Life and Growth

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“Certain events quite within the realm of possibility, such as a major asteroid collision, global bioterrorism, abrupt global warming — even certain lab accidents— could have unimaginably terrible consequences up to and including the extinction of the human race... I am not a Green, an alarmist, an apocalyptic visionary, a catastrophist, a Chicken Little, a Luddite, an anticapitalist, or even a pessimist. But... I have come to believe that what I shall be calling the ‘catastrophic risks’ are real and growing...”
Should we switch on the Large Hadron Collider?

- Physicists have considered the possibility that colliding particles together at energies not seen since the Big Bang could cause a major disaster (mini black hole, strangelets).

- Conclude that the probability is tiny.

- But how large does it have to be before we would not take the risk?

- As economic growth makes us richer, should our decision change?
Growth involves costs as well as benefits

- **Benefits | Costs**
  - Nuclear power | Nuclear holocaust
  - Biotechnology | Bioterror
  - Nanotechnology | Nano-weapons
  - Coal power | Global warming
  - Internal combustion engine | Pollution
  - Radium, thalidomide, lead paint, asbestos

- Technologies (new pharmaceuticals, medical equipment, airbags, pollution scrubbers) can also save lives

**How do considerations of life and death affect the theory of economic growth and technological change?**
The “Russian Roulette” Model

New ideas raise consumption,
but a tiny probability of a disaster...
Simple Model

- Single agent born at the start of each period
- Endowed with stock of ideas $\Rightarrow$ consumption $c$, utility $u(c)$
- Only decision: to research or not to research
  - Research:
    - With (high) probability $1 - \pi$, get a new idea that raises consumption by growth rate $\bar{g}$.
    - But, with small probability $\pi$, disaster kills the agent.
  - Stop: Consumption stays at $c$, no disaster.
• Expected utility for the two options:

\[
\begin{align*}
U^{\text{Research}} &= (1 - \pi)u(c_1) + \pi \cdot 0 = (1 - \pi)u(c_1), \quad c_1 = c(1 + \bar{g}) \\
U^{\text{Stop}} &= u(c)
\end{align*}
\]

• Taking a first-order Taylor expansion around \(u(c)\), agent undertakes research if

\[
(1 - \pi)u'(c)\bar{g}c > \pi u(c)
\]

• Rearranging:

\[
\bar{g} > \frac{\pi}{1 - \pi} \cdot \frac{u(c)}{u'(c)c}
\]
Three Cases

- Consider CRRA utility:

\[ u(c) = \bar{u} + \frac{c^{1-\gamma}}{1-\gamma} \]

\( \bar{u} \) is a key parameter

- Three cases:
  - \( 0 < \gamma < 1 \)
  - \( \gamma > 1 \)
  - log utility (\( \gamma = 1 \))
Case 1: $0 < \gamma < 1$

\[
\bar{g} > \frac{\pi}{1 - \pi} \cdot \frac{u(c)}{u'(c)c}
\]

- The value of life relative to consumption:

\[
\frac{u(c)}{u'(c)c} = \bar{u}c^{\gamma-1} + \frac{1}{1 - \gamma}.
\]

- $\bar{u}$ not important, so set $\bar{u} = 0$

\[
\Rightarrow \frac{u(c)}{u'(c)c} = 1/(1 - \gamma)
\]

Exponential growth, with rare disasters.
Case 2: $\gamma > 1$

- Notice that we’ve implicitly normalized the utility from death to be zero (in writing the lifetime expected utility function)
  - So flow utility must be positive for consumer to prefer life

- But $\gamma > 1$ implies $u(c)$ negative if $\bar{u} = 0$:

$$u(c) = \bar{u} + \frac{c^{1-\gamma}}{1-\gamma}$$

Example: $\gamma = 2$ implies $u(c) = -1/c$.

- Therefore $\bar{u} > 0$ is required in this case.
Case 2: $\gamma > 1$ (continued)

\[
\bar{g} > \frac{\pi}{1 - \pi} \cdot \frac{u(c)}{u'(c)c}
\]

- With $\gamma > 1$, the value of life rises relative to consumption!

\[
\frac{u(c)}{u'(c)c} = \bar{u}c^{\gamma - 1} + \frac{1}{1 - \gamma}.
\]

Eventually, people are rich enough that the risk to life of Russian Roulette is too great and growth ceases.
The Research Decision when \( \gamma > 1 \)

\[
U_{research} = (1 - \pi)u(c(1 + \bar{g}))
\]

\[
U_{stop} = u(c)
\]

Continue research \hspace{2cm} Stop research

\( \bar{u} \)

(1 - \pi)\bar{u}

Consumption, \( c \)

0

\( c^* \)
Case 3: log utility ($\gamma = 1$)

- Flow utility is unbounded in this case
- But the value of life relative to consumption still rises

$$\frac{u(c)}{u'(c)c} = \bar{u} + \log c$$

- So growth eventually ceases in this case as well.
Microfoundations in a Growth Model

Hall and Jones (2007) meet Acemoglu (Direction TechChg)
Production

\[ C_t = \left( \int_0^A x_{it}^{1/\alpha} di \right)^\alpha, \quad H_t = \left( \int_0^B z_{it}^{1/\alpha} di \right)^\alpha \]

Ideas

\[ \dot{A}_t = \bar{a} S_{at} A_t^\phi, \quad \dot{B}_t = \bar{b} S_{bt} B_t^\phi \]

RC (labor)

\[ L_{ct} + L_{ht} \leq L_t, \quad L_{ct} \equiv \int_0^A x_{it} di, \quad L_{ht} \equiv \int_0^B z_{it} di \]

RC (scientists, pop)

\[ S_{at} + S_{bt} \leq S_t, \quad S_t + L_t \leq N_t \]

Mortality

\[ \delta_t = h_t^{-\beta}, \quad h_t \equiv H_t / N_t \]

Utility

\[ U = \int_0^\infty e^{-\rho t} u(c_t) M_t dt, \quad \dot{M}_t = -\delta_t M_t \]

Flow util.

\[ u(c_t) = \bar{u} + \frac{c_t^{1-\gamma}}{1-\gamma}, \quad c_t \equiv C_t / N_t \]

Pop growth

\[ \dot{N}_t = \bar{n} N_t \]
Allocating Resources

- 14 unknowns, 11 equations (not counting utility)
  - $C_t, H_t, c_t, h_t, A_t, B_t, x_{it}, z_{it}, S_{at}, S_{bt}, S_t, L_t, N_t, \delta_t$

- Three allocative decisions to be made
  - (s<sub>t</sub>) Scientists: $S_{at} = s_t S_t$
  - (l<sub>t</sub>) Workers: $L_{ct} = \ell_t L_t$
  - (s<sub>t</sub>) People: $S_t = \sigma_t N_t$

- Rule of Thumb allocation: $s_t = \bar{s}$, $\ell_t = \bar{\ell}$, and $\sigma_t = \bar{\sigma}$
PROPOSITION 1: As $t \to \infty$, there exists an asymptotic balanced growth path such that growth is given by

\[
g_A^* = g_B^* = \frac{\lambda \bar{n}}{1 - \phi}
\]

\[
\delta^* = 0
\]

\[
g_c^* = g_h^* = \alpha g_A^* = \alpha g_B^* = \bar{g} = \frac{\alpha \lambda \bar{n}}{1 - \phi}.
\]
The Optimal Allocation

\[
\max_{\{s_t, \ell_t, \sigma_t\}} \quad U = \int_0^\infty M_t u(c_t)e^{-\rho t} \, dt \quad \text{s.t.}
\]

\[
c_t = A_t^\alpha \ell_t (1 - \sigma_t)
\]

\[
h_t = B_t^\alpha (1 - \ell_t)(1 - \sigma_t)
\]

\[
\dot{A}_t = \bar{a}s_t^\lambda \sigma_t^\lambda N_t^\lambda A_t^\phi
\]

\[
\dot{B}_t = \bar{b}(1 - s_t)^\lambda \sigma_t^\lambda N_t^\lambda B_t^\phi
\]

\[
\dot{M}_t = -\delta_t M_t, \quad \delta_t = h_t^{-\beta}
\]
Hamiltonian

- In solving, useful to define

\[ H = M_t u(c_t) + p_{at} \tilde{a} s_t^\lambda \sigma_t^\lambda N_t^\lambda A_t^\phi + p_{bt} \tilde{b} (1 - s_t)^\lambda \sigma_t^\lambda N_t^\lambda B_t^\phi \]

\[-v_t \delta_t M_t\]

- Co-state variables:
  - \( p_{at} \): shadow value of a consumption idea
  - \( p_{bt} \): shadow value of a life-saving idea
  - \( v_t \): shadow value of an extra person
Optimal Growth with $\gamma > 1 + \beta$

**PROPOSITION 2:** Assume $\gamma > 1 + \beta$. There is an asymptotic balanced growth path such that $\ell_t$ and $s_t$ both fall to zero at constant exponential rates, and

\[
g_s^* = g^* = \frac{-\bar{g}(\gamma - 1 - \beta)}{1 + (\gamma - 1)(1 + \frac{\alpha \lambda}{1 - \phi})} < 0
\]

\[
g^*_A = \frac{\lambda(\bar{n} + g^*_s)}{1 - \phi}, \quad g^*_B = \frac{\lambda \bar{n}}{1 - \phi} > g^*_A
\]

\[
g^*_\delta = -\beta \bar{g}, \quad g^*_h = \bar{g}
\]

\[
g^*_c = \alpha g^*_A + g^*_\ell = \bar{g} \cdot \frac{1 + \beta(1 + \frac{\alpha \lambda}{1 - \phi})}{1 + (\gamma - 1)(1 + \frac{\alpha \lambda}{1 - \phi})}
\]
Intuition

\[ \dot{A}_t = \bar{a} s_t^\lambda \sigma_t^\lambda N_t^\lambda A_\phi^\lambda \quad \text{and} \quad \dot{B}_t = \bar{b} (1 - s_t)^\lambda \sigma_t^\lambda N_t^\lambda B_\phi^\lambda. \]

• \(1 - s_t \to 1\), but \(s_t\) falls exponentially, slowing growth in \(A_t\).

• Why? The FOC for allocating \(\ell_t\) is

\[ \frac{1 - \ell_t}{\ell_t} = \beta \frac{\delta_t v_t}{u'(c_t)c_t} = \delta_t \tilde{v}_t \]

where \(v_t\) is the shadow value of a life, from the Hamiltonian.

• Numerator is extra lives that can be saved, denominator is extra consumption that can be produced.

• Race!
Optimal Growth with $\gamma < 1 + \beta$

**Proposition 3:** Assume $1 < \gamma < 1 + \beta$. There is an asymptotic balanced growth path such that $\tilde{\ell}_t \equiv 1 - \ell_t$ and $\tilde{s}_t \equiv 1 - s_t$ both fall to zero at constant exponential rates, and

$$g^*_A = \frac{\lambda \bar{n}}{1 - \phi}, \quad g^*_B = \frac{\lambda(\bar{n} + g^*_s)}{1 - \phi} < g^*_A$$

$$g^*_c = \bar{g}, \quad g^*_\delta = -\beta g^*_h.$$ 

$$g^*_s = g^*_\ell = \frac{-\bar{g}(\beta + 1 - \gamma)}{1 + \beta(1 + \frac{\alpha\lambda}{1 - \phi})} < 0$$

$$g^*_h = g^*_c \cdot \left(\frac{1 + (\gamma - 1)(1 + \frac{\alpha\lambda}{1 - \phi})}{1 + \beta(1 + \frac{\alpha\lambda}{1 - \phi})}\right) < g^*_c.$$
Optimal Growth with $\gamma = 1 + \beta$

**Proposition 4:** Assume $1 < \gamma = 1 + \beta$. There is an asymptotic balanced growth path such that $\ell_t$ and $s_t$ settle down to constants strictly between 0 and 1, and

$$g_A^* = g_B^* = \frac{\lambda \bar{n}}{1 - \phi}$$

$$g_c^* = g_h^* = \bar{g}, \quad g_{\delta}^* = -\beta \bar{g}.$$
Empirical Evidence
Empirical Evidence

- $\gamma - 1$ versus $\beta$
  - $\gamma > 1$ is the “normal” case
  - Evidence from health “production functions” suggests $\beta$ is relatively small

- Trends in R&D: Toward health

- Quantifying the “growth slowdown”
Evidence on $\beta$  (Recall: $\delta_t = h_t^{-\beta}$)

- Plausible upper bound compares trends in mortality to trends in health spending
  - Attributes all decline in mortality to real health spending
  - Minimal quality adjustment reinforces “upper bound” view for $\beta$

- Numbers for 1960 – 2007
  - Age-adjusted mortality rates fell at 1.2% per year
  - CPI-deflated health spending grew at 4.1% per year

$$\Rightarrow \beta_{\text{upper bound}} \approx \frac{1.2}{4.1} \approx 0.3$$

- Hall and Jones (2007) more careful analysis along these lines finds age-specific estimates of between .10 and .25.
Evidence on $\gamma$

- Risk aversion evidence suggests $\gamma > 1$
  - Asset pricing (Lucas 1994), Labor supply (Chetty 2006)

- Intertemporal substitution elasticity $1/\gamma$
  - Traditional view is EIS<1 $\Rightarrow \gamma > 1$ (Hall 1988)
  - Other recent work finds some evidence for EIS>1 $\Rightarrow \gamma < 1$ (Vissing-Jorgensen and Attanasio 2003, Gruber 2006)

- Mixed evidence.
Evidence on the Value of Life

- Same force as in Hall and Jones (2007) on health spending
  - Consumption runs into sharply diminishing returns: $u'(c)$
  - While life becomes increasingly valuable: $u(c)$

- Evidence on value of life?
  - Nearly all is cross sectional

- Other evidence? Safety standards?
The Changing Composition of U.S. R&D Spending

Health Share of R&D (percent)

- CMMS + NSF Recent
- NIH estimates
- CMMS + NSF IRIS
- Non-commercial health research (CMMS)

Year

The Changing Composition of OECD R&D Spending

Health Share of R&D (percent)

Year

0 2 4 6 8 10 12 14 16 18 20 22

U.S. OECD
Fraction of Patents for Medical Eq. or Pharma

Source: Jeffrey Clemens

Life and Growth – p.31/36
Health and Consumption

Real quantity per person (1950=100)

- Health (pce) 4.67%
- Health (official) 3.10%
- Non-health consumption (pce, official) 1.84%

Year


Life and Growth – p.33/36
The Growth Drag: Ratio of $g_c$ to $g_h$

<table>
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A Future Slowdown?

• Calibration to past growth suggests \( \frac{\alpha \lambda}{1 - \phi} < 2 \), so that \( \bar{g} < 2\% \).
  - Therefore growth in \( h \) must slowdown significantly from its \( 4 + \% \) rate.
  - And \( g_c \approx \frac{1}{2} g_h < 1\% \) suggests a slowdown of consumption growth as well.

• Intuition: \( h \) has been growing much faster than its steady state rate because of the rising share of research devoted to life-saving technologies.
Conclusions

- Including “life and death” considerations in growth models can have first order consequences.

- For a large class of preferences, safety is a luxury good.
  - Diminishing returns to consumption on any given day means that additional days of life become increasingly valuable.
  - R&D may tilt toward life-saving technologies and away from standard consumption goods.
  - Consumption growth may be substantially slower than what is feasible, possibly even slowing all the way to zero.