Gridlock and Inefficient Policy Instruments$^1$

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December 21, 2016

$^1$This paper has benefited from the comments of seminar participants at at Banco De Espana, CERGE-EI, Prague, Columbia University, Nottingham University, University of Oslo, Princeton University, University of Rochester, and Washington University. In particular, we are grateful to Vincent Anesi, Dan Bernhardt, Alessandra Casella, Ying Chen, John Duggan, Tassos Kalandrakis, Navin Kartik, Carlo Prato, Daniel Seidmann, Michael Ting and Jan Zapal. David Austen-Smith is also grateful for support under the IDEX Chair, "Information, Deliberation and Collective Choice" at IAST, Toulouse. All responsibility for any errors or views herein, however, lies exclusively with the authors.
Abstract

Why would rational politicians choose inefficient policy instruments? Environmental regulation, for example, often takes the form of technology standards and quotas even when cost-effective Pigovian taxes are available. In our model with multiple legislative veto players, inefficient policy instruments are politically easier than efficient instruments to repeal in dynamic environments subject to policy-relevant stochastic shocks. Anticipating this, heterogeneous legislators agree more readily on an inefficient policy instrument. We show when inefficient instruments are likely to be chosen, and predict that they will be used more often in polarized political environments and for issues where the fundamentals change relatively fast.
1 Introduction

Despite the sometimes considerable differences in perspective, economists often agree on the optimal policy instrument with which to address a particular problem. A well-known example is the regulation of environmental problems. Economists from the left to the right tend to recommend Pigou taxes because they are cost-effective, require little information, and offer a "double dividend" whereby emission taxes generate public revenues that allow governments to reduce other, distortionary, taxes.¹ It is thus ‘a mystery’, according to some economists, why (at the time of writing) the Republican Party in the US has blocked such a market-based policy, since doing so effectively led to the quantity controls on power plants introduced by the Obama administration in 2015.²

The gap between economically efficient and politically feasible policy interventions can be very costly. Minimizing these costs requires identifying second-best alternatives, that is, policy recommendations that are efficient subject to being politically feasible. Therefore, as is widely appreciated in the literature,³ it is as important to understand the political realities constraining policy choice as it is to identify (politically unconstrained) first-best economic recommendations. In this paper, we develop a parsimonious theory of the choice between using an efficient and an inefficient instrument to manage political or economic shocks.

The model rests on two characteristics. First, we recognize that policy-making frequently involves multiple pivotal players. Enacting or repealing regulation requires the consent of veto players who typically have different opinions on the desirability of the policy. Second, our model is dynamic, admitting an endogenous status quo. The policy agreed upon in one period becomes the status quo in the next. Combined, these two ingredients imply that the pivotal legislator for introducing any regulation is distinct from the legislator pivotal for any subsequent repeal of that regulation. Consequently, anticipating the potential loss of

¹Weitzman (1974) compared quotas and taxes in a setting with incomplete information and zero value of the tax revenues. But starting with Tullock (1967), there is a large literature in economics on the double dividend. While a "strong" version of it is controversial, the "weak" version—that the revenues reduce overall distortions compared to a setting without these revenues—is generally accepted. Only the weak version is required for the argument we make here. For surveys on the literature on the double dividend, see Bovenberg (1999), Sandmo (2000), Goulder (2002), or Jorgensen et al. (2013). In part because of the double dividend, all but four of fifty one prominent economists surveyed in 2011 agreed that a carbon tax would be the less expensive way to reduce carbon-dioxide emissions. (http://www.igmchicago.org/igm-economic-experts-panel/poll-results?SurveyID=SV_9Rezb430SESUA4Y)

²On this ‘mystery,’ see: http://www.nytimes.com/2015/07/01/business/energy-environment/us-leaves-the-markets-out-in-the-fight-against-carbon-emissions.html. While Pigou taxes are relatively rare also internationally, a famous exception is British Columbia, which introduced a carbon tax in 2008. Although initially controversial, the tax has gained support from all important stakeholders thanks to the rebates in other taxes that the revenues permit (http://www.nytimes.com/2016/03/02/business/does-a-carbon-tax-work-ask-british-columbia.html?smid=pl-share&_r=0).

³See, for example, Dewatripont and Roland (1995).
political influence, the pivotal legislator for implementing a policy has an incentive not to introduce the regulation in the first place. Hence, there is more gridlock and status-quo bias in the dynamic model than in the static case.

The contribution of this paper is to explore how gridlock varies systematically with the choice of policy instrument. In particular, an efficient and attractive regulatory instrument is difficult to repeal, even if the regulatory problem and the need for regulation is diminished. Anticipating the persistence of efficient instruments, the most reluctantly interventionist pivotal legislator may veto an appropriate policy intervention that would have been acceptable in a static environment. In contrast, a policy instrument that is considered less attractive by everyone, and which would thus have been Pareto dominated in a static setting, is easier to repeal should circumstances change. For this reason, the most efficient instrument might not be politically feasible, yet the less efficient instrument may still be approved by all veto players. In terms of the example above with emission taxes vs. quotas, it is exactly the benefit of the tax instrument—the double dividend—that makes the instrument hard to repeal once implemented and, therefore, less likely than the quota policy to be approved today.

The theory also sheds light on the mapping from the political system to the choice of policy instrument. If decisions require super-majorities or must pass multiple legislative chambers, then there is a larger set of veto players and the ideological distance between them is widened. Gridlock and status-quo bias can be severe in these circumstances, and it is then particularly likely that only the less efficient instrument will be approved. Furthermore, the inefficient instrument is more likely to be preferred and approved if the economic environment is volatile and future policy preferences are uncertain, since it is the relative ease with which inefficient interventions today may be repealed later if warranted.

These positive results are derived and formalized below. While a serious empirical investigation must await future research, note that our results can also be interpreted normatively: we find, in contrast to a static setting, conditions under which all veto players are better off in a dynamic environment by including an inefficient instrument as an available policy option.

Environmental regulation is not the only example of an economically efficient intervention being foregone in favor of an inefficient, but apparently more politically feasible, policy. For example, taxation is argued to be more efficient at curbing systemic risk, but we tend to see non-price regulation. Responding to fiscal need, we see governments resorting to increases in distortionary taxation instead of taxing negative externalities or removing inefficient loopholes. Similarly, redistributive subsidies are often distorting rather than being lump-sum. The prevalence of such inefficient instruments has proved a puzzle, especially in view of the argument that political competition should eliminate inefficiencies (Becker (1976, 1983),
Wittman (1989)). Consequently, there is a literature exploring a variety of reasons for the widespread use of inefficient policies to which we now turn.

**Related literature.** While our model is unique in the way it studies the choice of instrument in a dynamic game, there is a growing literature on dynamic policy making with an endogenous status quo. Most of this literature analyzes models in which preferences do not evolve over time, and policy dynamics occur because the proposer changes (e.g., Baron 1996, Kalandrakis 2004, Bowen et. al. 2014, Anesi and Seidmann 2015, Buisseret and Bernhardt 2016a), or because the same proposer seeks the support of different coalitions over time (e.g., Bernheim, Rangel and Rayo 2006, Diermeier and Fong 2006). In contrast, in our model, each proposer always seeks the support of the same policy maker and the qualitative nature of policy inefficiencies is, by and large, independent of the allocation of bargaining power. More closely related to this paper, Riboni and Ruge Murcia (2008), Zapal (2011), Duggan and Kalandrakis (2012), Bowen, Chen, Eraslan and Zapal (2015), and Dziuda and Loeper (2015, 2016) consider models of dynamic bargaining with policy-preference shocks. In particular, Dziuda and Loeper (2016) analyze a model that is similar to ours except that they study only one type of policy. They show that policy disagreement occurs for a relatively large set of states. In such states, the status quo can persist even when Pareto dominated. In their setting, however, any inefficiency is due to status quo inertia, or gridlock, with any policy change always being a Pareto improvement. By allowing for multiple instruments, the current paper shows that equilibrium behavior can involve Pareto-dominated policy changes. At the same time, it turns out that the availability of inefficient policy instruments can mitigate the gridlock, leading to more responsive policy decision making.

Unlike the current paper, much of the existing explanations for the use of inefficient policies focus on characteristics of policy-making other than the institutional logic of legislative choice per se. An exception is Spolaore (2004) who assesses the relative efficiency of policy interventions across three stylized organizational forms in a continuous time model. Unlike in our model, legislators’ policy preferences are fixed over time and every policy is efficient. Any inefficiencies that arise in Spolaore’s framework are due to costly delays in enacting some intervention, i.e. gridlock with respect to which of several efficient policies to implement at any time, rather than to a strategic choice of an inefficient instrument. There

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4This literature, as this paper, assumes that the implemented policy becomes the next status quo but does not affect the evolution of the state. A different strand of literature considers instead an exogenous status quo and assumes that the policy influences future states (see Hassler et al., 2003, for example). In Strulovici (2010), an endogenous status-quo bias arises if a majority learns that the new policy is beneficial for them. Concerned about this situation, it may be difficult to agree on trying out new policies in the first place. Baldursson and von der Fehr (2007) is closer to our story, as they argue that a relatively "brown" party may prefer quotas rather than taxes, because the relatively inefficient quotas are essentially property rights that are difficult to tighten or remove later.
is also a political science literature that explores inefficient policy making due to structural characteristics of legislative decisionmaking (the filibuster, for instance). In particular, Krehbiel (1998) and Brady and Volden (2006), among others, explore models of gridlock and government inaction, but do not focus on any strategic choice of inefficient policies.

Of necessity, explanations that consider policy choice by a unitary actor, whether a single legislator or a fully coordinated party or group, invoke aspects of the political economic environment other than legislative design. Coate and Morris (1995) and Acemoglu and Robinson (2001) both offer explanations of inefficient policy choices as arising from an incumbent legislator’s efforts to retain office. Whereas the Coate and Morris (1995) account rests on asymmetric information and the strategic use of policy choice as a signal of the incumbent’s motivations, Acemoglu and Robinson (2001) generate policy inefficiency from a model in which a group structures distortionary transfer payments to counter the group’s declining political support and influence. Tullock (1993), Grossman and Helpman (1994), Becker and Mulligan (2003) and Drazen and Limao (2008) also focus on groups and argue in various ways that any resource transfer increases wasteful lobbying (rent-seeking) activity by those groups. By committing itself to inefficient transfers, the government can reduce the level of wasteful lobbying.

More generally, there is an extensive literature on policy distortions induced through special interest groups’ lobbying and campaign contribution activities (see Wright 1996 and Grossman and Helpman 2001, for overviews of this literature). Similarly, Aidt (2003) claims that inefficient command-and-control instruments are more bureaucracy intensive and, to the extent that bureaucrats influence policy design and derive value from implementing policy, such interventions are favored by bureaucrats. At the electoral level, Canes-Wrone, Herron and Shotts (2001) consider elections with asymmetric information and provide conditions where incumbents seeking reelection can be expected to select policies contrary to voters’ (common value) interests. Similarly, Buisseret and Bernhardt (2016b) show that when current agreement distorts voters’ incentives about the identity of the future governments, efficient agreements may fail to realize. We abstract from electoral considerations in order to highlight the inefficiency of the legislative process per se. Alesina and Passarelli (2014) and Masciandaro and Passarelli (2013) offer explanations of socially suboptimal policies that hinge on the median voter failing to internalize the costs and benefits to others when policies have different distributional consequences. However, both available policy choices are Pareto optimal in these papers. In contrast to these approaches, inefficiency in our model does not depend on groups, informational asymmetries, or reelection concerns.
2 Example: Prices vs. Quantities

In this section we present a simple, stylized example to illustrate the key mechanism and to preview some of our results. The mechanism requires two periods and two pivotal players, or legislators, \( L \) and \( R \), both of whom must approve any change in environmental policy for that change to be implemented. Legislators can move away from no intervention \((n)\) by introducing either an emission quota \((q)\) or an emission tax, i.e., a price instrument \((p)\). The emission reduction gives everyone a benefit \( \theta \), and the costs associated with \( p \) and \( q \) are denoted \( w_i \) and \( w_i + e_i \), respectively, for \( i \in \{L, R\} \).

To microfound this preference parameterization, suppose that a representative firm can cut emissions from two to one unit, but the firm’s cost of doing so is one. Let \( \pi_i > 0 \) be the weight that legislator \( i \in \{L, R\} \) assigns to the firm’s profit; in particular, assume \( \pi_R > \pi_L \) so \( R \) is more business-friendly than \( L \). With an emission tax, the tax must be (at least) of size one to induce the firm to cut emissions. In this case, the cost to the firm is two, because the firm cuts emissions by one unit and pays taxes on the remaining unit. The tax revenue of one is valuable to the government since other taxes can be reduced accordingly. If the value of public revenues equals some \( \lambda > \max_i \pi_i \), the (net) cost of the tax is \( w_i = 2\pi_i - \lambda \) while the cost of the quota is \( \pi_i = w_i + e_i \), where \( e_i \equiv \lambda - \pi_i > 0 \), \( i \in \{L, R\} \).

Since \( e_i > 0 \), both players prefer \( p \) to \( q \) in a static setting because of the ‘double dividend’ associated with the tax revenues. For the same reason, policy \( p \) is more likely to be introduced than \( q \), since it is accepted by \( i \) whenever \( \theta \geq w_i \), while \( q \) is accepted only when \( \theta \geq w_i + e_i \). We can immediately make some simple observations:

**Proposition 0** Suppose there is only one period.

(i) Emissions are regulated for a larger set of benefits \( \theta \) when the policy menu is \( \{n, p\} \) than when the menu is \( \{n, q\} \).

(ii) With menu \( \{n, p, q\} \), every player proposing regulation proposes \( p \), and never \( q \).

(iii) Both players prefer menu \( \{n, p\} \) to \( \{n, q\} \), while \( \{n, p\} \) and \( \{n, p, q\} \) are equivalent since \( q \) is never used.

As a comparison, suppose now that there are two periods and let \( \delta \geq 0 \) be the common discount factor. The status quo in the first period is \( n \) and the first-period policy becomes the status quo in the second period. Since the legislators have different preferences, there are states in which the players agree on the need for a policy, and there are states in which they disagree. For simplicity, consider an environment in which, in each period, \( \theta \in \{\underline{\theta}, \bar{\theta}\} \), where \( \theta = \bar{\theta} > w_R + e_R \) with probability \( 1 - \rho \) and, with probability \( \rho \), \( \theta = \underline{\theta} \in (w_L, \min\{w_L + e_L, w_R\}) \). By construction, if \( \theta = \bar{\theta} \) both players prefer \( p \) to \( q \), and
q to n; if $\theta = \bar{\theta}$, however, player $R$ prefers $n$ to $p$, and $p$ to $q$, while $L$ prefers $p$ to $n$, and $n$ to $q$. Thus, if $\theta = \bar{\theta}$ in the second period, a first-period intervention using a quota would be repealed, but a first-period intervention using the price instrument remains in effect.

If the policy menu is $\{n, q\}$, so that $p$ is not available, then $q$ would be agreed on when $\theta = \bar{\theta}$ and $n$ when $\theta = \bar{\theta}$, regardless of the status quo. If instead the policy menu is $\{n, p\}$ and the first-period policy remains the status quo in the next period, then $R$ recognizes that, by accepting $p$ in the first period, $p$ remains in effect regardless of the second-period state.

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If, however, the first-period policy is $n$, the two players agree on $p$ in the second period if and only if $\theta = \bar{\theta}$. Anticipating these possibilities, $R$ accepts $p$ in the first period only if the benefit of such a policy today is larger than the expected cost tomorrow:

$$\bar{\theta} - w_R \geq \delta \rho (w_R - \bar{\theta}) \iff \delta \leq \hat{\delta}_{np} \equiv \frac{\bar{\theta} - w_R}{\rho (w_R - \bar{\theta})}.$$

Thus, if the future is sufficiently important (specifically, $\delta > \hat{\delta}_{np}$), then $R$ does not accept $p$ even though $R$ would have accepted the less efficient instrument, $q$. In this situation, the menu $\{n, q\}$ is clearly giving both players a higher utility than the menu $\{n, p\}$.

Suppose now that the policy menu is $\{n, p, q\}$, so that all three policies are available. If $\theta = \bar{\theta}$ in the second period, both players agree on policy $p$. If $\theta = \bar{\theta}$ in the second period, the players cannot agree on a change in policy if the status quo is either $n$, where $R$ blocks change, or $p$, where $L$ blocks change. However, if the first-period policy is $q$, then $L$ prefers a change to $p$, while $R$ prefers a change to $n$. Suppose $b_L \in [0, 1]$ is the probability that $L$ has the authority to make a take-it-or-leave-it policy proposal in the second period, and $1 - b_L$ is the probability that $R$ has authority to make such a proposal. Then, $R$ prefers $q$ to $n$ in the first period only if the benefit of $q$ today is larger than its expected cost tomorrow:

$$\bar{\theta} - w_R - e_R \geq \delta b_L (w_R - \bar{\theta}) \iff \delta \leq \hat{\delta}_{npq} \equiv \frac{\bar{\theta} - w_R - e_R}{\rho b_L (w_R - \bar{\theta})}.$$

Hence, if $\delta > \hat{\delta}_{npq}$, $R$ would agree to $q$ in the first period if $\theta = \bar{\theta}$ and $p$ were not available, but not when $p$ is also on the policy menu. Figure 1 illustrates $\hat{\delta}_{npq}$ and $\hat{\delta}_{np}$. Summarizing the preceding discussion, we have:

**Proposition 1** Suppose there are two periods and, in the first period, $\theta = \bar{\theta} > w_R + e_R$.

(i) Emissions are always regulated when the policy menu is $\{n, q\}$, but not when the menu is $\{n, p\}$ if also $\delta > \hat{\delta}_{np}$.

(ii) With menu $\{n, p, q\}$, both players always prefer and propose $q$ rather than $p$ if $\delta \in$
Policy $q$ may be easier to accept than policy $p$ if $\delta \leq \delta_{np}$, while no regulation can be approved if $\delta > \max \{\delta_{np}, \delta_{npq}\}$.

(iii) Both players prefer menu $\{n, q\}$ to $\{n, p\}$ if $\delta > \delta_{np}$, and both players prefer menu $\{n, q\}$ to $\{n, p, q\}$ if $\delta > \max \{\delta_{np}, \delta_{npq}\}$.

In other words, even when every player prefers the price instrument in a one-period model, it may be politically impossible for both players to agree on using this instrument in a dynamic model since an efficient policy will be harder to repeal later on. Thus, both legislators may propose, and prefer, the less efficient emission quota. Note that it is exactly the additional benefit of $p$, the double dividend, that makes player $R$ unwilling to approve it, since an attractive policy is harder to repeal. As part (iii) of the proposition states, it may not only be beneficial to include the inefficient instrument on the menu, it may also be desirable to remove the most efficient instrument. The reason, as mentioned, is that $R$ fears that by introducing $q$ today, $L$ may later pressure $R$ to accept policy $p$ in a situation where both $R$ and $L$ would have preferred $n$ to $q$.

Since the length of a period (and thus the size of $1/\delta$) can be interpreted as the time it takes for $\theta$ to be drawn anew, Proposition 1 also suggests that when the state is more volatile (and changes even for short period lengths), the players are more likely to propose and prefer the quota rather than the tax.

The results above are both important and new to the literature, but they are derived in a stylized example which raises a number of questions. For instance, while instrument
q is attractive today in part because the players will never use the inefficient instrument in the last period, what is the desirability of q in a dynamic model where there is no last period? Furthermore, while the example assumed that instrument p would never be repealed once implemented, why should the players agree on q in a more general model where both instruments can eventually be repealed? To answer these and other questions in depth, the following section generalizes the model to an infinite number of periods and considerably more general distributions of states.

3 A Dynamic Model of Instrument Choice

Policies, payoffs, and players: There are two pivotal players and three policy alternatives. We refer to policy n as "no policy", while policies p and q are alternative regulatory instruments. Since the purpose of this paper is to investigate when inefficient policy instruments will be chosen, we assume that everyone always prefers policy p (the efficient instrument) to q (the inefficient instrument) in a static or isolated setting. Thus, we let $e_i > 0$ measure player i’s value of policy p relative to policy q.\(^5\)

Although the ranking of instruments may be clear, whether any form of intervention is needed at all depends on the state of the world, $\theta \in \mathbb{R}$, a common preference shock that measures everyone’s benefit of intervention. If $w_i$ measures some individual-specific cost of the instrument, player i’s utility of p relative to n is:

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\begin{align*}
U_i(\theta, p) - U_i(\theta, n) &= \theta - w_i, \text{ and} \\
U_i(\theta, p) - U_i(\theta, q) &= e_i,
\end{align*}
$$

where $U_i(\theta, x)$ denotes the payoff to player i in state $\theta$ with policy $x \in \{n, p, q\}$. In a static setting, player i would prefer policy p to policy n when $\theta \geq w_i$, while q would be preferred to n only when $\theta \geq w_i + e_i$. A key simplifying assumption in specification (1) is that the utility difference between intervening with p rather than q for player i is independent of $\theta$. This is often natural, as illustrated by the example in the previous section.

Since we let $L$ refer to the pivotal player who is most pro-intervention, while $R$ is the pivotal player who is least likely to prefer intervention, we have $w_L < w_R$. We do not find it

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\(^5\)Despite the interpretation of p and q in the earlier Example, hereafter we always use p (respectively, q) to denote the efficient (respectively, inefficient) instrument, leaving any substantive meaning beyond relative efficiency dependent on the application. Further, we henceforth also use the terms “policy instrument” and “intervention” interchangeably to refer exclusively to alternatives p and q. The term “policy” may refer to any of the available alternatives, including n. This looseness should cause no confusion. Also, we say that intervention p or q is “repealed” when that intervention is the status quo and players agree to replace it by n.
necessary to assume any ranking of $e_L$ and $e_R$, however. Figure 2 illustrates an example of the two players’ static preferences as a function of $\theta$.

**Timing of the game:** Every period $t \in \mathbb{N}$ starts with some status quo $s_t \in \{n, p, q\}$, with $s_0 = n$. At the beginning of period $t$, both players observe the state $\theta_t \in \mathbb{R}$. The process $\{\theta_t : t \geq 0\}$ captures the evolution of the environment. For example, benefits (or costs) of intervention vary with the business cycle and the future environmental sensitivity may be unknown today. And insofar as the players in the game are legislators, $\theta_t$ can be interpreted as voters’ concern about regulation rather than the severity of the problem, *per se*. For simplicity, we assume $\{\theta_t : t \geq 0\}$ are distributed identically and independently over time according to some continuous cumulative distribution function $F$ with full support on $\mathbb{R}$. Section 5 relaxes the i.i.d. assumption and permits correlation across periods.

After $\theta_t$ is observed, $L$ and $R$ must agree to pick a new policy from the set $\{n, p, q\}$. To simplify, we assume that one player can make a take-it-or-leave-it offer to the other regarding which policy intervention, if any, to implement. The recognition probability for player $i$ is given by $b_i(\theta_t, s_t)$, perhaps dependent on $\theta_t$ and $s_t$. Throughout the paper we assume that the recognition probability of each player is bounded away from zero: for every $i \in \{L, R\}$ and all $(\theta, s)$, $b_i(\theta, s) < (b, 1 - b)$ for some $b > 0$. The recognized proposer offers a policy $y_t \in \{n, p, q\}$. If the other player, the veto-player, accepts this proposal, then $y_t$ is implemented; otherwise, the status quo $s_t$ stays in place. The policy implemented in $t$, whether the proposal $y_t$ or the status quo $s_t$, generates the flow payoff for that period, as measured by (1) and becomes the status quo in the next period, $t + 1$. Each player is infinitely lived and thus interested in maximizing its expected discounted payoff over the infinite horizon. The common discount factor is $\delta \in (0, 1)$.

**Equilibrium concept:** We denote the infinite horizon game by $\Gamma$ and restrict attention to stationary Markov-perfect equilibria (referred to as simply the "equilibria" in what follows). To define such equilibria in $\Gamma$, note first that a history of the game at a given action
node includes the identity of all previous proposers, proposals, veto decisions, realized states, and implemented policies. A strategy is Markov if it depends only on the payoff relevant history which, at the proposer nodes, is the current status quo and state and, at the veto-player nodes, is the current status quo, state, and proposal. A Markov strategy is further stationary if it does not depend on calendar time. Thus, a stationary Markov strategy for player \( i \in \{L, R\} \) consists of two contingent actions. First, a function that maps the current state and status quo into a policy proposal conditional on \( i \) being the proposer. Second, a function that maps the current state, status quo, and proposal into choice over accepting or rejecting the proposal conditional on \( i \) being the veto-player.\(^6\) A stationary Markov-perfect equilibrium is then defined as a subgame-perfect equilibrium in which players use stationary Markov strategies. We let \( \sigma_i \) denote \( i \)'s stationary strategy and write \( \sigma = (\sigma_L, \sigma_R) \).

**Remarks on the assumptions:** Some of the assumptions are made to simplify and relaxing them may not change the result. For example, binary policy levels are not necessary for the results, as we explain in the concluding section. However, three assumptions are crucial for the results. First, the mechanics of the model rests on multiple veto players. With a unicameral legislature taking decision under simple majority rule, the policy maker \( i \) with the median \( w_i \) would be the unique pivotal decision maker. However, multiple veto players are natural in politics. Bicameralism, supermajority requirements, or presidential veto power imply the existence of a set of veto-players, or pivotal policy makers, whose approval is necessary and sufficient to enact a policy change. In the case of a unicameral legislature taking decisions under a qualified majority \( m \in (1/2, 1] \), player \( L \) is such that exactly a fraction \( m \) of the \( w_i \)'s are larger than \( w_L \) and, for player \( R \), exactly \( m \) of the \( w_i \)'s are smaller than \( w_R \). Thus, the degree of heterogeneity, or polarization, \( w_R - w_L \), increases in the majority requirement \( m \) and player \( L \) is the more interventionist player: since \( w_L < w_R \), \( L \) prefers policy \( p \) to \( n \) for a larger set of states (i.e. all states \( \theta \geq w_L \)) than player \( R \).

Second, we assume away explicit side payments. If the players could make unlimited side-payments then only efficient instruments can be expected in equilibrium. In particular, policy \( p \) would be implemented in every period in which \( \theta > (w_L + w_R) / 2 \), and policy \( n \) would be implemented otherwise. Although the assumption that side-payments are unavailable is predicated on the difficulty, or even legality in some polities, of enforcing such transfers, it should be seen as a theoretical limitation on the results.

Third, if the status quo were exogenously given at the start of every period, making any policy decision transient, the environment would be a sequence of static games. In this case,

\(^6\)Mixed strategies are admissible. More formally, writing \( \Delta S \) for the set of probability distributions over a set \( S \), \( i \)'s proposal strategy takes \( \mathbb{R} \times \{n, q, p\} \) into \( \Delta \{n, q, p\} \); and \( i \)'s veto strategy takes \( \mathbb{R} \times \{n, q, p\}^2 \) into \( \Delta \{\text{accept, reject}\} \).
a status quo with $p$ or $n$ would prevail if $\theta_t \in (w_L, w_R)$, while for larger (smaller) realizations of the state, the policy would be $p$ ($n$). In this case, the inefficient policy $q$ would not be proposed or agreed upon. Therefore, the possibility of sunset clauses, clauses that specify a period after which a legislative act automatically expires, and temporary legislation call into question the applicability of the theory developed below. But most laws and policies enacted by the U.S. Congress, for example, are permanent: they remain in effect until a new legislative action is taken. This is the case for mandatory spending policies, which include all entitlements, currently about 60% of total federal spending (Austin and Levit 2010), constitutional amendments, most statutes in the U.S. code, the Senate’s rules of proceedings, and international treaties. Likewise, changes to the tax code are permanent unless legislators decide to attach a sunset provision, which historically has been the exception rather than the norm. Moreover, in practice, policies subject to a sunset clause are typically extended multiple times or made permanent. Indeed, sunset provisions and temporary legislation have been argued to generate economically detrimental legislative uncertainty, to increase compliance costs, and to diminish the incentive and preemptive effects of the law (Posner and Vermeule 2002 pp. 1670-2, Kysar 2011 pp. 1063-5) and, by requiring periodic reviews, sunset clauses increase administrative and bargaining costs (Behn 1977, Kysar 2011 pp. 1056-9). In the following, therefore, we assume any chosen policy stays in place until it is actively changed.

4 Analysis

In the following subsections, we characterize equilibria in which players intervene only with the efficient instrument and equilibria in which they also intervene with the inefficient instrument; describe the conditions under which the inefficient instrument is used in all equilibria; demonstrate how political polarization is crucial for the use of inefficient instruments; and argue that both players may benefit from having the inefficient instrument available. It is useful to begin, however, with a basic property on the players’ continuation payoffs for any equilibrium strategy profile.

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7 It is less clear, however, what happens if the players could choose between an institution in which policy decisions are transient, or one in which policy decisions persist unless both players agree to a change. Party $L$ prefers policy $p$ as the status quo and would propose (and have accepted) a persistent policy if $\theta$ were large, while player $R$ would propose (and have accepted) a transitory policy over time.

8 See, e.g., Posner and Verneule (2002, pages 1672, 1694, and 1701) on the permanent nature of statutes, the Senate’s internal rules, or international treaties. As for tax legislation, prior to the Bush administration, sunsets applied mainly to relatively small provisions known as “tax extenders” (Gale and Orszag 2003). As Mooney (2004) puts it, “the use of sweeping sunset provisions in the tax code under the Bush Administration represents a massive departure from previous tax policy.”
For every stationary strategy profile \( \sigma, i \in \{L, R\} \), \( \theta \in \mathbb{R} \), and \( x \in \{n, p, q\} \), let \( V_i^\sigma(\theta, x) \) be the continuation value for player \( i \) of implementing policy \( x \) in some period \( t \in \mathbb{N} \), conditional on \( \theta_t = \theta \) and on continuation play \( \sigma \).

Lemma 1 There exist equilibria in \( \Gamma \). Moreover, for any equilibrium strategy profile \( \sigma \), there exists \((w_L^\sigma, w_R^\sigma, e_L^\sigma, e_R^\sigma) \in \mathbb{R}^4 \) such that, for all \( i \in \{L, R\} \), \( \theta \in \mathbb{R} \), the continuation value function \( V_i^\sigma \) satisfies

\[
V_i^\sigma(\theta, p) - V_i^\sigma(\theta, n) = \theta - w_i^\sigma, \\
V_i^\sigma(\theta, p) - V_i^\sigma(\theta, q) = e_i^\sigma.
\] (2)

All proofs are in the Appendix. By comparing (1) and (2), we see that players’ continuation payoffs \( V^\sigma = (V_L^\sigma, V_R^\sigma) \) have the same structure as their flow payoffs \( U = (U_L, U_R) \), but with parameters \((w_L^\sigma, w_R^\sigma, e_L^\sigma, e_R^\sigma) \) instead of \((w_L, w_R, e_L, e_R)\). \( \text{Call} \) \((w_L^\sigma, w_R^\sigma, e_L^\sigma, e_R^\sigma) \) the continuation payoff parameters induced by \( \sigma \). Generically, the profile \((w_L^\sigma, w_R^\sigma, e_L^\sigma, e_R^\sigma) \) differs from \((w_L, w_R, e_L, e_R) \) because today’s policy affects not only today’s payoff, but also tomorrow’s status quo policy. The difference between flow and continuation payoff parameters, therefore, captures players’ forward-looking preferences over the next status quo.

It is convenient, in Section 4.1, to characterize the equilibria of \( \Gamma \) in terms of their continuation payoff parameters, since \((w_L^\sigma, w_R^\sigma, e_L^\sigma, e_R^\sigma) \) (almost) uniquely determine players’ behavior. Specifically, in any period \( t \), the veto-player \( i \) accepts (rejects) any proposal \( x \) for which \( V_i^\sigma(\theta_t, x) > (\leq) V_i^\sigma(\theta_t, s_i) \) and the proposer \( j \) proposes the policy that gives \( j \) the greatest \( V_i^\sigma(\theta_t, x) \) among those policies that are accepted by the veto-player \( i \). \( \text{For example, suppose} \) \( s_i = n, e_L^\sigma > 0 > e_R^\sigma, w_L^\sigma + e_L^\sigma < w_R^\sigma + e_R^\sigma \) and \( \theta_t \in (w_R^\sigma + e_R^\sigma, w_L^\sigma) \). Then \( L \) prefers \( p \) to \( q \) and \( q \) to \( n \), but \( R \) prefers \( q \) to \( n \) and \( n \) to \( p \). Hence, both players propose or (as appropriate) accept intervening with \( q \) on the equilibrium path, but only \( L \) would accept a proposal to intervene with \( p \). More generally, the greater is \( w_i^\sigma \) on the equilibrium path, the more likely is player \( i \) to accept or (as appropriate) propose intervening; and the greater is

\( \text{This result relies on} \{\theta_t\} \) being i.i.d. and on the stationarity of \( \sigma \) as follows. If \( \sigma \) was nonstationary, the same expression would hold but the function \( V_i^\sigma \) and the parameter \( w_i^\sigma \) would have to be indexed by the period \( t \) from which the continuation payoff is computed. Likewise, if \( \{\theta_t\} \) was not i.i.d., \( w_i^\sigma \) would have to be a function of \( \theta_t \).

\( \text{Duggan and Kalandrakis} \ (2012) \) provide a very general existence result for dynamic bargaining games with endogenous status quo. But to apply their result directly here requires violating our assumption that \( U(\theta, p) - U(\theta, q) \) is constant in \( \theta \). Although introducing some noise to the payoffs, to ensure the difference is not locally constant, and letting that noise tend to zero is possible, the result would be a correlated equilibrium that obscures the particular tradeoffs of interest here.

\( \text{The precise characterization of the equilibria of} \ \Gamma \) is a bit more involved when a player \( i \) is indifferent between two alternatives for a nonnegligible set of states of nature, which happens when \( e_i^\sigma = 0 \). This possibility is taken into account in the Appendix, but for the sake of clarity, we abstract away from it when describing the equilibria of \( \Gamma \) in the main text.
on the equilibrium path, the more likely is player \( i \) to accept or (as appropriate) propose intervening via \( p \) rather than \( q \).

### 4.1 Description of the Equilibria

This subsection distinguishes between equilibria in which the inefficient intervention is never enacted from those in which it is enacted with positive probability. We also characterize these types of equilibria here but postpone discussing their existence to the next subsection.

**Definition 1** Let \( \sigma \) be an equilibrium strategy profile. Then, \( \sigma \) is an **instrument efficient equilibrium** (EE) if \( q \) is implemented with probability-zero along the equilibrium path; \( \sigma \) is an **instrument inefficient equilibrium** (IE) otherwise.

Note that an instrument inefficient equilibrium is inefficient in a strong sense: \( q \) is implemented with positive probability even when it is not the status quo. Note also that instrument efficient equilibria can exhibit inefficiencies, but only insofar as any inefficiency is due to gridlock.

**Proposition 2** If \( \sigma \) is an EE, then \( p \) replaces \( n \) if \( \theta > w^\sigma_R \), while \( n \) replaces \( p \) if \( \theta < w^\sigma_L \), and:

\[
\begin{align*}
e^\sigma_L & > 0, \quad e^\sigma_R \geq 0, \text{ and} \\
w^\sigma_L & < w_L < w_R < w^\sigma_R.
\end{align*}
\]

That \( e^\sigma_L > 0 \) and \( e^\sigma_R \geq 0 \) simply means that both players always get a greater continuation payoff from implementing \( p \) than from implementing \( q \) in an EE. As a result, they behave as if \( n \) and \( p \) were the only two policies available.\(^\text{12}\) Proposition 2 further states that, when comparing \( n \) and \( p \), dynamic considerations lead players to behave in a more polarized way relative to the static environment. Specifically, the inequality \( w^\sigma_L < w_L \) states that forward-looking considerations lead the interventionist player \( L \) to bias her behavior, relative to \( L \)'s primitive preference, in favor of the intervention \( p \); conversely, \( w^\sigma_R > w_R \) means that the laissez-faire player \( R \)'s behavior is similarly biased in favor of no intervention (see Dziuda and Loeper (2016) for a similar result in a two-alternative model).

To see the intuition for the result, suppose, for example, that \( p \) is the status quo for some period \( t \) and that the realization of \( \theta_t \) is such that \( R \), the laissez-faire player, would surely repeal \( p \) if \( R \) was the only policymaker. But replacing \( p \) by \( n \) in \( \Gamma \) also requires acquiescence

\(^{12}\)When \( e^\sigma_R = 0 \), and only then, \( R \) is indifferent between \( q \) and \( p \) and therefore may veto a change away from \( q \) when it is the status quo. However, since \( s_0 = n \), \( q \) is never implemented on the equilibrium path.
by the player least willing to do so, the interventionist player \( L \). Consequently, anticipating \( L \)’s relative reluctance to overturn an interventionist status quo, \( R \) prefers not to support an efficient intervention for some realizations of the state in which \( R \) would prefer otherwise. The intuition for \( L \)’s relatively increased bias is symmetric.

As remarked earlier, EE can exhibit inefficiencies even if players use only the efficient instrument \( p \). When \( s_t = p \) and \( \theta_t \in (w_L^p, w_L) \), policy \( p \) remains in place despite both players receiving strictly higher payoffs from \( n \) in period \( t \). Similarly, when \( s_t = n \) and \( \theta_t \in (w_R^p, w_R^p) \), policy \( n \) remains despite both players receiving strictly higher payoffs from \( p \) in period \( t \). Thus inefficiency in an EE is only due to gridlock: although a status quo may stay in place when it ceases to be statically Pareto efficient, a change in the status quo can only occur if the change is statically Pareto efficient.\(^{13}\)

Now consider instrument inefficient equilibria. Recall that in an equilibrium \( \sigma \), player \( i \) is indifferent between \( p \) and \( n \) at \( \theta = w_i^p \), and is indifferent between \( q \) and \( n \) at \( \theta = w_i^q + e_i^q \). So both players are willing to repeal the efficient intervention \( p \) when \( \theta < \min \{ w_L^p, w_R^p \} \), and at least one player would veto replacing \( p \) by \( n \) when the reverse inequality holds. Similarly, both players are willing to repeal the inefficient intervention \( q \) when \( \theta < \min \{ w_L^q + e_L^q, w_R^q + e_R^q \} \), and at least one player would veto replacing \( q \) with \( n \) when the reverse inequality holds. Proposition 3 below states that the former threshold is smaller than the latter in any IE, so the inefficient intervention \( q \) is repealed in favor of \( n \) for a larger set of states than the efficient intervention \( p \). The fact that inefficient interventions are less "sticky" than efficient interventions is what makes the inefficient intervention more attractive.

Clearly, for \( q \) to be implemented on the equilibrium path, at least one player must prefer \( q \) to \( p \), so \( \min \{ e_L^q, e_R^q \} < 0 \). Proposition 3 below states that in any IE exactly one player prefers \( q \) to \( p \). It may seem immediate that this player is the less interventionist agent, \( R \), since the relative ease of repealing \( q \) mitigates \( R \)’s concerns about being able to repeal any intervention for lower realizations of the state in the future. The characterization result below, Proposition 3, shows that this is indeed the case in some IEs, but there may also exist IEs in which the opposite holds.

**Proposition 3** If \( \sigma \) is an IE, then players disagree on the choice of instruments,

\[
\min \{ e_L^q, e_R^q \} \leq 0 < \max \{ e_L^q, e_R^q \},
\]

\(^{13}\)A sequence of policies \( (x_t)_{t \in \mathbb{N}} \) is statically Pareto efficient if, in each period \( t \in \mathbb{N} \), there is no other policy \( y \) that is strictly preferred by both players to \( x_t \), keeping all other policies in the sequence unchanged.
and $q$ is always repealed for a larger set of states than $p$,

$$\min \{w^a_L, w^a_R\} < \min \{w^a_L + e^a_L, w^a_R + e^a_R\}.$$  

Furthermore, an IE is one of three types:

**IE-A:** $e^a_R \leq 0 < e^a_L$, $w^a_L < w^a_R$, and $w^a_L + e^a_L < w^a_R$,

**IE-B:** $e^a_R \leq 0 < e^a_L$, $w^a_L < w^a_R$, and $w^a_L + e^a_L \geq w^a_R$,

**IE-C:** $e^a_R > 0 \geq e^a_L$ and $w^a_L > w^a_R$.

In IE-A and IE-B, the statically less interventionist player $R$ remains less interventionist in equilibrium ($w^a_L < w^a_R$) and, when intervening, she prefers to use the inefficient instrument $q$ ($e^a_R \leq 0 < e^a_L$). Clearly, for sufficiently high states $\theta$, $n$ is very costly and hence both players prefer to intervene with any instrument. Exactly which instrument is used, however, depends on the identity of the proposer: $R$ suggests $q$ and $L$ approves, while $L$ proposes $p$ and $R$ approves. For more intermediate states, however, each player is willing to intervene only with her preferred instrument. Two alternative situations may arise, as described by the difference between IE-A and IE-B.

An IE-A is illustrated in Figure 3. The figure illustrates the policy change as a function of both the state (measured on the horizontal axis) and the status quo (indicated on the vertical axis). If there is no arrow, then there is no change, so $x = s$. If $\theta > w^a_L$, only instrument $q$ can be repealed. Since $R$ anticipates this, $R$ prefers $n$ to $p$ but prefers $q$ to $n$ when $\theta \in (\max \{w^a_L + e^a_L, w^a_R + e^a_R\}, w^a_R)$. Thus, for these states, $L$ indeed prefers $q$ to $n$ so both $L$ and $R$ propose $q$. In other words, both players propose the inefficient policy when $\theta \in (\max \{w^a_L + e^a_L, w^a_R + e^a_R\}, w^a_R)$.

**Corollary 1** Given a status quo $n$, for some states, any proposer offers (and the other player accepts) intervening with the inefficient instrument in any IE-A.
Furthermore, regardless of whether the status quo policy is \( p, q \) or \( n \) in the equilibrium, the status quo persists for all \( \theta \in (\min \{ w^\sigma_L + e^\sigma_L \}, \max \{ w^\sigma_R + e^\sigma_R \}) \), the gridlock interval for IE-A.

Now consider IE-B, illustrated in Figure 4. In this case we have \( w^\sigma_L + e^\sigma_L > w^\sigma_R \), implying \( \max \{ w^\sigma_L + e^\sigma_L, w^\sigma_R + e^\sigma_R \} > w^\sigma_R \). Thus, when \( \theta \in (w^\sigma_R, w^\sigma_L + e^\sigma_L) \), \( R \) prefers intervening with any instrument \( n \), but \( L \) strictly prefers \( n \) to the inefficient instrument \( q \); hence \( p \) is implemented for any allocation of proposal power. However, \( R \) prefers \( q \) to \( p \), since \( q \) is repealed for a larger set of states. Thus, \( R \) proposes \( q \) and \( L \) accepts this when \( \theta > w^\sigma_L + e^\sigma_L \).\(^{14}\)

Finally, consider IE-C. That equilibria of this sort can exist may be somewhat surprising. This is because \( w^\sigma_R < w^\sigma_L \) implies that the interventionist player is willing to accept the efficient intervention less frequently than the laissez-faire player. The intuition for IE-C is as follows. If \( e_R \) is large enough (see Proposition 5 below), \( R \) prefers to intervene with \( p \) rather than \( q \), if at all. As a result, \( R \) is concerned about the following situation: the status quo is \( n \) and the realization of \( \theta \) sufficiently large that \( R \) has no choice but to accept a proposal \( q \) by \( L \). The only way \( R \) can avoid such situations is by making the status quo \( n \) less likely, and thus by being biased against \( n \) when the status quo is \( p \). If this bias is large enough, \( R \) can become more reluctant than \( L \) to repeal \( p \), which explains why \( w^\sigma_R < w^\sigma_L \). Anticipating \( R \)'s reluctance to repeal \( p \), \( L \) prefers to intervene via the less sticky instrument \( q \), which rationalizes \( R \)'s aforementioned concern with respect to having to accept a proposal \( q \) by \( L \).

It is worth noting that the kind of inefficiency that occurs in an IE and in an EE are qualitatively different. In an EE, inefficiency occurs because one player unilaterally blocks any policy change from his preferred status quo. This type of inefficiency has already been highlighted in the existing literature on dynamic policymaking with an endogenous status quo summarized earlier. Policy inertia can also occur in an IE, but in addition, when the

\(^{14}\)The description of the mapping from the state and status quo to the policy outcome in Figures 3 and 4 is valid only for pure strategy IE. The mapping is slightly different for a mixed strategy IE, which can happen when \( e^\sigma_i = 0 \) and \( e^\sigma_j > 0 \). A mixed IE differs from a pure IE only in that \( q \) can be replaced directly by \( p \) with positive probability when \( \theta > \min \{ w^\sigma_L + e^\sigma_L \} \). We allow for such mixed equilibria when proving our results in the Appendix, but for the sake of clarity, we focus on pure strategy IE when describing the intuition for our results.
state \( \theta \) is sufficiently large, players can unanimously agree to replace status quo \( n \) with a policy that is Pareto dominated in all states of nature.

### 4.2 Two Results on the Existence of IE

While the previous subsection described the different types of equilibria, we now investigate which one can (co)exist as a function of the parameters of the model. As a start, the following proposition provides relatively weak conditions under which an IE exists for any payoff parameters \((w_L, w_R, e_L, e_R)\). In particular, no matter how inefficient is the policy instrument \( q \) relative to \( p \), all equilibria may be IE.

**Proposition 4** Let \( G \) be a c.d.f. with mean 0 and variance 1. For any \((w_L, w_R, e_L, e_R)\), for all \( \delta \) sufficiently close to 1, there exists a set of \( m \in \mathbb{R} \) and \( v > 0 \) with a non-empty interior such that, for \( F(\theta) \equiv G(\frac{\theta-m}{v}) \), all equilibria of \( \Gamma \) are IE.

The sufficient condition on the distribution of states here essentially requires that relatively little probability mass is concentrated in the tails. To see why, observe that if \( \theta \) is likely to be either very large or very small, players would agree most of the time on whether or not to intervene. As a result, their preferences over which instrument to use would not be distorted by their expectation of future disagreements on when to repeal any intervention.

More specifically, recall Figure 3, above. In this case, player \( R \)'s preference for implementing \( q \) instead of \( p \) increases with the likelihood of states in which \( q \) is repealed but a status quo \( p \) remains, that is, in states \( \theta \in (w_L^\alpha, \min \{w_L^\alpha + e_L^\alpha, w_R^\alpha + e_R^\alpha\}) \). Similarly, because, all else equal, \( p \) generates a greater flow payoff for \( R \), \( R \)'s preference for implementing \( q \) instead of \( p \) decreases in the likelihood of states where both interventions persist, that is, in the relatively extreme states \( \theta > w_R^\alpha \) in Figure 3. Thus, the condition on \( F \) used in Proposition 4 ensures, for the example of Figure 3, that states in \((w_L^\alpha, \min \{w_L^\alpha + e_L^\alpha, w_R^\alpha + e_R^\alpha\})\) are sufficiently likely, and states greater than \( w_R^\alpha \) sufficiently unlikely, to guarantee that an IE exists.\(^{15}\)

It is worth remarking that the proof to the result also yields that \( q \) can be implemented arbitrarily more frequently than \( p \) for at least some distributions \( G \). For example, if \( G \) is

\(^{15}\)A similar result for the exclusive existence of EE is also available. The only difference (\textit{mutatis mutandis}) is that the choice of \( m \) for insuring only EE is distinct from that for insuring only IE; for both cases, it suffices to consider \( v \) vanishing small to insure that, most of the time, states are in the neighbourhood of \( m \). Choosing \( m = w_L + \eta \) for \( \eta > 0 \) and sufficiently small, implies players disagree between \( n \) and \( p \) in the typical state \( m \), but agree that \( q \) is preferred to \( n \). Hence, for high \( \theta \), \( R \) prefers intervening with \( q \) to limit the expected persistence of the intervention for states around \( m \), yielding IE. On the other hand, by choosing \( m \notin [w_L, w_R] \), players also agree between \( n \) and \( p \) in the typical state \( m \), so \( R \) is unconcerned that efficient interventions persist and agrees to use \( p \).
normal then, for any $\epsilon > 0$, there is an $m$ and $v$ such that, under status quo $n$ and conditional on some intervention being implemented, the probability that $q$ is implemented relative to $p$ is greater than $(1 - \epsilon)$.\(^\text{16}\)

With respect to which type of IE that exists, we can show that if an IE-C exists, then there also exists an equilibrium that is not IE-C. Formally, we have the following result.

**Proposition 5** If an IE-C exists, then also an IE-A, an IE-B, or an EE exists. Moreover, IE-C cannot exist if $e_L \leq e_R$ and IE-B cannot exist if $e_L \geq e_R$. Furthermore, if $\frac{|e_R - e_L|}{(w_R - w_L)} < (1 - \delta)^3$, any equilibrium is either EE or IE-A.

The second and third sentences of the proposition state that the IE is always IE-A if $e_R$ and $e_L$ are sufficiently close to each other. Moreover, IE-A is the only type of IE for which existence does not depend on the ranking of $e_L$ and $e_R$. Given the somewhat unintuitive properties of IE-C, and given that they are never the unique equilibrium, in the interest of clarity, when discussing the intuition for the remaining results, we focus on the other types of equilibria, that is, on equilibria $\sigma$ such that $w'_R < w'_L$. We would like to stress however that we are not making any formal equilibrium selection beyond Markov perfection.

### 4.3 The Effect of Polarization

This subsection describes how the use of inefficient instruments hinges on political polarization and other parameters characterizing the political environment.

**Proposition 6** Fix $\delta \in (0, 1)$, the c.d.f. $F$ and any allocation of bargaining power $b = (b_L, b_R)$. Then

(i) For all $(w_L, w_R)$, if all equilibria are IE for some $(e_L, e_R)$, then all equilibria are IE for all $(e'_L, e'_R)$ such that $e'_L \geq e_L$ and $e'_R \leq e_R$.

(ii) For every $w_R, w_L$ and $e_L > 0$, there exists $\epsilon > 0$ such that all equilibria are IE for any $e_R \leq \epsilon$.

(iii) For all $(e_L, e_R)$ and fixed mean ideology $(w_L + w_R)/2$, there exists an EE as $(w_R - w_L) \to 0$. Furthermore, all equilibria are EE as $(w_R - w_L) \to \infty$.

The comparative static in Proposition 6(i) on $e_R$ is intuitive. If, for some $(e_L, e_R)$, $R$ is willing to accept the inefficient intervention $q$ in exchange for an increase in the likelihood that

\(^\text{16}\)This result extends to many of the standard distributions other than the normal. See the proof to Proposition 4 in the appendix for the exact restrictions on $G$ under which the claim holds.
the intervention is repealed in the future, $R$ is also willing to accept $q$ in such circumstances for lower degrees of inefficiency. The claim in Proposition 6(i) regarding changes in $e_L$, however, is less obvious, since an increase in $e_L$ has two effects. On the one hand, $L$ is more willing to repeal a status quo $q$ when $e_L$ is large. This, in turn, increases the strategic value of $q$ for $R$. On the other hand, since a larger $e_L$ implies that $L$’s payoff from $q$ is smaller, $L$’s willingness to accept any proposal to implement $q$ decreases as well. However, regardless of the value of $e_L$, for $\theta$ large enough, $L$ prefers $q$ to a status quo $n$, so $R$ can be sure $q$ is accepted and implemented on the equilibrium path.

It is informative to reformulate Proposition 6(i) in terms of player polarization. Recall that $w_i$ and $w_i + e_i$ can be interpreted as the ideological position of player $i$ on how often to intervene when using policy $p$ and $q$, respectively. For a fixed $(w_L, w_R)$, as $e_L$ increases and $e_R$ falls, the gap between the players’ thresholds on policy $p$ remains unaffected; for policy $q$, however, the difference between the two players decreases. Proposition 6(i) then says that as $e_L$ increases and $e_R$ falls, all else equal, players become more inclined to use the inefficient, but more consensual, policy instrument $q$. In particular, the inefficient policy instrument is used because it is costly for player $L$; not because it is costly for player $R$. Thus, as part (ii) of the proposition states, the inefficient instrument is always chosen (for sufficiently high states) if $e_R$ is small enough (where "small enough" depends on $w_R$, $w_L$ and $e_L$).

Part (iii) of Proposition 6 concerns how the equilibria change with alterations in legislative polarization. Given a mean ideology, existence of an instrument-efficient equilibrium is assured for sufficiently small polarization. Since the players’ preferences are essentially aligned in this case, muting any concerns with future status quo bias, the result is intuitive. On the other hand, when polarization is sufficiently large, the players are rarely aligned with respect to whether intervention is warranted. Thus, any policy is unlikely to be repealed and the players perceive the decision as permanent. In common with the players’ behavior in a static setting, both agree to an efficient intervention if they agree to intervene at all. In sum, therefore, part (iii) of the proposition yields the following prediction.

**Corollary 2** If an inefficient instrument is used in any equilibrium, then political polarization cannot be too small or too large.

Political polarization can, as observed earlier, be a function of the political system. With only one political chamber under simple majority rule, the median voter is pivotal in every decision and there can be no polarization between multiple pivotal legislators. With a super-majority requirement, however, there is almost always some ideological distance between the veto players. In this case, legislative players can use inefficient policy instruments to avoid gridlock.
4.4 The Value of Inefficient Instruments

Empirically, the menu of available policy instruments can be endogenous. For example, if environmental policy is chosen at the local level, or by a regulatory body, then the instruments that are available may be defined ex ante by the federal government and, in such cases, it is not at all clear whether an inefficient policy instrument would, or should, be made available to decision makers. By definition of the two interventions, efficient $p$ and inefficient $q$, there is no social welfare gain to be had from the existence of $q$ in a static environment; the question, then, is whether this holds for the dynamic setting.

Because players are free to ignore the inefficient instrument, having instrument $q$ available seems at least welfare-irrelevant. And although instrument $q$ can be abused by player $R$ to the extent that $R$ proposes $q$ solely to improve $R$’s future bargaining position, it turns out that $q$ can have a positive strategic value when an efficient intervention is politically infeasible, because of $R$’s fears that an efficient policy will not be repealed when $R$ prefers so. The next result states that, under certain conditions, both players are strictly better off with access to the inefficient instrument $q$, than if they are constrained to choosing only between $n$ and $p$. In other words, there are environments in which a statically inefficient policy instrument supports Pareto improvements in the dynamic game.

Let $\Gamma (n, p, q)$ denote the original game, $\Gamma (n, p)$ the game in which the inefficient instrument is unavailable, and $\Gamma (n, q)$ the game in which the efficient instrument is unavailable.

**Proposition 7** For any $(w_L, w_R, e_L, e_R)$ and for all $\delta$ sufficiently close to one, there exists an $F$ under which both players are strictly better off in any equilibrium of $\Gamma (n, p, q)$ than in any equilibrium of $\Gamma (n, p)$, and both players are strictly better off in any equilibrium of $\Gamma (n, q)$ than in any equilibrium of $\Gamma (n, p)$.

The intuition for Proposition 7 is similar to that of Proposition 4. If the c.d.f. $F$ puts enough weight on $(w_L, \min \{w_L + e_L, w_R\})$, then $L$ and $R$ are likely to disagree over $n$ and $p$, but are unlikely to disagree over $n$ and $q$. Consequently, in the game $\Gamma (n, p)$, sufficiently patient players become very biased in favor of the policy each prefers most on average. In particular, $w_R^p$ increases in $\delta$, so $R$ never agrees to intervene and the initial status quo $n$ persists indefinitely. The availability of $q$, however, provides room for players to intervene for states $\theta > \min \{w_L + e_L, w_R + e_R\}$, since the likelihood of agreement on repealing the intervention is greater with $q$ than with $p$. This greater flexibility leaves both players better-off in equilibria in $\Gamma (n, p, q)$ than in $\Gamma (n, p)$. Similarly, since it is harder, others things equal, to repeal an efficient intervention than an inefficient intervention, the same relative welfare property applies when comparing $\Gamma (n, q)$ to $\Gamma (n, p)$. 

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5 Persistent States

To this point, we have assumed that states, and thereby players’ state-contingent preferences, are i.i.d. draws over time. However, in many applications, states might persist for several consecutive periods before a period of relative volatility. There may be periods of relatively stability when players do not expect to change their positions quickly, and there may also be periods in which new information about the desirability of an intervention arrives frequently, resulting in frequent revisions of the relevant policy preferences.\textsuperscript{17} In this section, we ask how expectations about the volatility or persistence of states affect players’ strategic policy decisions.

To capture the possibility (in an analytically tractable way) that states $\theta$ may or may not persist across periods, and that players’ expectations regarding the persistence of the current state may vary over time, consider, for every period $t$, the tuple $(\theta_t, v_t) \in \mathbb{R} \times [0, 1]$. Assume the evolution of such tuples across periods satisfies the following transition property: for all $t$,

$$ (\theta_{t+1}, v_{t+1}) = \begin{cases} (\theta_t, v_t) \text{ with probability } 1 - v_t \\ (\theta_{t+1}, v_{t+1}) \sim H \text{ with probability } v_t \end{cases} $$

As before, $\theta_t$ is the underlying policy-relevant state. We interpret the additional variable $v_t$ as a measure of volatility of the current policy-relevant state $\theta_t$ or, equivalently, of players’ period $t$ expectations over $\theta_{t+1}$. In each period $t$, the state $\theta_t$ persists into period $t + 1$ with probability $(1 - v_t)$ and, for simplicity, we assume that the volatility $v_t$ also persists into period $t + 1$; with probability $v_t$, $\theta_{t+1}$ and $v_{t+1}$ are drawn according to some joint c.d.f. $H$. Thus, the volatility of future state-contingent policy preferences is redrawn if and only if the state-contingent policy preferences are redrawn.

Note that this evolution of the state collapses to the basic i.i.d. model if $v_t \equiv 1$ for all $t$. Similarly, $v_t \equiv 0$ for all $t$ implies preferences never change and $v_t \equiv v \in (0, 1)$ for all $t$ implies the degree of volatility is fixed.

The continuation payoff function $V^\sigma$ is defined as follows. For every stationary strategy profile $\sigma$, let $V_i^\sigma$ be the continuation payoff for player $i \in \{L, R\}$ of implementing policy $x \in \{n, p, q\}$ in some period $t \in \mathbb{N}$ with state $(\theta_t, v_t) = (\theta, v) \in \mathbb{R} \times [0, 1]$ until the state is redrawn, and play $\sigma$ thereafter.

\textsuperscript{17}A painful example is the US Congressional response to the 2008 fiscal collapse. For some years before the 2008 fiscal collapse, the US economy was growing strongly and atypical Congressional economic interventions were minimal. The fall of Lehman Brothers and the subsequent turmoil in much of the global economy led to serious Congressional disagreement regarding the appropriate level and duration of any extraordinary intervention, from whether to bail out banks or the car industry, to extensions of unemployment and welfare benefits.
Lemma 2 There exist \((w_L, w_R, e_L, e_R) \in \mathbb{R}^4\) such that, for any equilibrium strategy profile \(\sigma\) and for all \(i \in \{L, R\}, \theta \in \mathbb{R}, \text{and } v \in [0, 1]\),

\[
(1 - \delta (1 - v)) (V_i^\sigma (\theta, v, p) - V_i^\sigma (\theta, v, n)) = \theta - (1 - v) w_i - v w_i^\sigma \\
(1 - \delta (1 - v)) (V_i^\sigma (\theta, v, p) - V_i^\sigma (\theta, v, q)) = (1 - v) e_i + v e_i^\sigma.
\] (3)

Hence, as in the basic model, players continuation payoff functions \(V^\sigma\) have the same shape as their flow payoff functions in (1), but the flow payoff parameters \(w_i\) and \(e_i\) are replaced by \((1 - v) w_i + v w_i^\sigma\) and \((1 - v) e_i + v e_i^\sigma\), respectively.\(^{18}\) As before, the difference between the flow payoff parameters and the continuation payoff parameters, namely \(v (w_i^\sigma - w_i)\) and \(v (e_i^\sigma - e_i)\), captures the way players distort their equilibrium behavior relative to their flow payoff. Note that in this extended model, such distortion varies with \(v\) in a systematic way. When \(v = 0\), players expect their preferences to remain constant in the future. In this case, each player behaves as in a one period-model and the difference in the continuation payoffs coincides with the difference in the static flow payoffs (1). As \(v\) increases, the (state-contingent) preferences are more likely to change in the future and, therefore, the status quo becomes more salient, driving a wedge between the difference in the continuation payoffs and the difference in the static flow payoffs.

Recall that the inefficient instrument \(q\) is implemented for some realizations of the policy-relevant state \(\theta\) in some equilibrium \(\sigma\) when \(V_i^\sigma (\theta, v, p) < V_i^\sigma (\theta, v, q)\) for some player \(i\), whereas \(q\) is not implemented when the opposite inequality holds for both players. From (3), if \(e_i^\sigma > 0\) for \(i = L, R\), for all \(v < 1\), there is no realization of \(\theta\) at which \(V_i^\sigma (\theta, v, p) < V_i^\sigma (\theta, v, q)\); hence, \(q\) is implemented with probability 0 on the equilibrium path. In environments where \(e_i^\sigma < 0\) for some \(i\), however, from (3), for \(v\) sufficiently close to 1, \(V_i^\sigma (\theta, v, p) < V_i^\sigma (\theta, v, q)\) for some nonnegligible set of realizations of \(\theta\), so \(q\) is implemented with positive probability on the equilibrium path. The same logic as in Proposition 4 implies that for any profile of payoff parameters \((w_L, w_R, e_L, e_R)\), one can find a distribution \(H\) such that \(e_i^\sigma < 0\) for some \(i\), yielding the following result.

Proposition 8 In any equilibrium \(\sigma\), there exists \(\bar{v} \in (0, 1]\) such that \(q\) is never implemented when the realization of volatility is such that \(v < \bar{v}\) and \(q\) is implemented for a nonnegligible set of realizations of \(\theta\) if \(v > \bar{v}\). Moreover, for any \((w_L, w_R, e_L, e_R)\), for all \(\delta\) sufficiently close to 1, there exists a distribution \(H\) with full support on \(\mathbb{R} \times [0, 1]\) such that, for all equilibria, \(\bar{v} < 1\).

Proposition 8 states that \(q\) is implemented on the equilibrium path only in sufficiently

\(^{18}\)The common scaling factor \(1 - \delta (1 - v)\) does not affect the sign of the expressions in (3), and thus does not affect which policy parties prefer to implement on the equilibrium path.
volatile environments. Intuitively, when players expect the state to remain fairly stable over time \( (v_t \leq \bar{v}) \), strategic concerns regarding the possibility of conflict over repealing today’s intervention tomorrow, say, are muted and any intervention is efficient. When the state is expected to be sufficiently volatile, however \( (v_t > \bar{v}) \), today’s choice is likely to need revision in the next period, making salient exactly the sorts of strategic consideration underlying the use of inefficient interventions.

6 Conclusion

The continued and widespread use of inefficient policy instruments in more-or-less democratic political systems is a puzzle. For example, while economists uniformly recommend regulating emissions with Pigou taxes, technology and quantity controls are the most adopted instruments in reality. Why would rational politicians agree on the use of Pareto dominated policy instruments? A first guess may be that legislative history and vested interests in a changing environment, inevitably lead to policy inefficiencies over time. In our model, however, an inefficient policy intervention may arise even when there is no salient legislative history or vested interest.\(^{19}\) Rather, an inefficient policy may be chosen precisely because it is inefficient and the environment is expected to change in the future. From this perspective, the puzzle alluded to above can be understood without pointing to informational asymmetries, interest group influence, or differential distributional implications of alternative policy instruments among the electorate at large.

With a heterogeneous legislature and multiple veto players, inefficient policy instruments are politically easier than efficient instruments to repeal in dynamic environments subject to policy-relevant stochastic shocks. And since inefficient instruments are easier to repeal, heterogeneous veto players, differentiated only by the threshold shocks beyond which they judge some policy intervention to be warranted, can be more willing to agree on responding to a sufficiently severe downside shock with an inefficient instrument. As a consequence, inefficient instruments are more likely to be used in polarized political environments and for issues where the fundamentals are subject to change over time. These qualitative predictions seem to be consistent with reality, whereby technology standards and emission quotas are indeed frequently adjusted.

Our analysis and results hinge on a number of crucial assumptions. We assume that today’s policy constitutes the status quo for the next period; that players cannot use side payments in decision-making; and that the player relatively reluctant to intervene in any period, is likely to be similarly reluctant in subsequent periods. Each of these assumptions

\(^{19}\) We are grateful to Dan Bernhardt for this observation.
is crucial for our results, but we also believe that they are reasonable and satisfied in most democracies. As argued in Section 3, few policies come with expiration dates and explicit side payments are rare in politics. Furthermore, parties that are left-wing today have been left-wing for decades.

We also assume that the intervention level is binary: an instrument is either imposed or it is not. This assumption is for simplicity, however, and permitting the tax or quota to be any real number may not remove the benefit of using the inefficient instrument. The intuition for this is that the amount of policy intervention (e.g., cutting emission in half) can often be separated from the choice of instrument (price or quota) that will be used to achieve this. To illustrate the argument, suppose it is possible to set a tax or policy that is only a fraction of the full intervention we have considered, and that all costs and benefits are proportional to this fraction. In a two-period example, the inefficient instrument will not be used in the last period and the set of states under which a tax is repealed will be independent of the level of the tax. For this reason, no fraction in $(0, 1)$ will be used in an equilibrium when utilities are linear. With nonlinear utilities, the analysis becomes more complicated and must be left for future research.

While we have tried to keep the model as general as possible in other respects, we believe future research should make more specific (rather than general) assumptions on the available instruments and the legislative process. Specific assumptions will more likely yield sharp testable predictions that can be taken to the data, for example regarding how the choice of particular instruments depends on various details of the political system. A better understanding of the political constraints is immensely important in the search for efficient policies that both should and can be implemented.
7 Appendix

Throughout this appendix, for any player specific variable, the variable without the player subscript refers to the vector of this variable for each player. For instance, \( w \) refers to \( (w_L, w_R) \).

**Notation 1** For a given stationary Markov strategy profile \( \sigma \), the policy outcome in some period \( t \in \mathbb{N} \) with status quo \( s(t) \in \{n, p, q\} \) depends on the realization of \( \theta(t) \), the identity of the proposer in period \( t \), and possibly players’ private randomization devices if the equilibrium is mixed.\(^{20}\) Let \( v(t) \) denote the random variable that encodes all this information. We refer to \( v(t) \) as the state of the world in period \( t \). Let \( \Upsilon \) denote the set of possible states of the world.

Note that \( \{v(t) : t \in \mathbb{N}\} \) is i.i.d.. Let \( \mu \) denote its probability distribution. For any state of the world \( v \in \Upsilon \), \( \theta(v) \) denotes the corresponding realization of the state of nature.

For all \( s, x \in \{n, p, q\} \), \( \Upsilon^o(s, x) \) denotes the set of realizations of the state of the world for which status quo \( s \) leads to outcome \( x \).

We first prove the following lemma, which is instrumental in proving Lemma 1.

**Lemma 3 (Continuation Payoff)** Let \( \sigma \) and \( \sigma^* \) be two Markov strategy profiles, and let \( V_{i,\sigma^*}^\sigma(\theta, x) \) denote the expected continuation payoff for player \( i \in \{L, R\} \) from implementing policy \( x \in \{n, p, q\} \) in period 0 conditional on \( \theta(0) = \theta \), and on players playing \( \sigma \) in period 1 and playing \( \sigma^* \) from period 2 onwards. Then there exist \( w^i_{\sigma,\sigma^*}, e^i_{\sigma,\sigma^*} \in \mathbb{R} \) such that, for all \( \theta \in \mathbb{R} \),

\[
V_{i,\sigma^*}^\sigma(\theta, p) - V_{i,\sigma^*}^\sigma(\theta, n) = \theta - w^i_{\sigma,\sigma^*},
\]

\[
V_{i,\sigma^*}^\sigma(\theta, p) - V_{i,\sigma^*}^\sigma(\theta, q) = e^i_{\sigma,\sigma^*}.
\]

**Proof.** Using the notations of the lemma, \( V_{i,\sigma^*}^\sigma(\theta, p) - V_{i,\sigma^*}^\sigma(\theta, n) \) is given by the flow payoff gain from implementing \( p \) instead of \( n \) in \( t = 0 \), which is \( \theta - w, \) plus \( \delta \) times the continuation payoff gain from period 1 onwards from having \( s(1) = p \) instead of \( s(1) = n \), given continuation play \( \sigma \) in \( t = 1 \) and \( \sigma^* \) in \( t \geq 2 \). Let \( V^\sigma \) denote the continuation value function for the strategy profile \( \sigma^* \) as defined at the beginning of Section 4. Using Notation

\(^{20}\)Formally, suppose that each player \( i \) observes in each period \( t \) a random variable \( \rho_i(t) \) uniformly distributed on \([0, 1]\), and that these random variables are independent across players and periods. Then a mixed action for proposer (veto-player) \( i \) in period \( t \) can be represented as a piecewise constant function from \([0, 1]\) to \( \{n, p, q\} \) (\{accept, reject\}) which maps the realization of \( \rho_i(t) \) into a proposal (veto) decision for player \( i \) in period \( t \).
1, the above reasoning implies that

\[ V_{i}^{\sigma, \sigma^*} (\theta, p) - V_{i}^{\sigma, \sigma^*} (\theta, n) \]

\[ = \theta - w_{i} + \delta \sum_{x,y\in \{p,q\}} \left( \int_{Y^\sigma (p,x) \cap Y^\sigma (p,y)} (V_{i}^{\sigma^*} (\theta (v), x) - V_{i}^{\sigma^*} (\theta (v), y)) d\mu (v) \right). \]

Therefore, to obtain the desired expression for \( V_{i}^{\sigma, \sigma^*} (\theta, p) - V_{i}^{\sigma, \sigma^*} (\theta, n) \), it suffices to set

\[ w_{i}^{\sigma, \sigma^*} \equiv w_{i} - \delta \sum_{x,y\in \{p,q\}} \left( \int_{Y^\sigma (p,x) \cap Y^\sigma (p,y)} (V_{i}^{\sigma^*} (\theta (v), x) - V_{i}^{\sigma^*} (\theta (v), y)) d\mu (v) \right). \] (4)

An analogous reasoning on the continuation payoff gain from implementing \( p \) instead of \( q \) implies that

\[ e_{i}^{\sigma, \sigma^*} = e_{i} + \delta \sum_{x,y\in \{p,q\}} \left( \int_{Y^\sigma (p,x) \cap Y^\sigma (q,y)} (V_{i}^{\sigma^*} (\theta (v), x) - V_{i}^{\sigma^*} (\theta (v), y)) d\mu (v) \right). \] (5)

Proof. If we set \( \sigma = \sigma^* \) in Lemma 3, we obtain the second claim of Lemma 1: the continuation payoffs for any strategy profile \( \sigma \) are given by (2) with \( w_{i} = w_{i}^{\sigma, \sigma} \) and \( e_{i} = e_{i}^{\sigma, \sigma} \). The first claim, namely that an equilibrium exists, follows from Lemma 6 below, which proves the existence of an equilibrium with particular properties (these properties will be useful in the proof of Proposition 5). Note that Lemmas 4, 5 and 6 only use the second claim of Lemma 1, which we have just proven, so there is no circularity.

In what follows, we say that a Markov strategy profile \( \sigma \) is stage undominated for some continuation payoff parameters \((w^*, e^*) \in \mathbb{R}^4\), if \( \sigma \) is a subgame-perfect equilibrium of the game in which players play only one period of the infinite horizon game \( \Gamma (n, p, q) \) and their payoff are given by the continuation parameters \((w^*, e^*) \) as follows: for all \( \theta \in \mathbb{R} \), \( U_{i} (\theta, p) - U_{i} (\theta, n) = \theta - w_{i}^{*} \) and \( U_{i} (\theta, p) - U_{i} (\theta, q) = e_{i}^{*} \).

Lemma 4. A Markov strategy profile \( \sigma \) is an equilibrium if and only if \( \sigma \) is stage undominated for the continuation payoff parameters \((w^*, e^*) \).

Proof. The proof of that lemma is straightforward and omitted for brevity.

The following lemma derives some properties of the parameters \( w_{i}^{\sigma, \sigma^*} \) and \( e_{i}^{\sigma, \sigma^*} \) introduced in Lemma 3. These properties will be helpful when characterizing players’ equilibrium behavior—which corresponds to the case \( \sigma = \sigma^* \)—and when proving the existence of an equilibrium—which we do by letting \( \sigma \) be a stage-undominated response given continuation play \( \sigma^* \), and by finding a fixed point of the corresponding mapping.
Lemma 5 (Best Response) The parameters \( w^i_{\sigma*} \) and \( e^i_{\sigma*} \) introduced in Lemma 3 depend on the strategy profile \( \sigma^* \) only through the continuation payoff parameters \( w^i_{\sigma^*} \) and \( e^i_{\sigma^*} \), so we can write \( w^i_{\sigma*} = W^i_\sigma (w^i_{\sigma^*}, e^i_{\sigma^*}) \) and \( e^i_{\sigma*} = E^i_\sigma (w^i_{\sigma^*}, e^i_{\sigma^*}) \).

Moreover, for any profile of continuation payoff parameters \( (w^*, e^*) \in \mathbb{R}^4 \),

\[ (i) \quad w^* = W^\sigma (w^*, e^*) \text{ and } e^* = E^\sigma (w^*, e^*) \text{ if and only if } w^* = w^\sigma \text{ and } e^* = e^\sigma. \]

\[ (ii) \quad \text{If the actions prescribed by } \sigma \text{ are stage undominated given continuation payoff parameters } (w^*, e^*), \text{ then for all } i \in \{L, R\}, \]

\[
W^i_\sigma (w^i_*, e^i_*) = w_i + \delta \int_{Y^\sigma(p,p) \cap Y^\sigma(n,n)} (w^i_* - \theta(v)) d\mu(v) - \delta \int_{Y^\sigma(p,p) \cap Y^\sigma(n,q)} e^i_* d\mu(v), \quad (6)
\]

and

\[
E^i_\sigma (w^i_*, e^i_*) = e_i + \delta \int_{Y^\sigma(p,p) \cap Y^\sigma(q,q)} e^i_* d\mu(v) + \delta \int_{Y^\sigma(p,p) \cap Y^\sigma(q,n)} (\theta(v) - w^i_*) d\mu(v) \quad (7)
\]

\[ + \delta \int_{Y^\sigma(p,n) \cap Y^\sigma(q,q)} (w^i_* + e^i_* - \theta(v)) d\mu(v). \]

**Proof.** Step 1: \( (w^i_{\sigma*}, e^i_{\sigma*}) \) depends on \( \sigma^* \) only through \( (w^i_{\sigma*}, e^i_{\sigma*}) \), and \( W^i_\sigma (w^i_{\sigma*}, e^i_{\sigma*}) \) and \( E^i_\sigma (w^i_{\sigma*}, e^i_{\sigma*}) \) are \( \delta \)-contractions in \( (w^i_{\sigma*}, e^i_{\sigma*}) \).

From (2), depending on the policies \( x, y \in X \), the terms \( V^i_{\sigma^*} (\theta(v), x) - V^i_{\sigma^*} (\theta(v), y) \) in (4) and (5) are equal to 0 when \( x = y \), \( \pm e^i_{\sigma^*} \) when \( \{x, y\} = \{p, q\} \), \( \pm (\theta(v) - w^i_{\sigma^*}) \) when \( \{x, y\} = \{n, p\} \), or \( \pm (\theta(v) - w^i_{\sigma^*} - e^i_{\sigma^*}) \) when \( \{x, y\} = \{n, q\} \), which proves the first claim of Step 1. Moreover, these terms are bounded by \( \delta (E(|\theta|) + |w^i_{\sigma^*}| + |e^i_{\sigma^*}|) \), and in the formula (4) and (5), they are integrated over disjoint sets of states of the world, which proves the second claim of Step 1.

**Step 2: Proof of Part (i).**

Clearly, if players expect continuation payoff parameters \( (w^\sigma, e^\sigma) \) in period 2 and play \( \sigma \) in period 1, then they expect continuation payoff \( (w^\sigma, e^\sigma) \) in period 0. By definition of \( W^\sigma \) and \( E^\sigma \), this means that \( w^\sigma = W^\sigma (w^\sigma, e^\sigma) \) and \( e^\sigma = E^\sigma (w^\sigma, e^\sigma) \). Conversely, suppose that \( (w^*, e^*) \in \mathbb{R}^4 \) is such that \( w^* = W^\sigma (w^*, e^*) \) and \( e^* = E^\sigma (w^*, e^*) \). Then \( (w^*, e^*) \) and \( (w^\sigma, e^\sigma) \) are both fixed points of the mapping \( (\omega, \varepsilon) \rightarrow (W^\sigma (\omega, \varepsilon), E^\sigma (\omega, \varepsilon)) \). From Step 1, this mapping has a unique fixed point, so \( (w^*, e^*) = (w^\sigma, e^\sigma) \).

**Step 3: Proof of (ii).**

Consider the summands \( e^i_{\sigma*} (x,y) \equiv \int_{Y^\sigma(p,x) \cap Y^\sigma(q,y)} (V^i_{\sigma*} (\theta(v), x) - V^i_{\sigma^*} (\theta(v), y)) d\mu(v) \) on the R.H.S. of (5) for all possible policies \( x \) and \( y \) that can be implemented under status quo \( p \) and \( q \), respectively. Observe first that \( e^i_{\sigma*} (x,y) = 0 \) when \( x = y \). Consider next
the case \((x, y) = (q, n)\), that is, status quo \(p\) is replaced by \(q\), and status quo \(q\) is replaced by \(n\). Since \(\sigma\) is stage undominated given continuation payoff parameters \((w^*, e^*)\), for all \(v \in \Upsilon^\sigma(p, q) \cap \Upsilon^\sigma(q, n)\), both players must weakly prefer to implement \(q\) to \(p\) and \(n\) to \(q\) in period 1. Moreover, one of them must be indifferent between implementing \(n\) and \(q\), because if both players strictly preferred to implement \(n\) to \(q\) in state \(v\), then under status quo \(p\), either veto player would accept \(n\), so proposing \(n\) instead of \(q\) in that period would be a profitable deviation for the proposer. Therefore, for all \(v \in \Upsilon^\sigma(p, q) \cap \Upsilon^\sigma(q, n)\), \(\theta(v) - w^\sigma_i - e^\sigma_i = 0\) for some player \(i\). Since \(F\) is continuous, this implies that \(\mu(\Upsilon^\sigma(p, q) \cap \Upsilon^\sigma(q, n)) = 0\), so \(e_i^\sigma(q, n) = 0\). For the case \((x, y) = (p, n)\), an analogous reasoning implies that \(\mu(\Upsilon^\sigma(q, p) \cap \Upsilon^\sigma(p, n)) = 0\) so \(e_i^\sigma(q, p) = 0\). Consider now the case \((x, y) = (q, p)\), that is, status quo \(p\) is replaced by \(q\) and vice versa. If \(\Upsilon^\sigma(p, q) \cap \Upsilon^\sigma(q, p)\) is empty, \(e_i^\sigma(q, p) = 0\), and if it is not empty, stage undomination implies that \(e_i^\sigma = e^\sigma_R = 0\), so \(e_i^\sigma(q, p) = 0\) as well. The only remaining cases left are \((x, y)\) equal to \((p, q), (p, n)\), and \((n, q)\), so (5) can be simplified as follows:

\[
\frac{E^\sigma_i(w^*, e^*) - e_i}{\delta} = \int_{\Upsilon^\sigma(p, q) \cap \Upsilon^\sigma(q, n)} \left( V^\sigma_i(\theta(v), p) - V^\sigma_i(\theta(v), q) \right) d\mu(v) \\
+ \int_{\Upsilon^\sigma(p, q) \cap \Upsilon^\sigma(n, p)} \left( V^\sigma_i(\theta(v), p) - V^\sigma_i(\theta(v), n) \right) d\mu(v) \\
+ \int_{\Upsilon^\sigma(p, n) \cap \Upsilon^\sigma(q, q)} \left( V^\sigma_i(\theta(v), n) - V^\sigma_i(\theta(v), q) \right) d\mu(v).
\]

Substituting (2) into the above expression, we obtain (7).

**Step 4: Proof of (6).**

This proofs follows a similar logic as the proof of Step 3. The details are as follows. Consider the summands \(w_i^\sigma e_i^\sigma(x, y) \equiv \int_{\Upsilon^\sigma(x, y)} (V^\sigma_i(\theta(v), x) - V^\sigma_i(\theta(v), y)) d\mu(v)\) on the R.H.S. of (4) for all possible policies \(x\) and \(y\) that can be implemented under status quo \(p\) and \(n\), respectively. Using the same steps as the ones we used to prove \(\mu(\Upsilon^\sigma(p, q) \cap \Upsilon^\sigma(q, n)) = 0\) and reversing the role of \(p\) and \(q\), we obtain that \(\mu(\Upsilon^\sigma(p, n) \cap \Upsilon^\sigma(n, q)) = 0\), so \(w_i^\sigma e_i^\sigma(n, q) = 0\). An analogous reasoning implies that \(\mu(\Upsilon^\sigma(p, q) \cap \Upsilon^\sigma(n, p)) = 0\), so \(w_i^\sigma e_i^\sigma(q, p) = 0\). Moreover, for all \(v \in \Upsilon^\sigma(p, n) \cap \Upsilon^\sigma(n, p)\), either player \(i\) must be indifferent between implementing \(n\) and \(p\), so \(\theta(v) - w_i^\sigma - e_i^\sigma = 0\). Since \(F\) is continuous, this implies that \(\mu(\Upsilon^\sigma(p, n) \cap \Upsilon^\sigma(n, p)) = 0\), so \(w_i^\sigma e_i^\sigma(p, n) = 0\). The only remaining cases left are \((x, y)\) equal to \((p, n), (p, q)\), and \((q, n)\). Equation (6) follows then from substituting these three cases into the right-hand side of (4) and using (2).

**Lemma 6 (Equilibrium Existence)** There exists an equilibrium such that \(w^\sigma_L < w^\sigma_R\) and \(e^\sigma_L > 0\).
**Sketch of the proof.** The proof, whose details can be found in the online appendix, proceeds as follows. Consider an arbitrary strategy profile σ such that \( w_L^\sigma < w_R^\sigma \) and \( e_L^\sigma < 0 \). We first construct a stage-undominated response \( \sigma^* \) to \( \sigma \) such that (i) \( w_L^{\sigma^*} < w_R^{\sigma^*} \) and (ii) \( e_L^{\sigma^*} < 0 \), and (iii) \( \sigma^* \) depends on \( \sigma \) only through \( w^\sigma \) and \( e^\sigma \). We then show that a fixed point of the mapping \( (w^\sigma, e^\sigma) \rightarrow (w^{\sigma^*}, e^{\sigma^*}) \) characterizes an equilibrium with the desired properties. We finally show that this mapping satisfies the condition of Brower’s fixed point theorem. Special care is given to player’s behavior when indifferent between implementing properties. We finally show that this mapping satisfies the condition of Brower’s fixed point theorem.

**Proof of Proposition 2.** Proposition 2 follows from Lemma 7 part A) below.

**Lemma 7 (Necessary and Sufficient Conditions for EE)**

A) Let \( \sigma \) be an EE. Then \( e_L^\sigma > 0, e_R^\sigma \geq 0, w_L^\sigma < w_R < w_R^\sigma \), and for all \( i \in \{L, R\} \),

\[
  w_i^\sigma = w_i + \delta \int_{w_i^\sigma}^{w_R^\sigma} (w_i^\sigma - \theta) dF(\theta). \tag{8}
\]

B) For any EE \( \sigma \), there exists an EE \( \sigma' \) such that players never accept nor propose \( q \) under any status quo, \( w^{\sigma'} = w^\sigma \) and for all \( i \in \{L, R\} \),

\[
  e_i^{\sigma'} = e_i + \delta \int_{w_L^\sigma}^{\min\{w_L^{\sigma'}, w_R^\sigma\}} b_R(\theta, q)(\theta - w_i^\sigma) dF(\theta). \tag{9}
\]

C) Reciprocally, if there exists \( (w^*, e^*) \in \mathbb{R}^4 \) which satisfy \( e_R^* \geq 0 \) and the same conditions as \( (w^\sigma, e^\sigma) \) in (8) and (9), then there exists an EE \( \sigma \) such that \( (w^\sigma, e^\sigma) = (w^*, e^*) \).

**Proof.** Part A: Let \( \sigma \) be an arbitrary EE.

**Step A1:** \( w_L^\sigma < w_R^\sigma \) and for all \( i \in \{L, R\} \),

\[
  w_i^\sigma = w_i + \delta \int_{\mathcal{T}^\sigma(p, p) \cap \mathcal{T}^\sigma(n, n)} (w_i^\sigma - \theta(v)) d\mu(v). \tag{10}
\]

From Lemma 4, \( \sigma \) prescribes stage undominated actions given continuation play \( \sigma \). Therefore, Lemma 5 Part (i) implies that \( w_i^\sigma = W_i^\sigma(w_i^\sigma, e_i^\sigma) \), and Lemma 5 Part (ii) further implies that \( W_i^\sigma(w_i^\sigma, e_i^\sigma) \) is given by (6) where \( (w_i^*, e_i^*) = (w_i^{\sigma^*}, e_i^{\sigma^*}) \). Since \( \sigma \) is an EE, \( \mu(\mathcal{T}^\sigma(n, q)) = 0 \). Substituting the latter equality into (7), we obtain (10). Taking differences across players in (10) and solving for \( w_R^\sigma - w_L^\sigma \), we obtain \( w_R^\sigma - w_L^\sigma = \frac{w_R - w_L}{1 - \delta\mu(\mathcal{T}^\sigma(p, p) \cap \mathcal{T}^\sigma(n, n))} < 0 \).

**Step A2:** \( e_L^\sigma \geq 0 \) and \( e_R^\sigma \geq 0 \)

Suppose that \( e_i^\sigma < 0 \) for some \( i \in \{L, R\} \). Then in any period \( t \) in which \( s(t) = n \), the
proposer is $i$, and $\theta(t) > \max_{k \in \{L,R\}} \{w^\sigma_k + e^*_k\}$, stage undomination implies that the other player $j$ must accept proposal $q$. Since $q$ is the outcome that gives the greatest continuation payoff to $i$ in that state, the only stage undominated action for $i$ is to propose $q$, which contradicts the assumption that $\sigma$ is an EE.

**Step A3:** modulo a zero measure set, $\Upsilon^\sigma(p,p) = \{v \in \Upsilon : \theta(v) > w^\sigma_L\}$ and $\Upsilon^\sigma(n,n) = \{v \in \Upsilon : \theta(v) < w^\sigma_R\}$. For all $v \in \Upsilon$ such that $\theta(v) > w^\sigma_L$, $L$ strictly prefers to implement $p$ to $n$, so stage undomination implies that $v \notin \Upsilon^\sigma(p,n)$. Since $\sigma$ is an EE, $\mu(\Upsilon^\sigma(p,q)) = 0$. By definition of $\Upsilon^\sigma$, \((\Upsilon^\sigma(x), x \in \{n,p,q\}\) is a partition of $\Upsilon$, so necessarily, $v \in \Upsilon^\sigma(p,p)$. Conversely, for all $v \in \Upsilon$ such that $\theta(v) < w^\sigma_L$, from Step A1, $\theta(v) < w^\sigma_R$, so both players strictly prefer to implement $n$ to $p$. Therefore, stage undomination implies that $v \notin \Upsilon^\sigma(p,p)$. Since $F$ is continuous, the set of $v$ such that $\theta(v) = w^\sigma_L$ has probability 0, which completes the proof of the first equality in Step A3.

For all $v \in \Upsilon$ such that $\theta(v) < w^\sigma_R$, $R$ strictly prefers to implement $n$ to $p$, so stage undomination implies that $v \notin \Upsilon^\sigma(n,p)$. Since $\sigma$ is an EE, $\mu(\Upsilon^\sigma(n,q)) = 0$. Since $(\Upsilon^\sigma(x), x \in \{n,p,q\})$ is a partition of $\Upsilon$, necessarily, $v \in \Upsilon^\sigma(n,n)$. Conversely, for all $v \in \Upsilon$ such that $\theta(v) > w^\sigma_R$ and, from Step A1, $\theta(v) > w^\sigma_L$, both players strictly prefer to implement $p$ to $n$. Therefore, stage undomination implies that $v \notin \Upsilon^\sigma(n,n)$. The second equality in Step A3 follows then from the fact that $F$ is continuous.

**Step A4:** $(w^\sigma_L, w^\sigma_R)$ satisfies (8) and $w^\sigma_L < w^\sigma_R < w^\sigma_R$. From Step A3, modulo a zero measure set, $\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(n,n)$ is equal to the set of $v \in \Upsilon$ such that $\theta(v) \in (w^\sigma_L, w^\sigma_R)$. Substituting this equality into (10), we obtain (8). Step A1 and the assumption that $F$ has full support imply

$$\int_{w^\sigma_L}^{w^\sigma_R} (w^\sigma_R - \theta) dF(\theta) < 0 < \int_{w^\sigma_L}^{w^\sigma_R} (w^\sigma_R - \theta) dF(\theta),$$

so (8) implies in turn that $w^\sigma_L < w^\sigma_R < w^\sigma_R$.

**Step A5:** $e^*_L > 0$

Let us first prove that $\mu(\Upsilon^\sigma(p,n) \cap \Upsilon^\sigma(q,q)) = 0$. Suppose by contradiction that the latter probability is positive. Then for some $v \in \Upsilon^\sigma(p,n) \cap \Upsilon^\sigma(q,q)$, both players strictly prefer to implement $n$ to $p$ and one player weakly prefers to implement $q$ to $n$, so $\min_{i} \{w^\sigma_i + e^*_i\} \leq \theta(v) < w^\sigma_L$ and, therefore, $\min_{i} \{w^\sigma_i + e^*_i\} < w^\sigma_L$, a contradiction with Step A1 and A2.

From Lemma 4, $\sigma$ prescribes stage undominated actions given continuation play $\sigma$. So Lemma 5 Part (i) implies that $e^*_i = E^*_i(u^*_i, e^*_i)$, and Lemma 5 Part (ii) further implies that $E^*_i(u^*_i, e^*_i)$ is given by (7) with $(u^*_i, e^*_i) = (u^\sigma_i, e^\sigma_i)$. Substituting $\mu(\Upsilon^\sigma(p,n) \cap \Upsilon^\sigma(q,q)) = 0$
into (7), we obtain
\[ e_i^\sigma = e_i + \delta \int_{\mathcal{Y}^\sigma(p,p) \cap \mathcal{Y}^\sigma(q,q)} e_i^\sigma d\mu(v) + \delta \int_{\mathcal{Y}^\sigma(p,p) \cap \mathcal{Y}^\sigma(q,n)} (\theta(v) - w_i^\sigma) d\mu(v). \] (11)

From Step A3, for almost all \( v \in \mathcal{Y}^\sigma(p,p) \), \( \theta(v) > w_i^\sigma \), so (11) implies that \( e_i^\sigma > e_L > 0 \).

**Step A6: \( \mathcal{Y}^\sigma(q,q) \subseteq \mathcal{Y}^\sigma(p,p) \)**

For all \( v \in \mathcal{Y}^\sigma(q,q) \), stage undomination implies that one player weakly prefers to implement \( q \) to \( n \), so \( \theta(v) \geq \max\{w_L^\sigma + e_L^\sigma, w_R^\sigma + e_R^\sigma\} \). From Step A1, A2, and A5, this implies that \( \theta(v) > w_L^\sigma \) and so, as shown in Step A3, \( v \in \mathcal{Y}^\sigma(p,p) \).

**Part B:**

This step basically shows that for any EE \( \sigma \), there exists an "equivalent" EE \( \sigma' \) such that (i) under status quo \( q \), \( \sigma' \) prescribes players to play pure strategies which never implement \( q \), and (ii) under status quo \( n \) or \( p \), \( \sigma' \) prescribe actions which lead to the same path of play as \( \sigma \). We construct this strategy profile \( \sigma' \) as the limit of a sequence of strategy profiles \( (\sigma^k)_{k \in \mathbb{N}} \) which we define recursively as follows: \( \sigma^0 = \sigma \), and for all \( k \in \mathbb{N} \), \( \sigma^{k+1} \) is stage-undominated given continuation payoff \( (w_{\sigma^k}^\sigma, e_{\sigma^k}) \), and when indifferent between two actions that lead to outcome \( p \) or \( q \), \( \sigma^{k+1} \) always prescribes players to play the action that lead to \( p \). The details can be found in the online appendix.

**Part C:** See the online appendix.

The following two lemmas derive some properties that are common to all IE, and to all equilibria, respectively.

**Lemma 8 (Properties of IE)** Let \( \sigma \) be an IE and let \( \Lambda, \varrho \in \{L, R\} \) be such that \( w_{\Lambda}^\sigma \leq w_{\varrho}^\sigma \). Then \( e_{\varrho}^\sigma \leq 0 < e_{\Lambda} < e_{\varrho}^\sigma \), \( w_{\Lambda}^\sigma < w_{\varrho}^\sigma \), \( w_{\varrho}^\sigma < \min_{i \in \{L,R\}} \{w_i^\sigma + e_i^\sigma\} \), and \( \mathcal{Y}^\sigma(q,q) \subseteq \mathcal{Y}^\sigma(p,p) \).

**Proof.** Let \( \sigma \) be an arbitrary IE.

**Step 1:** \( \mu(\mathcal{Y}^\sigma(p,p) \cap \mathcal{Y}^\sigma(q,n)) > 0 \).

Since \( \sigma \) is an equilibrium, the actions prescribed by \( \sigma \) are stage undominated given continuation payoff \( (w_{\sigma}^\sigma, e_{\sigma}) \). Using Lemma 5 Parts (i) and (ii) for \( (w_i^\sigma, e_i^\sigma) = (w_i^\sigma, e_i^\sigma) \), we obtain
\[
e_i^\sigma = E_i^\sigma(\sigma, w^\sigma, e^\sigma) = e_i + \delta \int_{\mathcal{Y}^\sigma(p,p) \cap \mathcal{Y}^\sigma(q,q)} e_i^\sigma d\mu(v) + \delta \int_{\mathcal{Y}^\sigma(p,p) \cap \mathcal{Y}^\sigma(q,n)} (\theta(v) - w_i^\sigma) d\mu(v) \\
+ \delta \int_{\mathcal{Y}^\sigma(p,n) \cap \mathcal{Y}^\sigma(q,q)} (w_i^\sigma + e_i^\sigma - \theta(v)) d\mu(v). \] (12)

In any state \( v \in \mathcal{Y}^\sigma(p,n) \), each player \( i \) must prefer implementing \( n \) to \( p \) so \( \theta(v) \leq w_i^\sigma \) and, therefore, \( w_i^\sigma + e_i^\sigma - \theta(v) \geq e_i^\sigma \). Substituting the latter inequality in the above equation, we
obtain
\[
e^\sigma_i \geq e_i + \delta \left( \int_{\mathcal{Y}^\sigma(p,p) \cup \mathcal{Y}^\sigma(p,n) \cap \mathcal{Y}^\sigma(q,q)} e^\sigma_i d\mu(v) + \int_{\mathcal{Y}^\sigma(p,n) \cap \mathcal{Y}^\sigma(q,q)} (\theta(v) - w^\sigma_i) d\mu(v) \right).
\]
Regrouping the terms in factor of \( e^\sigma_i \) yields
\[
e^\sigma_i \geq e_i + \delta \int_{\mathcal{Y}^\sigma(p,p) \cap \mathcal{Y}^\sigma(q,q)} (\theta(v) - w^\sigma_i) d\mu(v)
\]
\[
\frac{1}{1 - \delta \mu((\mathcal{Y}^\sigma(p,p) \cup \mathcal{Y}^\sigma(p,n)) \cap \mathcal{Y}^\sigma(q,q))}.
\]
(13)
Suppose by contradiction that \( \mu(\mathcal{Y}^\sigma(p,p) \cap \mathcal{Y}^\sigma(q,q)) = 0 \). Then from (13), \( e^\sigma_L > 0 \) and \( e^\sigma_R > 0 \), that is, both players always strictly prefer to implement \( p \) to \( q \). But then \( q \) is never implemented on the equilibrium path, which is impossible since \( \sigma \) is an IE.

Step 2: \( w^\sigma_q < \min_{i \in \{L,R\}} (w^\sigma_i + e^\sigma_i) \).

For all \( v \in \mathcal{Y}^\sigma(p,p) \cap \mathcal{Y}^\sigma(q,q) \), both players weakly prefer to implement \( n \) to \( q \), i.e., \( \theta(v) \leq w^\sigma_i + e^\sigma_i \), and at least one player weakly prefers to implement \( p \) to \( n \), i.e., \( \theta(v) \geq w^\sigma_i \), so
\[
\min_{i \in \{L,R\}} w^\sigma_i \leq \theta(v) \leq \min_{i \in \{L,R\}} (w^\sigma_i + e^\sigma_i).
\]
From step 1, \( \mu(\mathcal{Y}^\sigma(p,p) \cap \mathcal{Y}^\sigma(q,q)) > 0 \), so the above inequalities must hold strictly for some \( v \in \mathcal{Y}^\sigma(p,p) \cap \mathcal{Y}^\sigma(q,q) \), which implies Step 2.

Step 3: \( e^\sigma_o \leq 0 \) and \( e^\sigma_A > e^\sigma_o > 0 \).

For all \( v \in \mathcal{Y}^\sigma(p,p) \), at least one player weakly prefers to implement \( p \) to \( n \), so \( \theta(v) \geq w^\sigma_A \). Since \( F \) is continuous, the latter inequality is strict for almost all \( v \in \mathcal{Y}^\sigma(p,p) \cap \mathcal{Y}^\sigma(q,q) \). Substituting this inequality and Step 1 into (13), we obtain \( e^\sigma_o > e^\sigma_A > 0 \). As argued in Step 1, since \( \sigma \) is an IE, \( e^\sigma_A > 0 \) implies \( e^\sigma_o \leq 0 \).

Step 4: \( \mathcal{Y}^\sigma(q,q) \subset \mathcal{Y}^\sigma(p,p) \).

Let \( v \in \mathcal{Y}^\sigma(q,q) \). Since \( \sigma \) is an equilibrium, in state \( \theta(v) \), one player must weakly prefer to implement \( q \) to \( n \), so \( \theta(v) \geq \min_{i \in \{L,R\}} \{w^\sigma_i + e^\sigma_i\} \). From Step 2, this implies that \( \theta(v) > \min_{i \in \{L,R\}} \{w^\sigma_i\} \), in which case one player must strictly prefer to implement \( p \) to \( n \) so \( v \notin \mathcal{Y}^\sigma(p,n) \). From Step 3, \( e^\sigma_A > 0 \) so \( v \notin \mathcal{Y}^\sigma(p,q) \). Since \( (\mathcal{Y}^\sigma(x,p))_{x \in \{n,p,q\}} \) is a partition of \( \mathcal{Y} \), this implies that \( v \in \mathcal{Y}^\sigma(p,p) \), as needed.

Step 5: \( w^\sigma_A < w^\sigma_o \).

From Step 3, \( e^\sigma_o \leq 0 \), so (13) implies that \( \int_{\mathcal{Y}^\sigma(p,p) \cap \mathcal{Y}^\sigma(q,q)} (\theta(v) - w^\sigma_o) d\mu(v) < 0 \). Therefore, there exists \( \mathcal{Y}^o \subset \mathcal{Y}^\sigma(p,p) \cap \mathcal{Y}^\sigma(q,q) \) such that \( \mu(\mathcal{Y}^o) > 0 \) and, for all \( v^o \in \mathcal{Y}^o \), \( \theta(v^o) < w^\sigma_o \), that is, \( g \) strictly prefers to implement \( n \) to \( p \) in state \( v^o \). Since \( \mathcal{Y}^o \subset \mathcal{Y}^\sigma(p,p) \) and since \( \sigma \) is an equilibrium, \( A \) must weakly prefer to implement \( p \) to \( n \) in state \( v^o \), i.e., \( \theta(v^o) - w^\sigma_A \geq 0 \),
which implies $w^\sigma_\Lambda < w^\sigma_\delta$. ■

**Proof of Proposition 3.** Let $\sigma$ be an IE. Lemma 8 directly implies that $\min \{e^\sigma_L, e^\sigma_R\} \leq 0 < \max \{e^\sigma_L, e^\sigma_R\}$, $\min \{w^\sigma_L, w^\sigma_R\} < \min \{w^\sigma_L + e^\sigma_L, w^\sigma_R + e^\sigma_R\}$, and $\Upsilon^\sigma (q, q) \subset \Upsilon^\sigma (p, p)$. The latter inclusion shows formally that $q$ is repealed for a larger set of states than $p$.

We now prove the characterization of the three kinds of IE. Using the notations of Lemma 8, suppose first that $\Lambda = L$, $\varrho = R$ and $w^\sigma_L + e^\sigma_L < w^\sigma_R$. Then Lemma 8 implies that $\sigma$ satisfies all the properties of an IE-A. Suppose now that $\Lambda = L$, $\varrho = R$ and $w^\sigma_L + e^\sigma_L \geq w^\sigma_R$. Then Lemma 8 implies that $\sigma$ satisfies all the properties of an IE-B. Finally, the only remaining possibility is that $\Lambda = R$ and $\varrho = L$. In that case, once again, Lemma 8 implies that $\sigma$ satisfies all the properties of an IE-C. ■

**Proof of Proposition 5.** See the online appendix. ■

**Proof of Proposition 6.** *Step 0: Notations.*

Throughout the proof, we fix $\delta$, $b$, and $F$. Consider first the necessary condition (8) for an EE. This condition depends on $(w, e)$ only through $w$, so let $W(w)$ denote the set of $w^\sigma \in \mathbb{R}^2$ that satisfy (8) for $i = L, R$. Consider then the necessary condition (9) for the type of EE described in Lemma 7 part B for $i = L$:

$$e^\sigma_L = e_L + \delta \int_{w^\sigma_L}^{\min \{w^\sigma_L + e^\sigma_L, w^\sigma_R\}} b_R (\theta, q) (\theta - w^\sigma_L) dF (\theta) , \tag{14}$$

For any $w^\sigma \in \mathbb{R}^2$ and $e_L \in \mathbb{R}$, (14) can viewed as a fixed point in $e^\sigma_L$. It depends on $(w, e)$ only through $w^\sigma$ and $e_L$, so let $E_L (w^\sigma, e_L)$ denote the set of solutions $e^\sigma_L$ to (14). Since the R.H.S. of (14) is continuous and bounded in $e^\sigma_L$, $E_L (w^\sigma, e_L)$ is not empty and closed, so let $e^\sigma_{\underline{L}} (w^\sigma, e_L)$ denote its smallest element. Finally, condition (9) for $i = R$ gives

$$e^\sigma_R = e_R + \delta \int_{w^\sigma_L}^{\min \{w^\sigma_L + e^\sigma_L, w^\sigma_R\}} b_R (\theta, q) (\theta - w^\sigma_R) dF (\theta) . \tag{15}$$

Let $e^\sigma_R (w^\sigma, e^\sigma_L, e_R)$ denote the R.H.S. of (15).

*Step 1: for all $w^\sigma \in W(w)$, $e^\sigma_{\underline{L}} (w^\sigma, e_L)$ is weakly increasing in $e_L$, and $e^\sigma_R (w^\sigma, e^\sigma_L, e_R)$ is weakly increasing in $e_R$, and weakly decreasing in $e^\sigma_L$. Since the R.H.S. of (14) is continuous in $(e^\sigma_L, e^\sigma_R)$ and weakly increasing in $e_L$, from Villas Boas (1997, Theorem 1), the smallest fixed point $e^\sigma_{\underline{L}} (w^\sigma, e_L)$ of (14) is weakly increasing in $e_L$. The comparative statics on $e^\sigma_R (\cdot)$ follow readily from (15) and the fact that for all $w^\sigma \in W(w)$, $w^\sigma_L < w^\sigma_R$ (see Step A1 in the proof of Lemma 7).

*Step 2: no EE exists if and only if for all $w^\sigma \in W(w)$, $e^\sigma_R \left[ w^\sigma, e^\sigma_L (w^\sigma, e_L), e_R \right] < 0$.

From Lemma 7, an EE exists if and only if there exists $(w^\sigma, e^\sigma) \in \mathbb{R}^4$ which satisfy the same
condition as \( w^\sigma \) and \( e^\sigma \) in (8) and (9), and \( e^\sigma_R \geq 0 \). Using the notations of Step 0, this means that an EE exists if and only if there exists \( w^\sigma \in W(w) \) and \( e^\sigma_L \in E_L(w^\sigma, e_L) \) such that 
\[
e^\sigma_R (w^\sigma, e^\sigma_L, e_R) \geq 0.
\]
From Step 1, this is the case if and only if 
\[
e^\sigma_R \left( w^\sigma, e^\sigma_L (w^\sigma, e^\sigma_L), e_R \right) \geq 0
\]
for some \( w^\sigma \in W(w) \).

**Step 3: Proof of Parts (i) and (ii) of Proposition 6.**

An equilibrium is either an EE or an IE, so all equilibria are IE if and only if no EE exists. Therefore, Part (i) follows from Step 2, together with the comparative statics established in Step 1. We now prove Part (ii). Let \( w^\sigma \in W(w) \). As shown in Step A1 in the proof of Lemma 7, \( w^a_L < w^a_R \), so (14) implies that 
\[
e^\sigma_L (w^\sigma, e_L) \geq e_L > 0.
\]
Finally, (15) implies that
\[
\lim_{e_R \to 0} e^\sigma_R \left( w^\sigma, e^\sigma_L (w^\sigma, e_L), e_R \right) = \delta \int_{w^\sigma_L}^{\min \left\{ w^\sigma_L + e^\sigma_L (w^\sigma, e_L), w^\sigma_R \right\}} b_R(\theta, q) \left( \theta - w^\sigma_R \right) dF(\theta).
\]
Since \( F \) has full support and \( b_R(\theta, q) > b \) for all \( \theta \in \mathbb{R} \), the R.H.S. of the above equation is strictly negative. From step 2, this implies that, for \( e_R \) sufficiently small, no EE exists.

**Step 4: Proof of Part (iii) of Proposition 6.**

The case \( w_R - w_L \to +\infty \) in Part (iii) is treated in the online appendix. Consider now the case \( w_R - w_L \to 0 \), and suppose Proposition 6 is false. Then from step 2, there exists a sequence \( (w^k)_{k \in \mathbb{N}} \) such that \( w^k_R - w^k_L \to 0 \) and for all \( k \in \mathbb{N} \), for all \( \hat{w}^k \in W(w^k) \),
\[
e^\sigma_R \left( \hat{w}^k, e^\sigma_L (\hat{w}^k, e_L), e_R \right) < 0.
\]
Taking differences across players in (8), we obtain
\[
\hat{w}^k_R - \hat{w}^k_L = \frac{w^k_R - w^k_L}{1 - \delta \left( F(\hat{w}^k_R) - F(\hat{w}^k_L) \right)} \leq \frac{w^k_R - w^k_L}{1 - \delta},
\]
so \( \hat{w}^k_R - \hat{w}^k_L \to k \to 0 \). Since \( \hat{w}^k \in W(w^k) \), \( \hat{w}^k_R > \hat{w}^k_L \), so (15) implies
\[
e^\sigma_R \left( \hat{w}^k, e^\sigma_L (\hat{w}^k, e_L), e_R \right) \geq e_R + \delta \int_{\hat{w}^k_L}^{\hat{w}^k_R} b_R(\theta, q) \left( \theta - \hat{w}^k \right) dF(\theta).
\]
Since \( \hat{w}^k_R - \hat{w}^k_L \to k \to 0 \) and \( F \) is continuous, the above inequality implies
\[
e^\sigma_R \left( \hat{w}^k, e^\sigma_L (\hat{w}^k, e_L), e_R \right) \to k \to 0 e_R > 0,
\]
contradicting the assumption that 
\[
e^\sigma_R \left( \hat{w}^k, e^\sigma_L (\hat{w}^k, e_L), e_R \right) < 0.
\]
Proof of Propositions 4 and 7. Let \( \pi \in [0, 1] \), and \( \theta_{\text{low}}, \theta_{\text{high}} \in \mathbb{R} \) be such that

\[
\begin{align*}
    w_L < \theta_{\text{low}} < (1 - \pi) \theta_{\text{low}} + \pi \theta_{\text{high}} < & \min_{i \in \{L,R\}} \{w_i + e_i\} \leq \max_{i \in \{L,R\}} \{w_i + e_i\} < \theta_{\text{high}}, \\
\pi < & \frac{w_R - (1 - \delta) \theta_{\text{high}} - \delta \theta_{\text{low}}}{\delta (\theta_{\text{high}} - \theta_{\text{low}})}, \\
\pi < & \frac{w_L + (1 - \delta) e_L - \theta_{\text{low}}}{\delta (\theta_{\text{high}} - \theta_{\text{low}})}, \\
\theta_{\text{high}} > & w_R + e_R + \frac{\delta (1 - \pi) (1 - b)}{1 - \delta (1 - \pi) (1 - b)} (w_R + e_R - \theta_{\text{low}}),
\end{align*}
\]

where \( b \) is the lower bound on the probability that either player is the proposer. Condition (16) implies that both players get a greater flow payoff from \( n \) than from \( q \) in state \( \theta_{\text{low}} \), and vice versa in state \( \theta_{\text{high}} \), whereas \( L \) (\( R \)) gets a greater flow payoff from \( n \) (\( p \)) in either states. The meaning of the other conditions will be clarified as we use them below.

Consider the following distribution for \( \theta(t) \). With probability \( 1 - \pi \), \( \theta(t) = \theta_{\text{low}} \), and with the remaining probability \( \pi \), \( \theta(t) = \theta_{\text{high}} \). For all \( X \subseteq \{n, p, q\} \), let \( \Gamma(X) \) denote the game \( \Gamma \) in which the set of available policies is \( X \). Below, we characterize the equilibria of \( \Gamma(X) \) for this degenerate distribution, show that the equilibrium correspondence is continuous as we approximate it with a full support distribution, and use these results to prove Propositions 4 and 7.

Step 0: for \( \delta \) sufficiently close to 1, there exists \( \tilde{\theta}_{\text{low}}, \tilde{\theta}_{\text{high}}, \tilde{\theta}_{\text{high}}, \bar{\pi} \in \mathbb{R} \) such that \( \tilde{\theta}_{\text{low}} > w_L, \tilde{\theta}_{\text{high}} < \tilde{\theta}_{\text{high}}, \bar{\pi} > 0 \), and for all \( \theta_{\text{low}} \in (w_L, \tilde{\theta}_{\text{low}}), \theta_{\text{high}} \in (\tilde{\theta}_{\text{high}}, \tilde{\theta}_{\text{high}}), \) and \( \pi \in [0, \bar{\pi}) \), conditions (16) to (19) are satisfied.

Let

\[
\begin{align*}
    \tilde{\theta}_{\text{low}} & \equiv \min \{w_L + (1 - \delta) e_L, w_R\}, \\
    \tilde{\theta}_{\text{high}} & \equiv \max \left\{ w_L + e_L, w_R + e_R + \frac{1 - b}{b} (w_R + e_R - \tilde{\theta}_{\text{low}}) \right\}, \\
    \tilde{\theta}_{\text{high}} & \equiv w_R + \frac{\delta}{1 - \delta} (w_R - \tilde{\theta}_{\text{low}}).
\end{align*}
\]

Observe that as \( \delta \to 1, \tilde{\theta}_{\text{high}} \to +\infty \) so \( \tilde{\theta}_{\text{high}} < \tilde{\theta}_{\text{high}} \). Let \( \theta_{\text{low}} \in (w_L, \tilde{\theta}_{\text{low}}) \) and \( \theta_{\text{high}} \in (\tilde{\theta}_{\text{high}}, \tilde{\theta}_{\text{high}}) \). By definition of \( \tilde{\theta}_{\text{low}} \) and \( \tilde{\theta}_{\text{high}} \), \( \theta_{\text{low}} \) and \( \theta_{\text{high}} \) satisfy (16) for \( \pi \) sufficiently small. Note also that the R.H.S. of (19) is increasing in the term \( \delta (1 - \pi) (1 - b) \), so the definition of \( \tilde{\theta}_{\text{high}} \) guarantees that (19) is satisfied for any \( \pi \in [0, 1] \). Finally, the definition of \( \tilde{\theta}_{\text{low}} \) and \( \tilde{\theta}_{\text{high}} \) imply that the R.H.S. of (17) and (18) are positive.

Step 1: \( \Gamma(\{n, p\}) \) has a unique equilibrium \( \sigma(\{n, p\}) \). In that equilibrium, the initial status quo \( n \) stays in place in all future periods.
Let \( \sigma(\{n,p\}) \) be an equilibrium of \( \Gamma(\{n,p\}) \). From (16), \( w_L < \theta_{\text{low}} < \theta_{\text{high}} \) so \( p \) gives a strictly greater flow payoff than \( n \) to \( L \) in both states. Therefore, if \( s(t) = p, \sigma(\{n,p\}) \) must prescribe \( L \) to veto any policy change, so the equilibrium path stays at \( p \) forever after. Therefore, the continuation payoff for \( R \) of implementing \( p \) in state \( \theta_{\text{high}} \) is

\[
\theta_{\text{high}} - w_R + \frac{\delta}{1 - \delta} \left[ (1 - \pi) \theta_{\text{low}} + \pi \theta_{\text{high}} - w_R \right].
\]

Note also that implementing \( n \) yields an expected continuation payoff of at least 0, because \( R \) can unilaterally impose to stay at \( n \) forever. Therefore, \( R \) prefers implementing \( n \) to \( p \) in state \( \theta_{\text{high}} \) when the above expression is strictly negative. Simple algebra shows that this is equivalent to (17). When (17) is satisfied, \( R \) prefers implementing \( n \) to \( p \) also in state \( \theta_{\text{low}} \), which proves Step 1.

**Step 2:** \( \Gamma(\{n,q\}) \) has a unique equilibrium \( \sigma(\{n,q\}) \). In that equilibrium, players implement \( n \) (\( p \)) in state \( \theta_{\text{low}} \) (\( \theta_{\text{high}} \)).

Let \( \sigma(\{n,q\}) \) be an equilibrium of \( \Gamma(\{n,q\}) \). Suppose some player \( i \) weakly prefers implementing \( q \) to \( n \) in state \( \theta_{\text{low}} \). Since the state is i.i.d. and \( \theta_{\text{low}} < \theta_{\text{high}} \), \( i \) must strictly prefer implementing \( q \) to \( n \) in state \( \theta_{\text{high}} \). A simple induction on \( t \) implies then that \( i \) must strictly prefer implementing \( q \) in all periods to the equilibrium path with initial status quo \( n \). Since \( \sigma(\{n,q\}) \) is an equilibrium, \( i \) also prefers the equilibrium path with initial status quo \( n \) to having \( n \) in all periods, since \( i \) can unilaterally impose the latter outcome. However, from (16), \( (1 - \pi) \theta_{\text{low}} + \pi \theta_{\text{high}} < w_i + e_i \), so ex-ante, \( i \) strictly prefers implementing \( n \) forever to \( q \) forever, a contradiction. Therefore, both players prefer implementing \( n \) to \( q \) in state \( \theta_{\text{low}} \), so \( n \) is implemented with probability 1 in that state irrespective of the status quo. From (16), \( \theta_{\text{high}} > \min_{i \in \{L,R\}} \{ w_i + e_i \} \), so \( q \) gives a strictly greater flow payoff than \( n \) to both players in state \( \theta_{\text{high}} \). From what precedes, having status quo \( q \) will not prevent then from implementing \( n \) in state \( \theta_{\text{low}} \), so both players must prefer implementing \( q \) to \( n \) in state \( \theta_{\text{high}} \). Therefore, \( q \) must be implemented with probability 1 in state \( \theta_{\text{high}} \) irrespective of the status quo.

**Step 3:** \( \Gamma(\{n,p,q\}) \) has a unique equilibrium \( \sigma(\{n,p,q\}) \). This equilibrium is outcome equivalent to \( \sigma(\{n,q\}) \).

Let \( \sigma(\{n,p,q\}) \) be an equilibrium of \( \Gamma(\{n,p,q\}) \). From (16), \( p \) gives a strictly greater flow payoff than \( n \) and \( q \) to \( L \) in both states. Therefore, if \( s(t) = p, \sigma(\{n,p,q\}) \) must prescribe \( L \) to veto any policy change, so the equilibrium path stays at \( p \) forever after. Note also that implementing \( n \) yields an expected continuation payoff of at least 0 to \( R \). Therefore, as explained in Step 1, (17) implies that if \( s(t) = n \), in either state of nature, \( R \) strictly prefers to implement \( n \) to \( p \).
Consider first the Markov state in which the state is \( \theta_{\text{low}} \), the status quo is \( q \) and \( R \) has proposed \( n \). If \( L \) accepts \( n \), \( L \) gets a continuation payoff of at least 0. If \( L \) refuses \( n \), \( q \) is implemented and \( L \) gets at most the flow payoff from \( q \), i.e., \( \theta_{\text{low}} - w_L - e_L \), plus the continuation payoff from having his most preferred policy \( p \) in all subsequent periods, i.e.,

\[
\frac{\delta}{1-\delta} [(1 - \pi)(\theta_{\text{low}}) + \pi \theta_{\text{high}} - w_L].
\]

So if we assume that

\[
0 > \theta_{\text{low}} - w_L - e_L + \frac{\delta}{1-\delta} [(1 - \pi)(\theta_{\text{low}}) + \pi \theta_{\text{high}} - w_L],
\]

then \( \sigma (\{n,p,q\}) \) must prescribe \( L \) to accept proposal \( n \) when \( s(t) = q \) and \( \theta(t) = \theta_{\text{low}} \). Simple algebra shows that the above inequality is equivalent to (18).

Suppose \( R \) weakly prefers implementing \( p \) to \( q \). Since under status quo \( n \), in states \( \theta_{\text{low}} \) and \( \theta_{\text{high}} \), \( R \) strictly prefers vetoing \( p \), he must also veto \( q \). Therefore, the path of play of \( \sigma (\{n,p,q\}) \) must stay at the initial status quo \( n \) forever. Consider then the Markov state in which the status quo is \( n \), the state is \( \theta_{\text{high}} \), \( L \) has proposed \( q \), and consider the following deviation: \( R \) accepts \( q \), vetoes any departure from \( q \) until the first period \( t' > t \) such that \( \theta(t') = \theta_{\text{low}} \) and \( R \) is the proposer. In \( t' \), \( R \) proposes \( n \), which we just showed will be accepted by \( L \), so the path of play of the deviation returns to the path of play of \( \sigma (\{n,p,q\}) \) from \( t' \) onwards. The expected payoff gain \( \Pi_R \) of that deviation is

\[
\Pi_R = \theta_{\text{high}} - w_R - e_R + \delta \left[ \pi \Pi_R + (1 - \pi) \left( (1 - b_R(\theta_{\text{low}}, q)) (\Pi_R + \theta_{\text{low}} - \theta_{\text{high}}) \right) \right]
\]

\[= \frac{\theta_{\text{high}} - w_R - e_R - \delta (1 - \pi) (1 - b_R(\theta_{\text{low}}, q)) (\theta_{\text{high}} - \theta_{\text{low}})}{1 - \delta (\pi + (1 - \pi) (1 - b_R(\theta_{\text{low}}, q)))}.\]

Condition (19) implies \( \Pi_R > 0 \), a contradiction. Hence, \( R \) must strictly prefer implementing \( q \) to \( p \), so \( p \) is never implemented under status quo \( q \). Since it is never implemented under status quo \( n \), \( p \) is never implemented on the equilibrium path. So when comparing \( n \) and \( q \), players’ incentives are identical to their incentives in \( \Gamma (\{n,q\}) \). Step 3 follows then from Step 2.

Step 4: if \( \pi > 0 \), \( L \) and \( R \) get a strictly greater expected payoff from \( \sigma (\{n,p,q\}) \) and \( \sigma (\{n,q\}) \) than from \( \sigma (\{n,p\}) \).

In \( \sigma (\{n,q\}) \), either \( L \) or \( R \) could unilaterally decide to stay forever at the initial status quo \( n \) as in \( \sigma (\{n,p\}) \). But as argued in Step 2, both strictly prefer implementing \( q \) in state \( \theta_{\text{high}} \). To complete the proof, note that from Step 3, \( \sigma (\{n,p,q\}) \) is outcome equivalent to \( \sigma (\{n,q\}) \).

Step 5: For all \( v > 0 \), let \( \theta(v,t) = \theta(t) + v \epsilon(t) \), where \( \{\epsilon(t) : t \geq 0\} \) is a sequence of i.i.d., real-valued random variables with c.d.f. \( G \), and \( \{\theta(t) : t \geq 0\} \) is the degenerate process considered in Steps 1 to 4, and for all \( X \subseteq \{n,p,q\} \), let \( \Gamma (v,X) \) denote same game as \( \Gamma (X) \).
in which the state is given by \( \{ \theta(v, t) : t \geq 0 \} \). Then the equilibria of \( \Gamma(v, X) \) tend to the equilibrium of \( \Gamma(X) \) as \( v \to 0 \).

For all \( v > 0 \), let \( \sigma(v, X) \) be an equilibrium of \( \Gamma(v, X) \). Note that in the equilibrium \( \sigma(X) \) of \( \Gamma(X) \), in either states \( \theta_{\text{low}} \) and \( \theta_{\text{high}} \), both players have strict preferences between implementing any two distinct policies \( x, y \in X \). Since \( \theta(v, t) \) converges in probability to \( \theta(t) \) as \( v \) tends to 0, by continuity, for \( v \) sufficiently small, players must have the same strict preferences between implementing \( x \) or \( y \) in \( \sigma(v, X) \) when \( \theta(v, t) \) is close enough to \( \theta_{\text{low}} \) or \( \theta_{\text{high}} \). So in any period \( t \), the probability that the realization of \( \theta(v, t) \) is such that \( \sigma(v, X) \) prescribes the same actions as \( \sigma(X) \) tends to 1 as \( v \) tends to 0.

**Step 6: Proof of Propositions 4 and 7**

Proposition 7 follows readily from Steps 4 and 5 (using some \( \pi > 0 \), which is possible from Step 0). To prove Proposition 4, observe that Step 0 implies that for \( \delta \) sufficiently close to 1, there exists a compact interval \( I \) and \( \theta_{\text{high}} \in \mathbb{R} \) such that if we set \( \pi = 0 \), for all \( \theta_{\text{low}} \in I \), conditions (16) to (19) are satisfied for \( \pi = 0 \). Since \( \pi = 0 \), the c.d.f. of \( \theta(v, t) \) is \( G\left(\frac{\theta - \theta_{\text{low}}}{v}\right) \), as required by Proposition 4. Step 5 implies then that for any \( \theta_{\text{low}} \in I \), there exists \( \bar{v} > 0 \) such that for all \( v \leq \bar{v} \), in all the equilibria of \( \Gamma(v, \{n, p, q\}) \), \( q \) is implemented when the state is \( \theta_{\text{high}} \) and the status quo is \( n \), so all equilibria are IE. Since \( I \) is compact, a standard continuity argument implies that \( \bar{v} \) can be chosen independently of \( \theta_{\text{low}} \in I \), so the set of \( \theta_{\text{low}} \) and \( v \) such that all equilibria are IE has a nonempty interior.

Let us finally argue that, as claimed in the text after Proposition 4, one can choose \( \theta_{\text{low}} \) and \( v > 0 \) (together with \( \pi = 0 \)) so that \( q \) is implemented arbitrarily more frequently than \( p \). For all \( v > 0 \), let \( \sigma(v) \) be an equilibrium of \( \Gamma(v, \{n, p, q\}) \) and let \( \theta_q(v) \equiv \max \left\{ w^{(v)}_L + e^{(v)}_L, w^{(v)}_R + e^{(v)}_R \right\} \) and \( \theta_p(v) \equiv w^{(v)}_R \). As argued in Step 5, \( \theta_q(v) \) and \( \theta_p(v) \) tend to \( \max_i \left\{ w^{(n,p,q)}_i + e^{(n,p,q)}_i \right\} \) and \( w^{(n,p,q)}_R \), respectively, as \( v \to 0 \). From Step 3, in \( \sigma(\{n, p, q\}) \), players strictly prefer implementing \( q \) to \( n \) in state \( \theta_{\text{high}} \) and \( R \) strictly prefer implementing \( n \) to \( p \), so

\[
\max_i \left\{ w^{(n,p,q)}_i + e^{(n,p,q)}_i \right\} < \theta_{\text{high}} < w^{(n,p,q)}_R.
\]

The probability \( \Pi(v) \) that \( p \) is implemented conditional on some intervention being implemented under status quo \( n \) on the path of play of \( \sigma(v) \) is

\[
\Pi(v) = \frac{1 - G\left(\frac{\theta_p(v) - \theta_{\text{low}}}{v}\right)}{1 - G\left(\frac{\theta_q(v) - \theta_{\text{low}}}{v}\right)}.
\]

One can easily show that in the case of the normal distribution, for any two sequences \( (a_n)_{n \in \mathbb{N}} \) and \( (b_n)_{n \in \mathbb{N}} \) that tend to infinity, \( 1 - G(a_n + b_n) \) is negligible relative to \( 1 - G(a_n) \).
as \( n \to \infty \), so \( \Pi(v) \to 0 \) as \( v \to 0 \), as needed.  

**Proof of Lemma 2.**

We use the same notations as in Notation 1, with the exception that a state of the world \( v \in \Upsilon \) now also includes the realization of the volatility \( v \). For any strategy profile \( \sigma \), for all \( v \in \Upsilon \) and \( s \in \{n, p, q\} \), let \( X^\sigma (v, s) \) denote the policy outcome in a period in which the state of the world is \( v \) and the status quo is \( s \). By definition of \( V^\sigma_i \),

\[
V^\sigma_i (\theta, v, p) - V^\sigma_i (\theta, v, n) = \theta - w_i + \delta \left\{ (1 - v) (V^\sigma (\theta, v, p) - V^\sigma (\theta, v, n)) + v \int_{v \in \Upsilon} (V^\sigma (\theta, v, X^\sigma (v, p)) - V^\sigma (\theta, v, X^\sigma (v, n))) d\mu (v) \right\} \\
= \theta - (1 - v) w_i - v \left( w_i - \delta \int_{v \in \Upsilon} (V^\sigma (\theta, v, X^\sigma (v, p)) - V^\sigma (\theta, v, X^\sigma (v, n))) d\mu (v) \right)  \\
= \frac{1 - \delta (1 - v)}{1 - \delta (1 - v)}. 
\]

Note that if we set \( w_i^\sigma \) equal to the term inside the brackets on the numerator of the above fraction, then \( w_i^\sigma \) depends neither \( \theta \) nor on \( v \), which proves the first line of (3). The proof for the second line of (3) follows an analogous argument and is omitted for brevity. ■

**Proof of Proposition 8.** The existence of \( \tilde{v} \) follows readily from Lemma 2. See the online appendix for the proof of the second claim. ■
References


Online Appendix to "Gridlock and Inefficient Policy Instruments"

**Proof of Lemma 6.** In this proof, we consider an arbitrary strategy profile $\sigma$ whose continuation payoff parameters $(w^\sigma, e^\sigma)$ satisfy the conditions of the lemma, construct a stage-undominated response to $\sigma$, show that this mapping satisfies Brower’s theorem and that, to any fixed point of this mapping, there corresponds an equilibrium with the desired properties.

Let $\sigma$ be such that $e^\sigma_L \geq 0$ and $w^\sigma_L \leq w^\sigma_R$, and let $\pi \in [0, 1]$ be such that $\pi = 0$ if $e^\sigma_R < 0$ and $\pi = 1$ if $e^\sigma_R > 0$. As will be clear from what follows, $\pi$ will be used as a tie-breaking rule when player $R$ is indifferent between implementing $q$ and $p$, i.e., when $e^\sigma_R = 0$. Below, we construct a strategy profile $\phi(\pi, w^\sigma, e^\sigma)$ which is stage undominated given continuation payoff parameters $(w^\sigma, e^\sigma)$. This construction is quite intuitive, but we describe it in detail below to show that $\phi(\pi, w^\sigma, e^\sigma)$ can be chosen to be continuous in $(\pi, w^\sigma, e^\sigma)$.

**Veto-player’s strategy:** In any Markov state in which the veto player $i \in \{L, R\}$ has the choice between $n$ and $p$ ($q$), $\phi$ prescribes $i$ to choose $n$ when $\theta \leq w^\sigma_i$ (when $\theta \leq w^\sigma_i + e^\sigma_i$), and $p$ ($q$) otherwise. When the veto player has to choose between $p$ and $q$, $\phi$ prescribes $L$ to always choose $p$, and $R$ to choose $p$ with probability $\pi$ and $q$ with probability $1 - \pi$. This behavior is clearly stage undominated given continuation play $\sigma$ because, by assumption, $e^\sigma_L \geq 0$, $\pi = 0$ when $e^\sigma_R < 0$ and $\pi = 1$ when $e^\sigma_R > 0$. Finally, when the status quo is proposed, the action prescribed by $\phi$ is irrelevant so $\phi$ prescribes both players to accept the proposal.

**Proposer’s strategy under status quo** $p$: By assumption, $e^\sigma_L \geq 0$, and by construction, $\phi$ prescribes $L$ always to reject proposal $q$. Therefore, proposal $q$ is always weakly dominated by proposal $p$ irrespective of the identity of the proposer. So we can restrict attention to proposals $n$ or $p$.

When $\theta \leq w^\sigma_L$, since $w^\sigma_L \leq w^\sigma_R$, both players prefer to implement $n$ to $p$ and, by construction of $\phi$, proposal $n$ is accepted by both players. So we set $\phi$ to prescribe both players to propose $n$.

When $\theta > w^\sigma_L$, $L$ strictly prefers to implement $p$ to $n$ and rejects proposal $n$. So we set $\phi$ to prescribe both players to propose $p$.

**Proposer’s strategy under status quo** $n$: Since $w^\sigma_R \geq w^\sigma_L$, one can easily check that when $\theta \leq \min \{\max_i (w^\sigma_i + e^\sigma_i), w^\sigma_R\}$, there is no alternative that is accepted by one player and that gives a strictly greater continuation payoff than $n$ to the other player. Therefore, we set $\phi$ to prescribe both players to propose $n$.

When $\max_i (w^\sigma_i + e^\sigma_i) < \theta \leq w^\sigma_R$ (see Figure 3), $\phi$ prescribes $R$ to reject proposal $p$ and $R$ prefers to implement $n$ to $p$, so proposal $p$ is weakly dominated by proposal $n$ for both
players. Moreover, $\phi$ prescribes both players to accept proposal $q$ and both players strictly prefer to implement $q$ to $n$. Therefore, we set $\phi$ to prescribe both players to propose $q$. When $w_{i}^{R} < \theta \leq \max_{i} \{ w_{i}^{R} + e_{i}^{R} \}$ (see Figure 4), the same configuration arises in which the role of $q$ and $p$ reversed, so we set $\phi$ to prescribe both players to propose $p$. When $\theta > \max \{ \max_{i} \{ w_{i}^{R} + e_{i}^{R} \}, w_{R}^{p} \}$, both players accept $q$ and $p$. So it is stage undominated for proposer $i \in \{ L, R \}$ to propose the policy that gives her the greatest continuation payoff, which is $p$ if $e_{i}^{R} \geq 0$ and $q$ if $e_{i}^{R} \leq 0$. Therefore, we set $\phi(\pi, \sigma)$ to prescribe $L$ to propose $p$ with probability 1, and $R$ to propose $p$ with probability $\pi$ and $q$ with probability $1 - \pi$. $L$'s proposal strategy under status quo $q$: Given how we have set $\phi$ in the Markov states of the veto player $R$, the continuation payoff gain for $L$ of proposing $p$ instead of $q$ is $\pi e_{L}^{q}$, which is nonnegative. So one can restrict attention to strategies in which $L$ only proposes $n$ or $p$. Note that the continuation payoff gain for $L$ of proposing $p$ instead of $n$ is $\pi e_{L}^{q}$ when $n$ is not accepted by $R$ (i.e., when $\theta > w_{R}^{q} + e_{R}^{q}$), and it is $\pi (\theta - w_{L}^{q}) + (1 - \pi) (\theta - w_{L}^{q} - e_{L}^{q}) = \theta - w_{L}^{q} - \pi e_{L}^{q}$ when $n$ is accepted by $R$ (i.e., when $\theta \leq w_{R}^{q} + e_{R}^{q}$). When $\theta \leq \min \{ w_{L}^{q} + \pi e_{L}^{q}, w_{R}^{q} + e_{R}^{q} \}$, from what precedes, $n$ is accepted by $R$ and $L$ prefers proposal $n$ to $p$, so we set $\phi$ to prescribe $L$ to propose $n$. When $\theta > \min \{ w_{L}^{q} + \pi e_{L}^{q}, w_{R}^{q} + e_{R}^{q} \}$, either $\theta > w_{R}^{q} + e_{R}^{q}$, in which case proposal $n$ is not accepted by $R$ and thus yields outcome $q$, so $L$ weakly prefers to propose $p$ (since $e_{L}^{q} \geq 0$), or $w_{L}^{q} + \pi e_{L}^{q} < \theta \leq w_{R}^{q} + e_{R}^{q}$, in which case $R$ accepts $n$, but as argued above, since $\theta > w_{L}^{q} + \pi e_{L}^{q}$, $L$ is better off proposing $p$ than $n$, so we set $\phi$ to prescribe $L$ to propose $p$. $R$'s proposal strategy under status quo $q$: By construction of $\phi$, when the status quo is $q$ proposals $q$ and $p$ are both accepted with probability 1 by $L$ so, by definition of $\pi$, proposing $p$ with probability $\pi$ and $q$ with probability $1 - \pi$ always weakly dominates any other proposal which mixes between $q$ and $p$. In what follows, $P^{\pi}$ refers to the latter proposal strategy. Thus, we can restrict attention to proposals $n$ or $P^{\pi}$. The continuation payoff gain of proposing $P^{\pi}$ instead of $n$ is $\pi (\theta - w_{R}^{q}) + (1 - \pi) (\theta - w_{R}^{q} - e_{R}^{q}) = \theta - w_{R}^{q} - \pi e_{R}^{q}$ when proposal $n$ is accepted by $L$ (i.e., when $\theta \leq w_{L}^{q} + e_{L}^{q}$), and it is $\pi e_{R}^{q}$ otherwise. When $\theta \leq \min \{ w_{R}^{q} + \pi e_{R}^{q}, w_{L}^{q} + e_{L}^{q} \}$, $L$ accepts proposal $n$ so, from what precedes, $R$ prefers proposal $P^{\pi}$ to $n$, so we set $\phi$ to prescribe $R$ to propose $n$, which is stage undominated. When $\theta > \min \{ w_{R}^{q} + \pi e_{R}^{q}, w_{L}^{q} + e_{L}^{q} \}$, then either $\theta > w_{L}^{q} + e_{L}^{q}$, in which case proposal $n$ is not accepted by $L$, so $R$ weakly prefers proposal $P^{\pi}$ because, by definition of $\pi$, $\pi e_{R}^{q} \geq 0$; or $w_{R}^{q} + \pi e_{R}^{q} < \theta \leq w_{L}^{q} + e_{L}^{q}$, in which case proposal $n$ is accepted by $L$ but, as argued above, since $w_{R}^{q} + \pi e_{R}^{q} < \theta$, $R$ prefers proposal $P^{\pi}$ to proposal $n$. Thus, we set $\phi$ to prescribe $P^{\pi}$, which is stage undominated. This completes the definition of $\phi$. 45
Recall that \( \sigma \) is such that \( w^a_L \leq w^a_R, e^a_L \geq 0 \) and \( \pi \) is such that \( \pi = 0 \) when \( e^*_{R} < 0 \) and \( \pi = 1 \) when \( e^*_{R} > 0 \). By construction, \( \phi(\pi, w^a, e^a) \) is stage undominated given continuation payoff \( (w^a, e^a) \). Therefore, if

\[
(w^a, e^a) = \left( w^{\phi(\pi, w^a, e^a)}, e^{\phi(\pi, w^a, e^a)} \right),
\]

then Lemma 4 implies that \( \sigma \) is an equilibrium. Using Lemma 5 Part (i), the above condition is equivalent to

\[
(\pi, w^a, e^a) = \left( \pi, W^{\phi(\pi, w^a, e^a)} (w^a, e^a), E^{\phi(\pi, w^a, e^a)} (w^a, e^a) \right).
\]

(20)

Thus, letting \( \Phi(\pi, w^a, e^a) \) denote the R.H.S. of (20), to prove the lemma, it suffices to show that \( \Phi \) has a fixed point \((\pi^*, w^*, e^*)\) such that \( w^a_R \leq w^a_R, e^a_L \geq 0, \pi^* = 0 \) if \( e^a_R < 0 \) and \( \pi^* = 1 \) if \( e^a_R > 0 \). Hence, it is enough to show that \( \Phi \) has a fixed point in \( D \), where

\[
D \equiv \left\{ (\pi^*, w^*, e^*) \in [0, 1] \times [-B, B]^4 : w^a_L \leq w^a_R, e^a_L \geq 0, \pi^* = \begin{cases} 0 & \text{if } e^a_R < 0 \\ 1 & \text{if } e^a_R > 0 \end{cases} \right\},
\]

and \( B \equiv \max_{i, j} \{ |w_i| + |e_j| + E(\theta) \} \) is a bound on continuation payoff parameters. Since \( D \) is a compact space, Brower’s theorem implies the existence of a fixed point of \( \Phi \) in \( D \) if \( \Phi \) is continuous and \( \Phi(D) \subseteq D \).

Let us first prove that \( \Phi \) is continuous. One can easily see from the definition of \( \phi \) that for all \( x, y \in \{ n, p, q \} \) and \((\pi^*, w^*, e^*) \in D \), the probability that \( \phi(\pi^*, w^*, e^*) \) prescribes to replace status quo \( x \) by \( y \) in state \( \theta \) is piece-wise constant in \( \theta \). Moreover, on each of the interval on which this probability is constant in \( \theta \), it is continuous in \((\pi^*, w^*, e^*)\). Finally, the bounds of these intervals depend continuously on \((\pi^*, w^*, e^*)\). Therefore, if we substitute \( \pi = \phi(\pi^*, w^*, e^*) \) in (6) and (7), the continuity of \( F \) implies that the integrals on the R.H.S. of these equations must be continuous in \((\pi^*, w^*, e^*)\), which implies the continuity of \( \Phi \).

To complete the proof, it remains to show that \( \Phi(D) \subseteq D \). To do so, the only non-trivial condition to check is that if \( w^a_L \leq w^a_R \) and \( e^a_L \geq 0 \), then \( W^{\phi(\pi^*, w^*, e^*)} (w^a_R, e^a) \leq W^{\phi(\pi^*, w^*, e^*)} (w^a_L, e^a) \) and \( E^{\phi(\pi^*, w^*, e^*)} (w^a_R, e^a) \geq 0 \). In what follows, for notational convenience, \( \phi(\pi, w^a, e^a) \) is denoted \( \phi \). Since \( \phi \) is stage undominated given continuation payoff parameters
\((w^*, e^*)\), Lemma 5 Part (ii) implies that
\[
\frac{E_i^\phi (w^*, e^*) - e_i}{\delta} = \int_{\text{\(\mathcal{Y}^\phi(p,p)\cap \mathcal{Y}^\phi(q,q)\)}} e_i^* d\mu (v) + \int_{\text{\(\mathcal{Y}^\phi(p,p)\cap \mathcal{Y}^\phi(q,q)\)}} (\theta (v) - w_i^*) d\mu (v)
\]
+ \int_{\text{\(\mathcal{Y}^\phi(p,n)\cap \mathcal{Y}^\phi(q,q)\)}} (w_i^* + e_i^* - \theta (v)) d\mu (v).
\]

By construction of \(\phi\), for all \(v \in \mathcal{Y}^\phi (p, n)\), \(\theta (v) \leq w_i^*\), so \(w_i^* + e_i^* - \theta (v) \geq e_i^*\). Substituting the latter inequality inside the last integral on the R.H.S. of the above equation, we obtain
\[
\frac{E_i^\phi (w^*, e^*) - e_i}{\delta} \geq \int_{\text{\(\mathcal{Y}^\phi(p,p)\cap \mathcal{Y}^\phi(q,q)\)}} e_i^* d\mu (v) + \int_{\text{\(\mathcal{Y}^\phi(p,p)\cap \mathcal{Y}^\phi(q,q)\)}} (\theta (v) - w_i^*) d\mu (v).
\]

By construction of \(\phi\), when \(\theta < w_L^*\), status quo \(p\) is always replaced by \(n\), so for all \(v \in \mathcal{Y}^\phi (p, p)\), \(\theta (v) \geq w_L^*\). Therefore, the second integral on the R.H.S. of the above equation must be weakly positive for \(i = L\). By assumption, \(e_L^* \geq 0\) implying the first integral is also weakly positive; thus \(E_L^\phi (w^*, e^*) \geq e_L^* > 0\).

Since \(\phi\) is stage undominated given continuation payoff parameters \((w^*, e^*)\), Lemma 5 Part (ii) implies
\[
\frac{W_i^\phi (w^*, e^*) - w_i}{\delta} = \int_{v \in \mathcal{Y}^\phi(p,p)\cap \mathcal{Y}^\phi(n,n)} (w_i^* - \theta (v)) d\mu (v) - \int_{v \in \mathcal{Y}^\phi(p,n)\cap \mathcal{Y}^\phi(n,q)} e_i^* d\mu (v),
\]
so
\[
\frac{W_R^\phi (w^*, e^*) - W_L^\phi (w^*, e^*) - (w_R - w_L)}{\delta} = \int_{v \in \mathcal{Y}^\phi(p,p)\cap \mathcal{Y}^\phi(n,n)} (w_R^* - w_L^*) d\mu (v) - \int_{v \in \mathcal{Y}^\phi(p,n)\cap \mathcal{Y}^\phi(n,q)} (e_R^* - e_L^*) d\mu (v).
\]

If \(e_R^* \leq 0\), then \(e_R^* \leq e_L^*\). Substituting this inequality, \(w_L^* \leq w_R^*\), and \(w_L < w_R\), into the above equation, we obtain \(W_R^\phi (w^*, e^*) > W_L^\phi (w^*, e^*)\). If \(e_R^* > 0\), then by construction of \(\phi\), \(\mathcal{Y}^\phi (n, q) = \emptyset\) so the above equation also implies that \(W_R^\phi (w^*, e^*) > W_L^\phi (w^*, e^*)\). 

**Proof of Lemma 7 parts B and C.** Part B: We construct the strategy profile \(\sigma'\) of the lemma as the limit of a sequence of strategy profiles \((\sigma^k)_{k \in \mathbb{N}}\) which we define recursively as follows.
Step B1: construction of the sequence of strategy profiles \((\sigma^k)_{k \in \mathbb{N}}\) such that for all \(k \in \mathbb{N}\), \(w^{\sigma^k} = w^\sigma\).

We set \(\sigma^0 = \sigma\). Suppose now that we have constructed \(\sigma^0, \ldots, \sigma^k\) for some \(k \in \mathbb{N}\). Consider first a Markov state in which the veto player moves. We set \(\sigma^{k+1}\) to prescribe the same actions as \(\sigma\) if neither the status quo nor the proposal is \(q\). When comparing \(p\) and \(q\), we set \(\sigma^{k+1}\) to prescribe the veto player to vote in favor of \(p\). Finally, when comparing \(n\) and \(q\), we set \(\sigma^{k+1}\) to prescribe the veto player to vote in favor of \(q\) if and only if \(\theta(v) > w_i^\sigma + e_i^{\sigma^k}\).

Now consider the Markov states in which the proposer moves. We set \(\sigma^{k+1}\) to prescribe the same actions as \(\sigma\) when the status quo is not \(q\). When the status quo is \(q\), \(\sigma^{k+1}\) prescribes \(L\) to propose \(n\) when \(\theta \leq w_i^\sigma\), and \(p\) when \(\theta > w_i^\sigma\), and \(\sigma^{k+1}\) prescribes \(R\) to propose \(n\) when \(\theta \leq \min\left\{w_i^\sigma + e_i^{\sigma^k}, w_i^\sigma\right\}\) and \(p\) when \(\theta > \min\left\{w_i^\sigma + e_i^{\sigma^k}, w_i^\sigma\right\}\).

By construction, for all \(k \in \mathbb{N}\), \(\sigma^k\) prescribes the same actions as \(\sigma\) when neither the status quo nor the proposal is \(q\). Since \(\sigma\) is an EE, this implies that \(q\) is never implemented on the path of play of \(\sigma^k\), and therefore that \(w^{\sigma^k} = w^\sigma\).

Step B2: statements of the properties satisfied by \((\sigma^k)_{k \in \mathbb{N}}\)

In the following steps, we show by induction on \(k\) that for all \(k \in \mathbb{N}\), \(\sigma^k\) satisfies the following properties: \((i)\) the actions prescribed by \(\sigma^k\) are stage undominated given continuation play \(\sigma^{k-1}\), \((ii)\) \(0 < e_i^{\sigma^k} \leq e_i^{\sigma^{k-1}}\), \(e_i^{\sigma^k} \geq e_i^{\sigma^{k-1}}\geq 0\), and \((iii)\) for all \(i \in \{L, R\}\),

\[
e_i^{\sigma^k} = e_i + \delta \int_{w_i^\sigma}^{\min\{w_i^\sigma + e_i^{\sigma^{k-1}}, w_i^\sigma\}} b_R(\theta, q) (\theta - w_i^\sigma) \, dF(\theta). \tag{21}
\]

Note first that since \(\sigma^0 = \sigma\), from step A2 and A5, we have \(e_i^{\sigma^0} > 0\) and \(e_i^{\sigma^0} \geq 0\), so property \((ii)\) is satisfied, which is the only condition we have to check for \(k = 0\). In what follows, we assume that for some \(k \in \mathbb{N}\), for all all \(k' = 1, \ldots, k\), \(\sigma^{k'}\) satisfies properties \((i)\), \((ii)\), and \((iii)\), and prove that \(\sigma^{k+1}\) satisfies the same properties.

Step B3: \(\sigma^{k+1}\) satisfies property \((i)\).

Consider first a Markov state in which the veto player moves and neither the status quo nor the proposal is \(q\). From Step B1, \(w^{\sigma^k} = w^\sigma\), so it is stage undominated for the veto player to play \(\sigma\) (or equivalently \(\sigma^{k+1}\)) given continuation play \(\sigma^k\).

Consider now a Markov state in which the veto player \(i \in \{L, R\}\) must choose between \(p\) and \(q\). By the induction hypothesis, \(e_i^{\sigma^k} \geq 0\), so it is stage undominated for \(i\) to vote for \(p\), given continuation play \(\sigma^k\).

In the Markov states in which the veto player \(i\) must choose between \(n\) and \(q\), it is stage undominated for \(i\) to vote for \(q\) if and only if \(\theta(v) > w_i^\sigma + e_i^{\sigma^k}\), as prescribed by \(\sigma^{k+1}\).

Consider now a Markov state in which proposer \(i \in \{L, R\}\) moves and the status quo is
not \( q \). Since \( \sigma^{k+1} \) coincides with \( \sigma \) on such Markov states, and since \( \sigma \) is an EE, the only potentially profitable deviations we need to rule out are deviations in which \( i \) proposes \( q \). Suppose by contradiction that this deviation is profitable for \( i \) given continuation play \( \sigma^k \). From the induction hypothesis, \( e_i^{\sigma^k} \geq 0 \), so \( p \) gives a weakly greater continuation payoff than \( q \) to \( i \). Since \( q \) is a profitable deviation, the outcome prescribed by \( \sigma^k \) cannot be \( p \), so it must be \( n \), and \( i \) must strictly prefer implementing \( q \) to \( n \). Since \( e_i^{\sigma^k} \geq 0 \), she must also strictly prefer implementing \( p \) to \( n \), and since \( p \) is not the outcome prescribed by \( \sigma^k \), the status quo must be \( n \), and the veto player \( j \) must veto proposal \( p \) under status quo \( n \). For the deviation to be profitable to \( i \), \( j \) must also accept proposal \( q \) under status quo \( n \). Since \( e_j^{\sigma^k} \geq 0 \), this implies that \( j \) is indifferent between implementing \( n \), \( p \), and \( q \), so \( \theta = w_{1}^\sigma + e_j^{\sigma^k} \).

By construction of \( \sigma^k \), we have assumed that in such states of nature, \( j \) vetoes proposal \( q \), a contradiction.

Consider then the Markov states in which the status quo is \( q \) and proposer \( L \) moves. Since \( e_i^{\sigma^k} \geq 0 \), for all \( \theta \leq w_{1}^\sigma \), we have \( \theta \leq w_{1}^\sigma + e_i^{\sigma^k} \) so, given continuation play \( e_i^{\sigma^k} \), \( n \) is the alternative that gives \( L \) the greatest continuation payoff and, since \( w_{1}^\sigma < w_{1}^\sigma \), we have \( \theta < w_{1}^\sigma \leq w_{1}^\sigma + e_i^{\sigma^k} \); by construction of \( \sigma^{k+1} \), therefore, \( R \) accepts proposal \( n \). Hence, it is stage undominated for \( L \) to propose \( n \) when \( \theta \leq w_{1}^\sigma \), as prescribed by \( \sigma^{k+1} \). When \( \theta > w_{1}^\sigma \), \( p \) is the alternative that gives \( L \) the greatest continuation payoff, and \( R \) accepts it. So it is stage undominated for \( L \) to propose \( p \), as prescribed by \( \sigma^{k+1} \).

Consider finally the Markov states in which the status quo is \( q \) and proposer \( R \) moves. When \( \theta \leq w_{1}^\sigma \), we have \( \theta \leq w_{1}^\sigma + e_i^{\sigma^k} \) so, given continuation play \( \sigma^k \), \( n \) is the alternative that gives \( R \) the greatest continuation payoff and \( \sigma^{k+1} \) prescribes \( L \) to accept proposal \( n \). So when \( \theta \leq \min \{ w_{1}^\sigma + e_i^{\sigma^k}, w_{1}^\sigma \} \), it is stage undominated for \( R \) to propose \( n \). When \( \theta > w_{1}^\sigma + e_i^{\sigma^k} \), \( \sigma^{k+1} \) prescribes \( L \) to veto \( n \), so \( n \) leads to outcome \( q \), which gives \( R \) a weakly smaller continuation payoff than proposing \( p \). Therefore, it is stage undominated for \( R \) to propose \( p \). When \( \theta > w_{1}^\sigma \), \( p \) is the alternative that gives the greatest continuation payoff to \( R \) and, since \( \theta > w_{1}^\sigma > w_{1}^\sigma \), \( \sigma^{k+1} \) prescribes \( L \) to accept \( p \), it is also stage undominated for \( R \) to propose \( p \). Thus, we have shown that it is stage undominated for \( R \) to propose \( n \) when \( \theta \leq \min \{ w_{1}^\sigma + e_i^{\sigma^k}, w_{1}^\sigma \} \), and \( p \) when \( \theta > \min \{ w_{1}^\sigma + e_i^{\sigma^k}, w_{1}^\sigma \} \), as prescribed by \( \sigma^{k+1} \).

Step B4: \( \sigma^{k+1} \) satisfies property (iii).

By definition, \( e_i^{\sigma^{k+1}} \) is the relative gain in continuation payoff of implementing \( p \) instead of \( q \) given continuation play \( e_i^{\sigma^k} \). It is equal to \( e_i^+ \) plus \( \delta \) times the expected gain from having status quo \( p \) instead of \( q \) in the next period. To compute the latter expected gain, note that by construction of \( \sigma^k \), when \( \theta < w_{1}^\sigma \), status quo \( q \) and \( p \) both lead to outcome \( n \); when \( \theta > w_{1}^\sigma \), status quo \( q \) and \( p \) both lead to outcome \( p \); when \( \theta \in (w_{1}^\sigma, w_{1}^\sigma) \) and \( L \) is the proposer, status quo \( q \) and \( p \) both lead to outcome \( p \); when \( \theta \in \{ \min \{ w_{1}^\sigma + e_i^{\sigma^k}, w_{1}^\sigma \} \} \) and \( R \) is
the proposer, status quo \( q \) and \( p \) both lead to outcome \( p \). Thus, in all the aforementioned cases, the expected gain in continuation payoff from having status quo \( p \) instead of \( q \) is 0. Modulo a zero measure states of nature, the only remaining case to consider is when \( R \) is the proposer and \( \theta \in \left( w^q_L, \min \left\{ w^q_L + e^q_L, w^q_R \right\} \right) \). In this case, status quo \( q \) leads to outcome \( n \) while status quo \( p \) stays in place and the gain in continuation payoff from having status quo \( p \) instead of \( q \) is \( \theta - w^q_i \), which proves property \((iii)\).

**Step B5:** \( \sigma^{k+1} \) satisfies property \((ii)\).

From Step B4 and the induction hypothesis, both \( \sigma^k \) and \( \sigma^k + 1 \) satisfy property \((iii)\), so
\[
e^i_{\sigma^{k+1}} - e^i_{\sigma^k} = -\delta \int_{\min \left\{ w^q_L + e^q_L, w^q_R \right\}}^{\min \left\{ w^q_L + e^q_L, w^q_R \right\}} b_R(\theta, q) (\theta - w^q_i) \, dF(\theta). \tag{22}
\]

From the induction hypothesis, \( \sigma^k \) satisfy property \((ii)\), so
\[
w^\sigma_L \leq \min \left\{ w^q_L + e^q_L, w^q_R \right\} \leq \min \left\{ w^q_L + e^{q-1}_L, w^q_R \right\} \leq w^q_R.
\]

The above inequalities imply that the right-hand side of \(22\) is negative for \( i = L \) and positive for \( i = R \), which proves that \( e^L_{\sigma^{k+1}} \leq e^L_{\sigma^k} \) and \( e^R_{\sigma^{k+1}} \leq e^R_{\sigma^k} \). Finally, \(21\) implies \( e^L_{\sigma^{k+1}} \geq e_L > 0 \).

**Step B6:** \( (\sigma^k)_{k \in \mathbb{N}} \) has a limit \( \sigma' \) which satisfies the properties stated in part B) of the lemma.

We have shown by induction that for all \( k \in \mathbb{N} \), \( \sigma^k \) satisfies property \((ii)\), so \( (e^\sigma_L)_{k \in \mathbb{N}} \) is decreasing and bounded below. As such, it converges to some limit \( e^\infty_L \). By construction, \( \sigma^{k+1} \) depends only and continuously on \( e^k_L \) (see Step B1), so \( (\sigma^k)_{k \in \mathbb{N}} \) converges as well to some limit \( \sigma' \). Since for all \( k \in \mathbb{N} \), \( \sigma^k \) satisfies property \((i)\), by continuity, the actions prescribed by \( \sigma' \) must be stage undominated given continuation play \( \sigma' \). Taking the limit in \(21\), we obtain that \( \sigma' \) satisfies \((9)\), \( e^\sigma_L > 0 \) and \( e^\sigma_R \geq 0 \). By construction of \( (\sigma^k)_{k \in \mathbb{N}} \), \( \sigma' \) never leads to outcome \( q \) irrespective of the status quo, as needed.

**Part C:** In this part, we assume that there exists \( (w^*, e^*) \in \mathbb{R}^4 \) which satisfies \( e^*_R \geq 0 \), and the same conditions as \( w^\sigma \) and \( e^\sigma \) in \((8)\) and \((9)\), and we construct an EE \( \sigma \) such that \( (w^\sigma, e^\sigma) = (w^*, e^*) \). Let \( \sigma \) be a strategy profile which is stage undominated when players expect continuation payoff parameters \( (w^*, e^*) \) and such that, whenever a player \( i \) is indifferent between \( q \) and \( p \) given continuation payoff \( (w^*, e^*) \), \( \sigma \) prescribes \( i \) to break the indifference in favor of \( p \). The strategy profile \( \phi(\pi, w^\sigma, e^\sigma) \) constructed in the proof of Lemma 6, with parameters \( (\pi, w^\sigma, e^\sigma) = (1, w^*, e^*) \) satisfies these properties.

One can see from \((9)\) that \( e^*_L \geq e_L \), so \( e^*_L > 0 \). Together with the assumption that \( e^*_R \geq 0 \) and the definition of \( \sigma \), this implies that \( q \) is never implemented on the path of play of \( \sigma \).
Therefore, to complete the proof, it suffices to show that \( \sigma \) is an equilibrium. From Lemma 4, it is equivalent to show that \( \sigma \) prescribes stage undominated actions given continuation payoff parameters \( (w^\sigma, e^\sigma) \). Note that, by construction of \( \sigma \), this is the case if \( (w^\sigma, e^\sigma) = (w^*, e^*) \). From Lemma 5 Part (i), \( (w^\sigma, e^\sigma) = (w^*, e^*) \) if \( w^* = W^\sigma(w^*, e^*) \) and \( e^* = E^\sigma(w^*, e^*) \). To complete the proof, we show below that the latter two equations are satisfied.

**Step C1:** \( w^*_L < w^*_R \) and \( w^* = W^\sigma(w^*, e^*) \).

By construction of \( \sigma, q \) is never implemented on the path of play of \( \sigma \), so \( \Upsilon^\sigma(n, q) = \emptyset \) and \( \sigma \) is stage undominated given continuation payoff parameters \( (w^*, e^*) \). By Lemma 5 Part (ii) implies that

\[
W^\sigma_i(w^*, e^*) = w_i + \delta \int_{\Upsilon^\sigma(p, p) \cap \Upsilon^\sigma(n, n)} (w^*_i - \theta(v)) d\mu(v).
\] (23)

As in Step A1, taking differences across players in (8) and solving for \( w^*_R - w^*_L \), we obtain that \( w^*_L < w^*_R \). By construction, \( \sigma \) is stage undominated given continuation payoff parameters \( (w^*, e^*) \) so the same reasoning as in Step A3 implies that, modulo a zero measure set, \( \Upsilon^\sigma(p, p) = \{v \in \Upsilon : \theta(v) > w^*_L\} \) and \( \Upsilon^\sigma(n, n) = \{v \in \Upsilon : \theta(v) < w^*_R\} \). Together with (23), this implies that

\[
W^\sigma_i(w^*, e^*) = w_i + \delta \int_{w^*_L}^{w^*_R} (w^*_i - \theta) dF(\theta).
\]

Since \( w^* \) satisfies (8), the above equation implies that \( w^*_i = W^\sigma_i(w^*_i, e^*_i) \).

**Step C2:** modulo a zero measure set of states of the world, \( \Upsilon^\sigma(p, p) \cap \Upsilon^\sigma(q, n) \) is the set of \( v \in \Upsilon \) such that \( \theta(v) \in (w^*_L, \min \{w^*_L + e^*_L, w^*_R\}) \) and \( R \) is the proposer.

Let \( v \in \Upsilon \).

Case 1: \( \theta(v) < w^*_L \). From Step C1, \( \theta(v) < w^*_R \), so both players get a strictly greater continuation payoff from \( n \) than from \( p \), which implies that \( v \notin \Upsilon^\sigma(p, p) \), and thus \( v \notin \Upsilon^\sigma(p, p) \cap \Upsilon^\sigma(q, n) \).

Case 2: \( \theta(v) > w^*_R \). From Step C1, \( \theta(v) > w^*_L \), so both players get a strictly greater continuation payoff from \( p \) than from \( n \), which implies that \( v \notin \Upsilon^\sigma(q, n) \), and thus \( v \notin \Upsilon^\sigma(p, p) \cap \Upsilon^\sigma(q, n) \).

Case 3: \( \theta(v) \in (w^*_L, w^*_R) \) and \( L \) is the proposer. In this case, since \( \theta(v) > w^*_L \), \( L \) strictly prefers to implement \( p \) to \( n \) and, since \( \sigma \) prescribes \( L \) to behave as if \( L \) strictly prefers \( p \) to \( q \), \( L \) proposes \( p \) whenever it is accepted. By construction of \( \sigma \), \( R \) always accepts \( p \) under status quo \( q \), so \( v \in \Upsilon^\sigma(q, p) \), which implies that \( v \notin \Upsilon^\sigma(q, n) \) and thus \( v \notin \Upsilon^\sigma(p, p) \cap \Upsilon^\sigma(q, n) \).

Case 4: \( \theta(v) \in (\min \{w^*_L + e^*_L, w^*_R\}, w^*_R) \) and \( R \) is the proposer. In that case, \( \theta(v) > w^*_L + e^*_L \), so \( \sigma \) prescribes \( L \) to reject \( n \) under status quo \( q \), so \( v \notin \Upsilon^\sigma(p, p) \cap \Upsilon^\sigma(q, n) \).

Case 5: \( \theta(v) \in (w^*_L, \min \{w^*_L + e^*_L, w^*_R\}) \) and \( R \) is the proposer. As argued in Step C1, \( \theta(v) > w^*_L \) implies that \( v \in \Upsilon^\sigma(p, p) \). Since \( \theta(v) < w^*_R \), \( \theta(v) < w^*_R + e^*_R \), so \( R \) gets a greater continuation payoff from \( n \) than from \( p \) or \( q \). Since \( \theta(v) < w^*_L + e^*_L \), \( \sigma \) prescribes \( L \) to accept
obtain (24).

From Steps 1 and 2, successively using Lemma 5 Part (ii), by construction, Step C2 follows then from the observation that \( v \in \mathcal{Y}^\sigma (p, p) \cap \mathcal{Y}^\sigma (q, n) \) only in case 5.

**Step C3:** \( e^* = E^\sigma (w^*, e^*) \).

By construction of \( \sigma \), \( q \) is never implemented on the path of play of \( \sigma \), so \( \mathcal{Y}^\sigma (q, q) = \emptyset \). By construction, \( \sigma \) is stage undominated given continuation payoff parameters \( (w^*, e^*) \). Successively using Lemma 5 Part (ii), \( \mathcal{Y}^\sigma (q, q) = \emptyset \) and Step C2, we obtain

\[
E_i^\sigma (w^*, e^*) = e_i + \delta \int_{\mathcal{Y}^\sigma (p, p) \cap \mathcal{Y}^\sigma (q, n)} (\theta (v) - w^*_i) \, d\mu (v)
\]

\[
= e_i + \delta \int_{w^*_L}^{\min \{w^*_L + e^*_L, w^*_R\}} b_R (\theta, q) (\theta - w^*_i) \, dF (\theta).
\]

Since \( e^*_i \) satisfies (9), the above equation implies that \( e^*_i = E_i^\sigma (w^*, e^*) \). ■

**Lemma 9 (Properties common to EE and IE)** For any equilibrium \( \sigma \), \( \max_{i \in [L,R]} \{ e_i^\sigma \} > 0 \), \( \min_{i \in [L,R]} \{ w_i^\sigma \} < \min_{i \in [L,R]} \{ w_i^\sigma + e_i^\sigma \} \), and

\[
e_i^\sigma = e_i + \delta \left( \int_{\mathcal{Y}^\sigma (q, n)} e_i^\sigma \, d\mu (v) + \int_{\mathcal{Y}^\sigma (p, p) \cap \mathcal{Y}^\sigma (q, n)} (\theta (v) - w_i^\sigma) \, d\mu (v) \right), \tag{24}
\]

\[
w_i^\sigma = w_i + \delta \left( \int_{\mathcal{Y}^\sigma (p, p) \cap \mathcal{Y}^\sigma (n, n)} (w_i^\sigma - \theta (v)) \, d\mu (v) - \int_{\mathcal{Y}^\sigma (n, q)} e_i^\sigma \, d\mu (v) \right). \tag{25}
\]

Moreover, \( \mathcal{Y}^\sigma (q, q) \subset \mathcal{Y}^\sigma (p, p) \), \( \mathcal{Y}^\sigma (p, q) = \emptyset \), and \( \mathcal{Y}^\sigma (n, q) \subset \mathcal{Y}^\sigma (p, p) \).

**Proof.** Step 1: if \( \sigma \) is an EE, then \( \max_{i \in [L,R]} \{ e_i^\sigma \} > 0 \), \( \min_{i \in [L,R]} \{ w_i^\sigma \} < \min_{i \in [L,R]} \{ w_i^\sigma + e_i^\sigma \} \), (24), and \( \mathcal{Y}^\sigma (q, q) \subset \mathcal{Y}^\sigma (p, p) \).

Step A6 and Equation (11) in the proof of Lemma 7 imply \( \mathcal{Y}^\sigma (q, q) \subset \mathcal{Y}^\sigma (p, p) \) and (24), respectively. The inequalities \( \max_{i \in [L,R]} \{ e_i^\sigma \} > 0 \) and \( \min_{i \in [L,R]} \{ w_i^\sigma \} < \min_{i \in [L,R]} \{ w_i^\sigma + e_i^\sigma \} \) follow readily from \( w_i^L < w_i^R \), \( e_i^L > 0 \), and \( e_i^R \geq 0 \), as established in Lemma 7 part A.

Step 2: if \( \sigma \) is an IE, then \( \max_{i \in [L,R]} \{ e_i^\sigma \} > 0 \), \( \min_{i \in [L,R]} \{ w_i^\sigma \} < \min_{i \in [L,R]} \{ w_i^\sigma + e_i^\sigma \} \), (24), and \( \mathcal{Y}^\sigma (q, q) \subset \mathcal{Y}^\sigma (p, p) \).

The inequalities \( \max_{i \in [L,R]} \{ e_i^\sigma \} > 0 \) and \( \min_{i \in [L,R]} \{ w_i^\sigma \} < \min_{i \in [L,R]} \{ w_i^\sigma + e_i^\sigma \} \) follow from Lemma 8, and \( \mathcal{Y}^\sigma (q, q) \subset \mathcal{Y}^\sigma (p, p) \) follows Lemma 8. By definition of \( \mathcal{Y}^\sigma \), \( \mathcal{Y}^\sigma (p, n) \cap \mathcal{Y}^\sigma (p, p) = \emptyset \), so \( \mathcal{Y}^\sigma (p, n) \cap \mathcal{Y}^\sigma (q, q) = \emptyset \). Substituting the latter identities into (12), we obtain (24).

Step 3: If \( \sigma \) is an equilibrium, then \( \mathcal{Y}^\sigma (p, q) = \emptyset \), and \( \mathcal{Y}^\sigma (n, q) \subset \mathcal{Y}^\sigma (p, p) \).

From Steps 1 and 2, \( \max_{i \in [L,R]} \{ e_i^\sigma \} > 0 \), that is, one player always strictly prefer to im-
plement \( p \) to \( q \), so \( \Upsilon^\sigma(p,q) = \emptyset \). To show that \( \Upsilon^\sigma(n,q) \subset \Upsilon^\sigma(p,p) \) note, that in any state \( v \in \Upsilon^\sigma(n,q) \), both players weakly prefer to implement \( q \) to \( n \). Hence, \( \theta(v) \geq \max_{i \in \{L,R\}} \{w^\sigma_i + e^\sigma_i\} \). From Steps 1 and 2, this implies that \( \theta(v) > \min_{i \in \{L,R\}} \{w^\sigma_i\} \), that is, one player strictly prefers to implement \( p \) to \( n \), and therefore \( v \not\in \Upsilon^\sigma(p,n) \). Since \( \Upsilon^\sigma(p,q) = \emptyset \), and since \((\Upsilon^\sigma(p,x))_{x \in \{n,p,q\}}\) form a partition of \( \Upsilon \), necessarily, \( v \in \Upsilon^\sigma(p,p) \), as needed.

**Step 4:** If \( \sigma \) is an equilibrium then it satisfies (25).

If \( \sigma \) is an equilibrium, then from Lemma 4, the actions prescribed by \( \sigma \) are stage undominated given continuation payoff \((w^\sigma, e^\sigma)\). Successively using Lemma 5 Part (i) and Part (ii ) for \((w^\sigma_i, e^\sigma) = (w^\sigma_i, e^\sigma)\), we obtain

\[
w^\sigma_i = W_i(\sigma, w^\sigma, e^\sigma) = w_i + \delta \int_{\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(n,n)} (w^\sigma_i - \theta(v)) d\mu(v) - \int_{\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(n,q)} e^\sigma_i d\mu(v) .
\]

From Step 3, \( \Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(n,q) = \Upsilon^\sigma(n,q) \). Substituting the last equality into the above equation gives (25).

**Lemma 10** For all \( x \in \{n,p,q\} \), and all strategy profile \( \sigma \), define \( \Upsilon^\sigma(x,\{n,q\}) \equiv \Upsilon^\sigma(x,n) \cup \Upsilon^\sigma(x,q) \) and

\[
D(\sigma) \equiv (1 - \delta \mu(\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(n,n))) (1 - \delta \mu(\Upsilon^\sigma(q,q))) - \delta^2 \mu(\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)) \mu(\Upsilon^\sigma(n,q)) .
\]

Then if \( \sigma \) is an equilibrium, we have \((1 - \delta)^2 \leq D(\sigma) \leq 1\) and

\[
\begin{align*}
W^\sigma_p - W^\sigma_L &= \frac{(1-\delta \mu(\Upsilon^\sigma(q,q)))[w^\sigma_R - w^\sigma_L] - \delta \mu(\Upsilon^\sigma(n,q))[e^\sigma_R - e^\sigma_L]}{D(\sigma)} , \\
e^\sigma_p - e^\sigma_L &= \frac{(1-\delta \mu(\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)))[e^\sigma_R - e^\sigma_L] - \delta \mu(\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n))[w^\sigma_R - w^\sigma_L]}{D(\sigma)} , \\
w^\sigma_R + e^\sigma_R - w^\sigma_L - e^\sigma_L &= \frac{(1-\delta \mu(\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n))[w^\sigma_R - w^\sigma_L] + (1-\delta \mu(\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(n,q))[e^\sigma_R - e^\sigma_L])}{D(\sigma)} .
\end{align*}
\]

**Proof.** Subtracting (25) for \( i = R \) from (25) for \( i = L \), and doing the same for (24), we get

\[
\begin{align*}
&\left\{\begin{array}{l}
w^\sigma_R - w^\sigma_L = w_R - w_L + \delta \mu(\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(n,n)) (w^\sigma_R - w^\sigma_L) - \delta \mu(\Upsilon^\sigma(n,q))(e^\sigma_R - e^\sigma_L) , \\
e^\sigma_R - e^\sigma_L = e_R - e_L + \delta \mu(\Upsilon^\sigma(q,q))(e^\sigma_R - e^\sigma_L) - \delta \mu(\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n))(w^\sigma_R - w^\sigma_L) .
\end{array}\right.
\end{align*}
\]

The above equations can be viewed as a linear system in \( w^\sigma_R - w^\sigma_L \) and \( e^\sigma_R - e^\sigma_L \). Straightforward algebra shows that its solution is given by the first two lines of (26).
From Lemma 9, \( \mathcal{Y}^{\sigma} (q, q) \subset \mathcal{Y}^{\sigma} (p, p) \) and, by definition of \( \mathcal{Y}^{\sigma} \), \( \mathcal{Y}^{\sigma} (q, n) \) and \( \mathcal{Y}^{\sigma} (q, q) \) are disjoint. Thus, using the notations of the Lemma,

\[
\mu (\mathcal{Y}^{\sigma} (p, p) \cap \mathcal{Y}^{\sigma} (q, n)) \leq \mu (\mathcal{Y}^{\sigma} (p, p) \cap \mathcal{Y}^{\sigma} (q, n)) = \mu (\mathcal{Y}^{\sigma} (p, p) \cap \mathcal{Y}^{\sigma} (q, \{n, q\})). \tag{27}
\]

From Lemma 9, \( \mathcal{Y}^{\sigma} (n, q) \subset \mathcal{Y}^{\sigma} (p, p) \) and, by definition of \( \mathcal{Y}^{\sigma} \), \( \mathcal{Y}^{\sigma} (n, n) \) and \( \mathcal{Y}^{\sigma} (n, q) \) are disjoint, so

\[
\mu (\mathcal{Y}^{\sigma} (p, p) \cap \mathcal{Y}^{\sigma} (n, n)) + \mu (\mathcal{Y}^{\sigma} (n, q)) = \mu (\mathcal{Y}^{\sigma} (p, p) \cap \mathcal{Y}^{\sigma} (n, \{n, q\})). \tag{28}
\]

Adding up the first two lines of (26) and substituting (27) and (28) into the corresponding expression for \( w_{\text{R}} - e_{\text{R}} - w_{\text{L}} - e_{\text{L}} \), we obtain the third line of (26).

The inequality \( D (\sigma) \leq 1 \) is obvious from the definition of \( D (\sigma) \). Let us now prove \( D (\sigma) \geq (1 - \delta)^2 \). Observe that \( \mathcal{Y}^{\sigma} (q, q) \) and \( \mathcal{Y}^{\sigma} (p, p) \cap \mathcal{Y}^{\sigma} (q, n) \) are disjoint. So \( \mathcal{Y}^{\sigma} (q, q) \) is included into the complement of \( \mathcal{Y}^{\sigma} (p, p) \cap \mathcal{Y}^{\sigma} (q, n) \). Likewise, \( \mathcal{Y}^{\sigma} (p, p) \cap \mathcal{Y}^{\sigma} (n, n) \) and \( \mathcal{Y}^{\sigma} (n, q) \) are disjoint, so \( \mathcal{Y}^{\sigma} (p, p) \cap \mathcal{Y}^{\sigma} (n, n) \) is included into the complement of \( \mathcal{Y}^{\sigma} (q, n) \). Therefore,

\[
D (\sigma) = (1 - \delta \mu (\mathcal{Y}^{\sigma} (p, p) \cap \mathcal{Y}^{\sigma} (n, n))) (1 - \delta \mu (\mathcal{Y}^{\sigma} (q, q)))
- \delta^2 \mu (\mathcal{Y}^{\sigma} (p, p) \cap \mathcal{Y}^{\sigma} (q, n)) \mu (\mathcal{Y}^{\sigma} (n, q))
\geq (1 - \delta (1 - \mu (\mathcal{Y}^{\sigma} (n, n)))) (1 - \delta (1 - \mu (\mathcal{Y}^{\sigma} (p, p) \cap \mathcal{Y}^{\sigma} (q, n))))
- \delta^2 \mu (\mathcal{Y}^{\sigma} (p, p) \cap \mathcal{Y}^{\sigma} (q, n)) \mu (\mathcal{Y}^{\sigma} (n, q))
= (1 - \delta) (1 - \delta + \mu (\mathcal{Y}^{\sigma} (n, q)) + \mu (\mathcal{Y}^{\sigma} (p, p) \cap \mathcal{Y}^{\sigma} (q, n)))
\geq (1 - \delta)^2.
\]

**Proof of Proposition 5.** To prove the first claim in Proposition 5, simply note that the equilibrium \( \sigma \) constructed in Lemma 6 is such that \( w_{\text{L}} > w_{\text{R}} \), so there always exists an equilibrium that is not an IE-C.

**Step 1:** if \( e_{\text{R}} - e_{\text{L}} < \frac{(1 - \delta)^2}{\delta} (w_{\text{R}} - w_{\text{L}}) \) (which is satisfied in particular when \( e_{\text{R}} \leq e_{\text{L}} \)), then for all equilibria \( \sigma, w_{\text{R}} < w_{\text{R}} \), and hence \( \sigma \) cannot be IE-C.

Successively using (26), \( e_{\text{R}} - e_{\text{L}} \leq \frac{(1 - \delta)^2}{\delta} (w_{\text{R}} - w_{\text{L}}) \) and \( (1 - \delta)^2 \leq D (\sigma) \leq 1 \) (see Lemma
10), we have

\[ w^\sigma_R - w^\sigma_L = \frac{(1 - \delta \mu(Y^\sigma(q, q)))}{D(\sigma)} (w_R - w_L) - \frac{\delta \mu(Y^\sigma(n, q))}{D(\sigma)} (e_R - e_L) \]

\[
> (1 - \delta)(w_R - w_L) - \frac{\delta \mu(Y^\sigma(n, q)) (1 - \delta)^3}{D(\sigma)} (w_R - w_L) 
\]

\[
\geq (1 - \delta)(w_R - w_L) - \frac{\delta}{(1 - \delta)^2} (1 - \delta)^3 (w_R - w_L) \geq 0.
\]

Step 2: if \( e_R - e_L > -(1 - \delta)^3 (w_R - w_L) \) (which is satisfied in particular when \( e_R \geq e_L \)), then for all equilibria, \( w^\sigma_L + e^\sigma_L < w^\sigma_R + e^\sigma_R \), and hence \( \sigma \) cannot be IE-B.

Successively using (26), \( e_R - e_L > (1 - \delta)^3 (w_R - w_L) \) and \( (1 - \delta)^2 \leq D(\sigma) \leq 1 \) (see Lemma 10), we have

\[
\frac{w^\sigma_R + e^\sigma_R - w^\sigma_L - e^\sigma_L}{1 - \delta \mu(Y^\sigma(p, p) \cap Y^\sigma(q, \{n, q\}))} (w_R - w_L) + \frac{1 - \delta \mu(Y^\sigma(p, p) \cap Y^\sigma(n, \{n, q\}))}{D(\sigma)} (e_R - e_L)
\]

\[
\geq \left( \frac{1 - \delta \mu(Y^\sigma(p, p) \cap Y^\sigma(q, \{n, q\}))}{D(\sigma)} - \frac{1 - \delta \mu(Y^\sigma(p, p) \cap Y^\sigma(n, \{n, q\}))}{D(\sigma)} \right) (1 - \delta)^3 (w_R - w_L)
\]

\[
\geq \left( (1 - \delta) - \frac{1}{(1 - \delta)^2} (1 - \delta)^3 \right) (w_R - w_L) \geq 0.
\]

If \( \sigma \) is an IE-B, then \( e^\sigma_L \leq 0 < e^\sigma_R \) and, from above, \( w^\sigma_L + e^\sigma_L < w^\sigma_R + e^\sigma_R \). Together, these inequalities imply that \( w^\sigma_L < w^\sigma_L + e^\sigma_L < w^\sigma_R + e^\sigma_R \leq w^\sigma_R \), contradicting the definition of an IE-B.

Step 3: if \( |e_R - e_L| < (1 - \delta)^3 (w_R - w_L) \), then any equilibrium must be EE or IE-A.

Let \( \sigma \) be an equilibrium. If \( \sigma \) is an EE, we are done. If \( \sigma \) is an IE then, since \( (1 - \delta)^3 < \frac{1 - \delta^3}{\delta} \), Step 1 implies that \( \sigma \) cannot be IE-C and Step 2 implies that \( \sigma \) cannot be IE-B. By elimination, \( \sigma \) can only be IE-A.

Proof of Proposition 6 Part (iii) case \( w_R - w_L \to +\infty \). Suppose by contradiction that Part (iii) is false in the case \( w_R - w_L \to +\infty \). Then there exists \( e \in (0, +\infty)^2 \), \( m \in \mathbb{R} \) and two sequences \((w^k)_{k \in \mathbb{N}} \) and \((\sigma(k))_{k \in \mathbb{N}} \) such that \( w^k_R - w^k_L \to +\infty \) and, for all \( k \in \mathbb{N} \), \( w^k_L + w^k_R = m \) and \( \sigma(k) \) is an IE for the flow payoff parameters \((e, w^k)\).

Since \( w^k_R - w^k_L \to +\infty \), Equation (26) and inequality \( D(\sigma) \geq (1 - \delta)^2 \) in Lemma 10 imply that, for \( k \) sufficiently large,

\[
w^\sigma_R(k) - w^\sigma_L(k) \geq (1 - \delta) (w^k_R - w^k_L) - |e_R - e_L|.
\]
Hence, \( w_R^{σ(k)} - w_L^{σ(k)} → +∞ \) and, therefore, \( w_R^{σ(k)} > w_L^{σ(k)} \) for \( k \) sufficiently large. Since \( σ(k) \) is an IE, the latter inequality, together with Lemma 8, implies that, for \( k \) sufficiently large, \( e_L^{σ(k)} > 0 \geq e_R^{σ(k)} \). By the same token, Lemma 10 implies that for \( k \) sufficiently large,

\[
w_R^{σ(k)} + e_R^{σ(k)} - w_L^{σ(k)} - e_L^{σ(k)} ≥ (1 - δ) \left( w_R^{k} - w_L^{k} \right) - |e_R - e_L|.
\]

Hence, \( w_R^{σ(k)} + e_R^{σ(k)} - w_L^{σ(k)} - e_L^{σ(k)} → +∞ \). Together with \( e_L^{σ(k)} > 0 \geq e_R^{σ(k)} \), this implies that for \( k \) sufficiently large,

\[
w_L^{σ(k)} < w_L^{σ(k)} + e_L^{σ(k)} < w_R^{σ(k)} + e_R^{σ(k)} \leq w_R^{σ(k)}.
\] (29)

Let \( v ∈ Υ^{σ(k)}(p, p) ∩ Υ^{σ(k)}(q, n) \). Since \( σ(k) \) is an equilibrium, in state \( v \), both players must weakly prefer to implement \( n \) to \( q \) and one player must prefer to implement \( p \) to \( n \). Thus,

\[
\min_{i ∈ \{L, R\}} w_i^{σ(k)} ≤ θ(v) ≤ \min_{i ∈ \{L, R\}} \left( w_i^{σ(k)} + e_i^{σ(k)} \right) \quad \text{and} \quad (29) \implies
\]

\[
Υ^{σ(k)}(p, p) ∩ Υ^{σ(k)}(q, n) ⊆ \left\{ v ∈ Υ : w_L^{σ(k)} ≤ θ(v) ≤ w_L^{σ(k)} + e_L^{σ(k)} \right\}.
\] (30)

From (30), for all \( v ∈ Υ^{σ(k)}(p, p) ∩ Υ^{σ(k)}(q, n) \), \( θ(v) - w_L^{σ(k)} ≤ e_L^{σ(k)} \). Substituting this inequality into (24), we obtain

\[
e_L^{σ(k)} ≤ e_L + δμ(Υ^{σ(k)}(q, q)) e_L^{σ(k)} + δμ(Υ^{σ(k)}(p, p) ∩ Υ^{σ(k)}(q, n)) e_L^{σ(k)}
\]

and therefore that \( e_L^{σ(k)} ≤ e_L / (1 - δ) \). Since \( e_L^{σ(k)} > 0 \), the latter inequality implies that \( e_L^{σ(k)} \) is bounded. From (29) and (30), for all \( v ∈ Υ^{σ(k)}(p, p) ∩ Υ^{σ(k)}(q, n) \), \( 0 ≥ θ(v) - w_R^{σ(k)} \geq w_L^{σ(k)} - w_R^{σ(k)} \). The preceding inequality and (30) imply that

\[
\int_{Υ^{σ(k)}(p, p) ∩ Υ^{σ(k)}(q, n)} \left( θ(v) - w_R^{σ(k)} \right) dμ(v) ≥ \int_{Υ^{σ(k)}(p, p) ∩ Υ^{σ(k)}(q, n)} \left( w_L^{σ(k)} - w_R^{σ(k)} \right) dμ(v)
\]

\[
≥ \int_{w_L^{σ(k)} + e_L^{σ(k)}}^{w_R^{σ(k)} + e_L^{σ(k)}} \left( w_L^{σ(k)} - w_R^{σ(k)} \right) dF(θ).
\]

Substituting the above inequality into (24), we obtain

\[
e_R^{σ(k)} ≥ e_R + δμ(Υ^{σ(k)}(q, q)) e_R^{σ(k)}
\]

\[
+ δ \int_{w_L^{σ(k)}}^{w_R^{σ(k)}} w_R^{σ(k)} dF(θ) - δ \int_{w_L^{σ(k)}}^{w_R^{σ(k)}} w_R^{σ(k)} dF(θ).
\] (31)
Since $e^\sigma(k)_L$ is bounded and $w^\sigma(k)_L \to -\infty$ as $k \to \infty$, the integrability of $F$ implies that the term $\int_{w^\sigma(k)_L}^{w^\sigma(k)_R} w^\sigma(k)_L dF(\theta)$ in (31) tends to 0. If we can show that $w^\sigma(k)_R \sim |w^\sigma(k)_L|$ then, by the same token, $\int_{w^\sigma(k)_L}^{w^\sigma(k)_R} w^\sigma(k)_R dF(\theta)$ tends to 0 and so (31) implies that, for $k$ sufficiently large, $e^\sigma(k)_R > 0$. This last inequality, together with $e^\sigma(k)_L > 0$ and the assumption that $\sigma(k)$ is an IE, yields the desired contradiction. To complete the proof of Step 5, it suffices to show that $w^\sigma(k)_R \sim \left|w^\sigma(k)_L\right|$.

Summing (24) and (25) across players and collecting the terms in factor of $w^\sigma(k)_L + w^\sigma(k)_R$ and $e^\sigma(k)_L + e^\sigma(k)_R$, we obtain:

\[
\left(1 - \delta \mu \left(\Upsilon^{\sigma(k)}(p, p) \cap \Upsilon^{\sigma(k)}(n, n)\right) \delta \mu \left(\Upsilon^{\sigma(k)}(n, q)\right) \delta \mu \left(\Upsilon^{\sigma(k)}(q, q)\right) \right) \left(w^\sigma(k)_L + w^\sigma(k)_R\right) + \left(e^\sigma(k)_L + e^\sigma(k)_R\right) = \begin{pmatrix}
1 - \delta \mu \left(\Upsilon^{\sigma(k)}(p, p) \cap \Upsilon^{\sigma(k)}(n, n)\right) \delta \mu \left(\Upsilon^{\sigma(k)}(n, q)\right) & \delta \mu \left(\Upsilon^{\sigma(k)}(q, q)\right) \\
\delta \mu \left(\Upsilon^{\sigma(k)}(p, p) \cap \Upsilon^{\sigma(k)}(n, n)\right) & 1 - \delta \mu \left(\Upsilon^{\sigma(k)}(n, q)\right)
\end{pmatrix} \begin{pmatrix}
w^\sigma(k)_L \\
w^\sigma(k)_R
\end{pmatrix} + \begin{pmatrix}
e^\sigma(k)_L \\
e^\sigma(k)_R
\end{pmatrix}.
\]

From Lemma 10, the determinant of that system, which is $D(\delta)$, is bounded away from 0 as $k \to \infty$. Moreover, all the coefficients of the above system are bounded. Therefore, the solution $w^\sigma(k)_L + w^\sigma(k)_R$ must be bounded as $k \to \infty$. Since $w^\sigma(k)_L \to -\infty$, this implies that $w^\sigma(k)_R \sim \left|w^\sigma(k)_L\right|$, as needed. ■

**Proof of the second claim of Proposition 8.** Consider a sequence of c.d.f. $(H_k)_{k \in \mathbb{N}}$ such that, for all $k \in \mathbb{N}$, $H_k(v, \theta) = G_k(v) F(\theta)$, where $G_k$ has full support and tends to the degenerate distribution which puts probability 1 on $v = 1$ as $k \to \infty$. For all $k \in \mathbb{N}$, let $\sigma(k)$ be an equilibrium for the c.d.f. $H_k$. Suppose the second claim of Proposition 8 is false. Then for some $\delta$ arbitrarily close to 1, one can choose $\sigma(k)$ such that $e^\sigma(k)_L \geq 0$ and $e^\sigma(k)_R \geq 0$. Since $w$, $e$ and $\delta$ are fixed, $w^\sigma(k)$ and $e^\sigma(k)$ are bounded, we can extract a subsequence such that $w^\sigma(k)$ and $e^\sigma(k)$ converge. Since $e^\sigma(k)_L \geq 0$ and $e^\sigma(k)_R \geq 0$, one can easily check from (3) that for all $v < 1$, for almost all $\theta$, for any two distinct policies $x, y \in \{n, p, q\}$, $V_i^\sigma(k)(\theta, v, x) \neq V_i^\sigma(k)(\theta, v, y)$, that is, players are not indifferent between implementing any two policies. Thus, for all $v < 1$, each player has a unique and pure stage undominated action at any Markov states for almost all $\theta$. If $e^\sigma(k)_i = 0$ for some player $i$, then for $v = 1$, player $i$ is indifferent between implementing $p$ and $q$ for all $\theta \in \mathbb{R}$. But we can assume w.l.o.g. that in this case, $\sigma(k)$ prescribes $i$ to behave as if she strictly prefers implementing $p$ to $q$. Since $G_k$ puts probability 0 on $v = 1$, this deviation from $\sigma(k)$ does not affect the continuation payoff parameter $w^\sigma(k)$ and $e^\sigma(k)$, and it is therefore still an equilibrium.

Given this restriction on $\sigma(k)$, for all $v \in [0, 1]$, the parameters $w^\sigma(k)$ and $e^\sigma(k)$ uniquely pin down the equilibrium behavior prescribed by $\sigma(k)$ for almost all $\theta \in [0, 1]$. Moreover,
they do so in a continuous way in the sense that, for all $v \in [0, 1]$, the set of realizations of $\theta$ for which a given action is prescribed to the veto player or the proposer depend continuously on $w^{\sigma(k)}$ and $e^{\sigma(k)}$ (for instance, veto player $i$ must veto proposal $p$ under status quo $n$ when $\theta < (1 - v) w_i + v w_i^{\sigma(k)}$ and accept it when the reverse inequality holds). Since $w^{\sigma(k)}$ and $e^{\sigma(k)}$ converge, this implies that for all $v \in [0, 1]$, $\sigma(k)$ converge as $k \to \infty$ and, by continuity, this limit must be an equilibrium for the game with the limit distribution $H_\infty(v, \theta)$, which puts probability 1 on $v = 1$. This game is equivalent to the game considered in Proposition 4. Since $\delta$ can be chosen arbitrarily close to 1, and since $F$ can be chosen arbitrarily, we obtain a contradiction with Proposition 4. ■