The Coordination of Intermediation\

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Abstract

We study decentralized trading among financial intermediaries (i.e., dealers) and market fragility in a dynamic model of asset markets. When interdealer trading is active, the intermediation chain is longer, dealers provide more liquidity by holding more inventory, and the market is more liquid. A dealer is more likely to provide liquidity if other dealers do so, leading to coordination motives and multiple equilibria. This implies market fragility in the sense of a shutdown of interdealer trading and liquidity drop without fundamental shocks. A low search friction among dealers reduces fragility. Surprisingly, an improvement in customer search technology may increase fragility.

Keywords: intermediation, interdealer trading, liquidity, coordination, multiple equilibria, fragility

JEL: D02, G01, G11, G12, G21, G23

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1 Introduction

Many over-the-counter (OTC) asset markets are intermediated in the sense that buyers and sellers cannot trade with each other directly but only through an intermediary (i.e., a dealer). Most intermediaries do not only stand as middlemen but also trade with each other bilaterally in a purely decentralized fashion. Theoretically, it is well-known that trading among intermediaries allows them to manage inventory more efficiently (e.g., Ho and Stoll, 1983, Viswanathan and Wang, 2004), potentially leading to better risk-sharing and liquidity provision for the whole market. These ideas are supported by evidence in various economically important markets, such as the corporate bond (Di Maggio, Kermani and Song, 2017), municipal bond (Green, Hollifield and Schurhoff, 2006, Li and Schurhoff, 2018), asset-backed security (Hollifield, Neklyudov and Spatt, 2016), mortgage-backed security (Gao, Schultz and Song, 2017), and credit default swap (Eisfeldt et al., 2018) markets, among others.

However, various OTC asset markets have recently experienced sharp reduction and fluctuation in trading among intermediaries associated with declined market liquidity, but without a clear negative fundamental shock. For instance, Bessembinder et al. (2018) document that interdealer trading as a proportion of total trading volume over the U.S. corporate bond market had declined from 25% during the financial crisis to 16% in 2013 despite that the fundamental actually improved in this period. Recent research has shown that the Volcker Rule that prevented dealer proprietary trading had contributed to dealer balance sheet space cost, leading to decline in interdealer trading and market liquidity (e.g., Duffie, 2016, Bao, O’Hara and Zhou, 2017, Bessembinder et al., 2018, Dick-Nielsen and Rossi, 2018). But regulation does not provide a complete explanation because the Volcker Rule had not yet been implemented until 2014.\footnote{As part of the Dodd-Frank Act, passed in 2010, section 13 (i.e., the Volcker Rule) was added to the Bank Holding Company Act of 1956. The Volcker Rule generally prohibits banking entities, many of which are corporate bond dealers, from using their balance sheet in proprietary trading. Implementation of the Volcker Rule, however, was not immediate and followed a laborious process. After long delays, final regulations were issued in late 2013 and implemented in early 2014.} Thus, it still remains a question as to whether, theoretically, intermediaries may stop trading with each other even without a fundamental shock.

The main contribution of this study is to offer a new strategic perspective to analyze trading among intermediaries, its liquidity implications, and in particular, the associated
market fragility. We formally define market fragility as a shutdown of interdealer trading and liquidity drop in an asset market without a fundamental shock. Built upon the standard modern OTC asset pricing literature (mostly following the paradigm of Duffie, Garleanu and Pedersen, 2005) that typically considers a unique equilibrium, our model shows that multiple equilibria and market fragility can generically arise due to a coordination failure in intermediaries’ liquidity provision decisions. This coordination failure is reminiscent of the same coordination failure in the bank run literature (e.g., Diamond and Dybvig, 1983) that depends on a depositor’s belief about other depositors’ run decisions. Similarly, in our framework, if one dealer is worrying about other dealers not providing liquidity, he may stop providing liquidity. In this sense, our theory provides one plausible and complementary explanation for why, for example, interdealer trading in the corporate bond market declined as dealer banks prepared for the to-be-implemented Volcker Rule before 2014.\(^2\)

In this paper, we formulate the endogenous emergence of the interdealer market and its potential fragility in a dynamic, search-based intermediated asset market à la Rubinstein and Wolinsky (1987) and Duffie, Garleanu and Pedersen (2005). In our model, buyers and sellers, that is, customers, can trade an asset only through a competitive dealer sector. Customers randomly arrive in the market, seeking to buy or sell a unit of identical asset. To model interdealer trading and its fragility, we explicitly relax the \(\{0,1\}\) asset position restriction commonly seen in the literature. Specifically, at any time, each dealer can hold 1) a high inventory or 2) a low inventory to help provide liquidity, or 3) does not hold any inventory at all, and the inventory cost function is convex. (Looking forward, we will show that allowing at least three inventory levels is necessary in generating interdealer trading and intermediation chains as observed in reality.) When meeting a dealer, a customer negotiates a price with the dealer through bargaining, completes the transaction, and leaves the market. Since a dealer can only hold a non-negative inventory (i.e., the dealer cannot short), this trading protocol also implies that the asset will flow from a seller to the dealer sector first, and then to a buyer.

\(^2\)Clearly, we are neither dismissing the findings in the recent literature such as Bao, O’Hara and Zhou (2017), Bessembinder et al. (2018), Dick-Nielsen and Rossi (2018) nor arguing that the higher balance sheet space cost caused by the Volcker Rule and other regulatory policies were unimportant. Rather, we view our mechanism as a complementary mechanism that helps to rationalize the recent large drop in interdealer trading and in market liquidity over various OTC markets.
In this model, an interdealer market where dealers trade with each other may endoge-
ously emerge (or not) in equilibrium. Consider a dealer who already holds a low inventory. When the asset fundamental is high (low) relative to dealer inventory cost, this dealer is more (less) willing to buy the asset from sellers, hold a high inventory temporarily, and then intermediate the asset later. This implies a higher (lower) dealer inventory on average and a larger (smaller) dispersion of inventory distribution among dealers. Since the gains from a potential interdealer trade are positive (zero) when the dispersion of inventory distribution is sufficiently large (small), the interdealer market becomes active (inactive), the implied intermediation chain is longer (shorter), and the market is more (less) liquid in equilibrium.

The key idea of our paper is that the two equilibria described above may generically co-exist in the same economy. When equilibrium multiplicity arises, a switch between the two equilibria may result from non-fundamental reasons as those featured in, for example, the bank run literature (e.g., Diamond and Dybvig, 1983), leading to market fragility.

To understand how, we illustrate the coordination motive behind dealers’ liquidity pro-
vision decisions, that is, the decision to buy the asset from the sellers, in three steps.

First, without active interdealer trading, the inventory cost limits a dealer’s ability to provide liquidity. This is because, if this dealer buys from a seller, the only way for it to offload the increased inventory is to trade with a buyer. But if interdealer trading were active, a dealer with a high inventory (after buying from a seller) could have offloaded the additional inventory to another dealer who holds no inventory. The absence of interdealer trading and the resulted difficulty in offloading inventory holdings limits a dealer’s inventory management capacity, discouraging the dealer’s willingness to buy from sellers.

Then, when the inventory cost is within an intermediate range relative to the asset fundamental, a dealer may be more willing to provide liquidity and hold a higher level of inventory if other dealers do so. This implies a coordination motive in dealers’ liquidity provision decision. To see this, consider a specific dealer’s willingness to buy from a seller. When other dealers all buy from sellers and hold a high inventory, the market becomes more liquid. This implies two competing effects on 1) markup per trade and 2) customer-dealer trading volume per unit of time. Naturally, markup per trade (i.e., the difference between the buyer-dealer price and the seller-dealer price) decreases due to intensified dealer competition.
But trading volume per unit of time increases thanks to a higher dealer inventory level, which in turn implies that more units of assets are intermediated per unit of time. Thus, if the latter effect dominates, that is, if trading volume per unit of time increases sufficiently fast, the specific dealer in question will be willing to join other dealers to provide liquidity, enjoying a higher markup per unit of time. This coordination motive suggests potential coordination failures in dealers’ liquidity provision decisions.

Lastly, we relate equilibrium multiplicity to the economic environment, in particular, to the search friction among dealers (recall that most OTC interdealer markets are purely decentralized). We show that a lower search friction among dealers can mitigate the concern of coordination failures. An active interdealer market allows an dealer that holds a high inventory to trade with a dealer without any inventory, giving the former an alternative way to offload its costly inventory. In this sense, the lower interdealer search friction effectively lowers dealer inventory cost in equilibrium, encourages the dealer to buy from a seller, and importantly makes the dealer’s liquidity provision decision less sensitive to other dealers’. We further show that, if the search friction among dealers is sufficiently small, the endogenously emerged interdealer market can completely eliminate the coordination motive among dealers’ liquidity provision decisions, thereby eliminate any possible coordination failures. On the flip side, as the search friction among dealers becomes higher, it is more likely for the coordination motive to emerge even if the interdealer market emerges.

Our model generates empirically plausible implications along many dimensions. In comparative statics, we find that market fragility is more likely to happen when the level of dealer inventory cost becomes higher, or when the inventory cost function becomes more convex. These results suggest adverse effects on asset market liquidity due to certain post-crisis regulatory policies aiming to reduce financial stability risks but effectively increasing intermediaries’ inventory costs, for example, the Volcker Rule. Particularly, the coordination failure mechanism in our theory helps explain, for example, why the decline in interdealer trading and market liquidity took place right after the announcement of the the Volcker Rule but well before its implementation.

Perhaps surprisingly, we also find that market fragility may be more likely to happen when the search friction between customers and dealers becomes smaller (e.g., due to customer
technological improvements). This is surprising; after all, we would have thought a better customer search technology leading to less market fragility. In our model, this results derives from that a better customer search technology improves the trading volume per unit of time, and thus ultimately increases a dealer’s markup per unit of time if the markup per trade does not decrease too much. This is consistent with the observation in Philippon (2015) that a reduction in trading frictions between customers and dealers in the recent decades improve liquidity but may also lead to more market fragility due to the escalated competition among customers.

**Contribution and related literature.** The main contribution of our paper is to formulate equilibrium multiplicity and the implied fragility of decentralized trading among intermediaries in an otherwise standard search-based asset pricing model following Rubinstein and Wolinsky (1987) and Duffie, Garleanu and Pedersen (2005).3

This asset pricing literature typically delivers a unique equilibrium,4 with a few exceptions that feature different economic forces from ours. Di Maggio (2016) builds on Lagos, Rocheteau, and Weill (2011) and considers how speculators in financial markets trade in anticipation of a future random negative shock. He shows that two equilibria, that the speculators may either jointly buy or jointly sell the asset before the shock, may co-exist.5 Farboodi, Jarosch and Menzio (2018) consider intermediation as a rent extraction activity: they allow traders to acquire a costly commitment technology before trading, and show that there may be multiple equilibria in which different fractions of agents acquire the commitment technology.6 Gu et al. (2018) explore three different ways of modeling financial

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3In the OTC asset pricing literature, some papers feature an explicit but reduced-form (usually modeled as centralized) interdealer market to facilitate the analysis of other aspects. For example, Garleanu (2009), Lagos and Rocheteau (2009) and subsequent papers built on them consider unlimited asset holding positions by investors. Babus and Parlatore (2018) consider how strategic traders choose a dealer to trade with.

4There are other papers in the broader search literature that feature multiple equilibria, but get equilibrium multiplicity for quite different reasons from ours. For example, Diamond (1982) get equilibrium multiplicity in a production economy by directly assuming increasing returns to matching, while we have constant returns to matching. In Kiyotaki and Wright (1989), multiple equilibria co-exist with different goods acting as money, depending on what the authors call “extrinsic beliefs” that what one agent thinks other agents are accepting as money. Trejos and Wright (2016) show that allowing for non-linear utility functions, as opposed to the linear utility used in the Duffie, Garleanu and Pedersen (2005) paradigm, can also generate multiple equilibria in search models.

5Thus, his multiplicity is reminiscent of the notion of “market runs” in Bernardo and Welch (2004).

6As the authors noted, this multiplicity does not come from the effect through changes in the composition of searchers, which instead plays a role in our model. Also, after acquiring the commitment technology, their “market equilibrium”, which is the counterpart of our trading equilibrium, is always unique.
intermediaries. They show that when intermediaries derive a positive return from holding the asset (as opposed to incur inventory costs), multiple equilibria may arise because intermediaries may either intermediate or just hoard them.

Several papers in this OTC asset pricing literature consider related but different notions of market fragility. Weill (2007) and Lagos, Rocheteau, and Weill (2011) consider how dealers provide liquidity during an exogenously specified financial crisis. He and Milbradt (2014) consider the feedback loop between corporate bond default and secondary market illiquidity and show that they reinforce each other. These papers do not have multiple equilibria or trading among intermediaries.

Our paper is also related to the literature that explores the the bright and dark sides of intermediation chains in OTC asset markets. Glode and Opp (2016) argue that longer intermediation chains between a buyer and a seller with market power may help mitigate adverse selection and lead to more efficient trading. Babus and Hu (2016) argue that intermediation chains and the resulting trading networks can help incentivize better monitoring. On the other hand, a longer intermediation chain can lead to higher intermediation costs (Gofman, 2014) and real inefficiencies (Philippon, 2015). Ours differs by offering a more balanced message in a single: longer intermediation chains are desirable in term of liquidity provision, but the the intermediation chain itself can be fragile.

On modeling contributions, our framework features a fully decentralized and two-tier market structure with separate customer-to-dealer and interdealer markets, capturing real intermediation chains and networks as observed in reality. At the same time, we relax the commonly used \{0, 1\} asset holding restrictions in most papers built upon Duffie, Garleanu and Pedersen (2005),\footnote{Garleanu (2009) and Lagos and Rocheteau (2009) are among the first to allow agents to have unrestricted asset holdings positions but require centralized interdealer trading. Afonso and Lagos (2015) and Ushu (2017) consider decentralized trading and allow for a finite \( N \) and unrestricted asset holding positions, respectively. But they do not specifically distinguish between customer-to-dealer and interdealer trading as we do, and their focuses are also different from ours.} aiming to capture various patterns of decentralized trading among intermediaries. Thus, our paper also contributes to a burgeoning branch of literature exploring the endogenous emergence of market structures in OTC asset markets. In this new literature, a wave of papers focuses on the emergence of the core-periphery trading networks observed in various OTC markets, asking why some traders become customers while others
become dealers. These models start either from identical traders (Wang, 2017) or heterogeneous traders along the dimension of initial asset positions (Afonso and Lagos, 2015), of asset valuations and trading needs (Chang and Zhang, 2016, Shen, Wei and Yan, 2016), of trading technologies (Neklyudov, 2014, Farboodi, Jarosch and Shimer, 2017), or of both trading needs and technologies (Uslu, 2017). Atkeson, Eisfeldt and Weill (2015), Neklyudov and Sambalaibat (2015) and Colliard, Foucault and Hoffmann (2018) further combine the OTC search and the network literatures to consider how exogenously specified network structures affect dealers’ entry, exit and inventory holding decisions. In terms of the market structure, Hugonnier, Lester and Weill (2018) also use a fully decentralized two-tier market structure but with ex-ante heterogenous dealers to explore endogenous intermediation chains. They do not focus on multiple equilibria or market fragility. Our work complement these papers in that we have a particular focus on the emergence of multiple equilibria and fragility of the interdealer market and the implied intermediation chains. In doing so, we fix the roles of dealers and customers but otherwise use a rich setting where the dealers are ex-ante identical and the trading, pricing, and inventory holding decisions of market participants are all endogenous.

2 The model

Timeline, asset, and preferences. We fix a probability space satisfying the usual conditions. Time is continuous and runs infinitely: \( t \in [0, \infty) \). Consider a market for a single indivisible asset. There are three types of agents: buyers, dealers, and sellers. All agents are risk-neutral with time preferences determined by a constant discount rate \( r > 0 \). The monetary value of a unit of the asset is \( \theta > 0 \) for a buyer and is normalized as 0 for a seller and for a dealer. We call \( \theta \) as the fundamental of the asset. We also call buyers and sellers jointly as customers.

Buyers and sellers (customers). We specify the arrival rate of new buyers and of new sellers to be \( n \).\(^8\) When arriving, a seller brings 1 unit of asset to the market while the buyer

\(^8\)The model can be extended to handle different arrival rates of new buyers and sellers without changing the main economic mechanisms, but doing so significantly complicates the calculation.
has nothing. We focus on intermediated asset markets in the sense that buyers and sellers cannot trade with each other directly but only through dealers. At any time \( t \), there are \( b_t \) active buyers and \( s_t \) active sellers present on the market searching for dealers, which will be endogenous determined. We denote by \( z_t = b_t + s_t \) the total mass of searching customers, or equivalently, the mass of customers who are waiting to be served. A seller or a buyer who has successfully met and transacted with an dealer leaves the market and consumes the monetary value he gets. We also assume that at probability rate \( \eta > 0 \) a searching customer departs the market without a successful transaction. This departure shock captures that a buyer or seller may lose it trading opportunity if cannot find an intermediary in a timely manner. For example, a junk bond mutual fund running out of cash rushes to find a dealer to sell its illiquid bonds, but it defaults and departs the market if cannot find a dealer timely.

**Dealers.** Dealers are homogeneous and we normalize the mass of dealers to 1. At any time \( t \), each dealer can hold 0, 1, or 2 units of the asset. As standard in the literature (e.g., Amihud and Mendelson, 1980), we assume a weakly convex inventory cost structure: \( C(0) = 0, C(1) = c > 0 \) whether \( c \) captures the level of inventory costs, and \( C(2) = \rho c \) where \( \rho \geq 2 \) captures the curvature of the cost structure. We denote the endogenous distribution of dealers who hold 0, 1, and 2 units of the asset \( d_{0t}, d_{1t}, \) and \( d_{2t} \), respectively, where \( d_{0t} + d_{1t} + d_{2t} = 1 \). For the ease of exposition, we also call them type-0, type-1, and type-2 dealers, with the type indicating an dealer’s inventory holding status.

It is worth noting that we purposefully relax the common assumption in the literature that limits the agents to hold either 0 or 1 unit of the asset. By allowing a dealer to hold 0, 1, or 2 units of the asset, we parsimoniously capture that a dealer may hold no inventory at all, or a low level of inventory, or a high level of inventory. As we show later, this deviation from the literature is important in generating gains from trade between intermediaries and thereby driving our results. As we will also discuss later, our results and the underlying intuition are robust to allowing for more asset holding positions. Thus, we choose the current setting to best flesh out the mechanism.

**Searching and matching protocols.** At any time \( t \), agents search for counter-parties for a potential trade. To make the model flexible and more realistic, we assume customers and dealers have access to different searching and matching protocols. These matching
protocols are consistent with and can be micro-founded by the continuous time random matching framework in Duffie, Qiao and Sun (2017).

First, on the customer-to-dealer market, a customer contacts a random dealer at Poisson rate $\alpha > 0$. Economically, $\alpha$ captures the friction of the customer-to-dealer market: a larger $\alpha$ implies a better technology held by customers and thus a smaller customer-to-dealer search friction. This setting immediately implies that a dealer is contacted by a random customer at endogenous Poisson rate $\beta_t = \alpha (b_t + s_t)$, which depends on the mass of searching customers. Intuitively, a dealer will be contacted faster when more customers are actively searching.

Second, on the interdealer market, a dealer contacts another random dealer according to an independent Poisson process with rate $\lambda$, which is also independent to a dealer’s inventory holdings. Similarly, $\lambda$ captures the friction of the interdealer market: a larger $\lambda$ suggests that intermediaries are easier to meet each other.

**Bilateral trading.** Trading prices are determined through generalized Nash bargaining as standard in the literature. As a benchmark, we assume that all agents, when meet bilaterally, have equal bargaining powers.\(^9\) Hence, agents who meet always equally share the gains from trade, provided the gains from trade are positive.

### 3 Equilibrium analysis

As standard in the literature, we focus on steady-state (i.e., stationary) trading equilibria, that is, equilibria in which the mass of each type of agents is constant, and trade happens. Thus, we suppress the time argument $t$ in equilibrium variables, and we call a steady-state equilibrium simply an equilibrium below.

Given our focus on trading among intermediaries, we first give a necessary condition for the interdealer market being active:

**Lemma 1.** *In any steady-state trading equilibrium, interdealer trade happens only if $d_2 > 0$.****

Economically, Lemma 1 amounts to saying that interdealer trading happens only when the dispersion of dealers’ inventory holdings is large enough, that is, only when there are some

\[^9\]The predictions of our model are robust to more sophisticated bargaining protocols that may give rise to unequal relative bargaining powers between the two bargaining parties.
dealers holding a relatively high level of inventory in equilibrium. Intuitively, if there were only type-0 and type-1 dealers in the market, no interdealer trade would happen because of the lack of gains from trade. Instead, a type-2 dealer may potentially trade with a type-0 dealer and both become type-1 dealers, because the gains from such a trade may be positive. Therefore, below we will use whether \( d_2 \) is positive in equilibrium to organize the discussion of the two types of trading equilibria: with or without an active interdealer market. Importantly, we will further show that interdealer trading must happen when a type-2 dealer meets a type-0 dealer. In other words, \( d_2 > 0 \) is also a sufficient condition for the interdealer market being active.

We also note a useful result that at any steady state, the inflow-outflow balance implies that the mass of searching buyers must equal that of searching sellers, that is, \( b = y \). To see this, notice that inflow-outflow balance of the dealer sector implies \( n - \eta b = n - \eta s \). We highlight that this is not an assumption but instead an equilibrium outcome.

### 3.1 Trading equilibrium with interdealer market: \( d_2 > 0 \)

In view of Lemma 1, we first characterize a type of equilibria where \( d_2 > 0 \). We characterize the equilibrium conditions and show that the interdealer market is active in the sense that when a type-2 dealer meets a type-0 dealer, a transaction must happen and thus both become type-1 dealers. Figure 1 below illustrates this type of equilibria, where solid arrows indicate the flows of assets and dashed arrows indicate changes in the distribution of agents, that is, the flows of customers in and out of the market as well as dealer’s changes in inventory-holding status.

In a steady state, the asset inflows to the buyers must equal the asset outflows from type-1 and type-2 dealers:

\[
    n - \eta b = b\alpha(d_1 + d_2),
\]

and similarly, the asset outflows from the sellers must equal the asset inflows taken by type-0 and type-1 dealers:

\[
    n - \eta s = s\alpha(d_0 + d_1).
\]
Recall that $b = y$ under any steady state, these two accounting identities (3.1) and (3.2) imply that
\[
d_0 = d_2,
\]
that is, the mass of type-0 dealers must equal that of type-2 dealers. In what follows, we define $d_I = d_0 = d_2$ when refer to the equilibrium with an interdealer market.

Denote by $V^1_b$, $V^1_s$, $V^1_0$, $V^1_1$, and $V^1_2$ the equilibrium value functions of an active buyer, seller, type-0 dealer, type-1 dealer, and type-2 dealer, respectively, where the superscript 1 indicates that these value functions are evaluated under the distribution of agents in an equilibrium with an active interdealer market. We derive the agents' equilibrium Hamilton-Jacobi-Bellman (HJB) equations, taking into account the endogenous matching and bargaining outcomes:

\begin{align*}
(\eta + r)V^1_b &= \alpha \left[d_2 \frac{1}{2}(\theta + V^1_0 - V^1_2 - V^1_b) + d_1 \frac{1}{2}(\theta + V^1_0 - V^1_1 - V^1_b)\right], \quad (3.4) \\
(\eta + r)V^1_s &= \alpha \left[d_0 \frac{1}{2}(V^1_1 - V^1_0 - V^1_s) + d_1 \frac{1}{2}(V^1_2 - V^1_1 - V^1_s)\right], \quad (3.5) \\
rV^1_0 &= \beta \frac{s}{b + s} \frac{1}{2}(V^1_1 - V^1_0 - V^1_s) + \lambda d_2 \frac{1}{2}(2V^1_1 - V^1_0 - V^1_2), \quad (3.6) \\
rV^1_1 &= \beta \left[\frac{b}{b + s} \frac{1}{2}(\theta + V^1_0 - V^1_1 - V^1_b) + \frac{s}{b + s} \frac{1}{2}(V^1_2 - V^1_1 - V^1_s)\right] - c, \quad (3.7) \\
rV^1_2 &= \beta \left[\frac{b}{b + s} \frac{1}{2}(\theta + V^1_1 - V^1_2 - V^1_b) + \lambda d_0 \frac{1}{2}(2V^1_1 - V^1_0 - V^1_2)\right] - \rho c. \quad (3.8)
\end{align*}
These HJB equations are intuitive. First, since all agents have equal bargaining power, they always equally share the potential gains from trade, if positive. The bargaining process also determines the trading prices in every bilateral trading.

Second, a buyer may buy a unit of asset either from a type-2 or a type-1 dealers, while a seller may sell to either a type-0 or type 2 dealer. This is reflected in the buyer’s and seller’s HJB equations (3.4) and (3.5).

Finally, for the three types of dealers, they all have their unique pattern of trading, reflected by (3.6), (3.7), and (3.8). A type-0 dealer can either buy a unit of asset from a seller or from a type-2 dealer, a type-1 dealer can either buy from a seller or sell to a buyer, while a type-2 dealer can either sell to a buyer or to a type-0 dealer. Consistent with our earlier argument, only the type-0 and type-2 dealers are potential candidates for an interdealer trade.

Analyzing these value functions, we first show that an interdealer trade must happen as long as a type-0 dealer meets a type-2 dealer.

**Lemma 2.** In any equilibrium with $d_2 > 0$, $2V^1_1 - V^1_0 - V^1_2 > 0$ holds. That is, the gains from trade between a type-0 and a type-2 dealer are always strictly positive and thus the interdealer market is active.

Lemma 2 implies that a trade must happen between a type-0 and a type-2 dealer. In the proof for Lemma 2, we show that the gains from trade from a potential trade between a type-0 and a type-2 dealer can be expressed as

$$2V^1_1 - V^1_0 - V^1_2 = \frac{(\rho - 2)c + \kappa_1 \beta}{\kappa_2},$$

where $\kappa_1 > 0$ and $\kappa_2 > 0$ are two strictly positive constants that are determined in equilibrium. It shows that the potential gains from trade are captured by two independent terms. The first term captures an instantaneous effect: it shows that an interdealer trade allows the two dealers to jointly save their inventory costs, consistent with the classical view of the inventory sharing. The second term captures a continuation effect: it implies that an interdealer trade allows the two dealers, who subsequently become two type-1 dealers, to jointly intermediate more assets between buyers and sellers. Since $\beta$ is always strictly positive in
any equilibrium, an interdealer trade happens as long as the second effect dominates, even if the inventory cost is not convex. In other words, the inventory cost structure being weakly convex is one (weak and economically relevant) sufficient condition for the interdealer market being active, but not a necessary condition.

So far, Lemmas 1 and 2 show that $d_2 > 0$ is both a sufficient and a necessary condition for the interdealer market being active. This verifies our equilibrium categorization: the equilibrium features an active interdealer market when $d_2 > 0$ while the interdealer market is not active when $d_2 = 0$.

A natural and important question is when an equilibrium with an active interdealer market exists. In principle, the existence of such an equilibrium requires all the five relevant trades as illustrated by the five solid arrows in Figure 1 happen. In other words, the gains from trade as shown in the right hand sides of value functions (3.4), (3.5), (3.6), (3.7), and (3.8) must be weakly positive. It is not surprising, however, that many of these constraints will be slack in equilibrium. The following proposition shows that the trade between a type-1 dealer and a seller is sufficient to determine the equilibrium existence:

**Proposition 1.** A trading equilibrium with an active interdealer market exists if

\[ V_2^1 - V_1^1 - V_s^1 \geq 0, \quad (3.9) \]

where $V_2^1$, $V_1^1$ and $V_s^1$ satisfy the value functions (3.4), (3.5), (3.6), (3.7), and (3.8).

Proposition 1 implies that the only criterion for the emergence of the interdealer market is that a type-1 dealer is willing to buy from a seller, given the corresponding distribution of agents. In other words, other relevant trades must happen as the trade between a type-1 dealer and a seller happens.

The intuition is as follows. First, holding a higher inventory is more costly to a dealer due to the weakly convex inventory cost structure. Thus, that a type-1 dealer, who already hold a low level of inventory, find it profitable to increase it inventory implies that a type-0 dealer must find it profitable to increase it inventory. This implies that the trade between a type-0 dealer and a seller must happen.
Second, dealers do not have any ultimate interest in holding the asset. Thus, that a
dealer finds it profitable to buy from a seller implies that she must also find it profitable
to sell to a buyer later, regardless of it type. This implies that the trade between a type-1
dealer and a buyer and that between a type-2 dealer and a buyer must happen.

Finally, notice that Lemma 2 already guarantees that the trade between a type-0 and a
type-2 dealer, that is, interdealer trading, must happen as $d_2 > 0$.

Following Proposition 1, we may further formulate the equilibrium criterion (3.9) with
respect to the fundamental of the asset $\theta$:

**Corollary 1.** There exists a lower threshold $\theta_2$ such that for all $\theta > \theta_2$, a trading equilibrium
with an active interdealer market exists.

Corollary 1 implies that the interdealer market will endogenously emerge when the asset
fundamental is high enough given other model parameters. Note that corollary 1 can be also
stated with a higher threshold of inventory cost $c$ in the sense that a trading equilibrium with
an active interdealer market exists when the inventory cost $c$ is lower than the threshold.
Intuitively, when the asset fundamental is high enough relative to dealers’ inventory cost,
dealers are more likely to hold a high level of inventory in equilibrium and accordingly trade
with each other to better manage their inventory holdings.

It is straightforward to formulate two empirically relevant equilibrium outcomes: the
average length of intermediation chains and the aggregate inventory held by the dealer
sector. In our framework, we define the length of intermediation chains $l$ as the number of
dealers through which an asset is intermediated from a seller to a buyer. Note that $l$ is a
random variable under equilibrium. We define the aggregate inventory $v$ as the amount of
assets held by all the types of dealers. Given the trading pattern in Figure 1, we have the
following result, where the superscript $1$ indicates that these variables are evaluated under
the distribution of agents in an equilibrium with an active interdealer market.

**Corollary 2.** In a trading equilibrium with an active interdealer market, the average
length of intermediation chains is strictly larger than 1, that is, $\mathbb{E}[l^1] > 1$, and the aggregate
inventory is $v^1 = 1$. 

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In Corollary 2, $E[l^1] > 1$ is a straightforward implication of the interdealer market being active, and $v^1 = 1$ follows from the inflow-outflow balance, in particular, (3.3). Later on, we will compare them to their counterparts in the equilibrium without an interdealer market. Note that our goal is not to directly mapping these equilibrium outcomes to data but rather to qualitatively compare them across the two types of equilibria.

3.2 Trading equilibrium without interdealer market: $d_2 = 0$

Next, we characterize a type of equilibria where $d_2 = 0$. In view of Lemmas 1 and 2, we have already known that $d_2 > 0$ is a sufficient and necessary condition for the interdealer market being active. Equivalently, $d_2 = 0$ is also a sufficient and necessary condition for the interdealer market being inactive. Figure 2 below illustrates this type of equilibria.

Under the equilibrium without an active interdealer market, inflow-outflow balance implies

$$n - \eta b = ba d_1, \quad (3.10)$$

$$n - \eta b = s \alpha d_0. \quad (3.11)$$

Recall that $b = s$ under any equilibrium, (3.10) and (3.11) together imply

$$d_0 = d_1 = \frac{1}{2}, \quad (3.12)$$

that is, the mass of type-0 dealers must equal that of type-1 dealers under the equilibrium without an interdealer market.

Denote by $V_b^0$, $V_s^0$, $V_0^0$, and $V_1^0$ the equilibrium value functions of an active buyer, seller, type-0 dealer, and type-1 dealer, respectively, where the superscript 0 indicates that the value functions are evaluated under the distribution of agents in an equilibrium without an interdealer market. We can similarly derive the agents’ equilibrium HJB equations, taking
Figure 2: Equilibrium without the interdealer market

Solid arrows indicate the flows of assets. Dashed arrows indicate changes in the distribution of agents, that is, the flows of customers in and out of the market as well as dealer’s changes in inventory-holding status.

Into account the endogenous matching and bargaining outcomes:

\[
(\eta + r)V_b^0 = \alpha d_1 \frac{1}{2} (\theta + V_0^0 - V_1^0 - V_b^0)^+, \tag{3.13}
\]

\[
(\eta + r)V_s^0 = \alpha d_0 \frac{1}{2} (V_1^0 - V_0^0 - V_s^0)^+, \tag{3.14}
\]

\[
rV_0^0 = \beta \frac{s}{b + s} \frac{1}{2} (V_1^0 - V_0^0 - V_s^0)^+, \tag{3.15}
\]

\[
rV_1^0 = \beta \frac{b}{b + s} \frac{1}{2} (\theta + V_0^0 - V_1^0 - V_b^0)^+ - c. \tag{3.16}
\]

The following proposition characterizes the conditions under which an equilibrium without an active interdealer market exists:

**Proposition 2.** A trading equilibrium without an interdealer market exists if \( V_0^0 \geq 0 \) and

\[
V_2^0 - V_1^0 - V_s^0 \leq 0, \tag{3.17}
\]

where the hypothetical value function \( V_2^0 \) is given by

\[
rV_2^0 = \beta \frac{b}{b + s} \frac{1}{2} (\theta + V_1^0 - V_2^0 - V_b^0)^+ + \lambda d_0 \frac{1}{2} (2V_1^0 - V_0^0 - V_2^0)^+ - \rho c, \tag{3.18}
\]

and \( V_1s \) and \( V_s \) satisfy the value functions (3.13), (3.14), (3.15), and (3.16).
Proposition 2 specifies two conditions. First, the value function of a type-0 dealer under the corresponding distribution of agents, $V_0^0$, must be weakly positive. To see the intuition, recall that we need to verify the gains from trade for all relevant trades to be positive, that is, all relevant trades in Figure 2 must happen. When $V_0^0 \geq 0$, it must be that a type-0 dealer is willing to buy from a seller since this is the only trade that a type-0 dealer can conduct. Then, because dealers do not have any ultimate interest in holding the asset, that a type-0 dealer finds it profitable to buy from a seller implies that she must also find it profitable to sell to a buyer later (i.e., when she becomes a type-1 dealer). This implies that the trade between a type-1 dealer and a buyer must happen.

Second, we need to further verify that a type-1 dealer would not find it profitable if it were to buy from an active seller. In other words, a type-1 dealer would never deviate from such an equilibrium by buying from a seller. This is captured by the additional condition (3.18), where the hypothetical value functional for a type-2 dealer is instead evaluated under the distribution without an active interdealer market.

We highlight that conditions (3.9) and (3.17) regarding the trade between a type-1 dealer and a seller are not mutually exclusive because these value functions are evaluated under different distributions. In other words, conditions (3.9) and (3.17) may both hold, or are both violated. This observation has direct implication on potential equilibrium multiplicity, which we elaborate below.

Similar to Corollary 1, we may formulate the equilibrium criterion (3.17) with respect to the asset fundamental:

**Corollary 3.** Suppose $V_0^0 \geq 0$. There exists a upper threshold $\theta$ such that for all $\theta \leq \theta$, a trading equilibrium without an active interdealer market exists.

Also similar to Corollary 2, we calculate the length of intermediation chains and aggregate inventory under equilibrium, where the superscript 0 indicates that these variables are evaluated under the distribution of agents in an equilibrium without an interdealer market.

**Corollary 4.** In a trading equilibrium with an active interdealer market, the length of intermediation chains is $l^0 = 1$, and the aggregate inventory is $v^0 = \frac{1}{2}$. 

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Compared to Corollary 2, Corollary 4 thus suggests that the intermediation chain is strictly shorter and dealers holder a lower level of inventory on average under an equilibrium without an interdealer market.

4 Coordination of intermediation

Having characterized all the possible types of trading equilibria, we consider the potential for equilibrium multiplicity and the underlying strategic interactions. We also analyze market liquidity in the two types of equilibria, which is not only empirically relevant but also useful in illustrating the source of the coordination motives behind dealers’ liquidity provision decisions. Given our focus on trading among intermediaries, below we focus on the parameter regions where \( V^0_0 \geq 0 \) is satisfied.

4.1 Equilibrium multiplicity

The analysis of equilibrium multiplicity hinges on two important observations from Section 3. First, Propositions 1 and 2 suggest that whether a type-1 dealer is willing buy from a seller and to effectively increase it inventory, captured by \( V_2 - V_1 - V_s \), is the sole criterion to determine the equilibrium, given the distribution of other agents in the corresponding equilibrium. Second, in turn, the distribution of other agents is solely determined by whether other type-1 dealers buy from sellers and to effectively increase their inventory. Since we focus on steady-state equilibria, these observations allows us to use a two-strategy game among type-1 dealers to represent the strategic interaction embedded in the dynamic framework.

**Proposition 3.** *For a given set of model parameters, if*

\[
\begin{align*}
 V^1_2 - V^1_1 - V^1_s > 0, \\
 V^0_2 - V^0_1 - V^0_s < 0, 
\end{align*}
\]

*then an equilibrium with \( d_2 > 0 \) and an equilibrium with \( d_2 = 0 \) co-exist.*

Proposition 3 is intuitive and follows from Propositions 1 and 2. Condition (4.1) implies that a type-1 dealer is willing to buy from a seller and to become a type-2 dealer when other
type-1 dealers also buy from sellers (and thus the trading pattern and distribution of agents follow from Figure 1). According to Proposition 1, this implies that an equilibrium with an interdealer market exists. In parallel, Condition (4.2) implies that a type-1 dealer is not willing to buy from a seller and to become a type-2 dealer when other type-1 dealers also refuse to trade with sellers (and thus the trading pattern and distribution of agents follow from Figure 2). According to Proposition 2, this implies that an equilibrium without an interdealer market exists. When (4.1) and (4.2) are both satisfied, both equilibria co-exist accordingly.

The idea underlying Proposition 3 resembles the classic notion of coordination in complete information static games such as the bank run models (e.g., Diamond and Dybvig, 1983). Whether a type-1 dealer trades with a seller and becomes a type-2 dealer would depend on its belief about other type-1 dealers’ strategies. Thus, we can focus on type-1 dealer’s this given strategy, and below we intuitively call it type-1 dealers’ liquidity provision strategy, that is, whether a type-1 dealer provide liquidity or not.

As usual in standard coordination games, we allow the agents to play mixed strategies. Specifically, we allow the probability at which a trade between a type-1 dealer and a seller happens to be $\phi \in (0, 1)$. In a mixed-strategy equilibrium, a given type-1 dealer has probability $\phi$ to trade with a seller when other type-1 dealers also trade with sellers conditional on a meeting with probability $\phi$.\footnote{Note that this description of mixed-strategy equilibria also accommodate the pure-strategy equilibria we already consider above.} In this case, an interdealer market emerges with probability $\phi$, which is determined by

$$V^\phi_2 - V^\phi_1 - V^\phi_s = 0$$

(4.3)
as well as other value functions as prescribed by (3.4), (3.5), (3.6), (3.7), and (3.8). Appendix C provides a detailed micro-foundation from which $\phi$ can be constructed from the staged bargaining game imbedded in our setting, and Appendix D provides the formal procedure to solve for the mixed-strategy equilibria, if any.

We provide an numerical example to illustrate Proposition 3. In this example, we choose $n = 1$, $\eta = 1$, $r = 1$, $c = 0.2$, $\rho = 2.5$, $\alpha = 5$, and $\lambda = 0.01$.

The top panel of Figure 3 plots $V^1_2 - V^1_1 - V^1_s$ and $V^0_2 - V^0_1 - V^0_s$, the payoff gains of...
Figure 3: The correspondence of payoff gains and equilibrium multiplicity

The top panel plots the gains from trade between a type-1 dealer and a seller against asset fundamental $\theta$, under the distribution when all other type-1 dealers trade with sellers (black) and under the distribution when all other type-1 dealers do not trade with sellers (blue). The bottom panel plots the equilibrium probability of the trade between a type-1 dealer and a seller against asset fundamental $\theta$. Parameters: $a = 1$, $\eta = 1$, $r = 1$, $c = 0.2$, $\rho = 2.5$, $\alpha = 5$, and $\lambda = 0.01$.

a type-1 dealer trading with a seller given other type-1 dealers’ strategy, against different values of $\theta$. It shows that in the given economic environment, type-1 dealers’ liquidity provision decisions exhibit coordination motives for intermediate levels of $\theta$. In particular, for intermediate levels of $\theta$, $V_1^1 - V_1^1 - V_s^1$ is positive while $V_2^0 - V_1^0 - V_s^0$ is negative. Therefore, Proposition 3 suggests equilibrium multiplicity within that intermediate range of $\theta$. This is confirmed by the bottom panel of Figure 3, which plots the equilibrium emergence of the interdealer market given different values of $\theta$. Intuitively, when $\theta$ is sufficiently large (small), the interdealer market endogenously emerges (closes). When $\theta$ is intermediate, whether the interdealer market will emerges depends on a type-1 dealer’s belief about other dealers’
liquidity provision decisions.

An important question is where type-1 dealers’ coordination motives do come from. We devote the next subsection to this question.

4.2 Liquidity implications and source of coordination motives

To address the source of coordination motives behind type-1 dealers’ liquidity provision decisions, we define and analyze two empirically relevant notions of market liquidity. These two notions capture two different but tightly linked aspects of market liquidity. As shown shortly, they are important in helping understand why a dealer’s revenue from liquidity provision is sensitive to other dealers’ liquidity provision decisions.

The first notion of market liquidity is customer-dealer trading volume, which captures the amount of assets being intermediated. It is one of the most commonly used liquidity measure in reality.

**Definition 1.** The trading volume per unit of time is defined by the unit of asset intermediated per unit of time:

\[ q = n - \eta b. \]  

(4.4)

The following proposition suggests that trading among intermediaries improve market liquidity by increasing trading volume:

**Proposition 4.** When multiplicity happens, trading volume per unit of time \( q \) is higher in an equilibrium with interdealer trading than the comparable equilibrium without interdealer trading.

Proposition 4 is intuitive. In view of Corollaries 2 and 4, the aggregate dealer sector jointly holds a higher inventory under the equilibrium with interdealer trading. This allows them to attract more flows from the customers and hence intermediate more assets during any given period of time.

We then consider the second notion of market liquidity: markup per unit of trade. It corresponds to the bid-ask spread, which is another commonly used liquidity measure in reality:
**Definition 2.** The markup per unit of trade $\Delta_i$ is defined by the difference between the buyer-dealer price $p_{b,i+1}$ and the seller-dealer price $p_{s,i}$:

$$\Delta_i = p_{b,i+1} - p_{s,i}, \quad (4.5)$$

where $i \in \{0, 1\}$ denotes dealer types.

As Definition 2 indicates, markup in principle depends on the dealer type. However, we can show that the markup per unit of time can be re-expressed as:

$$\Delta = \frac{\theta - (V_b + V_s)}{2},$$

regardless of dealer type. This expression is intuitive. The numerator indicates that what the dealer captures from each trade he intermediates is negatively related to $V_b + V_s$, that is, what the buyer and seller of this trade jointly capture. The denominator captures the relative bargain power of the dealer, which is set to be $\frac{1}{2}$ in this model. More generally, the higher relative bargaining power the dealer has, the higher markup per unit of trade he enjoys. We then show that $V_b + V_s$ is lower under the equilibrium with interdealer trading because the intensified dealer competition allows the customers to jointly capture more surplus from an intermediated trade. These arguments are summarized by the following proposition, which suggests that trading among intermediaries also improve market liquidity by lowering markup per trade:

**Proposition 5.** When multiplicity happens, markup per unit of trade $\Delta$ is lower in an equilibrium with interdealer trading than the comparable equilibrium without interdealer trading.

One interesting and important observation from Propositions 4 and 5 is that although both notions of market liquidity unambiguously increase as dealers provide more liquidity in the equilibrium with interdealer trading, the impact on dealer revenue per unit of time is ambiguous. Formally, dealer profitability per unit of time can be defined as:

**Definition 3.** The dealer revenue per unit of time $\pi$ is defined as the product of 1) the
trading volume per unit of time and 2) the markup per trade:

\[
R = q\Delta. \tag{4.6}
\]

Definition 3 implies that how a specific dealer’s revenue changes depends on how the trading volume and markup change. When all other type-1 dealers buy from sellers and thus provide liquidity, Proposition 4 suggests that trading volume per unit of time increases. On the other hand, Proposition 5 suggests that markup from providing such additional liquidity decreases. Hence, if the former trading volume effect dominates the latter market effect, the type-1 dealer in question’s revenue from buying from sellers will increases if he joins other type-1 dealers. This ultimately leads to the coordination motives behind dealers’ liquidity provision decisions, which further leads to multiple equilibria.

5 Market fragility

A natural question is how market fragility relates to the friction of the interdealer market: when it becomes easier for the intermediaries to meet each other, would it become more or less likely for multiple equilibria to occur?

For this purpose, we explicitly separate fundamental characteristics of our economy, including potential gains from trade \( \theta \) and inventory costs \( c \) and \( \rho \), from the meeting rate of the interdealer market \( \lambda \). We formally define market fragility as follows.

**Definition 4.** For a family of economies parameterized by \( \theta, c \) and \( \rho \), it is fragile if there exists a non-zero subset \( \{\theta, c, \rho\} \) of the space \( \mathbb{R}_+^2 \times (1, +\infty) \) in which multiple equilibria occur.

Economically, the definition captures the idea that if market liquidity may jump when fundamental does not change, the market is fragile. The next two subsections explore how market fragility evolves with \( \lambda \).

5.1 Interdealer market friction and market fragility

We then explore the case when the interdealer market friction is large. To help illustrate the mechanism, we consider what happens when it is extremely hard for dealers to meet each
other, that is, $\lambda$ is close to 0. The following proposition characterizes when market fragility happens.

**Proposition 6.** For any family of economies parameterized by $\theta, c$ and $\rho$, there exists a threshold $\Lambda$ such that the family of economies is fragile when $\lambda < \Lambda$.

The economic intuition underlying Proposition 6 is the coordination in dealer liquidity provision. When it is hard for dealers to meet and trade with each other, a type-1 dealer will be reluctant to buy from a seller due to the difficulty or absence of interdealer trading. Only when other type-1 dealers are willing to provide liquidity and sufficiently increase the potential trade volume, this type-1 dealer in question can enjoy a high enough revenue and thus is willing to provide liquidity. This effectively makes type-1 dealers’ liquidity provision decisions sensitive to each other’s, leading to coordination motives and ultimately multiple equilibria.

### 5.2 A frictionless benchmark

We then consider a benchmark under which dealers can meet each other frictionlessly. Concretely, we consider the possibility of market fragility when $\lambda \to \infty$. This resembles the modeling of a centralized interdealer market in the literature, for example, Garleanu (2009) and Lagos and Rocheteau (2009). The following proposition suggests that market fragility never happens under this frictionless benchmark.

**Proposition 7.** For any family of economies parameterized by $\theta, c$ and $\rho$, there exists a threshold $\bar{\lambda}$ such that the family of economies is not fragile when $\lambda > \bar{\lambda}$.

Proposition 7 suggests that the equilibrium with an interdealer market is the unique equilibrium regardless of the fundamentals $\theta, c$ and $\rho$, as long as the friction of the interdealer market is sufficiently small. Intuitively, a more efficient interdealer market implies that it is easier for dealers to offload their inventory holdings. This makes every dealer more willing to hold more inventory regardless of other dealers’ liquidity provision decisions. If the search friction among dealers is sufficiently small, the endogenously emerged interdealer market can completely eliminate the coordination motive among dealers’ liquidity provision decisions, thereby eliminate any possible coordination failures.
6 Comparative statics

To provide more visual guidance and intuition regarding market fragility, we provide several numerical comparative statics to illustrate under what conditions market fragility is more likely to occur, and what are the impact on trading volume. To focus on trading among intermediaries, we restrict our attention to the parameter region where dealers are willing to participate under any equilibrium. We fix the following common parameters: \( n = 1 \), and \( r = 1 \).

6.1 Interdealer market friction and market fragility

One of the key ideas of our model is that equilibrium multiplicity may arise due to the coordination motives of dealers in inventory holding and liquidity provision decisions. The coordination motives are in turn affected by the friction of the interdealer market: when dealers are easier to meet and thus trade with each other, one dealer’s inventory holding and liquidity provision decisions depend less on other dealers’, thereby weaken the coordination motives and eventually reduces potential market fragility. The following Figure 4 illustrates this idea, using the inventory cost concavity parameter \( \rho = 2.5 \), search friction parameter \( \alpha = 5 \), and exit shock parameter \( \eta = 1 \).

The four panels in Figure 4 show that, as the friction of the interdealer market becomes less severe, captured by a reduced \( \lambda \), the area where multiplicity happens over the \((\theta, c)\)–region becomes smaller. Specifically, as \( \lambda \) takes values from 0 to 0.15 and then to 0.3, the family of economics is always fragile, but becomes less fragile in the sense that multiplicity is less likely to happen. When the interdealer market becomes sufficiently efficient as \( \lambda = 0.45 \), the coordination motives among dealer’ inventory holding decisions is completely eliminated, and thus multiplicity cannot happen for any admissible parameters.

There are two important take-aways from Figure 4. First, the possibility of market fragility, measured by the area in the \((\theta, c)\)–region where multiplicity happens, strictly decreases as the friction of the interdealer market becomes smaller. This is consistent with the theoretical predictions of Propositions 6 and 7.

Second, when the family of economics is fragile for a given level of interdealer market
friction, equilibrium multiplicity always happens when the ratio of the potential gains from trade to the inventory cost lies in an intermediate range. Only in such cases, dealers’ inventory holding exhibits coordination motives. This is consistent with the theoretical predictions of Propositions 1 and 3.

6.2 Inventory cost, market fragility, and trading volume

We also explore the impacts of inventory costs on market fragility and trading volume across different parameter regions. The results are illustrated in Figures 5 and 6 below.

First, Figure 5 show the results when the interdealer market friction $\lambda$ and the level of inventory cost $c$ vary, under the same inventory cost concavity $\rho = 2.5$, search friction parameter $\alpha = 5$, and exit shock friction $\eta = 1$. Specifically, the left three panels in Figure
Figure 5: Equilibrium trading volume and fragility when $\lambda$ and $c$ vary

Each panel plots equilibrium trading volume $q$ against the asset fundamental $\theta$. Equilibrium multiplicity arises in the left panels. Common parameters: $n = 1$, $\eta = 1$, $r = 1$, $\rho = 2.5$, and $\alpha = 5$.

5 features a family of economics where fragility happens. Here, the interdealer market is extremely inefficient in the sense that dealers cannot meet with each other at all (i.e., $\lambda = 0$). As the level of inventory cost $c$ becomes higher from 0.1 to 0.2 and then to 0.4, the equilibrium without an active interdealer market is more likely to happen. In particular, multiplicity is more likely to happen as the inventory cost becomes higher. Interestingly, when the inventory cost is higher, multiplicity is more likely to happen when the potential gain from trade per unit of asset is higher.

In contrast, the right three panels in Figure 5 features a family of economics where
fragility does not happen. Here, the interdealer market is efficient in the sense that dealers can meet with each other at a sufficiently large rate (i.e., $\lambda = 0.4$). As the inventory cost becomes higher from 0.1 to 0.2 and then to 0.4, the equilibrium without an active interdealer market is more likely to happen. However, multiplicity never happens. These results are consistent with the theoretical predictions of Propositions 6 and 7.

![Graphs showing equilibrium trading volume and fragility when $\lambda$ and $\rho$ vary](image)

Figure 6: Equilibrium trading volume and fragility when $\lambda$ and $\rho$ vary

Each panel plots equilibrium trading volume $q$ against the asset fundamental $\theta$. Equilibrium multiplicity arises in the left panels. Common parameters: $n = 1$, $\eta = 1$, $r = 1$, $c = 0.2$, and $\alpha = 5$.

In a similar fashion, Figure 6 shows the results when the interdealer market friction $\lambda$ and the inventory cost concavity $\rho$ vary, under the same level of inventory cost $c = 0.1$ and search friction $\eta = 1$. As the inventory cost concavity parameter $\rho$ becomes higher from 2 to 4 and then to 6, the equilibrium without an active interdealer market is more likely to
happen, but market fragility happens only when the interdealer market friction is higher (i.e., $\lambda = 0$). In particular, multiplicity is more likely to happen as the inventory cost structure becomes more convex. These results are again consistent with the theoretical predictions of Propositions 6 and 7.

6.3 Customer search friction, market fragility, and trading volume

We next explore the impacts of the search friction between customers and dealers on market fragility and trading volume across different parameter regions. The results are illustrated in Figure 7 below.

![Figure 7: Equilibrium trading volume and fragility when $\lambda$ and $\alpha$ vary](image)

Each panel plots equilibrium trading volume $q$ against the asset fundamental $\theta$. Equilibrium multiplicity arises in the left panels. Common parameters: $n = 1$, $\eta = 1$, $r = 1$, $c = 0.2$, and $\rho = 2.5$. 

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Here, we consider varying the interdealer market friction $\lambda$ and the search friction parameter $k$ between customers and dealers, under the same inventory structure $c = 0.1$ and $\rho = 2.5$, and the exit shock parameter $\eta = 1$. Similar, market fragility happens only when the interdealer market friction is higher (i.e., $\lambda = 0$). As it becomes easier for customers to contact dealers, that is, as the search friction becomes higher from 5 to 5.5 and then to 5.75, the equilibrium evolution has two additional interesting features.

First, equilibrium trading volume becomes universally higher regardless the type of equilibrium. This is economically intuitive because a lower search friction makes it easier for customers to contact dealers, improving trading volume.

Perhaps surprisingly, market fragility in the sense of the possibility of multiplicity (happens when $\lambda$ is close to 0) becomes higher as the search friction between customers and dealers becomes lower. However, this result is still economically intuitive due to coordination motives among dealers in our framework. When it becomes easier for customers to contact the dealers, customers complete more fiercely for dealers. In other words, dealer competition becomes relatively weaker. This implies that the markup effect becomes weaker relative to the trading volume effect. Thus, the coordination motives among dealers to provide liquidity become higher, under the same level of interdealer market friction. This is broadly consistent with the pattern that a decline in trading frictions between customers and dealers in the recent decades may instead lead to a higher market fragility due to the escalated competition among customers who use financial intermediaries (e.g., Philippon, 2015).

7 Conclusion

We propose a search-based asset pricing model to study decentralized and endogenous trading among financial intermediaries and the implied intermediation chains. In particular, we focus on the emergence of multiple equilibria and the implied market fragility, which is formally defined as the possibility of a liquidity drop in an asset market without a fundamental shock. In an equilibrium where interdealer trading is active (inactive), the intermediation chain is longer (shorter), dealers provide more (less) liquidity by holding more (less) inven-
tory, and the market is more (less) liquid. Importantly, a dealer is more likely to provide liquidity if other dealers do so and the market becomes more liquid, leading to coordination motives among dealers. The coordination motives in turn lead to multiple equilibria and suggest market fragility. Market fragility is more likely to happen when the level of dealer inventory cost becomes higher, when the inventory cost structure becomes more convex, and when it is easier for the customers to contact dealers (e.g., due to customer technological improvements). A low search friction among dealers effectively reduces dealers’ intermediation cost, making a dealer’s willingness to provide liquidity less sensitive to other dealers’ decisions and thereby reducing market fragility.

References


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Appendix

A Additional results

We first prove an additional result that implying an equilibrium restriction on the distribution of agents. This result will be repeatedly used in other proofs.

**Lemma 3.** In any trading equilibrium with \( d_2 > 0 \), there must be that \( 0 < d_I \leq \frac{1}{3} \). In other words, \( d_1 \geq \frac{1}{3} \).

**Proof of Lemma 3.** Notice that, in any steady state equilibrium, the amount of type-0 dealers that buy from a seller and become type-1 dealers must equal the amount of type-1 dealers that sell to a buyer and become type-0 dealers:

\[
\begin{align*}
d_0 \left( \beta \frac{s}{b + s} + \frac{d_2}{d} \right) &= d_1 \beta \frac{b}{b + s}.
\end{align*}
\]

Because \( b = s \) in any equilibrium, it follows that

\[
\frac{\beta}{2} d_0 + \lambda \frac{d_0 d_2}{d} = \frac{\beta}{2} d_1. \tag{A.1}
\]

Then by equation (3.3), condition (A.1) thus becomes

\[
\lambda d^2_I + \frac{3}{2} \beta d_I - \frac{\beta}{2} = 0. \tag{A.2}
\]

Solving (A.2) and picking the positive root yields:

\[
d_I = \frac{-3 \beta + \sqrt{9 \beta^2 + 2 \lambda \beta}}{2 \lambda} \in \left[ 0, \frac{1}{3} \right],
\]

completing the proof.

In particular, Lemma 3 implies

\[
\begin{align*}
&\begin{cases}
  d_I = 0 & \text{if } \lambda \to \infty, \\
  d_I = \frac{1}{3} & \text{if } \lambda = 0.
\end{cases}
\end{align*}
\]

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B Proofs omitted from the main text

Proof of Lemma 2. First, (3.7) minus (3.6) gives

\[
\frac{1}{2}(V_1^1 - V_0^1)(2r + \beta + \lambda d_I) = \frac{\beta}{4} (\theta - V_0) + \frac{1}{2}(V_2^1 - V_1^1) \left( \frac{\beta}{2} + \lambda d_I \right) - c, \quad (B.1)
\]

and (3.8) minus (3.7) gives

\[
\frac{1}{2}(V_2^1 - V_1^1)(2r + \beta + \lambda d_I) = \frac{\beta}{4} V_0 + \frac{1}{2}(V_1^1 - V_0^1) \left( \frac{\beta}{2} + \lambda d_I \right) - (\rho - 1)c. \quad (B.2)
\]

Combining (3.4) and (B.1) yields

\[
\frac{1}{2}(V_1^1 - V_0^1)(2r + \beta + \lambda d_I) = \frac{\beta}{4} (\eta + r) + \frac{\alpha d_I}{2} (V_2^1 - V_1^1) + \frac{\alpha(1 - 2d_I)}{2} (V_1^1 - V_0^1) \frac{1}{\eta + r + \frac{\alpha d_I}{2}(1 - d_I)} + \frac{1}{2}(V_2^1 - V_1^1) \left( \frac{\beta}{2} + \lambda d_I \right) - c. 
\]

Define

\[
B = \beta + \lambda d_I + \beta \frac{\alpha d_I}{2(\eta + r) + \alpha(1 - d_I)},
\]

and

\[
Q = \frac{\beta(\eta + r)}{2(\eta + r) + \alpha(1 - d_I)}.
\]

It follows that

\[
\frac{1}{2}(V_1^1 - V_0^1)(2r + B + Q) = \frac{1}{2}(V_2^1 - V_1^1)B + \frac{\theta}{2}Q - c. \quad (B.3)
\]

Similarly, combining (3.5) and (B.2) yields

\[
\frac{1}{2}(V_2^1 - V_1^1) \left( 2r + \beta + \lambda d_I - \frac{\beta}{2} \frac{\alpha(1 - 2d_I)}{2(\eta + r) + \alpha(1 - d_I)} \right) = \frac{1}{2}(V_1^1 - V_0^1) \left( \frac{\beta}{2} + \lambda d_I + \beta \frac{\alpha d_I}{2(\eta + r) + \alpha(1 - d_I)} \right) - (\rho - 1)c,
\]

that is,

\[
\frac{1}{2}(V_2^1 - V_1^1)(2r + B + Q) = \frac{1}{2}(V_1^1 - V_0^1)B - (\rho - 1)c. \quad (B.4)
\]
Hence, (B.3) minus (B.4) implies

\[ \frac{1}{2}(2V_1^1 - V_2^1 - V_0^1)(2r + 2B + Q) = \frac{\theta}{2} Q + (\rho - 2)c. \quad (B.5) \]

Because \( B > 0 \) and \( Q > 0 \), this implies that \( 2V_1^1 - V_2^1 - V_0^1 > 0 \). Therefore, interdealer trading must happen as \( d_2 > 0 \), concluding the proof.

**Proof of Proposition 1.** Following the proof for Lemma 2, (B.4) and (B.5) imply

\[ \frac{1}{2}(V_1^1 - V_1^1) = (2r + Q)^{-1} \left( \frac{B}{2r + 2B + Q} \left( Q^2 + (\rho - 2)c \right) - (\rho - 1)c \right). \quad (B.6) \]

Conditions (B.6) and (B.3) then imply

\[ \frac{1}{2}(V_1^1 - V_0^1)(2r + B + Q) = B \left( \frac{1}{2}(V_1^1 - V_0^1) - \frac{\theta}{2} Q + (\rho - 2)c \right) + \frac{\theta}{2} Q - c. \quad (B.7) \]

Therefore,

\[ \frac{1}{2}(V_1^1 - V_0^1) = (2r + Q)^{-1} \frac{2r + B + Q)(\frac{\theta}{2} Q - c) - B(\rho - 1)c}{2r + 2B + Q}. \quad (B.8) \]

Similarly, (B.6) can be rewritten as

\[ \frac{1}{2}(V_2^1 - V_1^1) = (2r + Q)^{-1} \frac{B(\frac{\theta}{2} Q - c) - (2r + B + Q)(\rho - 1)c}{2r + 2B + Q}. \quad (B.9) \]

The symmetry between (B.8) and (B.9) implies

\[
\begin{align*}
\alpha(1 - 2t) \frac{1}{2}(V_2^1 - V_1^1 - V_s^1) \\
= \left( \eta + r + \frac{\alpha d_I}{2} \right) V_s^1 - \alpha d_I \frac{1}{2}(V_1^1 - V_0^1) \\
= \frac{2(\eta + r) + \alpha d_I}{2(\eta + r) + \alpha(1 - d_I)} \left( \alpha d_I \frac{1}{2}(V_1^1 - V_0^1) + \alpha(1 - 2d_I) \frac{1}{2}(V_2^1 - V_1^1) \right) - \alpha d_I \frac{1}{2}(V_1^1 - V_0^1) \\
= -\alpha(1 - 2d_I) \alpha d_I \frac{1}{2}(V_1^1 - V_0^1) + (2(\eta + r) + \alpha d_I) \alpha(1 - 2d_I) \frac{1}{2}(V_2^1 - V_1^1) \\
\right)
\end{align*}
\]

\[ \frac{2(\eta + r) + \alpha(1 - d_I)}{2(\eta + r) + \alpha(1 - d_I)} \]

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completing the proof.

**Proof of Corollary 1.** The equilibrium with an interdealer market exists when

\[ V_2^1 - V_1^1 - V_s^1 \geq 0. \quad (B.10) \]

Note that Lemma 3 implies that \( d_I \in [0, \frac{1}{3}] \). Therefore, \( \alpha(1 - 2d_I) > 0 \) and

\[ 2(\eta + r) + \alpha(1 - d_I) > 0. \]

Hence, (B.10) holds if and only if

\[ 2(\eta + r)\frac{1}{2}(V_2^1 - V_1^1) \geq \alpha d_I \frac{1}{2}(2V_1^1 - V_2^1 - V_0^1), \quad (B.11) \]

and by (B.9) and (B.5), we have

\[ \frac{2(\eta + r) B (\frac{\theta}{2} Q - c) - (2r + B + Q)(\rho - 1)c}{2r + Q} \geq \frac{\alpha d_I (\frac{\theta}{2} Q + (\rho - 2)c)}{2r + 2B + Q}. \]

Since \( B > 0 \) and \( Q > 0 \), we also have

\[ 2(\eta + r) \left( B \left( \frac{\theta}{2} Q - c \right) - (2r + B + Q)(\rho - 1)c \right) \geq (2r + Q)\alpha d_I \left( \frac{\theta}{2} Q + (\rho - 2)c \right). \quad (B.12) \]

Note that (B.12) holds if and only if

\[ \left( \frac{\theta}{2} Q + (\rho - 2)c \right)(2(\eta + r)B - (2r + Q)\alpha d_I) \geq 2(\eta + r)(\rho - 1)c(2r + 2B + Q), \quad (B.13) \]

implying

\[ 2(\eta + r)B - (2r + Q)\alpha d_I > 0. \quad (B.14) \]

Plugging the expression of \( B \) and \( Q \) into (B.14), we have

\[ 2(\eta + r) \left( \frac{\beta}{2} + \lambda d_I + \frac{\beta}{2} \frac{\alpha d_I}{2(\eta + r) + \alpha(1 - d_I)} \right) > \alpha d_I \left( 2r + \frac{\beta(\eta + r)}{2(\eta + r) + \alpha(1 - d_I)} \right), \]

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that is,
\[ \left( \frac{2r}{\eta + r} \alpha - 2\lambda \right) d_I < \beta . \] (B.15)

Note that
\[ (\beta + 2\lambda d_I)(2r + Q) - 2r B > 0 , \] (B.16)
therefore, there exists \( \bar{\theta} \) such that \( V_0 \geq 0 \) for all \( \theta \geq \bar{\theta} \), where \( \bar{\theta} \) is defined such that \( rV_0^1 = 0 \).
This completes the proof.

**PROOF OF PROPOSITION 2.** To begin, conditions (3.13) and (3.16) imply
\[ rV_1^0 = \frac{\beta(\eta + r)V_0^0}{\alpha} - c . \] (B.17)
and conditions (3.14) and (3.15) imply
\[ rV_0^0 = \frac{\beta(\eta + r)V_0^0}{\alpha} . \] (B.18)
Moreover, (3.16) minus (3.15) implies
\[ r(V_1^0 - V_0^0) = -\beta \frac{1}{2}(V_1^0 - V_0^0) + \frac{\beta}{2} \left( \frac{1}{2}(\theta - V_0^0) + \frac{1}{2}V_0^0 \right) - c \] (B.19)
\[ = -\beta \frac{1}{2}(V_1^0 - V_0^0) + \frac{\beta}{4} (\theta + (V_2^0 - V_0^0)) - c , \]
implying
\[ r(V_1^0 - V_0^0) = \frac{\beta}{\alpha}(\eta + r)(V_0^0 - V_0^0) - c . \]
Therefore,
\[ V_0^0 - V_0^0 = \frac{\alpha}{\beta(\eta + r)} (-r(V_1^0 - V_0^0) - c) . \] (B.20)
Then, (B.19) and (B.20) imply
\[ r(V_1^0 - V_0^0) = -\beta \frac{1}{2}(V_1^0 - V_0^0) + \frac{\beta}{4} \theta + \frac{\alpha}{4(\eta + r)} (-r(V_1^0 - V_0^0) - c) - c . \]
Therefore,

\[ V_1^0 - V_0^0 = \left( r + \frac{\beta}{2} + \frac{r\alpha}{4(\eta + r)} \right)^{-1} \left( \frac{\beta}{4} \theta - (c_1 - c_0)(1 + \frac{\alpha}{4(\eta + r)}) \right). \]  \hspace{1cm} (B.21)

On the other hand, (B.18) implies

\[ V_s^0 = \frac{\alpha}{\beta(\eta + r)} r V_0^0. \]  \hspace{1cm} (B.22)

Also note that (B.22) and (3.15) imply

\[ r V_0^0 = \frac{\beta}{4} (V_1^0 - V_0^0) - \frac{\beta}{4} V_s^0 = \frac{\beta}{4} (V_1^0 - V_0^0) - \frac{\alpha}{4(\eta + r)} r V_0^0. \]

Therefore,

\[ r \left( 1 + \frac{\alpha}{4(\eta + r)} \right) V_0^0 = \frac{\beta}{4} (V_1^0 - V_0^0). \]  \hspace{1cm} (B.23)

Define

\[ D = 1 + \frac{\alpha}{4(\eta + r)}. \]

Then (B.21) becomes

\[ V_1^0 - V_0^0 = \frac{\beta}{4} \frac{\theta - Dc}{\frac{\theta}{2} + rD}. \]  \hspace{1cm} (B.24)

Conditions (B.23) and (B.24) imply

\[ r D V_0^0 = \frac{\beta}{4} \frac{\theta - Dc}{\frac{\theta}{2} + rD}. \]  \hspace{1cm} (B.25)

As a final requirement for the equilibrium, we need \( V_0^0 > 0 \). By (B.25) and the fact that \( D > 0 \), we have

\[ \theta \geq 4\beta^{-1} Dc. \]  \hspace{1cm} (B.26)

completing the proof.

**Proof of Proposition 4.** Denote by \( q^\phi \) and \( x^\phi, \phi \in \{0, 1\}, \) the equilibrium trading
volume per unit of time and mass of buyers under the two types of equilibria (without or
with an interdealer market). Consider the equilibrium with an interdealer market first. The inflow-outflow balance condition (3.1) implies that

$$q^1 = n - \eta b^1 = \alpha b^1(1 - d_I).$$

(B.27)

On the other hand, under the equilibrium without an interdealer market, inflow-outflow balance (3.10) implies that

$$q^0 = n - \eta b^0 = \frac{1}{2} \alpha b^0.$$  

(B.28)

Notice that Lemma 3 implies that $d_I \leq \frac{1}{3}$, that is, $1 - d_I \geq \frac{2}{3} > \frac{1}{2}$. Hence, conditions (B.27) and (B.28) jointly imply that $b^1 < b^0$. This immediately implies $q^1 > q^0$ by definition, concluding the proof.

PROOF OF PROPOSITION 5. Regardless of the type of the equilibrium (i.e., whether an equilibrium with interdealer trading or without happens), direct calculation based on the definition yields:

$$\Delta = p_{h,i+1} - p_{s,i}$$

$$= \frac{1}{2} (V_{i+1} - V_i - V_s + \theta + V_i - V_b - V_{i+1})$$

$$= \frac{1}{2} (\theta - V_b - V_s),$$

hence comparing $\Delta^1$ and $\Delta^0$ is equivalent to comparing $V^1_b + V^1_s$ and $V^0_b + V^0_s$.

Consider $V^1_b + V^1_s$. Lemma 3 implies that $d_I \leq \frac{1}{3}$. Thus, combining the two value functions 3.4 and 3.5 yields

$$V^1_b + V^1_s \geq \frac{\alpha \theta}{3(\eta + r) + \alpha}.$$

On the other hand, combining the two value functions 3.13 and 3.14 yields

$$V^0_b + V^0_s = \frac{\alpha \theta}{4(\eta + r) + \alpha},$$

which is clearly smaller than $V^1_b + V^1_s$. This concludes the proof.
**Proof of Proposition 6.** We consider the limit as $\lambda \to 0$. Under the equilibrium with an active interdealer market, the inflow-outflow balance implies

$$b = \frac{3n}{2\alpha + 3\eta}.$$  \hspace{1cm} (B.29)

It follows that

$$\beta = 2\alpha b = \frac{6\alpha n}{2\alpha + 3\eta}.$$  \hspace{1cm} (B.30)

Recall the definition of $B$ and $Q$ in the proof for Proposition 2. Direct calculation yields:

$$\lim_{\lambda \to 0} B = \frac{3\alpha n}{2\alpha + 3\eta} \frac{6(r + \eta) + 3\alpha}{6(r + \eta) + 2\alpha},$$  \hspace{1cm} (B.31)

$$\lim_{\lambda \to 0} Q = \frac{3\alpha n}{2\alpha + 3\eta} \frac{3(r + \eta)}{6(r + \eta) + 2\alpha}.$$  \hspace{1cm} (B.32)

Hence, $V_2^1 - V_1^1 - V_s^1$ is increasing in $\theta$ if and only if

$$3n > \left(\frac{2}{3\alpha + \eta}\right) \frac{2r}{\eta + r},$$  \hspace{1cm} (B.33)

completing the proof by continuity.

**Proof of Proposition 7.** We consider the limit as $\lambda \to \infty$. First, under the equilibrium with an active interdealer market, the inflow-outflow balance implies

$$b = \frac{n}{\alpha + \eta}.$$  \hspace{1cm} (B.34)

It follows that

$$\beta = 2\alpha b = \frac{2\alpha n}{\alpha + \eta}.$$  \hspace{1cm} (B.35)

At the same time,

$$\lim_{\lambda \to \infty} \lambda d_t = \infty,$$

suggesting that the interdealer market is active with the equilibrium mass of type-0 and
type-2 dealers being 0. Intuitively, because dealers can contact each other infinitely quickly, any type-2 dealer will immediately trade with a type-0 dealer and then both become type-1 dealers.

Recall the definition of \( B \) and \( Q \) in the proof for Proposition 2. Direct calculation yields:

\[
\lim_{\lambda \to \infty} B = \infty , \tag{B.36}
\]

\[
\lim_{\lambda \to \infty} Q = \frac{2\alpha n (r + \eta)}{(\alpha + \eta)(2(r + \eta) + \alpha)} . \tag{B.37}
\]

Following the argument in the the proof for Proposition 2,

\[
\lim_{\lambda \to \infty} V_2^0 - V_1^1 - V_s^1 \geq 0
\]

if and only if

\[
\theta \geq \theta = \frac{2\rho c(\alpha + \eta)(2(r + \eta) + \alpha)}{2n\alpha(r + \eta)} , \tag{B.38}
\]

suggesting that an equilibrium with an active interdealer market must exist when the asset fundamental is high enough.

Then, consider the equilibrium without an interdealer market. The inflow-outflow balance implies

\[
b = \frac{2n}{\alpha + 2\eta} . \tag{B.39}
\]

It follows that

\[
\beta = 2\alpha b = \frac{4\alpha n}{\alpha + 2\eta} . \tag{B.40}
\]

Similar calculation following the argument in the the proof for Proposition 2 shows that

\[
\lim_{\lambda \to \infty} V_2^0 - V_1^0 - V_s^0 > 0 , \tag{B.41}
\]

regardless of \( \theta \). Thus, an equilibrium without an interdealer market never exists. By continuity, this concludes the proof.
C Strategy representation of the bargaining game

This appendix presents a strategy representation of the staged bargaining game embedded in our dynamic model. Since the game played by agents in our framework is essentially a complete information dynamic bargaining game, it is clear that at any steady-state equilibrium, the sub-game played by two meeting agents $j$ and $k$ can be summarized by the following two-strategy (sub-)game:

$$
\begin{array}{c|cc}
 & \text{Accept} & \text{Reject} \\
\hline
\text{Agent } j & G(\{V_j\}) \cdot G(\{V_j\}) & 0, 0 \\
\text{Reject} & 0, 0 & 0, 0 \\
\end{array}
$$

where $G(\{V_i\})$ denotes the potential gains from trade between the two meeting agents $j$ and $k$, which are in turn determined endogenously by all the agents’ value functions given the steady state of the dynamic game as well as agents’ rational expectations of achieving the corresponding steady state. Intuitively, only when the two meeting agents both choose “Accept”, trade will happen. In turn, only when the potential gains from trade are positive, the two meetings agents will choose “Accept” simultaneously. On the flip side, at least one agent will choose “Reject” when the potential gains from trade are negative, and thus a trade will not happen. When the potential gains from trade are zero, agents may play mixed strategies, where their equilibrium mixed strategies will be determined by the steady-state distribution of the mass of agents as well as their value functions.

Below we focus on the staged bargaining game played by a type-1 dealer and a seller conditional on a meeting. First, note that Propositions 1 and 2 suggest that whether a type-1 dealer is willing to buy from a seller and to effectively increase its inventory is the solely important criterion to determine which type of equilibria is sustainable, given the equilibrium distribution and values of other agents in the corresponding equilibrium. To formulate type-1 dealers’ strategy as well as their willingness to trade requires us to analyze the bargaining (sub-)game between an type-1 dealer and an active seller, when they meet each other:
As suggested by the bargaining (sub-)game above, the type-1 dealer’s inventory decision of whether or not to buy from a seller and increase his inventory holding is solely determined by whether $V_2 - V_1 - V_s$ is positive or negative, given the endogenously determined value functions in the corresponding equilibrium.

We explicitly show how the trading probability $\phi$ between a type-1 dealer and a seller, conditional on a meeting, can be constructed from the staged bargaining (sub-)game. Specifically, a type-1 dealer’s strategy in the above bargaining (sub-)game is $(p, 1-p)$ while a seller’s strategy is $(q, 1-q)$, whenever they meet each other. In this case, a trade between a type-1 dealer and a seller happens with probability $\phi = pq$ when they meet. Notice that the bargaining (sub-)game itself is not sufficient to determine the equilibrium profile $(p, q)$. Rather, the equilibrium probability $pq$ at which a trade between a type-1 dealer and a seller happens will be determined by the condition $V_2 - V_1 - V_s = 0$ as well as other value functions as prescribed by (3.4), (3.5), (3.6), (3.7), and (3.8).

**D Derivation of the mixed-strategy equilibria**

This appendix explicitly derive the mixed-strategy equilibria we consider in the main text. In any candidate mixed-strategy equilibria with $\phi \in (0, 1)$,

Under a mixed-strategy equilibrium, the inflow-outflow balance of the dealer sector implies

$$b = s,$$

$$b\alpha(d_1 + d_2) = n - \eta b,$$  \hspace{1cm} (D.1)

and

$$s\alpha(\phi d_1 + d_0) = n - \eta b.$$  \hspace{1cm} (D.2)

$$s\alpha(\phi d_1 + d_0) = n - \eta b.$$  \hspace{1cm} (D.3)
Conditions (D.1), (D.2), and (D.3) together imply

\[ d_1 + d_2 = \phi d_1 + d_0. \]  

(D.4)

At the same time, the inflow-outflow balance of type-0 dealers suggests

\[ d_0 \left( \beta \frac{s}{b + s} + \lambda d_2 \right) = d_1 \beta \frac{b}{b + s}, \]  

(D.5)

while the inflow-outflow balance of type-2 dealers suggests

\[ d_2 \left( \frac{b}{b + s} + \lambda d_0 \right) = d_1 \beta \frac{\phi s}{b + s}, \]  

(D.6)

First, condition (D.4) implies \( \phi d_1 + d_0 = 1 - d_0 \), that is,

\[ d_1 = \phi^{-1}(1 - 2d_0) \]  

(D.7)

and consequently,

\[ d_2 = 1 - d_0 - d_1 = 1 - d_0 - \phi^{-1}(1 - 2d_0). \]  

(D.8)

Thus, conditions (D.1), (D.5), (D.7), and (D.8) jointly imply

\[ (2 - \phi)\lambda d_0^2 + \left( \left( 1 + \frac{\phi}{2} \right) \beta - (1 - \phi)\lambda \right) d_0 - \frac{\beta}{2} = 0, \]  

(D.9)

which determine \( d_0 \) under a mixed-strategy equilibrium with \( \phi \).

To check (D.9) has meaningful solutions, define

\[ D = \left( \left( 1 + \frac{\phi}{2} \right) \beta - (1 - \phi)\lambda \right)^2 + 2\beta(2 - \phi)\lambda \]

\[ = \left( 1 + \frac{\phi}{2} \right)^2 \beta^2 + (1 - \phi)^2\lambda^2 - 2 \left( 1 + \frac{\phi}{2} \right) \beta(1 - \phi)\lambda + 2\beta(2 - \phi)\lambda, \]
where the sum of the last two terms

$$-2 \left(1 + \frac{\phi}{2}\right) \beta (1 - \phi) \lambda + 2 \beta (2 - \phi) \lambda = 2 \beta \lambda \left(1 - \frac{\phi}{4}\right)^2 + \frac{7}{16} \phi^2 > 0.$$

Therefore, $D > 0$ and

$$d_0 = \frac{(1 - \phi) \lambda - \left(1 + \frac{\phi}{2}\right) \beta + \sqrt{(1 + \frac{\phi}{2})^2 \beta^2 + (1 - \phi)^2 \lambda^2 + 2 \beta \lambda \left(\frac{\phi^2}{2} - \frac{\phi}{2} + 1\right)}}{2(2 - \phi) \lambda}. \quad (D.10)$$

Note that the negative solution is dropped since $d_0 \in [0, 1]$. By (D.7) and (D.8), this fully pins down the distribution of the dealer sector, and further of the customers by (D.5) and (D.6).