A Theory of Crowdfunding
- a mechanism design approach with demand uncertainty and moral hazard

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Abstract

Crowdfunding provides the innovation that, already before the investment, entrepreneurs can interact with consumers. It, thereby, reduces demand uncertainty and improves screening. Entrepreneurial moral hazard threatens these benefits. After formally characterizing optimal contracts between consumers and an entrepreneur who is susceptible to with moral hazard, this paper argues that current crowdfunding schemes reflect their salient features. Efficiency is sustainable only if expected returns exceed investment costs by a margin reflecting the degree of moral hazard. Constrained efficient mechanisms exhibit underinvestment. Crowdfunding destroys the classical separation between finance and marketing, but complements rather than substitutes traditional entrepreneurial financing.

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1 Introduction

Crowdfunding has, in recent years, attracted much attention as a new mode of entrepreneurial financing: through the internet many individuals — the crowd — provide funds directly to the entrepreneur.\footnote{Time Magazine (2010) lists crowdfunding as one of the “Best Inventions of 2010”, while The Economist (2012) reports that the “talk of crowdfunding as a short-lived fad has largely ceased”. On the policy side, the JOBS Act from 2012 and SEC (2015) are meant to boost crowdfunding in the US by setting the foundations to raise capital through securities offerings using the internet.} In the narrower context of “reward-crowdfunding”, this crowd consists of the very consumers whom the entrepreneur intends to target with her product.

Hence, crowdfunding provides the economic innovation that, in contrast to traditional modes of financing, entrepreneurs can contract with their future consumers already before the investment decision. Focusing on this ability, this paper argues that crowdfunding leads to a more effective screening of projects, because it allows the investment decision to be conditioned on consumers’ private information about demand. Before illustrating this effect by a concrete example, we first describe how reward-crowdfunding works in practice. The description clarifies the features by which crowdfunding schemes elicit consumers’ private information, reduce demand uncertainty, and achieve a more effective screening of entrepreneurial projects.

Attracting pledges of more than 2 billion dollars, the most successful crowdfunding platform to date is Kickstarter.\footnote{Kickstarter reports its statistics on www.kickstarter.com/help/stats.} It implements crowdfunding as follows. First, the entrepreneur describes her project, consisting of the following three elements: 1) a description of the reward to the consumer, which is typically the entrepreneur’s final product; 2) a “pledge level” \( p \); and 3) a “target level” \( T \). After describing these elements, a number, say \( \hat{n} \), of consumers pledge contributions. If the sum of pledges exceed the target level, i.e. if \( \hat{n} \cdot p \geq T \), the entrepreneur receives the contribution \( p \) from each of the \( \hat{n} \) pledging consumers and in return delivers to each of them the promised reward. If the pledged contributions lie below the target level, \( \hat{n} \cdot p < T \), then the project is cancelled; consumers withdraw their pledges and the entrepreneur has no obligations towards them. Hence, given a specified reward, the pair \((p, T)\) defines the crowdfunding scheme.
For a simple illustration of how a crowdfunding scheme \((p, T)\) screens for valuable projects, consider a “crowd” of only a single representative consumer.\(^3\) Suppose that this consumer’s willingness to pay for the good is either high, \(v_h = 1\), or low, \(v_l = 0\), each with probability \(\frac{1}{2}\). Let \(I = \frac{3}{4}\) represent the development costs which need to be invested before the good can be produced (with no further production costs). The project, therefore, has a positive value of \(\frac{1}{4}\) in the state \(v_h\), a negative value \(-\frac{3}{4}\) in the state \(v_l\), and a negative expected value of \(-\frac{1}{4}\) in expectation.

Hence, even if the entrepreneur had the required cash, she would not invest if she does not learn the consumer’s valuation before hand. A venture capitalist reviewing the entrepreneur’s business plan faces the same problem. The crowdfunding scheme \((p, T) = (1, 1)\), however, elicits the consumer’s private information naturally and leads to an investment only in state \(v_h\). Indeed, facing the scheme \((p, T) = (1, 1)\), only the consumer with a high value \(v_h = 1\) considers it optimal to pledge, and, hence, the investment is triggered only in the high valuation state. The scheme therefore induces an efficient outcome and, moreover, allows the entrepreneur to extract the project’s surplus.

This simple example not only illustrates the main efficiency effect of crowdfunding, but also identifies the three ingredients that are crucial for generating it: 1) the presence of fixed development costs; 2) uncertainty about whether the demand of consumers is large enough to recover these development costs; and 3) a trigger level that enables conditional investment. The first two ingredients are defining features of entrepreneurial financing. The third ingredient is the defining feature of a so-called “all-or-nothing” crowdfunding platform such as used by Kickstarter.\(^4\)

Crowdfunding, however, also seems to exhibit a clear economic disadvantage as compared to more traditional modes of financing. More specifically, its replacement of financial intermediaries as investors by an uncoordinated crowd raises important con-

\(^3\)A single agent illustrates well the main efficiency property of crowdfunding, but hides its other effective properties such as mitigating strategic uncertainty and coordination problems.

\(^4\)Platforms using an “all-or-nothing” pledge schemes are, for instance, Kickstarter, Sellaband, and PledgeMusic. The “keep-what-you-raise” model, where pledges are triggered even if the target level is not reached is popular for platforms that focus on non-profit projects (e.g. GoFundMe). Kickstarter (www.kickstarter.com/help/stats) reports that less than 40% of the projects meet the trigger level, which confirms that the trigger level plays a crucial role in crowdfunding.
cerns about entrepreneurial moral hazard. Economic theory provides clear efficiency arguments in favor of a specialized financial intermediary. In particular, Diamond (1984) points out that by coordinating investment through a single financial intermediary, free-riding problems associated with monitoring the borrower’s behavior are circumvented. Indeed, monitoring to limit moral hazard seems especially important for entrepreneurial financing. Entrepreneurs are typically new players in the market, who, in contrast to well-established firms, have not yet had the ability to build up a reputation to demonstrate their trustworthiness.

An analysis of crowdfunding without an explicit consideration of moral hazard seems therefore lopsided. As it turns out, this is even more so, because the crowdfunding scheme’s reduction in demand uncertainty interacts directly with the moral hazard problem: an elimination of demand uncertainty intensifies moral hazard. Hence, in the presence of both demand uncertainty and moral hazard, a non-trivial trade-off results concerning the informativeness of optimal mechanisms.

Using the generalized mechanism design framework of Myerson (1982), we explicitly address this trade-off. More generally, we characterize mechanisms that optimally address the problem of both demand uncertainty and moral hazard. Myerson’s generalized framework assumes the presence of a mediator who coordinates the communication between economic agents. One insight from our analysis is that the crowdfunding platform plays exactly the role of a mediator in the sense of Myerson.

In a nutshell, our characterization of the optimal contract yields the following results: 1) Optimal crowdfunding schemes are reward-based, i.e., consumers do not obtain a monetary return from funding the entrepreneur. 2) Optimal crowdfunding schemes condition the entrepreneur’s investment decision on the sum of reported consumer valuations. 3) To reduce the threat of moral hazard, optimal crowdfunding schemes defer payments to the entrepreneur. 4) Because the moral hazard problem interacts with the reduction in demand uncertainty, optimal crowdfunding schemes resolve demand uncertainty only partially. 5) Because the moral hazard problem stands in conflict with the entrepreneur’s need for capital, optimal crowdfunding

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5Mollick (2014) considers the funding of ventures “without standard financial intermediaries” as a defining feature of crowdfunding. Agrawal et al. (2014) stress and illustrate the problem of moral hazard in crowdfunding.
schemes achieve first-best efficient outcomes only if the ex ante expected returns of the project exceed the entrepreneur’s ex ante expected capital costs by a margin that is proportional to the threat of moral hazard. 6) Constrained efficient crowdfunding schemes display underinvestment but not overinvestment.

Whilst results 5 and 6 are normative, the first four findings are of a positive nature and we can judge their applicability by comparing them to the crowdfunding schemes that are used in practise. As discussed, all current reward-crowdfunding platforms conform to the first theoretical finding that optimal crowdfunding schemes do not require crowdfundingers to become investors. “All-or-nothing” crowdfunding platforms also conform to the second finding that optimal crowdfunding schemes induce conditional investment that is triggered on the sum of collected pledges. Moveover, some crowdfunding platforms such as PledgeMusic also reflect the third finding that they use deferred payout systems to reduce the threat of moral hazard.

In contrast, all current crowdfunding platforms announce publicly the total amount of pledges, implying that none of them support the fourth theoretical finding. An extension of our analysis reveals however that this fourth characteristic becomes important only if crowdfunding reaches a large enough share of all potential consumers. If a crowdfunding entrepreneur can reach only a limited number of consumers then the prospect to sell the good to non-crowdfunding consumers acts as a direct substitute for deferred payments. This finding is consistent with the observation that, despite popular warnings of the opposite, currently moral hazard in crowdfunding does not seem a major problem (e.g. Mollick, 2014). It leads to the normative prediction that if the popularity of crowdfunding increases so that the moral hazard problem intensifies, then more platforms will follow the example of PledgeMusic to introduce deferred payouts or even limit the platform’s transparency as suggested by our analysis.

The rest of the paper is organized as follows. The next section discusses the related literature. Section 3 introduces the setup and takes an intuitive approach that identifies the main trade-offs. Section 4 sets up the problem as one of mechanism design. Section 5 characterizes constrained efficient mechanisms. Section 6 relates optimal mechanisms to real-life crowdfunding mechanisms and examines extensions. Section 7 concludes. All formal proofs are collected in the appendix.
2 Related literature

Being a relatively new phenomenon, the literature on crowdfunding is still small and primarily of an empirical and case-based nature. Concerning crowdfunding’s economic underpinnings, Agrawal et al. (2014) provide a broad introduction that highlights the main issues. They emphasize entrepreneurial moral hazard with explicit quotes from the popular press. They also explicitly mention that crowdfunding can reduce demand uncertainty, but do not discuss the features of crowdfunding schemes that are especially helpful in this respect.

Belleflamme et al. (2014) is one of the few theoretical studies that deals specifically with crowdfunding. It addresses the question whether a crowdfunding entrepreneur is better off raising her capital by reward-crowdfunding or by equity-crowdfunding. Since the authors abstract from aggregate demand uncertainty, they do not identify the economic benefits of reward-crowdfunding in screening out projects. Instead, the benefit of crowdfunding stems from their assumption that consumers obtain an extra utility from participating in the crowdfunding scheme. In particular, they study the extent to which the different crowdfunding schemes enable the monopolistic entrepreneur to extract this additional utility.

The economic literature on demand uncertainty has mostly focused on its effect on equilibrium prices rather than on its effect on investment decisions (e.g., Klemperer and Meyer 1989, Deneckere and Peck 1995, Dana 1999). An exception is Jovanovic and Rob (1987), who study the dynamics of innovation, when firms can acquire information about the consumers’ evolving tastes and introduce product innovations that cater to them. Even though these randomly evolving changes express demand uncertainty, the paper is only tentatively related to the current study, as it does not consider direct revelation mechanisms of the consumer’s preferences.

In contrast, the marketing literature explicitly addresses a firm’s ability to reduce demand uncertainty through market research such as consumer surveys (e.g., Lauga and Ofek 2009). Ding (2007) however points out that marketing research relies on voluntary, non-incentivized consumer surveys. He emphasizes that consumers need to be given explicit incentives for revealing their information truthfully. Current crowdfunding schemes provide such incentives naturally.
Ordanini et al. (2011) present a marketing-based, qualitative case study on crowdfunding. They note that crowdfunding blurs the boundaries between marketing and finance and view the consumers’ investment support as the foundational trait of crowdfunding that sets it apart from other marketing theories. They do not focus on reward crowdfunding but mainly study two equity-crowdfunding schemes and a pure donation one.

Empirical studies of crowdfunding try to identify the crucial features of crowdfunding projects. Studies such as Agrawal et al. (2011) and Mollick (2014) focus on the geographic origin of consumers relative to the entrepreneur. Kuppuswamy and Bayus (2013) show that social information (i.e., other crowdfunders’ funding decisions) plays a key role in the success of a project. Focusing on equity-crowdfunding, Hildebrand, et al. (2013) identifies an increased problem of moral hazard.

3 Crowdfunding and the Information Trade-off

This section introduces the framework. It considers an entrepreneur, who can, prior to the investment stage, directly interact with consumers, who have private information about whether they value the product. We first introduce this model of demand uncertainty and subsequently introduce the problem of moral hazard.

The entrepreneur

We consider a penniless entrepreneur, who needs an upfront investment of $I > 0$ from investors to develop her product. After developing it, the entrepreneur can produce the good at some marginal cost $c \in [0, 1)$. The entrepreneur is crucial for realizing the project and cannot sell her idea to outsiders and let them develop it. We normalize interest rates to zero and abstract from discounting.

The crowd

We further consider a total of $n$ consumers and denote a specific consumer by the index $i = 1, \ldots, n$. A consumer $i$ either values the good, $v_i = 1$, or not, $v_i = 0$. In order to model an uncoordinated crowd, we assume that the consumers’ valuations

\footnote{The binary structure ensures that demand uncertainty expresses itself only concerning the question whether the entrepreneur should invest without affecting actual pricing decisions. This clarifies that the model’s driving force is not price discrimination. Section 6.3 addresses price discrimination.}
are iid with $\Pr\{v = 1\} = \nu$ and $\Pr\{v = 0\} = 1 - \nu$.\footnote{The next section allows these probabilities to be consumer specific, but, in line with standard mechanism design, upholds the independence assumption.} As a result, the number of consumers with value $v = 1$, which we express by $n_1$, is binomially distributed:

$$\Pr\{n_1\} = \binom{n}{n_1} \nu^{n_1} (1 - \nu)^{n - n_1}.$$ 

Since the marginal costs $c$ are smaller than 1, we can take $n_1$ as the potential demand of the entrepreneur’s good and its randomness expresses the demand uncertainty.

**Investing without demand uncertainty**

Consider as a benchmark the case of perfect information, where the realized demand $n_1$ is observable so that the investment decision can directly condition on it. It is socially optimal that the entrepreneur invests if it is large enough to cover the costs of production $I + n_1 c$, i.e. if

$$n_1 \geq \bar{n} \equiv \frac{I}{1 - c}.$$ 

In this case, the project generates an ex ante expected aggregate surplus of

$$S^* = \sum_{n_1 = \bar{n}}^{n} \Pr\{n_1\}[(1 - c)n_1 - I].$$ 

Note that by investing whenever $n_1 \geq \bar{n}$ and, subsequently, selling the good at a price $p = 1$, the entrepreneur can appropriate the full surplus. Given that the entrepreneur obtains the funds, this behavior represents an optimal strategy. Anticipating optimal behavior, a competitive credit market is willing to lend the required amount $I$ at the normalized interest rate of zero. Hence, perfect information yields an efficient outcome.

**Investing with demand uncertainty**

Next consider the setup with demand uncertainty, i.e. the entrepreneur must decide to invest $I$ without knowing $n_1$ if she wants to sell the good at some price $p$. If she does invest, it remains clearly optimal to sell the good at a price $p = 1$. Hence, expected profits from investing are

$$\bar{\Pi} = \left(\sum_{n_1 = 0}^{n} \Pr\{n_1\}(1 - c)n_1\right) - I.$$ 

It is therefore profitable to invest only if $\bar{\Pi} \geq 0$. As the price $p = 1$ does not leave any consumer rents, the entrepreneur’s profits coincide with aggregate welfare, but in comparison to the case of perfect demand information, either under- or over-investment results. For parameter constellations such that $\bar{\Pi} < 0$, the entrepreneur will not invest and, hence, under-investment results, because the good is not produced for any $n_1 > \bar{n}$, where it would be efficient to produce. For the parameter constellation $\bar{\Pi} \geq 0$, the entrepreneur does invest $I$, but this implies over-investment, since she produces the good also when it turns out that $n_1 < \bar{n}$.

**Crowdfunding**

We next consider the case of demand uncertainty but with crowdfunding the investment through consumers. This means that the entrepreneur commits to a pair $(p, T)$, where, as explained in the introduction, $p$ is the pledge level and $T$ the target level. The interpretation of this contract is that if $\hat{n}$ consumers make a pledge and the total amount of pledged funds, $P \equiv \hat{n}p$, exceeds $T$, then the entrepreneur obtains $P$, invests, and produces a good for each consumer who pledged. If the total amount of pledges $P$ falls short of $T$, then the pledges are not triggered and the entrepreneur does not invest. Hence, investment takes place when at least $T/p$ consumers pledge.

It is straightforward to see that crowdfunding enables the entrepreneur to extract the maximum aggregate surplus $S^*$ and, thereby, achieve an efficient outcome. Indeed, for any $p \in (0, 1]$, it is optimal for the consumer to pledge $p$ if and only if $\nu = 1$. As a result, exactly $n_1$ consumers sign up so that the sum of pledges equals $P = n_1p$. Hence, the project is triggered whenever $T \leq n_1 p$. We conclude that the crowdfunding scheme $(p, T)$ with $p \in (0, 1]$ yields the entrepreneur an expected profit

$$\Pi^c(p, T) = \sum_{n_1=T/p}^{n} \Pr\{n_1\}[(1 - c)p - I].$$

Price $p = 1$ and target level $P = \bar{n}$ maximize these profits, enabling the entrepreneur to extract the associated expected surplus of $S^*$ and yielding an efficient outcome.

In comparison to the single consumer example of the introduction, it is worthwhile to point out two additional features of the crowdfunding scheme. First, even without any active coordination between consumers, it circumvents any potential coordination problems. This is because of the scheme’s second feature: it eliminates any strategic uncertainty concerning both the behavior and the private information
of other consumers. In other words, the scheme’s conditional pledge system leads to a game between the consumers, in which it is a (weakly) dominant strategy for each individual consumer \( i \) to pledge if and only if \( v_i = 1 \).

**Moral hazard**

The setup until now abstracted from any problems of moral hazard. Consumers are sure to obtain the good as promised if their pledge is triggered. In practice, consumers may however worry about whether the entrepreneur will in the end deliver a good that meets the initial specifications, or whether they will receive some good at all.

We capture the problem of moral hazard by assuming that, after the entrepreneur obtains the money from the crowdfunding platform, she can make a run for it with a share \( \alpha \in [0, 1] \). When the entrepreneur “runs”, she does not incur any investment or production costs and consumers do not obtain their valuable goods. The parameter \( \alpha \) measures the strength of the moral hazard problem. For the extreme \( \alpha = 0 \), there is effectively no moral hazard, whereas for the other extreme, \( \alpha = 1 \), the principal can simply keep all the pledges without incurring any further costs.

The entrepreneur’s “running” captures several types of moral hazard problems. First, we can take the running literally: the entrepreneur is able to flee with the share \( \alpha P \) without being caught, or run with the amount \( P \) but with an expected fine of \((1 - \alpha)P\). Second, at a reduced cost of \((1 - \alpha)P < I - \tilde{c} \) the entrepreneur can provide the consumer a product that matches the formal description but is still worthless to the consumer. Third, by a cost \((1 - \alpha)P\), the entrepreneur can convincingly claim that the project failed so that, without fearing any legal repercussions, she need not deliver the product.

Considering the entrepreneur’s behavior, she obtains a profit \( P - I - cP/p \) from investing. An aggregated pledge level \( P \), therefore, induces the entrepreneur to run if

\[
\alpha P > P - I - cP/p.
\]

This holds for any \( \alpha > (p - c)/p \) and, in particular, for the extreme \( \alpha = 1 \). In this case, consumers rationally expect that the entrepreneur will not deliver the product so that they will not be willing to participate in this crowdfunding scheme.

One intuitive way to mitigate this problem is to change the crowdfunding scheme
such that the entrepreneur obtains the consumer’s pledges only after having produced the good. Because the penniless entrepreneur needs at least the amount $I$ to develop the product, such a delay in payments is possible only up to the amount $I$.

Hence, a first, ad hoc step to address the moral hazard problem is to adjust the crowdfunding scheme $(p, T)$ and introduce deferred payments as follows. As before, the price $p$ represents the pledge-level of an individual consumer and $T$ the target level which the sum of pledges, $P$, has to meet before the investment is triggered. Different from before however, the entrepreneur, after learning $P$, first obtains only the required amount $I$ for developing the product and only receives the remaining part $P - I$ after delivering the good to consumers.\footnote{As explicitly stated on their website, PledgeMusic, a reward-crowdfunding site specialized in raising funds for musicians, uses a scheme with deferred payments to prevent fraud.}

In order to characterize crowdfunding schemes with deferred payments that prevent moral hazard, note that the entrepreneur now obtains only the payoff $\alpha I$ from a run and the payoff $P - I - cP/p$ from realizing the project. Hence, she has no incentive to run if

$$\alpha I \leq P - I - cP/p \Rightarrow P \geq \bar{P} \equiv \frac{(1 + \alpha)IP}{p - c}. \quad (1)$$

In particular, the crowdfunding scheme with deferred payments $(p, T) = (1, (1 + \alpha)I/(1 - c))$ does not induce any moral hazard. Given this scheme, a consumer with value $v = 1$ is willing to pledge $p = 1$ and the scheme leads to an equilibrium outcome in which all consumers with $v = 1$ pledge and the project is triggered when at least $T$ consumers have the willingness to pay of 1, i.e. if $n_1 > (1 + \alpha)I/(1 - c)$. Although the scheme does prevent the moral hazard problem, it, for any $\alpha > 0$, does not attain the efficient outcome, because its target level is larger than the socially efficient one.

**The information trade-off**

We argued that a crowdfunding scheme with delayed final payments can circumvent the moral hazard problem to some extent. Since this delayed scheme does not yield an efficient outcome, the question arises whether even more sophisticated crowdfunding schemes exist that do better. To already give an indication of this, note that even though we praised the role of crowdfunding as a device to reduce demand uncertainty, the considered crowdfunding scheme actually reduces it too much when there is also
a moral hazard problem. Indeed, with respect to choosing the efficient investment decision, the entrepreneur only needs to know whether \( n_1 \) is above or below \( \bar{n} \). The exact value of \( n_1 \) is immaterial.

Yet, as inequality (1) reveals, the moral hazard problem intensifies if the entrepreneur obtains full information about \( P \). As discussed, this inequality has to hold for any possible realization of \( P \geq T \) in order to prevent the entrepreneur from running. Because the constraint is most stringent for \( P = T \), a crowdfunding scheme \((p, T)\) prevents moral hazard if and only if \( T \geq \bar{P} \). In contrast, if the entrepreneur would only learn that \( P \) exceeds \( T \), but not the exact value of \( P \) itself, then she rationally anticipates an expected payoff

\[
E[P|P \geq T] - I - cE[P|P \geq T]/p
\]

from not running with the money. Since, logically, the conditional expectation \( E[P|P \geq T] \) exceeds \( T \), a crowdfunding scheme that reveals only whether \( P \) exceeds \( T \) can deal with the moral hazard problem more efficiently.

Hence, in the presence of both demand uncertainty and moral hazard the information extraction problem becomes a sophisticated one. One neither wants too much nor too little information revelation. In order to find out the optimal amount of information revelation, we need to resort to the tools of mechanism design.

For this reason, the next section sets up the crowdfunding problem as one of mechanism design. It formally proves that the payout-deferred and information-restricted reward-crowdfunding scheme \((p, T)\) towards which we argued intuitively is indeed (constrained) efficient. More specifically, we can view such a crowdfunding scheme as an indirect implementation of an optimal direct mechanism.

4 Crowdfunding and Mechanism Design

In this section we cast the entrepreneur’s economic problem into a problem of mechanism design and characterize optimal mechanisms. In order to treat the entrepreneur’s moral hazard, we use the framework of Myerson (1982). This generalized framework introduces a mediator, who coordinates the communication between economic agents and gives incentive compatible recommendations concerning the unobservable
actions that lead to moral hazard. One of the insights from this analysis is that crowdfunding platforms play exactly the role of a mediator in the sense of Myerson (1982). The section’s main result is to confirm formally that the payout-deferred and information-restricted reward-crowdfunding scheme as identified in the previous section is a (constrained) efficient mechanism.

**Economic Allocations**

In order to cast the entrepreneur’s investment problem in a framework of mechanism design, we must first make precise the feasible economic allocations. Crowdfunding seeks to implement an allocation between one cash-constrained entrepreneur, player 0, and n consumers, players 1 to n. It involves monetary transfers and production decisions. Concerning monetary transfers, consumers can make transfers to the entrepreneur both before and after the entrepreneur’s investment decision. We denote the ex ante transfer from consumer i to the entrepreneur by \( t_a^i \) and the ex post transfer by \( t_p^i \). Concerning the production decisions, the allocation describes whether the entrepreneur invests, \( x_0 = 1 \), or not, \( x_0 = 0 \), and whether the entrepreneur produces a good for consumer i, \( x_i = 1 \), or not, \( x_i = 0 \). Consequently, an economic allocation is a collection \( a = (t, x) \) of transfers \( t = (t_a^1, \ldots, t_a^n, t_p^1, \ldots, t_p^n) \in \mathbb{R}^{2n} \) and outputs \( x = (x_0, \ldots, x_n) \in X \equiv \{0, 1\}^{n+1} \).

**Feasible Allocations**

By the very nature of the crowdfunding problem, the entrepreneur does not have the resources to finance the required investment \( I > 0 \); she is financially constrained. These constraints translate in the following restrictions on feasible allocations. First, if the entrepreneur invests \( x_0 = 1 \), the transfers of the consumers must be enough to cover the investment costs \( I \). Moreover, the entrepreneur can also not make any net positive ex ante transfers to consumers if she does not invest \( x_0 = 0 \). Second, aggregate payments must be enough to cover the entrepreneur’s investment and production costs. To express these two feasibility requirements, we say that an allocation \( a = (t, x) \) is budget-feasible if

\[
\sum_{i=1}^{n} t_a^i \geq I x_0 \quad \text{and} \quad \sum_{i=1}^{n} t_a^i + t_p^i \geq I x_0 + c \sum_{i} x_i. \tag{2}
\]

In addition, an entrepreneur can only produce a good to a consumer if she developed it. To express this feasibility requirement, we say that an allocation \( a = (t, x) \) is
development feasible if, whenever the good is produced for at least one consumer, the entrepreneur invested in its development:

$$\exists i : x_i = 1 \Rightarrow x_0 = 1.$$  \hspace{1cm} (3)

This condition logically implies that if $x_0 = 0$ then $x_i = 0$ for all $i$.

Let the set $A \subset \mathbb{R}^{2n} \times \{0, 1\}^{n+1}$ denote the set of budget- and development-feasible allocations, i.e. allocations that satisfy (2) and (3).

**Payoffs**

Let $v = (v_1, \ldots, v_n) \in V = \{0, 1\}^n$ represent the willingness to pay of the individual consumers and let $p(v)$ represent the probability of $v \in V$. We assume that $v_i$'s are drawn independently and identically: $Pr\{v_i = 1\} = Pr\{v_j = 1\}$ for all $i, j$ so that $p_i(v_i|v_{-i}) = p_i(v_i|v'_{-i}) = p_i(v_i)$ for all $v_{-i}, v'_{-i} \in V_{-i} \equiv \{0, 1\}^{n-1}$. As a consequence we can write this latter probability as $p_i(v_{-i})$.

A feasible allocation $a \in A$ yields a consumer $i$ with value $v_i$ the payoff

$$u_i(a|v_i) = v_i x_i - t^a_i - t^p_i;$$

and the entrepreneur the payoff

$$\pi(a) = \sum_{i=1}^{n} (t^a_i + t^p_i) - c \cdot \sum_{i=1}^{n} x_i - I x_0 \geq 0,$$

where the inequality follows directly from the second inequality in (2), implying that any feasible allocation yields the entrepreneur a non-negative payoff.

**Efficiency**

An output schedule $x \in X$ is Pareto efficient in state $v$ if and only if it maximizes the aggregate net surplus

$$S(x|v) \equiv \pi(a) + \sum_{i=1}^{n} u_i(a|v_i) = \sum_{i=1}^{n} (v_i - c) x_i - I x_0.$$ 

With respect to efficiency, two different types of production decisions matter: the overall investment decision $x_0$ and the individual production decisions $x_i$. Given $v_l = 0 < c < v_h = 1$, efficiency with respect to the individual allocations requires $x_i = v_i$. This yields a surplus of $\sum_i v_i(1 - c) - I$.  

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Defining
\[ \bar{n} \equiv \frac{I}{1-c}; \ V^0 \equiv \{ v : \sum_i v < \bar{n} \}; \ V^1 \equiv \{ v : \sum_i v \geq \bar{n} \}; \] and \[ p^* \equiv \sum_{v \in V^1} p(v), \]
we can fully characterize the Pareto efficient output schedule \( x^*(v) \) as follows. For \( v \in V^0 \), it exhibits \( x^*_0 = x^*_i = 0 \) for all \( i \). For \( v \in V^1 \), it exhibits \( x^*_0 = 1 \) and \( x^*_i = v_i \) for all \( i \).

Under an efficient output schedule, the entrepreneur invests only if \( v \in V^1 \), implying that \( p^* \) expresses the ex ante probability that the project is executed.

Although transfers are immaterial for Pareto efficiency, we must nevertheless ensure that the efficient output schedule \( x^*(v) \) can indeed be made part of some feasible allocation \( a \in A \). In order to specify one such feasible allocation, we define the first best allocation \( a^*(v) \) as follows. For \( v \in V^1 \), it exhibits \( x^*_i = t^a_i = v_i = 1 \) and \( t^p_i = 0 \). For \( v \in V^0 \), \( a^*(v) \) is defined by \( x^*_i = t^a_i = x^*_i = t^a_i = t^p_i = 0 \). By construction \( a^*(v) \) is feasible and yields an ex ante expected gross surplus (gross of investment costs) of \( W^* \) and an expected net surplus of \( S^* \), where

\[ W^* = \sum_{v \in V^1} \sum_i^n p(v)v_i(1-c) \] and \[ S^* = W^* - p^*I. \] (4)

We further say that an output schedule \( x : V \to X \) is development efficient if

\[ x_0(v) = 1 \Rightarrow \exists i : x_i(v) = 1. \] (5)

This condition is the converse of development feasibility (3). If it does not hold, it implies the inefficiency that there is a state \( v \) in which the entrepreneur invests \( I \) but no consumer consumes the good. Although technically feasible, a schedule that is not development efficient is not Pareto efficient, since it wastes the investment \( I > 0 \).

For future reference, the following lemma summarizes these considerations.

**Lemma 1** The first best allocation \( a^*(v) \) is feasible and exhibits an output schedule that is development efficient. It yields an expected net surplus of \( S^* \).

**Mechanisms**

We next turn to mechanisms. A mechanism \( \Gamma \) is a set of rules between the entrepreneur and the \( n \) consumers that induces a game. Its outcome is an allocation
\( a \in A \) with payoffs \( \pi(a) \) and \( u_i(a|v_i) \). In line with Myerson (1982), we interpret the crowdfunding platform as the mediator, who runs the mechanism; it credibly commits to enforce the rules of the game which the mechanism specifies, and it coordinates the communication between participants.

A **direct mechanism** is a function \( \gamma : V \rightarrow A \), which induces the following game. First, consumers simultaneously and independently send a (confidential) report \( v_i^r \) about their values to the platform. Based on the collected reports \( v^r \) and in line with the rules \( \gamma \), the platform collects the funds \( T_1 = \sum_i t_i^a(v^r) \) from the consumers and transfers them all to the entrepreneur together with the recommendation \( x_0(v^r) \) about whether to invest \( I \). To capture the moral hazard problem, we explicitly assume that the platform cannot coerce the entrepreneur into following the recommendation. That is, the entrepreneur is free to follow or reject it. If, however, the entrepreneur follows the recommendation, the platform enforces the production schedule \( x(v^r) = (x_1(v^r), \ldots, x_n(v^r)) \) and the transfers \( t_i^p(v^r) \). If the entrepreneur does not follow the recommendation, but runs, then individual production schedules are 0, and no ex post transfers flow, i.e. \( x_i = t_i^p = 0 \). Moreover, consumers forfeit their ex ante transfers \( t_i^a \).

A direct mechanism \( \gamma \) is **incentive compatible** if its induced game as described above has a perfect Bayesian equilibrium in which 1) consumers are **truthful** in that they reveal their values honestly, i.e. \( v_i^r = v_i \), and 2) the entrepreneur is **obedient** in that she follows the recommendation, i.e. \( x_0 = x_0(v^r) \).

To formalize the notion of truthful revelation, we define

\[
X_i(v_i) \equiv \sum_{v_{-i} \in V_{-i}} x_i(v_i, v_{-i}) p_i(v_{-i});
\]

and

\[
T_i(v_i) \equiv \sum_{v_{-i} \in V_{-i}} (t_i^a(v_i, v_{-i}) + t_i^p(v_i, v_{-i})) p_i(v_{-i}).
\]

Consequently, we say that a direct mechanism \( \gamma \) is **truthful** if

\[
v_i X_i(v_i) - T_i(v_i) \geq v_i X_i(v_i') - T_i(v_i') \text{ for all } i \in I \text{ and } v_i, v_i' \in V_i.
\]

To formalize the notion of obedience, we define for a direct mechanism \( \gamma \) the set \( T^a \) as the set of possible aggregate ex ante transfers which the mechanism can induce
conditional on recommending investment:

\[ \mathcal{T}^a \equiv \{ T^a | \exists v \in V : \sum_{i=1}^{n} t_i^a(v) = T^a \land x_0(v) = 1 \} \]

Given this set we define for any \( T^a \in \mathcal{T}^a \) the set \( V(T^a) \) which comprises all states that induce a recommendation to invest together with a total transfer \( T^a \):

\[ V(T^a) \equiv \{ v \in V | x_0(v) = 1 \land \sum_{i} t_i^a(v) = T^a \} \]

Upon receiving a recommendation to invest, the entrepreneur has received some transfer \( T^a \in \mathcal{T}^a \) and has a belief \( p(v|T^a) \) that the state is \( v \). These beliefs are Bayes’ consistent whenever

\[
p(v|T^a) \equiv \begin{cases} 
p(v) \sum_{v' \in V(T^a)} p(v') & \text{if } v \in V(T^a); \\
0 & \text{otherwise.}
\end{cases}
\]

We say that a direct mechanism \( \gamma \) is obedient if for any \( T^a \in \mathcal{T}^a \) and after obtaining the recommendation to invest, \( x_0 = 1 \), the entrepreneur, given her updated belief \( p(v|T^a) \), is better off investing than taking the money and run:

\[
\sum_{v \in V} \sum_{i=1}^{n} p(v|T^a)(t_i^a(v) - cx_i(v)) + T^a - I \geq \alpha T^a, \text{ for all } T^a \in \mathcal{T}^a.
\]

(7)

We say that a direct mechanism is incentive compatible if and only if it is truthful and obedient.

Note that crowdfunding schemes, which hand all transfers to the entrepreneur up-front, exhibits \( t_i^a(v) = 0 \) for all \( i \) and \( v \). Such schemes necessarily violate condition (7) for any \( T^a \in \mathcal{T}^a \). This formally confirms our informal discussion that such schemes are unable to handle the extreme kind of moral hazard problems that we consider here.

By its nature, participation in the crowdfunding mechanism is voluntary so that it must yield the consumers and the entrepreneur at least their outside option. Taking these outside options as 0, the entrepreneur’s participation is not an issue, because, as argued, any feasible allocation yields the entrepreneur a non-negative payoff. In contrast, a consumer’s participation in an incentive compatible direct mechanism is individual rational only if

\[
v_i X_i(v_i) - T_i(v_i) \geq 0 \text{ for all } i \in I \text{ and } v_i \in V_i.
\]

(8)
We say that a direct mechanism $\gamma$ is \textit{feasible}, if it is incentive compatible and individual rational for each consumer.\footnote{This implicitly assumes that the mechanism has “perfect consumer reach” in that every consumer is aware and can participate in the mechanism. As an extension that yields important additional insights, Subsection 6.2 studies the effect of imperfect consumer reach.} A feasible direct mechanism yields consumer $i$ with valuation $v_i$ the utility
\begin{equation}
    u_i(v_i) \equiv v_iX_i(v_i) - T_i(v_i).
\end{equation}
and the entrepreneur an expected payoff
\begin{equation}
    \pi = \sum_{v \in V} p(v) \pi(\gamma(v)).
\end{equation}

We say that two feasible direct mechanisms $\gamma = (t, x)$ and $\gamma' = (t', x')$ are \textit{payoff-equivalent} if they lead to identical payoffs to each consumer $i$:
\begin{equation}
    \sum_{v_{-i} \in V_{-i}} p(v_{-i}) u_i(\gamma(v), v_i) = \sum_{v_{-i} \in V_{-i}} p(v_{-i}) u_i(\gamma'(v), v_i) \ \forall i, v_i;
\end{equation}
and the entrepreneur:
\begin{equation}
    \sum_{v \in V} p(v) \pi(\gamma(v)) = \sum_{v \in V} p(v) \pi(\gamma'(v)).
\end{equation}

\section*{Implementability}
An \textit{allocation function} $f : V \rightarrow A$ specifies for any value profile $v$ an allocation $a \in A$. It is \textit{implementable} if there exists a mechanism $\Gamma$ such that the induced game has a perfect Bayesian equilibrium outcome in which the induced allocation coincides with $f(v)$ for every $v \in V$. In this case, we say $\Gamma$ \textit{implements} $f$.

Likewise, an \textit{output schedule} $x : V \rightarrow X$ specifies for any value profile $v$ an output schedule $x \in X$. It is \textit{implementable} if there exists a mechanism $\Gamma$ such that the induced game has a perfect Bayesian equilibrium outcome in which the induced output coincides with $x(v)$ for every $v \in V$. In this case, we say $\Gamma$ implements output schedule $x(\cdot)$.

By the revelation principle, an allocation function $f(\cdot)$ is implementable if and only if there exists a feasible direct mechanism $\gamma$ with $\gamma(v) = f(v)$ for any $v \in V$. Likewise, an output schedule $x(\cdot)$ is implementable if and only if there exists a direct mechanism...
\( \gamma = (x_\gamma, t_\gamma) \) such that \( x_\gamma(v) = x(v) \) for any \( v \in V \). Hence, the revelation principle, as usual, motivates incentive compatibility as one of the defining requirements of feasibility. A first question that arises is whether an efficient output schedule is always implementable. Considering a specific version of the model, the next proposition demonstrates that this is not the case:

**Proposition 1** For \( I = n - 1/2, \alpha = 1, \) and \( c = 0 \), the efficient output schedule \( x^*(v) \) is not implementable.

The proposition implies that, in general, the efficient output is not implementable. The main driver behind this inefficiency result is a tension between the entrepreneur’s budget constraint and her moral hazard problem. For consumers to make sure that the entrepreneur realizes her project, it does not suffice to give her simply the required amount \( I \) to invest. Due to the moral hazard problem, she must also be given an incentive to actually invest this money. The proposition shows that solving one problem, in general, precludes solving the other.

The proposition raises questions about which output schedules are generally implementable and about the conditions under which the efficient schedule is implementable. To answer these questions we investigate the mechanism design problem further. The following lemma shows that with respect to development-efficient allocations, we may reduce the class of feasible direct mechanisms further.

**Lemma 2** If \( \gamma = (t, x) \) is feasible and \( x \) is development-efficient then there is a feasible and pay-off equivalent direct mechanism \( \gamma = (\hat{t}, x) \) with

\[
\sum_i \hat{t}_i(v) = Ix_0(v), \forall v \in V.
\]  

The lemma implies that with respect to development-efficient mechanisms there is no loss of generality in restricting attention to feasible direct mechanisms that satisfy (11). Hence, we only need to consider mechanisms that give the entrepreneur exactly the amount \( I \) if the entrepreneur is to develop the product. This also means that the lemma makes precise the suggestion of the previous section that a mechanism should provide the entrepreneur with the minimal amount of information for reducing demand uncertainty; effectively, she should only be told that the demand of consumers
ensures that the project has a positive NPV, but not more. The main step in proving this result is to show that obedience remains satisfied when we replace different aggregate levels of ex ante payments by a single one.\(^{11}\)

The lemma simplifies the mechanism design problem in two respects. First, under condition (11), condition (2) reduces to

$$\sum_{i=1}^{n} t_i^p(v) \geq c \sum_{i} x_i(v). \quad (12)$$

Second, under condition (11), we have $T^a = \{I\}$ so that the obedience constraint (7) must only be respected with regard to $I$:

$$\sum_{v \in V} \sum_{i=1}^{n} p(v|I)(t_i^p(v) - cx_i(v)) \geq \alpha I. \quad (13)$$

## 5 Second-best crowdfunding schemes

In this section we characterize second best mechanisms $\gamma^{sb} = (x^{sb}, t^{sb})$ that maximize aggregate surplus in the presence of demand uncertainty and moral hazard. We are especially interested in determining and understanding the circumstances under which these second best mechanisms do not implement the efficient output schedule $x^*$.

Recall that a feasible direct mechanism $\gamma$ yields a surplus of

$$\sum_{v \in V} p(v)S(x(v)|v) = \sum_{v \in V} p(v) \left[ \sum_{i}(v_i - c)x_i(v) - Ix_0(v) \right]. \quad (14)$$

Clearly $\gamma^{sb}$ cannot yield more than the surplus $S^*$ that is generated under the efficient output schedule $x^*$. Indeed, Proposition 1 showed that, in general, we cannot guarantee that $\gamma^{sb}$ attains $S^*$. In this case, the second best output schedule $x^{sb}$ does not coincide with $x^*$ and will display distortions.

As $\gamma^{sb}$ is necessarily development-efficient, we can find $\gamma^{sb}$ by maximizing (14) subject to the constraints (6), (8), (11), (12), and (13), because these constraints characterize the set of implementable allocation functions that are development-efficient.

A straightforward consideration of this maximization problem yields the following partial characterization of $\gamma^{sb}$:

\(^{11}\)The lemma fails for specific development-inefficient mechanisms so that we cannot dispense with the restriction to development-efficient mechanisms.
Lemma 3  The individual rationality constraint of consumers with the high value \( v_i = 1 \) does not restrict the second best mechanism \( \gamma^{sb} \). The second best mechanism exhibits \( x_i(0, v_{-i}) = X_i(0) = T_i(0) = 0, \) and \( T_i(1) = X_i(1) \) for all \( i = 1, \ldots, n \).

It follows from the previous lemma that the second best mechanism \( \gamma^{sb} \) is a solution to the problem \( \mathcal{P} \):

\[
\mathcal{P} : \max_{x(.), t(.)} \sum_{v \in V} p(v) \left[ \sum_{i} (v_i - c)x_i(v) - Ix_0(v) \right]
\]

s.t. \( T_i(1) = X_i(1) \) for all \( i ; \)  
\( \sum_{v \in V} \sum_{i=1}^{n} p(v|I)(t_i^p(v) - cx_i(v)) \geq \alpha I ; \)  
\( T_i(0) = 0 \) for all \( i ; \)  
\( \sum_{i=1}^{n} t_i^a(v) = Ix_0(v) ; \)  
\( \sum_{i=1}^{n} t_i^p(v) \geq \sum_{i} cx_i(v) ; \)  
\( x_i(v) = 1 \Rightarrow x_0(v) = 1 ; \)  
\( x_i(0, v_{-i}) = 0, \forall v_{-i} \in V_{-i} . \)

Recalling that \( p^* \) represents the ex ante probability that the project is executed under the efficient schedule \( x^* \), we obtain the following result.

**Proposition 2**  The efficient output schedule \( x^* \) is implementable if and only if \( W^* \geq (1 + \alpha)p^*I \).

Proposition 2 makes precise the parameter constellation under which the first best \( x^* \) is implementable: only if the efficient production schedule \( x^* \) generates a surplus that exceeds the ex ante expected investment costs by \( (1 + \alpha) \) times.

As argued before, the main driver behind the inefficiencies is a tension between the entrepreneur’s budget constraint and her moral hazard problem. For consumers to make sure that the entrepreneur realizes her project, it does not suffice to give her simply the required amount \( I \) to invest. Due to the moral hazard problem, she must also be given an incentive to actually invest this money. As the proposition shows, this effectively requires consumers to pay the entrepreneur the run-away payoff \( \alpha I \).
consumers, realizing the project is therefore only worthwhile if the project’s revenue recovers the augmented investment cost \((1 + \alpha)I\).

Effectively, the proposition shows that the combination of the entrepreneur’s budget constraint and her moral hazard problem increases investment costs by a factor \(\alpha\). It prevents first best outcomes if the expected gross surplus \(W^*\) is too small.

Whenever the ex ante gross surplus does not exceed the expected investment costs by the factor \(\alpha\), the efficient output schedule, \(x^*\), is not implementable so that the second best output schedule \(x^{sb}\) does not coincide with \(x^*\). We next derive a partial characterization of the second best and, more importantly, characterize the type of efficiencies it exhibits.

**Proposition 3** For \(W^* < (1 + \alpha)p^*I\), the constrained efficient output schedule \(x^{sb}\) exhibits \(x^{sb}_i(v) = v_i\) whenever \(x^{sb}_0(v) = 1\); and \(x^{sb}_0(v) = 0\) whenever \(x^*_0(v) = 0\). Moreover, \(x^*_0(v) = 1\) whenever \(\sum v_i > (1 + \alpha)I/(1 - c)\). There exists a \(T \in R_+\) such that \(x^{sb}_0(v) = 1\) iff \(\sum v_i \geq T\).

The first part of the proposition shows that the constrained efficient output schedules are only distorted with respect to the investment decision but not to the individual assignments. The second part of the proposition shows that the second best output schedule is distorted downwards rather than upwards. The third part shows that at most the allocations for which aggregate valuations lie in the range between \(I/(1 - c)\) and \((1 + \alpha)I/(1 - c)\) are downward distorted. Exactly which of these are distorted downwards depends on the specific parameter constellation. The final statement implies that for the constrained efficient output schedule it matters only whether the sum of valuations exceed a target level \(T\). As a result, the second best scheme can be implemented indirectly by a crowdfunding scheme \((1, T)\).

6 Interpretation and Discussion

This section interprets the optimal direct mechanisms as derived in the previous sections and relate them to crowdfunding schemes in practise. It further discusses extensions and robustness of the results.
6.1 Comparison to real-life crowdfunding schemes

A first notable feature of optimal direct mechanisms is that they explicitly condition the entrepreneur’s investment decision on the aggregate reported valuations rather than each consumer’s report individually. This result confirms the intuitive ideas developed in Section 3. It is not only consistent with the “all-or-nothing” pledge schemes of the popular reward-crowdfunding platform Kickstarter, but also many others such as the music crowdfunding platforms Sellaband, which was already established in 2006, and PledgeMusic. We can interpret such schemes as indirect mechanisms that implement such conditional investment optimally.

Interestingly, the “keep-what-you-raise” model, where pledges are triggered even if the target level is not reached, is an alternative scheme that is popular for platforms that are less oriented towards for-profit causes such as the donation platform GoFundMe. Indiegogo, which markets itself as both a for-profit and non-profit crowdfunding platform, lets the project’s initiator decide between the two options. Anecdotal evidence suggests that for-profit projects are more prone to select the “all-or-nothing” scheme.

A second feature of optimal direct mechanisms is that they do not exhibit negative transfers. Hence, at no point in time does the entrepreneur pay consumers any money. This means that the entrepreneur does not share any of her revenue or profits after the investment. Consequently, optimal direct mechanisms do not turn consumers into investors; they are not equity- or debt-based. This feature is consistent with current reward-crowdfunding in that independently of the entrepreneur’s final revenues, a crowdfunding consumer receives only a fixed, non-monetary reward for his pledged contribution. Reward-crowdfunding schemes such as Kickstarter actually explicitly prohibit financial incentives like equity or repayment to crowdfunders.\(^{12}\) The next subsection points out however that optimal mechanism may require negative transfers if the consumer reach of the platform is limited.

A third feature of optimal direct mechanisms is the use of deferred payments to prevent moral hazard. Some but definitely not all crowdfunding platforms do so. For instance, PledgeMusic, a reward-crowdfunding site specialized in raising funds for

recordings, music videos, and concerts, explicitly states on its Website that it uses a sophisticated scheme with deferred payouts to prevent fraud.\textsuperscript{13} For its “direct-to-fan campaigns”, which represent its reward-crowdfunding schemes, it actually uses three payout phases: For a project that exceeds its target level, it pays 75\% of the target level (minus commissions) directly after the crowdfunding stage ends successfully. The remaining 25\% of the target level is paid out upon delivery of the digital album, while all funds in excess of the target level are paid out only upon successful delivery of all other types of rewards.

The final notable feature of optimal direct mechanisms is that they provide only information about whether the sum of pledges exceeds the target and not the total sum of pledges. In line with Lemma 2 any additional information is not needed to implement (constrained) efficient outcomes. Schemes that provide more information may exacerbate the moral hazard problem. Current crowdfunding platforms do not reflect this feature. Currently all crowdfunding platforms are fully transparent and announce publicly the total amount of pledges rather than just whether the target level was reached.\textsuperscript{14}

Despite explicit concerns by the press, practitioners, and also the crowdfunding platforms themselves, moral hazard currently does not seem a major issue for crowdfunding platforms. Studying failure rates in funded Kickstarter projects, Mollick (2013) reports a rate well below 5\% and explicitly notes that cases of fraud were “very rare”. The study does report a tendency of entrepreneurs to delay the delivery of rewards, but attributes this to genuine difficulties in production rather than fraud (see also Cowley et al. 2012).

As we discuss in the next subsection, one reason for this is the prospect of sales from non-crowdfunders. Due to the fact that crowdfunding is still a rather new phenomenon and does not reach all potential consumers, a successful entrepreneur can expect a substantial after-crowdfunding market and sell her products to consumers who did not participate in crowdfunding. The entrepreneur’s prospect to sell her

\textsuperscript{13}See http://www.pledgemusic.com/blog/220-preventing-fraud (last retrieved 20.07.2015)

\textsuperscript{14}If crowdfunders deduce the project’s quality from the pledges of others, then suppressing such information can actually be harmful. Studying the dynamics of crowdfunding contributions, Kuppuswamy and Bayus (2013), however, do not find support for such learning or other herding effects.
goods to non-crowdfunding consumers acts as a substitute for deferred payments and, therefore, mitigates moral hazard. As crowdfunding becomes more popular and the after-crowdfunding market decreases, this substitution effect diminishes. We investigate this aspect more closely in the next subsection.

6.2 Consumer reach and Crowdinvestors

In our formal analysis, consumers could only acquire the product by participating in the mechanism and, by assumption, the mechanism is able to reach every potential consumer. Given this latter assumption, the assumption that consumers can only acquire the product through the mechanism is, by the revelation principle, without loss of generality. This changes however when, for some exogenous reason, a mechanism’s consumer reach is imperfect in that not all consumers can participate in it. In practice this seems a reasonable concern, because a share of consumers may, for example, fail to notice the crowdfunding scheme, not have access to the internet, or only arrive in the market after the product has been developed. Hence, a relevant extension of our framework is to consider mechanisms, which, for some exogenous reason, have an imperfect consumer reach.

Consider first a model with imperfect consumer reach, in which only a share of $\beta \in (0, 1)$ can take part in the mechanism. Already the pure proportional case that a consumer’s ability to participate is independent of his valuation, yields new insights. For the pure proportional case, the crowdfunding scheme is still able to perfectly elicit consumer demand; a pledge by $\tilde{n}$ consumers means that there are in fact $n_1 = \tilde{n}/\beta$ who value the product. Consequently, investment is efficient if and only if

$$\tilde{n}/\beta \geq I/(1-c) \Rightarrow \tilde{n} \geq \tilde{n}(\beta) \equiv \beta I/(1-c).$$

It is straightforward to see that the previous analysis still applies when we factor in $\beta$. In particular, the efficient output scheme is implementable for $W^* \geq (1 + \alpha)p^*I\beta.15$

Even though our analysis readily extends to this proportional case, the interest-

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15 This “proportionality” property holds because the derived efficient scheme extracts all rents from consumers and the entrepreneur can implement the efficient outcome by using the scheme as derived and set a price $p = 1$ to the $(1 - \beta)n$ consumers who can only participate after the good has been developed.
ing economic effect arises that consumers become active investors when the share of crowdfunding consumers $\beta$ is small. To see this, note that, because the entrepreneur needs the amount $I$ to develop the product, the (average) ex ante transfer of a pledging consumer needs to be at least $I/\tilde{n}$. When $\beta$ is small in the sense that $\tilde{n}(\beta)$ is smaller than 1, it follows that for $\tilde{n}$ close to $\tilde{n}(\beta)$, the consumer’s ex ante transfer exceeds his willingness to pay. Individual rationality then implies that the ex post transfer to the consumer must be negative in order to make it worthwhile for the consumer to participate.

Since a negative ex post transfer means that the entrepreneur returns part of his ex ante contribution after the project is executed, such transfers imply that the consumer effectively becomes an investor in the usual sense that he receives a monetary return on his initial layout. Hence, with limited consumer reach, efficient crowdfunding schemes may require consumers to become actual investors.

As noted, reward-crowdfunding schemes such as Kickstarter explicitly prohibit financial incentives like equity or repayment to crowdfunders. Our formal analysis confirms that this is indeed not needed if the investment $I$ is small compared to the number of crowdfunding consumers, but for large investments and crowdfunding schemes with a relatively small consumer reach, such restrictions may matter.\textsuperscript{16} Probably the main reason that reward-crowdfunding platforms do not allow financial incentives is due to regulation. Without any monetary flows from the entrepreneur to crowdfunders, crowdfunding in the US is not an investment vehicle and does therefore not fall under SEC regulation.

\textsuperscript{16}Ordanini et al. (2011) report the case of Cameesa, a Chicago based clothing company which in 2008 introduced an “all-or-nothing” crowdfunding model, but also shared revenue with its crowdfunders. The company accepted pledges with a minimum of $10 from “Supporters” for the production of T-shirts designs and a target level of $500, which cumulative pledges needed to reach before the T-shirt was produced. Any Supporter who pledged in a failing design got their money back, while Supporters of a successful design not only obtained the shirt, but also shared in some of the revenue of its future sales. (see http://www.cnet.com/news/cameesa-a-threadless-where-customers-are-also-investors/, last retrieved 22.7.2015).
6.3 Elastic demand

Our formal model assumes that consumers’ valuations are of a binary nature. Consumers either do not value the good \((v = 0)\) or value it at the same positive amount \((v = 1)\). This assumption yields an inelastic demand structure and, more importantly, a framework in which demand uncertainty expresses itself only concerning the question whether the entrepreneur should invest or not. It enabled us to clearly illustrate crowdfunding as an economic institution that creates economic value by reducing demand uncertainty and, thereby, identify the projects with a positive value. Moreover, it allowed us to clarify that the “all-or-nothing”-pledge system of current crowdfunding platforms is in fact a crucial feature for enabling an effective screening of projects.

An obvious modeling extension is to consider consumer valuations that are drawn from more than two states or with different supports. In this case, aggregate demand will be elastic and demand uncertainty expresses itself not only in the question of whether to invest but also in the question of what price (or pledge level) to set. Hence, crowdfunding can then also be used to reduce uncertainty about picking the right price. The Economist (2010) reports the concrete example of a book publisher planning to fund a renewed publication of a sold-out book: “his efforts to tease out lenders’ price sensitivity from previous Kickstarter projects showed that a $50 contribution was the most popular amount. It also proved the largest dollar component for the highest-grossing Kickstarter projects.”

6.4 Crowdfunding vs. Venture Capitalists

By enabling direct interactions with consumers before the investment and financing stage, crowdfunding leads to a transformation of the entrepreneurial business model. The transformation takes place at a fundamental level in that it upsets the traditional separation of finance and marketing.\(^{17}\) Figure 6.4 illustrates this transformation. In the traditional model, venture capitalists (or banks) attract capital from consumers to finance entrepreneurs, who subsequently use this capital to produce goods and market

\(^{17}\)In contrast, “equity-crowdfunding” upholds the traditional separation between finance and marketing, because the consumers and the crowd-investors are typically not the same economic agents.
them to consumers. In this traditional model, finance and marketing are naturally separated and run along different channels. In contrast under reward-crowdfunding, finance and marketing run along the same channel: the crowdfunding platform.

We stress however that crowdfunding and venture capital financing are not mutually exclusive. There is no reason why a venture capitalist may not use crowdfunding to learn about demand and there is no reason why after a successful crowdfunding campaign an entrepreneur may not approach a venture capitalist. Dingman (2013) reports that this already occurred for the Pebble Smart Watch. Venture capitalist decided to support the entrepreneur’s project only after a successful crowdfunding campaign on Kickstarter. Quoting a managing partner of a venture capitalist firm: “What venture capital always wants is to get validation, and with Kickstarter, he [i.e. the entrepreneur] could prove there was a market.”

7 Conclusion

Crowdfunding provides the innovation that, already before the product is developed, an entrepreneur can directly interact with her future consumers. This enables the entrepreneur to condition her investment decision on a revelation of consumer’s demand. Optimal mechanisms use this ability by committing the entrepreneur to invest only if consumers reveal enough demand. Current all-or-nothing reward-crowdfunding schemes such as Kickstarter reflect this feature of optimal mechanisms. They are therefore consistent with the idea that these schemes are used to reduce demand uncertainty and improve a screening of entrepreneurial projects.

Despite the effectiveness of reward-crowdfunding schemes in eliciting demand information, their susceptibility to entrepreneurial moral hazard may prevent the imple-
mentation of fully efficient outcomes. In the presence of moral hazard, crowdfunding can attain fully efficient outcomes only if the project’s ex ante expected gross return exceeds its ex ante expected investment costs by a markup whose size reflects the severeness of the moral hazard problem. Constrained efficient mechanisms exhibit underinvestment, but still reflect crucial features of current all-or-nothing reward-crowdfunding schemes.

Because crowdfunding schemes by themselves are, in the presence of moral hazard, unable to attain efficiency in general, we see them as complements rather than substitutes for traditional venture capital. We therefore expect a convergence of the two financing forms. Current policy measures such as the US JOBS Act and its implementation in SEC (2015) will make such mixed forms of crowdfunding and more traditional venture capitalism easier to develop so that one can take advantage of their respective strengths. The website of the crowdfunding platform Rockethub already explicitly mentions this possible effect of the JOBS Act.18

Appendix

This appendix collects the formal proofs.

Proof of Lemma 1: Follows directly from the text Q.E.D.

Proof of Proposition 1: Let \( 1^n \) denote the vector \((1,\ldots,1)\) \(\in\mathbb{R}^n\). Since \(\bar{n} = I/(1-c) = n - 1/2\), it follows \(V^1 = \{1^n\}\) and \(V^0 = V \setminus V^1\) so that the efficient output schedule \(x^*(v)\) exhibits \(x^*_0(v) = x^*_i(v) = 0\) for \(v \neq 1^n\), and \(x^*_0(v) = x^*_i(v) = 1\) for \(v = 1^n\). We show, by contradiction, that a feasible direct mechanism \(\gamma^*\) that implements \(x^*(v)\) does not exist.

For suppose to the contrary that such a direct mechanism does exist, then there exists a transfer schedule \(t\) so that the direct mechanism \(\gamma^* = (x^*,t)\) is feasible. Since \(x^*_0(v) = 1\) implies \(v = 1^n\), it follows that \(T^a\) is a singleton and for all \(T^a \in T^a\), it holds \(V(T^a) = \{1^n\}\). Consequently, \(p(1^n|T^a) = 1\) and \(p(v|T^a) = 0\) for all \(v \neq 1^n\). Since \(\alpha = 1\) and \(c = 0\), (7) rewrites after multiplying by \(p(1^n)\) therefore as

\[
\sum_{i=1}^{n} t_i^0(1^n)p(1^n) \geq Ip(1^n). \tag{22}
\]

Since \(x_0(1^n) = 1\) the first inequality in (2) implies after multiplying with \(p(1^n)\)

\[
\sum_{i=1}^{n} t_i^0(1^n)p(1^n) \geq Ip(1^n). \tag{23}
\]

Note further that the second inequality in (2) for each \(v \neq 1^n\) implies

\[
\sum_{i=1}^{n} t_i^a(v) + t_i^p(v) \geq 0 \tag{24}
\]

Multiplying with \(p(v)\) and adding over all \(v \neq 1^n\) yields

\[
\sum_{v \neq 1^n} \sum_{i=1}^{n} (t_i^a(v) + t_i^p(v))p(v) \geq 0 \tag{25}
\]

Combining (22), (23), and (25) yields

\[
\sum_{i} \sum_{v \in V} (t_i^a(v) + t_i^p(v))p(v) \geq 2Ip(1^n) = (2n - 1)p(1^n), \tag{26}
\]

where the equality uses \(I = n - 1/2\).

We now show that (26) contradicts (8). First note that (8) for \(v_i = 0\) implies after a multiplication by \(p_i(0)\) for each \(i\)

\[
\sum_{v_{-i} \in V_{-i}} (t_i^a(0,v_{-i}) + t_i^p(0,v_{-i}))p(0,v_{-i}) \leq 0. \tag{27}
\]
Summing over $i$ it follows

$$\sum_i \sum_{v_{-i} \in V_{-i}} (t_i^a(0, v_{-i}) + t_i^p(0, v_{-i}))p(0, v_{-i}) \leq 0. \quad (28)$$

Likewise, since $X_i(1) = p_i(1^{n-1})$, (8) for $v_i = 1$ implies after a multiplication with $p_i(1)$ and using $p_i(1) = p(1, v_{-i})$ that for each $i$

$$\sum_{v_{-i} \in V_{-i}} (t_i^a(1, v_{-i}) + t_i^p(1, v_{-i}))p(1, v_{-i}) \leq p(1^n). \quad (29)$$

Summing over $i$ yields

$$\sum_i \sum_{v_{-i} \in V_{-i}} (t_i^a(1, v_{-i}) + t_i^p(1, v_{-i}))p(1, v_{-i}) \leq p(1^n)n. \quad (30)$$

Combining (28) and (30) yields

$$\sum_i \sum_{v \in V} (t_i^a(v) + t_i^p(v))p(v) \leq p(1^n)n. \quad (31)$$

But since $2n - 1 > n$, this contradicts (26).

**Proof of Lemma 2:** Fix a feasible $\tilde{\gamma} = (\tilde{t}, \tilde{x})$ with $\tilde{x}$ development efficient and define for each $v$,

$$\tilde{K}(v) = \sum_i \tilde{t}_i^a(v) - I\tilde{x}_0(v).$$

Feasibility of $\tilde{\gamma}$ means $\tilde{x}(v)$ satisfies (2) for all $v \in V$, and therefore $\tilde{K}(v) \geq 0$ for all $v \in V$. For any state $v$, let $\tilde{n}(v) \equiv \sum_i \tilde{x}_i(v)$ represent, for a given $v$, the total number of consumers with $x_i = 1$. For any state $v$ with $\tilde{x}_0(v) = 0$, define $\tilde{t}_i^a(v) \equiv 0$ and $\tilde{t}_i^p(v) \equiv \tilde{t}_i^p(v) + \tilde{t}_i^a(v)$. For $\tilde{x}_0(v) = 1$ define $\tilde{t}_i^a(v) \equiv \tilde{t}_i^a(v) - \tilde{x}_i(v)\tilde{K}(v)/\tilde{n}(v)$ and $\tilde{t}_i^p(v) \equiv \tilde{t}_i^p(v) + \tilde{x}_i(v)\tilde{K}(v)/\tilde{n}(v)$. Since $\tilde{x}$ is feasible and development efficient, it holds $\tilde{n}(v) > 0$ if and only if $\tilde{x}_0(v) = 1$. Hence, the transformed transfer schedule $\tilde{t}$ is well-defined. By construction, we have $\tilde{t}_i^a(v) + \tilde{t}_i^p(v) = \tilde{t}_i^a(v) + \tilde{t}_i^p(v)$ for all $v$, and $\sum_i \tilde{t}_i^a(v) = 0$ for any $v$ with $x_0(v) = 0$, and $\sum_i \tilde{t}_i^a(v) = \sum_i \tilde{t}_i^a(v) - \tilde{x}_i(v)\tilde{K}(v)/\tilde{n}(v) = \sum_i \tilde{t}_i^a(v) - \tilde{K}(v) = I$ for any $v$ with $x_0(v) = 1$. Hence, the allocation $(\tilde{t}(v), \tilde{x}(v))$ satisfies (11). Because the allocation $(\tilde{t}(v), \tilde{x}(v))$ is development feasible, also the allocation $(\tilde{t}(v), \tilde{x}(v))$ is development feasible. Moreover, from $\tilde{t}_i^a(v) + \tilde{t}_i^p(v) = \tilde{t}_i^a(v) + \tilde{t}_i^p(v)$ it follows that $(\tilde{t}, \tilde{x})$ is also budget-feasible, truthful, and individual rational, given that $(\tilde{t}, \tilde{x})$ is so by assumption.
In order to show that \((\bar{t}, \bar{x})\) is feasible, it only remains to show that it is obedient, i.e., satisfies (7). To show this, define for \(\bar{T}^a \in \mathcal{T}^a\)

\[
\bar{P}(\bar{T}^a) = \sum_{v \in V(\bar{T}^a)} p(v),
\]

Now since, \(\bar{\gamma} = (\bar{t}, \bar{x})\) is obedient by assumption, (7) holds for any \(\bar{T}^a \in \mathcal{T}^a\). Given that \(\bar{T}^a = \sum_i \bar{t}^a_i(v)\) for any \(v\) such that \(\bar{p}(v|\bar{T}^a) > 0\), we can rewrite (7) as

\[
\sum_{v \in V} \sum_{i=1}^n \bar{p}(v|\bar{T}^a)(\bar{p}^a_i(v) - c\bar{x}_i(v) + \hat{\bar{p}}_i(v)) - I \geq \alpha \bar{T}^a, \text{ for all } \bar{T}^a \in \bar{\mathcal{T}}^a. \tag{32}
\]

From \(\hat{\bar{t}}^a_i(v) + \hat{\bar{p}}_i(v) = \bar{t}^a_i(v) + \hat{\bar{p}}_i(v)\), this rewrites as

\[
\sum_{v \in V} \sum_{i=1}^n \bar{p}(v|\bar{T}^a)(\bar{p}^a_i(v) - c\bar{x}_i(v) + \hat{\bar{p}}_i(v)) - I \geq \alpha \bar{T}^a, \text{ for all } \bar{T}^a \in \bar{\mathcal{T}}^a. \tag{33}
\]

Because, by construction \(\sum_i \bar{t}^a_i(v) = I\) for \(v\) such that \(\bar{p}(v|\bar{T}^a) > 0\), this rewrites as

\[
\sum_{v \in V} \sum_{i=1}^n \bar{p}(v|\bar{T}^a)(\bar{t}^a_i(v) - c\bar{x}_i(v)) \geq \alpha I, \text{ for all } \bar{T}^a \in \bar{\mathcal{T}}^a. \tag{34}
\]

Moreover, since feasibility implies that \(\bar{T}^a \geq I\), the previous inequality implies that

\[
\sum_{v \in V} \sum_{i=1}^n \bar{p}(v|\bar{T}^a)(\bar{t}^a_i(v) - c\bar{x}_i(v)) \geq \alpha I, \text{ for all } \bar{T}^a \in \bar{\mathcal{T}}^a. \tag{35}
\]

It follows after a further multiplication by \(\bar{P}(\bar{T}^a)\) that

\[
\sum_{v \in V} \sum_{i=1}^n \bar{p}(v|\bar{T}^a)\bar{P}(\bar{T}^a)(\bar{t}^a_i(v) - c\bar{x}_i(v)) \geq \alpha I \cdot \bar{P}(\bar{T}^a), \text{ for all } \bar{T}^a \in \bar{\mathcal{T}}^a. \tag{36}
\]

By definition of \(\bar{p}(v|\bar{T}^a)\), we have \(\bar{p}(v|\bar{T}^a)\bar{P}(\bar{T}^a) = p(v)\mathbf{1}_{v \in \bar{V}(\bar{T}^a)}\), where \(\mathbf{1}_A\) is the indicator function which is 1 if the statement \(A\) is true and 0 otherwise. Thus we may rewrite the former inequality as

\[
\sum_{v \in V} \sum_{i=1}^n p(v)\mathbf{1}_{v \in \bar{V}(\bar{T}^a)}(\bar{t}^a_i(v) - c\bar{x}_i(v)) \geq \alpha I \cdot \bar{P}(\bar{T}^a), \text{ for all } \bar{T}^a \in \bar{\mathcal{T}}^a. \tag{37}
\]

Summing over all \(\bar{T}^a \in \bar{\mathcal{T}}^a\), we obtain

\[
\sum_{\bar{T}^a \in \bar{\mathcal{T}}^a} \sum_{v \in V} \sum_{i=1}^n p(v)\mathbf{1}_{v \in \bar{V}(\bar{T}^a)}(\bar{t}^a_i(v) - c\bar{x}_i(v)) \geq \sum_{\bar{T}^a \in \bar{\mathcal{T}}^a} \alpha I \cdot \bar{P}(\bar{T}^a). \tag{38}
\]
Denoting by $\hat{V}(\cdot)$ and $\hat{P}(\cdot)$ under the mechanism $\hat{\gamma}$ the corresponding sets $\hat{V}(\cdot)$ and probabilities $\hat{P}(\cdot)$ under the mechanism $\hat{\gamma}$, we can, after noting that $\hat{T}^a = \{I\}$ and $\hat{V}(I) = \{v|\bar{x}_0(v) = 1\} = \cup_{T^a \in T^a} \hat{V}(T^a)$, rewrite the previous inequality as

$$\sum_{v \in V} \sum_{i=1}^n p(v) 1_{v \in \hat{V}(I)}(\hat{t}_i(v) - c\bar{x}_i(v)) \geq \alpha I \cdot \hat{P}(\cup_{T^a \in T^a} \hat{T}^a), \quad (39)$$

which we can further rewrite as

$$\sum_{v \in V} \sum_{i=1}^n p(v) 1_{(\bar{x}_0(v)=1 \wedge \sum_i \hat{t}_i(v) = I)}(\hat{t}_i(v) - c\bar{x}_i(v)) \geq \alpha I \cdot \hat{P}(I), \quad (40)$$

but, since for $\hat{\gamma}$ we have $\hat{T}^a = \{I\}$, this is equivalent to

$$\sum_{v \in V} \sum_{i=1}^n p(v) 1_{(\bar{x}_0(v)=1 \wedge \sum_i \hat{t}_i(v) = I)}(\hat{t}_i(v) - c\bar{x}_i(v)) - \hat{T}^a - I \geq \alpha I \cdot \hat{P}(I), \text{ for all } \hat{T}^a \in \hat{T}^a. \quad (41)$$

Hence, $\hat{\gamma}$ satisfies (7) so that $\hat{\gamma} = (\hat{t}, \hat{x})$ is obedient. To complete the proof note that since $\hat{t}_i(v) + \hat{p}_i(v) = \bar{p}_i(v) + \bar{p}_i(v)$, the feasible direct mechanism $\gamma = (\hat{t}, \hat{x})$ is payoff equivalent to original mechanism $\hat{\gamma} = (\bar{t}, \bar{x})$.

**Proof of lemma 3**: The first statement follows because the incentive constraint (6) for $v_i = 1$, and the individual rationality (8) of a consumer with value $v = 0$ imply the individual rationality (8) for $v_i = 1$. That is, $1 \cdot X_i(1) - T_i(1) \geq 1 \cdot X_i(0) - T_i(0) \geq 0 \cdot X_i(0) - T_i(0) \geq 0$.

To see $x_i(0, v_{-i}) = 0$, note that if not, then $x_i(0, v_{-i}) = 1$. But then lowering it to 0 raises the objective (14) by $p(0, v_{-i})c$. This change is feasible, because it keeps constraints (6) for $v_i = 0$, (8), and (11) unaffected, while relaxing the constraints (6) for $v_i = 1$, (12), and (13). The statement $X_i(0) = 0$ then follows as a corollary.

To see $T_i(0) = 0$, note that (8) implies $T_i(0) \leq 0$. But if $T_i(0) < 0$, then raising each $t_i^p(0, v_{-i})$ and $\bar{t}_i(1, v_{-i})$ by $T_i(0)/p_i(v_{-i})$ for each $v_{-i} \in V_{-i}$ leads to a feasible mechanism with $T_i(0) = 0$ and the same value for the objective (14). The adapted mechanism is feasible since the change does not affect (6) and (11), and, by construction, satisfies (8) for $v_i = 0$ so that, by the first argument of this lemma, it also satisfies (8) for $v_i = 1$. The raises in $t_i^p(v)$ further relaxes (12) and (13). Consequently, there is no loss of generality in assuming that, at the optimum, $T_i(0) = 0$.

To see $T_i(1) = X_i(1)$, note that (6) for $v_i = 1$ together with $X_i(0) = T_i(0) = 0$ imply $T_i(1) \leq X_i(1)$. But if $T_i(1) < X_i(1)$, then we can raise all $t_i^p(1, v_{-i})$ by $\varepsilon > 0$
such that $T_i(1) = X_i(1)$. The increase is feasible and does not affect the objective (14). To see that the change is feasible, note that it relaxes constraint (6) for $v_i = 0$ and, by construction, satisfies (6) for $v_i = 1$. It further does not affect (8) for $v_i = 0$ and, by the first part of the lemma, the constraint (8) for $v_i = 1$ is redundant. It also does not affect (11), while relaxing (12) and (13). Consequently, there is no loss of generality in assuming that, at the optimum, $T_i(1) = X_i(1)$.

Q.E.D.

**Proof of Proposition 2:** Recalling that $p^* = \sum_{v \in V} p(v)$, define

$$p^*(v) \equiv \begin{cases} p(v)/p^* & \text{if } x_0^*(v) = 1; \\ 0 & \text{otherwise,} \end{cases}$$

The proposition’s condition $W^* \geq (1 + \alpha)p^*I$ is then equivalent to

$$\sum_{v \in V} \sum_i p^*(v)v_i(1 - c) \geq (1 + \alpha)I. \quad (42)$$

We first prove that under condition (42) the first best is implementable by constructing a transfer schedule $\hat{t}$ such that the direct mechanism $\gamma^* = (\hat{t}, x^*)$ is feasible and therefore implements $x^*$. For any $v$ such that $x_0^*(v) = 0$, set $\hat{t}_i^0(v) = \hat{t}_i^p(v) = 0$. For any $v$ such that $x_0^*(v) = 1$, let $\bar{x}^*(v) \equiv \sum_i x_i^*(v) > 0$ represents the efficient number of goods to be produced in state $v$.\footnote{\bar{x}^*(v) is greater than 0, since $x_0^*(v) = 1$ and $x^*$ is development-efficient.} Set $\bar{I}_i(v) = x_i^*(v)I/\bar{x}^*(v)$ and $\hat{t}_i^p(v) = x_i^*(v)(1 - I/\bar{x}^*(v))$.

We show that the resulting mechanism $\gamma^* = (\hat{t}, x^*)$ is direct and feasible. More specifically, for each $v \in V$ the allocation $\gamma^*(v)$ satisfies (2) so that $\gamma^*$ is direct (it trivially satisfies (3), since $x^*$ does so by construction). Moreover, the direct mechanism $\gamma^*$ satisfies (6), (7), and (8) for each $v \in V$.

To show (2) for $v$ such that $x_0^*(v) = 0$, note that $\sum_i \bar{I}_i(v) = 0 = Ix_0^*(v)$, and that

$$\sum_i \bar{I}_i^0(v) + \hat{t}_i^p(v) = 0 = Ix_0^*(v) + c \sum_i x_i^*(v),$$

since $x_i^*(v) = 0$ for all $i$ whenever $x_0^*(v) = 0$. To show (2) for $v$ such that $x_0^*(v) = 1$, note that $\sum_i \bar{I}_i(v) = \sum_i x_i^*(v)I/\bar{x}^*(v) = I = Ix_0^*(0)$ and

$$\sum_i \bar{I}_i^0(v) + \hat{t}_i^p(v) = \sum_{\{i:x_i^*(v) = 1\}}(\bar{I}_i^0(v) + \hat{t}_i^p(v)) + \sum_{\{i:x_i^*(v) = 0\}}(\bar{I}_i^0(v) + \hat{t}_i^p(v)) = \sum_{\{i:x_i^*(v) = 1\}} + \sum_{\{i:x_i^*(v) = 0\}} 0 = \sum_i v_i \geq I + \sum_i cv_i = Ix_0^*(v) + c \sum_i x_i^*(v),$$

where the inequality holds because $x^*(0) = 1$ is efficient by assumption so that $\sum_i v_i \geq I + \sum_i cv_i$. Hence, $\gamma(v) \in A$ for all $v$ so that the mechanism $\gamma^*$ is direct.
To show (6) and (8) note that \( x_i^*(0) = 0 \) implies \( X_i^*(0) = 0 \) and, by construction of \( \hat{t} \), also \( T_i^*(0) = 0 \). Moreover, \( X_i^*(1) \geq 0 \) and \( T_i^*(1) \geq 0 \). For \( v_i = 0 \), it therefore follows \( v_iX_i^*(v) - T_i^*(v) = 0 \cdot X_i^*(0) - T_i^*(0) = 0 \leq -T_i^*(1) = 0 \cdot X_i^*(1) - T_i^*(1) \) so that (6) and (8) are satisfied for \( v_i = 0 \). To see that they are also satisfied for \( v_i = 1 \), note that \( 1 \cdot X_i^*(1) - T_i^*(1) = \sum_{v_{-i}} p_i(v_{-i})[x_i^*(1, v_{-i}) - \hat{t}_i^p(1, v_{-i}) - \hat{p}_i^p(1, v_{-i})] = 0 = 1 \cdot X_i^*(0) - T_i^*(0) \).

Finally, to show (7), first note that for \( \gamma^* \) we have \( \mathcal{T}^* = \{ I \} \) and \( p(v|I) = p^*(v) \) so that we only need to show \( \sum_{v \in V} \sum_{i=1}^n p^*(v)[\hat{p}_i^p(v) - cx_i^*(v)] \geq \alpha I \), which follows from

\[
\sum_{v \in V} \sum_{i=1}^n p^*(v)[\hat{p}_i^p(v) - cx_i^*(v)] = \sum_{i=1}^n \sum_{v : x_i^*(v) = 1} p^*(v)[1 - I/\bar{x}^*(v) - c] = (43)
\]

\[
= \sum_{i=1}^n \sum_{v : x_i^*(v) = 1} p^*(v)(1 - c) - I = \sum_{i=1}^n \sum_{v \in V} p^*(v)v_i(1 - c) - I \geq \alpha I, \tag{44}
\]

where the inequality uses (42).

We next show that if condition (42) is violated so that

\[
\sum_{v \in V} \sum_{i} p^*(v)v_i(1 - c) < (1 + \alpha)I, \tag{45}
\]

then there does not exist a transfer schedule \( \hat{t} \) such that the direct mechanism \( \gamma = (\hat{t}, x^*) \) is feasible. In particular, we show there does not exist a transfer schedule \( \hat{t} \) such that \( (\hat{t}, x^*) \) satisfies (15)-(21).

For the efficient output schedule \( x^* \) it holds \( V^1 = \{ v | x_0^*(v) = 1 \} \) and \( V^0 = \{ v | x_0^*(v) = 0 \} \) and \( V = V^1 \cup V^0 \).

For \( v \in V^0 \), it therefore holds \( x_i(v) = 0 \) so that conditions (18) and (19) taken together imply \( \sum_i t_i^q(v) + t_i^p(v) \geq 0 \). Multiplying by \( p(v) \) and summing up over all \( v \) in \( V^0 \) yields

\[
\sum_{v \in V^0} \sum_i p(v)[t_i^q(v) + t_i^p(v)] \geq 0 \tag{46}
\]

For \( v \in V^1 \), (18) implies \( \sum_i t_i^q(v) = I \). Multiplying by \( p(v) \) and summing up over all \( v \) in \( V^1 \) yields

\[
\sum_{v \in V^1} \sum_i p(v)t_i^q(v) = \sum_{v \in V^1} p(v)I = p^*I \tag{47}
\]
Since (18) implies \( \mathcal{T}^* = \{I\} \), it follows that \( p^* \cdot p(v|I) = p(v) \) for \( v \in V^1 \) and \( p(v|I) = 0 \) for all \( v \in V^0 \). Hence, after a multiplication by \( p^* \) we can rewrite (16) as

\[
\sum_{v \in V^1} \sum_i p(v) t^p_i(v) \geq \alpha p^* I + \sum_{v \in V^1} \sum_i p(v) cx^*_i(v) \tag{48}
\]

Combining (47) and (48) yields

\[
\sum_{v \in V^1} \sum_i p(v) [t^a_i(v) + t^p_i(v)] \geq (1 + \alpha) p^* I + \sum_{v \in V^1} \sum_i p(v) cx^*_i(v) \tag{49}
\]

Since \( x^*_i(v) = 0 \) for \( v \in V^0 \), (49) together with (46) imply

\[
\sum_{v \in V} \sum_i p(v)[t^a_i(v) + t^p_i(v)] \geq (1 + \alpha) p^* I + \sum_{v \in V} \sum_i p(v) cx^*_i(v) \tag{50}
\]

Since \( x^*_i(v) = n_i \) for \( v \in V^1 \) and \( x^*_i(v) = 0 \) for \( v \in V^0 \), multiplying (45) by \( p^* \) and rearranging terms yields

\[
(1 + \alpha) p^* I + \sum_{v \in V} \sum_i p(v) cx^*_i(v) > \sum_{v \in V} \sum_i p(v)x^*_i(v). \tag{51}
\]

Combining this latter inequality with inequality (50) yields

\[
\sum_{v \in V} \sum_i p(v)[t^a_i(v) + t^p_i(v)] > \sum_{v \in V} \sum_i p(v)x^*_i(v). \tag{52}
\]

Condition (15) implies after multiplying by \( p_i(1) \) and summing over all \( i \)

\[
\sum_{i} \sum_{v_{i-j}} p(1, v_{i-j}) [t^a_i(1, v_{i-j}) + t^p_i(1, v_{i-j})] = \sum_{i} \sum_{v_{i-j}} p(1, v_{i-j}) x^*_i(1, v_{i-j}). \tag{53}
\]

Similarly, (17) implies after multiplying by \( p_i(0) \) and summing over all \( i \)

\[
\sum_{i} \sum_{v_{i-j}} p(0, v_{i-j}) [t^a_i(0, v_{i-j}) + t^p_i(0, v_{i-j})] = 0 = \sum_{i} \sum_{v_{i-j}} p(0, v_{i-j}) x^*_i(0, v_{i-j}), \tag{54}
\]

because \( x^*_i(0, v_{i-j}) = 0 \).

Combining the latter two inequalities yields

\[
\sum_{i} \sum_{v \in V} p(v)[t^a_i(v) + t^p_i(v)] = \sum_{i} \sum_{v \in V} p(v)x^*_i(v), \tag{55}
\]

but this contradicts (52). Hence, under (45) there does not exist a direct mechanism \( \gamma = (t, x^*) \) that satisfies (15)-(21) and, hence, \( x^* \) is not implementable. Q.E.D.
Proof of Proposition 3: Consider a maximizer $\hat{\gamma} = (\hat{t}, \hat{x})$ of problem $\mathcal{P}$.

To show that it satisfies the first statement, note that (21) directly implies that for $v_i = 0$ it holds $x_i(v_i, v_{-i}) = v_i$. So it is left to prove $x_0^0(1, v_{-i}) = 1 \Rightarrow \hat{x}_i(1, v_{-i}) = 1$.

Suppose to the contrary that there exists some $\bar{v} \in V$ with some $\bar{v}_i = 1$ so that $\bar{x}_0(\bar{v}) = 1$ and $\bar{x}_i(1, \bar{v}_{-i}) = 0$. Then by raising both $\bar{x}_i(1, \bar{v}_{-i})$ and the corresponding $\bar{u}_i(1, \bar{v}_{-i})$ by 1, the objective (14) is raised by $p(1, \bar{v}_{-i})(1-c) > 0$, while the constraints (15), (17), (18), (20), and (21) are unaffected, and (16) and (19) are relaxed.

To show the second statement, suppose to the contrary that $\hat{\gamma} = (\hat{t}, \hat{x})$ exhibits $\bar{x}_0(\bar{v}) = 1$, while $x_0^0(\bar{v}) = 0$ for some $\bar{v} = (\bar{v}_1, \ldots, \bar{v}_n)$. Define $I^1 = \{i | \bar{x}_i(\bar{v}) = 1\}$ as the set of consumers who receive the good under $\hat{\gamma}$ and the value realization $\bar{v}$.

Since $\hat{\gamma}$ is, by assumption, a maximizer of $\mathcal{P}$, it must hold that $I^1$ is non-empty and, due to (21), for all $i \in I^1$ it holds $v_i = 1$. But since $x_0^0(v) = 0$, it follows $\sum_{i \in I^1} \bar{v}_i(1-c) \leq \sum_{i \in I} \bar{v}_i(1-c) < I$. Now consider an alternative mechanism $\hat{\gamma} = (\hat{t}, \hat{x})$ that is identical to $\hat{\gamma}$ except that $\hat{x}_0(\bar{v}) = \hat{x}_i(\bar{v}) = 0$ and for all $i \in I^1$ it exhibits $\hat{x}_i(\bar{v}) = 0$, $\hat{u}_i(\bar{v}) = \hat{u}_i(\bar{v}) - c$, and $\hat{u}_i(\bar{v}) = \hat{u}_i(\bar{v}) - 1 + c$. First note that a comparison of the objective (14) evaluated at $\hat{\gamma}$ and $\hat{\gamma}$ yields a difference of $p(\bar{v})[I - \sum_{i \in I^1} (1-c)]$, which is positive. Hence, $\hat{\gamma}$ is not a solution to $\mathcal{P}$ if $\hat{\gamma}$ is feasible. In order to see that $\hat{\gamma}$ is feasible, we verify that it satisfies (2), (3), (6), (7), and (8) using that $\hat{\gamma}$ satisfies these constraints by assumption.

To verify the first inequality in (2), note $\sum_i \hat{u}_i(\bar{v}) = \sum_{i \in I^1} (\hat{u}_i(\bar{v}) - 1 + c) + \sum_{i \not\in I^1} \hat{u}_i(\bar{v}) \geq I - \sum_{i \in I^1} (1-c) \geq 0 = I\bar{x}_0(\bar{v})$, where the first inequality follows because $\hat{\gamma}$ satisfies (2) and the second inequality was already established above.

To verify the second inequality in (2) note $\sum_i (\hat{u}_i(\bar{v}) + \hat{u}_i(\bar{v})) = \sum_{i \in I^1} (\hat{u}_i(\bar{v}) + \hat{u}_i(\bar{v}) - 1) + \sum_{i \not\in I^1} (\hat{u}_i(\bar{v}) + \hat{u}_i(\bar{v})) \geq I + c \sum_{i \in I^1} \hat{x}_i(\bar{v}) - \sum_{i \in I^1} 1 = I - \sum_{i \in I^1} (1-c) \geq 0 = I\bar{x}_0(\bar{v}) + c \sum_i \hat{x}_i(\bar{v})$, where the first inequality follows because $\hat{\gamma}$ satisfies (2).

Noting that, because $\hat{\gamma}$ satisfies (3), it trivially follows that also $\hat{\gamma}$ satisfies (3), we continue to verify (6) and (8). Note that, by construction, $\hat{x}_i(\bar{v}) - \hat{u}_i(\bar{v}) - \hat{u}_i(\bar{v}) = \bar{x}_i(\bar{v}) - \bar{u}_i(\bar{v}) - \bar{u}_i(\bar{v})$ so that $\hat{X}_i(v_i) - \hat{T}_i(v_i) = \bar{X}_i(v_i) - \bar{T}_i(v_i) = 1$. Because $\hat{\gamma}$ satisfies (6) and (8), therefore, also $\hat{\gamma}$.

Finally, to verify (7) note that for $\hat{\gamma}$ we have $\mathcal{T}^a = \{I\}$ so that this is also the case for $\hat{\gamma}$. Hence, (7) reduces to (17). To see that $\hat{\gamma}$ satisfies this constraint, note that $\sum_{v \in V} \sum_{i = 1}^n p(v|I)(\hat{u}_i(v) - c\hat{x}_i(v)) = \sum_{v \not\in I} \sum_{i = 1}^n p(v|I)(\hat{u}_i(v) - c\hat{x}_i(v)) + \sum_{v \in I} \sum_{i = 1}^n p(v|I)(\hat{u}_i(v) - c\hat{x}_i(v))$.
\[ \sum_{1} p(\bar{v}|I)(\hat{P}_i(v) - c\bar{x}_i(v)) = \sum_{v \neq \hat{v}} \sum_{1} p(v|I)(\hat{P}_i(v) - c\bar{x}_i(v)) + \sum_{1} p(\bar{v}|I)\hat{P}_i(\bar{v}) = \sum_{v \neq \bar{v}} \sum_{1} p(v|I)(\hat{P}_i(v) - c\bar{x}_i(v)) + \sum_{1} p(\bar{v}|I)\hat{P}_i(\bar{v}) = \sum_{1} p(v|I)(\hat{P}_i(v) - c\bar{x}_i(v)) + \sum_{1} p(\bar{v}|I)\hat{P}_i(\bar{v}) = \sum_{1} p(v|I)(\hat{P}_i(v) - c\bar{x}_i(v)) = \sum_{v \neq \bar{v}} \sum_{1} p(v|I)(\hat{P}_i(v) - c\bar{x}_i(v)) + \sum_{1} p(\bar{v}|I)\hat{P}_i(\bar{v}) = \sum_{1} p(v|I)(\hat{P}_i(v) - c\bar{x}_i(v)) \geq \alpha I. \]

To show the proposition’s third statement, consider a mechanism \( \bar{\gamma} = (\bar{x}, \tilde{t}) \) which satisfies (15)-(21) and there is a \( \bar{v} \) such that \( \bar{x}_0(\bar{v}) = 0 \), while \( \sum_{i} \bar{v}_i > (1 + \alpha)I/(1 - c) \). We show that \( \bar{\gamma} \) is not a solution to \( \mathcal{P} \), because there exists a \( (\hat{x}, \hat{t}) \) that also satisfies (15)-(21) but yields a strictly higher surplus that \( \bar{\gamma} \). More specifically, let \( (\hat{x}, \hat{t}) \) be identical to \( (\bar{x}, \tilde{t}) \) except that \( \hat{x}_i(\bar{v}) = \tilde{v}_i \), \( \hat{P}_i(v) = \tilde{P}_i(\bar{v}) + \tilde{v}_i \cdot I/\sum_{j} \tilde{v}_j \), and \( \hat{P}_i(v) = \hat{P}_i(\bar{v}) + \tilde{v}_i(1 - I/\sum_{j} \tilde{v}_j) \).

Note first that the difference in surplus between \( (\hat{x}, \hat{t}) \) and \( (\bar{x}, \bar{t}) \) is \( p(\bar{v})[(1 - c)\sum_{j} \tilde{v}_j - I] > 0 \). It remains to be checked that \( (\hat{x}, \hat{t}) \) satisfies (15)-(21). That it satisfies (15), (17), (20), and (21) follows directly, because \( (\bar{x}, \tilde{t}) \) satisfies these constraints by assumption and \( (\hat{x}, \hat{t}) \) is a transformation of \( (\bar{x}, \tilde{t}) \) which preserves them.

Since (18) holds for \( (\bar{x}, \tilde{t}) \), \( (\hat{x}, \hat{t}) \) trivially satisfies it for all \( v \neq \bar{v} \). It, however, also holds for \( \hat{v} \), since \( \sum_{1} \hat{P}_i(v) = \sum_{1} \hat{P}_i(v) + \sum_{1} \tilde{v}_i \cdot I/\sum_{j} \tilde{v}_j = I\bar{x}_0(\bar{v}) + I = I = I\hat{x}_0(\bar{v}) \). Similarly (19) holds for all \( v \neq \bar{v} \), while for \( \bar{v} \) it follows \( \sum_{1} \hat{P}_i(v) = \sum_{1} \hat{P}_i(v) + \sum_{1} \tilde{v}_i(1 - I/\sum_{j} \tilde{v}_j) \geq \sum_{1} \hat{x}_i(\bar{v})(1 - I/\sum_{j} \tilde{v}_j) > \sum_{1} \hat{x}_i(\bar{v})c \), where the first inequality uses that \( (\bar{x}, \tilde{t}) \) satisfies (19), and the second inequality follows from the proposition’s presumption that \( \sum_{j} \tilde{v}_j > (1 + \alpha)I/(1 - c) \), as this implies \( c < 1 - I/\sum_{j} \tilde{v}_j \).

Finally, to see that \( (\hat{x}, \hat{t}) \) satisfies (16) because \( (\bar{x}, \tilde{t}) \) does so, first define

\[ \hat{R}(v) = \sum_{1} [\hat{P}_i(v) - c\bar{x}_i(v)] \text{ and } \tilde{R}(v) = \sum_{1} [\tilde{P}_i(v) - c\bar{x}_i(v)]. \]

It holds \( \hat{R}(v) = \tilde{R}(v) \) for all \( v \neq \bar{v} \), while for \( \bar{v} \) it follows \( \tilde{R}(\bar{v}) = \sum_{1} [\tilde{P}_i(v) - c\bar{x}_i(v)] = \sum_{1} [\tilde{P}_i(v) + \tilde{v}_i(1 - I/\sum_{j} \tilde{v}_j) - c\bar{v}_i] \geq \sum_{1} \tilde{v}_i(1 - I/\sum_{j} \tilde{v}_j - c) = \sum_{1} \tilde{v}_i(1 - c) - I > \alpha I, \)

where the first inequality uses that \( (\bar{x}, \tilde{t}) \) satisfies (19), and the final inequality uses the proposition’s presumption that \( \sum_{j} \tilde{v}_j > (1 + \alpha)I/(1 - c) \).

Since \( (\bar{x}, \tilde{t}) \) satisfies (16), the definition of \( p(v|I) \) implies that it holds

\[ \sum_{v: \bar{x}_0(v) = 1} p(v) \tilde{R}(v) \geq \alpha I \cdot \sum_{v: \bar{x}_0(v) = 1} p(v). \]

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Combining this with the previously established inequality \( \hat{R}(\bar{v}) > \alpha I \), it follows
\[
p(\bar{v}) \hat{R}(\bar{v}) + \sum_{\{v : \bar{x}_0(v) = 1\}} p(v) \hat{R}(v) \geq \alpha I [p(\bar{v}) + \sum_{\{v : \bar{x}_0(v) = 1\}} p(v)].
\]
But since \( \{v : \hat{x}_0(v) = 1\} = \{v : \bar{x}_0(v) = 1\} \cup \{\bar{v}\} \), this is equivalent to
\[
\sum_{\{v : \bar{x}_0(v) = 1\}} p(v) \hat{R}(v) \geq \alpha I \sum_{\{v : \bar{x}_0(v) = 1\}} p(v),
\]
which is equivalent to saying that \((\hat{x}, \hat{t})\) satisfies (16).

To show the proposition’s last statement, suppose that, to the contrary, there exists a solution \( \bar{\gamma} = (\bar{x}, \bar{t}) \) of problem \( \mathcal{P} \) for which such a \( T \) does not exist. In this case, there exist a valuation profile \( \bar{v} \) and \( \hat{v} \) with \( \sum_j \bar{v}_j > \sum_j \hat{v}_j \) such that \( \bar{x}_0(\bar{v}) = 0 \) and \( \bar{x}_0(\hat{v}) = 1 \). Since \( \sum_j \bar{v}_j > \sum_j \hat{v}_j \), we can find a bijective correspondence \( k : \{1, \ldots, n\} \rightarrow \{1, \ldots, n\} \) such that \( \hat{v}_j = 1 \) implies \( \bar{v}_{k(j)} = 1 \). Fix the correspondence \( k \) and its inverse \( k^{-1} \) and define the mechanism \( \hat{\gamma} = (\hat{x}, \hat{t}) \) by \( \hat{x}_0(v) = \bar{x}_0(v) \) for all \( v \neq \bar{v}, \hat{v} \), \( \hat{x}_0(\bar{v}) = 1 \) and \( \hat{x}_0(\hat{v}) = 0 \), and \( \hat{x}_i(v) = \bar{x}_{k^{-1}(i)}(v), \hat{t}_i(v) = \bar{t}_{k^{-1}(i)}(v), \hat{p}_i(v) = \bar{p}_{k^{-1}(i)}(v) \). Since \( \hat{\gamma} \) satisfies by assumption all constraints (15)-(21) of problem \( \mathcal{P} \), so does \( \hat{\gamma} \). They also yield the same objective (14). But since \( \sum_j \bar{v}_j > \sum_j \hat{v}_j \), mechanism \( \hat{\gamma} \) exhibits at least one \( i \) such that \( \bar{v}_i = 1 \) and \( x_i(\bar{v}) = 0 \). By the first statement of this proposition, \( \hat{\gamma} \) is not optimal, since there exists a feasible \( \bar{\gamma} \) which yields a strictly larger surplus. Consequently, we obtain the contradiction that \( \bar{\gamma} \) is not optimal.

Q.E.D.
References


