Estimating Airlines’ Dynamic Price Competition

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Abstract

Dynamic pricing has become a common practice in many industries, but its effect under competition is uncertain due to the potential for the Prisoner’s dilemma. The paper studies profit and welfare implications of competitive dynamic pricing in the context of the airline industry. The paper develops a structural dynamic oligopoly model where firms compete in selling limited capacities when facing demand fluctuations. The supply and demand are jointly estimated using a unique daily-level data on airfares and capacity utilization. The identification leverages a natural experiment of carrier exit. The estimates show that air travel demand exhibits a large degree of temporal heterogeneity and stochastic variability. The counterfactuals show that the ability to perform dynamic pricing increases total welfare. In particular, (i) price discrimination (charging late-arriving consumers higher prices) softens competition in the late market and increases profits substantially and (ii) revenue management (pricing on remaining capacities) intensifies competition and does not increase profits.

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1. Introduction

Dynamic pricing has become a common practice in many industries. These include transportation, hospitality, online retailing, event tickets, entertainment, tourism and so on. In these markets, the distribution of consumers may change over time—consumers entering the markets at different points in time may have different preferences. Firms can use time-dependent prices to price discriminate against consumers. Another common feature among these industries is the presence of capacity constraint—supply is limited whereas demand may fluctuate over time. Thus, firms may reprice in real time based on current availabilities and expected future demand.

Many of these industries are competitive. Not surprisingly, more and more dynamic pricing solutions are designed to help firms compete in prices in real time.\(^1\) Under competition, the effect of dynamic pricing on firms' profits is uncertain. A monopoly firm is likely to benefit from the ability to cut prices in "low states"—(i) when consumers are more price sensitive; (ii) when bad past sales cause excessive idle capacity.\(^2\) In a competitive setting, the ability to cut prices in these "low states" may potentially intensify price competition and lower firms’ profits as in the prisoner’s dilemma.\(^3\)

This paper attempts to provide a better understanding of profit and welfare implications of dynamic pricing in the competitive setting. The consequences of dynamic pricing practices have clear managerial and regulatory implications. These consequences are theoretically ambiguous or unknown, so I approach this question empirically. My analysis focuses on the airline industry. The industry is an oligopoly with sophisticated dynamic pricing firms. Airlines pioneered the widely known dynamic pricing technique called revenue management that re-optimizes based on real-time capacities. In addition, airlines price the same seat more than two times higher one week before departure as opposed to five weeks before departure. This increasing price pattern may be a result of price discrimination.\(^4\) Hence, I estimate a rich demand and supply model that encompasses these two forces. Subsequently, I use the estimated

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\(^1\)For instance, IBM Dynamic Pricing “helps online retailers respond in real time to changes in competitive prices, product demand and market conditions [...] offering automatically recommends an online retailer’s optimal response.” See Chen et al. (2016) for Amazon sellers’ use of dynamic pricing algorithms.

\(^2\)With patient consumers, a durable-good monopoly may be worse off with the ability of dynamic pricing (Coase (1972)). Related to the current paper, Bagnoli et al. (1989) and Board and Pycia (2014) show, respectively, Coarse conjecture fails with finite consumer pool or the presence of outside option. The current paper abstract away from patient consumers.


\(^4\)The increasing price path is in contrast with pure capacity-based pricing, in which the value of a seat should decrease as the departure date approaches, because of the diminishing option value to sell it in the future (see Theorem 1 in Gallego and Van Ryzin (1994)).
model to perform counterfactual simulations quantifying the competitive effects of these dynamic pricing techniques on airlines’ profits and consumer welfare.

To answer my research question, I manually collect real-time airfares and capacity utilization from the websites of several major airlines. The information on real-time prices and capacity utilization enables me to separate the pricing driven by revenue management from that driven by price discrimination. To obtain exogenous price variations, I leverage a natural experiment of carrier exit, which resulted in a change in the market structure. To control for the demand trend before and after the exit, I supply the market (a treated market) with a control market. The control market is a duopoly market that had the closest parallel trend in average price with the treated market before the exit event. Consequently, I implement a structural version of difference-in-differences research design with four conditions (control/treatment route × before/after the exit). The research design allows the exit decision to be correlated with the aggregate demand trend that was common with the control route.

The exit can be a strategic decision based on the firm’s belief on route-specific long-run demand trend. To address this, I only look at flights departing 15 days before and after the exit event. If the route-specific demand trend is smooth in time, it will vanish when the time interval gets small in a similar way as the regression discontinuity design. Conveniently, the exit was scheduled one year before it took place. If the firm was unable to predict demand shocks within a 30-day window one year beforehand, the exact timing of the exit was “as good as random.”

I justify my research design with empirical evidence. I show that the average prices before the exit event followed a common trend for the control and the treated market. Next, I demonstrate that after the exit, the average price increased significantly in the treated market but changed little in the control market. I show that the treatment effect was heterogeneous across different numbers of days to departure. In particular, monopolization increases the middle market prices the most. The reduced form analysis also demonstrates rich intertemporal variation in prices, quantities and capacity utilization.

The estimation results show that the demand elasticity varies significantly across time as the departure date approaches. For a typical flight, own demand elasticity is 1.2-1.4, measured seven weeks before departure. The elasticity decreases to approximately 1 when measured one week before departure. Industry demand elasticity also decreases over time from 1 to 0.6. The estimates suggest that late-arriving consumers have higher valuation for the tickets and substitute less to the outside option than early-arriving consumers. Thus, firms raise prices to price discriminate against late-arriving consumers. Cross-price elasticity ranges from 0.2-0.5. The level of the cross-price elasticity indicates that flights by different carriers departing on the same day are imperfect substitutes. Firms are vertically differentiated – Alaska has larger market
power among high willingness-to-pay (high type) consumers, whereas JetBlue dominates the segment of low willingness-to-pay (low type) consumers. Lastly, the model prediction is consistent with the reduced form evidence that the monopolization increases the middle-market prices the most. In the early (late) market, the firms have the common incentive to price low (high) because of low (high) consumer valuations. Thus the exit of the opponent has relatively small effect on the remaining firm's price. However, in the middle market, firms' incentives are misaligned. The firm that has large low-type consumer segment wants to keep low prices. This creates an externality that forces the opponent to price lower than it desires. When the exit removed this externality, the opponent's price increased dramatically.

I use these estimates to perform a series of counterfactual simulations to understand how airlines' dynamic price competition affect consumers' surplus and firms' profits. I consider three different pricing regimes: (i) full dynamic pricing when firms can price on the number of days to departure as well as own and the opponent's remaining capacities; (ii) fixed-path pricing when firms price only on the number of days to departure and do not adjust their prices based on any demand fluctuation; (iii) constant pricing when firms can only charge constant prices.

I first study the effect of revenue management. I compare the case of full dynamic pricing with the case of fixed-path pricing. I find that when airlines are unable to adjust prices based on stochastic demand, surprisingly, the industry profit does not decrease. However, total consumer surplus decreases by as much as 14.4%. These findings on firms' profits and consumers' surplus are driven by two factors: (i) inefficient allocation of seats to consumers and (ii) reduced competition. The variance of demand is large, thus, without capacity throttling, the probability of selling out “too soon” is high. This possibility lowers firms' incentives to increase sales by competing in early market prices. As a result, early market prices are higher, and early-arriving consumers are worse off. Moreover, because firms cannot throttle capacity, some flights do sell out “too soon,” thus, the supply to high type consumers is under-allocated. Consequently, late-arriving customers (high types) are worse off as well.

I then investigate the effect of price discrimination. I compare the case of constant pricing with the case of fixed-path pricing. I find that price discrimination decreases consumers' surplus by 9.4%. It increases industry profits by 9%. The result suggests that price discrimination is aligned with airlines' collective incentives to raise prices in the late market. It softens competition and helps airlines extract more consumer surplus. All in all, the ability to price discriminate has a much larger impact on profits than the ability to throttle capacity. This has significant managerial implications, since both scientific and popular literature on dynamic pricing tend to emphasize capacity throttling over price discrimination. My results suggest that increasing emphasis on price discrimination may be beneficial for firms.

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5It increases by a small margin of 0.5%.
This paper develops an equilibrium model of dynamic oligopoly following Maskin and Tirole (1988) and Ericson and Pakes (1995). To the best of my knowledge, the most related structural model is Sweeting (2015) who investigates dynamic price competition with demand uncertainty in the event ticket resale market from a platform design perspective. Sweeting (2015)'s model is not immediately applicable to the airline context. First, the event ticket resale markets typically have large numbers of sellers with no market power, whereas the airline markets typically have a small number of players with (potentially) considerable market power. Second, Sweeting (2015) abstracts away from consumer heterogeneity and price discrimination. This choice reflects the reality of the resale market for event tickets, but is likely to be inadequate for modeling the airline industry. My model incorporates these stylized features in the airline industry in a parsimonious way. In the model, oligopoly firms compete in selling limited capacities when facing demand uncertainty. The firms can set their prices depending on remaining time and remaining capacities. Consumers arrive as a stochastic process. The probability distribution of the arrival is known to the firms but the realization is not observed. In the beginning of each period, the firms compete on expected residual demand and update their expectations over time as demand uncertainty resolves gradually. Consumers may differ in both their price sensitivities and their valuations towards each firm. Their tastes follow a time-variant random distribution. The firms do not observe the consumers’ types, but they know how the distribution of the consumers’ types changes over time. So they can price discriminate consumers based on arrival times.

To account for price endogeneity, the demand model allows for product specific demand shocks observed by firms but not by researchers. When doing so, the paper cannot apply the standard tool for demand estimation, that is, BLP market share inversion (Berry et al. (1995)). The market share inversion works well in cases where the sales data is aggregated, usually over time, so that firms’ markets shares contain little measurement error. However, since the airline demand and supply changes daily and the capture of these changes is a goal of this paper, normal levels of aggregation can be excessive in my case. An appropriate daily level of aggregation that retains the pricing patterns results in mismeasured market shares. These market shares are frequently equal to zero or are even undefined if no firms sells during a particular day. Instead of applying BLP directly, this paper considers an alternative generalized method of moments (GMM) estimator in which demand and supply are estimated jointly using a nested fixed point approach reminiscent of Rust (1987). In the inner loop, I solve the dynamic system of demand and supply into its reduced form for a selected set of equilibrium outcome variables of interest conditional on all observed and unobserved states. In the intermediate loop, I integrate out

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6Sweeting (2015) uses a different equilibrium concept, Non-stationary Oblivious Equilibrium (Weintraub et al. (2008)).
all unobserved states, including unobserved demand and supply shocks.\footnote{The possibility of unobserved demand and supply shocks makes two-step methods, such as Bajari et al. (2007), difficult to apply. Note that Bajari et al. (2007)’s first-stage estimator would be a hedonic price regression, which is known to fail if unobserved demand characteristics are present.} In the outer loop, I match the moments from the model to the corresponding moments from the data. Such an approach requires me to solve a dynamic stochastic game with large state space, including unobserved states. To lower the computation burden, I use Gauss-Hermite quadrature to numerically integrate out unobserved demand and supply shocks. To further reduce the state space, I use cubic interpolation to interpolate firms’ value functions over capacity states. Finally, I use Julia’s parallel computation on a multi-processor server.

### 1.1. Related Literature

This paper is related to several streams of literature. Firstly, this paper contributes to the understanding of airlines’ dynamic price competition. The airline industry is essentially an oligopoly market. Its price competitions have received great attention from the policy makers. Curiously, not many structural papers have examined airlines’ price competition. As a notable exception, Berry and Jia (2010) treat airline price competition as a static problem under BLP framework. Recently, Lazarev (2013) and Williams (2013) extend airline demand estimations into monopoly dynamic pricing frameworks. The current paper is the first structural paper on oligopoly airline dynamic pricing, and it provides a useful picture of demand and supply dynamics underlying oligopoly airline markets.

Secondly, this paper adds to the empirical literature on competitive price discrimination, for instance, Besanko et al. (2003), Villas-Boas (2009), Hendel and Nevo (2013), etc.\footnote{See Stole (2007) for a summary.} In the current investigation, consumers can differ in both their valuations and their brand preferences. The former determines the industry-demand elasticities, i.e., the tendency to drop out of the market. The latter determines cross-demand elasticities, i.e., the tendency to switch between competitors. Without capacity constraint, the ability to price discriminate may increase or decrease airlines’ profits and social welfare depending on the ratio of the two (Holmes (1989) and Corts (1998)). This paper shows empirically that early-arriving consumers’ cross-demand elasticities are low relative to their industry elasticities. Thus airlines’ price discrimination is more motivated by their collective incentives to sell to more travelers and fill in capacities instead of private incentives to undercut and steal business from each other. Therefore, oligopoly airlines’ price discrimination benefits airlines and increases social welfare.

Thirdly, this paper relates to the literature on dynamic competition with capacity constraint. IO theorists have long recognized that quantities are limited and that the capacity constraint...
has important implications for firms’ strategic interactions (Edgeworth (1925)). Related to the current work, theorists have looked at multistage price and quantity games (Kreps and Scheinkman (1983) and Davidson and Deneckere (1986)) and repeated pricing games where firms face capacity constraints and demand uncertainty (Staiger and Wolak (1992)). Deneckere et al. (1996) and Deneckere et al. (1997) found that price commitment may increase social welfare by decreasing destructive competition under demand uncertainty. Recent empirical work has studied capacity constrained dynamic auction games (Jofre-Bonet and Pesendorfer (2003) and Jeziorski and Krasnokutskaya (2016)) and demand fluctuation in concrete industry (Collard-Wexler (2013)). Finally, this paper is related to the revenue management literature. The literature has traditionally focused on monopoly cases. As a notable exception, Gallego and Hu (2014) theoretically analyze a revenue management game similar to the current one.⁹

The plan of this paper is as followings: Section 2 presents the empirical setting with some reduced form evidence that motivates the structural model. Section 3 presents the research design. Section 4 sets up the model. Section 5 discusses the empirical strategies and identifications. Section 6 shows estimation results. Section 7 performs counterfactual analysis. Section 8 concludes.

2. **Empirical Setting**

2.1. *Airline Pricing*

The airline industry is an important contributor towards economic development. As in the year 2015, the industry generated $767 billion revenue and transported $3.3 billion passengers (the International Air Transport Association, IATA). In addition to the massive economic scale, airlines are also common textbook examples of dynamic pricing.

Historically in U.S., air travel was viewed as a public good, and the industry operated under governmental subsidies and regulations. To avoid destructive competition, the Civil Aeronautics Board (CAB) controlled airfares. “Discounts and promotions were typically disallowed on the grounds that they disadvantaged competitors or were unduly discriminatory across passengers” (Borenstein and Rose (2014)). After the 1970-1974 Domestic Passenger Fare Investigation, CAB developed the so-called Standard Industry Fare Level (SIFL). SIFL was a nonlinear distance-based formula for setting fares based approximately on industry average cost. Not until Airline Deregulation Act of 1978 did airlines begin their innovation on sophisticated dynamic pricing. The market liberalization removed government-imposed entry and price restrictions and spurred fierce price competition (Kahn (1988)).

⁹See also Xu and Hopp (2006).
**Dynamic pricing**  Airlines are capacity constrained. Aircrafts are assigned well before departure date. For each given flight, the cost of adjusting capacities are very high.\(^\text{10}\) Air tickets are perishable goods by nature. If a seat is not sold before its departure date, it has no value. Air travel demand is uncertain. Each day when setting prices, airlines do not know how many consumers will actually buy. Airlines frequently adjust prices based on real-time fluctuations in supply and demand. In particular, I single out two key components of airlines’ dynamic pricing – revenue management and price discrimination. It is worth pointing out that airlines’ pricing problem involves many other strategic considerations. To keep the model simple, many departures are made from precise institutional features, such as multi-fare-class pricing, demand learning, multi-product pricing, network pricing, and so on. These features are important by themselves and can be viewed as separate questions. I leave them to future research.\(^\text{11}\)

(i) *Revenue Management.* Airlines pioneered the development of revenue management. Revenue management was originally called yield management, although now it is outdated. The meaning of revenue management can be very broad in some cases. However, the fundamental concept of revenue management has not changed, that is, to optimize profits contingent on product availability, i.e. “the actual number of passengers who are currently booked on a specific flight”. In 1977, shortly before the deregulation of U.S. airline industry, American Airlines initiated an inventory-based pricing system called Dynamic Inventory Optimization and Maintenance Optimizer (DINAMO). This was the first large-scale dynamic pricing system. DINAMO gave American Airlines a large competitive advantage against their competitors. Over a three-year period starting around 1988, the system generated $1.4 billion in additional incremental revenue (Smith et al. (1992)). Meanwhile it also caused the bankruptcy of American Airlines’ direct competitor People Express. Thanks to the innovation in information technology and computational capability, the practice of revenue management is both prevalent and mature in today’s airline

\(^{10}\)For most airlines, the schedule design process begins 12 months prior to departure date. Based on this flight schedule, fleet (aircraft type) assignment is done about 12 weeks in advance (Bae (2010)). Last minute aircraft swaps happen mostly because of unscheduled maintenance, bad weather, air traffic control (ATC) delays, etc. As for American Airlines, aircraft swaps are covered in their contract of carriage and thus warrant free ticket cancellation (https://www.aa.com/i18n/Tariffs/AA1.html). There is active research on the concept of demand-driven dispatch (\(D^3\), see Berge and Hopperstad (1993) for the early work). Currently, it is still very expensive and complicated to put \(D^3\) into practice. It requires coordination of network development, schedule construction, fleet assignment, aircraft routing, crew scheduling and ground resource optimization (Shebalov (2009)). There is no \(D^3\) in my estimation data.

\(^{11}\)For example, in practice airlines offer a discrete distribution of airfares at each point in time (Alderighi et al. (2016)). The pricing department is responsible for creating buckets of fares. The revenue management department will adjust the availabilities of these fares dynamically. My model assumes that airlines choose one price from a continuous set. This is a good approximation if (1) There are many prices. In my data, the number of unique prices observed for each route×firm is 13, 22, 38 and 44. (2) Each period is short enough. In my data, 74% of the time the number of tickets sold in one period is no bigger than 1.
industry (Alderighi et al. (2012) and Escobari (2012)). Revenue management is viewed as critical to running a modern airline profitably.\(^\text{12}\)

(ii) Price Discrimination. As a well-known industry regularities, airfares are on average more expensive closer to departure date.\(^\text{13}\) This is viewed as evidence of intertemporal price discrimination. In 1970, British Airways offered “early bird” discounts to consumers who bought tickets at least four months in advance. Generally, leisure consumers with higher price elasticities arrive early, whereas business travelers who are less price sensitive arrive late. Airlines thus are able to screen consumers based on the times of their arrivals.

**Competition and Product Differentiation** There has been much discussion on whether or not the industry is subject to “excessive competition”. Airlines claim that the industry is too competitive. Indeed, since 1978 there have been well over 100 bankruptcy filings in the U.S airline industry. Every major US interstate airline at the time of deregulation in 1978 has since filed a bankruptcy request. A well-known argument is the problem of industry over-capacity. The internet has facilitated great transparency in airfares, and consumers can search them at minimal cost. Moreover, the marginal cost of selling to an additional passenger is low compared to the marginal consumer’s willingness to pay. Thus if flight tickets are not very differentiated across airlines and thus air travel demand is homogeneous, one would expect airlines’ competition to be fierce.

Therefore, the degree of airline differentiation is a crucial determinant of the competitiveness of the market. If flights are sufficiently differentiated, monopoly powers will arise. In an effort to mitigate competition, airlines introduced a loyalty-inducing marketing device called frequent flyer program (FFP). In 1981, American Airlines introduced the first FFP. Soon after 1986, FFP has spread to all major airlines. FFP reduces travelers’ cross-price elasticities by encouraging them to buy tickets from a single airline. As a result, it increases brand loyalty and switching cost (Borenstein (1992)). Should market power be a significant public policy concern in the airline industry? Concerns about airlines’ market power have waxed and waned considerably in the past three decades. For instance, in order to promote competition among its airlines Norway banned domestic FFP in 2002, but lifted the ban later in 2013.\(^\text{14}\) On the other hand, the industry’s profitability has fluctuated dramatically due to cyclical demand, sticky fixed costs,

\(^{12}\) According to Robert Crandall, the CEO of American Airlines, “[revenue management is] the single most important technical development in transportation management since we entered deregulation.” It has become a common practice in many other industries of perishable products including energy, hospitality, entertainment, broadcasting, transportation, etc. According to a recent industry report, the revenue management market is estimated to grow from $9.27 billion in 2015 to $21.92 billion by 2020 at a compound annual growth rate of 18.8 % during the forecast period.

\(^{13}\) This is well documented in the literature. The pattern also holds in my data. Alderighi et al. (2012) presents cases when the price pattern can be different.

\(^{14}\) http://www.aftenposten.no
and repeated disruptive business innovations. Factors like entry/exit and short-run profitability tell us little about airlines’ market power. To understand the competition in the industry, it is essential to estimate its demand primitives. The current paper provides empirical insights on airlines industry’s competitiveness by estimating a dynamic oligopoly model in differentiated product markets.

**Americans' Value Pricing** Sophisticated pricing system clearly benefits a monopoly airline. However, its impact is unclear under competition. As noted in Borenstein and Rose (2014), “many industries have learned from the sophistication airlines have developed in peak-load pricing, price discrimination, and revenue management. But the airlines themselves remain uncertain, and often in fundamental disagreement, over how much price segmentation is optimal and precisely how to accomplish it.” In April 1992, American Airlines announced its “value pricing” strategy to replace the complicated yield management system with a simple, four-tier price system. The goal was to simplify its existing fare system by eliminating 430,000 out of 500,000 fares in its computerized reservation system – an 86 percent drop. As the industry leader, American Airlines assumed that its competitor would follow. However, their major competitors responded to their simplified fare system by aggressive undercutting. The industry went into a full-fledged price war. In October 1992, Robert L. Crandall admitted publicly that his plan had failed. Americans completely abandoned its “value pricing,” after losing $251 million in just two quarters. The industry as a whole lost $1.53 billion in that year (Morrison et al. (1996)).

### 2.2. Data Source

I collect a high-frequency panel data of posted prices and seat maps from airlines’ websites. In the airline literature, the widely used data set is the “Airline Origin and Destination Survey” collected by U.S. Department of Transportation (called DB1B). This data report a 10% random sample of all domestic airline tickets at quarter-route level. This data set is highly aggregated, and it does not report the date at which a ticket is priced/purchased and the date at which the flight departs. As a result, it does not contain the intertemporal variations to reflect dynamics in demand and supply. Recently, researchers have obtained higher frequency prices and sales data. Lazarev (2013) use a high-frequency data set of daily-flight-level prices to approximate transaction prices. Escobari (2012) uses a dynamic panel of seat maps obtained from the internet to approximate daily sales.

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Posted daily-flight-level prices have been used to approximate transaction prices for several years (McAfee and Te Velde (2006)). Seat maps are used only recently as proxies for sales and inventories.\textsuperscript{17} To the best of my knowledge, currently seat maps are the best proxies for airlines’ remaining capacities.\textsuperscript{18} Technically, airlines hold their real-time inventory information in their computer reservations systems (CRSs). Most airlines have outsourced their CRSs to global distribution system (GDS) companies, such as Amadeus, Sabre, Galileo, and Worldspan. A seat map is a public interface connected to airlines’ real-time availability data. Reliable availability responses are crucial and inaccuracy could result in lost bookings (Barnhart and Smith (2012)).

As stated in Sabre’s website, the interactive seat map is to provide real-time “displays for more accurate views of remaining seat availability and seat location”.\textsuperscript{19} Nevertheless, this data is not ideal and has potential measurement errors. If a consumer did not select a seat at the time of the purchase, then the data may fail to indicate that the seat was sold.\textsuperscript{20} Williams (2013) compares collected seat map data with airlines’ reported loading factor and finds that the size of the measurement error seems acceptable.\textsuperscript{21} The current GMM estimator can account for random measurement errors.

Every day, I manually searched for all nonstop flights that departed within the next 100 days for a selected set of routes. I recorded the prices and the numbers of remaining seats from the source code of the airlines’ official websites. Figure 17 shows an example from one of the airlines. I used the lowest economy class price as my measure of price. I used the number of remaining economy class seats as my measure of remaining capacity. This is because my analysis focuses on economy class seats only. I admit the simplification I impose here. I do not explicitly model product differentiation within the economy class seats. In reality, airlines may offer different tiers of economy seats. This is less problematic if demand for different tiers of economy seats were more or less uncorrelated with time.

\textsuperscript{17}Seat map data has also been used in recent transportation, revenue management research, see Mumbower et al. (2014).
\textsuperscript{18}See Puller et al. (2012) and Li et al. (2014) for another potential data source for daily sales.
\textsuperscript{19}Similarly on Amadeus’s website– “Amadeus Interactive Seat Map allows airlines and travel agents to deliver superior service to passengers by displaying real-time and accurate seat information... Enhance customer service by providing them with seat choices based on accurate, real-time availability.”
\textsuperscript{20}Possibility of strategic seat blocking is not modeled. Ticket cancellation can cause measurement error in sales. The model assumes away ticket cancellations.
\textsuperscript{21}Seat maps understate reported load factor by an average of 2.3% at the flight level, with a range of 0-4%. The aggregate error at monthly level is 0.81% (Williams (2013)).
2.3. Route Selection

In order to obtain a robust research environment, I follow a systematic procedure for my route selection.\footnote{There was also a practical reason that I had to focus on a small set of routes. The cost of monitoring a large set of routes was high. Airlines’ websites usually respond very slowly to each request and frequently return errors. They block an IP if they sense too many “suspicious” searches. I also found that they change the source codes of their websites regularly. In many cases, the changes seemed meaningless but did create problems for data scraping. Since all the data is real-time, any missing data would be lost forever. Avoiding missing data is a costly problem even for firms that specialize in collecting online pricing data and selling this data to airlines (Mumbower et al. (2014)). To keep the data quality, I had to maintain my program in “real-time”. To minimize missing data, I scraped the same flights twice a day as a backup.} I use two major data sources to guide my route selection. Firstly, I analyze the DB1B data collected by DOT in the first quarter of 2014. It gives me summary statistics for most of U.S. domestic airline routes. Meanwhile, I manually collect a separate dataset from Google Flight API. I search for 17,392 randomly generated pairs of domestic airports. This covers more than 62% of all possible combinations among 237 major U.S. airports. I record a sample of nonstop tickets for these 17,392 routes.

**Step 1: preliminary** In the first step, I identify a set of routes that meet my route selection rules. These rules are intended to improve data quality and simplify the structural model. Below I explain each of the major criteria and the motivation behind them.\footnote{My selection procedure is similar to Williams (2013).}

(i) *Free seat assignment.* I only include routes where consumers can select seats for free at the time of purchase. This is because I use seat maps to approximate remaining capacities. Southwest does not allow any advance seat assignment. Some low-cost airlines, such as Spirit, charge consumers for advance seat reservation. I have to restrict my attention to routes with only a subset of the following carriers: American, Delta, JetBlue, Alaska, United, Hawaii, and Virginia. I noted at the time of data collection that some of these airlines, such as American, Delta and United, started to introduce basic economy class seats, and consumers could not reserve seats when purchasing this type of ticket. I manually checked to make sure that the selected routes were not affected by the new policy.

(ii) *Pricing of round-trip tickets.* I include routes where round-trip tickets are priced close to the corresponding one-way tickets. By doing so, the pricing and purchasing behavior of round-trip tickets can be approximately seen as independent one-way tickets. This criterion help simplify the model and allow me to abstract away from extra complication. Alaska and JetBlue price strictly at the segment level.

(iii) *High nonconnecting traffic.* I rank the routes by the ratio of traffic that is not connected to other cities. Then I select routes where this ratio is high. There are two reasons for this:
(i) when a seat was marked as occupied in a seat map, it was more certain that a nonstop ticket was purchased; (ii) the concern of network pricing is smaller. In the selected routes, the ratio is above 80%.\(^{24}\)

(iv) \textit{Minimal number of daily flights}. I choose oligopoly routes where the number of operating airlines and daily flights were as small as possible. I adopt a full solution method when estimating the model, and doing so reduces the computational burden while keeping the key ingredient of competition. In all the selected routes, there were two operating airlines. Each carrier has fewer than two daily flights. In most of the cases, each carrier had exactly one daily flight. To expand the sample, I allow a few exceptions when one firm has an extra flight during peak days.

The selection rules lead me to a final sample of 5 routes: Seattle-Tucson (SEA-TUS), San Diego-Boston (SAN-BOS), New York-Sarasota (NYC-SRQ), New York-Aguadilla (NYC-BQN) and New York-San Antonio (NYC-SAT). The operating airlines in these routes included JetBlue, Delta, Alaska, and United. These are among the six largest domestic airlines in U.S. I keep track of all nonstop flights in the selected five routes for a period of six months. My dataset covers 225,704 observations of daily prices and inventories from 4,550 flights.

\textbf{Step 2: exit route} I select a time window in which a route changes from duopoly to monopoly, namely Seattle-Tucson. From December, 2015 to March, 2016, both Alaska and Delta offer direct flights between Seattle and Tucson. Delta exited the market on March 31, 2016, and Alaska remains in the market. The market structure changes from duopoly to monopoly. My data collection period is chosen to contain this exit event.

Alaska has served this route since 2000. It offers daily non-stop flights year-round. Delta started to aggressively build its presence in Seattle airport in 2011 and announced Seattle's hub status in 2014. This created a “turf war” with Alaska, since Seattle airport is Alaska's headquarters and largest hub. Delta first entered the SEA-TUS market and offered a weekly nonstop flight from December 2014 to March 2015. They started to operate at daily level only since my data collection year, i.e. 2016. In the year 2016, they still operated in the December to March interval. The flight was operated under one of Delta's regional carriers, SkyWest Airlines, and Delta was responsible for the pricing and selling.

A concern is the potential of seasonality in demand. As will be clear in the next section, my identification assumption allows for potential seasonalities that are (i) “smooth” in time; (ii) “common with the control route”. It is nonetheless useful to have a sense of potential idiosyncratic “sharp” seasonality at either Seattle or Tucson airport. To do so, I discuss relevant

\(^{24}\)Specifically, 80.2% for SEA-TUS; 93.2% for SAN-BOS; NYC-SRQ for 87.4%; 90.9% for NYC-SAT; 83.5% for NYC-BQN.
air travel supply at the two airports. There is little direct evidence of idiosyncratic sharp seasonality at March for either Seattle airport or Tucson airport.

(i) **Tucson airport.** There were a total of 23 nonstop daily-level flights at Tucson airport. 4 of the 23 flights did not operate year-round. Delta’s SEA-TUS flight operated from December to March, and all of the other 3 flights started in November and ended in June.\(^{25}\)

(ii) **Seattle airport.** Delta had 60 nonstop flights at Seattle airport. 12 of the 60 flights did not operate year-round. 2 of the 12 flights ended in March, including the current one. 4 of the 12 flights ended in April. The other 6 flights ended in August.

**Step 3: control route** I supply the exit route with a control route, namely SAN-BOS. Both of the two routes are identified in Step 1 following the exact same set of criteria. The control route is chosen because it had the closest parallel trend in average price with the exit route before the exit event. Both the exit route and the control route are long-haul. Conveniently, Alaska operates in both of the routes. Although the identification does not require the two routes share the same set of carriers, this fact is still valuable for my reduced form analysis later. Both JetBlue and Alaska offer direct flights in the SAN-BOS route. JetBlue entered the route in 2007, and Alaska entered only since 2013.

**2.4. Data Patterns**

Table 2 reports the summary statistic of the estimation data. The data contains 10,290 observations of daily-flight-level prices and sales for 210 flights up to 49 days before departure dates. On average, a flight sells 1.11 seat each day. More than half of the days, a flight sells zero tickets. From day \( t + 1 \) to day \( t \), on average the price increases by $ 7.5. An average flight sells 47.39 seats 7 weeks before departure. The flight-level Gini coefficient in the bottom row captures the intertemporal price dispersion for a given flight. The mean of the flight-level Gini coefficient equals 0.21. This means that an expected absolute difference is 42% between two randomly selected prices for the same flight at two different pricing dates (Siegert and Ulbricht (2015)). The large dispersion of airfares is consistent with existing findings (Borenstein and Rose (1994)).

Figure 2 shows the path of prices and remaining seats for one particular flight. The red line shows that the seats were sold out gradually over time. 90 days before departure, the flight had more than 100 remaining seats. On the departure day, there were around 17 seats unsold. The blue line shows the dynamics of prices. The prices range from 150-600 dollars. The shaded

\(^{25}\)The other three flights were: Alaska’s Portland-Tucson flight, Delta’s Minneapolis-Tucson flight, Southwest’s Oakland-Tucson flight.
regions highlight some suggestive evidence of revenue management. In the light-blue area, seats were sold out quickly (red line dropped quickly), then the price increased. In the light yellow area, seats were sold out slowly (red line was flat), then the price dropped.

Figure 18 shows the average path of prices and loading factor by number of days to departure. Loading factor increases smoothly over time. The average loading factor on the departure date is 83%. The corresponding number is between 83%-84% for the U.S airline industry from 2011-2016.26 On average, price increases as the departure date approaches. Noticeably, the price jumps up at certain threshold such as 4 days, 1 week, 2 weeks, 3 weeks, etc. Prices are relatively stationary more than one month before departure date. The price path looks similar to existing literature.

3. Research Design

A common challenge in demand estimation is to find exogenous variations that can identify preference on endogenous variables, i.e. price. In order to address this, I leverage the exit event and pair the exit route with a control route that had the closest parallel trend in the average price before the exit event (see the previous section for route selection procedure and see the remainder of this section for the parallel trend discussion). I implement a structural version of (local) difference-in-differences design. In this section, I formally present the research design. I discuss the assumption under which the treatment (exit event) can be viewed as exogenous. I provide reduced form evidence supporting the assumption. I demonstrate that the treatment caused rich variations in prices.

Table 1: (local) difference-in-differences

<table>
<thead>
<tr>
<th>Date Range</th>
<th>Treatment route</th>
<th>Delta</th>
<th>Alaska</th>
</tr>
</thead>
<tbody>
<tr>
<td>03/17 — 03/31</td>
<td>Seattle-Tucson</td>
<td>N=2940</td>
<td>N=1470</td>
</tr>
<tr>
<td>04/01 — 04/15</td>
<td>Alaska</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Date Range</th>
<th>Control route</th>
<th>Alaska</th>
<th>JetBlue</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Boston-San Diego</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>N=2940</td>
<td>N=2940</td>
<td></td>
</tr>
</tbody>
</table>

Table 1 summarizes the research design. The identification condition is similar to the reg-

26http://airlines.org/dataset/annual-results-u-s-airlines-2/
ular reduced form difference-in-differences design. Recall that difference-in-differences allow the treatment and the control to be different as long as the differences are (i) persistent in time or (ii) random in time. If the differences are persistent in time; they are automatically controlled by the before and after design. It is valuable to allow consumers’ preferences and arrival patterns to be different between the two routes. If the time-varying differences are random, they are accounted by the law of large numbers. Importantly, difference-in-differences requires the systematic change after the treatment to be common across the routes. This is the well-known common trend assumption. A regular common trend assumption is admittedly unsatisfactory in my setting. This is because I use exit event as the treatment. Exit is typically viewed as a strategic decision. It is possibly based on the exit firm’s expectation on long-run demand in the route. If this is true, the treatment would be correlated with long-run route-specific demand trend that is known by Delta but not by the researcher.

To address this concern, I restrict my estimation sample to only a relatively small neighbor of the exit event, namely 15 days. That is, I only use flights that departed from March 17, 2016, to April 15, 2016, in the treatment route and the control route. In Table 1, each block of the 2-by-2 design has \( N = \#\text{departure-dates} \times \#\text{firms} \times \#\text{directions} \times \#\text{pricing-dates} \) observations. I assume that the common trend assumption holds in this 30-day time window.

**Assumption 3.1.** *Within the 30-day time window, the control route and the treatment route had a common demand trend.*

The (local) difference-in-differences design helps mitigate the concern on the endogeneity of the treatment. Firstly, if the route-specific demand trend is smooth in time, it will vanish when the time interval gets small in a similar way to the regression discontinuity design. Secondly, the exit was scheduled one year before it actually took place. If Delta was unable to predict demand shocks within a 30-day window 1 year beforehand, the exact timing of the exit can be “as good as random”. Lastly, the difference-in-differences design allows for one type of endogenous exit. It allows Delta’s exit decision to be correlated with aggregate demand trend that was common with the control route. This is valuable if Delta had limited prediction power for one specific route, but it had a prior for industry demand trend and acted on this prior.

Figure 3 shows suggestive evidence that the trends in route-level average price were indeed parallel across the two routes before the treatment on March 31, 2016. Each dot is an average price for flights departed on a given departure date in a given route. It is averaged over \( N = \#\text{firms} \times \#\text{directions} \times \#\text{pricing-dates} \) observations of prices. The lines are smoothed using Gaussian kernels. From March 01, 2016, to March 31, 2016, the price gap was stable over time between the two routes. The gap changed dramatically after Delta’s exit. Route-level mean prices increased substantially in the exit route but did not change much in the control route.
Note that average price in the treatment route could increase “mechanically” when the low-priced firm exited the market. This type of price variation is enough to identify consumer price sensitivity. Nonetheless, it is still valuable to single out how the remaining firm reacted to the exit event. Alaska Airlines operated in both the treatment and the control route before and after the exit event. Figure 4 shows Alaska’s average prices under the 2-by-2 treatment conditions. Each dot is an average price for flights departing on a given departure date in a given route. It is averaged over \( N = \#\text{directions} \times \#\text{pricing-dates} \) observations of prices. Alaska raised its price significantly in the treatment route. Alaska’s price did not change significantly in the control route.

Figure 4 shows a “point estimate” of the treatment effect. Figure 5 goes one step further and zooms in— it shows the heterogeneous treatment effects conditional on different numbers of days to departure. Each dot is an average price for flights departing under a given condition for a given number of days to departure. It is averaged over \( N = \#\text{directions} \times \#\text{departure-dates} \) observations of prices. The graph shows that there were rich variations in treatment effects across different numbers of days to departure. Alaska raised its prices significantly in the exit route after the exit event. Interestingly, most of the price increases happened 2-5 weeks before the departure date. The increases in prices are relatively small towards the two ends. Figure 6 shows that the model is successful in predicting these variations.

4. Model

In the following set up, products can be seen as non-stop economic class tickets for a directional city pair on a departure date, for instance, non-stop economy class tickets from San Diego to Boston on April 01, 2016. To simplify the notation, I omit subscripts for departure dates \( d \) and routes \( r \).

4.1. Demand

Let \( t = 1, 2, \ldots, T \) be the selling periods. Thus time is discrete and \( T \) is the deadline of selling, after which the product is assumed to have zero value. I assume that consumers live for only one period and will not wait. This assumption is motivated by price patterns observed in the data. The upper graph in Figure 18 shows that on average equilibrium price path increases as the departure date approaches. The lower graph shows that the possibility that price will drop tomorrow is only 0.11 on average. Therefore, consumers’ incentive to bet on prices is small.\(^{27}\)

\(^{27}\)The consumer who is indifferent between the best inside option and the outside option has the strongest incentive to wait. By waiting, she can still leave the market if the price goes up and buy a ticket if the price drops. Yet for this marginal consumer, the expected gain is only $5-$7. Using the estimated preference and assuming zero
That is, a forward-looking consumer would have behaved in a similar way as a myopic consumer. However, a consumer may wait if by doing so she can learn more about her preference (Dana (1998), Lazarev (2013)). To simplify the model and focus on the supply side, I abstract away from this in the current paper.\footnote{I note that the current set up can be extended to incorporate strategic consumers similar to Goettler and Gordon (2011).}

**Arrival Process** Let $I(t)$ be the random variable for the number of consumers arriving $t$ days before departure. I assume that $I(t)$ follows an exogenous Poisson stochastic process and the Poisson parameter takes some functional form $\lambda(t; \gamma_{\text{arrival}})$. Formally:

$$I(t) \sim \text{Pois} \left( \lambda \left( t; \gamma_{\text{arrival}} \right) \right),$$

where $\gamma_{\text{arrival}}$ is the parameters on arrival process to be estimated.

Modeling aggregate consumer arrival as a Poisson process is intuitive. The Poisson arrival process has been used widely in many fields, including economics and operations research (McGill and Van Ryzin (1999)). As opposed to Gershkov et al. (2016), I assume the parameters of the arrival process are known to the airlines. Importantly, the exact number of arriving consumers is unknown to airlines.\footnote{McGill and Van Ryzin (1999) discussed other arrival processes that can incorporate batch arrivals.}

**Choice Process** Each consumer who arrives $t$ days before the departure date faces a choice set $J_t$. $J_t$ includes (1) nonstop flight for a specific directional route on a specific departure date available $t$ days before departure and (2) an outside option. The subscript $t$ indicates that choice set may change over time depending on availability. If a consumer does not choose any flight, then she chooses the outside option labeled as 0. The outside option is a reduced form way of capturing all other possible alternatives. These include (1) different means of transportation and (2) different travel dates and/or destinations. By putting flights departing on other dates in the outside option, I abstract away from modeling airlines’ multi-product pricing.\footnote{See Li (2015) for dynamic pricing of complementary products in a monopoly market.}

Consumers make choices according to the standard discrete choice model. Consumer $i = 1, \ldots, I(t)$ arriving at time $t$ is endowed with preference $\{\alpha_{it}, \{\varepsilon_{ijt} \}_{j \in J_t} \}$, where $\alpha_{it}$ measures her preference over product characteristics (brand and price) and $\varepsilon_{ijt}$ is her idiosyncratic choice-specific preference shock. I assume that $\varepsilon_{ijt}$ follows type-I extreme value. Assume that $\alpha_{it}$ follows some exogenous distribution:

$$\alpha_{it} \sim F \left( \cdot ; t, \gamma_{\text{type}} \right), \ i = 1, \ldots, I(t),$$

waiting cost, one can show there are 1.5-2.2 these consumers each market.
where $\gamma$ are the parameters on random taste distributions to be estimated. Note that the preference distribution $F(\cdot; t, \gamma)$ is a function of time $t$. This characterization permits a demand structure that is heterogeneous within each period and non-stationary across periods. It thus allows a rich demand substitution pattern within each period and price discrimination across periods.

To account for price endogeneity, I allow for product-specific demand shock $\xi$ that is observed by market participants but not by researchers. The importance of controlling for unobserved product characteristic has been highlighted in previous literature (Villas-Boas and Winer (1999)). In practice, airlines gather real-time information on demand shocks. As a result, observed prices can be correlated with this type of information. This information, however, is not observed by researchers. Thus, without accounting for these unobserved demand shocks, price elasticities will likely be biased downwards (in absolute value).

I obtain the BLP utility specification that combines consumer heterogeneity through latent taste shocks and endogeneity through product-specific demand shocks:

$$u_{ijt} = \alpha_{iFirm1} \times 1_{j=1} + \alpha_{iFirm2} \times 1_{j=2} + \alpha_{iPrice} \times p_{jt} + \beta_{After} \times 1_{[Depart \ after \ treatment]} + \beta_{Weekend} \times 1_{[Depart \ on \ weekend]} + \xi_{jt} + \varepsilon_{ijt}$$

$$\equiv \tilde{u}_{ijt} + \varepsilon_{ijt}.$$

$\alpha_{it} \equiv \{\alpha_{iFirm1}, \alpha_{iFirm2}, \alpha_{iPrice}\}$ are the random coefficients, where $\alpha_{iFirm1}$ and $\alpha_{iFirm2}$ are consumer $i$'s preference towards firm 1 and firm 2 respectively and $\alpha_{iPrice}$ is consumer $i$'s price coefficient. Let $\alpha_{iFirm} \equiv \{\alpha_{iFirm1}, \alpha_{iFirm2}\}$ be the vector for consumer $i$'s brand preference. $\beta_{After}$ is a preference shock between before the exit event and after. This allows a common industry demand trend $\beta_{After}$ across the two routes. $\beta_{Weekend}$ is a dummy variable for weekends, and it allows consumers who travel on weekends to be different from those who travel on weekdays. Without loss of generality, normalize outside option such that $\bar{u}_{i0t} = 0$.

I make a simplifying assumption that consumers simply choose the product that maximizes her utility. If the product is sold out during a selling period, then a lottery is used to decide who get the remaining seats. However, consumers do not consider this possibility of sell out when make purchases.\textsuperscript{31} Finally, I obtain the familiar expression for market share:

\textsuperscript{31}Rationing rule is important in Bertrand pricing. See Davidson and Deneckere (1986).
\[ s_{jt} = \int_{\alpha_{jt}} \frac{\exp(\tilde{u}_{iit})}{1 + \sum_{j' \in J \setminus \{0\}} \exp(\tilde{u}_{ij't})} \times dF(\alpha_{it}; t, \gamma_{type}). \]

**Sales Process**  Let \( q_{jt} \) be the sales for firm \( j \)'s flight \( t \) days before its departure date. Proposition 1 states that a multinomial choice process conditional on a Poisson arrival process yields mutually independent Poisson sales process.

**Proposition 1.** We must have:

\[ q_{jt} \sim \text{Pois}\left( \lambda(t) \times s_{jt} \right), \]

\[ q_{jt} \perp q_{j't}. \]

This equivalence simplifies my model. To explain, consider the following simple example. If the number of arriving consumers follows \( \text{Pois}(4) \). An arriving consumer has 50% chance of choosing firm A from a choice set of firm A and firm B. It is trivial to show that the expected sales value of firm A is 2. If firm B’s sales is known to be 4, what do we expect firm A’s sales to be? The answer is 2. Proposition 1 says that conditional on demand information the sales of the two firms follow independent Poisson distributions. This result simply comes from the mathematical property of Poisson-Multinomial distribution. It makes the state transition more trackable and reduces the computation substantially.

### 4.2. Supply

In my model, firms play a dynamic pricing game under complete information. Same as in a single-agent dynamic programming problem, prices have to satisfy *intertemporal optimality condition*. Since sales are stochastic, the optimality condition changes constantly. Therefore, airlines continuously re-optimize to account of the changing shadow price of capacity. In addition to this monopolistic dynamic optimization, airlines also rationally expect that today’s pricing strategies not only affect all players’ sales today but also affect their capacities tomorrow. As a result, airlines adjust their prices dynamically contingent on time as well as each other’s remaining capacities. I assume that remaining capacities are common knowledge. This assumption is made in a few recent papers on competitive revenue management. See, for instance, Levin et al. (2009), Gallego and Hu (2014), etc.\(^{32}\)

\(^{32}\)In practise, real-time fares and inventories are indeed public information on GDS platform available to all its subscribers. According to the chief operating officer of Sabre—“At Sabre, we have worked jointly and diligently with airlines to come to mutual agreements that ensure efficient access to their inventory.” For related theories, see Vives (1984) and Shapiro (1986), etc. For recent structural work, see Asker et al. (2016).
States Firms’ payoff relevant state variables are summarized as \{t, c_t, \xi_t, \omega_t\}. \(c_jt\) denotes firm \(j\)’s remaining capacities \(t\) days before departure. \(\xi_jt\) denotes firm-specific demand shocks. \(\omega_jt\) denotes firm-specific shocks in marginal cost. One may think that airlines’ marginal costs are realized at the time of departure. In practice, airlines’ optimal prices account for their expected marginal costs at the pricing date. The expected marginal costs depend on current gas prices, the option value of a seat if sold as a connecting ticket, etc. These costs change over time. Thus I allow marginal cost shocks \(\omega_jt\) to have the same subscript \(t\) as observed prices \(p_jt\). Finally, \(\xi_jt\) and \(\omega_jt\) put together real-time demand and supply information known to firms but not to researchers.

Under capacity constraint, sales \(\tilde{q}_jt\) for firm \(j\) at time \(t\) follows truncated Poisson distribution. Let \(\tilde{q}_t\) be the \(J \times 1\) random vector for sales. The state transition of capacities are governed by the following stochastic sales process:

\[
\Pr_t(\tilde{q}_jt = \tilde{q}_jt | p_t, c_t, \xi_t) = \begin{cases}
    h(\tilde{q}_jt; \lambda(t) \times s_jt) & \text{if } \tilde{q}_jt < c_jt;
    \\
    c_jt - 1 - \sum_{k=1}^{c_jt-1} h(k; \lambda(t) \times s_jt) & \text{if } \tilde{q}_jt = c_jt,
\end{cases}
\]

where \(h(k; \lambda)\) is the Poisson probability mass function with parameter \(\lambda\) at \(k\). It follows that:

\[c_{jt+1} = c_{jt} - \tilde{q}_{jt}.\]

Timing of the Game In the beginning of each period \(t\), firms observe own and the opponent’s capacities \(c_t\) and the remaining time \(T - t\). Firms know the distributions of the Poisson arrival process as well as the distribution of consumer preference at any point in time. However, they do not observe how many consumers actually arrive, nor do they know the actual valuation of these consumers. They simultaneously choose prices that maximize their own expected total payoffs, which is the sum of the expected current period profits and the expected continuation value. In the end of the period, demand is realized and seats are filled. Then the game proceeds to the next period until \(t = T\). After the final period, all remaining capacities have zero value. A firm is out of the market if it runs out of capacity before the departure date.

Payoff Function I discuss the details of the supply model now. Marginal cost can be written as:

\[mc_{jt} = \eta_{Firm_1} \times 1_{\{j=1\}} + \eta_{Firm_2} \times 1_{\{j=2\}} + \omega_{jt},\]

where \(\omega_{jt}\) is IID cost shock, \(\eta = \{\eta_{Firm_1}, \eta_{Firm_2}\}\) are intercepts for firms’ marginal cost. Cost information is common knowledge.
In each period, the expected static payoff is given by:

$$\Pi_j(t)(p_t, c_t, \xi_t, \omega_t) = E_t\left[q_j(t) \times (p_j(t) - mc_j(t))\right].$$

The expected static payoff equals sales times margin. It is a function of firms’ current prices. It depends on capacity since the latter decides the maximal sales a firm can have. Current profit also depends on all supply and demand shocks. The expectation is taken over current stochastic sales. The expectation operator has subscript $t$ because the game is non-stationary.

**Markov Strategy** I consider non-stationary Markov pricing strategies. Airlines’ prices are contingent on all current payoff-relevant state variables. Specifically, their prices are contingent on the number of days to departure $t$, remaining capacities of each firm, demand shocks and cost shocks:

$$g_j : \mathbb{T} \times \mathbb{C}^J \times \mathbb{Z}^J \times \mathbb{Q}^J \rightarrow \mathbb{R}^+.$$

**Bellman Equation** Firm $j$’s Bellman equation is defined recursively:

$$V_j(t)(c_t, \xi_t, \omega_t|g) = \max_{p_{jt}} \left\{ \Pi_j(t)(p_t, c_t, \xi_t, \omega_t) + \delta \int \mathbb{E}V_{j+1}(c_{t+1}, \xi_{t+1}, \omega_{t+1}|g) \prod_{j=1,2} \text{dPr}_t\left(\tilde{q}_j(t) = \tilde{q}_j(t)|p_t, c_t, \xi_t\right) \right\},$$

where $\mathbb{E}V_{j+1}(c_{t+1}, \xi_{t+1}, \omega_{t+1}|g)$ the expected value function over future shocks satisfying:

$$\mathbb{E}V_{j+1}(c_{t+1}, \xi_{t+1}, \omega_{t+1}|g) = E_{\xi_{t+1}, \omega_{t+1}}\left[V_{j+1}(c_{t+1}, \xi_{t+1}, \omega_{t+1}|g)\right].$$

**Boundary Condition** To conclude the model, I specify boundary conditions for airlines’ dynamic problem. Naturally, perishability implies that all seats have no value after the end of the selling period.

$$V_{jT} = 0.$$

The second boundary conditions are driven by the existence of capacity constraint. $^{33}$

$$V_{jt} = 0, \text{ if } c_{jt} = 0.$$

$^{33}$Airlines’ overbooking strategy is well-known. In my estimation data, the situation of a plane being full but the airline still posting price is rare. It never happened in the treatment route, and happened 0.6% of the times in the control route.
**Discussion**  I have assumed that all error terms are IID in all their respective subscripts and uncorrelated with state variables. The conditional independence assumptions are standard and pragmatic in the context of dynamic models (Rust (1987)). Yet assuming that unobserved product-specific demand shocks are IID across time seems restrictive comparing to static demand estimations. For instance, BLP places little distributional assumptions on $\xi$. Allowing serially correlated demand shocks is conceptually simple, but it does not seem to add much to the current paper.

One may extend the model to incorporate the learning of demand. The idea is similar to Gershkov et al. (2016). Consider a general Markov process of arrival $\lambda(t, N(t))$ such that a firm can learn demand from cumulative sales. The key assumption is that arrival is orthogonal to valuations.

Finally, recent theoretical papers such as Board and Skrzypacz (2016) and Gershkov et al. (2016) have applied dynamic mechanism design for monopoly revenue management with strategic consumers. Their set ups are related to this current paper. This current paper only looks at simultaneous price posting.

4.3. **Equilibrium**

I look at Markov perfect Nash equilibrium for this dynamic stochastic game.

$$V_j(g^*_j; g^*_{-j}) \geq V_j(g_j; g^*_{-j}), \forall g_j.$$ 

Note that the game has a finite horizon, therefore one may attempt to argue backward by showing uniqueness for each static subgame. Although a finite horizon can alleviate concerns of the multiplicity of equilibria (Chen et al. (2009)), proving the uniqueness of this game is not easy. Caplin and Nalebuff (1991) establish a set of sufficient conditions for existence and uniqueness of price competition with differentiated products. Unfortunately, the results do not easily generalize to the BLP setting. In a recent paper, Pierson et al. (2013) provide another set of sufficient conditions for uniqueness of the pricing game under mixed multinomial logit demand. Pierson et al. (2013)’s proof mostly relies on restricting market concentrations and/or price spaces. To the best of my knowledge, there is no more general results.$^{34}$

Suppose the static pricing game is indeed unique, the next question is whether the uniqueness remains under a dynamic setting. Now the cost function needs to account for the option value of a sale and is thus more complicated. Without characterising the structure of the value functions, it is unclear how the standard techniques can possibly work. Although it may not be so hard to prove structures of monopoly dynamic programming problems (Gallego and

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Van Ryzin (1994)), it is significantly harder to do so in dynamic games. In fact, the structure of a RM game with stochastic demand is largely unknown (Lin and Sibdari (2009) and Gallego and Hu (2014)).

When the competition is mild enough, the equilibrium will be unique. In practice, I allow firms to play best response dynamics. At each state, the pair of optimal prices is found when the change of firms’ best responses is smaller than a tolerance. The pricing games’ solution is robust to different initial values and sequences of moves. I will discuss the numerical algorithm with more details in the appendix.

5. Estimation

There are two challenges in the estimation. The first one is how the unobserved product-specific errors in small “non-invertible” markets will be incorporated (I will be more precise later), and the second is how to keep the estimation computationally feasible. I adopt a nested fixed-point approach. In the first step, I solve the system of demand and supply into its reduced form for all market outcomes conditional on all observed and unobserved states. In the second step, I integrate out all unobserved states. In the final step, I match model predicted outcomes and empirical outcomes. I adopt a GMM estimator and interact the predicted errors with a set of covariates. I discuss identification at the end of this section.

5.1. Econometric Specification

I use a third order polynomial to approximate the Poisson aggregate arrival rate:

$$\lambda(t; \gamma_{\text{arrival}}) = \sum_{n=0}^{3} \gamma_{\text{arrival}}^{(n)} \times t^n.$$ 

I allow arrival rates to be different for the two routes.

I approximate the random coefficient demand model with discrete types. This is typical in economic literature. Examples include Berry and Jia (2010) on airline demand and Besanko et al. (2003) on price discrimination. The discretized random coefficient model captures the correlation of tastes but remains computationally cheap under certain circumstances. I allow for two vertical types of consumers \(\{H, L\}\) differing in their price sensitivities. In particular, let \(\alpha_{\text{Price}}^{H}\) be the price coefficient for the high type consumers and \(\alpha_{\text{Price}}^{L}\) for the low type. This is a parsimonious way of modeling traveler types as business travelers and leisure travelers.

I allow their arrival process to be correlated with time. The probability that a consumer who
arrives at time $t$ is a low type follows:

$$\Pr_L(t; \gamma_{\text{type}}) = \frac{1}{1 + \exp\left[\sum_{n=0}^{3} \gamma_{\text{type}}^{(n)} \times t^n\right]}.$$  

It follows that

$$\Pr_H(t; \gamma_{\text{type}}) = 1 - \Pr_L(t; \gamma_{\text{type}}).$$

Note that this parametric assumption is pragmatic in several ways. The logit transformation bounds probabilities in $[0, 1]$. Secondly, the third-degree polynomial flexibly captures the intertemporal variation in consumer types.

Within each vertical type of consumers, I further allow for two segments with possibly different horizontal brand preferences. Strictly speaking, I cannot identify brand preference from product preference, so I do not distinguish them in the paper. Let $\gamma_{\text{type}}^{H1}$ be the proportion of high type consumers that relatively prefer firm 1 and $\gamma_{\text{type}}^{L1}$ be the proportion of low type consumers that relatively prefer firm 1. Thus I have $2 \times 2$ discrete segments. Let $\alpha_{\text{Firm}}^{H1} = \{\alpha_{\text{Firm1}}^{H1}, \alpha_{\text{Firm2}}^{H1}\}$ be a $2 \times 1$ vector for brand preference of firm-1-leaning high type consumers, where $\alpha_{\text{Firm1}}^{H1}$ is firm-1-leaning high type consumers' preference for firm 1 and $\alpha_{\text{Firm2}}^{H1}$ is firm-1-leaning high type consumers' preference for firm 2. Define $\alpha_{\text{Firm}}^{H2} = \{\alpha_{\text{Firm1}}^{H2}, \alpha_{\text{Firm2}}^{H2}\}$, $\alpha_{\text{Firm}}^{L1} = \{\alpha_{\text{Firm1}}^{L1}, \alpha_{\text{Firm2}}^{L1}\}$, and $\alpha_{\text{Firm}}^{L2} = \{\alpha_{\text{Firm1}}^{L2}, \alpha_{\text{Firm2}}^{L2}\}$ accordingly.

Therefore, I can write out the probability distribution on the four discrete types of consumers:

$$F(\alpha_{i,t}; t, \gamma_{\text{type}}) = \begin{cases} 
\Pr_H(t; \gamma_{\text{type}}) \times \gamma_{\text{type}}^{H1} & \text{if } \alpha_{i,t}^{\text{Price}} = \alpha_{\text{H1}}, \alpha_{i,t}^{\text{Firm}} = \alpha_{\text{Firm1}}^{H1}, \\
\Pr_H(t; \gamma_{\text{type}}) \times [1 - \gamma_{\text{type}}^{H1}] & \text{if } \alpha_{i,t}^{\text{Price}} = \alpha_{\text{H1}}, \alpha_{i,t}^{\text{Firm}} = \alpha_{\text{Firm2}}^{H1}, \\
\Pr_L(t; \gamma_{\text{type}}) \times \gamma_{\text{type}}^{L1} & \text{if } \alpha_{i,t}^{\text{Price}} = \alpha_{\text{L1}}, \alpha_{i,t}^{\text{Firm}} = \alpha_{\text{Firm1}}^{L1}, \\
\Pr_L(t; \gamma_{\text{type}}) \times [1 - \gamma_{\text{type}}^{L1}] & \text{if } \alpha_{i,t}^{\text{Price}} = \alpha_{\text{L1}}, \alpha_{i,t}^{\text{Firm}} = \alpha_{\text{Firm2}}^{L1}.
\end{cases}$$

### 5.2. Endogeneity

Price endogeneity in a simultaneous demand and supply system has been studied for decades in economics. Ignoring the endogeneity problem will cause biased estimates on the price coefficient (Villas-Boas and Winer (1999)), since prices are often strategically chosen in response to demand errors unobserved by researchers, which violates identification conditions. If the demand function is aggregated from discrete choice and thus is nonlinear, a simple IV regression is not immediately applicable. The classic solution to this was developed in Berry (1994) and Berry.
The endogenous component in demand is assumed to be captured by an additive product shock $\xi_j$ observed by market players but not by researchers. The proposed solution is to linearize the demand equation and invert out the unobserved error $\xi_j$. Once back to a linear setting, the work remaining is to find appropriate instrument variables. Berry et al. (1995)’s inversion method works well when the market size goes to infinity at a certain speed such that observed market shares (1) can approximate choice probabilities, and (2) are bounded away from zero. Neither of this holds in the current setting. In each market (itinerary x departure-date x pricing-date) I observe on average fewer than two prices and two sales. This type of data is a deviation from the standard market-level data, but is not uncommon in dynamic pricing setting (see Sweeting (2015) for event ticket).

I adopt a “reduced form method” discussed in Berry (1994). Let $\mathcal{I}$ be all the relevant states for the joint system of demand and supply. Let $\mathcal{I} = \{\mathcal{I}^o, \mathcal{I}^u\}$ such that $\mathcal{I}^o$ is observed in the data while $\mathcal{I}^u$ is not. So $\mathcal{I}^u = \{\xi, \omega\}$. I first solve the game and translate the structured system into a reduced form. Let $\Psi(\mathcal{I}, \theta)$ be such an operator that takes structural parameters $\theta$ as well as all payoff relevant states $\mathcal{I}$ and returns a vector of market outcome (prices, quantities, prices interacting with quantities, etc). Then I integrate out unobserved shocks $\mathcal{I}^u$ and obtain the expected market equilibrium variables $\psi$ conditional on only observables $\mathcal{I}^o$ and the structural parameters $\theta$. Let $\theta = \{\theta^o, \theta^u\}$ be the structural parameters on observables and unobservables respectively. Note that the conditional independence assumption implies that $\Phi(\mathcal{I}^u|\mathcal{I}^o, \theta^u) = \Phi(\mathcal{I}^u|\theta^u)$. Therefore:

$$
\psi(\mathcal{I}^o|\theta) = \int_{\mathcal{I}^u} \Psi(\mathcal{I}^o|\theta^o) d\Phi(\mathcal{I}^u|\theta^u).
$$

Note that $\Psi$ takes $\xi$ and $\omega$ as its arguments and jointly solves demand and supply. It implicitly accounts for the dependence of prices on unobserved demand error $\xi$. This is different from integrating out $\xi$ separately for the demand equation while using observed prices. The latter approach, as pointed out by Berry (1994), is not consistent since it assumes that price does not respond to $\xi$.

The drawbacks of the reduced form method are discussed in Berry (1994). BLP does not impose any distributional assumption on $\xi$ and $\omega$. In practice, I assume that $\xi$ and $\omega$ are independent normal and use numerical integration method to calculate $\psi(\mathcal{I}, \theta)$. In addition, as in other full solution method, a stronger assumption is needed to address potential multiple equilibria. Lastly, in order to avoid the identification from functional form, the reduced form

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35 Control function is another approach, Petrin and Train (2010).
36 Outside of the dynamic pricing context, Goolsbee and Petrin (2004) looked at survey data with small (but non-zero) market size in the cable industry. They confirmed the importance of allowing unobserved demand shock and proposed a method for dealing with the measurement error in market shares.
37 Villas-Boas (2007) provides a more general perspective.
method by Berry (1994) requires an exclusion restriction – a variable that enters supply equation (supply shifter) but is excluded from the demand equation. I use current gas price as a supply shifter.

In practice, I solve for model predicted prices and sales and allow for the following interaction terms:

\[
\psi_{jtdr}(\mathcal{F}_{jtdi}^o | \theta) = \begin{bmatrix}
\hat{p}_{jtdr} \\
\hat{q}_{jtdr} \\
\hat{p}_{jtdr} \times \hat{q}_{jtdr} \\
\hat{p}_{jtdr} \times \hat{c}_{jtdr} \\
\hat{q}_{jtdr} \times \hat{c}_{jtdr} \\
\hat{p}_{jtdr} \times \hat{c}'_{jtdr} \\
\hat{q}_{jtdr} \times \hat{c}'_{jtdr}
\end{bmatrix}.
\]

5.3. **Identification**

I adopt a method of moments estimator by minimizing the differences between the model predicted market outcomes, \( \psi_{jtdr}(\mathcal{F}_{jtdi}^o | \theta) \), and the observed market outcomes, \( \psi_{jtdr} \). I match a set of observed moments with corresponding predicted moments jointly for the system of supply and demand. The prediction errors by construction have mean zero. To translate data variations into model parameters, I allow the market outcomes to interact with a set of observed covariates. Now I discuss the selection of these covariates.

(i) **Supply Covariates.** \( \mathbf{z}^{\text{supply}} \) include current gas price and dummy variables indicating the identities of the two firms. Note that the demand side allows product differentiation, and the supply side does not place restrictions on firms’ relative capacities. Thus firms can differ in size, market power, etc. Firms’ identity dummies help identify the relative position of the two firms.

(ii) **Demand Covariates.** \( \mathbf{z}^{\text{demand}} \) denote exogenous covariates in consumers’ utility function. Firstly, I let \( \mathbf{z}^{\text{demand}} \) include weekday/weekend dummies, which help identify the relative preference for weekend flights. This is one way to control for departure day fixed effects. Secondly, I allow \( \mathbf{z}^{\text{demand}} \) to contain dummies for weeks to departure, which helps identify temporal (cross-period) demand heterogeneities. The reduced form evidence shows rich cross-period heterogeneities. Moreover, these heterogeneities can be captured well with weekly time windows.

(iii) **Treatment Conditions.** To implement the difference-in-differences design, I use the treatment conditions \( \mathbf{z}^{\text{treatment}} \) as the third set of shifters. Thus \( \mathbf{z}^{\text{treatment}} \) includes interactions...
between treatment/control routes and before/after exit. This set of covariates (1) create exogenous variations in prices to identify within-period preferences and (2) infer route-specific demand structures.

Finally, I allow these covariates to interact with each other and obtain the full covariate vector $\mathbf{z}$. Thus, a covariate is a dummy vector that indicates, for instance, JetBlue’s weekday flight in the exit route before the exit five weeks before the departure:

$$
\mathbf{z}_{jtdr} = \mathbf{z}_j^{\text{supply}} \times \mathbf{z}_i^{\text{demand}} \times \mathbf{z}_r^{\text{treatment}}.
$$

The moment restriction is:

$$
g(\theta_0) = E_{[\xi, \omega, \tilde{q}]} \left[ (\psi(\theta_0) - \psi_{jtdr}) \left| \mathbf{z}_{jtdr} \right. \right] = 0.
$$

In the end, I have 469 moments and $|\theta| = 38$ parameters. I use a two-step generalized method of moments and assume that the necessary regularity condition holds for the GMM. As a first step, I estimate the model with each moment weighted by its empirical counterpart. This estimator is consistent but not efficient. With the first-step estimates $\hat{\theta}_1$, I can calculate the estimator for the optimal weighting matrix $\hat{W}_T(\hat{\theta}_1)$. With this weighting matrix, I update the estimators and calculate the standard error using the asymptotic variance matrix for the two-step feasible GMM estimator.

$$
\hat{\theta} = \arg\min_{\theta \in \Theta} \left[ \hat{g}_T(\theta) \right]^T \hat{W}_T(\hat{\theta}_1) \left[ \hat{g}_T(\theta) \right].
$$

The parameters are jointly identified from the demand and supply of the model. Price variations before and after the exit identify consumers’ price coefficients. In addition, the random demand fluctuations create variations in remaining capacities, which shift firms’ optimal prices through revenue management. These variations in prices will also help identify price coefficient. The mixture of consumer types is identified from violations of the IIA properties of simple logit demand. Temporal preference heterogeneity is identified by the temporal variations of high-frequency prices and quantities.

In many applications, market size $M$ is directly observed. For instance, researchers set $M$ for automobile and TV cable equal to the number of households in the whole population (Berry et al. (1995), Goolsbee and Petrin (2004)). When there is information in a number of markets, $M$ can be parameterized as depending on market level data (Berry (1990), Berry and Jia (2010), Lazarev (2013)). In my setting, the number of potential air travelers could potentially affect my inference on market competition and welfare. It is unobserved by the researcher. In a related dynamic setting, Nair (2007) looked at intertemporal price discrimination in monopoly video-game markets. Similarly, he does not assume the market size to be the whole population (all users for the game platform),
the current identification on market size can come from two sources. Firstly, the dependence of firms’ optimal prices on remaining capacities helps identify market size. In addition, the current difference-in-differences research design also helps. 39

6. Results

6.1. Estimates

In this section, I discuss the estimates from the structural model. Table 3 – Table 6 report the estimates from the structural model. In particular, Table 3 shows consumers’ preference parameters in the exit route. Table 4 shows consumers’ preference parameters in the control route. Table 5 shows consumers’ arrival rates and the distributions of their types for the two routes. Table 6 shows estimates on other common parameters for the two routes.

All the estimates are precise and significant at 1% level, suggesting that the model parameters are well identified by the data variations. The estimates seem intuitive. Early-arriving consumers’ price coefficients are about three times bigger (in absolute value) than late-arriving ones. This confirms one’s prior that late-arriving consumers are so-called “high types” – less sensitive to prices. Figure 7 plots the estimated Possion arrival rates for the two routes. The upper graph is for the exit route and the lower one is for the control route. The different colors indicate the distribution of different consumer types. This distribution indeed changes over time. It suggests that Alaska has a relatively larger loyal segment in the exit route than in the control route. In reality, Alaska is the relatively bigger player in the exit route but the smaller one in the control route. This is also consistent with the observation that Alaska is the incumbent in the exit route but the newer firm in the control route. Estimates of the ratios of each player’s loyal segments are consistent with their relative sales.

As shown in Figure 7, the estimated arrival patterns are similar for the two routes. On average, 4-7 travelers arrive at the market for a particular flight on a particular day. The number of arriving consumers is the lowest 3 weeks before departure. It then increases to its highest level just one week before departure. The estimated ratio of high-type travelers is increasing otherwise, the relatively small sales would imply that the game has almost zero market share at any period.

39 The change in relative sales in response to the change in relative prices across the two periods helps identify the price coefficient. This tells me the change of the utility for product 1 across the two periods. I also observe the change of the share for product 1 across two periods, since it is inferred from the change of product 1’s sales under the assumption that market size is constant. Thus I know (1) the change of the utility for product 1 (after identifying price coefficient); (2) the change of the utility for the outside option (equals zero by assumption); (3) the change of the share for good 1 (observed from sales); and I can infer the change of the share for the outside option. The absolute change of sales for the outside good is observed (from the absolute change of sales for the inside goods). This helps to infer market size. See the appendix illustrates more when demand is a simple logit.
with time. In particular, in the exit route, the ratio of high-type travelers starts from close to 7% seven weeks before departure. It increases gradually to more than 80 percent one week before departure. In the last day, about 87% of the arriving travelers are high-type travelers. Meanwhile, in the control route, the ratio of high-type travelers starts at around 23 percent. It then gradually increases to around 60% just one week before departure. On the last day, the ratio of high-type travelers is around 85%. Figure 7 also summarizes the probability distribution of consumers’ brand preferences conditional on the arrival time. In the exit route, the incumbent firm Alaska has a larger loyal segment for both high-type consumers and low-type ones. 88% of high-type consumers prefer the incumbent firm Alaska to Delta, whereas 84% of low-type consumers prefer the incumbent firm Alaska to Delta. In the control market, 26% of high-type consumers and 38% of low-type consumers prefer the incumbent firm JetBlue. In both routes, Alaska seems to have relatively larger loyal segments than its opponents in high-type consumer segments.

The estimates on the firm-specific constant terms in the consumer utility function confirm that there is great heterogeneity in consumers’ relative brand preferences. These estimates are reflected by firms’ relative average price levels. On average, high type consumers have stronger brand preference than low type consumers, as measured by the amount of money that can induce an “average” consumer to switch. In the exit route, an average Alaska-leaning high-type consumers would pay an extra of $457 to fly with their preferred provider Alaska instead of Delta. An average Delta-leaning high-type consumers would pay an extra of $768 to fly with their preferred provider Delta instead of Alaska. These numbers suggest that high-type travelers’ brand preferences are very strong. As a result, the competition in the late market is weak and the late market is close to monopoly – this is the reason why the exit had no effect on the prices one week before departure. On the other hand, in the control route, an average Alaska-leaning high type consumers would pay an extra of $168 to fly with their preferred provider Alaska instead of JetBlue. It suggests that although Alaska has a larger loyal segment of high-type consumers, the average degree of loyalty is not very high. An average JetBlue-leaning high type consumers would pay an extra of $678 to fly with their preferred provider JetBlue instead of Alaska.

The relative brand preference for leisure travelers differs substantially from only $5 to as much as $207. In the exit route, an average Alaska-leaning low-type consumer would pay an extra of $100 to fly with their preferred provider Alaska instead of Delta; an average Delta-leaning low-type consumers would pay an extra of $163 to fly with their preferred provider Delta instead of Alaska. In the control route, the low-cost carrier JetBlue dominates the low-type consumer segment. In fact, even the Alaska-leaning low type travelers are almost indifferent between Alaska and JetBlue – on average, it only takes $5 to make them switch to JetBlue. On the other
hand, JetBlue seems to have a “very loyal” low-type segment—on average it takes $207 to make them switch to the opponent Alaska.

6.2. Competition

In this section, I zoom in on one route and investigate its competitive landscape in more detail. Figure 8 shows the choice probabilities of the four segments choosing between Alaska and JetBlue. It is simulated using the empirical distribution of the initial capacities under the optimal dynamic pricing strategies. Loosely speaking, Alaska faces more competitive pressure than JetBlue. This is because Alaska’s loyal segments have lower loyalty than JetBlue’s in both the high-end and the low-end market.

Figure 8 also suggests that there is more substitution across firms in the middle periods than in the two ends. In the very early periods, consumers valuation is low and the firms try to fill in their capacities and price very low. It would be too costly for either one of them to price even lower in order to capture the opponent’s consumers. There is some sense that they are competing more against the outside option than against the opponent. In the very late market, it would also be very costly to attack the competitor. The reason is different. Late market has so much “fat” that attacking the opponent aggressively would lose ones’ own profit and thus is almost never optimal. Overall, both firms want to price high (low) in the late (early) market. However, in the middle market, the firm with more high-type consumers (Alaska) wants to increase its price, and the firm with more low-type consumers (JetBlue) wants to keep its low price. The price gap becomes large and it induces more substitution across firms. Figure 9 shows the ratio of consumers who choose JetBlue among all Alaska’s consumers who buy tickets. It shows that the ratio of Alaska’s consumers who choose JetBlue over Alaska changes as the departure date approaches. The ratio is relatively low in the early market. It increases in the middle market as the price gap between the two firms becomes wider. It decreases to the lowest level when JetBlue also jacks up its price in the late market.

Figure 10 shows own price elasticity and industry elasticity (in absolute values) using the empirical distribution of capacities. The elasticities are evaluated at the optimal dynamic prices. Own demand elasticity is defined as the percentage change of own sales with respect to a one percentage change in own price. Industry demand elasticity is defined as the percentage change of industry sales when both firms increase their prices by one percentage. The estimated values are consistent with previous literature. The commonly cited Gillen et al. (2003) suggests that airline travel demands elasticity ranges from 0.181-2.01 with a median of 1.3 for a sample of 85 city-pairs. Previous literature also points out that demand in long-haul routes like the current

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40 For simplicity and for consistency with the counterfactual analysis, I focus the discussion on the control route. I conduct similar examination in the treatment route, and the insight is qualitatively similar.
one is less elastic. The median own demand elasticity of long-haul domestic leisure consumer is 1.228. Similarly, my estimates range from 1-1.4.

My result also highlights the intertemporal variations in elasticity. Own demand elasticity decreases as departure date gets closer. Industry demand elasticity also decreases over time. It starts from just below 1.7 weeks before departure and starts to drop substantially at 3 weeks to departure. This confirms the belief that the demand in the late market is very inelastic. The gap between own demand elasticity and industry demand elasticity is informative. It suggests that the firms would collectively benefit if they can increase their prices together since the industry demand is much less elastic.

6.3. Fitting

Figure 19 shows the fitting errors in percentage for all the 469 moments. The mean fitting error in percentage is 11.5%. This suggests that the data can be reasonably well explained by the structural model. Figure 20 compares the model predicted price paths with their empirical counterparts conditional on the 2-by-2 treatment conditions. It suggests that the model captures the upward-slopping trend for prices. The estimates also reflect the relative price levels between firms. Moreover, the model captures differences across the 2-by-2 treatment conditions. It fits reasonably well the changes in prices caused by the change in market structure in the exit route. This is presented in Figure 6.

7. Counterfactuals

I present some results of my counterfactual simulations. The discussion focuses on the route where Alaska competes against the low-cost carrier JetBlue. Under my previous assumption of myopic consumers, the estimated arrival patterns would not change when airlines’ pricing policies change. This remains reasonable because in the equilibrium for each of the scenarios I consider below, the expected price path does not decrease as departure date approaches (in fact, it strictly increases except for the constant pricing regime). To reduce unnecessary computation, I fix unobserved demand errors $\xi$ and supply errors $\omega$ at zero and focus on isolating other forces.

I study how dynamic pricing affects airlines’ profits and consumer welfare. I consider possible policy interventions that fix admissible pricing policies. By comparing different pricing regimes I single out the effects of different dynamic pricing techniques such as price discrimination and revenue management. Let $c_0$ be firms’ initial capacities. In this exercise, I use the empirical distribution of capacities at period 0 (i.e., 49 days before departure). I assume that
this distribution of capacities is exogenous. I consider the following three different pricing regimes.

(i) **Static pricing** Each firm $j$ can only charge a constant price $p_j \in \mathbb{R}^+$. They choose $p_j$ simultaneously at period 0 before any realization of demand uncertainty. They (know that both of them) cannot change prices later on. In equilibrium, their prices are mutual best responses that maximize own expected profits given the opponent’s strategy. The expectation is taken over all future demand uncertainty.

$$p_j^* = \arg \max_{p_j \in \mathbb{R}^+} E_0 \left[ \sum_{t=0}^T \Pi_j^t \left( p_j, p_{-j}^*, c_t \right) \right], \quad j = 1, 2.$$  

The equilibrium constant prices are the fixed points $(p_1^*, p_2^*)$. Define $V_j^{CST}$ as the total expected discounted profit in the constant-pricing equilibrium.

(ii) **Fixed-path pricing** Firms can set prices conditional on the number of days to departure. At period 0, each firm submits a price policy that is a function of the number of days to departure:

$$g_j^{CMT} : \underbrace{T}_{\text{time to departure}} \rightarrow \underbrace{\mathbb{R}^+}_{\text{current price}}.$$  

The two firms (know that both of them) cannot deviate from their price policies. In equilibrium, their price policies are mutual best responses that maximize own expected profits given the opponent’s strategy. The expectation is taken over all future demand uncertainty.

$$g_j^{CMT*} = \arg \max_{g_j^{CMT}} E_0 \left[ \sum_{t=0}^T \Pi_j^t \left( g_j^{CMT} (t), g_{-j}^{CMT*} (t), c_t \right) \right], \quad j = 1, 2.$$  

The equilibrium price policies are the fixed points $(g_1^{CMT*}, g_2^{CMT*})$. Define $V_j^{CMT}$ as the total expected discounted profit in the equilibrium of fixed-path pricing.

(iii) **Full dynamic pricing** Firms can set prices conditional on the number of days to departure and can adjust prices based on each other’s cumulative sales (or remaining capacities).

At period 0, each firm submits a price policy that is a function of the number of days to

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41 Ideally, one can start to keep track of a flight once it opens up to sale. This requires a horizon of more than 300 days and is impractical. Another practical approach is to collapse all the periods more than 49 days before departure into one “big” period. As long as the “big” period fits well with the empirical data, the amount of demand uncertainty 49 days before departure shall be similar with my current treatment. Thus it will not change the current result qualitatively. As a robustness check, it will come in the next version of the paper.
departure and the remaining capacities of both players:

\[ g_j^{\text{dyn}} : \mathcal{T} \times \mathcal{C}^j \rightarrow \mathbb{R}_+^+ . \]

This game is subgame perfect. In equilibrium, firms’ price policies are mutual best responses that maximize own expected profits given the opponent’s strategy. The expectation is taken over all future demand uncertainty.

\[ g_j^{\text{dyn}*} = \arg\max_{g_j^{\text{dyn}}} \mathbb{E}_0 \left[ \sum_{t=0}^{T} \Pi^t \left( g_j^{\text{dyn}}(t, c_t), g_{-j}^{\text{dyn}*}(t, c_t), c_t \right) \bigg| g_{-j}^{\text{dyn}*}, c_0 \right] , \quad j = 1, 2. \]

The equilibrium price policies are the fixed points \( \left( g_j^{\text{dyn}*}, g_{-j}^{\text{dyn}*} \right) \). Define \( V_j^{\text{dyn}} \) as the total expected discounted profit in the equilibrium of full dynamic pricing.

### 7.1. Price Discrimination

The estimates show that late-arriving consumers are indeed less price sensitive. Thus airlines charge higher prices in the late market to price discriminate against these high-valuation consumers. The existing theoretical work shows that under competition price discrimination has ambiguous effects on consumer welfare and firms’ profits. In this section, I empirically quantify these effects. In particular, I consider two relevant policy interventions. In the first scenario, airlines can only charge constant prices over time. In the second one, airlines can charge time-dependent prices. By comparing these two, I single out the effects of price discrimination.\(^{42}\)

Figure 11 compares the optimal fixed price paths with the optimal constant prices. Not surprisingly, each airline’s optimal constant price lies in between its highest and lowest discriminatory prices. Specifically, Alaska charges $450 and JetBlue charges $319. The $131 price gap suggests that the airlines are vertically differentiated under constant pricing. JetBlue keeps a low price and sells mostly to low valuation consumers. Alaska focuses on high valuation consumers by charging a high price. This pattern is driven by the fact that JetBlue has bigger market power in the segment of low type consumers whereas Alaska has bigger market power in the segment of high type consumers.

In the fixed-path pricing scenario, Alaska’s price starts from $366 seven weeks before departure. It quickly increases to $415 around six weeks before departure. It rises above Alaska’s

\(^{42}\)Strictly speaking, when firms are able to charge time-dependent prices, their prices would also account for the time value of a seat. I performed similar analysis when assuming away the capacity constraint. The result is similar—price discrimination shifts a substantial amount of consumer welfare to industry profit.
constant price around two weeks before departure. JetBlue’s fixed price path starts at $288 seven weeks before departure. It stays roughly at this level and increases slowly over time. Until around two weeks before departure, JetBlue’s price jumps to around $600 for the last two weeks before departure. In my numerical analysis, I find that under price commitment, firms’ optimal price paths always converge to “step functions”. The result is robust and stable – A firm charges a low price in the early periods and increases its price slowly over time. Then at a certain point, the firm increases its price substantially. This jump in price is especially interesting since the consumers’ distribution is “smooth” in time. Indeed, this pattern disappears under full dynamic pricing – the average price paths under full dynamic pricing are smooth.

More than two weeks before departure, constant prices are lower than fixed price paths. This reflects that price discrimination intensifies price competition in the low-end market. The price gap between the two firms looks similar with and without price discrimination. Within the last two weeks before departure, JetBlue raises its price substantially in order to extract more profit from its high valuation consumers. Its price is even higher than Alaska’s. This is because although JetBlue has a smaller high-type loyal segment than its opponent, its high-type consumers are more “loyal” (willing to pay more for its tickets). In the late market, both airlines price higher under price discrimination than under constant pricing. All in all, price discrimination softens competition in the late market because of airlines’ collective incentives to jack up prices in the late market.

Figure 12 shows the airlines’ sales paths under constant pricing and under price discrimination. Alaska sells more seats under price discrimination. Figure 12 shows that the incremental sales of Alaska under price discrimination mainly comes from the late market. Interestingly, Alaska prices higher in the late market under price discrimination than under constant pricing. This is because price discrimination softens competition – under price discrimination, the airlines collectively raise their prices in the late market. After JetBlue jacks up its price in the late market, Alaska-leaning high-type consumers who choose JetBlue under constant pricing find it less appealing. Some of them switch back to Alaska (industry demand elasticity is relatively low).

In addition, price discrimination causes a misallocation of seats. More seats are allocated to low valuation consumers under price discrimination than under constant pricing. The left panel of Figure 12 shows that under price discrimination JetBlue allocates more seats to the early market and allocate fewer seats to the late market.

Table 7 summarizes the change of consumer welfare by consumer segments. Consistent with the standard intuition of price discrimination, high-type consumers suffer from price discrimination. The welfare of Alaska-leaning high-type consumers decreases by -9.7%. The welfare of JetBlue-leaning high-type consumers decreases by -12.9%. Although low-type consumers’
welfare increases substantially in percentage, their monetary gain is much smaller than the monetary loss of high valuation consumers. Overall, consumer welfare decreases by -9.4%.

Meanwhile, the airlines’ revenues increase considerably. Table 8 shows that Alaska’s revenue increases by +$1567.38 per flight, which amounts to +13.6% of its total revenues. JetBlue’s revenue increases by +$1155.57 per flight, which amounts to +6.2% of its total revenues. These gains in revenue are significant comparing to the magnitudes of airlines’ margins. According to IATA, airlines’ margin is around 8% of its revenue in the past three years. Thus, my estimates suggest that the amount of gains in profit from price discrimination is substantial.

Overall, price discrimination is aligned with airlines’ collective incentives to raise prices in the late market. It softens competition and helps airlines extract more consumer surplus. As a result, social welfare decreases by −1.6%, or −$1135.16.

7.2. Revenue management

The estimates suggest that air travel demand is uncertain and can fluctuate substantially. Therefore, revenue management (pricing on remaining capacities) may have significant welfare implications. This exercise attempts to single out the effect of revenue management. To do so, I compare two pricing regimes—full dynamic pricing and fixed-path pricing. Under full dynamic pricing, the airlines are able to adjust prices based on remaining capacities. Under fixed-path pricing, the airlines submit their pricing policies at period zero, and cannot change their prices regardless of how good or bad their sales turn out to be. This removes the airlines’ ability to smooth the impacts of demand fluctuations on capacity utilization.

Figure 13 compares the fixed pricing paths with the average price paths under full dynamic pricing. When the airlines cannot adjust prices conditional on remaining capacities, they increase prices in the early periods. The effect is especially strong for Alaska. This is because Alaska has a large segment of high valuation consumers and many of them arrive late. When revenue management is turned off, Alaska raises its price substantially in the early market to save seats for its late-arriving high type consumers. To highlight this incentive, Figure 14 shows that if each firm were to commit to its average price path of full dynamic pricing, Alaska would be twice more likely to sell out all its seats comparing to the case when each firm commits on its optimal fixed price path. Thus, low price in the early market hurts Alaska, since it increases the probability of selling out all seats. Figure 15 shows the sales path under the two pricing regimes. It suggests that Alaska indeed tries to reserve more seats for its high valuation consumers. Its sales decrease in the early market but do not change much in the late market.

43Globally, the numbers are 4.6% 8.5%, 8.8%,7.5% for 2014-2017. These recent numbers are much higher than historical numbers. In general, the North American airlines’ margins are higher than average.
Figure 16 shows the probability of selling out with and without revenue management. Under revenue management, the airlines are able to raise their prices when they are selling out too soon. Therefore, the probability of selling out all the seats early on is very low. Without revenue management, the probability of selling out all the seats very early on becomes nontrivial. Figure 16 shows that without revenue management JetBlue is twice more likely to sell out all its seats before departure. In particular, there is a positive probability that JetBlue may sell out all its seats one month before departure.

Table 9 summarizes the effect of revenue management on consumer welfare. It shows that revenue management increases welfare substantially for all the four consumer segments. This result is driven by two effects: lower prices and higher supply. The two consumer segments that benefit the most from revenue management are Alaska’s high type consumers and JetBlue’s low type consumers. JetBlue’s low type consumers benefit from lower prices in the early market. Alaska’s high type consumers benefit from higher supply because revenue management allows Alaska to manage capacity more efficiently. These “extra” seats are allocated to consumers in the early market.

Table 10 shows that overall consumer welfare increases by a substantial 14.4%. Interestingly, firms’ revenues change little under revenue management. This is the result of two competing forces. On one hand, airlines manage their capacities more efficiently. This enables the firms to expand the market and sell to more consumers. On the other hand, this incentive intensifies price competition in the early market. The extra seats that the airlines can supply to the market are sold at low prices. Overall, the positive effect of increased sales is canceled out by the negative effect of intensified competition.

8. Conclusion

The digitalized marketplace allows firms to monitor demand and supply in real time and to continuously re-price. Dynamic pricing has become a common practice in many industries. Under competition, the effect of dynamic pricing is unclear due to the prisoner’s dilemma. This paper empirically investigates the competitive effects of dynamic pricing in the context of oligopolistic airline markets. I develop a dynamic oligopoly model where firms sell limited capacities under demand fluctuations. I estimate the model and show that (i) price discrimination softens competition in the high-end market and increases profits substantially and (ii) revenue management (pricing on remaining capacities) intensifies competition and does not increase profits.

Future research may look at the competitive effect of information sharing. The current paper abstracts away from strategic information sharing, although the fixed-path pricing can be
viewed as an extreme case when all demand information is turned off. In addition, endogenizing capacity choices may also yield interesting results.

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Edgeworth, F. Y., 1925. The pure theory of monopoly. Papers relating to political economy 1, 111–142.


9. Figures

This graph shows average price path for JetBlue Flight 19, which departed from March 01, 2016, to June 01, 2016. The red line shows average price conditional on the number of days to departure. The light-blue area shows one standard deviation of prices.

Fig. 1. Average price path: an example
This graph shows the history of prices and remaining seats for a particular flight – JetBlue Flight 19, which departed on March 25, 2016. The red line shows the path of remaining seats. The blue line shows the path of prices. The shaded regions highlight some suggestive evidence of revenue management. In the light-blue area, seats were sold out quickly (the red line with circles dropped quickly), then the price (the blue line with squares) was increased. In the light-yellow area, seats were sold out slowly (the red line with circles was flat), then the price (the blue line with squares) dropped.

Fig. 2. Data on price and remaining capacity: an example
This graph provides suggestive evidence that the trends in average prices were parallel across the treatment route and the control route. Each dot is an average price for flights departing on a given departure date in a given route. It is averaged over $N = \#\text{firms} \times \#\text{directions} \times \#\text{pricing-dates}$ observations of prices. Each line is smoothed using Gaussian kernels. Before March 31, 2016, price gaps were stable over time across the two routes. The gap changed after Delta’s exit. Route-level mean prices increased substantially in the exit route but did not change as much in the control route.

Fig. 3. Parallel trend in average price for the treatment route and the control route before the exit event (local difference-in-differences design)
This graph singles out Alaska’s price response to Delta’s exit. It compares Alaska’s average prices under the 2-by-2 treatment conditions. Alaska raised its price significantly only in the route where its competitor exited. Each dot is an average price for flights departing on a given departure date in a given route. It is averaged over $N = \#\text{directions} \times \#\text{pricing-dates}$ observations of prices. The light blue areas are at 95% confidence interval ($N=15$, i.e. the number of departure dates).

Fig. 4. Treatment effect on Alaska's mean price (regression discontinuity design)
The graph shows Alaska's average price paths under the 2-by-2 treatment conditions. It illustrates heterogeneous treatment effects across different numbers of days to departure. Each dot is an average price for flights departing under a given condition for a given number of days to departure. It is averaged over $N = \#\text{directions} \times \#\text{departure-dates}$ observations of prices.

Fig. 5. Heterogeneous treatment effects on Alaska's average price path
Note: This graph shows how Delta's exit affects Alaska's price. The blue line with squares is the change in prices observed in the data. The red line with circles is the change in prices predicted by the model.

Fig. 6. Effect of exit on Alaska's price: predicted vs real
Note: The graph plots Poisson arrival rates for different segments of consumers in the two routes. The upper graph shows the arrival distribution in the exit route. The lower graph shows the arrival distribution in the control route.

Fig. 7. Estimated arrival by consumer segments
Note: This graph shows how estimated choice probabilities change over time by consumer types in the control route. The blue line shows the probability of choosing Alaska and the red line shows the probability of choosing JetBlue.

Fig. 8. Estimated choice probability by segments

Note: This graph shows the ratio of consumers who choose JetBlue among all Alaska's consumers who buy tickets. It is calculated as the number of Alaska's consumers that choose JetBlue divided by the number of Alaska's loyal consumers that choose inside options. For example, 7 weeks before departure around 32% of Alaska-leaning consumers who buy tickets choose the opponent. The ratio is low in the two ends.

Fig. 9. Business stealing by number of days to departure
Note: This graph shows own price elasticity and industry elasticity (in absolute values) using the empirical distribution of capacities. The elasticity is evaluated at the optimal dynamic prices. Own demand elasticity is defined as the percentage change of own sales with respect to one percentage change in own price. Industry demand elasticity is defined as the percentage change of industry sales when both firms increase their prices by one percentage.

Fig. 10. Estimated elasticities
Note: This graph compares the case of fixed-path pricing with the case of constant pricing. The solid lines are optimal fixed price paths and the the dashed lines are optimal constant prices. Alaska’s prices are in red, and JetBlue’s prices are in blue.

Fig. 11. Constant price vs price on time only
Note: this graph shows the average number of seats sold each day as departure date approaches. The solid line (blue) is for the case of fixed price path. The dashed line (yellow) is for the case of constant pricing. The left panel is for Alaska, and the right panel is for JetBlue.

Fig. 12. Sales: constant price vs price on time
Note: This graph compares the case of full dynamic pricing (with revenue management) with the case of fixed-path pricing (without revenue management). The solid lines are average price paths under full dynamic pricing and the dashed lines are optimal fixed price paths. Alaska's prices are in red, and JetBlue's prices are in blue.

Fig. 13. Price on time and capacities vs price on time only
Note: This graph shows the probability of selling out all seats under fixed-path pricing. It compares the case when firms commit on optimal price path (yellow-dashed) with the case when they commit on average price path of full dynamic pricing (blue-solid). Alaska is on the left, and JetBlue is on the right.

Fig. 14. Probability of selling out all seats: commit on optimal fixed price path vs commit on average price path of full dynamic pricing
Note: this graph shows the average number of seats sold each day as departure date approaches. The solid line (blue) is for the case of full dynamic pricing (with revenue management). The dashed line (yellow) is for the case of fixed-path pricing (without revenue management). The left panel is for Alaska, and the right panel is for JetBlue.

Fig. 15. Sales: price on time and capacities vs price on time
Note: This graph shows the probability of selling out all seats. It compares the case of full dynamic pricing (blue-solid) with the case of fixed-path pricing (yellow-dashed). Alaska is on the left, and JetBlue is on the right.

Fig. 16. Probability of selling out all seats: price on time and capacities vs price on time

Note: this graph shows a screenshot of some source codes of one major airline. From these data, one can learn the price, the characteristics and the status of each individual seat for a particular flight at any point in time. It is the information source for seat maps.

Fig. 17. Source code from an airline's official website
Note: this graph shows the dynamic patterns for prices and sales. The upper graph shows the average paths for price (red-circle) and loading factor (blue-square). The lower graph shows the probability of a price hike (red-circle) or drop (blue-square) in a given day.

Fig. 18. Dynamic patterns for prices and sales (estimation data)
Note: this graph shows the fitting errors for the GMM. It is calculated as the absolute value of the residuals divided by their empirical means.

Fig. 19. Fitted error for all (469) moments
Note: this graph compares the model predicted prices with the real prices conditional on the number of days to departure. It is organized by the 2-by-2 research design. The dashed lines are model predicted prices and the solid lines are the observed prices.

Fig. 20. Fitted price path
## 10. Tables

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Mean</th>
<th>Median</th>
<th>St. Dev.</th>
</tr>
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<tbody>
<tr>
<td><strong>Observations</strong> (N=10,290)</td>
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<td></td>
<td></td>
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<tr>
<td>Price ($)</td>
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<td>268.00</td>
<td>154.63</td>
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<tr>
<td>Number of seats sold daily</td>
<td>1.11</td>
<td>0</td>
<td>1.87</td>
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<td>0</td>
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<tr>
<td><strong>Products</strong> (N=210)</td>
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<td>Capacity</td>
<td>64.27</td>
<td>64.00</td>
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<td>Average total sales</td>
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<td>Load factor</td>
<td>0.85</td>
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<tr>
<td>Gini coefficient</td>
<td>0.21</td>
<td>0.22</td>
<td>0.08</td>
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</tbody>
</table>

Notes:

(i) Market: directional city pair × departure date. (N=4×30=120)
(ii) Product: market × firm. (N=120×1.75=210)
(iii) Observation: product × pricing date. (N=210×49=10,290)
Table 3: Consumers’ preferences in the exit route

<table>
<thead>
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<tr>
<td>Price coefficients</td>
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<tr>
<td>Alaska’s consumers’ preference to</td>
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<tr>
<td>Alaska</td>
<td>2.578</td>
</tr>
<tr>
<td></td>
<td>(0.042)</td>
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<tr>
<td>Delta</td>
<td>0.035</td>
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<td></td>
<td>(0.085)</td>
</tr>
<tr>
<td>Delta’s consumers’ preference to</td>
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</tr>
<tr>
<td>Alaska</td>
<td>0.131</td>
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<tr>
<td></td>
<td>(0.065)</td>
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<tr>
<td>Delta</td>
<td>4.400</td>
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<td>(0.049)</td>
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Note: Price coefficients on $100.
Table 4: Consumers’ preferences in the control route

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<tr>
<td>Price coefficients</td>
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<tr>
<td>−0.383 (0.001)</td>
<td>−1.535 (0.009)</td>
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<td>Alaska’s consumers’</td>
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<td>preference to</td>
<td>1.077 (0.010)</td>
<td>0.821 (0.017)</td>
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<tr>
<td>Alaska</td>
<td>0.434 (0.009)</td>
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<td>Jetblue’s consumers’</td>
<td>0.703 (0.026)</td>
<td>0.087 (0.019)</td>
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<td>preference to</td>
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<td>3.257 (0.027)</td>
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Note: Price coefficients on $100.
Table 5: Consumer arrival and segmentation

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<td><strong>Probability of preferring Alaska</strong></td>
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<td>(0.004)</td>
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<td>(0.008)</td>
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<td><strong>Poisson arrival</strong></td>
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<td>(0.013)</td>
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<td>$\gamma^1_{arrival}$</td>
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<td></td>
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<td>(0.009)</td>
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<td>$\gamma^2_{arrival}$</td>
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<td></td>
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<td>(0.004)</td>
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<td>(6e-4)</td>
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<td><strong>Probability on types</strong></td>
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<td>(2e-4)</td>
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<td>----------------------------</td>
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<tr>
<td><strong>Preference shock</strong></td>
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<td>After dummy</td>
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<td><strong>Weekend dummy</strong></td>
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<td><strong>Unobserved shock</strong></td>
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<td>$\sigma_\omega$</td>
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<td>$\sigma_\xi$</td>
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<td><strong>Cost parameter</strong></td>
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<td>Alaska</td>
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<td></td>
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<tr>
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<tr>
<td></td>
<td>(0.150)</td>
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</tr>
<tr>
<td>JetBlue</td>
<td>30.937</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.960)</td>
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Table 7: The effect of price discrimination on consumer welfare by segment

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<tr>
<th>Welfare Change</th>
<th>High Type</th>
<th>Low Type</th>
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<tbody>
<tr>
<td>Alaska-leaning</td>
<td>-$ 2082.88 (-9.7%)</td>
<td>+$47.3135 (+29.8%)</td>
</tr>
<tr>
<td>JetBlue-leaning</td>
<td>-$ 2087.95 (-12.9%)</td>
<td>+$265.382 (+15.9%)</td>
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Table 8: The effect of price discrimination on profits and welfare

<table>
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<th>Fixed-path pricing</th>
<th>Change</th>
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<tr>
<td>Alaska's Profits</td>
<td>$11503.38</td>
<td>$13070.76</td>
<td>+$1567.38 (+13.6%)</td>
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<tr>
<td>JetBlue's Profits</td>
<td>$18694.74</td>
<td>$19850.32</td>
<td>+$1155.57 (+6.2%)</td>
</tr>
<tr>
<td>Consumer Surplus</td>
<td>$ 39584.58</td>
<td>$35726.46</td>
<td>-$3858.12 (-9.4%)</td>
</tr>
<tr>
<td>Total Welfare</td>
<td>$69782.71</td>
<td>$68647.55</td>
<td>-$1135.16 (-1.6%)</td>
</tr>
</tbody>
</table>

Table 9: The effect of revenue management on consumer welfare by segment

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<tr>
<th>Welfare Change</th>
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<th>Low Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alaska-leaning</td>
<td>+$2601.22 (+13.3%)</td>
<td>+$581.16 (+282.3%)</td>
</tr>
<tr>
<td>JetBlue-leaning</td>
<td>+$951.67 (+6.7%)</td>
<td>+$1022.92 (+53.0%)</td>
</tr>
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</table>

Table 10: The effect of revenue management on profits and welfare

<table>
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<th>Full dynamic pricing</th>
<th>Change</th>
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</thead>
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<tr>
<td>Alaska's Profits</td>
<td>$13070.76</td>
<td>$13031.33</td>
<td>-$39.43 (-0.3%)</td>
</tr>
<tr>
<td>JetBlue's Profits</td>
<td>$19850.32</td>
<td>$19726.35</td>
<td>-$123.97 (-0.6%)</td>
</tr>
<tr>
<td>Consumer Surplus</td>
<td>$ 35726.46</td>
<td>$40883.43</td>
<td>+$5156.97 (+14.4%)</td>
</tr>
<tr>
<td>Total Welfare</td>
<td>$68647.55</td>
<td>$73641.11</td>
<td>+$4993.55 (+7.3%)</td>
</tr>
</tbody>
</table>
11. Appendix

11.1. Example: competitive price discrimination

The following toy model summarizes the ambiguous welfare and profit effects of competitive price discrimination in the context of airline price discrimination (Holmes (1989)). Consider that United and Delta compete for travelers from San Francisco to Boston. For a given flight, consumers can be segmented into two markets, i.e. late market and early market. For now, (1) ignore capacity constraint; (2) assume that each market is endowed with simple logit demand; (3) firms are symmetric. Late consumers are less price sensitive relative to early consumers. The impacts of price discrimination depend on the elasticity ratio. As shown in Figure 11.1, the signs of price discrimination on both total industry profit and social welfare are not certain. When the early market’s cross elasticity is high, price discrimination is more driven by excessive private incentives of airlines to undercut each other. This undercutting decreases both social welfare and their own profits. On the other hand, when the early market’s industry elasticity is high, price discrimination is more driven by airlines’ collective incentive. It is thus more likely to increase profits.

![Figure 21. Price discrimination on profit and welfare under competition](image)

11.2. Proposition 1

Proposition 1 follows immediately from the following result.

Claim 1. Let $X$ be the number of consumers that arrive and $Y_j$ be the number of arrived consumers who choose alternative $j$, with $X = \sum_{j \in \{-1,0,1,\ldots,J\}} Y_j$. Let $s_j$ be the probability that an arrived consumer chooses alternative $j$, with $\sum_{j \in \{-1,0,1,\ldots,J\}} s_j = 1$. The following two statements will generate the same joint distribution:

1. $X \sim \text{Pois} \left( \lambda \right)$ and $(Y_j \mid X = x) \sim \text{Multinorm} \left( x, s_j \right)$.

In this example, the price coefficients are $-0.005$ and $-0.015$ in late and early market respectively. The late market is twice the size of the early market. Utility intercept for consumers in the late market is fixed at zero.
Proof.

\[
\begin{align*}
\text{Let } Y_j \sim \text{Pois} \left( \lambda_j \right) \text{ and } Y_j \perp Y_{j'}, \forall j' \neq j.
\end{align*}
\]

That is, we must have that \( f^{(1)}(y_{-1}, y_0, y_1, \ldots, y_J) = f^{(2)}(y_{-1}, y_0, y_1, \ldots, y_J) \).


11.3. Market size identification

Consider a static duopoly pricing game with simple logit demand. Suppose we observe two realizations of market outcomes under an exogenous supply shock, \((\text{price}_1^A, \text{price}_2^A, \text{sale}_1^A, \text{sale}_2^A)\) and \((\text{price}_1^B, \text{price}_2^B, \text{sale}_1^B, \text{sale}_2^B)\). One can identify four demand parameters \(\alpha, \delta_1, \delta_2, M\), i.e., price coefficient, two product intercepts and market size. Without loss of generality, we normalize the outside option’s mean utility to zero.

- \(\alpha = \log \frac{\text{sale}_1^A / \text{sale}_1^B}{\text{sale}_2^A / \text{sale}_2^B} - \log \frac{\text{sale}_1^B / \text{sale}_1^A}{\text{sale}_2^B / \text{sale}_2^A} - \alpha \times (\text{price}_1^A - \text{price}_2^A).
\]

\[
\begin{align*}
\text{share}_1^A / \text{share}_1^B &= \frac{\text{sale}_1^A / \text{sale}_1^B}{\text{share}_1^A / \text{share}_1^B} = \frac{\text{share}_1^A / \text{share}_1^B}{\text{exp}(\alpha \times \text{price}_1^A + \delta_1)} \\
&= \frac{\text{exp}(\alpha \times \text{price}_1^A + \delta_1)}{\text{exp}(\alpha \times \text{price}_1^B + \delta_2)} \\
&= \frac{\text{exp}(\alpha \times (\text{price}_1^A - \text{price}_1^B) + (\delta_1 - \delta_2))}
\end{align*}
\]

- \(\delta_1 - \delta_2 = \log \frac{\text{sale}_1^A / \text{sale}_1^B}{\text{sale}_2^A / \text{sale}_2^B} - \alpha \times (\text{price}_1^A - \text{price}_2^A).
\]

\[
\begin{align*}
\text{sale}_1^A / \text{sale}_2^A &= \frac{\text{exp}(\alpha \times \text{price}_1^A + \delta_1)}{\text{exp}(\alpha \times \text{price}_2^A + \delta_2)} \\
&= \text{exp} \left[ \alpha \times (\text{price}_1^A - \text{price}_2^A) + (\delta_1 - \delta_2) \right]
\end{align*}
\]

- \(M = \text{sale}_1^B + \text{sale}_2^B + \frac{\text{sale}_1^A / \text{sale}_1^B - \text{sale}_1^A / \text{sale}_2^B}{\text{sale}_1^A / \text{sale}_1^B \times \text{exp} \left[ \alpha \times (\text{price}_1^B - \text{price}_1^A) \right]} - 1.
\]
Under condition A, outside option has “sales” $sale^A_0$:

$$sale^A_0 = share^A_0 \times M$$

$$= share^A_0 \times \frac{sale^A_1}{share^A_1}$$

$$= sale^A_1 \times \frac{share^A_0}{share^A_1}$$

$$= sale^A_1 \times \frac{1}{\exp(\alpha \times price^A_1 + \delta_1)}$$

Under condition B, outside option has “sales” $sale^B_0$:

$$sale^B_0 = share^B_0 \times M$$

$$= share^B_0 \times \frac{sale^B_1}{share^B_1}$$

$$= sale^B_1 \times \frac{share^B_0}{share^B_1}$$

$$= sale^B_1 \times \frac{1}{\exp(\alpha \times price^B_1 + \delta_1)}$$

Divide both sides of the two equations:

$$\frac{sale^A_0}{sale^B_0} = \frac{sale^A_1}{sale^B_1} \times \exp[\alpha \times (price^B_1 - price^A_1)]$$

Note that

$$\frac{sale^A_0}{sale^B_0} = \frac{M - sale^A_1 - sale^A_2}{M - sale^B_1 - sale^B_2}$$

Combine the two equations we must have:

$$\frac{M - sale^A_1 - sale^A_2}{M - sale^B_1 - sale^B_2} = \frac{sale^A_1}{sale^B_1} \times \exp[\alpha \times (price^B_1 - price^A_1)]$$

Note this is one linear equation wit one unknown. We can solve

$$M = sale^B_1 + sale^B_2 + \frac{sale^B_1 \times \left(\left(sale^B_1 + sale^B_2\right) - (sale^A_1 + sale^A_2)\right)}{sale^A_1 \times \exp[\alpha \times (price^B_1 - price^A_1)] - sale^B_1}$$

- It is easy to see that $\delta_1 = 0 = \log \left[\frac{sale^A_1}{M - sale^A_1 - sale^A_2}\right] - \alpha \times price^A_1$. 

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