Dynamic Adverse Selection
Time Varying Market Conditions and Endogenous Entry

Job Market Paper

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Abstract

In this paper I analyze the effect of time-varying market conditions and endogenous entry on equilibrium dynamics of markets plagued by adverse selection. I show that variation in gains from trade, stemming from marker conditions, creates an option value and distorts liquidity when current gains from trade are low. An improvement in market conditions triggers a wave of high quality deals due to the preceding illiquidity and lack of incentives to signal quality. When gains from trade are high, the market is fully liquid; high prices and no delay in trade attract low-grade assets, and the average quality deteriorates. My analysis also reveals that illiquidity can act as remedy as well as cause of inefficiency: partial illiquidity allows for screening of assets and restores efficient entry incentives. I demonstrate model implications using several applications: early stage financing, initial public offerings, and private equity buyouts.

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1 Introduction

Adverse selection is an important feature of financial markets. Recently, substantial progress has been made to understand dynamic adverse selection and study adverse selection in richer environments\(^1\). The general finding of this research is that, unlike in classic Akerlof (1970) model, trade does not necessarily break down. Owners of higher quality assets can signal the quality by accepting either a lower probability of trade (Chang (2011), Guerrieri and Shimer (2014)) or longer waiting times (Daley and Green (2012), Fuchs and Skrzypacz (2014)) in return for higher prices. While the possibility to signal better asset quality allows for all assets to be eventually traded, trade is inefficiently delayed.

The markets that involve dynamic adverse selection commonly share two important features that have not been explored in the literature. First, the quality of assets that enter the market is endogenous and is affected by the evolution of beliefs. Consider, for example, a market for early stage financing of high-growth firms, provided by venture capital (VC) and angel investors. When deciding whether to pursue an innovative idea a potential entrepreneur (privately informed about her own ability or about the quality of the idea) weighs the private cost of quitting a job or dropping out of college against a potential benefit of working on a startup. This decision is strongly influenced by the prevailing prices in the market for venture and angel capital as well as the time it takes to raise the funds. The potential entrepreneur enters the market for funding after observing how “hot” the market is. In other words, the decision to enter the funding market is strategic and depends not only on the quality of the idea but also on current market conditions. These strategic decisions affect the average project quality in the market and cause a feedback loop leading to adjustments in prices and incentives to signal quality.

Second, the markets are characterized by the time variation of market conditions. In the example of early stage financing, the cost of VC funding varies over time for reasons unrelated to the supply of innovative ideas. Gompers and Lerner (2000) and Diller and Kaserer (2009) show that higher capital inflows to the VC industry raise valuations of young ventures regardless of their quality, lowering the cost of funding for entrepreneurs. Variation in market conditions creates an option for entrepreneurs to optimally time their fund raising decision.

In this paper I incorporate these two features in a model of dynamic adverse selection

and explore how endogenous entry and time varying market conditions impact equilibrium market dynamics. In the context of early stage financing, I analyze the dynamics of average quality of projects receiving funding, resulting patterns in deal volume and the overall market efficiency. I build a dynamic model of adverse selection, in which entrepreneurs, who are privately informed about the quality of their ideas, enter the market over time and attempt to raise funds from a competitive uninformed investors. The variation in market conditions is driven by investors’ cost of capital, modeled as a discount rate, which ultimately affects gains from trade and incentives of entrepreneurs to raise funds and enter the market in the first place.

Although I use the market for early stage financing as a motivating example, the economic mechanism studied in this paper is quite general and can manifest itself in multiple markets, in which market conditions vary over time and adverse selection plays an important role, such as private equity and IPO markets, among others.

My first result characterizes the dynamics of equilibrium volume of deals. In particular, I find that improvement in market conditions triggers a wave of deals. The wave is driven by a combination of two factors: accumulation of unfunded projects in the market and subsequent deterioration of incentives to delay fund-raising. Low liquidity of the market during the period of high discount rates is caused by the desire of entrepreneurs with good projects to signal their type and raise funds at higher valuations. The delay results in a build up of inventories over time. When the discount rate falls unfunded entrepreneurs with good projects rush to the market for two reasons. First, the option value of waiting for the low discount rate disappears. Second, more importantly, because high valuations attract worse projects to the market, contaminating the pool. Strict preference for immediate trade results in an atom of fund-raising activity.

In contrast to the previous literature (see Pástor and Veronesi (2005) and Bustamante (2012)), I find that option to delay fund-raising results in a very high quality of funded projects in the beginning of the wave (even when compared to the projects raising funds in the “cold” market). It is a consequence of the wave being driven by the incentives of the high quality entrepreneurs. Thus, the “quantity adjustment” (Ritter and Welch (2002)) in my model comes from the top rather than from the bottom of the quality distribution.

My second set of results shows the variation in equilibrium quality of funded projects. I show that during good times, when supply of capital is high and the discount rate of investors is correspondingly low, funds are raised immediately upon entry. However, and perhaps surprisingly, the average quality of innovative projects is low. Conversely, during bad times,
when the discount rate of investors is high, a fraction of low quality entrepreneurs entering
the market is low, and the average quality of projects in the market is relatively high. Raising
funds, however, takes on average longer. Entrepreneurs with good projects relay fund-raising
in order to secure better terms, suffering lower underpricing, when investors’ discount rate is
high. An apparent mismatch between the time it takes to raise funds and quality of funded
projects is resolved via the following intuition. When investors’ discount rate is low, project
valuations are high regardless of quality. This reduces the desire of entrepreneurs with good
projects to signal their type, implying shorter fund-raising times. At the same time, ease of
obtaining funds and high valuations attract a lot of entrepreneurs with low quality projects
resulting in the low average quality.

High discount rates increase sensitivity of entrepreneurs with good projects to underpric-
ing. This observation, in combination with an option to wait for the fall of the discount rate
that entrepreneurs have, renders low price offers in bad times unattractive. In order to raise
funds at higher valuations entrepreneurs with good projects opt to delay fund-raising. This
makes a high pooling price less attractive to entrepreneurs with bad projects and incentivizes
a fraction of them to reduce the waiting time by raising funds at low separating valuations.
A resulting increase of the quality of the pool in the market allows investors to offer high
pooling price in the first place. Difficulty of raising funds and low valuations conditional
on the project type reduce entry of entrepreneurs with bad projects and, thus, improve the
average quality of projects receiving financing.

I also find that illiquidity can be both a source of and a remedy for inefficiency. On
the one hand, high liquidity is desirable, because positive NPV projects are funded imme-
diately and no value is lost due to time discounting. On the other hand, when funding is
raised immediately, prices reflect average (pooling) project quality and fail to reveal private
information. Pooling price is an effective subsidy to the entrepreneurs with bad projects.
It distorts incentives to enter the market and results in negative NPV projects obtaining
funding. Efficient incentives to enter are restored when the discount rate of investors is high.
Delay allows investors to partially screen the projects by type. Specifically, equilibrium pay-
off to the entrepreneur with a bad project fully reveals her private information, resulting in
only positive NPV projects being funded in equilibrium.

My model naturally leads itself to several empirical implications. First, the model predicts
that volume of deals is positively correlated with gains from trade. Lower gains from trade
caused by, for example, higher cost of investors’ capital cause not only the price adjustment
but also a quantity adjustment. This prediction is broadly consistent with finding in many
industries, for example, venture capital (Gompers and Lerner (2000)) and private equity buyouts (Axelson, Jenkinson, Strömberg and Weisbach (2009)). My model also generates a wave of deals, which is a definitive feature of IPO (Ritter and Welch (2002)) and private equity buyout (Kaplan and Stein (1993)) markets.

Second, my model predicts that quality of the projects receiving funding is non-monotone with respect to the supply of capital, proxied by investors’ discount rate, and the volume of deals. The quality is at its lowest when the discount rates have been low for a prolonged period of time; it is higher when discount rates are high, and it is the highest early on in the fund-raising wave. Empirically this prediction has been generally supported in several markets. In context of IPOs, Ritter and Welch (2002) write that “it is conventional wisdom among both academics and practitioners that the quality of firms going public deteriorates as a period of high issuing volume progresses”. This is consistent with the findings of my model and is empirically confirmed by Chang, Kim and Shim (2013), who show that firms going public early on in the hot markets are of higher quality than firms going public later. Similarly, Kaplan and Stein (1993) document that transactions completed in the late 1980’s (following a long period of cheap access to debt) were of poorer quality: among the largest buyouts roughly every third resulted in some form of financial distress with every fourth actually defaulting on debt and filing for Chapter 11.

Lastly, my model predicts that fund-raising takes more time when investors discount rate is high. Moreover, startups that raise funds with a delay receive a better price and are on average of higher quality. Empirically this prediction is harder to test since the time when entrepreneur or firm first enters the market for funding is difficult to observe. For younger firms, however, this naturally leads to implications about firm’s age at the time of receiving financing. For example, one could test whether the age of startups raising Series A (the first round of VC investment) covaries over time with VC fund flows. Specifically, whether older startups are more likely to raise Series A round when VC funding is scarce, and whether they are of higher quality and secure better terms.

1.1 Related Literature

IPO Waves. Accumulation of projects on the market during the times when gains from trade are low and subsequent high volume of deals when gains from trade increase, predicted by my model, resembles IPO waves. IPO waves have attracted a lot of attention both in empirical and theoretical literature (see for example Pástor and Veronesi (2005), Bustamante (2012), Yung, Çolak and Wei (2008), Alti (2005)). The underlying economic
mechanism for the occurrence of the “wave” in my model is very different from the previous literature. Pástor and Veronesi (2005) and Bustamante (2012) use the real option framework to explain IPO waves. In both models entrepreneurs withdraw from the market, when market conditions decline, due to option value of waiting and issuing at better terms later. In these papers deteriorating market conditions prevent entrepreneurs, ceteris paribus, with worse projects from issuing. Yung et al. (2008) consider a static model of adverse selection. In their paper a decrease in gains from trade also affects the volume of deals through the lower part of the distribution. In contrast, quantity adjustment in my model is driven by the entrepreneurs with better projects withdrawing from the market, when conditions (gains from trade) deteriorate, due to a dynamic lemons problem. Similarly, an improvement in market conditions causes the wave in my model through the incentives of entrepreneurs with good projects, resulting in initial increase of average quality of funded projects.

Dynamic Markets for Lemons. My paper contributes to theoretical literature on dynamic markets for lemons. In particular, I follow the line of Swinkels (1999), Daley and Green (2012) and Strebulaev et al. (2014) by assuming that investors do not observe previous offers received by entrepreneurs (private offers assumption). Unlike Swinkels (1999), who solves a model in which lemons condition is not binding, and Daley and Green (2012) and Strebulaev et al. (2014), who focus on slow revelation of information, my paper primarily investigates interaction between variation in gains from trade with endogenous quality of entry.

The differences between models with private and public offers have been studied in Horner and Vieille (2009) and Fuchs, Öry and Skrzypacz (2014). In my model private offers play a crucial role: they do not allow for a complete separation of entrepreneurs with good and bad projects. This leads to cross-subsidization in equilibrium. Cross-sectional distribution of the quality of projects affects the degree of cross-subsidization and through this channel has a profound effect on the equilibrium structure. In contrast, in models with public offers Noldeke and Van Damme (1990) and Guerrieri, Shimer and Wright (2010) equilibrium is distribution free and features delay or probabilistic trade even when the quality of assets has been inferred to be good.

Similar to Guerrieri et al. (2010), trade in my model can happen at several prices simultaneously and sellers are rationed at higher prices. However, the set of prices offered in equilibrium as well as expected time to trade at each particular price depends on the distribution of the projects in the market and expectations about future evolution of gains.
from trade and/or quality of entry. In Guerrieri et al. (2010), equilibrium equilibrium prices are distribution and expectation free, and depend only on buyers’ valuations.

The rest of the paper is organized as follows. Section 2 describes a model with a constant discount rate of investors and endogenous entry. Section 3 characterizes the steady state equilibrium of that model. Section 4 describes a model with time varying discount rate, characterizes the equilibrium, and explores its dynamic properties. Section 5 considers several applications and discusses empirical implications of the model. Section 6 concludes. All proofs are in Appendix.

2 Model Setup

In this section I consider a model with a constant discount rate of investors. This assumption is relaxed in Section 4.

2.1 Lemons Market for Projects

Projects. The model is set up in continuous time. There is a continuum of potential entrepreneurs indexed by \( i \in I \). Each entrepreneur \( i \) comes up with an idea of quality \( \theta^i \in \{g, b\} \) at time \( t^i \) and makes a one-time decision whether to start developing the idea (entry decision). In case of the positive entry decision the idea becomes a project and the entrepreneur loses the ‘potential’ prefix. The project requires investment \( I \) for successful completion which can be raised at any time after \( t^i \) from a competitive market using equity. Both the time of entry \( t^i \) and the quality of the project \( \theta^i \) are entrepreneur’s private information. Once investment \( I \) is made, the project generates a one-time payment \( X_{\theta^i} \) (\( X_g > X_b > 0 \)) with Poisson intensity \( \delta_X \). Prior to investment, information about project’s quality of a particular entrepreneur \( i \) becomes public with intensity \( \delta \). Entrepreneurs are risk neutral and discount future payoffs at a rate \( \rho \).

Every moment \( t \geq t^i \) since the time of entry, each entrepreneur receives private offers from investors.\(^2\) If current offers are unfavorable, the entrepreneur can reject them and

\(^2\)An alternative way (leading to the same equilibrium) to specify the model is similar to Guerrieri et al. (2010): at each moment in time there is a continuum of markets open indexed by the price \( v \) offered by investors and probability of obtaining funds. Entrepreneur \( i \) decides on the minimal acceptable price \( v_i^v \) and participates in all markets with \( v \geq v_i^v \). Every instant markets clear from the top down (highest prices to lowest) with entrepreneurs being rationed if supply of projects exceeds demand for projects at a particular price.
continue waiting for a better price or information revelation. As soon as funds are raised entrepreneur $i$ leaves the market.

**Investors.** There is a continuum of competitive and homogeneous risk neutral investors who discount future payoffs at rate $r \in (0, \rho)$. Thus, investor’s valuation of a $\theta$ quality project is

$$V_\theta = \frac{\delta X}{\delta X + r} X_\theta.$$  

**Payoffs.** If some investor offers to provide capital in return for a share $I/v$ in the project $i$, I will call $v$ investor’s valuation of project $i$ or, interchangeably, a price offer. If entrepreneur $i$ decides to accept an offer and raise funds at time $t$ at price $v$, then her expected discounted payoff is

$$e^{-\rho(t-t^i)} \frac{\delta X}{\delta X + \rho} X_{\theta^i} \left(1 - \frac{I}{v}\right),$$

where $1 - \frac{I}{v}$ is the entrepreneur’s share of the project and $e^{-\rho(t-t^i)}$ is her specific discount factor. Denote by $S_\theta$ the value of the project of quality $\theta$ to entrepreneur who decides to wait until full information revelation. When information is revealed all investors value the project at $V_\theta$. Since investors are homogeneous and competitive, they will offer financing at zero expected profit. Thus, entrepreneur’s payoff upon raising funds is $\frac{\delta X}{\delta X + \rho} X_\theta \left(1 - \frac{I}{\delta X + \rho} X_\theta\right)$. Taking expectation over the time of information arrival gives

$$S_\theta = \frac{\delta}{\delta + \rho} \frac{\delta X}{\delta X + \rho} X_\theta \left(1 - \frac{I}{\delta X + \rho} X_\theta\right).$$

I will assume that regardless of the project quality it is profitable to raise funds conditional on entry.

**Assumption 1.** The parameters of the model satisfy

$$\frac{\delta X}{\delta X + \rho} X_\theta > I \quad \theta \in \{g, b\}. \quad \text{(Profitability)}$$

Let $B_\theta(v)$ denote the expected payoff to entrepreneur with a $\theta$ quality project raising funding at a price $v$ immediately upon entry:

$$B_\theta(v) = \frac{\delta X}{\delta X + \rho} X_\theta \left(1 - \frac{I}{v}\right).$$  

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2.2 Entry of Entrepreneurs

Entrepreneurs arrive to the market starting at time 0. I assume that the supply of entrepreneurs with bad projects is more sensitive to the market conditions than supply of good projects. In particular, I make the following simplifying assumption: potential entrepreneurs with good ideas always enter the market as soon as they have an idea. Without loss of generality, I can normalize the rate of entry of entrepreneurs with good projects to $1/\dt$, i.e. at time $t$ the total mass of entrepreneurs with good projects who entered the market is $t$.

Entrepreneurs with bad projects are strategic about the entry decision. Their entry is affected by the valuations prevalent in the market and the ease of obtaining funding. Every potential entrepreneur $i$ with quality $\theta^i = b$ at time $t^i$ weighs the benefits from entering the market against a private cost $c^i$. The private cost can be interpreted broadly as an opportunity cost of engaging in some other activity, e.g. the cost of quitting a job or dropping out of college. Denote by $G(c)$ the measure\(^3\) of entrepreneurs with bad projects having private cost no greater than $c$. I assume that $G(\cdot)$ is continuous, strictly increasing with $G(0) = 0$ and $G(\infty) = \infty$. Denote by $c_t$ be the highest private cost of a potential entrepreneur with a bad idea willing to enter the market at time $t$, then $G(c_t)\dt$ is the rate of entry of entrepreneurs with bad projects at time $t$.

2.3 Strategies

Investors. Instead of defining investors’ information sets, strategies, and payoffs, I model them as a collection of stochastic processes $V = (V^i)_{i \in \mathcal{I}}$ with each $V^i = (V^i_t)_{t \geq 0}$ and $V^i_t$ denoting the highest valuation of entrepreneur’s $i$ project at time $t$ conditional on information about the project $i$ not being released yet.\(^4\) The stochastic component in the definition of

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\(^3\)For a precise definition of the index set $\mathcal{I}$ and distribution of private costs $c^i$ and potential entry times $t^i$ see Appendix 6.1.

\(^4\)Recall that when information is released, the project is priced at $\frac{\delta X}{\delta X + \tau} X_{\theta^i}$ and the entrepreneur raises funds immediately.
the price processes is needed to allow investors to play mixed strategies which will be crucial for equilibrium construction. The class of processes that I consider (see Definition 4 in the Appendix) allows for playing a pure strategy, mixing between different prices with positive probabilities, and mixing between prices with positive rates.\footnote{The latter mixing, for example, could be used for offering high prices with an exponential delay, similar to arrival of jumps of a Poisson process.}

To reflect the information available to investors I impose the following restrictions on the price processes.

**Assumption 2.** The collection of price offers \((V^i)_{i \in I}\) satisfies:

1. **Private Offers**

\[
\{V^i_s; s < t\} \text{ is independent from } \{V^i_s; s \geq t\} \quad \forall i \in I, t \geq t^i
\]

2. **Anonymity**

\[
\{V^i\}_{i \in I} \text{ are pairwise i.i.d}
\]

The first part of Assumption 2 captures the notion that investors do not observe previous offers received by entrepreneur \(i\). Therefore, they cannot condition their current and future offers on that information. The second part implies that investors cannot condition their offer on the identity of the entrepreneur (recall that \(t^i\) is entrepreneur’s private information). It also allows me to use the exact law of large numbers in the cross section of entrepreneurs, i.e. if investors mix between valuations \(v_1\) and \(v_2\) with equal probabilities then exactly a half of population of entrepreneurs will be offered \(v_1\) with another half being offered \(v_2\).\footnote{See Sun (2006) for more details.} It will be useful to denote the set of all offered valuations at time \(t\) as \(V_t = \text{supp}(V^i_t)\).

**Entrepreneurs.** At time \(t\) potential entrepreneurs with bad ideas and \(t^i = t\) face an entry decision, which will be captured by \(c_t\) – the highest private cost of entrepreneur willing to enter at time \(t\). Conditional on entry, each entrepreneur \(i\) observes all previously received offers. Hence, her private history is \(H^i_t = \{V^i_u; u \leq t\}\). In order to allow for mixing, I will define entrepreneur’s strategy \(F^i\) as a non-decreasing càdlàg stochastic process \(F^i = (F^i_t)_{t \geq t^i}\) adapted to private history \((H^i_t)_{t \geq t^i}\) such that \(0 \leq F^i_t \leq 1\) for all \(t \geq t^i\). Intuitively, \(F^i_t\) is a cumulative probability of entrepreneur \(i\) raising funds before or at time \(t\). Every strategy \(F^i\) induces a (possibly stochastic) time of trade for seller \(i\) which will be denoted by \(\tau^i\). Let \(F = (F^i)_{i \in I}\) denote the strategy profile of all entrepreneurs.
2.4 Market Belief

Since the investors do not observe either the quality of the projects they are evaluating or the time any particular entrepreneur has been on the market, they form beliefs based on aggregate quantities. Denote by $m^g_t$ ($m^b_t$) the mass of sellers with good (bad) projects in the market at time $t$. Then

$$m^g_t \equiv \mes \{ i : t^i \leq t \leq \tau^i \text{ and } \theta^i = g \}; \quad (4)$$

$$m^b_t \equiv \mes \{ i : t^i \leq t \leq \tau^i \text{ and } \theta^i = b \text{ and } c^i \leq c_t \}. \quad (5)$$

Let $\pi_t$ denote the average aggregate quality of assets in the market at time $t$, then

$$\pi_t = \begin{cases} \frac{m^g_{t^-} - m^b_{t^-}}{m^g_{t^-} + m^b_{t^-} + 1}, & \text{if } m^g_{t^-} + m^b_{t^-} > 0, \\ \frac{1}{1 + G(c_t)}, & \text{if } m^g_{t^-} + m^b_{t^-} = 0. \end{cases} \quad (6)$$

Although I do not model matching of investors and entrepreneurs explicitly, one can think of investors meeting a random entrepreneur every period $t$ with every entrepreneur meeting at least two investors. If there are currently $m^g_{t^-}$ entrepreneurs with good projects in the market and $m^b_{t^-}$ entrepreneurs with bad projects in the market, then chances that a randomly picked entrepreneur has a good project is exactly $m^g_{t^-} / (m^g_{t^-} + m^b_{t^-})$. If, however, all the projects in the past have already received funding ($m^g_{t^-} + m^b_{t^-} = 0$) then quality of a randomly picked project in the market equals to the average quality of the new projects entering the market $1/(1 + G(c_t))$.

2.5 Equilibrium

Every seller $i$ entering the market at time $t^i$ maximizes

$$\sup_{F^i} \left[ S_{\theta^i} + \int_{t^i}^{\infty} e^{-(\rho + \delta)(t - s)} (B_{\theta^i}(V^i_s) - S_{\theta^i})dF^i_t \right]. \quad (7)$$

One can think of expected value of investment post information arrival $S_{\theta^i}$ as an outside option that entrepreneur is endowed with at date $t^i$. If funds are raised at valuation $V^i_t$

\footnotetext[7]{As usual, $m^g_{t^-}$ stands for the least limit of $m^g$ at time $t$, i.e. $m^g_{t^-} = \lim_{s \uparrow t} m^g_s$.}

\footnotetext[8]{Second part of equation (6) requires conditioning on measure zero set in $\mathcal{I}$. Such conditional expectation is well defined due to to Radon-Nikodym.}
at time $\tau$ than she receives the value $B_\theta(V_i^\tau)$ but loses the option $S_\theta$. This payoff is discounted by $e^{-\rho(\tau-t^i)}$ due entrepreneur’s time preferences and by $e^{-\delta(\tau-t^i)}$ due to possibility of information arrival before time $\tau$. Finally the expectation is taken over all times $\tau$ which have a cumulative distribution function $F_i$.

For $t \geq t^i$ denote by $W_i^g$ entrepreneur $i$’s continuation value conditional on the observed private history $H_i^t$ and the fact that she has not raised funds yet

$$W_i^g = \sup_{F_i^t} \mathbb{E} \left[ S_\theta + \int_{t}^{\infty} e^{-(\rho+\delta)(\tau-t)} (B_\theta(V_i^\tau) - S_\theta) dF_i^\tau \mid H_i^t, \tau^i > t \right].$$

(8)

Define two auxiliary processes

$$W_i^g = \sup_{i: \tau^i \geq t, F_i^t \leq 1, \theta^i = g} W_i^g \quad \text{and} \quad W_i^b = \inf_{i: \tau^i \geq t, F_i^t \leq 1, \theta^i = b} W_i^g.$$  

(9)

$W_i^g$ ($W_i^b$) is the highest (lowest) continuation value of all entrepreneurs with good (bad) projects who are present in the market at time $t$ with positive probability.

**Definition 1.** An equilibrium of the game is a quadruple $(F, V, m^g, m^b)$ with induced continuation values $(W_i^g, W_i^b)$ that satisfies

1. **Seller Optimality.** Given $V_i$, $F_i$ solves sellers’ problem (7) for all $i$ and $t \geq t^i$ and the entry cut-off is given by

$$c_t = W_i^b.$$  

(10)

2. **Buyer Optimality.**

   (a) **Zero Profit.** For any valuation $v \in V_i$ offered at time $t$ either there does not exist $i$ such that $\tau^i = t$ and $V_i^t = v$, or

   $$v = \frac{\delta X}{\delta X + r} E \left( X_{\theta^i} \mid V_i^t = v, \tau^i = t \right).$$ 

(11)

   (b) **Market Clearing.**

   $$W_i^g \geq B_g \left( \frac{\delta X}{\delta X + r} (\pi_t X_g + (1 - \pi_t) X_b) \right) \quad \text{and} \quad W_i^b \geq B_b \left( \frac{\delta X}{\delta X + r} X_b \right).$$  

(12)

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9This condition prevents existence of out-of-equilibrium price offers that would yield positive profits to investors. It is similar to No Deals restriction of Daley and Green (2012) and Market Clearing restriction of Fuchs and Skrzypacz (2014).
3. Belief Consistency. Buyers’ belief about the proportion of good quality projects in the market is consistent with $m^g$ and $m^b$ induced by entry of new projects (characterized the entry cut-off $c_1$) and fund raising decisions induced by the sellers’ strategy $F$ and offered prices $V$.

The first part of the Market Clearing condition states that the (highest) expected continuation value of the entrepreneur with a good project should be greater or equal than the average quality of all the projects present in the market. If at some point expected continuation value would fall below the average quality of projects in the market, any investor could make profit by picking a random project and offering a valuation slightly below the market average. Similarly, if (the lowest) continuation value of the entrepreneur with bad asset falls below $B_b \left( \frac{\delta X}{\delta X + r} X_b \right)$, then a price offer $\frac{\delta X}{\delta X + r} X_b - \varepsilon$ would make profits with positive probability since it would attract entrepreneurs with bad projects.

**Remark 1.** Although I model an environment with entrepreneur (firm) raising a fixed amount of funds by issuing equity, the model is rich enough to incorporate other setups. Consider for example a market in which sellers who are privately informed about the quality of the assets (such as pools of mortgages or high yield corporate bonds) sell to uninformed competitive buyers. Suppose that for (unmodeled) reasons such as liquidity or hedging risks, seller’s value of holding the $\theta$ quality asset ad infinitum, $S_\theta$, is smaller than the buyer’s value of holding the asset ad infinitum, $V_\theta > S_\theta$. When a seller transacts at time $t$ at price $v$, she receives $B_\theta(v) = v$, but loses the future stream of dividends. Her payoff, therefore, is $S_\theta + e^{-\rho t} (B_\theta(v) - S_\theta)$, similar to (7). That is, the model can be used to describe markets where buyers offer a fixed amount of money in exchange for equity share of varying size, or markets where buyers obtain a fixed asset/equity stake of unknown quality for varying prices (and hybrid situations as well).

**Definition 2.** Equilibrium is in steady state if $V^i$ is a stationary process for all $i \in I$ and $(m^g_t, m^b_t)$ are constant over time.

The next section characterizes the steady state equilibria of the model.

3 Steady State Equilibria

3.1 Preliminary Analysis

Before fully characterizing the steady state, I describe properties of any equilibrium that greatly simplify the analysis.
I begin by showing that in any equilibrium entrepreneurs are using threshold strategies.

**Lemma 1. (Threshold Strategies)**

There exist two deterministic functions $r_g^t$ and $r_b^t$ such that any entrepreneur with $\theta$ quality project present in the market at time $t$ rejects all offers $v < r_g^t$ and raises funds with probability 1 if offered a valuation $v > r_g^t$.

The intuition behind this lemma strongly relies on the Private Offers assumption. Since future investors do not observe previous offers, continuation value for any entrepreneur does not depend either on the current valuation itself or on her acceptance decision (even when considering an off-equilibrium deviation). Thus, any valuation strictly higher than continuation value triggers acceptance and any valuation strictly lower than continuation value will be rejected.

Furthermore, these two thresholds can be ranked.

**Lemma 2. (Skimming Property)**

At any time $t$, 

$$r_g^t > r_b^t.$$  

(13)

Lemma 2 implies that if some price is attractive for the entrepreneur with a good project, then it will be accepted with probability 1 by the entrepreneur with a bad project. Without asymmetric information the good project is more valuable than the bad one, hence, $S_\theta$ serves as an option value of delaying investment until information revelation. This option is less valuable when the project is bad creating incentives for the entrepreneur to accept lower valuations.$^{10}$

The previous lemmas uniquely define entrepreneur’s best response to any valuation $v$ which is not equal to $r_g^t$ or $r_b^t$. When the valuation is equal to either of the respective thresholds an entrepreneur with a corresponding project is indifferent, nevertheless, in any equilibrium the action of an entrepreneur with a good project is uniquely pinned down by the following lemma.

**Lemma 3. (No mixing at $r_g^t$)**

Entrepreneur with good quality projects never plays a mixed strategy. In particular, she accepts all offers with valuations $v \geq r_g^t$.

$^{10}$Similarly to Kremer and Skrzypacz (2007), in my model single crossing arises not from costs but from benefits of delay.
Nobody raises funds Only bad projects accept funding Any project accepts funding

Figure 2: Types of projects willing to raise funds at valuation $v$

If the high type were mixing at some offer $v = r_t^g$, then the average quality of projects funded at this price would be below the current average quality of the project in the market. Recall that, on the one hand, $r_t^g$ equals to the expected continuation value of the entrepreneur with a good project and, on the other hand, investors break even at $v$. These two facts together imply that the first part of Market Clearing is violated.

Optimal behavior of entrepreneurs together with a break-even constraint for investors put strong discipline on the equilibrium set of offered valuations. In particular, any valuation $v > r_t^g$ or $v \in (r_t^b, r_t^g)$ would lose money for investors and therefore will not occur in equilibrium.

**Corollary 1.** In any equilibrium, any valuation $v \in V_t$ offered at time $t$ is either

**Pooling Offer** $v = \frac{\delta X}{\delta X + r_t}(\pi_t X_g + (1 - \pi_t) X_b)$, or

**Separating Offer** $v = \frac{\delta X}{\delta X + r_t} X_b$, or

**Losing Offer** $v < \frac{\delta X}{\delta X + r_t} X_b$.

For any equilibrium, in which losing offers are made, one can construct an equilibrium by replacing all loosing offers with a separating offer and adjusting probability of acceptance of the separating offer by the entrepreneurs with bad projects. Without loss of generality, I focus on equilibria in which only pooling and separating offers are made.

### 3.2 Equilibrium Construction

I construct a steady state equilibrium in two steps. First, I exogenously fix the entry rate $G(c)$ of entrepreneurs with bad projects to the market and solve for a steady state equilibrium. Then, I will endogenize the (constant) $c_t$ by tying the entry cut-off with the equilibrium continuation value using condition (10).
In the steady state equilibrium incentives of entrepreneurs with a good quality project to accept a pooling offer are driven by the following comparison:

\[
\frac{\rho}{\delta + \rho} \left[ \frac{\delta X}{\delta X + r} X_g - I \right] \quad \text{vs.} \quad \frac{(1 - \pi_t)I(X_g - X_b)}{\pi X_g + (1 - \pi)X_b}.
\] (14)

The left hand side represents the benefit due to early investment (recall that the signal about the project quality is revealed with intensity \(\delta\)), while the right hand side stands for underpricing costs. Underpricing costs are lower when investors’ belief \(\pi_t\) is higher (underpricing completely disappears when \(\pi_t = 1\)), when funding need \(I\) is lower, and when \(X_g - X_b\) is lower.

Whenever the right hand side of expression (14) is higher than the left hand side, entrepreneurs with good type would rather wait for information revelation than raise fund at the current pooling valuation. Similar to Akerlof (1970), the market for lemons develops. The dynamic continuation value of entrepreneurs plays a role of the seller’s cost from Akerlof’s model and precludes trade at the average price.

Denote by \(\hat{\pi}\) the value of \(\pi_t\) that equates the left and right-hand sides of (14). When the quality of newly arrived projects \(1/(1 + G(c_t))\) is above \(\hat{\pi}\), then immediate acceptance of a pooling offer for the entrepreneurs with good projects is incentive compatible.

However, when \(1/(1 + G(c_t))\) is below \(\hat{\pi}\) immediate pooling is not the best response, i.e. the lemons condition is binding. It cannot also be true that in equilibrium entrepreneurs with good projects never raise funds prior to information revelation, for if it were the case, then all entrepreneurs with bad projects would raise funding at the moment of entry. In arbitrarily small amount of time, the investor belief about remaining types in the market would reach \(\pi_t = 1\). The unique continuation equilibrium would then have immediate trade at the pooling offer, which is strictly higher than the low valuations just a few moments earlier. It would make it suboptimal for the bad types to raise funds immediately upon entry.

The only way bad and good projects would be able to raise funding in equilibrium with when \(1/(1 + G(c_t))\) is below \(\hat{\pi}\) is through delayed trade at the pooling valuation. Higher expected time to raise funds at pooling valuation makes it incentive compatible for entrepreneurs with bad projects to randomize in their acceptance of (an always standing) low valuation offer, thus, improving average quality of projects in the market and allowing investors to break even when offering a pooling valuation.

Denote by \(\alpha = \frac{1}{1+G(c)}\) the fraction of entrepreneurs with good projects entering the
market. The following proposition characterizes steady state equilibrium for an exogenously fixed $\alpha$.

**Proposition 1.**

1. If $\alpha > \hat{\pi}$, then there exists an essentially unique steady state equilibrium. Along the equilibrium path all projects are funded at pooling ($\alpha$) valuation upon entry.

2. If $\alpha < \hat{\pi}$, then there exists an essentially unique steady state equilibrium. Along the equilibrium path

   (a) good projects raise funds at pooling ($\hat{\pi}$) valuation;
   
   (b) bad projects raise funds at separating and pooling ($\hat{\pi}$) valuations;
   
   (c) supply is rationed at the pooling ($\hat{\pi}$) valuation.

3. If $\alpha = \hat{\pi}$, there exists a continuum of steady state equilibria. Along the equilibrium path funds are raised at pooling ($\hat{\pi}$) valuations and supply is rationed.

When $\alpha < \hat{\pi}$, funds are raised at pooling and separating valuations at the same time (see Figure 3). Through mixing on the investor side of the market at each time $t$ a fraction of entrepreneurs is offered a pooling valuation which both types accept. However, a vast majority of the investors offer to invest only at low (separating) valuation. Such offer is rejected by good types, while the bad types randomize between acceptance and rejection, with only a flow of bad types accepting, so that investor beliefs change continuously.

With a slight abuse of notation, let $B_{\theta}(\pi)$ denote the expected value of accepting a pooling offer for the entrepreneur with a $\theta$ quality project when investors belief is $\pi$. Then equilibrium payoff $W^b$ to the same entrepreneur is

$$W^b(\alpha) = \begin{cases} 
B_b(\alpha), & \text{if } \alpha = \frac{1}{1+G(c)} > \hat{\pi}; \\
S_b, & \text{if } \alpha = \frac{1}{1+G(c)} < \hat{\pi}; \\
[S_b, B_b(\hat{\pi})], & \text{if } \alpha = \frac{1}{1+G(c)} = \hat{\pi}.
\end{cases} \quad (15)$$

At $t_i$ each entrepreneur with a bad project weighs costs of entry $c^i$ and expected equilibrium payoff $W^b$. Those with cost below $W^b$ choose to enter, hence, entry rate of bad projects is $G(W^b)$. In order to solve for the steady state equilibrium with endogenous entry

\footnote{The equilibrium is unique up to (i) implementation of mixed strategy by a continuum of entrepreneurs and (ii) measure zero of entrepreneurs following an arbitrary strategy.}
the actual proportion of high quality projects needs to coincide with the one expected by investors.

**Proposition 2.** There exists an essentially unique steady state equilibrium. Proportion $\alpha^*$ of good projects entering the market every period is the unique root of

$$\frac{1}{1 + G(Wb(\alpha^*)))} = \alpha^*.$$  

(16)

Given $\alpha^*$, the equilibrium outcome is characterized by Proposition 1.

When equilibrium quality of entry $\alpha^*$ is below $\hat{\pi}$ and the lemons condition is binding then equilibrium in the funding market is inefficient. Since the private cost $c^i$ is sunk, Assumption 1 implies that conditional on entry all the projects should be financed immediately in the first best. However, in equilibrium it takes on average positive time to raise funds and efficiency is lost due to time discounting of gains from trade.

However, illiquidity in the fund-raising market has a second, welfare improving, side. Delayed funding at high prices serves as an imperfect screening mechanism that allows investors to separate entrepreneurs with good projects from (some) entrepreneurs with bad projects. Partial sorting of the projects implies that equilibrium payoff to entrepreneurs with bad projects equals to the true value of their idea, thus, their entry decisions are efficient.\footnote{Since I have assumed that entry of good types is inelastic, in this steady state the entry is efficient. However, entrepreneurs with good projects do not earn their true value. If their rate of entry depended on the expected return, it would be inefficient.}
On the other hand, when $\alpha^*$ is above $\hat{\pi}$ and equilibrium features immediate pooling, the entrepreneurs with bad projects get a payoff higher than the true value of their ideas, hence, the entry is inefficiently high.

A combination of observations discussed above leads to the following result.

**Proposition 3.** Steady state equilibrium is always inefficient: if $\alpha^* \leq \hat{\pi}$ then positive gains from trade are realized with a delay; if $\alpha^* \leq \hat{\pi}$ there is excessive entry of bad projects to the market.

### 3.3 Varying the Discount Rate

In this section I explore how the nature of the steady state equilibrium depends on the discount rates of investors.

**Proposition 4.** Equilibrium fraction $\alpha^*$ of good projects entering the market at every period is increasing in the discount rate of investors, $r$.

An increase in the discount rate of investors increases the cost of early financing of good projects in two ways. First, it leads to a decrease in the differential benefits of early
investment, because the NPV of the projects becomes smaller. Second, it leads to an increase in adverse selection costs, because investors demand a higher share for regardless of the project quality. Both of these factors increase $\hat{\pi}$. In addition, an increase in $r$ decreases the payoff to the entrepreneurs with bad projects conditional on pooling, $B_b(\pi)$. A lower payoff conditional on pooling and a decrease in willingness to pool of owners of good projects reduces incentives of entrepreneurs with bad projects to enter.

Both equilibrium quality of entry $\alpha^*$ and the lemons condition threshold $\hat{\pi}$ move in the same direction when investors’ discount rate $r$ changes. In order to rank them I will make a following parametric assumption.

**Assumption 3.** Let the parameters of the model satisfy

$$\frac{\rho}{\delta + \rho} \left( \frac{\delta_X}{\delta_X + \rho} X_g - I \right) < I \left( \frac{X_g}{X_b} - 1 \right) < \frac{\rho}{\delta + \rho} (X_g - I), \quad (17)$$

and

$$G \left( \frac{\delta_X}{\delta_X + \rho} X_b - I \right) > \frac{1}{\hat{\pi}_{r=\rho}} - 1, \quad (18)$$

where $\hat{\pi}_{r=\rho}$ is a solution of (14) with $r = \rho$.

The first part of Assumption 3 makes sure that there is enough variation in gains from trade between investors and entrepreneurs relative to adverse selection discount. The second part rules out the case when distribution of private costs of entry is so steep that not enough bad projects enter the market to make the lemons condition binding.

**Proposition 5.** If parameters satisfy Assumption 3 then there exist two thresholds $0 < \underline{r} < \overline{r} < \rho$ such that (i) for all $r < \underline{r}$ steady state equilibrium features $\alpha^* > \hat{\pi}$, (ii) for all $r > \overline{r}$ steady state equilibrium features $\alpha^* < \hat{\pi}$.

4 Transition Dynamics

In the previous section I have shown that a lower discount factor of investors increases incentives for entrepreneurs with bad projects to enter. As a result, it reduces the average quality of projects in the market. In this section, I explore implications of the discount rate variation over time for quality of projects receiving funding and the volume of funded projects in a dynamic model.

**State Process.** Dynamics of the discount rate is driven by a publicly observable Markov switching state process $Y = (Y_t)_{t \geq 0}$ which takes two values $Y_t \in \{0, 1\}$. Denote by $\lambda^1$ the
arrival intensity of state 1 conditional on \( Y_t = 0 \), and by \( \lambda^0 \) the arrival intensity of state 0 conditional on \( Y_t = 1 \). Let the interest rate of investors \( r(y) \) satisfy the following inequality:

\[
0 < r(1) < r < \tilde{r} < r(0) < \rho. \tag{19}
\]

Inequality (19) implies that in the steady state equilibrium the lemons condition is binding when \( Y = 0 \) and not binding when \( Y = 1 \). Intuitively, in state \( Y_t = 1 \) investors’ capital is in abundance, thus, it is cheaper to finance, while in \( Y_t = 0 \) the capital is scarce and funding is more costly.

**Histories and Strategies.** Definition of strategies and equilibrium from Section 2 needs to be augmented to allow for state contingency. Define the public history to be \( \mathcal{H} = (\mathcal{H}_t)_{t \geq 0} \) where \( \mathcal{H}_t \) is generated by \( \{Y_u; u \leq t\} \).

Private history of entrepreneur \( i \) now includes both the public history as well as all previously received price offers, i.e. \( \mathcal{H}^i_t = \sigma(\mathcal{H}_t; V_s^i, s \leq t) \). Similarly to Section 2 entrepreneur’s strategy \( F^i \) is a non-decreasing càdlàg stochastic process \( F^i = (F^i_t)_{t \geq t^i} \) adapted to private history \( (\mathcal{H}^i_t)_{t \geq t^i} \) such that \( 0 \leq F^i_t \leq 1 \) for all \( t \geq t^i \).

Price processes \( V^i \) are now allowed to depend on the public history, since the state process is observed by all investors. Private Offers and Anonymity conditions are easily adapted to incorporate state dependency with conditional independence. In particular, I will say that a collection of price offers \( (V^i)_{i \in I} \) satisfies Private Offers restriction if \( \sigma\{V^i_s; s < t\} \) is independent from \( \sigma\{V^i_s; s \geq t\} \) conditional on \( \mathcal{H}_t \) for all \( i \in I \) and \( t \geq t^i \); it satisfies Anonymity restriction if \( \{V^i\}_{i \in I} \) at i.i.d. conditional on \( \mathcal{H} \).

**Pay-offs.** Similarly to Section 2 define investor’s valuation of a \( \theta \) quality project conditional on state \( Y_t = y \) as

\[
V_\theta(y) = D_X(y)X_\theta, \tag{20}
\]

where the \( D_X(y) \) is expected discounted time until the project pay-out conditional on the current state being \( y \).\(^{13} \) If a type \( \theta \) entrepreneur raises funds in state \( Y_t = y \) at valuation \( v \), then her payoff is

\[
B_\theta(\pi; y) = \frac{\delta_X}{\delta_X + \rho} X_\theta \left( 1 - \frac{I}{v} \right). \tag{21}
\]

Let \( S_\theta(y) \) be expected payoff to a type \( \theta \) entrepreneur from obtaining funding upon infor-

\(^{13}\) The values \( D_X(y) \) uniquely solve a linear system \( (r(y) + \delta_X + \lambda^{1-y})D_X(y) = \delta_X + \lambda^{1-y}D_X(1-y) \) for \( y \in \{0, 1\} \).
information revelation. $S_\theta(y)$ can be written as
\[
S_\theta(y) = \frac{\delta}{\delta + \rho} \left[ p(1, y) B_\theta(V_\theta; 1) + (1 - p(1, y)) B_\theta(V_\theta; 0) \right],
\]
(22)
where $p(y', y)$ is the probability of state being $y'$ at the moment of information revelation, conditional on the current state being $y$.\footnote{Conditional probabilities $p(y', y)$ are the unique solution of the linear system $(r(y) + \delta + \lambda^{-1}y)p(y', y) = \delta 1(y' = y) + \lambda^{-1}y p(y', 1 - y)$ for $y', y \in \{0, 1\}$.}

In equilibrium every entrepreneur $i$ solves an optimal stopping problem similar to the one in Section 2:
\[
\sup_{F^i} \mathbb{E} \left[ S_{\theta^i}(Y_{ti}) + \int_0^\infty e^{-(\rho + \delta)(\tau - t)} (B_{\theta^i}(V_{\tau}, Y_\tau) - S_{\theta^i}(Y_\tau)) dF^i_{\tau} \middle| \mathcal{H}_t \right].
\]
(23)

I now define an equilibrium of the model with stochastic discount rate.

**Definition 3.** An equilibrium of the game is a quadruple $(F, V, m^g, m^b)$ with induced continuation values $(W^g_t, W^b_t)$ that satisfies

1. **Seller Optimality.** Given $V^i$, $F^i$ solves seller’s problem (23) for all $i$ and $t \geq t^i$ and the entry cut-off is given by
\[
c_t = W^b_t.
\]
(24)

2. **Buyer Optimality.**

   (a) **Zero Profit.** For any valuation $v \in V_t$ offered at time $t$ either there does not exist $i$ such that $\tau^i = t$ and $V^i_t = v$, or
   \[
v = D_X(Y_t) \cdot E \left( X_{\theta^i} \middle| V^i_t = v, \tau^i = t \right).
   \]
   (25)

   (b) **Market Clearing.**
   \[
   W^g_t \geq B_g \left( D_X(Y_t) \cdot (\pi_t X_g + (1 - \pi_t) X_b) \right) \quad \text{and} \quad W^b_t \geq B_b \left( D_X(Y_t) \cdot X_b \right).
   \]
   (26)

3. **Belief Consistency.** Buyers’ belief about the proportion of good quality projects in the market is consistent with $m^g$ and $m^b$ induced by entry of new projects (characterized the entry cut-off $c_t$) and fund raising decisions induced by the sellers’ strategy $F$ and offered prices $V$. 
The following proposition shows existence of dynamic equilibrium when state transitions do not happen too often.

**Proposition 6.** *For sufficiently small* $\lambda^1$ *and* $\lambda^0$ *there exists an equilibrium with stochastic discount rate.*

I now characterize the most salient properties of the equilibrium.

### 4.1 Wave of Deals

**Proposition 7.** *In the equilibrium of Proposition 6 when the discount rate of investors decreases from* $r(0)$ *to* $r(1)$, *a wave of deals follows. The average quality of the projects funded at the moment of transition is strictly higher than the average quality of projects raising funds at any other time.*

When the discount rate decreases, incentives to delay fund-raising disappear for two reasons. First, conditional on the average quality of projects in the market valuations are as high as they are ever going to be. Second, more importantly, higher valuations attract worse projects to the market, thus, in expectation the prices will be decreasing. Strict preferences for immediate trade of entrepreneurs with good projects together with the Market Clearing condition require for pooling valuation to be offered with probability 1. Accumulation of unfunded projects over the period of the high discount rate together with the previous observation imply that there is going to be an *atom* of trade (see Figure 5) when the discount rate falls back to $r(1)$.

The quality of the projects raising funds at the time of transition to $Y_t = 1$ is equal to the average quality of unfunded projects a second earlier. Due to illiquidity in state 0 the
average quality of unfunded projects is strictly better than the quality of projects entering the market, which, in turn, is better than the average quality of those receiving funding. Right after the transition the average quality significantly drops, since high valuations and ease of receiving financing attracts worse quality projects to the market.

The economic mechanism underlying the wave of deals in my model is novel and very different from those previously proposed in the literature. In particular, it leads to a different observable dynamics of the quality of funded projects in the hot markets. For example, Pástor and Veronesi (2005) and Bustamante (2012) use the real option framework to explain clustering of IPO deals over time. Optimal exercise threshold of a real option is a decreasing function of project’s quality, thus, an improvement in market conditions triggers owners of worse projects to exercise their options. As a result both papers predict that the quality of funded projects is the highest in cold markets and always lower in hot markets. A similar pattern arises from the static adverse selection model of Yung et al. (2008). In contrast, the quantity of funded projects in my model is the highest early in the wave, since the quantity adjustment comes from the top part of the distribution. It is the entrepreneurs with good projects who partially withdraw from the market in bad times and rush back, generating a wave of deals, when market conditions improve.

4.2 Average Quality of Funded Projects

Proposition 8. In the equilibrium of Proposition 6 the average quality of funded projects in state $Y_t = 0$ is higher than in state $Y_t = 1$ if the corresponding state lasts sufficiently long.

Figure 6 shows the dynamics of the average quality of funded projects over time. In state $Y_t = 1$ with the low discount rate funding is raised immediately. Therefore, the average quality of the projects conditional on receiving financing is fully determined by the quality of the projects entering the market. High prices and ease of obtaining funding attracts a pool of projects of low average quality.

In state $Y_t = 0$, however, projects are financed at both separating and pooling valuations. Conditional on raising funding at the separating valuation the quality of the project is always bad while those funded at the pooling valuation are on average of very high quality. The average quality conditional on getting financing, thus, is a weighted average of a low (separating) and high (pooling) project quality. The acceptance rate of the separating valuation is constant over time. At the same time, the acceptance rate of pooling valuation increases over time due to accumulation of projects in the market. Thus, the volume-weighted average
quality of funded projects is increasing over time. It eventually approaches the steady state quality, which is equal to the quality of the projects entering the market. In equilibrium entrepreneurs with bad projects are indifferent between accepting a separating price and waiting to get a pooling price, thus, \( W_t^b = B_b(D_X(0)X_b) \) when \( Y_t = 0 \). The high discount rate and lack of overpricing attract less entrepreneurs with bad projects to the market, resulting in \( \alpha^*_0 > \alpha^*_1 \).

4.3 Liquidity and Entry

**Proposition 9.** The equilibrium of Proposition 6 is never efficient: when \( Y_t = 0 \) the funds on average are raised with a positive delay, and when \( Y_t = 1 \) there is an excessive entry of entrepreneurs with bad projects.

When the discount rate is low (\( Y_t = 1 \)), every entrepreneur entering the market immediately raises funds at pooling valuation. This is not, however, the case in the state \( Y_t = 0 \). When the discount rate is high, immediate funding can be raised only at a separating valuation which is unacceptable for entrepreneurs with good projects, so only a fraction of bad projects is funded at low valuations. The vast majority of the market is attempting to raise funds at the pooling valuation which leads to a delay. Since pooling valuation is offered with intensity \( \lambda_p \), the rate of funding at this valuation is \( \lambda_p(m^g_t + m^b_t) \). At the time of arrival of \( Y_t = 0 \) both \( m^g_t \) and \( m^b_t \) are zero, and the inflow rate of new entrepreneurs arriving to the market is positive. Hence, inventory starts building up (see Figure 7).

On the other hand, illiquidity in state \( Y_t = 0 \) allows for partial screening of projects.
The equilibrium payoff of the entrepreneur entering the market, when discount rate is high, is exactly $B_b(D_X(0)X_b;0)$. It implies that the NPV of the marginal project entering the market, taking into account the private cost of entry $c_i$, is zero. Thus, equilibrium entry threshold $c_t$ is at the first best level, as shown in Figure 8. When the state is $Y_t = 1$, equilibrium features immediate pooling and equilibrium continuation value of entrepreneur with a bad project entering the market is $B_b(D_X(1)(\alpha^*_1X_g + (1 - \alpha^*_1)X_b);1)$. In addition to low discount rate, owners of bad projects enjoy the benefits of overpriced equity. Such an implicit subsidy distorts incentives to enter the market. Indeed, the NPV of the marginal project brought to the market in the state with low discount rate is

$$B_b(D_X(1)X_b;1) - c_t = B_b(D_X(1)X_b;1) - B_b(D_X(1)(\alpha^*_1X_g + (1 - \alpha^*_1)X_b);1) < 0.$$  

Hence, when $Y_t = 1$ the equilibrium is inefficient due to an excessive entry of entrepreneurs with bad projects.
5 Empirical Implications

The results based on the equilibrium of the model naturally lead to several empirical implications.

Volume of Deals. My model predicts that volume of deals and market prices are positively correlated with market conditions. Lower gains from trade caused by, for example, higher cost of investors’ capital, cause not only the price adjustment but also a quantity adjustment, because owners of high quality projects find it optimal to partially withdraw from the market. This prediction is broadly consistent with findings across multiple markets, in which adverse selection is an important concern. In the venture capital industry, fund inflows significantly affect the number of funded ventures. In addition (Gompers and Lerner (2000)) document that fund flows predict valuations of startups. Axelson et al. (2009) show that cost of debt drives the volume and valuation levels in private equity buyouts. Similarly, in IPO and M&A markets Ball, Chiu and Smith (2011) and Pagano, Panetta and Zingales (1998) find that the volume of deals is explained by market conditions. Secondary equity issuances seem to exhibit a similar pattern: Baker and Wurgler (2002) argue that timing of market conditions is an important determinant of firms’ capital structure, while Choe, Masulis and Nanda (1993) show that firms increase equity offerings (relative to debt offerings) in periods of economic expansion consistent with predictions of my model.

Additionally my model generates a wave of deals, which is a definitive feature of IPO (Ritter and Welch (2002)) and private equity buyout (Kaplan and Stein (1993)) markets.

Quality of Funded Projects. Quality of funded projects in my model is non-monotone with respect to market conditions (and volume of deals). The quality is low when the discount rates have been low for a prolonged period of time. Conversely, it is higher in times of high discount rates, and it is the highest early on in the fund-raising wave. This implies that empirically measured quality of funded projects might strongly depend on the definition of hot/cold markets. Nevertheless, this prediction have found support across several markets. In context of IPOs, Ritter and Welch (2002) write that “it is conventional wisdom among both academics and practitioners that the quality of firms going public deteriorates as a period of high issuing volume progresses”. This is consistent with my findings and is confirmed by Chang et al. (2013) who show that firms issuing earlier in the hot markets are of higher quality than firms issuing later. Similarly, Kaplan and Stein (1993) document that transactions completed in late 1980’s (following a long period of cheap access to debt) were
of poorer quality: among the largest buyouts roughly every third resulted in some form of financial distress with every fourth actually defaulting on debt and filing for Chapter 11. In the context of venture capital funding Nanda and Rhodes-Kropf (2013) find that startup raising funds in hot markets are more likely to fail but conditional on not failing are more successful. This could be driven by the time varying quality of projects raising funds predicted by my model: early on in the hot markets the average quality is the highest, however, it deteriorates very fast.

**Time on the Market.** Lastly, my model predicts that fund-raising takes more time when investors market conditions are bad. Moreover, startups that raise funds with a delay receive a better price and are of higher quality on average. Empirically, this prediction is harder to test, because the time when entrepreneur or firm first enters the funding market might not be observable. Thus, one should be careful taking this prediction to the data. For younger firms, however, this naturally leads to implications about firm’s age at the time of receiving financing. For example, one could test whether the age of startups raising Series A (first round of VC funding) covaries over time with VC fund flows, and whether older startups raising Series A round when VC funding is scarce are of higher quality and secure better terms.

6 Concluding Remarks

In this paper, I study the effects of time varying market conditions and endogenous entry on equilibrium dynamics of markets with adverse selection. In my leading application, early stage financing of high-growth firms, the variation in market conditions is driven by availability of VC and angel capital.

I analyze the dynamics of the quality of projects receiving financing, deal volume and market efficiency. In good times, when supply of VC capital is high, entrepreneurs raise funds without delay. At the same time, the average quality of funded projects is low. In bad times, when supply of VC capital is low and the discount rate of investors is correspondingly high, raising funds takes on average longer, but the average quality of funded projects is high.

Time variation in market conditions creates incentives for entrepreneurs with good projects to signal their type when investors’ discount rate is high. Signaling takes the form of delayed fund-raising, which leads to accumulation of unfunded projects in the market. Consequently,
an improvement in market conditions and deterioration of incentives to signal quality triggers a wave of fund-raising activity.

I also uncover a previously unnoticed *positive* side of illiquidity. Delay in bad times, although wasteful due to the lost time value of positive NPV projects, allows investors to partially screen entrepreneurs. Equilibrium payoff to entrepreneurs with low quality projects reflects the true value of their ideas, providing efficient entry incentives.

My model generates several empirical predictions which are broadly consistent with finding in early stage financing, IPO, private equity buyout, and secondary offering markets.
Appendix

6.1 Price Processes

Fix the probability space \((\Omega, \mathcal{F}, P)\). Let the index set be \(\mathcal{I} = (-\infty, 1] \times \mathbb{R}_+\). Index \(i = (x, t)\) stands for a potential entrepreneur \(i\) who comes up with an idea at time \(t\). Let \(\theta^i = g\) if the \(x\) coordinate of a potential entrepreneur’s index is positive and \(\theta^i = b\) if it is negative. In addition for potential entrepreneurs with bad ideas put the private cost of entry \(c^i = G^{-1}(-x)\). Define \(\text{mes}\) as an extension of the Lebesgue measure with a sigma algebra of measurable sets \(\mathcal{F}_\mathcal{I}\) such that \(((\mathcal{I} \times \Omega), \mathcal{F}_\mathcal{I} \otimes \mathcal{F}, \text{mes} \otimes P)\) is a rich Fubini extension of the usual product space.

Put \(T = \mathbb{R}_+\) to be the time indexing set. Let \(V\) be a real-valued measurable function on \(((\mathcal{I} \times \Omega) \times T), (\mathcal{F}_\mathcal{I} \otimes \mathcal{F} \times \mathcal{B}_T)\) which is a regular version of itself. Existence of such \(F\) with essentially pairwise i.i.d. processes \(V^i\) for arbitrary distribution of \(V^i\) is guaranteed by Sun (2006).

Below I will consider a particular class of processes \((V^i)_{i \in \mathcal{I}}\) satisfying Assumption 2 that will be sufficient to consider for this paper.

**Definition 4.** Let \(\{\xi^i_k\}_{k=0}^\infty\) be a sequence of independent bounded discrete random variables with finite support distributed according to \((F^i_{\xi,k})_{k=0}^\infty\) and \((t_k)_{k=0}^\infty\) be a an unbounded increasing sequence of non-negative constants. For each entrepreneur \(i\) define \((N^i_j)_{j=1}^n\) – a collection of independent (and independent from all the \(\xi\)’s) time non-homogeneous Poisson processes with deterministic intensities \((\lambda^i_{N,j}(t))_{j=1}^n\). Finally, let \((v_j(t))_{j=1}^n\) be a set of deterministic functions of finite variation. Then define \(V^i_t\) as

\[
V^i_t = \begin{cases} 
\xi^i_k, & \text{if } t = t_k; \\
\sum_{j=1}^n v_j(t)dN^i_j(t) \prod_{i \neq j} (1 - dN^i_j(t)) + \sum_{j=1}^n (1 - dN^i_j(t))v_0(t), & \text{if } t \neq t_k. 
\end{cases}
\]  

(27)

Such class of processes allows investors to play a pure strategy \((v_0)\), play mixed strategy with positive weights of mixing \((\xi\)’s), and play a mixed strategy with positive rates of mixing \((v_j\) for \(j \geq 1)\).

The private offers condition is satisfied since the Poisson processes have deterministic intensities and \(\xi\)’s are independent. The anonymity condition holds by construction due to independence of distributions \(F^i_{\xi}\)’s, times of mixing \(t_k\)’s, intensities of mixing \(\lambda^i_{N,j}\)’s, and offered prices \(v_j\)’s on the identity of the entrepreneur \(i\).
6.2 Proofs

Proof of Lemma 1. Consider any equilibrium and denote $W^i_t$ continuation value of entrepreneur $i$ present in the market at time $t$. Since investors do not observe previous offers and each entrepreneur is atomistic continuation value does not depend on the private history $H^i_t$ (value of offers made before or at time $t$), it depends only of the type of the project $\theta^i$ and calendar time $t$. Thus, any offer $v$ with $B_\theta(v)$ above $W^\theta_t$ will be accepted by entrepreneur with $\theta$ theta quality project and any valuation $v$ with $B_\theta(v)$ below $W^\theta_t$ will be rejected. □

Proof of Lemma 2. Recall that continuation value of entrepreneur with good quality project is given by

$$W^g_t = \sup_{F} E \left[ S_g + \int_t^\infty e^{-r(\tau-t)}(B_g(V_\tau) - S_g)dF_\tau \mid \tau > t \right]. \quad (28)$$

Index $i$ can be omitted since continuation value, expected future valuations and, hence, time of funding do not depend on the identity of the seller. Rewrite then

$$W^g_t = \sup_{F} E \left[ S_g + \int_t^\infty e^{-r(\tau-t)}(B_g(V_\tau) - S_g)dF_\tau \mid \tau > t \right]$$

$$= E \left[ S_g + \int_t^\infty e^{-r(\tau-t)}(B_g(V_\tau) - S_g)dF^*_\tau \mid \tau^* > t \right]$$

$$= E \left[ \int_t^\infty e^{-r(\tau-t)}B_g(V_\tau) + S_g(1 - e^{-r(\tau-t)})dF^*_\tau \mid \tau^* > t \right]$$

$$> E \left[ \int_t^\infty e^{-r(\tau-t)}B_b(V_\tau) + S_g(1 - e^{-r(\tau-t)})dF^*_\tau \mid \tau^* > t \right]$$

$$= E \left[ S_b + \int_t^\infty e^{-r(\tau-t)}B_b(V_\tau) - S_b)dF^*_\tau \mid \tau^* > t \right]$$

$$\geq \sup_{F} E \left[ S_b + \int_t^\infty e^{-r(\tau-t)}(B_b(V_\tau) - S_b)dF_\tau \mid \tau > t \right]$$

$$= W^b_t.$$  

The strict inequality follows from $B_g(v) > B_b(v)$ and $S_g > S_b$ while the second inequality follows from the fact that $F^*$ could be suboptimal for strategy for entrepreneur with bad project. □

Proof of Lemma 3. Suppose the contrary, i.e. that entrepreneur with good project quality accepts an offer $v$ with probability less than one. Since $W^g_t > W^b_t$ all entrepreneurs with bad projects will accept valuation $v.$
Zero profit condition implies that
\[ v = \frac{\delta_X}{\delta_X + r} E \left( X_{\theta^i} \mid V^i = v, \tau^i = t \right) < \frac{\delta_X}{\delta_X + r} (\pi_t X_g + (1 - \pi_t) X_b). \] (29)

Inequality above is precisely due to mixing of entrepreneurs with good projects.

Since entrepreneur with good project type is indifferent between accepting and not
\[ W_t^g = B_g(v) < B_g \left( \frac{\delta_X}{\delta_X + r} (\pi_t X_g + (1 - \pi_t) X_b) \right). \] (30)

Which violates Market Clearing condition. \qed

\textit{Proof of Corollary 1}. Easily follows from Lemmas 1-3 and Zero Profit condition. \qed

\textit{Proof of Proposition 1}. In the steady state equilibrium average quality of the assets in the market \( \pi_t \) is constant over time, which implies that \( \pi_t \) can never be below the proportion of good project entering the market.

Suppose that \( \pi_t > \hat{\pi} \). If it takes time to raise funds at a pooling valuation, then Market Clearing condition is violated since
\[ W_t^g < B_g \left( \frac{\delta_X}{\delta_X + r} (\pi_t X_g + (1 - \pi_t) X_b) \right). \] (31)

If there exists a steady state equilibrium with \( \pi_t > \hat{\pi} \) then it has to feature immediate acceptance of pooling offer. Since \( \pi_t \geq 1/(1 + G(c)) \) the unique equilibrium with \( 1/(1 + G(c)) > \hat{\pi} \) features immediate pooling.

Suppose that \( \pi_t < \hat{\pi} \). This implies that entrepreneurs with good projects never invest prior to information arrival, i.e. \( W_t^g = S_g \). But
\[ S_g = B_g \left( \frac{\delta_X}{\delta_X + r} (\hat{\pi}_t X_g + (1 - \hat{\pi}_t) X_b) \right) < B_g \left( \frac{\delta_X}{\delta_X + r} (\pi_t X_g + (1 - \pi_t) X_b) \right), \] (32)

which violates Market Clearing condition. Thus, \( \pi_t < \hat{\pi} \) is impossible.

When \( 1/(1 + G(c)) < \hat{\pi} \) the only possibility for \( \pi_t \) is to be equal to \( \hat{\pi} \). In order for \( \pi_t \) to stay constant separating offer has to be accepted, hence \( W_t^b = S_b \). Pooling offers need to be accepted as well, otherwise \( \pi_t = 1 \), thus \( W_t^g = B_g \left( \frac{\delta_X}{\delta_X + r} (\hat{\pi}_t X_g + (1 - \hat{\pi}_t) X_b) \right) \).

In order to keep \( W_t^g \) and \( W_t^b \) constant expected time to observing a pooling offer for individual entrepreneur has to be constant over time, i.e. distribution has to be memoryless.
Exponential distribution is the only one satisfying this condition. Pooling offer arrives with intensity $\lambda_p$ which solves

$$(r + \lambda_p + \delta_I)S_b = \lambda_p \left( \frac{\delta X}{\delta X + r} (\hat{\pi}_t X_g + (1 - \hat{\pi}_t)X_b) \right). \tag{33}$$

Continuum of equilibria with $1/(1 + G(c)) = \hat{\pi}$ can be constructed in the following way: pick any $\lambda > \lambda_p$ and put it to be the intensity of arrival of pooling offer for each entrepreneur, otherwise only separating offers are made. This price path together with entrepreneurs accepting offers no worse than the pooling one and entrepreneurs accepting offers only better than separating ones constitutes an equilibrium.

\[ \square \]

**Proof of Proposition 2.** First notice that $B_b(\alpha)$ is continuous and decreasing in $\alpha$ with

$$\frac{1}{1 + G(B_b(0))} > 0 \quad \frac{1}{1 + G(B_b(1))} < 1, \tag{34}$$

hence there exists a unique $\hat{\alpha}$ that solves $1/(1 + G(B_b(\alpha))) = \alpha$.

If $\hat{\alpha} > \hat{\pi}$, then $\alpha^* = \hat{\alpha}$ and uniqueness of equilibrium is guaranteed by Proposition 1.

If $\hat{\alpha} < \hat{\pi}$ and $1/(1 + G(B_b(0))) < \hat{\pi}$, then $\alpha^* = 1/(1 + G(B_b(0)))$ and uniqueness of equilibrium is guaranteed by Proposition 1.

Otherwise $\alpha^* = \hat{\pi}$. Continuation value of entrepreneurs with low quality assets solves

$$\frac{1}{1 + G(W^b_t)} = \hat{\pi}, \tag{35}$$

which pins down the arrival rate of the pooling offer $\lambda$:

$$(\lambda + \delta_I + r)W^b_t = \delta_I B_b(0) + \lambda B_b(\hat{\pi}). \tag{36}$$

\[ \square \]

**Proof of Proposition 4.** With a increase in $r$ the lemons condition threshold $\hat{\pi}$ is increasing, moreover, the whole curve $f(\pi) = \frac{1}{1 + G(B_b(\pi))}$ shifts up since for fixed $\pi$ $B_b(\pi)$ is decreasing in $r$.

\[ \square \]

**Proof of Proposition 5.** When $I \left( \frac{X_g}{X_b} - 1 \right) < \frac{\delta}{\delta_p} (X_g - I) \hat{\pi}$ then $\hat{\pi}$ that solves (14) is negative for $r = 0$, implying that lemons condition is not binding regardless of the market belief.
Proposition 2 ensures that the unique equilibrium in this case is immediate pooling and a fraction of good projects entering the market $\alpha^*_p=0$ is positive. Put $\underline{r}$ to be a solution of

$$\hat{\pi}|_{\underline{r}=\underline{r}} = \alpha^*|_{r=0}. \quad (37)$$

Then for all $\underline{r} <\underline{r}$ lemons condition threshold $\hat{\pi}|_{\underline{r}} < \hat{\pi}|_{\underline{r}=\underline{r}} = \alpha^*|_{r=0} < \alpha^*|_{\underline{r}}$.

If $\frac{\rho}{\delta+\rho} \left( \frac{\delta X}{\delta X+\rho} X_g - I \right) < I \left( \frac{\lambda X}{X_b} - 1 \right)$ then $\hat{\pi}|_{r=\rho} \in (0, 1)$. Additionally, the restriction on $G$ implies that $\alpha^*|_{r=\rho}$ is strictly below $\hat{\pi}|_{r=\rho}$. Define $\tau$ as a solution to

$$\hat{\pi}|_{\tau}=\alpha^*|_{r=\rho}. \quad (38)$$

Clearly $0 < \underline{r} < \tau < \rho$ and for all $r \in (\tau, \rho)$ the following chain of inequalities holds $\alpha^*|_r < \alpha^*|_{r=\rho} = \hat{\pi}|_{\tau=\tau} < \hat{\pi}|_r$ since both $\alpha^*$ and $\hat{\pi}$ are strictly increasing in $r$. \qed

Proof of Proposition 6. Assume that in times when $Y_\ell = 1$ there is immediate acceptance of the pooling offer, hence, continuation values for entrepreneurs seeking funding is

$$W^g_t(1) = B_g \left( D_X(1)(\alpha^*_p X_g + (1-\alpha^*_p) X_b) \right) \quad W^b_t(1) = B_b \left( D_X(1)(\alpha^*_p X_g + (1-\alpha^*_p) X_b) \right). \quad (39)$$

And suppose that in times $Y_\ell = 0$ pooling valuation $D_X(1)(\pi^* X_g + (1-\pi^*) X_b)$ arrives with intensity $\lambda_\pi$. Partial pooling outcome implies continuation values are given by

$$W^b_t(0) = B_b \left( D_X(0) X_g \right) \quad W^g_t(0) = B_g \left( D_X(0)(\pi^* X_g + (1-\pi^*) X_b) \right). \quad (40)$$

I will now prove that for sufficiently small $\lambda_1$ this can be sustained as an equilibrium. First define $\pi^*$ as a solution to

$$(\rho + \lambda_1 + \delta) B_g \left( D_X(0)(\pi^* X_g + (1-\pi^*) X_b) \right) = \lambda_1 B_g \left( D_X(1)(\pi^* X_g + (1-\pi^*) X_b) \right) + \delta B_g(V_g).$$

For small $\lambda_1$ the solution $\pi^*$ is well defined (i.e. it is less than 1), moreover, if $\lambda_1 = 0$ then $\pi^* = \alpha^*|_{r=0}$, otherwise $\pi^* > \alpha^*|_{r=0}$. Now, define $\lambda_\pi$ as a solution to

$$(\rho + \lambda_1 + \lambda_\pi) B_b(V_b) = \lambda_1 B_b \left( D_X(1)(\pi^* X_g + (1-\pi^*) X_b) \right) + \lambda_\pi B_b \left( D_X(0)(\pi^* X_g + (1-\pi^*) X_b) \right).$$

Existence of $\lambda_\pi > 0$ for small $\lambda_1$ is guaranteed by the fact that $\hat{\pi}|_{r= ‚0} > \alpha^*|_{r=0}$. These two equation make sure that in the state $Y_\ell = 0$ pooling offer is accepted by entrepreneurs with good and bad projects and entrepreneurs with bad projects are indifferent between
waiting for a pooling valuation (or information revelation) and accepting a separating offer immediately. Clearly Zero Profit and Market Clearing condition is satisfied in this state.

I now need to make sure that immediate pooling is equilibrium in state \(Y_t = 1\), given the continuation values for state \(Y_t = 0\). Since the price of pooling is not moving over time, the only potentially profitable deviation for entrepreneurs is wait all the way until \(Y_t = 0\). This yields expected payoff

\[
\frac{\delta}{\delta + \lambda_0 + \rho} B_g() + \frac{\lambda_0}{\delta + \lambda_0 + \rho} W_t^g(0) \quad \text{and} \quad \frac{\delta}{\delta + \lambda_0 + \rho} B_b(V_b) + \frac{\lambda_0}{\delta + \lambda_0 + \rho} W_t^b(0)
\]

for the entrepreneurs with good and bad projects respectively. Conjectured \(W_t^b(1)\) is higher than the pay-off from such deviation since it is higher component-wise. When \(\lambda^1 = 0\) then \(W_t^g(0)\) simply becomes \(\delta/(\delta + \rho)B_g(D_X(1)X_g)\), hence the pay-off to deviation for the owner of the good project is less than \(\delta/(\delta + \rho)B_g(V_g)\). Since \(\alpha^*\big|_{r=r(1)} > \hat{\pi}\big|_{r=r(1)}\) the following inequality must hold

\[
W_t^g(1) = B_g\left(D_X(1)(\alpha^*_1X_g + (1 - \alpha^*_1)X_b)\right) > \frac{\delta}{\delta + \rho} B_g(D_X(1)X_b), \quad (41)
\]

implying that \(W_t^g(1)\) is still higher then the pay-off from deviation from sufficiently small \(\lambda^1\). Zero Profit and Market Clearing conditions are trivially satisfied in \(Y_t = 1\) given the \(W_t^g(1)\).

Proof of Propositions 8, 7, and 9. The entry threshold \(c_t\) is determined by

\[
c_t \equiv c_t^* = B_b\left(D_X(1)(\alpha^*_1X_g + (1 - \alpha^*_1)X_b)\right) \quad \text{when} \ Y_t = 1 \quad (42)
\]

and

\[
c_t \equiv c_t^0 = B_b\left(D_X(0)X_b\right) \quad \text{when} \ Y_t = 0 \quad (43)
\]

Clearly, \(c_t^0 < c_t^*\) since \(D_X(0) < D_X(1)\) and \(\alpha^*_1 \in (0, 1)\), implying that quality of entry is worse in the \(Y_t = 1\) state.

When the state lasts for sufficiently long, the the average quality of funded projects is close to the average quality of projects entering the market, implying the statement of Proposition 8.

When \(Y_t = 0\) funds are raised at both pooling and separating valuations. Denote by \(q_t^0\)
and $q_t^s$ rates of fund-raising at separating and pooling valuations respectively, then

$$
\dot{m}_t^g = 1 - q_t^s \cdot \pi_t \quad \dot{m}_t^b = G(c_t) - q_t^s \cdot (1 - \pi_t) - q_t^0 \quad \text{when } Y_t = 0 \quad (44)
$$

Since $\pi_t$ is equal to $\pi^*$ when $Y_t = 0$, $G(c_t) - q_t^0 = \frac{1-\pi_t}{\pi^*}$. Each entrepreneur $i$ raises funds at the pooling valuation with intensity $\lambda_p$, thus, the rate of deals at this valuation is $q_t^p = \lambda_p (m_t^g + m_t^b)$. Plugging the values for $q_t^0$ and $q_t^p$ reveals that

$$
\dot{m}_t^g = 1 - \lambda_p m_t^g > 0 \quad \dot{m}_t^b = \frac{1-\pi^*}{\pi^*} - \lambda_p m_t^b > 0 \quad \text{when } Y_t = 0, \quad (45)
$$

implying that there is accumulation of unfunded projects in the market over time. When discount rate falls back to $r(1)$ immediate pooling implies at atom of deals,

The average quality of projects funded in the atom is $\pi^*$ which is higher than $\alpha_0^*$ by construction of equilibrium, hence, the statement of Proposition 7 follows.

Proof of Proposition 9 is given in the main text.
References


