FX Returns and Aggregate Risk

Adrien Verdelhan, Professor, MIT Sloan

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Adrien Verdelhan

August 2020
Why Study Exchange Rates?

- Arguably the single most important price in each country
  - Imports/exports, foreign assets/liabilities
- Largest financial market in the world
  - Foreign exchange market turnover is over $6 trillion a day
  - 18 times bigger than the trading volume on all equity markets in the world
Size of the Currency Market: Daily Turnover

Sources: BIS Triennial Surveys and World Federation of Exchanges.
Why Study Exchange Rates?

- Arguably the single most important price in each country
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- Largest financial market in the world
  - Foreign exchange market turnover is over $5 trillion a day
  - 18 times bigger than the trading volume on all equity markets in the world

- Lots of puzzles and open questions... i.e., OPPORTUNITIES!

- Warning: Vibrant field, but limited tour today due to time-constraints — I am going to focus on data issues and will miss many important contributions and potential topics, my apologies
Road Map

1. Exchange Rate Determinants (Hanno)
   - Exchange rates and short yields
   - Exchange rates and long yields
   - Exchange rates and convenience yields/balance sheet constraints

2. FX Returns and Aggregate Risk
   - From the U.I.P condition to the currency portfolios
     - Data and code to build currency portfolios
     - Empirical evidence on aggregate risk in exchange rates
   - Complete market benchmark
   - Open questions

3. FX Returns without Risk (Wenxin)
FX Returns and Aggregate Risk
Start from the definition of a currency excess return:

$$\mathbb{E}_t(r^e_{t+1}) = i^*_t - i_t - \mathbb{E}_t(\Delta s_{t+1})$$

where $S_t$: spot exchange rate in foreign currency per U.S. dollar (when $S_t \rightarrow^\uparrow$, the dollar appreciates and the foreign currency depreciates), $i_t$ is the U.S. risk-free interest rate, and $i^*_t$ is the foreign one.

Iterate forward to obtain:

$$s_t = \mathbb{E}_t \sum_{\tau=0}^{\infty} (i_{t+\tau} - i^*_{t+\tau}) + \mathbb{E}_t \sum_{\tau=0}^{\infty} r^e_{t+\tau} + \mathbb{E}_t [\lim_{\tau \rightarrow \infty} s_{t+\tau}]$$

Ignoring convenience yield & balance sheet constraints for now (Wenxin will focus on that)

How much do interest rates matter? Is there any empirical evidence of currency risk premia?
Classic Benchmark = No Risk Premia

- U.I.P.: $E_t (\Delta s_{t+1}) = i^*_t - i_t$. (Douglass, 1740) where
  - $\Delta s_{t+1}$ denotes the bilateral exchange rate in foreign currency per U.S. dollar (i.e. the foreign currency depreciates when $s$ increases)
  - $i^*_t - i_t$ is the interest rate difference between two countries.

- Rational expectations (R.E.): $\Delta s_{t+1} = E_t (\Delta s_{t+1}) + \varepsilon_{t+1}$

  \[ \Delta s_{t+1} = \alpha + \beta (i^*_t - i_t) + \varepsilon_{t+1} \]

- U.I.P. and R.E. implies that $\beta=1$.

- Classic result: $\beta$ less than 1, usually negative, $R^2$ around 0%.
  - More than 300 academic publications since 1990...
Fake Data: If U.I.P & R.E. Held...

Fictitious Changes in Exchange Rates (truncated to the -20% to +20% range)

Interest Rate Differences (truncated to the -20% to +20% range)
Actual Data

Changes in Exchange Rates (truncated to the -20% to +20% range)

Interest Rate Differences (truncated to the -20% to +20% range)
Monthly Changes in Exchange Rates — or Random Variables?

Expected Excess Returns

- U.I.P assumes risk-neutrality:
  
  U.I.P: $E_t(\Delta s_{t+1}) = i_t^* - i_t$

  Risk-Neutral: $E_t(r_{t+1}^e) = i_t^* - i_t - E_t(\Delta s_{t+1}) = 0$

- Nominal vs real:
  
  $E_t(r_{t+1}^e) = E_t (i_t^* - i_t - \Delta s_{t+1}) \simeq E_t (r_t^* - r_t - \Delta q_{t+1})$

- In U.I.P/R.E. empirical tests, if $\beta \neq 1$, expected excess returns are time-varying:
  
  $\Delta s_{t+1} = \alpha + \beta(i_t^* - i_t) + \epsilon_{t+1}$,

  $E_t(r_{t+1}^e) = i_t^* - i_t - E_t(\Delta s_{t+1}) = -\alpha + (1 - \beta)(i_t^* - i_t)$
From Time-Series to Cross-Sectional Evidence

- Idea: Extract information from currency markets using portfolios
  - Countries sorted by the level of short-term interest rates
  - Time-series vs cross-sectional evidence: Hassan and Mano (2019)

- In practice: How to build currency portfolios?
  - Sample of countries and time-window: Can foreign investors buy risk-free notes there?
    - Capital controls, default risk
  - Number of portfolios & portfolio allocation
    - Small number of countries/assets
    - Tradeoff between averaging out idiosyncratic risk and asset pricing power
    - More countries in last portfolio

- Weighting:
  - Value-weighted vs equally-weighted: GDP? Trade? FX flows?
Currency Portfolios: Annual Frequency

- Sort countries by interest rates, build portfolios of currency excess returns (Lustig and Verdelhan, 2007, 2011a)
- The higher the interest rate, the larger the average excess return
- At annual frequency: high (low) interest rate currencies tend to depreciate (appreciate) when consumption growth is low

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>-2.34</td>
<td>-0.87</td>
<td>-0.75</td>
<td>0.33</td>
<td>-0.15</td>
<td>-0.21</td>
<td>2.99</td>
<td>2.03</td>
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<tr>
<td><strong>SR</strong></td>
<td>-0.36</td>
<td>-0.13</td>
<td>-0.11</td>
<td>0.04</td>
<td>-0.02</td>
<td>-0.03</td>
<td>0.37</td>
<td>0.16</td>
</tr>
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</table>

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<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>-2.99</td>
<td>-0.01</td>
<td>-0.83</td>
<td>1.14</td>
<td>-0.69</td>
<td>-0.00</td>
<td>3.94</td>
<td>1.48</td>
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<tr>
<td><strong>SR</strong></td>
<td>-0.38</td>
<td>-0.00</td>
<td>-0.10</td>
<td>0.11</td>
<td>-0.07</td>
<td>-0.00</td>
<td>0.39</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Source: Lustig and Verdelhan, AER, 2007
**Consumption Betas of Carry Trades**

### Panel I: Simple Regression

<table>
<thead>
<tr>
<th></th>
<th>Panel A: Nondurables</th>
<th>Panel B: Durables</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta_{c}^{HML}$</td>
<td>$p(%)$</td>
</tr>
<tr>
<td>1953–2002</td>
<td>1.00 [0.44]</td>
<td>2.23</td>
</tr>
<tr>
<td>1971–2002</td>
<td>1.54 [0.52]</td>
<td>0.28</td>
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</tbody>
</table>

### Panel II: Multivariate Regression

<table>
<thead>
<tr>
<th></th>
<th>$\beta_{c}^{HML}$</th>
<th>$\beta_{d}^{HML}$</th>
<th>$\chi^2$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1953–2002</td>
<td>0.07 [0.68]</td>
<td>1.03 [0.62]</td>
<td>9.40</td>
<td>9.07</td>
</tr>
</tbody>
</table>

Source: LV, AER, 2011
At the monthly frequency, a factor structure appears:

- New risk factors: return on high- minus low-interest-rate portfolios ($HML_{FX}$) or changes in global equity volatility
- The higher the interest rate, the larger the loading on the risk factor
- High- (low-) interest-rate currencies tend to depreciate (appreciate) when global equity volatility is high

Good news: Datastream data requests and Matlab programs available to reproduce all the following results!
Data and Code

Spot and forward exchange rates:

\[ E_t(r_{t+1}^e) = f_t - s_{t+1} = f_t - s_t - \Delta s_{t+1} \]

\[ = E_t(i^*_t - i_t - \Delta s_{t+1}) \text{ if Covered Interest Rate Parity (C.I.P.) holds} \]

- Pros: actual investment opportunities (deliverable vs non-deliverable forwards), bid-ask spreads
- Cons: more limited sample, C.I.P. doesn’t hold post-financial crisis (carry more profitable on forward markets than on cash markets, more on that later)

Data and code:

- Import, clean, and merge Barclays (1983-) and Reuters (1998-) spot and forward FX series
- Import and clean MSCI equity return indices and U.S. interest rates
- Build portfolios of developed (≈ Barclays sample) and developed & emerging countries
- Run simple asset pricing experiment
Excess Returns and Betas at the Monthly Frequency

- Portfolios of countries sorted by interest rates
- Risk factors: a slope factor $HML_{FX}$, or a measure of global volatility on equity markets (no exchange rates)

<table>
<thead>
<tr>
<th>Mean</th>
<th>s.e.</th>
</tr>
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<tbody>
<tr>
<td>−1.54</td>
<td>[1.61]</td>
</tr>
<tr>
<td>0.13</td>
<td>[1.49]</td>
</tr>
<tr>
<td>1.48</td>
<td>[1.52]</td>
</tr>
<tr>
<td>3.90</td>
<td>[1.64]</td>
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<tr>
<td>4.23</td>
<td>[1.84]</td>
</tr>
<tr>
<td>7.17</td>
<td>[2.05]</td>
</tr>
</tbody>
</table>

Panel II: $HML_{FX}$ Betas

<table>
<thead>
<tr>
<th>$\beta_{HML}$</th>
<th>s.e.</th>
</tr>
</thead>
<tbody>
<tr>
<td>−0.39</td>
<td>[0.02]</td>
</tr>
<tr>
<td>−0.12</td>
<td>[0.03]</td>
</tr>
<tr>
<td>−0.13</td>
<td>[0.02]</td>
</tr>
<tr>
<td>−0.01</td>
<td>[0.03]</td>
</tr>
<tr>
<td>0.03</td>
<td>[0.03]</td>
</tr>
<tr>
<td>0.61</td>
<td>[0.02]</td>
</tr>
</tbody>
</table>

Panel III: Global Equity Vol Betas

<table>
<thead>
<tr>
<th>$\beta_{Vol}$</th>
<th>s.e.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.19</td>
<td>[0.07]</td>
</tr>
<tr>
<td>0.13</td>
<td>[0.06]</td>
</tr>
<tr>
<td>0.05</td>
<td>[0.08]</td>
</tr>
<tr>
<td>0.02</td>
<td>[0.08]</td>
</tr>
<tr>
<td>0.03</td>
<td>[0.08]</td>
</tr>
<tr>
<td>−0.42</td>
<td>[0.15]</td>
</tr>
</tbody>
</table>

Source: Lustig, Roussanov, and Verdelhan (2011)
Expected and Realized Carry Trade Excess Returns:

Expected and Realized Carry Trade Excess Returns:

Average vs High Vol Shocks (11/1983–7/2020, sample of developed countries)

Average Currency Excess Returns

Average Currency Excess Returns in Times of High Global Equity Volatility
Average vs High Vol Shocks  
(11/1983–7/2020, sample of developed and emerging countries)
Carry Trades and Equity Volatility
(11/1983–7/2020, sample of developed countries)
The countries are sorted by the slope of their yield curves into three portfolios. The holding period is one quarter. The returns are annualized. Data are monthly. Maturities are 4, 8, 12, 16, 20, 40, and 60 quarters. The dark shaded area corresponds to one-standard-error bands around the point estimates. The gray and light gray shaded areas correspond to the 90% and 95% confidence intervals. Standard deviations are obtained by bootstrapping 10,000 samples of non-overlapping returns. Source: Lustig, Stathopoulos, and Verdelhan (2019)
Taking Stock

▶ Recall the present value definition

\[ s_t = \mathbb{E}_t \sum_{\tau=0}^{\infty} (i_{t+\tau} - i^*_t) + \mathbb{E}_t \sum_{\tau=0}^{\infty} r^e_{t+\tau} + \mathbb{E}_t[\lim_{\tau \to \infty} s_{t+\tau}] \].

▶ Clear empirical evidence of currency risk premia

▶ Exchange rates are not random

▶ Immediately raises more questions, e.g.:

▶ Other conditioning variables and currency risk premia?
▶ Predictability of exchange rates?
▶ Sources of aggregate risk?
▶ Implications for households, financial intermediaries, firms, welfare?
What’s next?

► If there is a time-varying risk premium, then we have a potential driver of the exchange rate

► Is there more evidence of time-varying currency risk premia?
  

► How can we explain those risk premia? **Your job**
  
  ▶ With frictions in goods’ trading? Assets’ trading? **Your job**

► To guide your future work, let me simply remind you of another useful benchmark...
Exchange Rates in Complete Markets
A Theoretical Benchmark: Complete Markets

- Complete markets:
  - Suppose that there are $S$ possible states of nature tomorrow.
  - Financial markets are complete if investors can buy any contingent claim.
    - Each contingent claim is a security that pays one dollar in one state $s$ only tomorrow; $pc(s)$ is its price today.
  - Let $x(s)$ denote the asset’s payoff in state $s$, then the asset price $P(X)$ must be:
    \[
P(X) = \sum_{s} pc(s) x(s) = \sum_{s} \pi(s) \frac{pc(s)}{\pi(s)} x(s) = \mathbb{E}(MX)
    \]
    \[1 = \mathbb{E} \left( \frac{MX}{P} \right) = \mathbb{E}(MR)\]

- Link to risk-neutral probabilities:
  \[
P(X) = \mathbb{E}(MX) = \sum_{s} \pi(s) m(s) x(s) = \sum_{s} \pi^*(s) x(s) = \mathbb{E}^*(X)
  \]
Euler Equation

- Asset with price $P_t$ today and payoff $X_{t+1}$ next period; $\xi$ the amount of the asset the investor chooses to buy.

$$Max_{\xi} \quad u(C_t) + \mathbb{E}_t[\beta u(C_{t+1})]$$

$$C_t = Q_t - P_t \xi,$$

$$C_{t+1} = Q_{t+1} + X_{t+1} \xi.$$  

- Substituting the constraints into the objective, and setting the derivative with respect to $\xi$ to zero, yields:

$$P_t u'(C_t) = \mathbb{E}_t[\beta u'(C_{t+1})X_{t+1}],$$

The Euler equation is thus:

$$P_t = \mathbb{E}_t[\beta \frac{u'(C_{t+1})}{u'(C_t)}X_{t+1}] = \mathbb{E}_t[M_{t+1}X_{t+1}],$$

$$1 = \mathbb{E}_t[M_{t+1} \frac{X_{t+1}}{P_t}] = \mathbb{E}_t[M_{t+1} R_{t+1}].$$

where $M_{t+1} \equiv \beta \frac{u'(C_{t+1})}{u'(C_t)}$. 
Exchange Rates in Complete Markets

Consider the foreign (domestic) investor’s Euler equation for investing in foreign asset with returns $R_{t+1}^*$:

$$\mathbb{E}_t (M_{t+1}^* R_{t+1}^*) = 1 = \mathbb{E}_t \left( M_{t+1} \frac{Q_t}{Q_{t+1}} R_{t+1}^* \right).$$

- $S_t$: spot exchange rate in foreign currency per U.S. dollar
- $Q_t = S_t \frac{P_t}{P_t^*}$: real exchange rate in foreign good per U.S. good; $P_t$ is the U.S. price index (in U.S. dollars per U.S. good)

In complete markets, the percentage rate of appreciation of foreign currency is:

$$\Delta q_{t+1} = m_{t+1} - m_t^*$$

- home currency appreciates in bad times for domestic investors

- Nominal vs real:
  $$\mathbb{E}_t (r_{t+1}^e) = \mathbb{E}_t (i_t^* - i_t - \Delta s_{t+1}) \approx \mathbb{E}_t (r_{t+1}^* - r_t - \Delta q_{t+1})$$
Expected Excess Returns

- If agents are not risk neutral, then \( \mathbb{E}_t [M_{t+1} R_{t+1}^{e,j}] = 0 \), implying that:

\[
\mathbb{E}_t [R_{t+1}^{e,j}] = -\frac{\text{var}_t[M_{t+1}]}{\mathbb{E}_t[M_{t+1}]} \cdot \frac{\text{cov}_t[M_{t+1}, R_{t+1}^{e,j}]}{\text{var}_t[M_{t+1}]}.
\]

- Intuition: Expected excess return compensate investors for low returns in bad times.
Log-Normal Case

Assuming log-normal stochastic discount factors, $\mathbb{E}_t(M_{t+1} R_t) = 1$ leads to:

$$r_t = - \log \mathbb{E}_t M_{t+1} = - \mathbb{E}_t \log M_{t+1} - \frac{1}{2} \text{Var}_t(\log M_{t+1}),$$

$$r^*_t = - \log \mathbb{E}_t M^*_{t+1} = - \mathbb{E}_t \log M^*_{t+1} - \frac{1}{2} \text{Var}_t(\log M^*_{t+1}).$$

In complete markets, the expected change in the exchange rate is:

$$- \mathbb{E}_t(\log \frac{Q_{t+1}}{Q_t}) = \mathbb{E}_t(\log M^*_{t+1}) - \mathbb{E}_t(\log M_{t+1})$$

$$= - r^*_t + r_t - \frac{1}{2} \text{Var}_t(\log M^*_{t+1}) + \frac{1}{2} \text{Var}_t(\log M_{t+1}).$$

Thus, the expected foreign currency log excess return is equal to (Bekaert, 1996, Bansal, 1997):

$$\mathbb{E}_t(r_{t+1}^e) = \mathbb{E}_t(r^*_t - r_t - \Delta q_{t+1}) = \frac{1}{2} \text{Var}_t(m_{t+1}) - \frac{1}{2} \text{Var}_t(m^*_{t+1}).$$
Log-Normal Case, cont.

- Expected foreign currency log excess return:

\[
\mathbb{E}_t(r_{t+1}^e) = \frac{1}{2} \text{Var}_t(m_{t+1}) - \frac{1}{2} \text{Var}_t(m_{t+1}^*).
\]

- NB, in levels:

\[
\mathbb{E}_t(r_{t+1}^e) + \frac{1}{2} \text{Var}_t(\Delta s_{t+1}) = \text{cov}_t(m_{t+1}, \Delta s_{t+1}).
\]

- Maximal Sharpe ratio:

\[
SR = \frac{|\mathbb{E}(R^i) - R^f|}{\sigma(R^i)} \leq \frac{\sigma_t(M_{t+1})}{\mathbb{E}_t(M_{t+1})} \simeq \sigma_t(m_{t+1}).
\]
Open Questions
How to Understand Exchange Rates?

- Disaster risk: Farhi and Gabaix (2015)
- Intermediary risk: Gabaix and Maggiori (2015)
- Commodity trade risk: Ready, Roussanov, and Ward (2016)
- Inflation risk: Fang, Liu, and Roussanov (2020)
- Preferred habitat: Gourinchas, Ray, and Vayanos (2020), Greenwood, Hanson, Stein, and Sunderam (2020)

(put your name here)

(and here)

(and here)
Open questions

\[ \Delta q_{t+1} = m_{t+1} - m^{\star}_{t+1} + \varepsilon_{t+1} \]

- New properties of exchange rates:
  - Are there other risk premia in currency markets? New risk factors? Link to micro/liquidity/macro shocks?
  - Interaction between FX and other risk premia?
  - New sources of predictability?

- Why systematic variations in exchange rates?
  - Why are SDFs moving differently in different countries?
  - What are the global shocks? Where do they come from?
  - What are the micro- and macro-economic sources of cross-country and time-series variations in the shares of systematic currency risk?
  - What are the key frictions that prevent perfect risk-sharing across countries? How do countries share risk in practice? What is the key friction for financial intermediaries?

- Consequences
  - Impact of monetary and fiscal policies on exchange rates? Optimal policies with FX risk? Optimal currency hedging?
What We Discussed So Far

1. Exchange Rate Determinants (Hanno)

2. FX Returns and Aggregate Risk
   - From the U.I.P condition to the currency portfolios
   - Data and code to build currency portfolios
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3. FX Returns Without Risk (Wenxin)
What We Discussed — in pictures :)  

- FX Returns and Aggregate Risk

- Average Currency Excess Returns

- Average Currency Excess Returns in Times of High Global Equity Volatility

- Cumulative Returns on Carry Trades

- Predicted Mean Excess Return vs. Actual Mean Excess Return

- Dollar Excess Returns
THANK YOU!