How CEO Contracts Address Multitasking: An Empirical Matching Approach

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Abstract

We explore multitasking as an explanation for the use of more than one performance measure in the incentive compensation of CEOs. We use detailed data on compensation contracts to describe how incentive pay for multiple performance measures varies across firms. We then use the compensation data to specify and estimate an equilibrium matching game between chief executives and firms. In the matching game, firms and executives sign linear incentive contracts based on both equity and profit performance under a model of hidden executive effort with multitasking. The estimated structural model is used to explore the counterfactual of requiring firms to base incentive pay on only one performance measure in order to quantify the role of current contract forms in ameliorating the efficiency losses from multitasking.

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1 Introduction

1.1 Lit Review

Our main interest is to study the multidimensional contract between the firm and CEO. These contracts are commonly used in practice and have multiple performance measures. Within each measure are performance targets with caps and floors. Furthermore, we assume the manager and firm both have multidimensional types. Managers differ in their abilities with respect to different measures. For example, some managers are better at increasing earnings and others are increasing stock price. Similarly, there may be some firms where increasing earnings is easier, and others where it is simpler to increase stock price. We allow separate parameters for firm and managerial ability, though in the model they are theoretically equivalent, as they capture the marginal return from the manager’s effort on any given measure.

Our core model is ultimately a two-layered multitasking model. The first layer captures effort that the CEO can exert to increase non-contractible firm value, and the second layer is a vector of actions that the executive can take to increase any one of many performance measures. On top of this, we add types of the firm and the manager, necessary for the matching game.

Our paper combines three separate literatures, and builds on each of them. The first is the contracting theory literature, which has provided much of the foundation for the contracting game in our model. This literature explores accounting and finance models of CEO pay using the LEN (linear contracts, exponential utility, and normal errors) framework popularized by Holmstrom and Milgrom (1987) for its tractability.

The second literature is matching theory, which asks how heterogeneous firms and workers match to one another in a decentralized market setting. Our specific context is
one-to-one matching, because each firm has only one CEO and each CEO can work for only one firm. In the standard transferable utility model, better managers can simply bid for better firms through their type (e.g. Becker 1973; Edmans et al. 2009; Kelso and Crawford 1982). While that intuition is still relevant here, the actual mechanics are more complex because of risk aversion. In a model with non transferable utility (NTU), the risk aversion of the CEO means the two parties cannot seamlessly transfer utility to equilibrate matches, thus making transfers between the firm and manager difficult. Indeed, Legros and Newman (2007), Alaei et al. (2016), and Galichon et al. (2017) consider imperfectly transferable matching, and Elliott (2015), Hatfield and Milgrom (2005), Manea (2016), Fleiner et al. (2017), and Hatfield et al. (2013) consider the contract as a trade in a network of buyers and sellers.

The third literature is the empirical CEO pay literature. We rely on data sources common to this literature, such as Execucomp data on total levels of ex-post executive pay, Incentive Lab for ex-ante CEO contracts, and CRSP data for stock returns. Because our contribution is to structurally test a model, we do not innovate on the measurement of CEO activities but simply use off the shelf measures that are considered standard in this literature, such as in Edmans et al. (2017) and Ales and Sleet (2016).

Our contribution is not necessarily to deepen any one of these three literatures independently but, rather, to show there is value in combining all three in a single analysis.

2 The Model

There are two stages to the model. The first stage is a matching stage where a heterogeneous firm matches with a heterogeneous manager. Each firm hires just one manager and each manager works for only one firm. In the second stage, after the firm and man-
ager have matched, they enter into a contract. The firm writes a contract to control the manager’s moral hazard problem based on unobservable effort.

To solve this, work backwards. In stage two, take a $\gamma$-firm and $\theta$-manager as given, and solve for the optimal effort and optimal contract. This will generate equilibrium payoffs to both the $\gamma$-firm and the $\theta$-manager. These equilibrium payoffs from the second stage will determine the match in the first stage.

2.1 The Contracting Stage

A risk neutral firm employs a risk averse CEO to generate output. The manager exerts effort across multiple dimensions. One dimension increases (unobservable) firm value.

The other dimension increases the value of the manager’s (observable) performance measures, on which his pay is based. As an example, the standard compensation contract for most public company executives today relies on multiple performance measures. The two main ones are accounting based measures, such as earnings or earnings per share, and stock price measures, such as return on stock price. According to the Incentive Lab database, over the horizon from 1998 to 2015, the average executive had $X\%$ of his compensation based on accounting measures. The second most frequent measure was stock price ($X\%$).
The $\theta$-manager exerts effort $e$ to increase firm value at a convex cost of effort $C(e) = \frac{c_0}{2}e^2$, where $c$ is the (unobservable) marginal cost of managerial effort. Firm value is

$$q = \gamma_0 \theta_0 e + \sum_{i=1}^{n} \gamma_i \theta_i a_i + \eta,$$

where $\eta$ is a mean zero disturbance term. Action $a_i$ exerted at cost $\frac{c_i}{2}a_i^2$ increases firm value if $\gamma_i \theta_i > 0$. Write $a = (a_i)_{i=1}^{n}$ for the vector of actions, and $c_0 = (c_i)_{i=0}^{n}$ for the vector of cost parameters.

Firm value is not contractible, but is measured through a vector of performance measures $m = (m_i)$ for $i = 1 \ldots n$, given by

$$m_i = \gamma_0 \theta_0 e + \gamma_i \theta_i a_i + \epsilon_i,$$

where each $\epsilon_i$ is a normally distributed random variable with mean zero and variance $\sigma_i^2$. Each $\epsilon_i, \epsilon_j$ are independent. We assume that firm and manager types equally affect both true firm value as well as the performance measures. In general, this will not be the case. One could model different impacts of ability/productivity on the performance measure or firm value. We have no intuition as to whether ability should have a greater or lesser impact on firm value versus the performance measures. Thus, we simply assume that the impact is the same for both. Observe that the moments of $m_i$ are

$$Em_i = \gamma_0 \theta_0 e + \gamma_i \theta_i a_i \quad \text{and} \quad Var(m_i) = \sigma_i^2.$$

The manager’s action on each measure increases the value of that measure, and may have some impact on firm value as well. For example, if the manager faces a contract
with an EPS (earnings per share) target, then taking an action like smoothing earnings from one period to the next will change his EPS performance measure but may have little effect on firm value. Repurchasing shares or acquiring outside companies may temporarily boost the stock price but have no effect on firm value. Finally, some actions (such as real earnings management) can boost net income and also increase firm value, and so can increase both measures. Ultimately, the shareholders care about increasing effort $e$, even though contracting on firm value is impossible.

The firm pays the manager a contract $(s, b)$ off of the performance measures, where $s$ is a fixed cash salary and $b = (b_i)$ is a bonus vector paying a bonus $b_i$ on each performance measure $m_i$. The wage of the manager is $w = s + \Sigma b_i m_i$, and has first and second moments given by

$$Ew = s + \sum_{i=1}^{n} b_i (\gamma_0 \theta_0 e + \gamma_i \theta_i a_i) \quad \text{and} \quad Var(w) = \sum_{i=1}^{n} b_i \sigma_i^2.$$  \hskip 1cm (4)

Assume the manager has mean-variance preferences with constant risk aversion (CARA) parameter $r$. Therefore the manager’s expected utility is given by $EU = Ew - \frac{r}{2} Var(w)$. Let $\theta = (\theta_0, \ldots, \theta_n, c_0, \ldots, c_n, r) \in \mathbb{R}^{2n+3}$ be the vector capturing all aspects of a CEO heterogeneity, and $\gamma = (\gamma_0, \ldots, \gamma_n) \in \mathbb{R}^{n+1}$ the CEO vector for the firm.

Each performance measure has mean $e + a_i$ and variance $\sigma_i^2$, so increasing managerial effort increases output on average across all measures, but the variance across measures may differ.  \hskip 0.5cm \text{1} Substituting the moments of the wage function into the manager expected

\hskip 0.5cm \text{1}This is a standard LEN model (linear contracts, exponential utility, and normal errors) of Holmstrom and Milgrom (1987). The innovation here is the inclusion of the types of the manager and firm, which will be necessary in the matching stage.
utility:
\[ EU = s + \sum_{i=1}^{n} b_i (\gamma_0 \theta_0 e + \gamma_i \theta_i a_i) - \frac{r}{2} \sum_{i=1}^{n} b_i^2 \sigma_i^2 - \frac{c_0}{2} e^2 - \sum_{i=1}^{n} \frac{c_i}{2} a_i^2 \]  

(5)

The manager maximizes his expected wage, minus the cost of effort, and minus the risk premium from his compensation scheme. The disutility of risk is a cost of the manager because of the manager’s risk aversion. The first order condition gives the incentive constraint:
\[ e = \frac{\gamma_0 \theta_0}{c_0} \sum_{i=1}^{n} b_i \quad \text{and} \quad a_j = \frac{\gamma_j \theta_j b_j}{c_j} \quad \text{for} \quad j = 1 \ldots n \]  

(IC)

Increasing the bonus on any one measure increases managerial effort and action on that measure, and (as expected) increasing the marginal cost of effort or action decreases effort and that action.

The firm imposes the standard participation constraint (PC) that the manager’s expected utility exceed his outside option \( \bar{u} \). The timeline is standard: the firm offers the contract \((s, b)\) to the manager, the manager decides to accept or reject, the manager exerts effort, and finally, output is realized and the firm pays out wages.

The firm maximizes output less wages, given by \( \pi = q - w \). The fixed salary does not affect incentives and therefore the firm will lower the salary until the participation constraint binds. Substituting in the binding participation constraint, the firm’s problem is therefore to pick a bonus vector to maximize expected profits:
\[ \max_{b_i} \gamma_0 \theta_0 e + \sum_{i=1}^{n} \gamma_i \theta_i a_i - \frac{r}{2} \sum_{i=1}^{n} b_i^2 \sigma_i^2 - \frac{c_0}{2} e^2 - \sum_{i=1}^{n} \frac{c_i}{2} a_i^2 - \bar{u}, \]  

(6)

subject to (IC). Because the firm holds the manager to his participation constraint, the firm therefore bears the cost of these risk premia in its optimization problem. Therefore,
the firm maximizes total surplus minus the risk premia from the compensation plan. Solving this problem leads to this first result.

**Proposition 1** The optimal bonus across the performance measures is given by

\[
\hat{b}_j = \frac{\alpha_j^2 c_0}{\alpha_0^2 c_j} + 1 - \sum_{i \neq j} b_i \\
1 + \frac{r c_0 \sigma_j^2}{\alpha_0^2} + \frac{\alpha_j^2 c_0}{\alpha_0^2 c_j}
\]

for \( j = 1 \ldots n \), where \( \alpha_j = \gamma_j \theta_j \).

(7)

This shows that increasing the cost of effort, level of risk aversion, or risk on any one performance measure will decrease the bonus. More importantly the bonuses are jointly determined, so increasing any one bonus \( b_j \), will decrease another bonus \( b_i \). The proof of Proposition 1 shows that we can now in fact rewrite the bonuses in terms of each other. Importantly, as risk on any one measure increases, the bonus for that measure decreases.

**Corollary 1** For \( n=2 \), the optimal bonus is

\[
b_i = \frac{\alpha_i^2 c_0 \left( \alpha_0^2 + r c_0 \sigma_j^2 + \frac{\alpha_j^2 c_0}{c_j} \right) + c_i \alpha_0^2 r c_0 \sigma_j^2}{\left( \alpha_0^2 + r c_0 \sigma_j^2 + \frac{\alpha_j^2 c_0}{c_j} \right) \left( \alpha_0^2 + r c_0 \sigma_i^2 + \frac{\alpha_i^2 c_0}{c_i} \right) c_i - \alpha_0^4 c_i}
\]

(8)

For \( n=3 \), we have

\[
b_i = \frac{L_j U_i + L_k U_i - L_j U_k - L_k U_j + L_j L_k + L_j L_k U_i}{L_i L_j + L_i L_k + L_j L_k + L_i L_j L_k}
\]

(9)

where

\[
L_x = \frac{r c_0 \sigma_x^2}{\alpha_0^2} + \frac{\alpha_x^2 c_0}{\alpha_0^2 c_x} \quad \text{and} \quad U_x = \frac{\alpha_x^2 c_0}{\alpha_0^2 c_x}.
\]

(10)
Corollary 2  Observe that the derivative is

$$
\frac{\partial b_j}{\partial \theta_j} = \frac{2 (1 - b_j) \frac{\alpha_i \gamma_j c_0}{c_j} - \sum_{i \neq j} \alpha_0^2 \frac{\partial b_i}{\partial \theta_j}}{r c_0 \sigma_j^2 + \frac{\alpha_0^2 c_0}{c_j} + \alpha_0^2}
$$

(11)

For $n=2$,

$$
\frac{\partial b_i}{\partial \theta_i} > 0, \quad \frac{\partial b_i}{\partial \gamma_i} > 0.
$$

(12)

$$
\frac{\partial b_i}{\partial \theta_j} < 0, \quad \frac{\partial b_i}{\partial \gamma_j} < 0.
$$

(13)

Increasing the marginal impact of a performance measure on firm value will cause the firm to increase the bonus on that measure, but decrease the bonus on other measures. Recall that the parameters $\theta_i$ and $\gamma_i$ are measuring how sensitive firm value is to the manager’s individual actions on each performance measure. This should be intuitive, because making firm value more sensitive to any one measure provides an incentive effect and makes the managers action more productive. Therefore, the firm optimally increases the bonus on that action and decreases it on other actions.

This optimal contract illustrates that the types of the manager and firm are embedded inside the contract. It is tedious but straightforward to show that the optimal bonus also exhibits complementarity between types, so $\frac{\partial b_i}{\partial \gamma_j \partial \theta_j} > 0$. Again, this follows from the incentive constraint. As more able managers are more productive at better firms, they exert more effort and therefore the firm optimally selects higher bonuses. Indeed, the bonus is itself a reflection of the joint productivity of the match, given by the joint parameter ($\alpha_j = \gamma_j \theta_j$).
2.2 The Matching Stage

In the matching stage, a population of firms matches with a population of managers. Each firm matches with only a single manager because each firm requires only one CEO. Moreover, each firm and manager pair will forecast their equilibrium payoffs, if they match, using the computations in the second stage contracting game. Specifically, a \( \gamma \)-firm matched with a \( \theta \)-manager accrues a surplus that includes an equilibrium profit to the firm and an equilibrium utility for the manager. The matching stage involves imperfectly transferable utility because the firm is risk neutral whereas the manager is risk averse. In other words, one dollar to the firm is worth less than a dollar to the manager because of his risk aversion. Because of this, the firms and managers cannot perfectly resolve conflicts through prices.

Under fully transferable utility, better managers could bid to work at better firms by offering higher transfers. The difference in risk aversion prevents this. However, Legros and Newman (2007) provide conditions for assortative matching under non-transferable utility, such as when one side of the market has different risk preferences than the other, such as here. In addition to the standard complementarity in types condition of Becker (1972), Legros and Newman (2007) requires monotonicity in type of the degree of transferability between agents. We show that our CEO context here satisfies the conditions of Legros and Newman (2007) to guarantee positive assortative matching. To do so, we will first need to establish the matching game and then discuss the core assumptions.

Let \( K \) be the set of firms on one side of the market and \( J \) be the set of managers on the other. The description of a specific economy includes an assignment to types via maps \( \rho : K \to P \), and \( \alpha : J \to A \), where \( P \subset \mathbb{R}^{n+1} \) and \( A \subset \mathbb{R}^{2n+3} \) are compact. To simplify the exposition we assume that \( K \) and \( J \) have the same cardinality.
The object of analytical interest is the utility possibility frontier for each possible pairing. This frontier will be represented by a bounded continuous function \( \phi : P \times A \times \mathbb{R}_+ \to \mathbb{R}_+ \). The function \( \phi(\gamma, \theta, \bar{u}) \) denotes the maximum utility generated by a type \( \gamma \in P \) in a match with a type \( \theta \in A \) who receives utility \( \bar{u} \). Let \( \psi(\theta, \gamma, \cdot) \) be the “quasi-inverse” of \( \phi(\gamma, \theta, \cdot) \): \( \psi(\theta, \gamma, \pi) \) is the maximum payoff to a type-\( \theta \) when his type-\( \gamma \) partner receives payoff \( \pi \).

The model encompasses the case of transferable utility (TU), in which there exists a production function \( h(\theta, \gamma) \) such that \( \phi(\gamma, \theta, \bar{u}) \) can be written as \( h(\gamma, \theta) - \bar{u} \) for \( \bar{u} \in [0, h(\gamma, \theta)] \). In all other cases we have nontransferable utility (NTU).

**Definition 1** Payoffs \((\pi, \bar{u})\) are feasible for \((\gamma, \theta) \in P \times A\) if \(\pi \leq \phi(\gamma, \theta, \bar{u})\) and \(\bar{u} \leq \psi(\theta, \gamma, 0)\).

**Definition 2** An equilibrium specifies a one-to-one matching function \(m : K \to J\) and payoff allocations \(\pi^* : K \to \mathbb{R}_+\) and \(\omega^* : J \to \mathbb{R}_+\) that satisfy the two following conditions.

(i) Feasibility of \((\pi^*, \omega^*)\) with respect to \(m\): for all \(k \in K\), \(\langle \pi^*(k), \omega^*(m(k)) \rangle\) is feasible for \(\langle \gamma(k), \alpha(m(k)) \rangle\).

(ii) Stability of \(m\) with respect to \((\pi^*, \omega^*)\): there do not exist \((k, j) \in K \times J\) and \(\bar{u} > \omega^*(j)\) such that \(\phi(\gamma(k), \alpha(j), \bar{u}) > \pi^*(k)\).

These definitions follow in a straightforward manner from Legros and Newman (2007). Observe that Legros and Newman (2007) has defined an underlying space of individual firms and managers, and then a higher level type space above that; the matching operates on the lower level space. Regardless, Legros and Newman (2007) assumes that the basis of the model is the space of individuals given by the sets \(K\) and \(J\), and the maps
connect the individuals to their types. Multiple individuals may have the same type. The matching function matches between individuals, but the matching correspondence matches between types. The matching function \( m : K \rightarrow J \) generates a matching correspondence on types \( \mathcal{M} : P \rightarrow A \):

\[
\mathcal{M}(\gamma) = \{ \alpha(j), j \in J : \exists k \in K, \rho(k) = \gamma \text{ and } j = m(k) \}.
\] (14)

We say that \( \mathcal{M} \) is stable given \((\pi, \omega)\) if there do not exist \((\gamma, \theta)\) and \(\bar{u} > \omega(\theta)\) such that \(\phi(\gamma, \theta, \bar{u}) > \pi(\gamma)\). The objective here is to generate a representation of the equilibrium profit function of the firm for a given level of utility promised to the manager. This representation will be necessary in the matching game. It is also the key analytical apparatus we will use for computing equilibria. Later, we demonstrate that in the unidimensional case, complementarity in the firm’s profit function is sufficient to generate assortative matching. The key analytical construct is \(\phi(\gamma, \theta, \bar{u})\), the equilibrium profit to the firm from a match of a gamma firm to a theta manager who received utility \(\bar{u}\). This equilibrium profit function is precisely the firm’s payoff from the contracting game. Specifically, it evaluates the firm’s profit at the optimal contract, in which the manager selects his equilibrium effort when offered the contract that maximizes firm profits.

This expression is simply the expected profits evaluated at the optimal contract. Therefore, from (A16), we can write this as

\[
\gamma_0 \theta_0 \hat{c} + \sum_{i=1}^{n} \gamma_i \theta_i \hat{a}_i - \frac{r}{2} \sum_{i=1}^{n} b_i^2 \sigma_i^2 - \frac{c_0}{2} \hat{c}^2 - \sum_{i=1}^{n} \frac{c_i}{2} \hat{a}_i^2 - \bar{u}
\] (15)

In this equation, the contract parameters, \((\hat{s}, \hat{b})\), are at the optimal values, and the effort choices by the manager, \((\hat{e}, \hat{a})\), are the optimal responses to this contract. Observe that this is the result of the binding participation and incentive constraints, which have
been inserted into the firm profit function. The empirical strategy will be to utilize this equilibrium profit function as the key tool in estimating the matching between firms and CEOs in a multidimensional context. Before we proceed, it is useful to examine the unidimensional benchmark, to sharpen intuition.

### 2.3 The Unidimensional Benchmark

To build intuition, it will be useful to consider the benchmark of a unidimensional contract. We assume that the manager can exert effort to increase firm value which is contractible. This model also follows the larger literature surveyed in Lambert (200x) and Prendergast (2002), which considers a single effort decision. The benefit of considering the unidimensional case is that it simplifies the analysis of the model, generates intuition, and provides a closed-form solution for the comparative statics. The unidimensional measure also allows the theory to consider positive assortative matching, i.e., whether high types match with high or low types. Assortative matching has a long tradition in the matching literature. A natural question is whether modelling productivity as in this model generates assortative matching.

Let $\gamma_i = \theta_i = 0$ for $i = 1 \ldots n$. Write $\gamma$ for $\gamma_0$ and $\theta$ for $\theta_0$. Suppose a $\gamma$-firm and $\theta$-manager match with one another. The manager exerts effort at cost of effort $c_0^2 e^2$. The output from a match of a $\gamma$-firm and $\theta$-manager is

$$q = \gamma \theta e + \varepsilon$$

(16)

The uncertainty term $\varepsilon$ is distributed normally with mean 0 and variance $\sigma^2$. The firm offers a linear contract $(s, b)$, consisting of a base salary $s$ and a bonus $b$ on output. Therefore, the manager’s wage is $w = s + bq$, his expected wage is $Ew = s + be$, and the
variance on his wage is $Vw = \hat{b}^2\sigma^2$.

The manager has mean variance preferences with C.A.R.A. parameter $r$. As before, the manager faces an outside option $\bar{u}$, and the firm chooses a salary such that participation binds. Solving this leads to the optimal contract:

**Proposition 2** The optimal contract for a $\gamma$-firm and $\theta$-manager is:

$$\hat{b} = \frac{\gamma^2\theta^2}{\gamma^2\theta^2 + r c_0\sigma^2} \quad \text{and} \quad \hat{s} = \frac{r}{2} \hat{b}^2\sigma^2 + \frac{\hat{b}^2\gamma^2\theta^2}{2c_0} + \bar{u} - \frac{\hat{b}\gamma\theta}{c_0}. \quad (17)$$

This optimal contract illustrates that the types of the manager and firm are embedded inside the contract. It is tedious but straightforward to show that the optimal bonus also exhibits complementarity between types, so $\frac{\partial \hat{b}}{\partial \gamma \partial \theta} > 0$. Again, this follows from the incentive constraint. As more able managers are more productive at better firms, they exert more effort and therefore the firm optimally selects higher bonuses. Indeed, the bonus is itself a reflection of the joint productivity of the match, given by the joint parameter ($\gamma\theta$).

The firm extracts the surplus through the salary. The optimal contract chosen by the firm will then lead to an optimal effort choice by the manager. Substituting this optimal contract and optimal effort into the firm’s profit function will generate the equilibrium profit to a $\gamma$-firm from a match with a $\theta$-manager. This will be the key expression in the matching model. Observe that in the optimal contract, the participation constrain binds, so the manager always receives, in equilibrium, his expected utility. Total surplus is $\hat{\pi} + \bar{u}$, the sum of the payoffs to both parties in equilibrium, the profits that accrue to the firm and the expected utility that accrues to the manager. This total surplus is the match surplus. Because the manager’s expected utility is constant across his type, all of the variation in surplus derives from the variation in the equilibrium profit function.
Thus, the behavior of the equilibrium profit function will determine the conditions for assortative matching in the matching stage.

**Proposition 3** The profit function \( \hat{\pi} \) is supermodular (exhibits complementarity) in \( \gamma \) and \( \theta \left( \frac{\partial^2 \hat{\pi}}{\partial \gamma \partial \theta} > 0 \right) \).

Under fully transferable utility, better managers could bid to work at better firms by offering higher transfers. The difference in risk aversion prevents this. However, Legros and Newman (2007) provide conditions for assortative matching under non-transferable utility, such as when one side of the market has different risk preferences than the other, such as here. In addition to the standard complementarity in types condition of Becker (1972), Legros and Newman (2007) requires monotonicity in type of the degree of transferability between agents. We show that our CEO context here satisfies the conditions of Legros and Newman (2007) to guarantee positive assortative matching.

Recall that the matching function \( m : K \rightarrow J \) generates a matching correspondence on types \( \mathfrak{M} : P \rightrightarrows A : \)

\[
\mathfrak{M} (\gamma) = \{ \alpha (j), j \in J : \exists k \in K, \rho (k) = \gamma \text{ and } j = m (k) \} .
\] (18)

We say that \( \mathfrak{M} \) is stable give \((\pi, \omega)\) if there do not exist \((\gamma, \theta)\) and \( \bar{u} > \omega (\theta) \) such that \( \phi (\gamma, \theta, \bar{u}) > \pi (\gamma) \). An equilibrium displays positive assortative matching (PAM) if \( \gamma > \gamma', \theta \in \mathfrak{M} (\gamma) ; \theta' \in \mathfrak{M} (\gamma') \Rightarrow \theta \geq \theta' \).

**Definition 3** \( \phi \) is type increasing if for all \((\gamma, \theta) \in P \times A, \pi, \bar{u} \in \mathbb{R}_+ \), \( \phi (\gamma, \theta, \bar{u}) \) is nondecreasing in \( \theta \) and \( \psi (\theta, \gamma, \pi) \) is nondecreasing in \( \gamma \).

The definition of positive assortative matching (PAM) relies on the properties of the matching correspondence. To apply this model faithfully in our context, we simply ob-
serve that the equilibrium payoffs $\hat{\pi}$ and $\bar{u}$ are the equilibrium profit of the firm and the equilibrium expected utility of the manager respectively. Importantly, the contracting stage involves a firm acting as a Stackelberg leader (offering a contract to a manager who must accept or reject). Like all contracting games, the party offering the contract has all the bargaining power and so the firm can hold the manager to his expected utility, extracting all of the surplus through his choice of salary.

The consequence for matching is that a $\theta$-manager’s payoff will be $\bar{u}$. Therefore, the surplus is exactly the equilibrium profit of the firm. So, the properties of the equilibrium profit function determine the properties of the match surplus function, which will determine positive assortative matching.

**Corollary 3** (Legros and Newman 2007) Suppose $\phi$ is type increasing and twice continuously differentiable. A sufficient condition for the economy to satisfy PAM is that for all $(\gamma, \theta, \bar{u})$,

$$\phi_{12}(\gamma, \theta, \bar{u}) \leq 0 \quad \text{and} \quad \phi_{13}(\gamma, \theta, \bar{u}) \leq 0. \quad (19)$$

To check the conditions of Legros and Newman (2007), observe that the first requirement is that the equilibrium payoff functions are non-decreasing in the other parties’ type. This means that the firm’s payoff must weakly increase in the type of the manager, and the manager’s payoff must weakly increase in the type of the firm. Because the manager receives his expected utility $\bar{u}$ in equilibrium, his payoff function is (trivially) non-decreasing in the firm’s type. For the firm, the computation is more complex and requires that $\frac{\partial \hat{\pi}}{\partial \gamma \theta} > 0$. Of course, we have already shown this in Proposition 2. Thus, we have all the tools necessary to prove positive assortative matching.

**Proposition 4** The unidimensional benchmark satisfies positive assortative matching.
3 Structural Model

We now turn to the structural model, which will specify the vector of manager attributes (such as risk aversion, cost of effort, and marginal productivities) as a function of the measured worker background characteristics and an unobservable term. Distinguishing between measures in our data and firm heterogeneity will be important when we take the model to data. We can write the manager’s expected utility to include match specific shocks. This captures the reasons, other than money, to form a match. When fitting the model to data, there may be CEOs with strong characteristics that sign unfavorable contracts. The non-wage utility benefit for this type of match can explain circumstances like these.

4 Empirical Specification

5 Data and empirical findings

Our sample consists of the intersection of Execucomp, IncentiveLab, CRSP, and Compustat. We use Execucomp to obtain characteristics of the CEO along with data on the CEO’s equity incentives. We use IncentiveLab to obtain data on the performance measures, targets, and potential payouts of the CEO’s accounting-based incentives. To obtain characteristics of the firm, we use the CRSP and Compustat databases.

Our sample starts in 2006 to account for the increased disclosure requirements of performance targets that started in that year. Our sample ends in 2014. Table 1 presents the number of firms by year. Overall, the sample consists of 2,630 firms-years
ranging from 102 in 2006 to 388 in 2014.2

To measure the CEO’s accounting-based incentives, we collect all performance plans in the IncentiveLab database that use accounting-based performance measures. We then convert all ratio-based measures (e.g., ROA, ROE, ROI...) into levels of accounting performance. Similarly, we convert earnings per share to earnings. We next convert any pre-tax measure (EBT, EBITDA, EBIT) to a post-tax measure using a rate of 35%. The distribution of accounting-based performance measures is presented in Table 2.

For plan we then estimate the slope between the targeted pay and the maximum payout for a one percent increase in the accounting-based performance measure. (If there are multiple accounting-based plans in the year, we weight them by the payout value at the targeted performance level.) Our accounting-based slope therefore represents the dollar change in payout to the CEO for a one percent change in accounting performance.

For equity incentives, we follow Core and Guay (1999) and calculate the dollar change in the CEO’s wealth for a one percent change in stock price. The first two columns Table 3 presents the annual means of the accounting and equity-based slopes. The overall average accounting slope is $59,015 and ranges from $35,385 in 2006 to $77,020 in 2014. The equity slope is larger with an overall mean of $351,083 and a range of $202,655 in 2008 and $530,609 in 2014.

In the last two columns, we present the means of total compensation and the ratio of equity compensation to total compensation. We calculate the CEO’s total compensation as the sum of salary, bonus, stock awards, option awards, non-equity compensation, payouts under long-term incentive plans, and other compensation. Average total compensation ranges from $6,228,932 in 2008 to $9,543,237 in 2014. Equity grants (stock and options) represent, on average, over half of total compensation. There are differences

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2In subsequent drafts, we will hand collect data and hand match the datasets to increase our sample size.
in how accounting slopes, equity slopes, and total compensation vary over our sample period. The mean accounting slope increases monotonically over the sample period. In contrast, equity slopes and total compensation reach their lowest levels in 2008 during the financial crisis. In our further empirical tests, we take the natural log of the slopes along with total compensation.

As shown in Table 4, the vast majority of CEOs in our sample face both accounting and equity slopes in their incentive plans. Zero accounting slopes only appear in 125 out of 2,630 firm years and zero equity slopes only appear in 304 firm years.

In our tests, we include several firm characteristics that can affect the CEO’s compensation package: the logarithm of assets to capture the firm’s size; return on assets to capture accounting performance; stock returns to capture equity performance; stock volatility to capture risk; and the firm’s book-to-market ratio to capture growth opportunities. This set of observables is commonly used in empirical studies of executive compensation and incentives (see, for example, Core and Guay 1999). In addition, we control for the firm’s industry as measured by two-digit SIC. With respect to the CEO, we include the log of tenure (in years) in the empirical tests to examine the relation between tenure and equity and accounting-based slopes.

Table 5 presents the correlations among variables. In contrast with our theoretical predictions, the equity and accounting slopes are positively correlated (0.19), emphasizing the need for a structural model. They are also positively correlated with total compensation (0.48, equity slope; 0.22, accounting slope). Equity slope is positively correlated with size, return on assets, stock returns, and the CEO’s tenure. It is negatively correlated with stock volatility and the firm’s growth opportunities. Accounting slope is positively correlated with size, return on assets, and the CEO’s tenure. It is uncorrelated with stock returns, but negatively correlated with stock volatility and the
firm’s growth opportunities.

Table 6 examines the relation between aspects of the CEO’s compensation and observables. We present ordinary least squares regressions of log accounting slope, log equity slope, and log total compensation on observables. All regressions include year fixed effects. We also present regressions with and without industry fixed effects (based on Two-digit SIC) to examine how much variation is at the industry-level.

The first two regressions use log accounting slope as the dependent variable. Accounting slope are agains positively correlated with size and the CEO’s tenure and negatively correlated with stock volatility. When we include industry fixed effects the adjusted $R^2$ increases from 0.10 to 0.17 suggesting that there is substantial accounting slope variation at the industry level.

The next two columns use log equity slope as the dependent variable. For these regressions, the coefficients on size, return on assets, and tenure are all positive and statistically significant, and the coefficients on stock volatility and the firm’s book-to-market ratio are negative and statistically significant. When we include industry fixed effects, the adjusted $R^2$ again increases but not to the extent as for accounting slope (0.41 to 0.46).

In additional tests, we correlate the residuals of columns (2) and (4) to examine whether accounting slopes and equity slopes remain positively correlated conditional on observables. The correlation between the two residuals is 0.02 with a p-value of 0.28, suggesting that the unconditional correlation is driven by observables.

In the last two columns, the dependent variable is log total compensation. For these regressions, the coefficients on size and return on assets are positive and statistically significant, and the coefficient on the firm’s book-to-market ratio is negative and statistically significant. When we include industry fixed effects, the adjusted $R^2$ again
increases and the increase is similar in magnitude as for equity slope (0.42 to 0.46).

Overall, these regressions show that accounting slope, equity slope, and total compensation have similar relations with observables further emphasizing the benefit of a structural model.

We next examine whether the accounting and equity slopes have predictive power for future performance. Table 7 presents regressions of future return on assets (one year out, two years out, and three years out) on log accounting slope and observables. The coefficient on accounting slope is not statistically significant for these regression, suggesting that the slopes do not contain information about future performance.\textsuperscript{3}

In Table 8, we examine whether equity slopes contain information about future stock returns. As with accounting slopes, the coefficients on equity slope are not significantly different from zero for one year ahead, two years ahead, and three years ahead stock returns. Overall, we find that neither accounting nor equity slopes contain information about future firm performance.

\textsuperscript{3}In additional tests, we replaced the dependent variable the log of income before extraordinary items. For this specification, the coefficients on accounting slope are again not significantly different from zero. Our preferred specification is to use return on assets as the dependent variable to allow for negative values.
This table presents the number firms per year in our sample. Our sample starts in 2006 to reflect the increase in disclosure requirements about accounting incentives that occurred. Our sample is the intersection of the Execucomp, IncentiveLab, CRSP, and Compustat databases.

<table>
<thead>
<tr>
<th>Year</th>
<th>Firms</th>
<th>Percent</th>
<th>Cum. percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>102</td>
<td>3.88</td>
<td>3.88</td>
</tr>
<tr>
<td>2007</td>
<td>208</td>
<td>7.91</td>
<td>11.79</td>
</tr>
<tr>
<td>2008</td>
<td>240</td>
<td>9.13</td>
<td>20.91</td>
</tr>
<tr>
<td>2009</td>
<td>274</td>
<td>10.42</td>
<td>31.33</td>
</tr>
<tr>
<td>2010</td>
<td>323</td>
<td>12.28</td>
<td>43.61</td>
</tr>
<tr>
<td>2011</td>
<td>365</td>
<td>13.88</td>
<td>57.49</td>
</tr>
<tr>
<td>2012</td>
<td>369</td>
<td>14.03</td>
<td>71.52</td>
</tr>
<tr>
<td>2013</td>
<td>361</td>
<td>13.73</td>
<td>85.25</td>
</tr>
<tr>
<td>2014</td>
<td>388</td>
<td>14.75</td>
<td>100.00</td>
</tr>
<tr>
<td>Total</td>
<td>2,630</td>
<td>100.00</td>
<td></td>
</tr>
</tbody>
</table>
Table 2: Performance measures used for accounting slopes

This table presents the accounting performance measures that underlie the CEO’s incentive plans. We take this data from IncentiveLab and use the data to estimate the CEO’s accounting slope, which is the dollar change in the CEO’s incentive plans for a one percent change in accounting performance. Some CEOs have more than one accounting-based incentive plan thereby leading to more accounting performance measures than firm years.

<table>
<thead>
<tr>
<th>Measure</th>
<th>Number of plans</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cashflow</td>
<td>752</td>
</tr>
<tr>
<td>Earning before interest and taxes</td>
<td>189</td>
</tr>
<tr>
<td>Earnings before interest, taxes, depreciation, and amortization</td>
<td>556</td>
</tr>
<tr>
<td>Earnings before taxes</td>
<td>207</td>
</tr>
<tr>
<td>Earnings per share</td>
<td>1,327</td>
</tr>
<tr>
<td>Earnings</td>
<td>389</td>
</tr>
<tr>
<td>Funds from operations</td>
<td>15</td>
</tr>
<tr>
<td>Operating income</td>
<td>844</td>
</tr>
<tr>
<td>Profit margin</td>
<td>191</td>
</tr>
<tr>
<td>Return on assets</td>
<td>102</td>
</tr>
<tr>
<td>Return on equity</td>
<td>93</td>
</tr>
<tr>
<td>Return on investment</td>
<td>26</td>
</tr>
<tr>
<td>Return on invested capital</td>
<td>515</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>5,206</strong></td>
</tr>
</tbody>
</table>
Table 3: Descriptive statistics

This table presents incentive compensation measures used in our study. Accounting slope is the dollar change in the CEO’s incentive plans for a one percent change in accounting performance. Equity slope is the dollar change in the CEO’s wealth for a one percent change in the firm’s stock price. Total compensation is the sum of salary, bonus, stock awards, option awards, non-equity compensation, payouts under long-term incentive plans, and other compensation. Equity ratio is the ratio of the value of equity compensation (i.e., stock awards and option awards) to the value of total compensation.

<table>
<thead>
<tr>
<th>Year</th>
<th>Accounting slope</th>
<th>Equity slope</th>
<th>Total compensation</th>
<th>Equity ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>35,385</td>
<td>530,609</td>
<td>7,541,868</td>
<td>0.52</td>
</tr>
<tr>
<td>2007</td>
<td>40,064</td>
<td>462,610</td>
<td>7,357,539</td>
<td>0.55</td>
</tr>
<tr>
<td>2008</td>
<td>45,987</td>
<td>202,655</td>
<td>6,228,932</td>
<td>0.56</td>
</tr>
<tr>
<td>2009</td>
<td>46,318</td>
<td>286,106</td>
<td>6,921,159</td>
<td>0.50</td>
</tr>
<tr>
<td>2010</td>
<td>50,837</td>
<td>342,763</td>
<td>7,902,609</td>
<td>0.55</td>
</tr>
<tr>
<td>2011</td>
<td>63,761</td>
<td>294,752</td>
<td>8,239,235</td>
<td>0.58</td>
</tr>
<tr>
<td>2012</td>
<td>61,207</td>
<td>330,432</td>
<td>8,308,853</td>
<td>0.60</td>
</tr>
<tr>
<td>2013</td>
<td>75,837</td>
<td>429,603</td>
<td>8,385,077</td>
<td>0.61</td>
</tr>
<tr>
<td>2014</td>
<td>77,020</td>
<td>393,179</td>
<td>9,543,237</td>
<td>0.62</td>
</tr>
<tr>
<td>Total</td>
<td>59,015</td>
<td>351,803</td>
<td>8,002,509</td>
<td>0.57</td>
</tr>
</tbody>
</table>
Table 4: Firms for which the CEO has zero accounting or equity slopes

This table presents the distributions of firm years in which the CEO had a zero accounting slope or a zero equity slope. Accounting slope is the dollar change in the CEO’s incentive plans for a one percent change in accounting performance. Equity slope is the dollar change in the CEO’s wealth for a one percent change in the firm’s stock price.

<table>
<thead>
<tr>
<th>Year</th>
<th>Accounting-based</th>
<th></th>
<th></th>
<th>Equity-based</th>
<th></th>
<th></th>
<th>Total</th>
<th>Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yes</td>
<td>No</td>
<td></td>
<td>Yes</td>
<td>No</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2006</td>
<td>94</td>
<td>8</td>
<td></td>
<td>99</td>
<td>3</td>
<td></td>
<td>102</td>
<td></td>
</tr>
<tr>
<td>2007</td>
<td>200</td>
<td>8</td>
<td></td>
<td>195</td>
<td>13</td>
<td></td>
<td>208</td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td>228</td>
<td>12</td>
<td></td>
<td>224</td>
<td>16</td>
<td></td>
<td>240</td>
<td></td>
</tr>
<tr>
<td>2009</td>
<td>258</td>
<td>16</td>
<td></td>
<td>257</td>
<td>17</td>
<td></td>
<td>274</td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td>313</td>
<td>10</td>
<td></td>
<td>299</td>
<td>24</td>
<td></td>
<td>323</td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>348</td>
<td>17</td>
<td></td>
<td>329</td>
<td>36</td>
<td></td>
<td>365</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>355</td>
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<td>2013</td>
<td>342</td>
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<td>51</td>
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<td>361</td>
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<td>2014</td>
<td>367</td>
<td>21</td>
<td></td>
<td>282</td>
<td>106</td>
<td></td>
<td>388</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>2,505</td>
<td>125</td>
<td></td>
<td>2,326</td>
<td>304</td>
<td></td>
<td>2,630</td>
<td></td>
</tr>
</tbody>
</table>
Table 5: Correlations

This table presents Pearson correlations of the measures used in this study. Accounting slope is the dollar change in the CEO’s incentive plans for a one percent change in accounting performance. Equity slope is the dollar change in the CEO’s wealth for a one percent change in the firm’s stock price. Total compensation is the sum of salary, bonus, stock awards, option awards, non-equity compensation, payouts under long-term incentive plans, and other compensation. Total assets is firm’s total assets in the year of compensation. Return on assets is the ratio of income before extraordinary items to total assets. Annual stock return in the annual return for the year in the year of compensation. Stock volatility is the standard deviation of daily returns in the year of compensation. Book-to-market is the ratio of the firm’s book value of equity to its market value of equity. Tenure is the number of years that the CEO has been in office.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td></td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>(2)</td>
<td>0.19</td>
<td></td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3)</td>
<td>0.48</td>
<td>0.22</td>
<td></td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>(4)</td>
<td>0.37</td>
<td>0.24</td>
<td>0.61</td>
<td></td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5)</td>
<td>0.29</td>
<td>0.09</td>
<td>0.14</td>
<td>−0.07</td>
<td></td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6)</td>
<td>0.11</td>
<td>−0.01</td>
<td>0.01</td>
<td>−0.02</td>
<td>0.01</td>
<td></td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(7)</td>
<td>−0.39</td>
<td>−0.19</td>
<td>−0.28</td>
<td>−0.25</td>
<td>−0.30</td>
<td>0.01</td>
<td></td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>(8)</td>
<td>−0.33</td>
<td>−0.07</td>
<td>−0.10</td>
<td>0.07</td>
<td>−0.36</td>
<td>−0.18</td>
<td>0.27</td>
<td></td>
<td>1.00</td>
</tr>
<tr>
<td>(9)</td>
<td>0.26</td>
<td>0.10</td>
<td>0.05</td>
<td>−0.00</td>
<td>0.03</td>
<td>−0.03</td>
<td>−0.04</td>
<td>0.01</td>
<td>1.00</td>
</tr>
</tbody>
</table>
This table presents ordinary least squares regressions of the CEO’s log accounting slope, log equity slope, and log total compensation on observables. Accounting slope is the dollar change in the CEO’s incentive plans for a one percent change in accounting performance. Equity slope is the dollar change in the CEO’s wealth for a one percent change in the firm’s stock price. Total compensation is the sum of salary, bonus, stock awards, option awards, non-equity compensation, payouts under long-term incentive plans, and other compensation. Total assets is firm’s total assets in the year of compensation. Return on assets is the ratio of income before extraordinary items to total assets. Annual stock return in the annual return for the year in the year of compensation. Stock volatility is the standard deviation of daily returns in the year of compensation. Book-to-market is the ratio of the firm’s book value of equity to its market value of equity. Tenure is the number of years that the CEO has been in office. Industry fixed effects are based on two-digit SIC. Standard errors are clustered at the firm-level.

<table>
<thead>
<tr>
<th></th>
<th>log(Accounting slope)</th>
<th>log(Equity slope)</th>
<th>log(Total compen.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(Total assets)</td>
<td>0.27***</td>
<td>0.40***</td>
<td>0.34***</td>
</tr>
<tr>
<td></td>
<td>(6.33)</td>
<td>(10.27)</td>
<td>(23.76)</td>
</tr>
<tr>
<td>Return on assets</td>
<td>0.75</td>
<td>2.69***</td>
<td>1.11***</td>
</tr>
<tr>
<td></td>
<td>(1.26)</td>
<td>(5.29)</td>
<td>(5.22)</td>
</tr>
<tr>
<td>Annual stock return</td>
<td>0.00</td>
<td>0.23</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(-0.35)</td>
<td>(1.93)</td>
<td>(0.53)</td>
</tr>
<tr>
<td>Stock volatility</td>
<td>-20.45***</td>
<td>-27.26***</td>
<td>-0.40</td>
</tr>
<tr>
<td></td>
<td>(-3.87)</td>
<td>(-4.98)</td>
<td>(-0.24)</td>
</tr>
<tr>
<td>log(Book-to-market)</td>
<td>-0.08</td>
<td>-0.42***</td>
<td>-0.07***</td>
</tr>
<tr>
<td></td>
<td>(-1.32)</td>
<td>(-6.64)</td>
<td>(-3.35)</td>
</tr>
<tr>
<td>log(Tenure)</td>
<td>0.20***</td>
<td>0.46***</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>(3.71)</td>
<td>(9.05)</td>
<td>(1.71)</td>
</tr>
<tr>
<td>Constant</td>
<td>7.06***</td>
<td>8.16***</td>
<td>12.48***</td>
</tr>
<tr>
<td></td>
<td>(15.32)</td>
<td>(20.10)</td>
<td>(87.34)</td>
</tr>
</tbody>
</table>

Year fixed effects: Yes, Industry fixed effects: No, Adj. R²: 0.10
Observations: 2,413

* p<0.05, ** p<0.01, *** p<0.001
This table presents ordinary least squares regressions of the firm’s future return on equity on the CEO’s log accounting slope and other observables. Accounting slope is the dollar change in the CEO’s incentive plans for a one percent change in accounting performance. Total assets is firm’s total assets in the year of compensation. Return on assets is the ratio of income before extraordinary items to total assets. Annual stock return is the annual return for the year in the year of compensation. Stock volatility is the standard deviation of daily returns in the year of compensation. Book-to-market is the ratio of the firm’s book value of equity to its market value of equity. Tenure is the number of years that the CEO has been in office. The sample is from 2006 through 2014. Industry fixed effects are based on two-digit SIC. Standard errors are clustered at the firm-level.

<table>
<thead>
<tr>
<th></th>
<th>One year</th>
<th>Two years</th>
<th>Three years</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(Accounting slope)</td>
<td>0.00</td>
<td>−0.00</td>
<td>−0.00</td>
</tr>
<tr>
<td></td>
<td>(0.24)</td>
<td>(−0.12)</td>
<td>(−0.02)</td>
</tr>
<tr>
<td>log(Total assets)</td>
<td>−0.00*</td>
<td>−0.00</td>
<td>−0.00</td>
</tr>
<tr>
<td></td>
<td>(−2.53)</td>
<td>(−1.24)</td>
<td>(−0.96)</td>
</tr>
<tr>
<td>Return on assets</td>
<td>0.38***</td>
<td>0.30***</td>
<td>0.25***</td>
</tr>
<tr>
<td></td>
<td>(9.84)</td>
<td>(8.71)</td>
<td>(4.66)</td>
</tr>
<tr>
<td>Annual stock return</td>
<td>0.02*</td>
<td>0.02*</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>(2.41)</td>
<td>(2.10)</td>
<td>(1.75)</td>
</tr>
<tr>
<td>Stock volatility</td>
<td>−1.61***</td>
<td>−0.94**</td>
<td>−1.54***</td>
</tr>
<tr>
<td></td>
<td>(−4.63)</td>
<td>(−3.07)</td>
<td>(−3.79)</td>
</tr>
<tr>
<td>log(Book-to-market)</td>
<td>−0.03***</td>
<td>−0.02***</td>
<td>−0.02***</td>
</tr>
<tr>
<td></td>
<td>(−6.32)</td>
<td>(−5.86)</td>
<td>(−4.33)</td>
</tr>
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<td>log(Tenure)</td>
<td>−0.00</td>
<td>0.00</td>
<td>−0.00</td>
</tr>
<tr>
<td></td>
<td>(−1.52)</td>
<td>(0.62)</td>
<td>(−0.11)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.10***</td>
<td>0.07**</td>
<td>0.06*</td>
</tr>
<tr>
<td></td>
<td>(4.38)</td>
<td>(2.66)</td>
<td>(2.28)</td>
</tr>
</tbody>
</table>

Year fixed effects: Yes, Yes, Yes
Industry fixed effects: Yes, Yes, Yes
Adj. R\(^2\): 0.360, 0.290, 0.224
Observations: 2,317, 1,841, 2,240

* p<0.05, ** p<0.01, *** p<0.001
Table 8: Relation between equity slopes and stock returns

This table presents ordinary least squares regressions of the firm’s future stock returns on the CEO’s log equity slope and other observables. Equity slope is the dollar change in the CEO’s wealth for a one percent change in the firm’s stock price. Total assets is firm’s total assets in the year of compensation. Return on assets is the ratio of income before extraordinary items to total assets. Annual stock return in the annual return for the year in the year of compensation. Stock volatility is the standard deviation of daily returns in the year of compensation. Book-to-market is the ratio of the firm’s book value of equity to its market value of equity. Tenure is the number of years that the CEO has been in office. The sample is from 2006 through 2014. Industry fixed effects are based on two-digit SIC. Standard errors are clustered at the firm-level.

<table>
<thead>
<tr>
<th></th>
<th>One year</th>
<th>Two years</th>
<th>Three years</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(Equity slope)</td>
<td>-0.01</td>
<td>0.01</td>
<td>-0.00</td>
</tr>
<tr>
<td></td>
<td>(-0.99)</td>
<td>(0.99)</td>
<td>(-0.62)</td>
</tr>
<tr>
<td>log(Total assets)</td>
<td>-0.01</td>
<td>-0.02**</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(-0.85)</td>
<td>(-3.04)</td>
<td>(-1.38)</td>
</tr>
<tr>
<td>Return on assets</td>
<td>0.11</td>
<td>-0.22</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td>(0.58)</td>
<td>(-1.64)</td>
<td>(-0.16)</td>
</tr>
<tr>
<td>Annual stock return</td>
<td>-0.07*</td>
<td>-0.02</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(-2.28)</td>
<td>(-1.50)</td>
<td>(1.58)</td>
</tr>
<tr>
<td>Stock volatility</td>
<td>8.70</td>
<td>-2.78</td>
<td>-3.89**</td>
</tr>
<tr>
<td></td>
<td>(1.96)</td>
<td>(-1.93)</td>
<td>(-2.99)</td>
</tr>
<tr>
<td>log(Book-to-market)</td>
<td>-0.02</td>
<td>-0.01</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>(-0.81)</td>
<td>(-0.98)</td>
<td>(0.99)</td>
</tr>
<tr>
<td>log(Tenure)</td>
<td>0.01</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(0.97)</td>
<td>(-0.94)</td>
<td>(-1.12)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.20</td>
<td>-0.31**</td>
<td>0.72***</td>
</tr>
<tr>
<td></td>
<td>(1.44)</td>
<td>(-3.22)</td>
<td>(5.56)</td>
</tr>
<tr>
<td>Year fixed effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Industry fixed effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Adj. R^2</td>
<td>0.287</td>
<td>0.284</td>
<td>0.202</td>
</tr>
<tr>
<td>Observations</td>
<td>2,231</td>
<td>2,180</td>
<td>1,863</td>
</tr>
</tbody>
</table>

* p<0.05, ** p<0.01, *** p<0.001
6 Conclusion

Our chief innovation is to combine matching and multitasking in CEO pay. These two literatures have largely proceeded independently of one another. Multitasking has developed as a subset of contract theory, where agents allocate effort across multiple tasks and the principal induces an optimal contract. The matching literature has largely been preoccupied with unidimensional matching and positive associative matching. In reality, executive contracts are complex and serve the dual role of matching a CEO to firms, as well as controlling moral hazard problems after the two parties sign a contract.

Further research in this area will develop even more detailed and structured models of executive behavior. For example, stock option grants are a major form of payment for CEOs. An explicit model of these grants, their vesting schedules, and their staggered deployment over time could provide further insight into CEO incentives. Second, CEO performance targets often have caps and floors, where the payout is linear in some intermediate regions but flat in the tails. Analytical models of these caps and floors rarely generate close-form solutions. Yet, they may provide more realism when computationally simulated. Finally, real-life executives are unlikely to exhibit constant absolute risk aversion. Decreasing absolute risk aversion hinders closed-form solutions but could generate more realistic results when simulated on a computer.

On the empirical side, data on CEO contracts should improve. These improvements can lead to better estimates of our model parameters. The Incentive Lab data only covers the top 750 firms of market capitalization. Any data beyond this limit requires hand collection, but could generate a more complete picture of the CEO’s actual choice problem. In particular, relative performance evaluation is a large portion of CEO pay. With the current limitations in the dataset, only some of the peer firms are in Incentive
Lab’s database. With a more comprehensive database that covers all firms (for example, the S&P 500), it would be possible to conduct a more comprehensive analysis of relative performance evaluation that this paper omits. We remain excited about opportunities to deploy structural methods to understand the incentives of the most important agent in the firm, the CEO.
7 References
8 Appendix: Proofs

Proof of Proposition 1: The firm’s problem is to solve (A16). Observe that from the incentive constraint,

\[
\frac{\partial a_j}{\partial b_j} = \frac{\gamma_j \theta_j}{c_j} \quad \text{and} \quad \frac{\partial e}{\partial b_j} = \frac{\gamma_0 \theta_0}{c_0}.
\]  

(A1)

Therefore, differentiating the firm’s expected profit \( E\pi \) with respect to bonus \( b_j \), generates the first order condition

\[
\frac{\partial E\pi}{\partial e} \frac{\partial e}{\partial b_j} + \sum_{i=1}^{n} \frac{\partial E\pi}{\partial a_i} \frac{\partial a_i}{\partial b_i} + \frac{\partial E\pi}{\partial b_j} = 0
\]  

(A2)

Let \( \alpha_j = \gamma_j \theta_j \). Evaluating,

\[
(\alpha_0 - ce) \frac{\alpha_0}{c_0} + (\alpha_j - c_j a_j) \frac{\alpha_j}{c_j} - rb_j \sigma_j^2 = 0.
\]  

(A3)

Differentiating again with respect to \( b_j \),

\[
-\frac{\alpha_0^2}{c_0} \frac{\alpha_j^2}{c_j} - r \sigma_j^2 < 0
\]  

(A4)

So the second order conditions hold. Inserting the incentive constraint into (A3):

\[
\left( \alpha_0 - \alpha_0 \sum_{i=1}^{n} b_i \right) \alpha_0 + (\alpha_j - \alpha_j b_j) \frac{\alpha_j c_0}{c_j} - rc b_j \sigma_j^2 = 0
\]  

(A5)

Re-arranging this generates

\[
b_j = \frac{\frac{\alpha_0^2 c_0}{\alpha_j^2 c_j} + 1 - \sum_{i \neq j} b_i}{1 + \frac{r c \sigma_j^2}{\alpha_0^2} + \frac{\alpha_j^2 c_0}{\alpha_j^2 c_j}}.
\]  

(A6)
Proof of Corollary 1: From (7),

$$b_i = \frac{\alpha^2 c_0}{\alpha_0^2 c_i} + 1 - b_j \quad \text{and} \quad b_j = \frac{\alpha^2 c_0}{\alpha_0^2 c_j} + 1 - b_i \quad \text{for} \quad i \neq j. \quad (A7)$$

From the second equation above for $b_j$,

$$1 - b_j = \frac{rc_0 \sigma_i^2 + b_i}{1 + \frac{rc_0 \sigma_i^2}{\alpha_0^2} + \frac{\alpha^2 c_0}{\alpha_0^2 c_j}} \quad (A8)$$

And inserting into the first equation from (A34) for $b_i$,

$$b_i \left(1 + \frac{rc_0 \sigma_i^2}{\alpha_0^2} + \frac{\alpha^2 c_0}{\alpha_0^2 c_i}\right) = \frac{\alpha^2 c_0}{\alpha_0^2 c_i} + \frac{rc_0 \sigma_i^2}{\alpha_0^2} + b_i \quad (A9)$$

Solving for $b_i$ gives the result for $n = 2$. Similarly for $n = 3$,

$$b_j = \frac{\alpha^2 c_0}{\alpha_0^2 c_j} + 1 - \sum_{i \neq j} b_i = \frac{U_j + 1 - \sum_{i \neq j} b_i}{1 + L_j} \quad (A10)$$

$$U_j = \frac{\alpha^2 c_0}{\alpha_0^2 c_j} \quad L_j = \frac{rc_0 \sigma_j^2}{\alpha_0^2} + \frac{\alpha^2 c_0}{\alpha_0^2 c_j} \quad (A11)$$
when $n = 3$,

\[
\begin{align*}
    b_i &= \frac{U_i + 1 - (b_j + b_k)}{1 + L_i} \quad \text{(A12)} \\
    b_k &= \frac{U_k + 1 - (b_i + b_j)}{1 + L_k} \quad \text{(A13)} \\
    b_j &= \frac{U_j + 1 - (b_i + b_k)}{1 + L_j} \quad \text{(A14)}
\end{align*}
\]

Then we have,

\[
\begin{align*}
    (1 + L_i) b_i &= U_i + 1 - (b_j + b_k) \quad \text{(A15)} \\
    (1 + L_j) b_j &= U_j + 1 - (b_i + b_k) \quad \text{(A16)}
\end{align*}
\]

Take difference, (A15) – (A16), we have,

\[
\begin{align*}
    (1 + L_i) b_i - (1 + L_j) b_j &= U_i + 1 - (b_j + b_k) - [U_j + 1 - (b_i + b_k)] \\
    b_i + L_i b_i - b_j - L_j b_j &= U_i - b_j - b_k - U_j + b_i + b_k \quad \text{(A17)} \\
    L_i b_i - L_j b_j &= U_i - U_j \quad \text{(A18)} \\
    b_j &= \frac{L_i b_i - U_i + U_j}{L_j} \quad \text{(A19)}
\end{align*}
\]

Similarly,

\[
\begin{align*}
    b_k &= \frac{L_i b_i - U_i + U_k}{L_k} \quad \text{(A20)}
\end{align*}
\]

Plug $b_j$ and $b_k$ into $b_i$,

\[
\begin{align*}
    b_i &= \frac{U_i + 1 - \left(\frac{L_i b_i - U_i + U_j}{L_j} + \frac{L_i b_i - U_i + U_k}{L_k}\right)}{1 + L_i} \quad \text{(A22)}
\end{align*}
\]
\[(1 + L_i) b_i = U_i + 1 - \frac{L_i b_i}{L_j} + \frac{U_i}{L_j} - \frac{U_i}{L_k} - \frac{L_i b_i + U_i}{L_k} - \frac{U_i}{L_k} \tag{A23}\]

\[ (L_i L_j L_k + L_j L_k) b_i = L_j L_k U_i + L_j L_k - L_i L_k b_i - L_i L_j b_i + U_i L_k - U_j L_k + U_i L_j - U_k L_j \tag{A24}\]

\[ (L_i L_j L_k + L_i L_k + L_i L_j) b_i = L_j L_k U_i + U_i L_k + U_i L_j - U_j L_k - U_i L_j - U_k L_j \tag{A25}\]

\[ b_i = \frac{L_j L_k U_i + L_j L_k + U_i L_k + U_i L_j - U_j L_k - U_i L_j - U_k L_j}{L_i L_j L_k + L_j L_k + L_i L_k + L_i L_j} \tag{A26}\]

\[\blacksquare\]

**Proof of Corollary 2:** Observe that we have the following derivative

\[ \frac{\partial \alpha_j}{\partial \theta_j} = \gamma_j \tag{A27}\]

Differentiate equation 5 on both sides with respect to \( \theta_j \)

\[
\left(-\alpha_0 \sum_{i=1}^{n} \frac{\partial b_i}{\partial \theta_j}\right)\alpha_0 + \left(\gamma_j - \left(\alpha_j \frac{\partial b_i}{\partial \theta_j} + b_j \gamma_j\right)\right) \frac{\alpha_j c_0}{c_j} + \left(\alpha_j - \alpha_j b_j\right) \frac{\gamma_j c_0}{c_j} - r c_0 \sigma_j^2 \frac{\partial b_j}{\partial \theta_j} = 0 \tag{A28}\]

Let \( b_{ij} = \frac{\partial b_i}{\partial \theta_j} \). We can rearrange this equation as

\[
- \left(\sum_{i,j=1 \atop i \neq j}^{n} b_{ij} + b_{jj}\right) \alpha_0^2 + \left(\gamma_j - \alpha_j b_{jj} - b_j \gamma_j\right) \frac{\alpha_j c_0}{c_j} + \left(1 - b_j\right) \frac{\alpha_j \gamma_j c_0}{c_j} - r c_0 \sigma_j^2 b_{jj} = 0 \tag{A29}\]

Simplifying further yields.

\[
- \sum_{i,j=1 \atop i \neq j}^{n} b_{ij} \alpha_0^2 + \left(1 - b_j\right) \frac{\alpha_j \gamma_j c_0}{c_j} + \left(1 - b_j\right) \frac{\alpha_j \gamma_j c_0}{c_j} = b_{jj} \left(r c_0 \sigma_j^2 + \frac{\alpha_j^2 c_0}{c_j} + \alpha_0^2\right) \tag{A30}\]

36
Rearranging further gives

\[
b_{jj} = \frac{2(1 - b_j) \frac{\alpha_j \gamma_j c_0}{c_j} - \sum_{i,j=1, i\neq j}^{n} b_{ij}\alpha_0^2}{rc_0\sigma_j^2 + \frac{\alpha_j^2 c_0}{c_j} + \alpha_0^2}
\]  \quad (A31)

Observe that if \( b_{ij} < 0 \) each i and j we must have \( b_{jj} > 0 \). For \( n = 2 \), let

\[
A_j = (\gamma_0 \theta_0)^2 + r c_0 \sigma_j^2 + \frac{(\gamma_j \theta_j)^2 c_0}{c_j}
\]  \quad (A32)

\[
A_i = (\gamma_0 \theta_0)^2 + r c_0 \sigma_i^2 + \frac{(\gamma_i \theta_i)^2 c_0}{c_i}
\]  \quad (A33)

Then

\[
b_i = \frac{(\gamma_i \theta_i)^2 c_0 A_j + c_i (\gamma_0 \theta_0)^2 r c_0 \sigma_j^2}{c_i A_i A_j - (\gamma_0 \theta_0)^4 c_i}
\]  \quad (A34)

\[
\frac{\partial A_i}{\partial \theta_i} = 2\gamma_i^2 \theta_i c_0 \frac{c_i}{c_i}
\]  \quad (A35)

\[
\frac{\partial A_i}{\partial \theta_j} = 0
\]  \quad (A36)

\[
\frac{\partial b_i}{\partial \theta_i} = \frac{2\gamma_i^2 \theta_i c_0 A_j [c_i A_i A_j - (\gamma_0 \theta_0)^4 c_i] - 2\gamma_i^2 \theta_i c_0 A_j [(\gamma_i \theta_i)^2 c_0 A_j + c_i (\gamma_0 \theta_0)^2 r c_0 \sigma_j^2]}{[c_i A_i A_j - (\gamma_0 \theta_0)^4 c_i]^2}
\]  \quad (A37)

\[
= \frac{2\gamma_i^2 \theta_i c_0 A_j}{[c_i A_i A_j - (\gamma_0 \theta_0)^4 c_i]^2} [c_i A_i A_j - (\gamma_0 \theta_0)^4 c_i - (\gamma_i \theta_i)^2 c_0 A_j - c_i (\gamma_0 \theta_0)^2 r c_0 \sigma_j^2]
\]  \quad (A38)

WTS \( \frac{\partial b_i}{\partial \theta_i} > 0 \). Just need to show

\[
A_i A_j - (\gamma_0 \theta_0)^4 \frac{c_0}{c_i} A_j - (\gamma_0 \theta_0)^2 r c_0 \sigma_j^2 > 0
\]  \quad (A39)
Now,

\[
A_i A_j - \left( \gamma_0 \theta_0 \right)^4 - \left( \gamma_i \theta_i \right)^2 c_0 \frac{A_j}{A_i} - \left( \gamma_0 \theta_0 \right)^2 r c_0 \sigma_j^2 > 0
\]

\[
= A_i A_j - \left( \gamma_0 \theta_0 \right)^2 \left[ \left( \gamma_i \theta_i \right)^2 c_0 + r c_0 \sigma_j^2 \right] + \left( \gamma_0 \theta_0 \right)^2 (\gamma_j \theta_j)^2 c_0 - \left( \gamma_i \theta_i \right)^2 c_0 \frac{A_j}{A_i}
\]  

(A40)

\[
= \left[ \left( \gamma_0 \theta_0 \right)^2 + r c_0 \sigma_i^2 \right] A_j - \left( \gamma_0 \theta_0 \right)^2 A_j - \left( \gamma_i \theta_i \right)^2 c_0 \frac{A_j}{A_i} + \left( \gamma_0 \theta_0 \right)^2 (\gamma_j \theta_j)^2 c_0 \frac{A_j}{c_j}
\]

(A41)

\[
= r c_0 \sigma_i^2 A_j + \frac{\left( \gamma_0 \theta_0 \right)^2 (\gamma_j \theta_j)^2 c_0}{c_j} > 0
\]

(A42)

So \( \frac{\partial b_i}{\partial \theta_i} > 0 \). Similarly, \( \frac{\partial b_i}{\partial \theta_j} > 0 \). Now,

\[
\frac{\partial b_i}{\partial \theta_j} = \left( \gamma_i \theta_i \right)^2 c_0 \frac{2 \gamma_j \theta_j c_0}{c_j} A_j - \left( \gamma_0 \theta_0 \right)^4 c_0 A_j - c_i \frac{2 \gamma_j \theta_j c_0}{c_j} A_j \left[ \left( \gamma_i \theta_i \right)^2 c_0 A_j + c_i \left( \gamma_0 \theta_0 \right)^2 r c_0 \sigma_j^2 \right]
\]

\[
\frac{1}{\left[ c_i A_j - \left( \gamma_0 \theta_0 \right)^4 c_i \right]^2}
\]

(A43)

\[
= \left( \gamma_i \theta_i \right)^2 c_i A_j - \left( \gamma_i \theta_i \right)^2 c_0 \frac{2 \gamma_j \theta_j c_0}{c_j} \left( \gamma_0 \theta_0 \right)^4 c_i - c_i A_j \frac{2 \gamma_j \theta_j c_0}{c_j} \left( \gamma_0 \theta_0 \right)^4 c_i - c_j \frac{2 \gamma_j \theta_j c_0}{c_j} \frac{\left( \gamma_i \theta_i \right)^2 A_j}{c_i}
\]

\[
= \left( \gamma_i \theta_i \right)^2 c_i A_j - \left( \gamma_0 \theta_0 \right)^4 c_i \left[ c_i A_j - \left( \gamma_0 \theta_0 \right)^4 c_i \right]^2
\]

(A44)

\[
= -\left( \gamma_i \theta_i \right)^2 c_0 \frac{c_j}{c_i} \left[ \left( \gamma_i \theta_i \right)^2 \left( \gamma_0 \theta_0 \right)^4 c_i A_j \left( \gamma_0 \theta_0 \right)^2 r c_0 \sigma_j^2 \right] < 0
\]

(A45)
So $\frac{\partial b_i}{\partial \theta_j} < 0$. Similarly, $\frac{\partial b_i}{\partial \gamma_j} < 0$.

**Proof of Proposition 2:** The manager’s expected utility is

$$EU = Ew - \frac{r}{2} V w - C(e).$$  \hfill (A46)

Given a contract $(s, b)$, the manager selects effort to maximize his expected utility. The solution to this generates the incentive constraint

$$\hat{e} = \frac{b \gamma \theta e}{c_0}$$  \hfill (IC)

Binding participation constraint guarantees

$$Ew = \frac{r}{2} V w + C(e) + \bar{u}.$$  \hfill (PC)

Since the firm chooses salary to guarantee participation, it selects the bonus to maximize expected profits, or

$$\max_b \gamma \theta e - \frac{r}{2} V w - C(e) - \bar{u} \text{ subject to (IC)}$$  \hfill (A47)

Substituting (IC) into (A47) and differentiating,

$$\gamma \theta \frac{\partial e}{\partial b} - 2rb \sigma^2 - C'(\hat{e}) \frac{\partial e}{\partial b} = 0.$$  \hfill (A48)
Now $\frac{\partial e}{\partial b} = \frac{\gamma \theta}{c_0}$ and $C'(\hat{e}) = \frac{b^2 \gamma^2 \theta^2}{2c_0}$, so

$$\hat{b} = \frac{\gamma^2 \theta^2}{\gamma^2 \theta^2 + rc_0 \sigma^2}.$$ (A49)

From binding (PC),

$$\hat{s} = \frac{r}{2} \hat{b}^2 \sigma^2 + C'(\hat{e}) + \bar{u} - b \gamma \theta \hat{e}$$ (A50)

Substituting in (IC) gives $\hat{s}$ in (17).

\[\blacksquare\]

**Proof of Proposition 3**: We can write the equilibrium profit equation as

$$\hat{\pi}(\gamma, \theta, \bar{u}) = \gamma \theta \hat{e} - E\hat{w}$$ (A51)

Substituting in the (IC) and (PC) generates profit as a function of the optimal contract:

$$\hat{\pi} = \gamma^2 \theta^2 \frac{\hat{b}}{c_0} - \frac{r}{2} \hat{b}^2 \sigma^2 - \frac{\hat{b}^2 \gamma^2 \theta^2}{2c_0} - \bar{u}$$ (A52)

For analytical convenience, multiply both sides by $2c$, so

$$2c_0 \hat{\pi} = 2\gamma^2 \theta^2 \hat{b} - r c_0 \hat{b}^2 \sigma^2 - \hat{b}^2 \gamma^2 \theta^2 - 2c_0 \bar{u}$$ (A53)

Observe that we have the following identities, which follow from the optimal contract given above:

$$z \equiv \gamma^2 \theta^2 + rc_0 \sigma^2$$ (A54)
\[ \hat{b} = \frac{\gamma^2 \theta^2}{z} \quad \text{and} \quad 1 - \hat{b} = \frac{r c_0 \sigma^2}{z} \]  
\hspace{10cm} (A55)

\[ b' = \frac{\partial \hat{b}}{\partial \gamma} = \frac{2 r c_0 \sigma^2 \gamma^2 \theta^2}{z^2} \quad \text{and} \quad \frac{\partial \hat{b}}{\partial \theta} = \frac{2 r c_0 \sigma^2 \gamma^2 \theta}{z^2} = \frac{\partial \hat{b}}{\partial \gamma} \frac{\gamma}{\partial \theta} \]  
\hspace{10cm} (A56)

\[ b'' = \frac{\partial^2 \hat{b}}{\partial \gamma^2} = \frac{4 \gamma \theta^2 c_0^2 \sigma^4}{z^3} = \frac{4 r^2 c_0^2 \sigma^4 \gamma \theta}{z^3} \quad \text{and} \quad (b')^2 = \frac{4 r^2 c_0^2 \sigma^4 \gamma^2 \theta^4}{z^4} = \frac{b''}{z^2} \gamma \theta^3 \]  
\hspace{10cm} (A57)

\[ \hat{b}b'' = \frac{4 r c_0 \sigma^2 \gamma^3 \theta^3}{z^4} \quad \text{and} \quad \hat{b} \frac{b'}{b} = \frac{\gamma^4 \theta^4}{z^2} \]  
\hspace{10cm} (A58)

\[ \hat{b}b' = \frac{2 r c_0 \sigma^2 \gamma^3 \theta^4}{z^3} \quad \text{and} \quad \hat{b} \frac{\partial b}{\partial \theta} = \frac{2 r c_0 \sigma^2 \gamma^4 \theta^3}{z^3} \]  
\hspace{10cm} (A59)

\[ C(\hat{e}) = \frac{\hat{b}^2 \gamma^2 \theta^2}{2 c_0} \]  
\hspace{10cm} (A60)

To save notation, write \( b \) for \( \hat{b} \). Differentiate \( \hat{\pi} \) with respect to \( \gamma \) to get

\[ 2 c_0 \frac{\partial \hat{\pi}}{\partial \gamma} = 2 \gamma^2 \theta^2 b' + 4 \gamma \theta^2 b - 2 r c_0 b \sigma^2 b' - b^2 2 \gamma \theta^2 - \gamma^2 \theta^2 2 b b' \]  
\hspace{10cm} (A61)

\[ = 2 \gamma^2 \theta^2 b' + 4 \gamma \theta^2 b - 2 b b' \left[ r c_0 \sigma^2 + \gamma^2 \theta^2 \right] - b^2 2 \gamma \theta^2 \]  
\hspace{10cm} (A62)

Differentiate again with respect to \( \theta \) to get

\[ 2 \hat{c}_0 \pi'' = 2 \gamma^2 \theta^2 b'' + b' 4 \gamma^2 \theta + 4 \gamma \theta^2 b' + b b' 8 \gamma \theta - 2 b b' \left[ \gamma^2 \partial \theta \right] \]  
\[ - 2 \left[ r c_0 \sigma^2 + \gamma^2 \theta^2 \right] \left[ b b'' + b' b \right] - 4 b^2 \gamma \theta - 4 \gamma \theta^2 b'b' \]  
\hspace{10cm} (A63)
Now,

\[ 2c_0\pi'' = 2\gamma^2\theta^2b'' + 4\gamma^2\theta b' + 4\gamma^2\theta b' + 8\gamma\theta b - 4\gamma^2\theta bb' - 2 \left[ r c_0\sigma^2 + \gamma^2\theta^2 \right] \left( bb'' + (b')^2 \frac{\gamma}{\theta} \right) 
- 4\gamma\theta b'^2 - 4\gamma^2\theta bb' \]

\[ = 2\gamma^2\theta^2b'' + 8\gamma^2\theta b' + 8\gamma\theta b - 8\gamma^2\theta bb' - 2z \left[ bb'' + (b')^2 \frac{\gamma}{\theta} \right] - 4\gamma\theta b^2 \]  
\[ \text{(A64)} \]

Thus,

\[ 2c_0\pi'' = 8\gamma^2\theta^2b'' + 8\gamma\theta b - 8\gamma^2\theta bb' - 2zbb'' - 4\gamma\theta b^2 > 0 \]  
\[ \text{(A65)} \]

iff \[ 16r c_0\sigma^2 \gamma^3 \theta^3 + \frac{8\gamma^3 \theta^3}{z^2} > \frac{16r c_0\sigma^2 \gamma^5 \theta^5}{z^3} + \frac{8r^2 c_0^2 \sigma^4 \gamma^3 \theta^3}{z^3} + \frac{4\gamma^5 \theta^5}{z^2} \]
\[ \text{(A67)} \]

iff \[ 16r c_0\sigma^2 \gamma^3 \theta^3 z + 8\gamma^3 \theta^3 z^2 > 16r c_0\sigma^2 \gamma^5 \theta^5 + 8r^2 c_0^2 \sigma^4 \gamma^3 \theta^3 + 4\gamma^5 \theta^5 z \]  
\[ \text{(A68)} \]

Now

\[ z = \gamma^2\theta^2 + r c_0\sigma^2, \quad z^2 = \gamma^4\theta^4 + r^2 c_0^2 \sigma^4 + 2\gamma^2\theta^2 r c_0\sigma^2, \quad \text{so} \quad \pi'' > 0 \]  
\[ \text{(A69)} \]

iff \[ 16r^2 c_0^2 \sigma^4 \gamma^3 \theta^3 + 8\gamma^7 \theta^7 + 16\gamma^5 \theta^5 r c_0\sigma^2 > 4\gamma^7 \theta^7 + 4\gamma^5 \theta^5 r c_0\sigma^2 \]  
\[ \text{(A70)} \]

iff \[ 16r^2 c_0^2 \sigma^4 \gamma^3 \theta^3 + 4\gamma^7 \theta^7 + 12\gamma^5 \theta^5 r c_0\sigma^2 > 0, \]  
\[ \text{(A71)} \]

which is true since all terms are positive.
Proof of Proposition 4: It is sufficient to check the conditions of Legros and Newman (2007). First, we need to show the equilibrium payoffs of both parties are non-decreasing in the other party’s type. Now,

\[
2c_0 \frac{\partial \pi}{\partial \theta} = 2\gamma \left[ \theta^2 \frac{\partial b}{\partial \theta} + b2\theta \right] - 2rc_0\sigma^2b \frac{\partial b}{\partial \theta} - \gamma^2 \left[ 2b^2\theta + 2\theta^2b \frac{\partial b}{\partial \theta} \right] \tag{A72}
\]

\[
= \gamma^2 \left[ 2\theta^2 \frac{\partial b}{\partial \theta} + 2\theta^2b \frac{\partial b}{\partial \theta} - 2b^2\theta \right] - 2rc_0\sigma^2b \frac{\partial b}{\partial \theta} \tag{A73}
\]

\[
= \gamma^2 \left[ 2\theta^2 \frac{\partial b}{\partial \theta} (1 - b) + \frac{4\gamma^2\theta^3}{z^3} - \frac{2\gamma^4\theta^5}{z^2} \right] - 2rc_0\sigma^2b \frac{\partial b}{\partial \theta} \tag{A74}
\]

\[
= \frac{\gamma^2}{z^3} \left[ 4r^2c_0^2 \sigma^4 \gamma^2 \theta^3 + 4\gamma^2 \theta^3 z^2 - 2\gamma^4 \theta^5 z \right] - \frac{4r^2c_0^2 \sigma^4 \gamma^4 \theta^3}{z^3} \tag{A75}
\]

\[
= \frac{\gamma^2}{z^2} \left( 4\gamma^2 \theta^3 z - 2\gamma^4 \theta^5 \right). \tag{A76}
\]

Plugging in \( z = \gamma^2 \theta^2 + rc_0\sigma^2 \),

\[
2c_0 \frac{\partial \pi}{\partial \theta} = \frac{\gamma^2}{z^2} \left( 4\gamma^4 \theta^5 + 4\gamma^4 \theta^3 rc_0\sigma^2 - 2\gamma^4 \theta^5 \right) \tag{A77}
\]

\[
= \frac{\gamma^2}{z^2} \left( 2\gamma^4 \theta^5 + 4\gamma^2 \theta^3 rc_0\sigma^2 \right) > 0, \tag{A78}
\]

since all terms are positive. So \( \pi(\gamma, \theta, \bar{u}) \) is nondecreasing in \( \theta \). For any profit level \( \pi \in \mathbb{R} \),

\[
\psi(\theta, \gamma, \pi) = \bar{E}U = \bar{u} \tag{A79}
\]

from binding (PC), and so is (trivially) nondecreasing in \( \gamma \).

Next, observe that the expected utility is always \( \bar{u} \), so the cross partial \( \frac{\partial \bar{u}}{\partial \gamma \theta} = 0 \). The last condition to check is that \( \frac{\partial \pi}{\partial \gamma \theta} > 0 \), but Proposition 2 shows this. Corollary 3 of Legros and Newman (2007) proves the claim. Ultimately, the transferable utility representation of our matching function is the reason assortative matching
occurs in this model. Indeed, Legros and Newman (2007) demonstrates that for the linear contracts, exponential utility, and the normal errors model a transferable utility representation always exists. Thus, the TU representation will generate assortative matching. Granted, the specific representation of the manager and firm types is new here, as types affect productivity, rather than cost of effort or risk aversion. However, the underlying intuition is the same. Because of the transferable utility representation, complementarity/super-modularity in types is sufficient to generate assortative matching, much like in the standard model.