpha = N_{\text{min}} \), because, by continuity, this equilibrium is the limiting equilibrium of a sequence of economies under no hoarding condition such that \( m \uparrow 0 \). In other words, pick the equilibrium such that \( r^*(\zeta, S^*, 0) = \lim_{m \downarrow 0} r^*(\zeta, S^*, -m^0) \).

Fix some \( \epsilon^m > 0 \). Since all the macroeconomic conditions considered here are close with respect to liquidity stress index, there exists some constants \( O, \epsilon^m > 0 \) such that for all \( \epsilon^m < \epsilon^m \),

\[
\left( r^*(\zeta, S^*, \epsilon^m) - r^*(\zeta, S^*, 0) \right) S^* \\
\geq (\xi D_{\text{min}} - \xi \lambda^{-1}) S(\zeta, D_{\text{min}}, \epsilon^m) - \frac{\xi}{2} S(\zeta, D_{\text{min}}, \epsilon^m)^2 - (\xi D_{\text{min}} - \xi \lambda^{-1}) S(\zeta, D_{\text{min}}, 0) + \frac{\xi}{2} S(\zeta, D_{\text{min}}, 0)^2 \\
+ \int_{S(\zeta, D_{\text{min}} : 0)}^{S^*} \Gamma(\zeta - Q - s ; \epsilon^m) - \Gamma(\zeta - Q - s ; 0) \ ds + \int_{S(\zeta, D_{\text{min}} : \epsilon)}^{S(\zeta, D_{\text{min}} : 0)} \Gamma(\zeta - Q - s ; \epsilon^m) \ ds - O \epsilon^m \\
= \int_{S(\zeta, D_{\text{min}} : 0)}^{S^*} \Gamma(\zeta - Q - s ; \epsilon^m) - \Gamma(\zeta - Q - s ; 0) \ ds - O \epsilon^m \\
+ \int_{S(\zeta, D_{\text{min}} : \epsilon)}^{S(\zeta, D_{\text{min}} : 0)} - (\xi D_{\text{min}} - \xi \lambda^{-1}) + \xi s + \Gamma(\zeta - Q - s ; \epsilon^m) \ ds.
\]

Since

\[
\Gamma\left(\zeta - Q - S(\zeta, D_{\text{min}} ; \epsilon^m) ; \epsilon^m\right) \geq \xi (D_{\text{min}} - \lambda^{-1}) - \xi S(\zeta, D_{\text{min}} ; \epsilon^m)
\]

and the mapping \( s \mapsto \Gamma(\zeta - Q - s ; \gamma) + \xi s \) is monotonically increasing in \( s \) for any \( \gamma \),

\[
\int_{S(\zeta, D_{\text{min}} : \epsilon)}^{S(\zeta, D_{\text{min}} : 0)} - (\xi D_{\text{min}} - \xi \lambda^{-1}) + \xi s + \Gamma(\zeta - Q - s ; \epsilon) \ ds \geq 0.
\]

Thus,

\[
\left( r^*(\zeta, S^*, \epsilon) - r^*(\zeta, S^*, 0) \right) S^* \geq \int_{S(\zeta, D_{\text{min}} : 0)}^{S^*} \Gamma(\zeta - Q - s ; \epsilon) - \Gamma(\zeta - Q - s ; 0) \ ds - O \epsilon^m. \tag{41}
\]
Lemma 23. For all \( e^m < \bar{e}^m \), there is some \( \delta_0(\zeta, s) \geq 0 \) and constant \( \mathcal{O}^1 \geq 0 \) such that

\[
\Gamma(\zeta - Q - s ; e^m) - \Gamma(\zeta - Q - s ; 0) > \delta_0(\zeta, s) - \mathcal{O}^1 e^m.
\]

In addition, whenever \( \zeta - Q - s \neq 0 \), \( \delta_0(\zeta, s) > 0 \).

Lemma 23 and Eq. (41) imply that for all \( e^m < \bar{e}^m \), there is some \( \delta(\zeta, S^*) \) such that \( r^*(\zeta, S^*, e^m) - r^*(\zeta, S^*, 0) > \delta(\zeta, S^*) - \mathcal{O}^2 e^m \) for some constant \( \mathcal{O}^2 \). This proves the Theorem 10.

Proof of Lemma 23. Recall that under no hoarding condition, \( a^*_j = \min((L_j + \alpha)^+, N_j) \) for some \( \alpha \geq N_{\min} \). Under no hoarding condition, \( a^*_j = \min(L_j^+, N_j) \). We discuss \( \Gamma(y ; e) - \Gamma(y ; 0) \) case by case.

First, when \( y \geq 0 \), by directly calculation the assumption that two macroeconomic conditions are close with respect to liquidity stress index, there is some constant \( \mathcal{O}^3 > 0 \) such that

\[
\Gamma(y ; e^m) - \Gamma(y ; 0) \geq \int_y^{y + \alpha} c - \psi \mathbb{P}(\min\{L_j + \alpha, N_j\} \leq n - y) dF_N(n) - \mathcal{O}^3 e^m.
\]

When \( y \geq N_{\min} \),

\[
\int_y^{y + \alpha} c - \psi \mathbb{P}(\min\{L_j + \alpha, N_j\} \leq n - y) dF_N(n) = \int_{y + \frac{N_{\min}}{2}}^{y + \alpha} c - \psi \mathbb{P}(\min\{L_j + \alpha, N_j\} \leq n - y) dF_N(n) + \int_{y + \frac{N_{\min}}{2}}^{y + \alpha} c - \psi \mathbb{P}(\min\{L_j + \alpha, N_j\} \leq n - y) dF_N(n).
\]

Since \( c - \psi \mathbb{P}(\min\{L_j + \alpha, N_j\} \leq \alpha) = 0 \) and \( \mathbb{P}(\min\{L_j + \alpha, N_j\} \leq \vartheta) \) strictly increases in \( \vartheta \), there is some \( \delta_1(\zeta, y) \) such that

\[
\int_{y + \frac{N_{\min}}{2}}^{y + \alpha} c - \psi \mathbb{P}(L_j + \alpha \leq \frac{N_{\min}}{2}) dF_N(n) > \delta_1(\zeta, y) > 0,
\]

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and

\[ \int_{y + \frac{N_{\min}}{2}}^{y + \alpha} c - \psi \mathbb{P}(\min\{L_j + \alpha, N_j\} \leq n - y) dF_N(n) \geq 0. \]

Thus,

\[ \int_{y}^{y + \alpha} c - \psi \mathbb{P}(\min\{(L_j + \alpha)^+, N_j\} \leq n - y) dF_N(n) > \delta_1(\zeta, y) > 0, \]

Hence, when \( y \geq N_{\min} \),

\[ \Gamma(y ; \epsilon^m) - \Gamma(y ; 0) \geq \delta_1(\zeta, y) - O^3 \epsilon^m. \]

When \( N_{\min} > y > 0 \),

\[ \Gamma(y ; \epsilon^m) - \Gamma(y ; 0) \geq \int_{N_{\min}}^{y + \alpha} c - \psi \mathbb{P}(\min\{(L_j + \alpha)^+, N_j\} \leq n - y) dF_N(n) - O^3 \epsilon^m \]

\[ \geq \int_{N_{\min}}^{y + \alpha} c - \psi \mathbb{P}(\min\{(L_j + \alpha)^+, N_j\} \leq n - N_{\min}) dF_N(n) - O^3 \epsilon^m. \]

When \( y = 0 \), clearly \( \Gamma(y ; \epsilon^m) - \Gamma(y ; 0) \geq -O^3 \epsilon^m \). When \( 0 > y \geq -\alpha \), \( \exists O^4 > 0 \) such that

\[ \Gamma(y ; \epsilon^m) - \Gamma(y ; 0) \geq \psi \mathbb{P}^m(\min(L_j^+ , N_j) \leq -y) - \psi \int_{0}^{y + \alpha} \mathbb{P}(a_j^* \leq n - y) dF_N(n) - c \mathbb{P}(N_i > y + \alpha) - O^4 \epsilon^m \]

\[ > \delta_2(\zeta, y) + c - \psi \int_{0}^{y + \alpha} \mathbb{P}(a_j^* \leq n - y ; T + \epsilon) dF_N(n) - c \mathbb{P}(N_i > y + \alpha) - O^4 \epsilon^m > \delta_2(\zeta, y) - O^4 \epsilon^m. \]

where the second inequality derives from the fact that under liquidity hoarding condition,

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\[ \psi \mathbb{P}^m(L_j \leq 0) > c. \] Finally, when \( y < -\alpha \), then
\[
\Gamma(y ; e^m) - \Gamma(y ; 0) = \psi \mathbb{P}^m(\min(L_j^+, N_j) \leq -y) - \psi \mathbb{P}(\min((L_j + \alpha)^+, N_j) \leq -y)
\]
\[
> \psi \mathbb{P}^m(\min(L_j^+, N_j) \leq -y) - \psi \mathbb{P}(\min(L_j^+, N_j) \leq -y)
\]
\[
+ \psi \left( \mathbb{P}(\min((L_j + \alpha)^+, N_j) \leq -y) - \psi \mathbb{P}(\min(L_j^+, N_j) \leq -y) \right)
\]
\[
> \psi \left( \mathbb{P}(\min((L_j + \alpha)^+, N_j) \leq -y) - \psi \mathbb{P}(\min((L_j)^+, N_j) \leq -y) \right) > \delta_3(\zeta, y)
\]

for some \( \delta_3(\zeta, y) > 0 \) regardless of \( \epsilon \). Let \( \delta_0(\zeta, \zeta - Q - y) \) be the corresponding \( \delta_i(\zeta, y) \) in each case corresponding to different \( y \). This finishes the proof. \( \square \)

### E.11 Proof of Lemma 5

Similar analysis as in the proof for Lemma 1 gives the following result:

**Lemma 24.** Given all other banks’ strategy \( a_{-i} \), the best response actions of bank \( i \) are of the form

\[ a_i = \min((L_i + z_i)^+, N_i), \]

where \( z_i \) is some non-negative random variable with support \( \mathcal{I}_z \). For any \( \nu \in \mathcal{I}_z \),

\[ \mathbb{P}(a_{-i} \leq \nu) \geq \frac{c}{\psi}. \]

Moreover, if there does not exist \( \nu^* \) such that \( \mathbb{P}(a_{-i} \leq \nu^*) = c/\psi \), then \( \mathcal{I}_z = \{ \nu^* \} \) is a singleton and \( \nu^* = \inf \{ \nu \geq 0, \mathbb{P}(a_{-i} \leq \nu) > \frac{c}{\psi} \} \). If there is an \( \nu^* \) such that \( \mathbb{P}(a_{-i} \leq \nu^*) = c/\psi \), then for any \( \nu \in \mathcal{I}_z \),

\[ \mathbb{P}(a_{-i} \leq \nu) = c/\psi. \]

**Lemma 25.** Suppose that \( \prod_{j \neq i} \mathbb{P}(L_j \leq 0) > \frac{c}{\psi} \) for all \( i \), then there is a unique pure strategy equilibrium. In this equilibrium, each bank \( i \) chooses the payment \( a_i^* = \min(L_i^+, N_i) \).

**Proof.** When \( \prod_{j \neq i} \mathbb{P}(L_j \leq 0) > \frac{c}{\psi} \) for all \( i \), then \( \mathbb{P}(\sum_{j \neq i} \min(L_j^+, N_j) \leq 0) > \frac{c}{\psi} \). This implies that \( \alpha_i = 0 \). If not, assume bank \( i \) makes payment \( \min((L_i + \alpha_i)^+, N_i) \) for some \( \alpha_i \geq 0 \). WLOG assume \( \alpha_1 = \max \{ \alpha_i \} > 0 \). Then \( \mathbb{P}(\sum_{j \neq i} \min((L_j + \alpha_j)^+, N_j) \leq (n-1)\alpha_1) \geq \mathbb{P}(\sum_{j \neq i} \min((L_j + \alpha_j)^+ - \alpha_1, N_j - \alpha_1) \leq 0) > \mathbb{P}(\sum_{j \neq i} \min(L_j^+, N_j) \leq 0) > \frac{c}{\psi}, \) a contradiction. \( \square \)

**Lemma 26.** Suppose that \( \prod_{j \neq i} \mathbb{P}(L_j \leq 0) < \frac{c}{\psi} \) for all \( i \), then there is a unique pure strategy equilibrium. In this equilibrium, each bank \( i \) chooses the payment \( a_i^* = \min((L_i + \alpha_i)^+, N_i) \). For each \( i, \alpha_i > 0 \) and \( \mathbb{P}(\sum_{j \neq i} \min((L_j + \alpha_j)^+, N_j) \leq \alpha_i) = \frac{c}{\psi}. \)

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Proof. Consider a map $T^\alpha : [0, F_N^{-1}(\frac{c}{\psi})]^n \to [0, F_N^{-1}(\frac{c}{\psi})]^n$, where $T^\alpha(x_1, x_2, \ldots, x_n) = (a_1, a_2, \ldots, a_n)$ such that

$$a_i = \inf \left\{ \vartheta \geq 0 : \mathbb{P}\left( \sum_{j \neq i} \min((L_j + x_j)^+, N_j) \leq (n-1)\vartheta \right) \geq \frac{c}{\psi} \right\}.$$ 

It is easy to check that $T^\alpha$ is continuous. By Schauder fixed-point theorem there is at least one fixed point $(\alpha_i)_{i=1}^n$ for $T^\alpha$. When $\mathbb{P}(L_j < 0) = 0$ for all $j$, then $(n-1)\alpha_i \geq N_{\min}$. To see that, suppose $\alpha_k = \min \{ \alpha_i \} < \frac{N_{\min}}{n-1}$. Then

$$\mathbb{P}\left( \sum_{j \neq i} \min((L_j + \alpha_j)^+, N_j) \leq (n-1)\alpha_k \right) = \mathbb{P}\left( \sum_{j \neq i} L_j + \alpha_j \leq (n-1)\alpha_k \right) < \mathbb{P}\left( \sum_{j \neq i} L_j \leq 0 \right) < \frac{c}{\psi},$$

a contradiction. Suppose there are two different fixed points, $(\alpha^1_i)_{i=1}^n$ and $(\alpha^2_i)_{i=1}^n$, for $T^\alpha$. Without loss of generality (WLOG), assume that $\alpha^1_1 < \alpha^2_1$. By assumption $N_j$ has continuous cdf, so

$$\mathbb{P}(\min((L_2 + \alpha^1_1)^+, N_2) + \sum_{j>2} \min((L_j + \alpha^1_j)^+, N_j) \leq (n-1)\alpha^1_1) = \frac{c}{\psi},$$

$$\mathbb{P}(\min((L_2 + \alpha^2_1)^+, N_2) + \sum_{j>2} \min((L_j + \alpha^2_j)^+, N_j) \leq (n-1)\alpha^2_1) = \frac{c}{\psi},$$

Thus, it cannot be the case that $\alpha^2_j = \alpha^1_j = 0$ for all $j > 1$. Assume that $\alpha^2_2, \alpha^1_2 > 0$, then

$$\mathbb{P}(\min((L_1 + \alpha^1_1)^+, N_1) + \sum_{j>2} \min((L_j + \alpha^1_j)^+, N_j) \leq (n-1)\alpha^1_2) = \frac{c}{\psi},$$

$$\mathbb{P}(\min((L_1 + \alpha^2_1)^+, N_1) + \sum_{j>2} \min((L_j + \alpha^2_j)^+, N_j) \leq (n-1)\alpha^2_2) = \frac{c}{\psi}.$$ 

Since $\{L_j\}, \{N_j\}$ are i.i.d, the above equations imply that $\alpha^2_j \geq \alpha^1_j$. Replacing $\alpha^2_2, \alpha^1_2$ with $\alpha^2_j, \alpha^1_j$, the same argument shows that $\alpha^2_j \geq \alpha^1_j$. WLOG, assume that $\alpha^1_1 - \alpha^1_1 = \max_j \{ \alpha^2_j - \alpha^1_j \}$. Then

$$\mathbb{P}\left( \sum_{j>1} \min((L_j + \alpha^1_j)^+, N_j) \leq (n-1)\alpha^1_1 \right) = \frac{c}{\psi} = \mathbb{P}\left( \sum_{j>1} \min((L_j + \alpha^2_j)^+, N_j) \leq (n-1)\alpha^2_1 \right) \Rightarrow$$

$$\mathbb{P}\left( \sum_{j>1} \min((L_j + \alpha^1_j)^+, N_j) + \alpha^1_1 - \alpha^1_1 \leq (n-1)\alpha^1_1 \right) = \mathbb{P}\left( \sum_{j>1} \min((L_j + \alpha^2_j)^+, N_j) \leq (n-1)\alpha^2_1 \right).$$

Note that $\min((L_j + \alpha^1_j)^+, N_j) + \alpha^1_1 - \alpha^1_1 \geq \min((L_j + \alpha^2_j)^+, N_j)$ almost surely. Suppose that $\mathbb{P}(L_j < 0) > 0$ for some $j$, then $\mathbb{P}(\min((L_j + \alpha^1_j)^+, N_j) + \alpha^1_1 - \alpha^1_1 > \min((L_j + \alpha^2_j)^+, N_j))$ and LHS < RHS. This is a contradiction. When $\mathbb{P}(L_j < 0) = 0$ for all $j$, then by the previous argument, $(n-1)\alpha^1_1 \geq N_{\min}$ and $(n-1)\alpha^2_1 \geq N_{\min}$. However, when this happens, since $F_N$ strictly increases,
LHS < RHS. This is a contradiction.

Suppose that there is a mixed strategy equilibrium such that \( a_i = \min((L_i + z_i)^+, N_i) \). Let \( I_z^i \) denote the support of \( z_i \). Since \( I_z^i \) and \( I_z^j \) are bounded, let \( v_i \overset{\text{def}}{=} \inf I_z^i \) and \( \overline{v}_i \overset{\text{def}}{=} \sup I_z^i \). At least one of \( I_z^i \) must have more than one element, for otherwise it is a pure strategy equilibrium. Say \( I_z^1 \) has at least two elements, then \( \overline{v}_1 > v_1 \) and for any \( v \in [v_1, \overline{v}_1) \), \( P(a_i \leq v) = \frac{v}{\psi} \).

**Lemma 27.** There is no mixed strategy equilibrium.

**Proof.** Suppose that there is a mixed strategy equilibrium. If \( v_1 \geq N_{\min} \) pick any \( N_{\min} \leq v'_1 < v''_1 \) in \( I_z^1 \). Since \( F_N \) strictly increases, for any \( j > 1, P(N_j \leq v''_1) > P(N_j \leq v'_1) \). Thus, if

\[
\mathbb{P}(a_{-i} \leq v''_i) = \mathbb{P}(a_{-i} \leq v'_i) \Rightarrow \\
\mathbb{P}(\sum_{j>2} \min((L_j + z_j)^+, N_j) \leq (n-1)v''_1) = \mathbb{P}(\sum_{j>2} \min((L_j + z_j)^+, N_j) \leq (n-1)v'_1).
\]

This is impossible, since bank \( j \) cannot choose her strategy based on the the state variable of bank \( k \neq j \). In other words, \( \{\min((L_j + z_j)^+, N_j)\} \) are independent of each other. Since \( N_j \) is strictly increasing, the cdf of \( \sum_{j>2} \min((L_j + z_j)^+, N_j) \) is strictly increasing. Thus, LHS > RHS.

Finally, similar analysis as in the proof for Lemma 1 shows that when \( L_i \) are i.i.d, then banks’ payment strategies are symmetric: \( \alpha_i = \alpha \) for all \( i \).

**E.12 Proof of Theorem 11**

Suppose that liquidity hoarding condition for \( n \) banks holds. We prove by contradiction that banks hoard liquidity in equilibrium at time 1. By Lemma 27 there is no mixed strategy equilibrium for the payment subgame. By Lemma 24 and Lemma 6, \( \Gamma^+_i(y, \alpha_i, \alpha_{-i}) \leq c \) for any \( y \geq 0 \) when \( \alpha_i = 0 \); \( \Gamma^+_i(y, \alpha_i, \alpha_{-i}) < c \) for any \( y \geq 0 \) when \( \alpha_i > 0 \). Assume that bank \( j \) does not hoard liquidity, i.e. bank \( j \) pays \( \min(L^+_j + \alpha_j, N_j) \) at time 1 for some \( \alpha_j > 0 \). WOLG, let
\( \alpha_j = \max \{ \alpha_i \} \). Since bank \( j \) is best responding, it must still hold that

\[
\mathbb{P}(a_{-j} \leq \alpha_j) = \mathbb{P}\left( \sum_{i \neq j} \min((L_i + \alpha_i)^+, N_i) \leq (n-1)\alpha_j \right) = \frac{c}{\psi} \Rightarrow \\
\mathbb{P}\left( \sum_{i \neq j} \min((L_i + \alpha_i)^+ - \alpha_j, N_i - \alpha_j) \leq 0 \right) = \frac{c}{\psi} \Rightarrow \\
\prod_{i \neq j} \mathbb{P}(L_i \leq 0) = \mathbb{P}\left( \sum_{i \neq j} \min(L_i^+, N_i) \leq 0 \right) < \mathbb{P}\left( \sum_{i \neq j} \min((L_i + \alpha_i)^+ - \alpha_j, N_i - \alpha_j) \leq 0 \right) = \frac{c}{\psi}.
\]

By Lemma 7,

\[
\mathbb{P}(L_i \leq 0) = \mathbb{P}\left( R_i - D_i - Q \leq -\frac{\Gamma_i^+(0, \alpha_i, \alpha_{-i})}{\xi} \right) \geq \mathbb{P}\left( R_i - D_i - Q \leq -\frac{c}{\xi} \right) > \left( \frac{c}{\psi} \right)^1/(n-1) \Rightarrow \\
\prod_{i \neq j} \mathbb{P}(L_i \leq 0) > \frac{c}{\psi},
\]
a contradiction. Thus, \( \alpha_i = 0 \) and the marginal value of liquidity functions for all banks are the same.

### E.13 Proof of Theorem 12

Suppose that Eq. (18) holds. By Lemma 27 there is no mixed strategy equilibrium for the payment subgame. Suppose that bank \( i \) pays \( a_i = \min((L_i + \alpha_i)^+, N_i) \), our goal is to show \( \alpha_i = \alpha_j, \forall i, j \). First, since \( \frac{c}{\psi} > \prod_{i \neq j} \mathbb{P}(L_i \leq 0) = \mathbb{P}\left( \sum_{i \neq j} \min(L_i^+, N_i) \leq 0 \right) \geq \mathbb{P}\left( \sum_{i \neq j} \min((L_i + \alpha_i)^+, N_i) \leq 0 \right), \alpha_i > 0 \) for all \( i \). Thus, by Lemma 24 and Lemma 7,

\[
\frac{c}{\psi} = \mathbb{P}(a_{-j} \leq \alpha_j) = \mathbb{P}(\min((L_i + \alpha_i)^+, N_i)) + \sum_{k \neq i, j} \min((L_k + \alpha_k)^+, N_k) \leq (n-1)\alpha_j.
\]

Let \( \tilde{K} = \sum_{k \neq i, j} \min((L_k + \alpha_k)^+, N_k) \), then

\[
\frac{c}{\psi} = \mathbb{P}(a_{-j} \leq \alpha_j) = \int_0^{(n-1)\alpha_j} \mathbb{P}(\min((L_i + \alpha_i)^+, N_i)) \leq (n-1)\alpha_j - \kappa) \ dF_\tilde{K}(\kappa)
\]

\[
= \int_0^{(n-1)\alpha_j} \mathbb{P}\left( R_i - D_i - Q \leq (n-1)\alpha_j - \alpha_i - \kappa - \frac{\Gamma_i((n-1)\alpha_j - \alpha_i - \kappa, \alpha_i, \alpha_{-i})}{\xi} \right) + \mathbb{P}(N_i \leq (n-1)\alpha_j - \kappa) \left( 1 - \mathbb{P}\left( R_i - D_i - Q \leq (n-1)\alpha_j - \alpha_i - \kappa - \frac{\Gamma_i((n-1)\alpha_j - \alpha_i - \kappa, \alpha_i, \alpha_{-i})}{\xi} \right) \right) dF_\tilde{K}(\kappa)
\]
Lemma 28. Suppose that \((R_i, D_i, N_i)\) and \((R_j, D_j, N_j)\) have the same distribution. If in equilibrium bank \(i\) pays \(a_i = \min((L_i + \alpha_i)^+, N_i)\) and bank \(j\) pays \(a_j = \min((L_j + \alpha_j)^+, N_j)\) in the payment subgame and \(\alpha_j > \alpha_i\), then \(\mathbb{P}(a_{-i} \leq \eta) \leq \mathbb{P}(a_{-j} \leq \eta)\) for all \(\eta \in [N_{\min}, \alpha_j]\).

The proof is omitted since it is similar to the proof for Lemma 19 with slight modifications. Lemma 28 implies that \(\Gamma_j((n - 1)\alpha_i - \alpha_j - \kappa, \alpha_j, \alpha_{-j}) \geq \Gamma_i((n - 1)\alpha_j - \alpha_i - \kappa, \alpha_i, \alpha_{-i})\), for any \(\kappa \in (0, (n - 1)\alpha_i]\). However, this means

\[
\frac{c}{\psi} > \int_0^{(n-1)\alpha_i} \mathbb{P}\left( R_i - D_i - Q \leq (n - 1)\alpha_i - \alpha_i - \kappa - \frac{\Gamma_i((n - 1)\alpha_j - \alpha_i - \kappa, \alpha_i, \alpha_{-i})}{\xi} \right) + \mathbb{P}(N_i \leq (n - 1)\alpha_j - \kappa) \\
\quad \times \left( 1 - \mathbb{P}\left( R_j - D_j - Q \leq (n - 1)\alpha_j - \alpha_j - \kappa - \frac{\Gamma_j((n - 1)\alpha_i - \alpha_j - \kappa, \alpha_j, \alpha_{-j})}{\xi} \right) \right) \ dF_K(\kappa)
\]

\[
\geq \int_0^{(n-1)\alpha_i} \mathbb{P}\left( R_j - D_j - Q \leq (n - 1)\alpha_i - \alpha_j - \kappa - \frac{\Gamma_j((n - 1)\alpha_i - \alpha_j - \kappa, \alpha_j, \alpha_{-j})}{\xi} \right) + \mathbb{P}(N_j \leq (n - 1)\alpha_i - \kappa) \\
\quad \times \left( 1 - \mathbb{P}\left( R_j - D_j - Q \leq (n - 1)\alpha_i - \alpha_j - \kappa - \frac{\Gamma_j((n - 1)\alpha_i - \alpha_j - \kappa, \alpha_j, \alpha_{-j})}{\xi} \right) \right) \ dF_K(\kappa) = \frac{c}{\psi},
\]

a contradiction. Thus, \(\alpha_i = \alpha_j\). Let \(\alpha_i = \alpha_j = \alpha\). This also implies that \(\Gamma_i = \Gamma_j = \Gamma\) for some function \(\Gamma\). Also, \(\Gamma\) and \(\alpha\) is jointly determined by Eq. (19).

Lemma 29. There is at least one pair of \(\alpha\) and \(\Gamma\) that satisfies Eq. (19).

The proof is omitted for it is similar to the proof of Lemma 20.

## F Appendix: Additional proofs

### F.1 Proof of Lemma 19

Fix any \(\alpha_i, \alpha_j \in [N_{\min}, \infty)\) such that \(\alpha_i < \alpha_j\). Assume bank \(i\) pays \(a_i = \min((L_i + \alpha_i)^+, N_i)\) and bank \(j\) pays \(a_j = \min((L_j + \alpha_j)^+, N_j)\). Banks optimize in the trading game so Lemma 4 holds.
Lemma 30. Fix any $x \geq y$ and $x > N_{\text{min}}$. If $y \leq N_{\text{min}}$ then $\mathbb{P}(a_i \leq x) \geq \mathbb{P}(a_j \leq y)$. If $y > N_{\text{min}}$ and $\forall n \in [N_{\text{min}}, y), \mathbb{P}(a_i \leq n - (y - \alpha_j)) \geq \mathbb{P}(a_j \leq n - (x - \alpha_i))$, then $\mathbb{P}(a_i \leq x) \geq \mathbb{P}(a_j \leq y)$.

Proof. By Lemma 9, in equilibrium $\psi \mathbb{P}(a_j \leq \alpha_i) \geq c$ and for any $\vartheta < \alpha_i$, $\psi \mathbb{P}(a_j \leq \vartheta) \leq c$. When $y \leq -\alpha_j$, then by Lemma 2,

$$\Gamma_j(y - \alpha_j, \alpha_j, \alpha_i) = \psi \mathbb{P}(a_i \leq -y) \geq c \geq \int_{n \in [N_{\text{min}}, x]} \psi \mathbb{P}(a_j \leq n - (x - \alpha_i)) \, dF_N(n) + c\mathbb{P}(N_i \geq x) = \Gamma_j(x - \alpha_i, \alpha_i, \alpha_j)$$

When $-\alpha_j < y \leq N_{\text{min}}$, then by Lemma 2,

$$\Gamma_j(y - \alpha_j, \alpha_j, \alpha_i) = c \geq \int_{n \in [N_{\text{min}}, x]} \psi \mathbb{P}(a_j \leq n - (x - \alpha_i)) \, dF_N(n) + c\mathbb{P}(N_i \geq x) = \Gamma_i(x - \alpha_i, \alpha_i, \alpha_j)$$

When $y > N_{\text{min}}$ and $\forall n \in [N_{\text{min}}, y),$

$$\mathbb{P}(a_i \leq n - (y - \alpha_j)) \geq \mathbb{P}(a_j \leq n - (x - \alpha_i)).$$

Then

$$\int_{n \in [N_{\text{min}}, y]} \psi \mathbb{P}(a_i \leq n - (y - \alpha_j)) \, dF_N(n) \geq \int_{n \in [N_{\text{min}}, y]} \psi \mathbb{P}(a_j \leq n - (x - \alpha_i)) \, dF_N(n).$$

Therefore, by Lemma 2

$$\Gamma_j(y - \alpha_j, \alpha_j, \alpha_i) = \int_{n \in [N_{\text{min}}, y]} \psi \mathbb{P}(a_i \leq n - (y - \alpha_j)) \, dF_N(n) + c\mathbb{P}(N_i \geq y) \geq \int_{n \in [N_{\text{min}}, x]} \psi \mathbb{P}(a_j \leq n - (x - \alpha_i)) \, dF_N(n) + c\mathbb{P}(N_i \geq x) = \Gamma_i(x - \alpha_i, \alpha_i, \alpha_j)$$

In any case,

$$x - \alpha_i - \frac{\Gamma_i(x - \alpha_i, \alpha_i, \alpha_j)}{\xi} \geq y - \alpha_j - \frac{\Gamma_j(y - \alpha_j, \alpha_j, \alpha_i)}{\xi}.$$
Since $R_i - D_i$ and $R_j - D_j$ have the same distribution,
\[
\mathbb{P}\left(R_i - D_i - Q + \lambda^{-1} \leq x - \alpha_i - \frac{\Gamma_i (x - \alpha_i, \alpha_i)}{\xi} \right) \geq \mathbb{P}\left(R_j - D_j - Q + \lambda^{-1} \leq y - \alpha_j - \frac{\Gamma_j (y - \alpha_j, \alpha_j)}{\xi} \right).
\]

**Lemma 4** and this inequality imply that $\mathbb{P}(L_i \leq x - \alpha_i) \geq \mathbb{P}(L_j \leq y - \alpha_j)$. Since $\mathbb{P}(N_i \leq x) \geq \mathbb{P}(N_j \leq y)$,
\[
\mathbb{P}(a_i \leq x) = \mathbb{P}(L_i + \alpha_i \leq x) + \mathbb{P}(N_i \leq x) - \mathbb{P}(N_i \leq x) \mathbb{P}(L_i + \alpha_i \leq x) \\
\geq \mathbb{P}(L_i + \alpha_i \leq x) + \mathbb{P}(N_i \leq y) - \mathbb{P}(N_i \leq y) \mathbb{P}(L_i + \alpha_i \leq x) \\
= \mathbb{P}(L_i + \alpha_i \leq x) + \mathbb{P}(N_i \leq y) - \mathbb{P}(N_i \leq y) \mathbb{P}(L_i + \alpha_i \leq x) \\
\geq \mathbb{P}(L_j + \alpha_j \leq y) + \mathbb{P}(N_i \leq y) - \mathbb{P}(N_i \leq y) \mathbb{P}(L_j + \alpha_j \leq y) \\
= \mathbb{P}(a_j \leq y).
\]

\[\square\]

Define set $\mathcal{I}_0 = [N_{\min}, \alpha_i)$, and functions $A_0, B_0 : \mathcal{I}_0 \to \mathbb{R}$ such that $A_0(n_0) = n_0$, and $B_0(n_0) = n_0$. Define correspondence $\mathcal{I}_1(n) = [N_{\min}, B_0(n))$, and functions $A_1(n_0, n_1) = n_1 - B_0(n_0) + \alpha_j$ and $B_1(n_0, n_1) = n_1 - A_0(n_0) + \alpha_i$ on domain $\mathcal{D}_1 = \{(n_0, n_1) \mid n_0 \in \mathcal{I}_0, n_1 \in \mathcal{I}_1(n_0)\}$. Iteratively, given $\ell \in \mathbb{N}_+$, $\mathcal{I}_\ell(n_0, n_1, \ldots, n_{\ell-1})$, $A_\ell(n_0, n_1, \ldots, n_\ell)$ and $B_\ell(n_0, n_1, \ldots, n_\ell)$, define
\[
\mathcal{I}_{\ell+1}(n_0, n_1, \ldots, n_\ell) = [N_{\min}, B_\ell(n_0, n_1, \ldots, n_\ell)).
\]

Then define functions $A_{\ell+1}(n_0, n_1, \ldots, n_\ell, n_{\ell+1})$, $B_{\ell+1}(n_0, n_1, \ldots, n_\ell, n_{\ell+1})$ on domain $\mathcal{D}_{\ell+1} = \{(n_0, n_1, \ldots, n_\ell, n_{\ell+1}) \mid n_0 \in \mathcal{I}_0, n_1 \in \mathcal{I}_1(n_0), \ldots, n_\ell \in \mathcal{I}_\ell(n_0, n_1, \ldots, n_{\ell-1}), n_{\ell+1} \in \mathcal{I}_{\ell+1}(n_0, n_1, \ldots, n_\ell)\}$ such that
\[
A_{\ell+1}(n_0, n_1, \ldots, n_\ell, n_{\ell+1}) = n_{\ell+1} - B_\ell + \alpha_j
\]
\[
B_{\ell+1}(n_0, n_1, \ldots, n_\ell, n_{\ell+1}) = n_{\ell+1} - A_\ell + \alpha_i.
\]

It is easy to check that
\[
A_\ell(n_0, n_1, \ldots, n_\ell) - B_\ell(n_0, n_1, \ldots, n_\ell) = \ell(\alpha_j - \alpha_i).
\]

In addition, $B_\ell(n_0, n_1, \ldots, n_\ell) \leq \alpha_i$ and $A_\ell(n_0, n_1, \ldots, n_\ell) \geq N_{\min}$ for all $\ell$ on their domain $\mathcal{D}_\ell$.

Because $A_\ell(n_0, n_1, \ldots, n_\ell) - B_\ell(n_0, n_1, \ldots, n_\ell) \to \infty$ as $\ell \to \infty$, there is one $T \in \mathbb{N}_+$ such
that

\[ \mathbb{P}(a_i \leq A_T(n_0, n_1, \cdots, n_T)) \geq \mathbb{P}(a_j \leq B_T(n_0, n_1, \cdots, n_T)) \]

on domain \( D_T \). Then by Lemma 30

\[ \mathbb{P}(a_i \leq A_{T-1}(n_0, n_1, \cdots, n_{T-1})) \geq \mathbb{P}(a_j \leq B_{T-1}(n_0, n_1, \cdots, n_{T-1})) \]

on domain \( D_{T-1} \). Apply Lemma 30 repeatedly, we will arrive

\[ \mathbb{P}(a_i \leq A_0(n_0)) = \mathbb{P}(a_i \leq n_0) \geq \mathbb{P}(a_j \leq B_0(n_0)) = \mathbb{P}(a_j \leq n_0) \]

for all \( n_0 \in [N_{\text{min}}, \alpha_i) \).

\subsection*{F.2 Proof of Lemma 20}

Let \( \mathcal{L} \) be the set of continuous decreasing functions defined on \(( -\infty, 0 ] \).

\[ \mathcal{L} = \{ f : (-\infty, 0] \to [0, c] \mid f(x) \text{ is decreasing and continuous.} \} \]

Consider a map \( \mathcal{T} : \mathcal{L} \to \mathcal{L} \) such that for any \( \Gamma_0 \in \mathcal{L} \), we have

\[ \mathcal{T}(\Gamma_0) = \Gamma. \]

\( \Gamma \) is characterized by a constant \( \alpha \), defined as

\[ \alpha = \inf \left\{ \vartheta \geq N_{\text{min}}, \ F_R \left( -\frac{\Gamma_0(0)}{\xi} \right) + F_N(\vartheta) \left( 1 - F_R \left( -\frac{\Gamma_0(0)}{\xi} \right) \right) \geq \frac{c}{\psi} \right\}, \]

in the following way: for all \(-\alpha + N_{\text{min}} \leq y \leq 0,

\[ \Gamma(y) = \int_{n \in (N_{\text{min}}, (y+\alpha))} \psi \left( F_N(n-y) + (1 - F_N(n-y))F_R(n-y - \alpha - \frac{\Gamma_0(n-y-\alpha)}{\xi}) \right) f_N(n)dn \]

+ \( c(1 - F_N(y + \alpha)) \)

and for all \( y \leq -\alpha + N_{\text{min}}, \)

\[ \Gamma(y) = c. \]
We first show that $\mathcal{T}$ is well-defined. Since $F_R$ is monotone, $F_R$ only has finite discontinuous point. Thus, when $\Gamma_0$ is continuous, $\Gamma$ is continuous.

In addition, by definition $\alpha \in [\hat{N}_{\min}, \hat{N}]$. Note that by construction,

$$F_R\left(-\frac{\Gamma_0(0)}{\xi}\right) + F_N(\vartheta) \left(1 - F_R\left(-\frac{\Gamma_0(0)}{\xi}\right)\right) < \frac{c}{\psi}$$

for all $\vartheta < \alpha$. Also note that when $n = y + \alpha$,

$$\psi \left(F_N(n - y) + (1 - F_N(n - y))F_R(n - y - \alpha - \frac{\Gamma_0(n - y - \alpha)}{\xi})\right) = c.$$ 

It follows that $\Gamma$ decreases on $\mathbb{R}^-$. Hence $\Gamma \in \mathcal{L}$ and $\mathcal{T}$ is well-defined.

Let $\mathcal{L}$ be endowed with $L^\infty$ norm. It follows that $\mathcal{L}$ is a closed and compact space, and $\mathcal{T}$ is continuous. Schauder fixed-point theorem implies that there exist at least one fixed point.
References


