Regulation and the Evolution of the Financial Sector

Vania Stavrakeva*
London Business School
PRELIMINARY DRAFT
February 10, 2016

Abstract

Bank regulation affects the size of the banking sector relative to the "shadow banking" sector, which, in turn, affects the effectiveness of bank regulation. Moreover, both affect the liquidity of financial markets. This paper evaluates the effect of the various new regulatory instruments imposed by Basel III and the Dodd Frank Act on the probability and severity of banking crises and on welfare, taking into account these feedback effects. The liquidity coverage ratio proposed by Basel III appears to be the most powerful instrument. Prohibiting banks from holding equity, as suggested by the Volcker rule, is also effective but at the cost of generating significant mis-pricing in equity markets. The optimal regulation and its impact on welfare differs significantly when the regulator takes into account the endogenous evolution of the financial sector relative to the case when she does not.

*Contact information: Vania Stavrakeva (vstavrakeva@london.edu)
1 Introduction

This paper focuses on the nexus between financial sector regulation and the resilience and evolution of the financial sector. While financial sector regulation has come to the forefront of the policy debate lately, there is little understanding of how and whether the new regulation will affect the stability of the financial sector and will improve welfare. This is particularly true if one takes into account important general equilibrium effects such as the endogenous composition of the financial sector.

Basel III and the Dodd Frank Act impose a wide range of new regulations on the financial sector such as a leverage ratio, liquidity coverage ratio (LCR) and a ban on commercial banks from engaging in proprietary trading. While the final goal is to minimize the probability of systemic banking crisis and improve welfare, the new regulation also affects the ex-ante profits of various agents in the financial sector such as commercial banks and "shadow banks". Systemic banking crises can be potentially good news for the "shadow" banking sector, which includes unlevered or less levered institutions such as hedge funds and asset managers, since they lead to firesales of both bank loans and equity and increase the profits of "shadow banks". Therefore, endogenously, regulation can change the size of the two financial sector industries in a way which can help or hurt its initial goal. Furthermore, since asset prices also directly and indirectly depend on bank regulation, the new regulatory environment will also affect the liquidity in asset markets, measured as the deviation of asset prices from their fundamental value.

In order to be able to answer the question how the new regulatory instruments will affect the stability of the financial sector, welfare and market liquidity, I build a model with three types of asset classes – equity (modelled as a Lucas tree), bank loans (modelled as a
linear production technology) and a safe asset (modelled as a storage technology). There are two types of risk neutral investors — levered bankers and unlevered asset managers — whose size is endogenous. There is also endogenous default and firesales. The model in this paper presents an alternative to the standard asset pricing portfolio choice problem. Instead of focusing on the expected return/risk trade-off, the portfolio allocation is driven by expected returns/liquidity traded-off. The various asset classes have different liquidity properties and, moreover, different investors perceive the liquidity properties of the same asset class differently. More specifically, bankers perceive loans as less liquid than equity and the safe asset since they might have to liquidate them prematurely at a price lower than their fundamental value. Since bankers face limited liability, they perceive equity and the safe asset as having the same liquidity properties since absent systemic bank default, equity trades at its fundamental value. This is not the case with the asset managers who profit from buying firesold loans in case of systemic bank default, at which point, equity trades below its fundamental value. This makes the levered bankers the natural holders of equity, which can potentially rationalize the large trading books of commercial banks prior to the crisis.

The model has three periods, where the only shock is a liquidity shock, modelled as deposit withdrawal from banks. I use the term deposits broadly to also include wholesale funding. Conditional on the realization of the liquidity shock, the equilibrium will be one of the following types. If the shock is very large, the total savings of the whole financial sector in the storage technology are not enough to be able to finance the withdrawals of depositors and there is a systemic bank default. For smaller values of the liquidity shock, there is no bank default, but the banking sector has to firesale illiquid loans. Finally, if the liquidity shock is small, there is no firesale of loans.

One of the key exogenous assumptions is that loans are more productive in the hands of the bankers (the originators of loans) than in the hands of the asset managers. This
assumption is commonly used in the banking literature and can be rationalized with more
effective monitoring power of the originator of loans (see the literature review in Acharya
and Yorulmazer (2008) for details). I also assume that bankers cannot issue new debt fast
enough in the middle of a crisis. This assumption is crucial in order for the model to have
firesales. It captures the fact that financial institutions tend to meet withdrawals by selling
assets rather than issuing new liabilities (see Shleifer and Vishny (2011) for theoretical and
empirical support regarding this assumption). To simplify the model and to make it more
realistic, I assume that there is deposit insurance. The externality in the model comes from
the fact that the infinitesimally small banker does not internalize the cost of the deposit
insurance and the fact that his actions affect the probability of systemic banking crisis,
which is a function of only aggregate variables.

I consider three regulatory instruments — a maximum leverage ratio, maximum holdings
of equity and a minimum holdings of safe assets relative debt. I calibrate the model to match
a number of US financial sector ratios as closely as possible, given the reduced form nature
of the model. After that, I calculate the effect of each one of the regulatory instruments
on welfare, default, the endogenous size of the two types of financial investors and market
liquidity. I study the question, for which regulatory instruments, the general equilibrium
effects help or hinder the goals of the regulator to increase welfare and to decrease the
probability of default. One of the most powerful instruments in terms of improving welfare
and decreasing the probability of default is the minimum safe assets holdings requirement.
While imposing no equity holdings on banks also helps improve welfare and decrease the
probability of default, it comes at a cost of significant mis-pricing of equity relative to its
fundamental value.

Finally, I ask the question of how and whether the CP’s allocation can be replicated
and how important it is for the CP to internalize the effect of regulation on the size of the
banking sector relative to the size of the "shadow" banking sector. The CP’s allocation which
takes into account general equilibrium effects can be replicated with bank capital to debt ratio of 11.6% and safe assets to debt ratio of 20% and no equity regulation. Such a policy will improve welfare by 4.6% relative to the benchmark case calibrated to the US economy. In contrast, if the regulator designs the policy without internalizing how it will affect the evolution of the financial sector, the result will be a long run improvement of welfare of only 0.4% relative to the benchmark case.

**Literature Review**

A number of papers explore the link between commercial banks and "shadow banks", where the definition of "shadow banks" varies across papers (for example, Gennaioli et al. (2013), Plantin (2015), Moreira and Savov (2016)).

Some of the more closely related papers include Begenau and Landvoigt (2015) and LeRoy and Singhania (2015). Begenau and Landvoigt (2015) study optimal capital regulation in an environment with commercial banks and levered "shadow banking" sector where safe deposits carry a liquidity premium. LeRoy and Singhania (2015) explore the link between the type of the deposit insurance and the size of the banking sector relative to the levered shadow banking sector. In contrast to both of these papers, the focus of this paper is on the two-way interaction between ex-ante regulation and the size of commercial banks relative to non levered financiers, where I consider different types of policy instruments. Furthermore, modelling-wise there are significant differences between the two papers and, as a result, the trade-offs driving the ex-ante size of the two financial sector types differ.

The model is also closely related to "the cash in the market pricing" literature since a systemic bank default arises if there does not exist a price vector for equity and loans that can clear the market and prevent period one systemic default. A number of papers

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1In this paper I use "shadow" banks as a synonym for un-levered or less levered financial firms such as asset managers. This definition is closer to the one used in policy circles. A number of papers use the term "shadow banks" as a synonym for structured investment vehicles (SIVs), which played an important part during the 2007 financial crisis.

2For example, in Begenau and Landvoigt (2015), the relative size is determined by the liquidity value of the deposits issued by each financial sector.
in that literature include Allen and Gale (1994), Allen and Gale (1998), Diamond and Rajan (2005), Begnau and Landvoigt (2015) and Acharya and Yorulmazer (2008). The most closely related paper is Acharya and Yorulmazer (2008), whose model also features a trade-off between banks hoarding safe assets in order to purchase firesold risky assets in a crisis versus investing in the risky asset ex-ante. Similarly, Acharya and Yorulmazer (2008)’s model has a sector of unlevered investors, but unlike in this paper, the size of the two types of financial sectors is exogenous. Moreover, Acharya and Yorulmazer (2008) focuses on the ex-post optimal resolution of financial crises while the focus of this paper is on ex-ante regulation, welfare and market liquidity.

This paper also relates to models of market liquidity and firesales, which feature mis-pricing of assets due to some form of market incompleteness. Prominent examples include Shleifer Vishney (1997), Brunnermeier and Pedersen (2008) and Shleifer and Vishny (2010) among many others. In these papers asset prices can be below their fundamental value for variety of reasons such as borrowing constraints, margin calls or exogenous market incompleteness. In addition to the presence of firesales due to exogenous market incompleteness, equity prices in this paper can also carry liquidity premium, which varies depending on who the marginal holder of equity is.

The first section presents the model set-up followed by the second section which describes the Central Planner’s problem. The third section presents the calibration and the key results and the last section concludes.

2 Model Set-Up

The model has three periods $t = 0, 1, 2$. There is a liquidity shock in $t = 1$, which is the only source of uncertainty in the economy. In the beginning of period zero there are two types of agents – financial experts and consumers – where each type is distributed

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3 According to Cash in the market occurs when the amount of cash in the financial system is insufficient to achieve the fair price of the asset.
uniformly on \([0, 1]\). In \(t = 0\), each financial expert can decide to be either a non-levered asset manager or a levered banker. All agents are risk neutral with a discount rate of one. There is a regulator, who has the ability to regulate only the banking sector, which also benefits from a deposit insurance. While the bankers and asset managers are ex-ante and ex-post homogeneous (they face only aggregate risk), the consumers are ex-ante homogeneous but ex-post heterogeneous. There are three types of assets — equity, modelled as a Lucas tree, safe asset, modelled as a storage technology and loans, modelled as constant returns to scale production technology.\(^4\) The model has bank default in equilibrium.

Upon birth, each financial expert is endowed with \(\Theta\) units of the Lucas tree and an exogenous endowment equal to \(e_0\). For simplicity, I assume that financial experts derive utility only from last period consumption. In \(t = 0\) each financial expert chooses whether she will be a banker or an asset manager by comparing the ex-ante expected welfare from taking each role. The size of the banking sector is given by \(\eta^b\), which implies that asset managers have a mass equal to \(\eta^a = 1 - \eta^b\), where \(\eta^a\) and \(\eta^b\) are endogenous variables.

If the financial expert chooses to be a banker, in \(t = 0\), he can give new loans which pay a return of \(A_2\) in period \(t = 2\). While the banker can not provide new loans in \(t = 1\), he can sell existing loans on the secondary market. Also the banker can buy shares in the Lucas tree or invest in the storage technology both in \(t = 0\) and \(t = 1\). The storage technology delivers one unit of the consumption good the next period while the Lucas tree pays dividends equal to \(Y_2\) in the last period. Both \(Y_2\) and \(A_2\) are deterministic. To finance his investments in \(t = 0\), the banker uses his own net worth and also collects deposits from consumers, where I assume that attracting new deposits is costly due to brick and mortar and advertising costs. I assume that in \(t = 1\) the banker cannot raise new deposits upon observing the liquidity shock. This is an important assumption which introduces loan firesale and bank failure in the model. One potential microfoundation, which I do not model explicitly, is that banks

\(^4\)I don’t formally model the lending of banks to firms. This is equivalent to ignoring any frictions in the bank-borrower contract.
can sell assets faster than they can adjust their liability side. \(^5\)

Conditional on choosing to be an asset manager, the financial expert can invest either in the Lucas tree or the storage technology in \(t = 0\) and \(t = 1\). In \(t = 1\), he can also purchase loans from the banking sector conditional on there being a fire sale where the loans in the hands of the asset managers deliver a return of \(a_2\). I assume that

\[
(1) \quad \text{Assumption 1 : } A_2 > 1 > a_2
\]

\(A_2 > a_2\) implies that the loans are more productive in the hands of the bankers than in the hands of the asset managers. Also the fact that \(1 > a_2\) will imply that the return on the loan is lower if the banker has to sell it in \(t = 1\) relative to the return on the storage technology, which introduces a non-trivial role for the storage technology in the model.

Finally, I assume that the regulator maximizes the equally weighted welfare of all agents. Since all agents are risk neutral, this is equivalent to maximizing total GDP. Before I proceed with solving for the SPE via backwards induction, I solve the representative consumer’s problem.

### 2.1 Consumer’s Problem

Each consumer receives a deterministic endowment \(e_i^c\) every period when alive. In \(t = 0\), he enters a debt contract with the banker, which promises a return \(r_0\) regardless of whether the deposit is withdrawn in period one or two. I assume that the consumers split their deposits evenly across all banks, which ensures that banks are ex-post homogeneous. While consumers are ex-ante identical, there is ex-post heterogeneity. In \(t = 1\) each banker

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\(^5\)In reality banks can access the interbank market or can obtain short term loans from the Central Bank. Here I do not model interbank lending, thereby focusing on crises where the interbank market either freezes or provides insufficient liquidity, which was the case during the last crisis, and CBs do not or cannot provide sufficient liquidity in a timely fashion, given the size of withdrawals.
learns that he is either an early consumer with preferences $\sum_{t=0}^{t=1} c_t^i$ or a late consumer with preferences $\sum_{t=0}^{t=2} c_t^i$. Early consumers die in period one while late consumers die in period two. The probability of being an early consumer is given by $\xi_1$ and the shock is iid across bankers. $\xi_1$ is unknown as of period zero. The CDF and PDF of $\xi_1$ are given by $F(\xi_1)$ and $f(\xi_1)$, respectively, where the support of $\xi_1$ is $[0, \xi]$. Since the late consumers are indifferent whether to withdraw their deposits in $t = 1$ or $t = 2$, I assume that they choose to keep them until $t = 2$. However, I assume that they cannot provide new deposits to the bankers. Finally, in $t = 1, 2$ the consumer might have to pay a lump sum tax if the banking sector defaults. The optimization problem of the representative consumer implies that $r_0 = 1$. For details on the consumer problem see Appendix, Section A.1.

2.2 Solving the Model

In this section I sketch the solution to the model via backwards induction and delegate detailed derivations to the Appendix Section A.2. In $t = 2$, asset manager $i$ consumes the return on his portfolio

$$c_{i,a}^2 = Y_2 x_{i_1}^a + a_2 k_{i_1}^a + s_{i_1}^a$$

where $x_{i_1}^a, k_{i_1}^a, s_{i_1}^a$ are his period one holdings of the Lucas tree, loans purchased in $t = 1$, if any, and the investment in the storage technology.

Conditional on no bank default in $t = 1$, in $t = 2$, banker $i$ consumes

$$c_{i,b}^2 = \max \left\{ A_2 \left( k_i^0 - k_{i,f}^1 \right) + Y_2 x_{i_1}^b + s_{i_1}^b - r_0 (1 - \xi_1) d_{i_0}^b, 0 \right\}$$

where $k_i^0$ is period zero loans given by the bank and $k_{i,f}^1$ is the amount of loans sold by the bank to the asset manager in $t = 1$. $x_{i_1}^b$ and $s_{i_1}^b$ are banker $i$’s period one holdings of the Lucas tree and investment in the storage technology. $d_{i_0}^b$ is the period zero debt chosen by
the banker and \((1 - \xi_1) d_0^{i,b}\) are the deposits of the late consumers.

In \(t = 1\), after the realization of \(\xi_1\), the asset manager maximizes

\[
\max_{k_1^{i,a},x_1^{i,a},s_1^{i,a}} c_2^{i,a} = \max_{k_1^{i,a},x_1^{i,a},s_1^{i,a}} (Y_2 x_1^{i,a} + a_2 k_1^{i,a} + s_1^{i,a})
\]

The optimization problem is subject to the period one budget constraint

\[
(2) \quad p_1 x_0^{i,a} + s_0^{i,a} \geq q_1 k_1^{i,a} + s_1^{i,a} + p_1 x_1^{i,a}
\]

where \(p_1\) is the period one price of equity and \(q_1\) is the secondary market period one price of loans. \(x_0^{i,a}\) and \(s_0^{i,a}\) are period zero investment in equity and in the storage technology. The optimization problem is also subject to the no short selling constraints, \(s_1^{i,a} \geq 0, k_1^{i,a} \geq 0\) and \(x_1^{i,a} \geq 0\).

Conditional on no bank default in \(t = 1\) and no expected default in \(t = 2\), banker \(i\) solves the following optimization problem

\[
\max_{k_1^{i,f},x_1^{i,b},s_1^{i,b}} c_2^{i,b} = \max_{k_1^{i,f},x_1^{i,b},s_1^{i,b}} \left( A_2 \left(k_0^i - k_1^{i,f}\right) + Y_2 x_1^{i,b} + s_1^{i,b} - r_0 d_1^{i,b}\right)
\]

subject to the period one budget constraint of the banker

\[
(3) \quad p_1 x_0^{i,b} + q_1 k_1^{i,f} + s_0^{i,b} \geq s_1^{i,b} + p_1 x_1^{i,b} + r_0 \xi_1 d_0^{i,b}
\]

where \(x_0^{i,b}\) and \(s_0^{i,b}\) are banker \(i^i\)’s period zero holdings of the Lucas tree and investment in the storage technology. The optimization problem is also subject to the no short selling constraints \(s_1^{i,b} \geq 0, k_1^{i,f} \geq 0\) and \(x_1^{i,b} \geq 0\).
The bank will default in $t = 1$ if there does not exist a vector of market clearing prices $\{p_1, q_1\}$ such that even if the banker sells all of his assets, he will be able to repay the early consumers. There will be period one default as long as

$$r_0 \xi_1 d_0^{i,b} > q_1 k_0^i + p_1 x_0^{i,b} + s_0^{i,b}.$$  

If there is default in $t = 1$, the regulator sells all of the bank assets and the proceeds go to the deposit insurance fund. If the bank is expected to default in $t = 2$ ($e_2^b = 0$) but does not default in $t = 1$, the government takes over the bank and continues operating it. In order to simplify the problem, I consider parametrization such that if $q_1 = a_2$ and $Y_2 = p_1$ then there will be no default in $t = 1$ (i.e. $r_0 \xi d_0^b < a_2 k_0 + Y_2 x_0^b + s_0^b$). Sufficient but not necessary condition for this to be the case will be $a_2 > \bar{\xi}_1$ and the brick and mortar cost of deposit collection (to be defined later on) not being very high.

The following lemma summarizes important features of the admissible equilibria. For example, in $t = 1$, the banker sells loans in order to repay depositors only after using all of his savings in the storage technology and selling all of his equity. Also the secondary market price of loans, $q_1$, is lower than $A_2$ which is the fundamental value of the loan if kept in the hands of the banker. This is the source of the deadweight loss associated with the sale of loans on the secondary market, which is what I refer to as a resale of loans in this model.

**Lemma 1:** In a symmetric equilibrium, $p_1 \leq Y_2$ and if $k_1^f > 0$, then $q_1 \leq a_1$. If there is a resale of loans, $k_1^f > 0$, then $x_1^b = s_1^b = 0$.

Proof: See Appendix, Part A.2.

The following proposition characterizes the period one equilibrium, conditional on the period zero endogenous state variables, $\{k_0, x_0^a, x_0^b, s_0^a, s_0^b, d_0^b\}$ and on the realization of the

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6This case is equivalent to the policy maker solving the same problem as the banker in $t = 1$ where the policy maker minimizes the losses on the deposit insurance fund. The set of first order conditions is identical to the period one problem of the banker conditional on no default, which is why I do not reproduce it again.
exogenous state variable $\xi_1$.

**Proposition 1**: Conditional on Assumption 1 and assuming a symmetric equilibrium, in $t = 1$ there are three types of equilibria:

**Type 1**) No loan firesale in $t = 1$: $\left(\xi_1 \leq \hat{\xi}\right)$  
$k_1^l = 0$, $p_1 = Y_2$

**Type 2**) Loan firesale and no default in $t = 1$:  
$k_1^f > 0$, $p_1 = Y_2$, $q_1 = a_2$

**Type 3**) Loan firesale and default in $t = 1$:  
$(\xi_1 > \xi^c)$  
$k_1^f > 0$, $p_1 = \frac{\eta^a s^a_t}{Y^2} \frac{\eta^b s^b_t}{Y^2} < Y_2$,  
$q_1 = \frac{p_1}{Y^2} a_2 < a_2$

where

\[
\xi^c = \min \left\{ \frac{S_0}{\eta^b p_0 d^b_0}, \frac{\hat{\xi}}{c} \right\}
\]

\[
\hat{\xi} = \min \left\{ Y^2 a^b_0 + s^b_t \frac{r_0 d^b_0}{r_0 d^b_0}, \xi^c \right\}
\]

and $S_t = \eta^a s^a_t + \eta^b s^b_t$.

**Proof**: See Appendix, See Appendix, Part A.2.

The equilibrium is such that there are three regions. If the equilibrium is of Type 1, the liquidity shock is fairly small and the banking sector does not sell any loans to the asset managers. The price of equity is equal to its fundamental value, making the banker and the asset manager indifferent as to whether they invest their period one profits in equity or in the storage technology. From Lemma 1 and Proposition 1, if the equilibrium is of Type 2, the banking sector does not default in $t = 1$ but there is a sale of loans from the banks to asset managers on the secondary market which leads to a welfare loss since the loans are less productive in the hands of the asset managers. Finally, if the equilibrium is of Type 3, the whole banking sector defaults. This will be the case only if the savings in the storage technology of the whole financial sector — bankers and asset managers — are insufficient to repay the early consumers. If there is default, all the banks’ assets are sold to the asset managers and the higher the period zero amount of bank loans and bank equity holdings
are, the lower the prices of loans and equity are. Also the higher the savings of the asset
managers are, the higher the secondary market asset prices are. Finally, all else equal, the
larger the size of the asset management sector is, the higher \( p_1 \) and \( q_1 \) are. Since ex-ante
regulation will affect all of these period zero endogenous state variables, it will have direct
effect on market liquidity in case of systemic banking crisis. Also ex-ante regulation will
affect the probability of the period one equilibrium being each one of the three types.

In \( t = 0 \) the asset manager takes into account his and other agents’ future best response
functions and solves the following optimization problem

\[
U^{i,a} = \max \int_0^{\xi^c} \left( Y_2 x_0^{i,a} + s_0^{i,a} \right) f(\xi_1) \, d\xi_1 + \int_{\xi^c}^{\hat{\xi}} \frac{Y_2}{\hat{p}_1} (\hat{p}_1 x_0^{i,a} + s_0^{i,a}) \, f(\xi_1) \, d\xi_1
\]

where \( \hat{\xi}_1 = \frac{\eta a_0 a_2 Y_2}{\eta b_0 k_0 + \eta^2 x_0^0} \) is the period one price of equity in the state of default. Since
\( \frac{Y_2}{\hat{p}_1} > 1 \), this will imply that the marginal value of asset manager’s wealth is higher in the
crisis state in \( t = 1 \) than in the no crisis state. The asset manager takes the aggregate
variables \( \xi^c \) and \( \hat{p}_1 \) as given. The optimization problem is also subject to the period zero
budget constraint

\[
e_0 + p_0 \Theta \geq p_0 x_0^{i,a} + s_0^{i,a}
\]

and the two short-selling constraints \( s_0^{i,a} \geq 0 \) and \( x_0^{i,a} \geq 0 \), where \( p_0 \) is the period zero price
of equity.

In \( t = 0 \), the probability of banker \( i \) not having to sell loans next period is determined
by the probability of \( \xi_1 \leq \xi^i \). The probability of banker \( i \) having to sell loans in \( t = 1 \) but
not defaulting either in periods one or two is given by the probability of \( \xi^i \geq \xi_1 > \xi^i \), where
the the cutoffs are determined by
\[ \xi^i = \min \left\{ \frac{Y_2 x_0^{i,b} + s_0^{i,b}}{r_0 d_0^{i,b}}, \xi^c \right\} \]

\[ \tilde{\xi}^i = \min \left\{ \frac{a_2 k_0^i + Y_2 x_0^{i,b} + s_0^{i,b} - \frac{a_2}{A_2} r_0 d_0^{i,b}}{(1 - \frac{a_2}{A_2}) r_0 d_0^{i,b}}, \xi^c \right\}. \]

Banker \( i \) internalizes the fact that \( \xi^i \) and \( \tilde{\xi}^i \) can depend on his own actions but he takes aggregate variables such as prices and the period one default cut-off, \( \xi^c \), as given. With that in mind, the objective function of the banker in \( t = 0 \) can be re-written as

\[
U^{i,b} = \max_{k_0^{i,b}, d_0^{i,b}, x_0^{i,b}, s_0^{i,b}} E_0 e_2^{i,b} = \max_{k_0^{i,b}, d_0^{i,b}, x_0^{i,b}, s_0^{i,b}} F \left( \tilde{\xi}^i \right) \left( A_2 k_0^i + Y_2 x_0^{i,b} + s_0^{i,b} - r_0 d_0^{i,b} \right) + \left( F \left( \xi^i \right) - F \left( \tilde{\xi}^i \right) \right) \left( \frac{A_2}{a_2} - 1 \right) \left( Y_2 x_0^{i,b} + s_0^{i,b} \right) - \int_{\xi^i}^{\tilde{\xi}^i} \left( \frac{A_2}{a_2} - 1 \right) r_0 c_1 d_0^{i,b} f \left( \xi_1 \right) d\xi_1
\]

The second term of the objective function captures the fact that if there is a loan firesale in \( t = 1 \) but no default, then larger holdings of equity and the safe asset improve banker \( i \)'s welfare by decreasing the deadweight loss from the firesale. The reverse is true with respect to debt which is captured by the last term. The optimization problem is subject to the period zero budget constraint

\[
k_0^i + C \left( d_0^{i,b} \right) \leq e_0^b + a_0^{i,b} - p_0 x_0^{i,b} - s_0^{i,b} - \left[ \lambda_0^{i,b} \right],
\]

where \( C \left( d_0^{i,b} \right) \) is the brick and mortar cost of attracting deposits and I assume that \( C' \left( d_0^{i,b} \right) > 0, C'' \left( d_0^{i,b} \right) > 0, C \left( 0 \right) = 0 \) and \( C' \left( 0 \right) = 0 \). The Lagrange multipliers are given in square brackets. The banker also takes into account the no short-selling constraints \( s_0^{i,b} \geq 0 \).
0, \( k^i_0 \geq 0 \), \( x^i_0 \geq 0 \) and \( d^i_0 \geq 0 \), and the regulatory constraints

\[
\begin{align*}
    s^i_0 & \geq d^i_0 \bar{s} \quad [\nu^i_s] \\
    x^i_0 & \leq \frac{\Theta}{\eta^b} \quad [\nu^i_x] \\
    \psi d^i_0 & \leq e_0 \quad [\nu^i_e]
\end{align*}
\]

where \( 0 \leq \bar{x} \leq 1 \). Assuming symmetric equilibrium for the bankers and the asset managers, the system of equations which determines the period zero endogenous variables \( \{ k_0, x^a_0, x^b_0, s^a_0, s^b_0, d^a_0 \} \), the price \( p_0 \) and the Lagrange multiplier \( \lambda^b_0 \) is given by the two budget constraints

\[
k_0 + C (d^b_0) = e^b_0 + d^b_0 - p_0 x^b_0 - s^b_0
\]

\[
e_0 + p_0 \Theta = p_0 x^a_0 + s^a_0
\]

the banker’s and the asset manager’s first order conditions

(5) \[
    x^i_0 : \frac{Y_2}{p_0} + \left( 1 - \frac{Y_2}{p_1} \right) \left( 1 - F (\xi^c) \right) - \mu^a_0 + \frac{\mu^a_0}{p_0} = 1
\]

\[
d^i_0 : F (\hat{\xi}) + \left( \frac{A_2}{a_2} - 1 \right) \int_{\xi}^{\hat{\xi}} \xi_1 f (\xi_1) d\xi_1 + \nu^a_0 \bar{s} + \nu^a_0 \psi = \lambda^b_0 - \lambda^b_0 C^a (d^a_0)
\]

(6) \[
k_0 : A_2 F (\hat{\xi}) + \mu^b_0 = \lambda^b_0
\]

14
\begin{align}
\left(x_0^i, b \right) & : \frac{Y_2}{p_0} \left[ F \left( \xi \right) + \left( F \left( \xi \right) - F \left( \xi \right) \right) \left( \frac{A_2}{a_2} - 1 \right) \right] + \frac{\mu_{0,x}^i}{p_0} + \frac{\nu_0}{p_0} + \lambda_0^b \\
\left(s_0^i, b \right) & : F \left( \xi \right) + \left( F \left( \xi \right) - F \left( \xi \right) \right) \left( \frac{A_2}{a_2} - 1 \right) + \mu_{0,s}^i + \nu_0^s = \lambda_0^b
\end{align}
and the market clearing condition

\[ \eta^a x_0^a + \eta^b x_0^b = \Theta \]

where \( \mu_{0,s}^a, \mu_{0,x}^a, \mu_{0,b}^a, \mu_{0,x}^b, \mu_{0,k}^b \) are the Lagrange multipliers on the short selling constraints from the asset manager’s and the banker’s problem where the superscript \( s \) stands for storage technology, \( x \) for equity and \( k \) for loans. In the Appendix I provide the complementarity slackness conditions which determined these Lagrange multipliers and in Lemma 1A in the Appendix I prove that the banker will always borrow positive amount of debt, \( d_0^b > 0 \).

The following Lemma establishes that the bankers are the natural holders of equity.

**Lemma 2:** Part 1) \( p_0 \leq Y_2 \)

Part 2) In \( t = 0 \) the asset managers will hold equity if there is a non zero probability of default in \( t = 1 \) only if \( p_0 < Y_2 \)

Part 3) In \( t = 0 \), if there is no ex-ante regulation and if \( p_0 < Y_2 \), the banker will prefer investing in equity over the storage technology and if \( Y_2 = p_0 \), he will be indifferent between the two assets.

**Proof:** See Appendix, Section A.2.

The intuition for Part 1) of Lemma 2 follows from the fact that both agents can invest in a storage technology with a gross return of one. This provides a lower bound on the return that equity investment has to provide. The intuition for Part 2) follows from the first order
condition with respect to $x^{i,a}_0$, which equates the marginal benefit of an extra dollar used to purchase equity to the marginal cost, which is equal to 1. Notice that if there is default in $t = 1$, then the marginal benefit of an extra dollar of equity is lower, which can been seen from the fact that \((1 - \frac{Y_2}{p_1}) < 0\). This is the case because, during a bank default, when the marginal value of asset manager’s wealth is higher, the resale value of equity is lower than its fundamental value, $Y_2$. In contrast, investing in the storage technology guarantees a return of one. Finally, Part 3) follows from the first order conditions with respect to $x^{i,b}_0$ and $s^{i,b}_0$.

With no ex-ante regulation, as long as $p_0 < Y_2$, the marginal benefit of investing in equity is higher relative to the marginal benefit from investing in the storage technology from the banker’s point of view. This is the case since the banker, unlike the asset manager, does not internalize the default states of nature given its limited liability and, unless there is default, the price of equity is equal to the fundamental value of the Lucas tree, $Y_2$. The banker is indifferent between the storage technology and equity if $Y_2 = p_0$ since he perceives the assets as perfect substitutes.

In summary, if the equilibrium solution is such that $p_0 < Y_2$, the banker will not hold any of the safe asset. In contrast, if the equilibrium is such that there is default in $t = 1$ and $p_0 = Y_2$, then the banker will hold all of the Lucas tree.

Finally the size of the asset management sector relative to the banking sector will be determined by equating the ex-ante welfare of the banker and the asset manager. $\eta^a$ will be determined by the following expression

$$U^b = (A_2(\xi_0) + Y_2 x^{b}_0 + s^{b}_0 - d^{b}_0) F(\xi) + \left( \frac{A_2}{a_2} - 1 \right) (Y_2 x^{b}_0 + s^{b}_0) \left( F(\xi) - F(\xi') \right)$$

$$- \left( \frac{A_2}{a_2} - 1 \right) \int_{\xi}^{\xi'} d^{b}_0 f(\xi_1) d\xi_1 \geq ((Y_2 - p_0) x^{a}_0 + e_0 + \Theta p_0) F(\xi') + (1 - F(\xi')) \left( a_2 \eta^{a} - k_0 + \frac{Y_2 \Theta}{\eta^{a}} \right) = U^a$$

where equation 8 takes into account the fact that if there is default in $t = 1$, all the equity and
loans are held by the asset managers. If equation 8 holds with an equality, then $0 < \eta^a < 1$ and if it holds with an inequality, then $\eta^a = 0$. In the following section, I define the Central Planner’s problem.

3 Central Planner’s Problem

In this section I define the Central Planner’s (CP’s) problem. The CP places equal weights on all agents. She chooses all of the banker’s variables and solves the problem backwards by taking into account the best response functions of consumers and asset managers and the market clearing conditions. I assume that the CP cannot change the regulatory environment in case there is bank default. This implies that the CP takes as given the fact that if there is default in $t = 1$ the regulator sells all of the banks’ assets and if there is expected default in $t = 2$, then the optimization problem is the same as the one solved by the regulator in $t = 1$, who minimizes the period two loss to the deposit insurance fund.\footnote{As already mentioned, the solution to this last problem coincides with the first order conditions of the banker in $t = 1$ if there is no default in $t = 1$.} Finally, the CP, takes as given the availability of the deposit insurance. As a result, the focus is on ex-ante externalities.

I consider two cases. In the first case the CP chooses the optimal allocation for an exogenous $\eta^a$ and in the second case he internalizes how his choices will affect the endogenous $\eta^a$ determined by equation 8.

The period $t = 2$ and $t = 1$ problems of the banker and the CP coincides. If $\xi^c \leq \bar{\xi}$, which is the case in all calibrations considered, in $t = 0$, the CP’s problem becomes equivalent to

$$U^{CP} = \max_{\mu_0^a, \mu_0^b, \nu_0, k_0, d_0, a_0, b_0} \eta^b \left( F(\xi^c) A_2 + a_2 (1 - F(\xi^c)) k_0 + Y_2 \Theta + \eta^a s_0^a + \eta^b s_0^b - \eta^b d_0^b \right)$$

$$+ \eta^b \left(1 - \frac{A_2}{a_2}\right) \int_{\xi^c}^{\bar{\xi}} (\xi_1 d_0^b - (Y_2 x_0^b + s_0^b)) f(\xi_1) d\xi_1$$

$$7$$
where the optimization problem is subject to the period zero budget constraint of the banker, the short selling constraints and the period zero first order conditions of the asset manager. For details see Appendix, Section A.3.\(^8\)

\section{Calibration and Results}

In this section, I solve the model numerically where I choose a calibration that matches certain empirical facts about the US financial sector. While this model is not complex enough to match the data in all dimensions, by calibrating it as closely to the data as possible, I ensure that the types of equilibria that occur in the simulations are potentially close to what we might expect in reality. In order to perform the simulations, I assume that \(\xi_1^{-1} U [0, \bar{\xi}]\) and \(C (d^b_0) = \frac{b}{2} (d^b_0)^2\).

Before I discuss the calibration, I describe some important general features of the model. If there is no default, then the CP’s and the decentralized equilibria coincide since the CP’s and the banker’s optimization problems coincide. Furthermore, if there is no default in \(t = 1\) and \(p_0 = Y_2\), multiple equilibria exist since the storage technology and the Lucas tree become perfect substitutes for both agents. When I report the results, if there are multiple equilibria, I choose the one with the smallest \(s^b_0\). This assumption is innocuous given that in cases with no period one default there is no need for ex-ante regulation.

In order to calibrate the model I use a specification with endogenous \(\eta^a\) and binding leverage regulation where \(\bar{\psi} = \frac{0.096}{1-0.996}\). \(\bar{\psi}\) is calibrated to match the equity to total assets of US commercial banks over the 1995-2014 period, which was equal to 9.6\%.\(^9\) It would be incorrect to calibrate the model to a specification without regulation given that banks were

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\(^{8}\)I omit the endowments of the consumers from the CP’s problem since they only scale the objective function without affecting the optimization problem. I do that so that I do not have to calibrate them later on.

\(^{9}\)\(\bar{\psi}\) is calculated assuming that the bank equity to total assets in the model equals \(\frac{e_0}{e_0 + d_0}\) which is approximately correct for low \(b\).
regulated over the period considered. The leverage regulation in this model is the closest to a minimum bank capital requirement, which was the main binding regulatory instrument over that period. Given that the liquidity coverage ratio (LCR) and the Volcker rule, which forbids commercial banks from engaging in proprietary trading, were not in effect over the sample I use to calibrate the model, I do not impose these regulatory instruments when matching the data.

I calibrate the model to match a number of averages and ratios in the data as closely as possible. $A_2$ is chosen to match the net interest margin (NIM) of US commercial banks between 1995 and 2014, which is equal to 3.7 percent.\textsuperscript{10} The corresponding quantity that I match in the model is $A_2 - 1$.\textsuperscript{11} One can attempt to match the return on equity as well, $\frac{Y_2 - p_0}{p_0}$, calculated as the average excess return of the S&P between 1995 and 2014, which is equal to 4.4 percent. However, the model will not be able to match the slightly higher equity risk premium relative to the net interest income of banks. The reason why is apparent from combining equations 7 and 6. Consider the case $\mu_{k,b} = \mu_{x,b} = \nu_0^r = 0$, which will be the relevant equilibrium case for the calibration considered here. If the probability of firesale of loans without default is zero, $F(\tilde{\xi}) - F(\hat{\xi}) = 0$, then $\frac{Y_2 - p_0}{p_0} = A_2 - 1$. If it is greater than zero, it will be the case that $\frac{Y_2 - p_0}{p_0} < A_2 - 1$ since equity receives a liquidity premium given that it insures the banker in states of nature with loan firesale and no default.\textsuperscript{12} According to Irani and Meisenzahl (2015), between 2007 and 2010, all banks supervised by the US regulators sold as much as 20% of their syndicated loans portfolio, which is the number I use to calibrate the upper bound on the liquidity shock, $\tilde{\xi}$. $b$ is calibrated to match US banks’ overhead cost to total assets, which was on average 3.1% from 1999 to 2013. The respective

\textsuperscript{10}See Appendix for details on data sources.

\textsuperscript{11}In the model the net rate of return on deposits is zero. Also the net safe rate, proxied by the return on the storage technology, is zero as well.

\textsuperscript{12}The leverage ratio does not map perfectly to a risk weighted minimum bank capital requirement. Since equity is riskier, banks have to hold more capital against it, relative to other safer assets. Allowing for risk weighted minimum bank capital requirement in the calibrated economy can potentially allow this model to match the historical equity premium as well.
metric in the model is given by \( \frac{C(d_0)}{k_0 + p_0 x_0 + s_0} \). However, since the overhead costs include more than just brick and mortar costs, this number is just suggestive of the upper bound of the brick and mortar costs. \( \Theta \) and \( a_2 \) are chosen to improve the fit. Note that the results are not particularly sensitive to varying either \( \Theta \) or \( a_2 \).

The final set of parameters that brings us as close as possible to the set of empirical moments is

<table>
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<th>( Y_2 )</th>
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<th>( a_2 )</th>
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<th>( \Theta )</th>
<th>( b )</th>
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Table 1:

where I assume that \( e_c^c \) is such that \( e_0^c \geq \eta_b d_0^b \) and \( e_1^c \) and \( e_2^c \) are larger than the shortfall of the deposit insurance in states of default. Below I show how well the calibrated model matches the moments considered

<table>
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<tr>
<th>( \frac{e_0}{e_0 + \tilde{d}_0} )</th>
<th>( A_2 - 1 )</th>
<th>( Y_2 - p_0 )</th>
<th>( \frac{C(d_0)}{k_0 + p_0 x_0 + s_0} )</th>
<th>( \bar{\xi} )</th>
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<td>0.037</td>
<td>0.013</td>
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<tr>
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<td>0.044</td>
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</table>

where the match with \( \frac{e_0}{e_0 + \tilde{d}_0} \), \( A_2 - 1 \) and \( \bar{\xi} \) is by construction and I already discussed why the model will not be able to match an equity risk premium higher than the return on loans. Finally, the brick and mortar costs are below the total bank operating cost as required. The equilibrium size of the asset management sector, which ensures that \( U^a = U^b \), is given by \( \eta^a = 0.488 \). I will discuss the rest of the endogenous variables of the benchmark calibrated economy later on.

Before studying the effect of regulation on default, welfare and on the evolution of the financial sector, in Graph 1, I present the comparative statics of the key endogenous variables with respect to \( \eta^a \) in the case with no ex-ante regulation. In all of the simulations considered, it is always the case that \( \bar{\xi} = \xi^c \). First consider the region \( \eta^a < 0.6 \). The banker holds all of
the equity and does not invest in the storage technology. As $\eta^a$ increases the probability of systemic banking crisis decreases while the probability of loan firesale without bank default is zero. The former can be explained by rising aggregate savings and falling total banking sector debt, $\eta^b d^b_0$. Since when $\eta^a < 0.6$, the probability of loan firesale without default is zero, equity does not have liquidity premium over loans from the perspective of the banker, and the long run returns on both asset classes are equalized and equal to 0.037. In that region also aggregate savings in the storage technology $S_0$ increase since the share of asset managers increases and they hold positive savings in the storage technology while bankers do not. The equity holdings per bank also increase because there are fewer banks to hold the Lucas tree. The welfare of the asset managers decreases while the welfare of the bankers increases monotonically.\footnote{The $U^a, U^b$ graph starts at $\eta^a = .4$ for visual purposes.}

In the region where $\eta^a \geq 0.6$, the probability of bank default is zero since savings in the whole financial sector are larger than the amount of deposits withdrawn in $t = 1$ even if the worst liquidity shock is realized, but the probability of loan firesale without default increases. The long run return on equity is lower than that on loans since the equity carries a liquidity risk premium due to the non-zero probability of loan firesales without default. Equity and the storage technology become perfect substitutes from the perspective of the banker and banks now hold positive amount of both. Since banks internalize the fact that there is a non zero probability of having a loan firesale without default they decrease their investment in loans and their leverage.
Next I focus on the effect of regulation on welfare, default and other equilibrium variables of interest. I present the results for the case where $\eta^a$ changes endogenously with regulation (the general equilibrium (GE) case). As a comparison, I also report the partial equilibrium
(PE) case, where I assume that the size of the two financial sectors does not change endogenously when the regulatory environment changes. Instead of solving the Ramsey problem, (i.e. solving for the optimal regulation given the instrument allowed), I set the regulatory ratios in a manner that is somewhat consistent with the new policy instruments introduced in Basel III and the Dodd Frank Act. I also report the constrained CP’s allocation and, later on, discuss how it can be decentralized with the instruments at hand.

Basel III introduced a maximum leverage ratio, which maps to the model as the following constraint \( a \leq \frac{e_0}{(d_0^e + e_0)} \). Currently, Basel III recommends \( a = 0.06 \) for systemically important banks. In the calibrations, I impose \( a = 0.1 \), which implies \( \bar{\psi} = \frac{0.1}{(1-0.1)} \), since the maximum leverage ratio does not bind when \( a = 0.6 \). The liquidity coverage ratio (LCR) is the second new regulatory instrument imposed by Basel III, which focuses on regulating the composition of the banks’ liability side. The idea behind the LCR is that the bank needs to hold enough safe assets to be able to cover 100% of projected withdrawals in a stress testing scenario. The only safe asset in the model is the storage technology. Therefore, the LCR maps to the minimum safe asset requirement in the model. I set \( \bar{s} = 0.15 \), which implies that the ratio of safe assets to deposits has to be greater than or equal to 15%. Finally, the Volcker rule which is a part of the Dodd Frank Act, postulates no holdings of equity by commercial banks, which maps to \( \bar{x} = 0 \).

A minimum bank capital requirement, which is also part of Basel III, is redundant in this model, given the presence of the other three regulatory instruments. However, given the larger heterogeneity of asset classes that banks hold in the real world, there might be an independent role for a risk based minimum bank capital requirement, which is not modelled here.

The three new regulatory instruments have not been fully implemented yet, which is why one cannot test the model’s predictions with respect to the change in regulation in the data.

\[ \text{Since the LCR is a function of complicated stress testing models, it is not trivial to know what is the exact ratio of safe assets over debt that will be imposed by regulators.} \]
yet. However, the calibrations presented here are suggestive of the expected effectiveness of the new regulatory instruments and of their impact on market liquidity.

I consider the following six scenarios, where the parametrization for each one of them is the same as in the calibrated economy and is given by Table 1.

Benchmark Case) Only maximum leverage requirement, where $\psi = \frac{0.096}{1-0.096}$ (same as calibrated economy);

Scenario 1) No ex-ante regulation;
Scenario 2) Only maximum leverage requirement, where $\psi = \frac{0.1}{1-0.1}$;
Scenario 3) Only maximum equity holding requirement where $x = 0$;
Scenario 4) Only minimum safe assets holding requirement where $s = 0.15$;
Scenario 5) All three instruments imposed jointly where $\psi = \frac{0.1}{1-0.1}$, $x = 0$ and $s = 0.15$;
Scenario 6) Optimal allocation of the constrained CP.

Tables 3 and 4 summarizes the main results of the paper while the graphs labelled "Scenario" 2 to 5 in the Appendix present the equilibrium variables as a function of $\eta^a$. Tables 3 and 4 also contain two additional specifications related to the decentralization of the CP’s problem, which I will define later on.

Since all agents are risk neutral and there is only one source of risk in the economy, the model will not be able to match quantities well such as the holdings of equity of levered versus unlevered financial investors. Notice that in the benchmark case all the equity is held by the bankers who are the natural holders of equity. However, the goal of this paper is to focus on the expected return/liquidity trade-off rather than on standard asset pricing channels.

In the no regulation case (Scenario 1), the CP’s welfare, given by $U^{CP}$, is lower relative to the benchmark case. Considering the PE case, which can be interpreted as the equilibrium one might observe in the short run if regulators eliminate all regulation, $\frac{\sigma_p}{\delta}$ decreases by 1.4 percentage points and the probability of default increases from 2.6% to 15.2%. Given
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<th>$U^{CP}$</th>
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</table>

BC stands for benchmark case.

the higher probability of default, the ex-ante profits of asset managers, $U^a$, will also increase relative to the banker’s ex-ante profits, $U^b$, which over time, will increase the entry of financial experts into the asset management business and the economy will converge to the equilibrium given by the GE case, which also features $U^{CP}$ lower than the benchmark case. In the GE case, the probability of default is only 2.5% but there is also a positive probability of loan firesale and no default, which is equal to 3%. The return on equity decreases since from the perspective of the banker, who holds all of the Lucas tree, equity carries a liquidity premium since the probability of firesale and no default is positive. This analysis suggests that since financial experts chase the highest returns, there is an equilibrating force which contains the probability of systemic banking crisis, even if there is no ex-ante regulation.

The second scenario features a slightly higher minimum bank capital to debt ratio.
Table 4:

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$1 - F(\xi^c)$</th>
<th>$F(\xi^c) - F(\xi)$</th>
<th>$s_0^b/d_0^b$</th>
<th>$e_0/d_0^b$</th>
<th>$(Y_2 - p_0)/p_0$</th>
<th>$\tilde{p}_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BC</strong></td>
<td>0.026</td>
<td>0.000</td>
<td>0.000</td>
<td>0.106</td>
<td>0.037</td>
<td>0.205</td>
</tr>
<tr>
<td><strong>GE</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.025</td>
<td>0.030</td>
<td>0.000</td>
<td>0.097</td>
<td>0.027</td>
<td>0.209</td>
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<tr>
<td>2</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.111</td>
<td>0.037</td>
<td>0.204</td>
</tr>
<tr>
<td>3</td>
<td>0.003</td>
<td>0.124</td>
<td>0.175</td>
<td>0.114</td>
<td>0.104</td>
<td>0.032</td>
</tr>
<tr>
<td>4</td>
<td>0.006</td>
<td>0.000</td>
<td>0.150</td>
<td>0.080</td>
<td>0.037</td>
<td>0.063</td>
</tr>
<tr>
<td>5</td>
<td>0.003</td>
<td>0.124</td>
<td>0.174</td>
<td>0.116</td>
<td>0.109</td>
<td>0.032</td>
</tr>
<tr>
<td>6</td>
<td>0.000</td>
<td>0.000</td>
<td>0.200</td>
<td>0.116</td>
<td>0.000</td>
<td></td>
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<tr>
<td>7</td>
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<td>0.000</td>
<td>0.200</td>
<td>0.116</td>
<td>0.037</td>
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<tr>
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<td>0.013</td>
<td>0.115</td>
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<td><strong>PE</strong></td>
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<td>1</td>
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<td>4</td>
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<td>0.005</td>
<td>0.150</td>
<td>0.085</td>
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<td>0.123</td>
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<tr>
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<td>0.000</td>
<td>0.000</td>
<td>0.013</td>
<td>0.115</td>
<td>0.000</td>
<td></td>
</tr>
</tbody>
</table>

BC stands for benchmark case.
$\tilde{p}_1$ is the price of equity in case of bank default.

Since in that scenarios banks are better capitalized in the short run, the probability of default drops significantly – down to 0.9% which also leads to a significant increase of the CP’s welfare. However, lower probability of default makes the asset management sector less profitable which leads to more financial experts joining the banking sector. As a result, in the long run the economy converges to the GE case which features a probability of default almost identical to the benchmark case and a very slight increase of the CP’s welfare.

Scenario 3 restricts banks from holding equity. Given that in most equilibria, bankers are the natural holders of equity, this leads to a significant drop in the price of equity. In the short run, $\frac{Y_2 - p_0}{p_0}$ increases to 30%. While banks substitute into investing in the storage
technology, it is not enough to offset the decrease of equity holdings, where both the storage technology and equity are valuable in states of nature with loan firesale but no default. The probability of those states of nature increases in the short run from 0 to 12.4% while the probability of systemic banking crisis decreases to 1%. Given the significant mis-pricing of equity, the ex-ante profits of asset managers increase substantially which encourages the entry into the asset management industry in the long run, which leads to a further decrease in the probability of default down to 0.3% in the GE case. In the long run, the equity premium decreases to 10%. However, in case of default, both the price of equity and capital collapse in this equilibrium, since asset managers invest most of their ex-ante wealth in equity rather than in the storage technology. The GE effects here also help in terms of leading to an even larger improvement of welfare relative to the benchmark case in the long run rather than in the short run, which is in contrast to Scenario 2.

Scenario 4 considers the case where the banker has to hold safe assets greater than a fraction of his debt. In the short run, this lowers the probability of default down to 0.2%, which also in the long run decreases the size of the asset management sector. In the long run more financial experts enter into the banking sector, leading to a long run probability of default of 0.6% and an even larger improvement of the welfare of the CP. This is the case because banks have sufficient amount of savings in the safe asset to be able to survive most withdrawal shocks even for higher level of loans originated relative to the other scenarios. Also the expected returns on equity and loans are equalized and equity doesn’t carry a liquidity premium.

Scenario 5 jointly imposes all three regulatory instruments. CP’s welfare improves relative to the benchmark case, the probability of systemic defaults decreases significantly but the probability of loan firesale without default increases. The equity risk premium increases to more than 10 percentage points in the long run (GE) and is much higher in the short run (PE), implying that the imposition of the Volcker rule will have significant
impact on equity markets given that it prevents bankers, who used to be the natural holders of equity, from holding any equity.

Given that the policy instruments were not set optimally in Scenarios 2 through 5, it is important to considered what is the optimal constrained CP’s allocation. It is given by Scenario 6. First notice that if the CP does not internalize the impact of regulation on the size of the asset management sector and the banking sector, she will choose the optimal allocation assuming that \( \eta^a = 0.488 \) and, as a result, will choose an allocation where the fraction of safe assets to debt, \( \bar{s} \), will be equal to 0.013 and of bank net worth to debt, \( \bar{\psi} \), to 0.115. The Lucas tree holdings are equal to 1.7. This ratios are very different from the ones that the CP will choose if she internalized the effect of regulation on \( \eta^a \). In that case, the optimal allocation will be such that \( \bar{s} = 0.2, \bar{\psi} = 0.116 \) and \( \bar{x} = 1 \), and in equilibrium, \( \eta^a = 0 \). The CP would optimally want to set the regulation in such a way so that all financial experts choose to become bankers who can originate loans, and the probability of default or loan firesale without default is zero. \(^{15}\) This result is not surprising given that the bankers have an access to a more productive technology than the storage technology conditional on no loan firesale or default.

Specifications 7 and 8 calculate the equilibrium long run allocation achieved if the regulator sets \( \bar{s} \) and \( \bar{\psi} \) according to the GE CP’s allocation (Scenario 7) and the PE CP’s allocation (Scenario 8), where the maximum holding of equity never binds if set equal to the CPs allocation.\(^{16}\) Using only two out of the three instruments and setting optimal regulation by taking into account how it impacts \( \eta^a \) allows the regulator to replicate the constrained CP’s allocation and achieve the highest attainable welfare — an improvement of welfare of 4.6% relative to the benchmark case. In contrast, if the regulator takes \( \eta^a \) as given and sets the regulatory instruments accordingly, the increase in welfare is equal only to 0.4% relative

\(^{15}\)The optimal CP’s allocation which incorporates the GE effects is not always a corner equilibrium such that \( \eta^a = 0 \). It depends on the parametrization considered.

\(^{16}\)Graphs Scenarios 7 and 8 in the Appendix provide more information on the two scenarios.
to the benchmark case.

5 Conclusion

In this paper I study the feedback loop between ex-ante policy regulation, welfare and the size of the commercial banking sector relative to the "shadow banking" sector, which is the natural buyer of bank loans on the secondary market. The environment is rich enough to endogenously generate systemic bank default and loan firesale without default and to have predictions about how regulation affects market liquidity. The results suggest that the general equilibrium effects of the changing size of the two sector can either help or hinder the goal of regulators and it is crucial to take them into account when setting optimal regulation.
A Appendix

A.1 Consumer Problem

\[
\max_{d_0} F(\xi_1) c_{1}^{c.e} + (1 - F(\xi_1)) \sum_{t=2} c_{t}^{c.l} + c_{0}^{c}
\]

\[
c_0^c \leq e_0^c - d_0^c
\]

\[
c_1^{c.e} \leq e_1^c + r_0 d_0^c - T_1; \quad c_1^{c.l} = e_1^c - T_1
\]

\[
c_2^{c.l} = e_2^c - T_2 + r_0 d_0^c
\]

where \(c_{t}^{c,e}\) stands for the consumption of early and late consumer, \(d_0^c\) is the amount of debt and \(T_t\) is lump sum tax used to finance the deposit insurance scheme in case the banking sector defaults. The solution to the optimization problem above becomes equivalent to the consumer solving the following simple problem

\[
\max_{d_0} r_0 d_0^c - d_0^c
\]

The first order condition with respect to debt implies that \(r_0 = 1\).

A.2 Asset Manager and Banker’s Problems

Here I solve the problem of both types of agents using backwards inductions.

In \(t = 1\), after the realization of the liquidity shock, \(\xi_1\), the asset manager solves the following problem

\[
\max_{k_{1,a}, x_{1,a}, s_{1,a}} c_{2}^{i,a} = \max_{k_{1,a}, x_{1,a}, s_{1,a}} (Y_2 x_{1,a} + a_2 k_{1,a} + s_{1,a})
\]

subject to
The first order conditions are given by

\[ k_{i,f}^{i,a} = a_2 + \mu_{1,i,f,a} = \lambda_{1,i} q_1 \]
\[ x_{i,a}^1 = Y_2 - \lambda_{i,a} p_1 + \mu_{1,i,x,a} = 0 \]
\[ s_{i,a}^1 = 1 - \lambda_{i,a} + \mu_{1,i,s,a} = 0 \]

In \( t = 1 \), conditional on no default either in \( t = 1 \) and no expected default in \( t = 2 \), banker \( i \) solves the following optimization problem.

\[
\max_{k_{i,f}^{i,b}, x_{i,b}^{i,b}, s_{i,b}^{i,b}} c_{i,b} = \max_{k_{i,f}^{i,b}, x_{i,b}^{i,b}, s_{i,b}^{i,b}} \left( A_2 \left( k_{0,i}^{i,f} - k_{1,i}^{i,f} \right) + Y_2 x_{i,b}^{i,b} + s_{i,b}^{i,b} - r_0 d_{1,i}^{i,b} \right)
\]
\[
p_1 x_{0}^{i,b} + q_1 k_{1,i}^{i,f} + s_{0}^{i,b} \geq s_{1}^{i,b} + p_1 x_{1}^{i,b} + r_0 d_{1,i}^{i,b} - \lambda_{i}^{i,b}
\]
\[
s_{1}^{i,b} \geq 0 \quad [\mu_{1,i,s,b}]
\]
\[
k_{1,i}^{i,b} \geq 0 \quad [\mu_{1,i,f,b}]
\]
\[
x_{1,i}^{i,b} \geq 0 \quad [\mu_{1,i,x,b}]
\]
\( k_1^{1,f} : A_2 = \lambda_1^{i,b} q_1 + \mu_1^{i,f,b} \)

\( x_1^{i,b} : Y_2 - \lambda_1^{i,b} p_1 + \mu_1^{i,x,b} = 0 \)

\( s_1^{i,b} : 1 - \lambda_1^{i,b} + \mu_1^{i,s,b} = 0 \)

**Proof of Lemma 1 in text:** The first order conditions of the asset manager imply that

\[
\frac{Y_2}{p_1} + \frac{\mu_1^{x,a}}{p_1} = \frac{a_2}{q_1} + \frac{\mu_1^{f,a}}{q_1} = \lambda_1^a \geq 1
\]

The first order conditions of the banker imply that

\[
\frac{Y_2}{p_1} + \frac{\mu_1^{x,b}}{p_1} = \frac{A_2}{q_1} - \frac{\mu_1^{f,b}}{q_1} = \lambda_1^b \geq 1
\]

Since the market clearing condition for equity needs to hold, either \( \mu_1^{x,b} = 0 \) or \( \mu_1^{x,a} = 0 \) or both, which implies \( p_1 \leq Y_2 \). If there is a firesale of loans in \( t = 1 \) then \( \mu_1^{f,b} = \mu_1^{f,a} = 0 \) and \( a_2 \geq q_1 \). During a loan firesale, from Assumption 1, \( A_2 > a_2 \), which implies \( \lambda_1^b > \lambda_1^a \geq 1 \) and, therefore \( \mu_1^{x,b} > \mu_1^{x,a} \geq 0 \). Since the market for equity has to clear this implies that \( \mu_1^{x,a} = 0 \). Therefore, if \( k_1^f > 0 \), then \( x_1^b = s_1^b = 0 \). □

**Proof of Proposition 1 in text:** I assume a symmetric equilibrium. By combining the two budget constraints, 2 and 3, we derive the law of motion for aggregate savings

\[
S_1 = S_0 - \eta^b r_0 \xi_1 d_0^b
\]
where

\[ S_t = \eta^a s_t^a + \eta^b s_t^b \]

For a given \( \xi_1 \), first, I prove that if \( S_1 > 0 \), then there is no default in \( t = 1 \). From A3 and A4, \( S_1 > 0 \) implies that either \( \lambda_t^a = 1 \) or \( \lambda_t^b = 1 \) and that \( p_1 = Y_2 \). If there is a loan resale, from Lemma 1, we also know that \( x_t^b = s_t^b = 0 \) which implies that \( s_t^a > 0 \) and \( x_t^a > 0 \), which implies that \( \frac{Y_2}{p_1} = \frac{a_2}{q_1} = 1 \) and, therefore, \( q_1 = a_2 \). I assumed that the parametrization is such that if \( q_1 = a_2 \) and \( Y_2 = p_1 \) then there is no bank default in \( t = 1 \). Therefore, this implies that if \( S_1 > 0 \), the financial sector has enough savings to repay early consumers, and there is no period one default. If \( S_0 < \eta^b r_0 \xi_1 d_0^b \), then it is impossible for both budget constraints, equations 2 and 3, to be jointly satisfied and there does not exist a vector of prices such that the banks can repay early consumers. Since I solve for the symmetric equilibrium, this implies that if \( \xi_1 > \xi^c \) where

\[ \xi^c = \min \left\{ \frac{S_0}{\eta^b r_0 d_0^b}, \tilde{\xi} \right\} \]

the whole banking sector defaults. Next I solve for the threshold above which there is loan resale, \( \hat{\xi} \). In Lemma 1 I proved that the bank will sell loans on the secondary market in \( t = 1 \) only after he sells all of his equity and uses all of his cash. Since if there is no loan resale then there is no default, it has to be the case that the no resale equilibrium is such that, \( S_1 > 0 \), which implies \( p_1 = Y_2 \). Therefore, there will be loan resale, \( k_1^f > 0 \), only if \( \xi_1 > \hat{\xi} \), where

\[ \hat{\xi} = \min \left\{ \frac{Y_2 x_0^b + s_0^b}{r_0 d_0^b}, \frac{S_0}{\eta^b r_0 d_0^b}, \tilde{\xi} \right\} \]

There will be a region with loan resale and no default only if \( \xi^c > \hat{\xi} \) which will be the case if \( \frac{\eta^a s_0^a}{\eta^b} \geq Y_2 x_0^b \).

Consider the three regions:
I) No loan resale, \( k^f_1 = 0, \) (\( \xi_1 \leq \hat{\xi} \))

This case implies that \( \mu^{f,a}_1 > 0 \) and \( \mu^{f,b}_1 > 0 \) \((k^a_1 = k^b_1 = 0)\). First, consider \( \xi_1 \) realizations such that \( S_1 > 0 \) which is captured by the three cases: \( \mu^{s,a}_1 > 0 \) and \( \mu^{s,b}_1 = 0 \) \((s^a_1 = 0, s^b_1 > 0)\); \( \mu^{s,a}_1 = 0 \) and \( \mu^{s,b}_1 > 0 \) \((s^a_1 > 0, s^b_1 = 0)\) or \( \mu^{s,a}_1 = 0 \) and \( \mu^{s,b}_1 = 0 \) \((s^a_1 > 0, s^b_1 > 0)\)

It will have to be the case either \( \lambda^a_1 = 1 \) or \( \lambda^b_1 = 1 \) or both, which implies that \( \frac{Y_2}{p_1} = 1 \).

In this case the banker and the asset manager are indifferent where to put their money and any allocation of \( s^a_1, s^b_1, x^a_1 \) and \( x^b_1 \) are feasible where the following system of three equations is satisfied

\[
Y_2 x^b_0 + s^b_0 - r_0 \xi_1 d^i_0 = s^b_1 + Y_2 x^b_1
\]
\[
Y_2 x^a_0 + s^a_0 = s^a_1 + Y_2 x^a_1
\]
\[
\eta^a x^a_1 + \eta^b x^b_1 = \Theta
\]

Regarding optimizing welfare it is not necessary to specify exactly how they split the allocation. Also note that since the marginal value of wealth is one of the bankers and the arbitrageurs there will be no optimal bail out if there is no resale since if the cost of the bail out is non zero the marginal cost of the bail out will be greater than one

If \( \hat{\xi} = \frac{S_0}{\eta^a r_0 d^0} \) and \( \xi_1 = \hat{\xi} \) which implies \( S_1 = 0 \), then the relevant equilibrium is \( \mu^{s,a}_1 > 0 \) and \( \mu^{s,b}_1 > 0 \) \((s^a_1 = s^b_1 = 0)\). In this case, \( p_1 \) is not uniquely determined. To make the problem continuous, I assume that the realized \( p_1 = Y_2 \) and then the equilibrium is determined by the system of equations above where \( s^b_1 = s^a_1 = 0 \).

II) Loan resale and no default in \( t = 1, \) (\( k^f_1 > 0 \)), \( \xi^c \geq \xi_1 > \hat{\xi} \)

This case implies that \( \mu^{f,a}_1 = 0 \) and \( \mu^{f,b}_1 = 0 \) \((\eta^a k^f_1 = \eta^b k^b_1 > 0)\)

First, consider \( \xi_1 \) realizations such that \( S_1 > 0 \) which is captured by the three cases: \( \mu^{s,a}_1 > 0 \) and \( \mu^{s,b}_1 = 0 ; \mu^{s,a}_1 = 0 \) and \( \mu^{s,b}_1 > 0 \) \((\mu^{s,a}_1 = 0 \) and \( \mu^{s,b}_1 = 0)\). From the first order
conditions

\[
\frac{1}{q_1} = \frac{\lambda_1^a}{a_2} - \frac{\lambda_1^b}{A_2}
\]

where since \(A_2 > a_2\), then \(\lambda_1^b > \lambda_1^a\) which implies also \(\mu_1^{x,b} > \mu_1^{x,a} \geq 0\) and \(\mu_1^{x,b} > \mu_1^{s,a} \geq 0\). Therefore, the only relevant case is \(\mu_1^{s,a} = 0\), \(\mu_1^{x,b} > 0\), \(\mu_1^{x,a} > 0\), \(\mu_1^{s,a} = 0\) and \(\mu_1^{x,a} = 0\) imply \(Y_2 = p_1\), \(a_2 = q_1\) and \(\lambda_1^a = 1\).

\[
\begin{align*}
\kappa^f_1 &= \frac{r_0\xi^0 - (Y_2 x^b_1 + s^b_1)}{a_2} \\
\sigma^a_1 &= \frac{S_0 - \eta^b r_0 \xi^0 d^b_0}{\eta^a}; \ s^b_1 = 0 \\
x^b_1 &= 0; \ x^a_1 = \frac{\Theta}{\eta^a}
\end{align*}
\]

(A6)

If \(\xi^c = \frac{S_0}{\eta^b r_0 a_0^c}\) and \(\xi_1 = \xi^c\), which implies \(S_1 = 0\), then the relevant equilibrium is \(\mu_1^{s,a} > 0\) and \(\mu_1^{x,b} > 0\) \((s^a_1 = s^b_1 = 0)\). In this case, \(p_1 = q_1 Y_2^a / a_2\) but \(p_1\) is not uniquely determined. In order for the problem to be continuous and consistent with the case where \(S_1 > 0\), I assume that the realized equilibrium is \(p_1 = Y_2\), which implies \(q_1 = a_2\). Then the equilibrium is determined by the system of equations above where \(s^b_1 = s^a_1 = 0\).

**III) Loan resale and default in** \(t = 1\), \((K^f_1 > 0)\), \(\xi_1 > \xi^c\)

Note that it is possible that there is no default if \(\xi^c = \bar{\xi}\). Consider the case where \(\xi^c < \bar{\xi}\). There will be default if

\[
S_0 < \eta^b r_0 \xi^0 d^b_0
\]

I assume that if the inequality above is satisfied, all banks default and the regulator auctions all of their assets. Since the asset manager has to hold the Lucas trees as well as the loans, then the expected returns on both need to be equalized.
\[ \frac{Y_2}{p_1} = \frac{a_2}{q_1} = \lambda^i_1 \geq 1 \]

Since \( s_1^a = s_1^b = 0 \), by using the market clearing conditions, \( x_1^a = \frac{\Theta}{\eta^a}; \eta^a k_1^a = \eta^b k_0 \) and the two budget constraints, we derive \( q_1 = \frac{\eta^a}{\eta^b} \frac{p_1 (x_0^a - \frac{a_2}{\eta^a}) + s_0^a}{k_0} \), which when combined with \( q_1 = p_1 \frac{a_2}{Y_2} \) implies

\[ \frac{p_1}{a_2^2} \frac{\eta^b}{Y_2 \eta^b} k_0 + \frac{\eta^b}{\eta^a} x_0^b = Y_2 \]

The last inequality follows from the fact that since I consider only parametrization such that if \( q_1 = a_2 \) and \( p_1 = Y_2 \) there is no default then

\[ a_2 k_0 \eta^b + \eta^b Y_2 x_0^b + \eta^b s_0^b \geq \xi \eta^b r_0 d_0 > S_0 \]

\[ a_2 k_0 \eta^b + \eta^b Y_2 x_0^b > s_0^a \]

which directly implies inequality A8.\( \Box \)

In \( t = 0 \), the asset manager solves the following optimization problem, where he takes into account \( t = 1, 2 \) best response functions

\[ \max_{s_0^i, x_0^i} E_0 c_2^{i,a} = \max_{s_0^i, x_0^i} E_0 \left( a_2 k_1^{i,a} + s_1^{i,a} + Y_2 x_1^{i,a} \right) \]

If \( \xi_1 \leq \xi^c \), the period one budget constraint can be re-written as

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If $\xi_1 > \xi^c$, the period one budget constraint can be re-written as

$$(A10) \quad \tilde{q}_1 k_1^{i,a} + s_1^{i,a} + \tilde{p}_1 x_1^{i,a} = \tilde{p}_1 x_0^{i,a} + s_0^{i,a}$$

where $\{\tilde{q}_1, \tilde{p}_1\}$ is the vector of prices if there is default (Type 3 equilibrium). The banker internalizes the fact that from equation A7, then $\frac{Y_2}{\tilde{p}_1} = \frac{a_2}{\tilde{q}_1} > 1$, which implies that if the banking sector defaults, then $s_1^{i,a} = 0$. Using equation A7, one can re-write equation A10 as

$$(A11) \quad a_2 k_1^{i,a} + Y_2 x_1^{i,a} = \frac{Y_2}{\tilde{p}_1} (\tilde{p}_1 x_0^{i,a} + s_0^{i,a})$$

Using equations A9 and A11 and the period zero budget constraint,

the objective function can be re-written as

$$\max_{x_0^{i,a}} \int_0^{\xi^c} \left( Y_2 x_0^{i,a} + p_0 \Theta + e_0 - p_0 x_0^{i,a} \right) f (\xi_1) d\xi_1 + \int_{\xi}^{\xi^c} \frac{Y_2}{\tilde{p}_1} \left( \tilde{p}_1 x_0^{i,a} + p_0 \Theta + e_0 - p_0 x_0^{i,a} \right) f (\xi_1) d\xi_1$$

where $\tilde{p}_1 = \frac{e_0}{Y_2 \frac{Y_2}{\tilde{p}_1} k_0 + \frac{a_2}{\tilde{q}_1} x_0^{i,a}}$. The banker takes as given aggregate variables such as prices and the default cut-off, $\xi^c$. The problem is subject to the the no short selling constraints,
where I have substituted in for the budget constraint,

\[
e_0 + p_0 \Theta - p_0 x_0^{i,a} \geq 0 \quad [\mu_0^{i,s,a}]
\]

\[
x_0^{i,a} \geq 0 \quad [\mu_0^{i,x,a}]
\]

The first order conditions with respect to \( x_0^{i,a} \) is given by

\[
(A12) \quad x_0^{i,a} : \frac{Y_2}{p_0} + \left(1 - \frac{Y_2}{p_1}\right) (1 - F (\xi_c)) - \mu_0^{i,s,a} + \frac{\mu_0^{i,x,a}}{p_0} = 1
\]

and the complementarity slackness conditions are

\[
(\mu_0^{i,s,a} > 0 \text{ and } \mu_0^{i,x,a} > 0).
\]

In Proposition 1 I proved that if there is loan resale but no default, then \( p_1 = Y_2 \) and
\( q_2 = a_2 \), which when combined with the first order conditions defined in A2, implies

\[
\frac{A_2}{a_2} = 1 + \frac{\mu_1^{i,i,b}}{Y_2} > 1
\]

\[
\frac{A_2}{a_2} = 1 + \mu_1^{i,s,b} > 1,
\]

Therefore, in \( t = 1 \) banker \( i \) will sell loans on the secondary market only after selling all of his equity and using all of his savings. Therefore, if \( \xi_1 \leq \bar{\xi}^i = \min \left\{ \frac{Y_2x_{0}^{i,b} + s_{0}^{i,b}}{r_0d_{0}^{i,b}}, \xi^c \right\} \), then banker \( i \) does not resell loans and his period one budget constraint can be written as

\[
Y_2x_{0}^{i,b} + s_{0}^{i,b} = s_{1}^{i,b} + Y_2x_{1}^{i,b} + r_0\xi_1d_{0}^{i,b}
\]

if \( \min \left\{ \frac{a_2k_0^{i,i,b} + Y_2x_{0}^{i,b} + s_{0}^{i,b} - \frac{a_2}{Y_2}r_0d_{0}^{i,b}}{(1 - \frac{a_2}{Y_2})r_0d_{0}^{i,b}}, \xi^c \right\} = \bar{\xi}^i \geq \xi_1 > \hat{\xi}^i \), banker \( i \) will have to resell loans but there will be no default either in \( t = 1 \) or \( t = 2 \) \( (c_2^{i,b} \geq 0) \). In that case, his period one budget constraint can be written as

\[
Y_2x_{0}^{i,b} + a_2k_1^{i,i,f} + s_{0}^{i,b} = r_0\xi_1d_{0}^{i,b}
\]

Since \( \bar{\xi}^i \geq \hat{\xi}^i \), the objective function of the banker in \( t = 0 \) can be re-written as

\[
\max_{k_0^{i,i,b},d_0^{i,b},x_{0}^{i,b},s_{0}^{i,b}} E_0c_{2}^{i,b} = \max_{k_0^{i,i,b},d_0^{i,b},x_{0}^{i,b},s_{0}^{i,b}} F\left( \bar{\xi}^i \right) \left( A_2k_0^{i,i} + Y_2x_{0}^{i,b} + s_{0}^{i,b} - r_0d_{0}^{i,b} \right)
\]

\[
+ \left( F\left( \bar{\xi}^i \right) - F\left( \hat{\xi}^i \right) \right) \left( \frac{A_2}{a_2} - 1 \right) \left( Y_2x_{0}^{i,b} + s_{0}^{i,b} \right)\]

\[
- \int_{\hat{\xi}^i}^{\bar{\xi}^i} \left( \frac{A_2}{a_2} - 1 \right) r_0\xi_1d_{0}^{i,b} f(\xi_1) d\xi_1
\]
\[
\tilde{\xi}^i = \min \left\{ \frac{Y_2 x_i^0 + s_i^b}{r_0 d_i^b}, \xi^c \right\}
\]

\[
\bar{\xi}^i = \min \left\{ \frac{\alpha_2 k_i^0 + Y_2 x_i^0 + s_i^b - \frac{\alpha_2}{A_2} r_0 d_i^b}{(1 - \frac{\alpha_2}{A_2}) r_0 d_i^b}, \xi^c \right\}
\]

The banker internalizes the fact that the cut-off above which there is a loan firesale \(\tilde{\xi}^i\) and above which the banker’s equity gets wiped out \(\bar{\xi}^i\), can depend on his own actions. The optimization problem is subject to the period zero budget constraint

\[
k_i^0 + C \left( d_i^0 \right) \leq e_i^b + d_i^0 - p_0 x_i^0 - s_i^b - \lambda_i^0
\]

the no short-selling conditions

\[
s_i^b \geq 0 \quad \left[ \mu_i^{s,b} \right]
\]

\[
k_i^0 \geq 0 \quad \left[ \mu_i^{k,b} \right]
\]

\[
x_i^0 \geq 0 \quad \left[ \mu_i^{x,b} \right]
\]

\[
d_i^0 \geq 0 \quad \left[ \mu_i^{d,b} \right]
\]

and the regulatory constraints

\[
s_i^b \geq \rho_i^b \left[ \nu_i^{s,b} \right]
\]

\[
x_i^0 \leq \frac{\Theta}{\eta_i^b} \left[ \nu_i^{x,b} \right]
\]

\[
\bar{\psi} d_i^0 \leq e_0 \left[ \nu_i^{c} \right]
\]
The first order conditions are given by

\[ d_0^{i,b} : = -r_0 F\left( \tilde{\xi}^i \right) + \left( A_2 k_0^{i} + Y_2 x_0^{i,b} + s_0^{i,b} - r_0 d_0^{i,b} \right) f \left( \tilde{\xi}^i \right) \frac{\partial \tilde{\xi}^i}{\partial d_0^{i,b}} + \left( \frac{A_2}{a_2} - 1 \right) \left( Y_2 x_0^{i,b} + s_0^{i,b} \right) \left( f \left( \tilde{\xi}^i \right) \frac{\partial \tilde{\xi}^i}{\partial d_0^{i,b}} - f \left( \tilde{\xi}^i \right) \frac{\partial \tilde{\xi}^i}{\partial d_0^{i,b}} \right) - \left( \frac{A_2}{a_2} - 1 \right) r_0 d_0^{i,b} \left( \xi^i f \left( \tilde{\xi}^i \right) \frac{\partial \tilde{\xi}^i}{\partial d_0^{i,b}} - \xi^i f \left( \tilde{\xi}^i \right) \frac{\partial \tilde{\xi}^i}{\partial d_0^{i,b}} \right) - \left( \frac{A_2}{a_2} - 1 \right) r_0 \int_{\tilde{\xi}^i}^{\tilde{\xi}^i} \xi_1 f \left( \xi_1 \right) d\xi_1 - \nu_0^{i,s} \hat{s} - \nu_0^{i,e} \hat{e} + \mu_0^{i,d,b} = -\lambda_0^{i,b} + \lambda_0^{i,b} C \left( d_0^{i,b} \right) \]

\[ k_0^i : = A_2 F \left( \tilde{\xi}^i \right) + \left( A_2 k_0^{i} + Y_2 x_0^{i,b} + s_0^{i,b} - r_0 d_0^{i,b} \right) f \left( \tilde{\xi}^i \right) \frac{\partial \tilde{\xi}^i}{\partial k_0^{i}} + \left( \frac{A_2}{a_2} - 1 \right) \left( Y_2 x_0^{i,b} + s_0^{i,b} \right) f \left( \tilde{\xi}^i \right) \frac{\partial \tilde{\xi}^i}{\partial k_0^{i}} - \left( \frac{A_2}{a_2} - 1 \right) r_0 d_0^{i,b} \xi^i f \left( \tilde{\xi}^i \right) \frac{\partial \tilde{\xi}^i}{\partial k_0^{i}} + \mu_0^{i,k,b} = \lambda_0^{i,b} \]

\[ x_0^{i,b} : = Y_2 F \left( \tilde{\xi}^i \right) + \left( A_2 k_0^{i} + Y_2 x_0^{i,b} + s_0^{i,b} - r_0 d_0^{i,b} \right) f \left( \tilde{\xi}^i \right) \frac{\partial \tilde{\xi}^i}{\partial x_0^{i,b}} + \left( F \left( \tilde{\xi}^i \right) - F \left( \tilde{\xi}^i \right) \right) \left( \frac{A_2}{a_2} - 1 \right) Y_2 + \left( \frac{A_2}{a_2} - 1 \right) \left( Y_2 x_0^{i,b} + s_0^{i,b} \right) \left( f \left( \tilde{\xi}^i \right) \frac{\partial \tilde{\xi}^i}{\partial x_0^{i,b}} - f \left( \tilde{\xi}^i \right) \frac{\partial \tilde{\xi}^i}{\partial x_0^{i,b}} \right) - \left( \frac{A_2}{a_2} - 1 \right) r_0 d_0^{i,b} \left( f \left( \tilde{\xi}^i \right) \frac{\partial \tilde{\xi}^i}{\partial x_0^{i,b}} - f \left( \tilde{\xi}^i \right) \frac{\partial \tilde{\xi}^i}{\partial x_0^{i,b}} \right) + \mu_0^{i,x,b} = \nu_0^{i,x} + p_0 \lambda_0^{i,b} \]
\[ s_0^{i,b} : F\left(\bar{\xi}^i\right) + \left(A_2k_0^i + Y_2x_0^{i,b} + s_0^{i,b} - r_0d_0^{i,b}\right) f \left(\bar{\xi}^i\right) \frac{\partial \bar{\xi}^i}{\partial s_0^{i,b}} \]
\[ + \left(F\left(\bar{\xi}^i\right) - F\left(\bar{\xi}_0^i\right)\right) \left(\frac{A_2}{a_2} - 1\right) + \left(\frac{A_2}{a_2} - 1\right) \left(Y_2x_0^{i,b} + s_0^{i,b}\right) \left(f \left(\bar{\xi}^i\right) \frac{\partial \bar{\xi}^i}{\partial s_0^{i,b}} - f \left(\bar{\xi}_0^i\right) \frac{\partial \bar{\xi}_0^i}{\partial s_0^{i,b}}\right) \]
\[ - \left(\frac{A_2}{a_2} - 1\right) r_0d_0^{i,b} \left(f \left(\bar{\xi}^i\right) \frac{\partial \bar{\xi}^i}{\partial s_0^{i,b}} - f \left(\bar{\xi}_0^i\right) \frac{\partial \bar{\xi}_0^i}{\partial s_0^{i,b}}\right) + \mu_0^{i,s,b} + \nu_0^i = \lambda_0^{i,b} \]

Simplifying the first order conditions,

(A13)
\[ d_0^{i,b} : -r_0F\left(\bar{\xi}^i\right) - \left(\frac{A_2}{a_2} - 1\right) r_0\int_{\xi}^{\bar{\xi}^i} \xi_1 f \left(\xi_1\right) d\xi_1 - \nu_0^{i,s} s - \nu_0^{i,e} \bar{v} + \mu_0^{i,d,b} = -\lambda_0^{i,b} + \lambda_0^{i,b} C' \left(d_0^{i,b}\right) \]

(A14)
\[ k_0^i : A_2F\left(\bar{\xi}^i\right) + \mu_0^{i,k,b} = \lambda_0^{i,b} \]

(A15)
\[ x_0^{i,b} : \frac{Y_2}{p_0} \left[F\left(\bar{\xi}^i\right) + \left(F\left(\bar{\xi}^i\right) - F\left(\bar{\xi}_0^i\right)\right) \left(\frac{A_2}{a_2} - 1\right)\right] + \frac{\mu_0^{i,x,b}}{p_0} = \frac{\nu_0^{i,x}}{p_0} + \lambda_0^{i,b} \]

(A16)
\[ s_0^{i,b} : F\left(\bar{\xi}^i\right) + \left(F\left(\bar{\xi}^i\right) - F\left(\bar{\xi}_0^i\right)\right) \left(\frac{A_2}{a_2} - 1\right) + \mu_0^{i,s,b} + \nu_0^i = \lambda_0^{i,b} \]

and the complementarity slackness conditions are
\[ s_{00} i;b_i \mu_0 = 0; \quad k_{00} i;k_i \mu_0 = 0 \]
\[ x_{00} i;x_i \mu_0 = 0; \quad d_{00} i;d_i \mu_0 = 0 \]

\[
\begin{align*}
(s_{00} \hat{d}_0 i;b i) \nu^{i,s} = 0; & \quad \left(\frac{\Theta}{\eta^b} - x_{00} i;b \right) \nu^{i,x} = 0 \\vspace{0.5em} \\
(e_{00} - \psi \hat{d}_0 i;b) \nu^{i,e} = 0
\end{align*}
\]

where \( i;b \quad 0 \geq 0, \mu_0 i;k \quad 0 \geq 0, \mu_0 i;d \quad 0 \geq 0, \nu_0 i,s \quad 0 \geq 0, \nu_0 i,x \quad 0 \geq 0, \nu_0 i,e \quad 0 \geq 0.\)

**Lemma 1A:** Imposing a symmetric equilibrium, given Assumption 1, then \( \mu_0 d;b = 0. \)

Proof: I prove by contradiction that \( \mu_0 d;b = 0 \). Assume that \( \mu_0 d;b > 0 \). From Assumption 1, \( A_2 > 1 \). Since \( d_0 = 0 \) implies \( \bar{x} = \bar{x} = \bar{x} \) and \( \nu_0 s = \nu_0 e = 0 \), then combining equations A13 and A14

\[ 1 - \mu_0 d;b = \lambda_b^b = A_2 + \mu_0 k;b > 1 \]

However, the inequality above cannot be satisfied since \( \mu_0 d;b > 0 \), which is a contradiction. Therefore, it is has to be the case that \( \mu_0 d;b = 0. \Box \)

**Proof of Lemma 2 in Text:** First, I prove Part 1) that \( p_0 \leq Y_2 \). By combining equations A15 and A16

\[ \frac{Y_2}{p_0} - 1 \left[ F(\bar{x}) + \left( F(\bar{x}) - F(\bar{x}) \right) \left( \frac{A_2}{a_2} - 1 \right) \right] = \mu_0 s;b + \frac{\nu^x}{p_0} + \nu_0 s^b - \frac{\mu_0 x;b}{p_0} \]

Therefore if the banker holds the Lucas tree in \( t = 0, \mu_0 x;b = 0 \), then \( Y_2 \geq p_0 \). Consider
the case where the asset manager is a marginal holder of the Lucas tree, \( \mu_{0}^{x,a} = 0 \). From equation A12

\[
(A18) \quad \frac{Y_2}{p_0} = \mu_{0}^{s,a} + 1 + \left( \frac{Y_2}{\tilde{p}_1} - 1 \right) (1 - F(\xi^c)) \geq 1
\]

Since it has to be the case that either \( \mu_{0}^{x,b} = 0 \) or \( \mu_{0}^{x,a} = 0 \) or both hold, then it has to be the case that \( p_0 \leq Y_2 \). Next I prove Part 2) that the asset managers will hold equity if there is default in \( t = 1 \), \( 1 - F(\xi^c) > 0 \), only if \( p_0 < Y_2 \). This is immediately clear from inequality A18 combined with the fact that in Part III of Proposition 1, I proved that \( \tilde{p}_1 < Y_2 \) where \( \tilde{p}_1 \) is the period one price of equity if there is bank default.

Finally, I prove part 3) of Lemma 2. No regulation implies \( v_{0}^{s} = v_{0}^{x} = 0 \) and equation A17 can be re-written as

\[
\left( \frac{Y_2}{p_0} - 1 \right) \left[ F(\bar{\xi}) + \left( F(\bar{\xi}) - F(\bar{\xi}) \right) \left( \frac{A_2}{a_2} - 1 \right) \right] = \mu_{0}^{s,b} - \frac{\mu_{0}^{x,b}}{p_0} \geq 0
\]

If \( Y_2 = p_0 \) then \( \mu_{0}^{s,b} - \frac{\mu_{0}^{x,b}}{p_0} = 0 \) and the first order conditions of the banker with respect to \( s_{0}^{i,b} \) and \( x_{0}^{i,b} \) coincide. In that case the assets are perfect substitutes from the perspective of the banker and he is indifferent which asset to invest in. If \( Y_2 > p_0 \), then \( \mu_{0}^{s,b} > \frac{\mu_{0}^{x,b}}{p_0} \geq 0 \) and the banker does not invest in the storage technology but he might invest in equity if \( \mu_{0}^{x,b} = 0 \). The assets are no longer perfect substitutes from the perspective of the banker. \( \square \)

### A.3 Central Planner

If \( \xi^c \leq \bar{\xi} \), in \( t = 0 \) the objective function of the CP is given by
where \( \hat{q}_1 = \hat{p}_1 \frac{s_0^a}{c_0} \) and \( \hat{p}_1 = \frac{s_0^a}{c_0} \frac{\hat{q}_1}{Y_2} \). The CP’s objective function can be re-written as

\[
\max_{\mu_0, \mu_0^a, \mu_0^a, \mu_0^a, k_0, k_0, x_0, x_0, s_0, s_0, k_0, d_0, x_0, s_0, d_0} \eta^b \left( (F(c)) A_2 + a_2 (1 - F(c)) \right) k_0 + Y_2 \Theta + \eta^a e_0 - p_0 \eta^b (\Theta - x_0^b) + \eta^b s_0^b - \eta^b d_0^b + \eta^b \left( 1 - \frac{A_2}{a_2} \right) \int_0^\xi (\xi_1 d_0^b - (Y_2 x_0^b + s_0^b)) f(\xi_1) d\xi_1
\]

and is subject to

\[
\begin{align*}
s_0^b & \geq 0 \\
k_0 & \geq 0 \\
x_0^b & \geq 0 \\
d_0^b & \geq 0
\end{align*}
\]

\[
k_0 + C (d_0^b) \leq e_0 + \Theta p_0 + d_0^b - p_0 x_0^b - s_0^b
\]
\[
\frac{Y_2}{p_0} + \left(1 - \frac{Y_2}{\bar{p}_1}\right) (1 - F(\xi^c)) - \mu_{0}^{s,a} + \frac{\mu_{0}^{x,a}}{p_0} \leq 1 \quad \left[ \mu_0^{CP,1} \right]
\]

\[
\left( e_0 \eta^a + \eta^b p_0 \left( b_0 - \Theta \right) \right) \mu_{0}^{s,a} = 0 \quad \left[ \mu_0^{CP,2} \right]
\]

\[
(\Theta - \eta^b x_0^b) \mu_{0}^{x,a} = 0 \quad \left[ \mu_0^{CP,3} \right]
\]

\[
\mu_{0}^{s,a} \geq 0 \quad \left[ \mu_0^{CP,4} \right]
\]

\[
\mu_{0}^{x,a} \geq 0 \quad \left[ \mu_0^{CP,5} \right]
\]

and equation 8 if the CP internalizes his effect on \( \eta^a \).

### A.4 Data description and sources

NIM is the net interest margin for all US banks calculated as the ratio of Tax-Adjusted Income to Average Earning Assets. Source: Federal Reserve Bank of St. Louis. If one uses only tax adjusted interest income the number is very similar but the data sample available is shorter.

The average excess return of the S&P is calculated as the annual S&P return minus the annual 3 month US government bill rate. After that averages are taken of the excess annual return over the specified period. Source: Datastream

The total equity to total assets for banks is from the Federal Reserve Bank of St. Louis.
B Graphs

Scenario 2: Only maximum leverage requirement
Scenario 3: Only no equity holdings
Scenario 4: Only minimum safe assets requirement
Scenario 5: All three instruments
Scenario 7: Decentralization of the GE CP’s allocation
Scenario 8: Decentralization of the PE CP’s allocation
REFERENCES


