Informed Trading and Intertemporal Substitution:
The Limits of the No-Trade Theorem

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Abstract

I examine the conditions for the no-trade theorem to hold in multiperiod consumption settings and show it no longer holds in many reasonable scenarios. In situations where agents have different concerns for intertemporal substitution, information-based trade can be mutually acceptable because it enables agents to readjust their consumption profiles based on future consumption shocks. I show that the existing literature that finds no-trade results in various multiperiod consumption settings crucially depends on specific preference assumptions that lead to risk aversion dominating concerns for intertemporal substitution. The no-trade theorem fails to hold when a wider range of utility functions with a more important role for intertemporal substitution are considered. Intertemporal substitution bridges information-based trading and consumption-based asset pricing. Consumption-based asset pricing models are natural candidates to analyze information-based trading, and information-based trading affects the volatility of individual consumption processes. Quantitative analysis demonstrates that besides asset pricing implications, information-based trading related to intertemporal consumption smoothing can also explain a significant part of the trading volume observed in financial markets.

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1 Introduction

Every financial market shares two fundamental features: price and trading volume. Though tons of studies focus on asset prices, little is known about trading volume. In the real world we observe significant volumes of trade in markets everyday. Figure 1 shows that the trading volume for NYSE listed stocks is comparable to the U.S. total consumption. Many studies also document various interesting trading patterns associated with asset returns or information (Morse, 1981; Lee and Swaminathan, 2000). This high level of trading is difficult to explain solely in terms of liquidity motives. This unexplained trading has been an ongoing puzzle in both finance and economic research.

![Figure 1: Trading Volume in NYSE Listed Stocks and U.S. Final Consumption Expenditure](image)

Figure 1: Trading Volume in NYSE Listed Stocks and U.S. Final Consumption Expenditure

One important reason that we know little about trading volume is the no-trade theorem. The no-trade theorem in the seminal paper by Milgrom and Stokey (1982) shows that when the market is state-contingent complete,¹ and the initial allocation is Pareto optimal, the addition of private information will not generate trade among rational agents. The no-trade theorem denies the possibility of analyzing information-based trading in a rational framework.

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¹In the no-trade literature, the state of the world can be decomposed into two components. Payoff-relevant events comprise one of the components and include events that affect agents’ endowments or tastes directly. The other component is based on signal or information events, which do not affect endowments or tastes directly but may be statistically related to payoff-relevant events. In this paper, the state-contingent complete market is defined as the market that is complete across all payoff-relevant events.
Possibly the most natural way to test the robustness of results in a single period consumption model is to consider its multiperiod consumption extensions. For the no-trade theorem, however, dynamic consumption extensions are widely believed to be trivial. Since consumption in different periods can be treated as different goods in agents’ consumption bundles, common sense suggests that any multiperiod extension should be isomorphic to a corresponding single period case when the market is state-contingent complete. Several studies (Tirole, 1982; Judd et al., 2003; Maurer and Tran, 2015) support this intuition, showing that the no-trade result is robust to various dynamic consumption extensions. Researchers provide many explanations for trades, including explanations involves “irrational trading” by noisy traders (Grossman and Stiglitz, 1980; Glosten and Milgrom, 1985; Kyle, 1985) or traders with heterogeneous beliefs (Harrison and Kreps, 1978; Morris, 1994). Explanations for rational trading are often based on market incompleteness across payoff-relevant states (Blume et al., 2006; Bond and Eraslan, 2010; Gottardi and Rahi, 2013; Lubensky and Smith, 2014; Edmans et al., 2015; Goldstein and Yang, 2015). Dynamic consumption has generally not been regarded as a potential source for information-based trading.

In this paper I study information-based trading in settings with multiperiod consumption. I show that dynamic consumption extensions are non-trivial. I also find that information generates trades with dynamic consumption profiles for many reasonable scenarios, primarily because dynamic consumption extensions change the structure of a state-contingent complete market. The nature of information filtration in the context of dynamic consumption world suggests that no security can pay different amounts of current consumption based on future payoff-relevant states being realized. In the single period consumption context, agents can adjust their consumption of different goods through ex ante trading when the market is state-contingent complete. In the multiperiod consumption scenario, even when the market is state-contingent complete, agents cannot achieve optimal current and future consumption levels through ex ante trading. Information-based trading is attractive because it enables agents to readjust their consumption profiles based on additional information about future payoff-relevant states. In other words, information-based trading can enrich the possible consumption reallocation space, and this expansion is Pareto-improving because agents want to smooth consumption.

Previous studies find that the no-trade result is robust to various dynamic consumption extensions (Tirole, 1982; Judd et al., 2003; Maurer and Tran, 2015). To understand why those studies fail to generate information-based trading, I discuss a dynamic version of the no-trade
theorem and show that all previous dynamic no-trade results implicitly rely on some more restrictive assumptions. Compared to Milgrom and Stokey (1982), which assumes concordant beliefs and concave utility functions, this new version of the no-trade theorem requires more restrictive conditions on both beliefs and preferences. The new assumption about beliefs can be interpreted as a natural extension of the original concordant beliefs assumption. The new preference condition, the risk-aversion-dominating condition, requires that risk aversion concerns dominate desires for intertemporal substitution. This assumption puts a significant restriction on preferences. Although this restriction on preferences is severe, many popular utility functions used in economics and finance research actually adhere to this restriction.

The concavity of the utility function determines both risk aversion and intertemporal substitution. Strong degrees of risk aversion relative to desires for intertemporal substitution support the no-trade result because agents with such preferences are focused more on ex ante trading to limit risk rather than dynamic trading to smooth consumption. Conversely, strong concerns with intertemporal substitution lead to dynamic trading because agents readjust their consumption profiles based on information about future consumption. In the single period consumption world, concave utility function only represents risk aversion. In the multiperiod consumption case, both risk aversion and intertemporal substitution are present. The risk-aversion-dominating condition assumes that for agents any arbitrary one-period risk aversion always dominates the need for intertemporal substitution. No reason exists for believing that this condition is a reasonable description of human behavior. However, I show that many popular utility functions used in economics and finance satisfy this condition. Previous work finds that no-trade results robust in various dynamic consumption settings, but this is because those studies all assume one or more of standard utility functions for tractability reasons. Since those utility descriptions fail to capture all potential effects of intertemporal substitution, the no-trade results that have been found before are special and do not generalize to situations involving less restrictive assumption on preferences.

The introduction of multiperiod consumption and intertemporal substitution bridges information-based trading and consumption-based asset pricing. The effect of consumption smoothing in dynamic consumption worlds have been widely studied in consumption-based asset pricing models, such as habit formation (Constantinides, 1990; Campbell and Cochrane, 1999) and heterogeneous agent (Guvenen, 2006, 2009; Garleanu and Panageas, 2015). Since intertemporal substitution,
the source of potential Pareto improvement, is the key reason to generate retrade, consumption-based asset pricing models are natural candidates to analyze information-based trading. In this paper, based on a rare disaster environment similar to Barro (2006), I study a simple two period example with habit formation preference and a dynamic heterogeneous agents model similar to Guvenen (2009). Both models show that the concern for potential rare disaster events alone can generate around 15%−35% of total consumption as the trading volume. Information-based trading also endogenously affects agents’ individual consumption processes. In the dynamic example I show that higher values on risk aversion parameters do not necessarily result in a higher equity premium. Similar to Garleanu and Panageas (2015), this result comes from the fact that agents’ consumption processes are endogenous. When agents are more risk averse, information-based trading becomes less attractive because it introduces additional uncertainty on consumption. In the equilibrium, the trading volume decreases, and each individual’s consumption process becomes less volatile.

Milgrom and Stokey (1982) and Blume et al. (2006) show that the no-trade result is robust to whether information is publicly or privately observed and whether the equilibrium concept is market competitive or game theoretic equilibrium concept. Based on a simple example with information-based trading and public information, I show that for the corresponding private information case, in the Perfect Bayesian equilibrium traders may strategically reveal their private information, leading to a different equilibrium outcome.

The paper is organized as follows. Section 2 reviews the literature. Section 3 compares two simple examples to illustrate the basic intuition. Section 4 introduces the model, and section 5 discusses the difference between the market structures in the single period and the multiperiod consumption setups. Section 6 proposes a dynamic version of no-trade theorem and section 7 discusses the risk-aversion-dominating condition. Section 8 presents the quantitative analysis. Section 9 investigates whether information being publicly or privately observed will affect the equilibrium outcome when agents are strategic players. The final section concludes.

2 Literature Review

The literature on no-trade result goes back to Aumann (1976) who introduces a mathematical formulation for the concept of common knowledge. Following Milgrom and Stokey (1982), a series of no-trade theorems overrule the commonsense intuition of information-based trading
(Kreps, 1977; Grossman and Stiglitz, 1980; Hakansson et al., 1982; Tirole, 1982; Holmstrom and Myerson, 1983; Fudenberg and Levine, 2005; Blume et al., 2006). Rubinstein and Wolinsky (1990) provides a unified analysis for “agreeing to disagree” type results, including no-trade theorems. I generalize the no-trade theorem to multiperiod consumption settings and show the theorem no longer holds for many reasonable scenarios.

Several studies (Tirole, 1982; Judd et al., 2003; Maurer and Tran, 2015) find that the no-trade result is robust to various dynamic consumption extensions. I show that their no-trade results crucially depends on the fact that risk aversion dominates intertemporal substitution in the specific utility functions they use. The no-trade theorem fails to hold when a wider range of utility functions with a more important role for intertemporal substitution are considered.

The paper contributes to the literature on information-based trading. Most explanations for information-based trades involves “irrational trading”. Following Grossman and Stiglitz (1980), Glosten and Milgrom (1985) and Kyle (1985), the vast majority of papers studying financial markets introduce noise traders to generate trade. Other studies follow Harrison and Kreps (1978) and Morris (1994) and assume traders have heterogeneous beliefs. My paper provides a rational channel to generate information-based trade.

Existing explanations for rational trading are based on market incompleteness across payoff-relevant states (Blume et al., 2006; Bond and Eraslan, 2010; Gottardi and Rahi, 2013; Lubensky and Smith, 2014; Edmans et al., 2015; Goldstein and Yang, 2015). In contrast to existing market incompleteness literature that focus on insurance opportunities, I introduce intertemporal substitution as the new source of potential Pareto improvement. While insurance opportunities exhaust when the market converges to the state-contingent complete one, the nature of time and information filtration guarantee that the effect of intertemporal substitution is still valid even when the market is state-contingent complete. In terms of quantitative results, since there is no good way to measure market incompleteness, market incomplete models often find it difficult to produce quantitative predictions. In this paper I show that quantitative analysis is possible for multiperiod extensions and my model can explain a significant part of the trading volume we observe in financial markets.

This paper is related to the literature on information value. Blackwell (1953) first discusses the welfare effect of early information releases. Hirshleifer (1971) argues that too early information releases reduce risk-sharing. Orosel (1996), Schlee (2001), Gottardi and Rahi (2014) and
Maurer and Tran (2015) extend the results of Hirshleifer (1971). My paper contributes to the literature by introducing the benefit of intertemporal substitution, and I show that information is valuable even when the market is state-contingent complete.

The quantitative analysis in this paper involves three strands of asset pricing literature, habit formation (Constantinides, 1990; Campbell and Cochrane, 1999), rare disaster (Rietz, 1988; Barro, 2006) and heterogeneous agent model (Guvenen, 2006, 2009; Garleanu and Panageas, 2015). I show that besides asset pricing predictions, those models also have the potential to analyze trading volume. My analysis also relates to the literature on allocations in heterogeneous agent models (Kan, 1995; Dumas, 1989; Dumas et al., 2000; Borovicka, 2015).

The trading result in my paper crucially relies on the fact that agents can not “travel” through time. Motivated by the same observation, Grenadier et al. (2015) study how organizations make timing decisions. In their model the inability to go back in time creates an implicit commitment device for the principal to follow the advisor’s recommendations and thereby improves communication. In my paper the inability to travel to the future shapes the structure of the market, affecting the space of feasible ex ante allocations.

My discussion on preferences is related to the literature on time non-additive preferences (Kreps and Porteus, 1978; Epstein and Zin, 1989; Weil, 1990). The consumption allocation and preferences has been widely studied in macroeconomics (Lucas and Stokey, 1984; Kan, 1995; Judd et al., 2003; Anderson, 2005; Berradaa et al., 2007). With risk-sensitive preferences formulated by Hansen and Sargent (1995), Anderson (2005) studies a social planner’s optimal allocation problem. Necessary and sufficient conditions are given for the existence and stability of steady states at which Pareto weights are time-invariant. My paper studies whether the Pareto optimal allocation is independent of signal states. Of course, Pareto weights being time-invariant is a (super strong) sufficient condition for the no-trade result to hold.

3 Two Examples

In this section I construct two simple examples to provide some basic intuition of the information-based trade result.

Consider a pure exchange economy with four dates \( \{0, 0.5, 1, 2\} \) and two agents \( \{A, B\} \). The state of the world consists of two components: two signal events \( \{x^1, x^2\} \) and two payoff-relevant events \( \{\theta^1, \theta^2\} \). Endowments only depend on payoff-relevant states, and signal states
do not affect them directly. However, $x^k$ and $\theta^x$ may be statistically related. The signal state is publicly observed at period 0.5, and the payoff-relevant state is realized at period 2. There are two consumption goods $\{C_1, C_2\}$. The market is state-contingent complete; that is, it is complete across $\{\theta^1, \theta^2\}$. Without loss of generality, agents can trade at time 0 before information arrival, and the market will reopen at time 0.5 afterward. The discount factor is normalized to 1.

The first example describes a single period consumption world:

**Example 1** (Single Period Consumption Case). *Suppose that agents consume both $\{C_1, C_2\}$ in the final period 2.*

Example 1 fits the model in Milgrom and Stokey (1982). With mild assumptions on agents’ beliefs and utility functions, Milgrom and Stokey (1982) show that there would be zero retrade at time 0.5.

The second example describes a dynamic consumption world:

**Example 2** (Multiperiod Consumption Case). *Suppose that agents consume $C_1$ at time 1 and $C_2$ in the final period 2.*

It is natural to think that this dynamic consumption extension would not change the no-trade result when the market is state-contingent complete because consumption at different times can be treated as different goods in the consumption bundle. However, this argument ignores that the market structure for the state-contingent complete market may also change in the multiperiod consumption case.

Table 1 shows the securities available in each example when the market is state-contingent complete. $S_{C_i}^{\theta^k}$ is a security that pays 1 unit of consumption good $C_i$ given that $\theta^k$ is realized. In example 2, agents consume goods $C_1$ at time 1. For any security, its current payoff can not
be conditional on states realized in the future. In this multiperiod consumption case, even when the market is state-contingent complete, the security for $C_1$ is independent of the realization of payoff-relevant events $\{\theta^1, \theta^2\}$.

This subtle difference in market structure affects the ex ante trade at time 0. Suppose in the single period consumption case after ex ante trade that agent $A$’s consumption allocation is $(c_1, c_2) = (1, 1)$ when $\theta^1$ is true and $(c_1, c_2) = (2, 2)$ when $\theta^2$ is realized. It is then impossible to reach the same consumption allocation through ex ante trade in the dynamic consumption case. To be more specific, because only one security exists for $C_1$, agent $A$ cannot condition her consumption of $C_1$ in different payoff-relevant states through ex ante trades. Formally, any consumption profile with $c_1(\theta^1) \neq c_1(\theta^2)$ is not feasible in the dynamic consumption case, although it may be feasible in the single period consumption case.

The restriction $c_1(\theta^1) = c_1(\theta^2)$ on the ex-ante allocations affects the retrade opportunity at time 0.5 after information arrival. For simplicity, assume the signal arrives at time 0.5 and perfectly reveals the true payoff-relevant state. After agents learn the signal, they can trade conditional on the information; that is, they can agree to make different trades given different realizations of $\theta^i$. In example 2, by retrading after the arrival of information, agent $A$ is now able to reach consumption profiles with $c_1(\theta^1) \neq c_1(\theta^2)$. That is, retrades at time 0.5 enable agents to reach allocations that they cannot reach through ex ante trade at time 0.

In the dynamic consumption case, information-based trade at time 0.5 expands the set of possible consumption allocations. This implies that information-based trade may also change the Pareto frontier. In contrast to existing literature (Blume et al., 2006) that emphasizes insurance opportunities, the source of potential Pareto improvement here is intertemporal substitution. Agents find information-based trades mutually acceptable because they enable agents to correlate their time 1 consumption with their time 2 consumption, and thus smooth their consumption over time.
These two examples illustrate the basic intuitions of the information-based trade result. For simplicity, most examples in the following analysis are based on example 2.

4 The Model

This section presents the model setup, for which I follow the notation used in Milgrom and Stokey (1982) whenever possible. Consider a pure exchange economy with $N$ traders and $T$ periods, where $T$ is finite. Let $\Omega$ be the set of finite possible states of the world, with generic element $\omega$. For convenience they can be rewritten as $\Omega = \Theta \times \mathcal{X}$ and $\omega = (\theta, x)$. Use $\Theta$ to denote the set of payoff-relevant events, and let $\mathcal{X}$ be the set of payoff-irrelevant events. Endowments and utility functions may depend on $\theta$, but they are not affected by $x$. Although $x$ does not affect endowments or utility functions directly, it may be statistically related to $\theta$ and can be interpreted as a signal about $\theta$. Let $\pi^i(\omega)$ be agent $i$’s subjective belief about the probability of state $\omega$. I assume $\pi^i(\omega) > 0$ for every $i$ and every $\omega$.

At each period $t$, agent $i$ receives some private information, denoted as event $f^i_t$ of partition $\mathcal{F}^i_t$ of set $\Omega$. Given the construction of $\Omega$, one can also rewrite the information structure as $\mathcal{F}^i_t = \Theta_t \times \mathcal{X}^i_t$ and $f^i_t = (\theta_t, x^i_t)$, where $\Theta_t$ is a partition of set $\Theta$ and $\mathcal{X}^i_t$ is a partition of set $\mathcal{X}$. It is natural to assume that partition $\mathcal{F}^i_t$ becomes finer over time, suggesting $\mathcal{F}^i_t \subseteq \mathcal{F}^i_{t+1}$, $\Theta_t \subseteq \Theta_{t+1}$ and $\mathcal{X}^i_t \subseteq \mathcal{X}^i_{t+1}$. The filtration $\mathcal{F}^i = \{\mathcal{F}^i_1, \ldots, \mathcal{F}^i_T\}$ describes agent $i$’s information process, and $\Theta^i = \{\Theta_1, \ldots, \Theta_T\}$, $\mathcal{X}^i = \{\mathcal{X}^i_1, \ldots, \mathcal{X}^i_T\}$ represent dynamic payoff-relevant and payoff-irrelevant information processes for her, respectively. Let $\mathcal{R}_t$ describe the meet of $\mathcal{F}^1_t, \ldots, \mathcal{F}^N_t$, which is the set of common knowledge at period $t$, and the filtration $\mathcal{R} = \{\mathcal{R}_1, \ldots, \mathcal{R}_T\}$ represents how common knowledge is revealed through time.

In this economy, there would be $l \equiv \sum_{t=1}^T l_t$ commodities. At time $t$, there are $l_t$ commodities, and each trader $i$ receives his endowment $e^i_t : \Theta_t \rightarrow R^l_t$. I assume no storage technology in this pure exchange economy.

A trade $z = (z^1, \ldots, z^N)$ is a function from $\Omega$ to $R^{NL}_+$, where $z^i(\omega)$ is agent $i$’s net trade of $l$ commodities in state $\omega_i$, and $z^i_t(\omega)$ is her net trade of $l$ commodities in state $\omega$ at period $t$. Since agents can only trade based on the information they have at the time, and trades are common

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2In a more general case, agents may have different partitions $\Theta^i_t$. This paper focuses on the case with a state-contingent complete market. The state-contingent market implies a common partition $\Theta_t$ for all traders.
knowledge, \( z_i^t \) must be a function measurable with respect to \( \mathcal{R}_t \). \( z \) is feasible if it satisfies

\[
e^i(\theta) + z^i(\theta, x^i) \in \mathbb{R}_+^l, \quad \forall \, i, x^i, \theta
\]

\[
\sum_{i=1}^N z^i(\theta, x^i) \in \mathbb{R}_-^l, \quad \forall \, x^i, \theta
\]

Agents should find the trade contract mutually-acceptable when they enter into the contract, but the realized trade process may not be mutually acceptable in every period. For example, traders may find trading a European call option contract mutually acceptable, but it is not mutually acceptable when the buyer wants to exercise the option.

At each time \( t \), agent \( i \)'s consumption is determined by \( c^i_t = e^i_t + z^i_t \in \mathbb{R}_+^l \). Let \( C^i_t = \{c^i_t, c^i_{t+1}, \ldots, c^i_T\} \) describes agent \( i \)'s consumption process starting at time \( t \).

Agent \( i \)'s utility function \( U^i(\theta, C^i_t) \) is a mapping from \( \Theta \times \mathbb{R}_+^l \) to \( \mathbb{R} \). It is assumed that \( U^i(\theta, \cdot) : \mathbb{R}_+^l \rightarrow \mathbb{R} \) is increasing and concave in \( C^i_t \) for all \( i, \theta \) and \( t \). \( U^i \) is not necessarily an expected utility function. For convenience I use \( E\{U^i|\mathcal{F}^i_t\} \) throughout the paper, but one can always replace the expected term with \( U^i(C^i_t) \) in the case of non expected utility functions.

Traders update their subjective beliefs about \( \omega \) according to \( \{\mathcal{F}^i_0, \mathcal{F}^i_1, \ldots\} \). Following Milgrom and Stokey (1982), traders’ beliefs are concordant, defined as

\[
\pi^1(x^i|\theta) = \pi^2(x^i|\theta) = \cdots = \pi^N(x^i|\theta), \quad \forall \, x^i, \theta
\]

Concordant beliefs condition says that conditional on the pay-off relevant state \( \theta \), agents agree on the distribution of signal \( x \). This is a more general condition than the common prior assumption.

## 5 Market Structure in the Dynamic World

In section 3 I used two examples to show how the evolution of \( \Theta_t \) and consumption opportunities cause a subtle but important change in the market structure in the dynamic consumption world. This section provides a formal analysis of the effects of market structure change.

The original no-trade theorem builds on the concept of \( \theta \)-trade, which is defined as

**Definition 1.** A trade is called a \( \theta \)-trade if it is a function from \( \Theta \) to \( \mathbb{R}_+^N \).

To understand why information-based trades emerge in the multiperiod consumption world,
I first present a dynamic version of Milgrom and Stokey’s no-trade theorem.

**Proposition 1.** Suppose that all traders are weakly concave in $C_i^t$, that the initial allocation $e = (e^1, \ldots, e^N)$ is Pareto-optimal relative to $\theta$-trades, that their beliefs are concordant, and that each agent $i$ observes the private information conveyed by the filtration $\{F^i_t\}$. At any time $t$, if it is common knowledge at $\omega$ that $z$ is a feasible trade and that each trader weakly prefers $z$ to the zero trade, then every agent is indifferent between $z$ and zero trade. If all agents are strictly concave in $C_i^t$ then $z$ is the zero trade.

**Proof.** See Appendix A. \qed

The proof is essentially the same as Milgrom and Stokey’s original one. This theorem states that any Pareto-superior information-based trade can be done ex-ante by $\theta$-trades. It is natural to think that any $\theta$-trade can be done ex-ante if we have a state-contingent complete market. As shown in the section 3, this intuition is not true in the multiperiod consumption world, although it holds in the single period consumption case. To understand this, I introduce a new concept $\theta_t$-trade.

**Definition 2.** A trade is called a $\theta_t$-trade if it is a function from $\Theta$ to $R^N_+$ such that at each time $t$, $z^i_t$ is a function measurable with respect to $\Theta_t$ for all $i$.

If a trade can be done ex ante through a state-contingent complete market, then it must be a $\theta_t$-trade. Since $\Theta_t \subseteq \Theta$, any $\theta_t$-trade is a $\theta$-trade, but a $\theta$-trade is not necessarily a $\theta_t$-trade. In other words, agents are not able to execute all $\theta$-trades even when the market is state-contingent complete. The original notion of efficiency, Pareto-optimal relative to $\theta$-trades, may not be feasible in the dynamic consumption world. The alternative notion of efficiency, Pareto-optimal relative to $\theta_t$-trades, works in the dynamic consumption setup but is more restrictive. When the allocations prior to information arrival are not Pareto-optimal relative to $\theta$-trades, information-based trades generically become mutually acceptable because they enable agents to smooth their consumption profiles overtime.

### 6 No-Trade Theorem with $\theta_t$-Trade

As argued in section 5, information-based trades generically become mutually acceptable in a multiperiod consumption world and they expand the set of possible consumption allocations.
However, several studies (Tirole, 1982; Judd et al., 2003; Maurer and Tran, 2015) explore the no-trade theorem in various dynamic consumption setups and show that the no-trade result is robust to dynamic consumption extensions. This finding raises the question of why those studies only find no-trade results in their dynamic consumption setups. This section answers this question by providing a general no-trade theorem in multiperiod consumption worlds.

I first introduce two concepts for analysis: extensions to the original concordant beliefs and concave utility functions assumptions in Milgrom and Stokey (1982), respectively.

**Definition 3.** Traders’ beliefs are dynamically concordant if $p^1(x_i^t|\theta_t) = \cdots = p^N(x_i^t|\theta_t)$ for all $i, t, x_i^t \in X_i^t$ and $\theta_t \in \Theta_t$.

Dynamically concordant beliefs are more restrictive than concordant beliefs. Since $\theta_T = \theta$, dynamically concordant beliefs are concordant beliefs. Intuitively, when traders have dynamically concordant beliefs, they agree on how to interpret payoff-irrelevant events at each period $t$. Concordant beliefs are not necessarily dynamically concordant beliefs, however, as shown in the following example.

**Example 3 (Dynamically Concordant Beliefs).** Consider the economy described in example 2. Suppose agents $\{A, B\}$ have heterogeneous beliefs on the possibility of potential payoff-relevant states: $\pi^A(\theta^1) = \pi^B(\theta^2) = p > 1 - p = \pi^B(\theta^1) = \pi^A(\theta^2)$. They believe that $\pi^A(\theta^k|x^k) = \pi^B(\theta^k|x^k) = 1, k = 1, 2$. So the public signal perfectly reveals the true payoff-relevant state.

In this example, $\pi^A(x^k|\theta^k) = \pi^B(x^k|\theta^k) = 1$, hence agents have concordant beliefs. However, at time 0.5 before the information arrival, $\pi^A(x^1|\theta_{0.5}) = \pi^A(x^1) = \pi^A(\theta^1) \neq \pi^B(\theta^1) = \pi^B(x^1) = \pi^B(x^1|\theta_{0.5})$, so the agents’ beliefs are not dynamically concordant.

Another factor that plays a role relates to preferences. In single period consumption models, the concavity of utility function only reflects risk aversion. In a dynamic consumption setup, the concavity of utility function introduces both risk aversion and intertemporal substitution. The next concept relates to how agents evaluate those two effects.

**Definition 4.** A trader’s preference is risk-aversion-dominating if for any consumption process $C_i^t \in R_+^t$, and any time $k$, she prefers the corresponding consumption process $\{c_i^1, \ldots, E(c_i^k|\theta_k), \ldots\}$. In the case of expected utility, $E[U(C_i^1)] \leq E[U(\{c_i^1, \ldots, E(c_i^k|\theta_k), \ldots\})]$. In the case of non-expected utility, $U(C_i^1) \leq U(\{c_i^1, \ldots, E(c_i^k|\theta_k), \ldots\})$. 

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An agent with risk-aversion-dominating utility finds her desire for less risk always dominates her desire for smoothing consumption overtime.

The following proposition shows that when a state-contingent complete market exists, the no-trade result is valid if agents have dynamically concordant beliefs and their preferences are risk-aversion-dominating.

Proposition 2. Suppose that all traders have risk-aversion-dominating preferences, that their beliefs are dynamically concordant, that the initial allocation \( e = (e_1, \ldots, e^N) \) is Pareto-optimal relative to \( \theta_t \)-trades and that each agent \( i \) observes the private information conveyed by the partition \( \{F^i_t\} \). At any time \( t \), if it is a common knowledge at \( \omega \) that \( z \) is a feasible trade and that each trader weakly prefers \( z \) to the zero trade. Then every agent is indifferent between \( z \) and zero trade. If all agents utility functions are strictly concave in \( C^i_1 \) then \( z \) is the zero trade.

Proof. See Appendix A.

The theorem confirms that when the market is state-contingent complete, dynamically concordant beliefs and risk-aversion-dominating preference are sufficient to generate no-trade results in multiperiod consumption worlds. The following two examples show that information-based trades are possible if either of those two conditions is absent.

Example 4 (Trade without Dynamically Concordant Beliefs). Consider the economy described in example 3. Suppose agents \( \{A, B\} \) have log utility. The initial endowments for both agents are \((1, 1, 1)\), where \((a, b, c)\) means the agent holds \(a\) units of security \(S_{C_1}\), \(b\) units of security \(S_{C_2}^{\theta_1}\) and \(c\) units of security \(S_{C_2}^{\theta_2}\). For simplicity it is assumed that \(0.5 < p < 0.75\).

In the equilibrium, after time 0 trades, agent \(A\) holds \((1, 4p - 1, 3 - 4p)\) and agent \(B\) holds \((1, 3 - 4p, 4p - 1)\). If \(x^1\) is observed, agent \(A\) holds \((2p, 2p, 3 - 4p)\) and agent \(B\) holds \((2 - 2p, 2 - 2p, 4p - 1)\) after retrades. If \(x^2\) arrives, agent \(A\) holds \((2 - 2p, 4p - 1, 2 - 2p)\) and agent \(B\) holds \((2p, 3 - 4p, 2p)\) after retrades.

This example satisfies all conditions required for the original no-trade theorem in Milgrom and Stokey (1982), but it does not satisfy the dynamically concordant beliefs condition. In the equilibrium, at time 0, agents reach symmetric asset portfolios \((1, 4p - 1, 3 - 4p)\) and \((1, 3 - 4p, 4p - 1)\). This outcome is intuitive since the two traders are symmetric except for their subjective priors on payoff-relevant states in the future; therefore, they hold larger positions in
the states they believe are more likely to be true. When the public news arrives, two agents agree on which payoff-relevant state will be realized in the future. They have the same position for consumption $C_1$ but different allocations of consumption $C_2$. To smooth consumption overtime, the rich agent (the “winner” that bets on the true state) would like to have more consumption $C_1$ while the poor one (the “loser” that bets on the other state) wants to increase her future consumption $C_2$. After the arrival of information, trading $C_1$ and $C_2$ is mutually acceptable, reaching the new positions $(2p, 2p)$ for the winner and $(2 - 2p, 2 - 2p)$ for the loser.\(^3\)

The other example explores the possibility of information-based trades when one agent’s preference is not risk-aversion-dominating.

**Example 5** (Trade without Risk-Aversion-Dominating Preferences). *Consider the economy described in example 2. Agent A and B believe that two payoff-relevant states \(\{\theta^1, \theta^2\}\) are equally likely to happen, and the time 0.5 signal \(\{x^1, x^2\}\) will reveal the true state. The initial endowments for both agents are \((2, 0.5, 2)\). Agent A has a habit-formation preference \(c_A^1 + \log(c_A^2 + 2 - c_A^1)\), and agent B is risk neutral.*

In the equilibrium, after time 0 trades, agent A holds \((1, 1, 3.5)\) and agent B holds \((3, 0, 0.5)\). If $x^1$ is observed, there would be no trade. If $x^2$ arrives, agent A holds \((2.25, 1, 2.25)\) and agent B holds \((1.75, 0, 1.75)\) after retrades.

In this example, because agents have common priors, their beliefs are dynamically concordant. Both agents’ utility functions are weakly concave and strictly increasing in their domains. Trades occur because agent A does not have risk-aversion-dominating utility. Her utility from consumption $c_A^2$ depends on both $c_A^2$ and her reference point $c_A^1$.

Intuitively, this example describes a world with a potential serious recession in the future. With habit-formation preferences, agent A is concerned about a possible low consumption state in the future and chooses to decrease her consumption $c_A^1$ to lower her reference point. At time 1, when the information reveals that no recession will occur, agent A knows her future consumption $c_A^2$ will be high and wants to increase $c_A^1$. The incentive to readjust her consumption allocations makes information-based trade desirable.

\(^3\)Here I omit the security for the state that will not be realized.
7 Risk-Aversion-Dominating Preferences

While the dynamically concordant beliefs condition is a natural extension of the original concordant beliefs condition, the risk-aversion-dominating utility condition restricts human behavior in a significant way. This section discusses this condition and shows that it fits many popular utility functions used in finance and economics studies.

Though both risk aversion and intertemporal substitution are often described by the concavity of utility functions, they affect the no-trade result differently. In Milgrom and Stokey (1982), only risk aversion plays a role in preferences. Risk aversion contributes to the no-trade result because for any information-based trade $z$, agents prefer the corresponding ex ante trade $z^{**} \equiv E[z|\theta].$ Examples in section 3 show that information about future consumption shocks in a dynamic consumption world enables agents to readjust their consumption allocations. Information-based trade is mutually acceptable because agents want to smooth consumption over time. In a multiperiod consumption world, to make sure that ex-ante trades are preferred to information-based trades, one needs the concern for risk aversion to dominate the concern for intertemporal substitution. This is exactly the definition of risk-aversion-dominating preferences.

Many popular utility functions used in finance and economics models satisfy this condition. For example, all standard time-additive expected utility functions $U(C) = \sum_{t=1}^{\infty} U_t(c_t)$ are risk-aversion-dominating.

The following lemma shows that many standard recursive utility functions are risk-aversion-dominating:

**Proposition 3.** Any utility function $U$ satisfying the following properties is risk-aversion-dominating:

1. At time $t$, $U$ only depends on $\{c_i^t, c_{i+1}^t, \ldots\}$;
2. For each time $t$, there exists a function $F(C_i^t)$ such that $U(C_i^t) = F(c_i^t, E[\mu_t(U(C_{i+1}^t))|\mathcal{F}_i^t])$.
3. $F$ is concave in $c_i^t$ and is increasing in $\mu_t(U(C_{i+1}^t))$. $\mu_t(U(C_{i+1}^t))$ is concave in $c_{i+1}^t$.

**Proof.** See Appendix A.

This is not the first paper to investigate information-based trading in a multiperiod consumption world. For tractability, all previous studies (Tirole, 1982; Judd et al., 2003; Maurer...
and Tran, 2015) assumed risk-aversion-dominating preferences, which explains why they found no-trade results in dynamic consumption setups. The risk-aversion-dominating condition relates to how agents trade off risk-aversion with intertemporal substitution. To generate information-based trading, one natural preference candidate is Epstein-Zin utility function (Epstein and Zin, 1989) with different parameters on risk-aversion and intertemporal substitution. Another candidate, as used in example 5, is habit formation preference.

There is no reason to expect that an arbitrary single period risk-aversion concern should always dominate the incentive to smooth consumption over the entire consumption path. The risk-aversion-dominating preferences condition suggests that researchers need to use preferences that characterize agents’ concerns for intertemporal substitution, for example Epstein-Zin utility and habit formation preferences, to analyze information-based trade in multiperiod consumption world.

8 Quantitative Analysis

This section presents the quantitative results for several examples. Based on the rare disaster example 5, I study a simple two period example with habit formation preference and a dynamic example with Epstein-Zin preferences to demonstrate that multiperiod extensions can explain a significant part of the trading volume we observe in financial markets.

Before the formal analysis, I first rephrase the no-trade theorem and my rational trading result in the framework of social planner’s optimal allocation problem. This discussion provides a good way to link the rational information-based trading problem to standard consumption-based asset pricing models. Now think about an endowment economy and a standard social planner’s optimal allocation problem with Arrow-Debreu market. That is to say, the market is complete across both endowment states (θ) and signal states (x). Let the state of the world be ω = (θ, x). Suppose at time t, the optimal allocation for agent i is $c_i^t(\theta_t, x_t)$. No-trade theorem basically says that in the single period consumption case with mild beliefs and preferences assumptions, $c_i^t(\theta_t, x_t) = c'_i(\theta_t, x'_t), \forall x_t, x'_t \in X_t$ and $\forall \theta_t \in \Theta$. That is to say, the optimal allocation is independent of signal states and signal events contingent claims are redundant. My rational trading result suggests that in the multiperiod consumption scenarios, with reasonable beliefs and preferences, $c_i^t(\theta_t, x_t) \neq c'_i(\theta_t, x'_t)$. Then in the Arrow-Debreu market, there would be no trade but agents will hold non-trivial positions in signal state contingent claims. If the market
is state-contingent complete, that is to say, only complete across endowment states, then there would be rational information-based trading and agents can achieve the optimal allocation by trading endowment contingent claims based on information.

Following this, in both models I first solve the social planner’s optimal allocation problem and check if \( c_i^t(\theta_t, x_t) \neq c_i^t(\theta_t, x_t') \). Then I find a trade \( z \) to implement this optimal allocation with endowment state contingent claims. In both cases the trades come from the fact that the optimal allocation depends on the realization of signals. I use the ratio of trading to total consumption as the measure of trading volume. As figure 1 shows, the trading volumes in financial markets are comparable to total consumption.

8.1 Two Period Example

The setup is similar to example 5. Consider the economy described in example 2. Agent \( A \) and \( B \) agree that the potential disaster state \( \theta^1 \) will happen with probability \( p \). The time 0.5 signal \( \{x^1, x^2\} \) will perfectly reveal the true state at time 2. The initial endowment for agent \( A \) is \((1, 1 - b, 1)\) and is \((1, 0, 1)\) for agent \( B \). Agent \( A \) has a habit formation preference:

\[
U^A(c^A_1, c^A_2) = \frac{(c^A_1)^{1-\rho}}{1-\rho} + \beta \frac{(c^A_2 - \alpha c^A_1)^{1-\rho}}{1-\rho}
\]  

(4)

where \( \beta \) is the time discount factor and the parameter \( \alpha \in [0, 1] \) measures the strength of habit formation. When \( \alpha \) is larger, the consumer receives less time 2 utility from a given amount of consumption. The utility function becomes CRRA utility when \( \alpha = 0 \). The parameter \( \rho \) measures the degree of relative risk aversion, and the agent \( A \)'s utility in example 5 can be viewed as a special case when \( \rho \) approaches 1. To compare with existing literature on habit formation, I use this more general form. Agent \( B \) is risk neutral and share the same time discount factor \( \beta \). One can interpret agent \( A \) as the household and agent \( B \) as the market maker or financial intermediaries. Agent \( B \) sets the prices and allows agent \( A \) to reallocate her consumption across different times and states, but subject to the constraint that in the disaster state the total endowment of the whole economy is limited.

One potential concern for the habit formation preference in the baseline model is that habit does not affect the time 1 utility. To address this concern, I also consider an extension in which
the agent $A$ has a habit formation preference with an exogenous time 1 habit:

$$U^A(c^A_1, c^A_2) = \left(\frac{c^A_1 - \alpha H}{1 - \rho}\right)^{1 - \rho} + \beta \left(\frac{c^A_2 - \alpha c^A_1}{1 - \rho}\right)^{1 - \rho}$$  \hspace{1cm} (5)$$

where $H$ is the exogenous time 1 habit.

8.1.1 Parametrization

For convenience, I assume that the constant risk-aversion coefficient $\rho$ for agent $A$ equals 3.5. The time discount factor $\beta$ is set to 0.98. As argued in example 3, this two period example can be viewed as a simplified rare disaster model. Following the estimation in Barro and Ursua (2012) and Barro and Jin (2011), the disaster state probability $p$ is assumed to be 3.83%, and the size of endowment contraction in disaster state $b$, is set to 20.8%. The exogenous habit $H$ is set to 0.7, which is very close to the total endowment in the disaster state $(1 - b) = 0.792$.

8.1.2 Calibration Results

The example is solved numerically for different values of habit formation parameter $\alpha$. Similar to example 5, information-based trading occurs when agents observe good news. Figure 3(a) shows the trading volume (in blue) and the ratio of information-based trading to time 1 consumption (in red) in the baseline model. Figure 3(b) reports the case when agent $A$ has a habit formation preference characterized by equation 5. Since the exogenous time 1 reference point $H = 0.7$, in the extension case the marginal utility for time 1 consumption increases and thus the trading volume declines.

The habit formation parameter $\alpha$ clearly plays an important role in information-based trading. In both cases the trading volume is increasing in $\alpha$. As suggested by theorem 2, the no-trade result holds in both cases when $\alpha = 0$. Existing literature require high value of $\alpha$ to explain the empirical regularities for which habit formation has been suggested as a solution. For example, Deaton (1987) shows that $\alpha$ must equal 0.78 to fully explain the excess smoothness of aggregate consumption. Carroll and Weil (1994) calculate that $\alpha$ would have to exceed 0.95 to explain the observed relationship between high aggregate income growth and subsequent periods of high aggregate saving. Finally, Constantinides (1990) shows that $\alpha$ must be approximately 0.80 to explain the historical equity premium. When the value of $\alpha$ is high ($> 0.75$), this simple two
period model can generate trading volume that is around 30% of total consumption (around 20% in the extension case). Some empirical study finds low value of \( \alpha \) in consumption data (Dynan, 2000). However, since the trading volume is a concave function in \( \alpha \), even when \( \alpha \) is low (\( \alpha = 0.2 \)), both cases still generate a significant amount of information-based trading (10% of total consumption).

![Graphs](image)

(a) Trading Volume: Baseline Model  
(b) Trading Volume: Extension  
(c) Consumption Profile before News: Baseline Model  
(d) Consumption Profile before News: Extension  
(e) Realized Consumption with Good News: Baseline  
(f) Realized Consumption with Good News: Extension

Figure 3: Trade and Consumption in the Habit Formation Models
8.2 Infinite Horizon Examples

In this subsection I extend example 5 to a dynamic discrete time model. Consider a pure exchange economy with two agents A and B and infinite horizon \( \{0, 1, 2, \ldots \} \). At each time \( t \), agents receive a total endowment of \( e_t \) and the endowment cannot be stored between periods. At each time \( t \geq 1 \), there are two potential endowment states \( \{ \theta^H, \theta^L \} \). Similar to the two period example, \( \theta^L \) can be viewed as the disaster state and \( \theta^H \) is the normal state. Let \( \theta_t \in \{ \theta^H, \theta^L \} \) be the realization of state at time \( t \). Similar to Barro (2006) and Barro and Jin (2011), the evolution of total endowment follows:

\[
\log e_{t+1} = \log e_t + \log (1 + g) + \log (1 - b) I_{\theta_{t+1} = \theta^L}
\]

where \( g \geq 0 \) represents the exogenous total endowment growth and \( 0 < b < 1 \) is the contraction of total endowment in the rare disaster event. \( I_{\theta_{t+1} = \theta^L} \) is the indicator function that equals one if and only if at time \( t + 1 \) the endowment state is \( \theta^L \), otherwise it would be 0. Let \( e^i_t \) be agent \( i \)'s endowment at time \( t \). Without loss of generality, the initial total endowment \( e_0 \) is normalized to be 1, and \( e^A_t = w^A e_t, e^B_t = w^B e_t, w^A = 1 - w^B \). \( W_0 = \{w^A, w^B \} \) represents the initial wealth distribution. I assume \( w^A \in (0, 1) \) to exclude the trivial “one takes all” scenarios.

The endowment state \( \theta_t \) follows a first-order Markov process. If the endowment state this period is \( \theta^i \) then the probability that the endowment state next period is \( \theta^j \) will be denoted by \( P_{ij} \). The Markovian transition matrix is:

\[
\begin{bmatrix}
P_{HH} & P_{HL} \\
P_{LH} & P_{LL}
\end{bmatrix}
\]

For simplicity I assume that at time 0 both agents believe that the probability that \( \theta_1 = \theta^H \) is \( P_0 = \frac{P_{HH}}{P_{HH} + P_{LH}} \). This is the steady state probability of this Markovian transition process. At time 0, for any period \( t \geq 1 \), both agents agree that the probability that \( \theta_t = \theta^H \) is \( P_0 \).

At any time \( t \geq 1 \), the history of realized endowment states is denoted by \( h^t = \{\theta_1, \theta_2 \ldots \theta_t\} \). Let \( \mathcal{H}^t \) be the set of all possible \( h^t \). I assume that the market is state-contingent complete, that is to say, the market is complete across all \( \theta \). For any \( h^t \in \mathcal{H}^t \), there exists a security \( S(h^t) \) that

\^[5\]In section 4 for simplicity I assume a finite \( T \). However, all analysis in this paper can be extend to the infinite horizon case.
pays 1 unit of consumption at time $t$ when the history is $h^t$. Let $S_0$ be the security of time 0 consumption.

At each time $t$, a signal $x_t \in \{H, L\}$ arrives. The signal perfectly reveals $\theta_{t+1}$. At time 0, agent can do one round ex ante trade before the arrival of single $x_0$. In each period agents can trade on the market after the arrival of $x_t$, and then consume. As in section 4, the state of the world $\Omega$ is characterized by $\Omega = \Theta \times X$. $\Theta$ is the set of endowment events and $X$ is the set of signal events.

A trade $z = (z_0, z_0, z_1, \ldots)$ is a function from $\Omega$ to $R^\infty$, where $z_0$ is agent $A$’s ex ante trade before the arrival of $x_0$. $z_t(\omega)$ is agent $A$’s time $t$ net trade in state $\omega \in \Omega$, and $-z_t(\omega)$ is agent $B$’s time $t$ net trade in state $\omega$. Since agents can only trade based on the available information. $z_t$ is assumed to be measurable with respect to $\Omega_t \equiv H^t \times \{H, L\}$. At each time $t \geq 0$, the price of security $S(h_k), k \geq t$ is denoted by $q_t(S(h_k)|\omega_t), \omega_t = (h^t, x_t)$. At each time $t$, the price vector of all securities $S(h_k), k \geq t$ is defined as $Q_t$. Let $z_0(S(h^t))$ be agent $A$’s ex ante net trade on security $S(h_k)$ and $z_t(\omega_t, S(h_k))$ be agent $A$’s time $t$ net trade on security $S(h_k)$, then $z$ is feasible if it satisfies

$$c^A_t(z, \omega_t) \equiv e^A_t(h^t) + z_0(S(h^t)) + \sum_{k=0}^t z_k(\omega_k, S(h^t)) \in R_+, \ \forall \ x, \theta, t$$  \hspace{1cm} (8)$$

$$c^B_t(z, \omega_t) \equiv e^B_t(h^t) - z_0(S(h^t)) - \sum_{k=0}^t z_k(\omega_k, S(h^t)) \in R_+, \ \forall \ x, \theta, t$$  \hspace{1cm} (9)$$

In each period $t$, the events flow can be summaries as the following: First the endowment state $\theta_t$ is realized. Then agents observe the signal $x_t$, and the market reopens and agents can trade. Finally agents consume their time $t$ consumption.

At each time $t$, agents have preferences that can be written as special cases of the following
Epstein-Zin recursive utility function:

\[ U_i^t = [(c_i^t)^{\rho_i} + \beta(E_t[U_{i+1}^{\frac{1}{1-\gamma_i}}])^{\frac{\rho_i}{1-\gamma_i}}]^{\frac{1}{\rho_i}} \]  \hspace{1cm} (10)

for \( i \in \{A, B\} \). \( \beta \in (0, 1] \) is the time discount factor, \( \gamma_i > 0 \) is the parameter that controls risk aversion, the elasticity of intertemporal substitution in consumption (EIS) is

\[ EIS_i^t = \frac{1}{1-\rho_i} \]  \hspace{1cm} (11)

Following Kan (1995), for each initial wealth distribution \( W_0 \), there exists a unique equilibrium. In this paper I focus on stationary recursive competitive equilibrium, which is defined as:

**Definition 5.** A stationary Markovian equilibrium is a trade \( z^* \), security prices \( Q^* \equiv \{Q_0^*, Q_1^*, Q_2^*, \ldots \} \) and the initial wealth distribution \( W_0^* \equiv \{w_A^*, w_B^*\} \) such that:

1. Given \( W_0^* \), \( z^* \), \( Q^* \), at each time \( t \), \( z_t^*(\omega_t) \) maximizes agent \( i \)'s expected utility conditional on \( \omega_t = (h^t, x_t) \);
2. Given \( W_0^* \), \( z^* \) is feasible;
3. For Agent \( i \), at any time \( t \geq 1 \), if \( \theta_t = \theta^j \) and \( x_t = k \), then \( c_i^t = \pi_{jk}^i c_t \).

Existing literature on the survival of heterogeneous agents with CRRA preferences in general find that the economy will converge to “one agent takes all” case (Dumas, 1989). Guvenen (2009), Borovicka (2015) and other recent studies show that this might not be the case when agents have Epstein-Zin preferences. I solve the model numerically as in the Appendix B and find that for parameters I tried the stationary recursive competitive equilibrium does exist for certain initial wealth distribution. In a world with a different initial wealth distribution, the economy might converge to this stationary recursive competitive equilibrium in the long run. The following lemma simplifies the analysis.

**Lemma 1.** In the stationary recursive competitive equilibrium, \( \pi_{jk}^i = \pi_k^i \) for any \( j \in \{H, L\} \).

*Proof. See Appendix B.*
Lemma 1 suggests that agent’s current consumption allocation is independent of current endowment state but depends on the signal about future endowment shocks.

Given the consumption plans \( \{\pi^i_k\} \) and \( W^* \) in the equilibrium, the optimal trade \( z^* \) may not be unique, and the trading volume will depend on how \( z^* \) is constructed. Because the fundamental source of information-based trading is \( \pi^i_H \neq \pi^i_L \), without loss of generality, I define the trading volume as \( T = |\pi^A_H - \pi^A_L| \) and the trade \( z^* \) is the one that at any time \( t \geq 1 \), agents trade if and only if they observed good news \( (x_t = H) \).

In the economy, at time \( t \geq 1 \), a risk free asset is a security that pays 1 unit of consumption at time \( t + 1 \) regardless of the signal \( x_{t+1} \). The return of the risk free asset at time \( t + 1 \) is defined as \( R_{f,t+1} = \frac{1}{q_{f,t}} \), where \( q_{f,t} \) is the price of the risk free asset at time \( t \) after the arrival of information \( x_t \). The equity is a claim on all current and future endowments in the economy. At time \( t \geq 1 \), the price of the equity after the arrival of information \( x_t \) is \( q_{e,t} \). The equity return at time \( t + 1 \) is:

\[
q_{e,t+1} = R_{e,t+1}(q_{e,t} - e_t)
\]  

**Baseline Model**  Following Barro (2006), in the baseline model I assume that the length of each period is the duration of one disaster, that is to say, each disaster will only exists for one period. The disaster events are i.i.d., that is to say, \( P_{HL} = P_{LL} \).

**Extension**  In the baseline model, similar to Barro (2006), each disaster event is considered as one period. This setup implicitly assumes that the negative shock happens immediately when the disaster event occurs. In the data the economic crises persist for varying numbers of years, and it may take years for the contraction to fully realize. In other words, the shock may be smoothed over the recession time. One may wonder that this “shock smoothing” may affect the trading volume. In this extension I study a regime switching model in which the length of each period is one year and \( P_{HL} \neq P_{LL} \). Agents in the recession state will face a contraction of total endowment each period, and will continue to suffer this until the state transit back to \( \theta^H \).

8.2.1 **Parametrization**

**Preferences**  Guvenen (2009) uses a two-agent macroeconomic model with heterogeneity in the elasticity of intertemporal substitution in consumption (EIS) to study asset prices. Here I just follow his parameter choices on preferences. The annualized time discount factor is set to 0.99. Let agent A and B be the “non-stockholders” and “stockholders” in Guvenen (2009),
respectively. Then $EIS^A = 0.1$ and $EIS^B = 0.3$ and $\gamma_A = \gamma_B = 6$.  

**Disasters** In the baseline model I assume the disaster parameters follow the estimations on GDP contractions in Barro and Jin (2011). The probability of a disaster event is $P_{HL} = P_{LL} = 3.83\%$. The exogenous endowment growth rate is set to $g = 2.5\%$. Barro and Jin (2011), Gabaix (2012) and Barro and Ursua (2012) show that the size of contraction varies in different disaster events. In the simple two period example, I used the arithmetic mean of disaster contractions 20.8%. Since the dynamic model features a permanent shocks on future endowments in disaster states, instead of the arithmetic mean, I use the geometric mean of disaster contractions. Barro and Jin (2011) show that the size of disasters follows a double power law distribution:

$$f(s) = \begin{cases} 
0, & \text{if } s < s_0 \\
B_d s^{-(\beta_d+1)}, & \text{if } s_0 \leq s < \delta_d \\
A_d s^{-(\alpha_d+1)}, & \text{if } s \geq \delta_d 
\end{cases} \quad (13)$$

where $s \equiv \frac{1}{1-b}$, and $A_d > 0$, $B_d > 0$. $\beta_d$, $\alpha_d > 0$ are exponent parameters, $s_0 > 0$ is the known threshold, and $\delta_d \geq s_0$ is the cutoff separating the lower and upper parts of the distribution. The conditions that the density integrate to 1 over $[s_0, \infty)$ and that the densities be equal just to the left and right of $\delta_d$ imply

$$\frac{1}{A_d} = \frac{\delta_d^{\beta_d-\alpha_d}}{\beta_d} (s_0^{\beta_d} - \delta_d^{\beta_d}) + \frac{\delta_d^{-\alpha_d}}{\alpha_d} \quad (14)$$

$$B_d = A_d \delta_d^{\beta_d-\alpha_d} \quad (15)$$

The geometric mean of disaster contractions is estimated to be $b = 21.8\%$. Barro (2006) argues that the rare disaster model works well with discrete periods with length corresponding to the duration of disasters (all with the same duration, say 3.5 years). In the baseline model I also assume that the length of discrete periods is 3.5 years.

In the extension model, the annualized contraction rate is set to be 6.67% to match the 21.8%
### Table 2: Model Parameter Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Baseline</th>
<th>Extension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annualized time preference</td>
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<td></td>
</tr>
<tr>
<td>EIS</td>
<td>$EIS^A$</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$EIS^B$</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$\gamma_A = \gamma_B$</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Annualized growth</td>
<td>$g$</td>
<td>2.5%</td>
<td></td>
</tr>
<tr>
<td>Transition probability</td>
<td>$P_{HL}$</td>
<td>3.83%</td>
<td>1.14%</td>
</tr>
<tr>
<td></td>
<td>$P_{LL}$</td>
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<td>71.43%</td>
</tr>
<tr>
<td>Period length (years)</td>
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<td>1</td>
</tr>
<tr>
<td>Size of contraction</td>
<td>$b$</td>
<td>21.8%</td>
<td>6.67%</td>
</tr>
</tbody>
</table>

This table presents the calibration parameter choices for the baseline model and the extension. The preference parameters follow the preferences setting in Guvenen (2009) and the disaster parameters is calibrated to match the estimations in Barro and Jin (2011).

contraction rate per disaster event and the average 3.5 years disaster duration. The expected duration of a disaster event is:

$$
\sum_{k=1}^{\infty} k P_{LL}^{k-1} P_{LH} = \sum_{k=1}^{\infty} k (1 - P_{LH})^{k-1} P_{LH}
$$

$$
= \frac{1}{P_{LH}}
$$

$P_{LH}$ is set to 28.57% to match the average 3.5 years duration. $P_{HL}$ is assumed to be 1.14% to make sure that the steady state probability $P_0$ is consistent with the 3.83% disaster rate in Barro and Jin (2011).

Table 2 summarizes the key parameter values used to solve the dynamic models.

### 8.2.2 Calibration Results

Models are solved numerically as in the Appendix B. Table 3 reports the calibration results. It is clear that all models generate significant amount of trading volume (14% to 35% of total consumption). Model (1) provides a benchmark. It predicts a fairly reasonable risk premium (6.00%) but very high risk-free rate (15.50%). In Guvenen (2009), he assumes low $EIS$, which mechanically causes the “risk free rate puzzle”. Guvenen (2009) solves this problem by abstracting his model from long run consumption growth. However, this is not the case in my model because in the rare disaster setup the long run endowment growth can not be 0, otherwise the
economy will collapse to 0 in the long run.

Table 3: Trading Volume, Consumption and Asset Returns

<table>
<thead>
<tr>
<th></th>
<th>Baseline Model</th>
<th>Extension</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td><strong>EIS^A/EIS^B</strong></td>
<td>0.1/0.3</td>
<td>0.1/0.3</td>
</tr>
<tr>
<td><strong>γ_A/γ_B</strong></td>
<td>6/6</td>
<td>12/6</td>
</tr>
<tr>
<td><strong>π^A_H</strong></td>
<td>0.7665</td>
<td>0.6247</td>
</tr>
<tr>
<td><strong>π^A_L</strong></td>
<td>0.5163</td>
<td>0.4800</td>
</tr>
<tr>
<td><strong>T</strong></td>
<td>25.02%</td>
<td>14.47%</td>
</tr>
<tr>
<td><strong>w_A</strong></td>
<td>55.24%</td>
<td>50.07%</td>
</tr>
<tr>
<td><strong>E(R_f)</strong></td>
<td>15.50%</td>
<td>11.35%</td>
</tr>
<tr>
<td><strong>E(R_e)</strong></td>
<td>21.50%</td>
<td>14.85%</td>
</tr>
<tr>
<td><strong>E(R_e - R_f)</strong></td>
<td>6.00%</td>
<td>3.50%</td>
</tr>
<tr>
<td><strong>E(P/D)</strong></td>
<td>2.529</td>
<td>3.008</td>
</tr>
</tbody>
</table>

This table shows how agents with heterogeneous IES may lead to information-based trading and risk premium. Both baseline and extension models extend Barro (2006) by introducing heterogeneous agents with Epstein-Zin preferences similar to Guvenen (2009). In the baseline model, the length of each period is the duration of a rare disaster (3.5 years) and the disaster events are i.i.d.. In the extension, the length of each period is 1 year and a rare disaster may last for more than one period. π_A^k is agent A’s share of current consumption given the signal x = k. T is the trading volume as a fraction of total current consumption. w_A is agent A’s initial share of total wealth. E(R_f) and E(R_e) are the unconditional risk free rate and equity return, respectively. E(R_e - R_f) is the equity premium and E(P/D) is the price dividend ratio. All returns are annualized.

Model (4) shows the equilibrium outcome in the Markov transition extension. Because the “shock smoothing” in disaster events (from 21.8% to 6.67%), the trading volume declines (from 25.02% to 15.93%). However, it is still quite significant compared to the 6.67% annual contraction rate. This is because agents know that the disaster may last more than 1 period, and the longer the disaster lasts, the severer the negative shock is. Similar to model (1), model (4) also generates a fairly high risk free rate 11.17%.

**Risk Aversion and Equity Premium** Standard asset pricing theories predict that a higher value of risk aversion parameter γ increases the equity premium. However, model (2) shows that it might not be the case in this dynamic model. When agent A becomes more risk averse (γ_A = 12), the equity premium declines (from 6.00% to 3.50%).

The reason behind this result is that different from standard asset pricing models that assume an exogenous consumption process, in this model the consumption process is endogenous.
Agents’ consumption profile depends on both the total endowment $e_t$ and information-based trading. The evolution of total endowment follows the rare disaster setup and is not affected by agents’ preferences. However, the information-based trading depends on agents’ preferences. As argued in section 7, when agents are more risk averse, information-based trading becomes less attractive because it introduces additional uncertainty in consumption. As agents become more risk averse, the trading volume decreases (from 25.02% to 14.47%), this implies a less volatile consumption process (for each agent), resulting in a lower risk premium. Figure 5 shows how trading volume and equity premium change as both agents become more risk averse.

The figure reports the trading volume and equity premium as the risk aversion parameter $\gamma = \gamma_A = \gamma_B$ increases. All other parameters are the same as in the model (1).

Figure 5: Risk Aversion, Trading Volume and Equity Premium

**Risk Free Rate** Following the preference parameters in Guvenen (2009), both model (1) and (4) generate very high risk free rates. One natural way is to assume higher $EIS$ parameters ($EIS^A = 0.3$, $EIS^B = 0.9$). Model (3) and (5) report the calibration results with a different set of preference parameters. With higher $EIS$, in both models the risk free rate is much lower (6.70% and 5.20%), while the trading volume is still significant and the equity premium is reasonably good.
8.3 Discussions

Both the simple two period model with habit formation preferences and the dynamic heterogeneous agent model explain a significant part of the trading volume we observe in financial markets. There are several reasons that those models may still underestimate the rational trading volume in the real world.

In the model the trading volume is defined as the trading volume on contingent claims. In the real markets traders may execute trades by trading different securities, such as stocks, options. The actual trading volume associated with the rational trading may be larger than what my model predicts.

Trading volume in my model is the consumption reallocation between agents $A$ and $B$. In the economy there may exist other traders in the market. A market maker or a high frequency trader may first trade with agent $A$ and then trade with agent $B$, resulting in “doubt counting” on trading volume.

In this model the only shock is the potential rare disaster. However, in the real world there are many other potential shocks and they may contribute to rational trading as well.

9 Private Information

Milgrom and Stokey (1982) and Blume et al. (2006) show that in any equilibrium with no-trade result the information revealed by price changes “swamps” each trader’s private information. Formally,

$$\pi^i(\theta|\mathcal{F}_t^i, q_t) = \pi^i(\theta|q_t), \quad \forall \ i, t, \theta$$ (17)

where $q_t$ is the price vector at time $t$. This result suggests that the no-trade equilibrium outcome is robust to both public and private information, as well as both the game equilibrium concept and the competitive market equilibrium concept. The following examples show that in a dynamic consumption world with possible information-based trades, the equilibrium outcome may depend on the equilibrium concept and the information structure.

Example 6 (Trade with Public Information). Consider an economy similar to the one described in example 5. Agents will observe $x^1$ with a probability of 0.25, and $x^2$ with a probability of 0.75. Signal $x^1$ reveals that $\theta = \theta^1$ for sure, while $x^2$ suggests that $P(\theta^1|x^2) = \frac{1}{3}$. Agent $A$’s utility
function is \( c_1^A + \log(c_2^A + \frac{5}{3} - c_1^A) \), and agent B is risk neutral.

Under the concept of competitive market equilibrium, a fully revealing equilibrium exists. In the equilibrium, similar to example 5, after time 0 trades, agent A holds \((1, 1, 3.5)\) and agent B holds \((3, 0, 0.5)\). If \(x^1\) is observed, there would be no trade. If \(x^2\) arrives, agent A holds \((\frac{5}{3}, 1, 2.5)\) and agent B holds \((\frac{2}{3}, 0, 1.5)\) after re trades.

If both agents are strategic players, then at each time each agent’s strategy space is the space of feasible trades. Using backward induction, one can solve the Subgame Perfect Nash Equilibrium, which is exactly the same as the competitive market equilibrium.

**Example 7 (Trade with Private Information).** Consider an economy similar to the one described in example 6. Now the signal \(x^i\) is Agent A’s private information.

In this example the signal is agent A’s private information. The fully revealing equilibrium still exists and is the same as in example 6.

If both agents are strategic players, then additional to the feasible trade space, at time 0.5 after observing the signal \(x^i\), agent A can report a signal \(\hat{x}\). Similar to standard asymmetric information models, using backward induction, one can solve the Perfect Bayesian equilibrium. Now the original equilibrium no longer holds because given the trading strategy, agent A will always report \(\hat{x} = x^2\). Focus on separating equilibrium, then the IC constraint implies that any trading strategy in the equilibrium must satisfies:

\[
c_0^A + \delta + \log(1 + \frac{5}{3} - c_0^A - \delta) \leq c_0^A + \log(1 + \frac{5}{3} - c_0^A),
\]

(18)

where \(\delta\) is the information-based trading when agent A reports \(\hat{x} = x^2\). In this equilibrium, agent A holds \((S_{C_1}, S_{C_2}^{\theta_1}, S_{C_2}^{\theta_2}) \approx (1.3688, 1, 2.7623)\). Agent A will truthfully report her private information \(x^i\), and will sell \(S_{C_2}^{\theta_2}\) to buy 0.5462 \(S_{C_1}\) when she observes \(x^2\). Clearly this separating equilibrium is ex ante Pareto-inferior to the original one in the example 6. The private information setup in fact introduces a commitment problem: agent A can not commit to report the true signal she observes, which in turn results in the second best outcome.
10 Conclusion

This paper studies information-based trading in multiperiod consumption worlds. In contrast to the celebrated no-trade theorem in single period consumption cases, information-based trades in multiperiod consumption world are mutually acceptable because the information about future consumption shocks enables agents to readjust their allocations to smooth consumption over time.

In the dynamic consumption world, the no-trade result requires risk-aversion-dominating preferences. There is no reason to expect that arbitrary single period risk aversion concern should always dominate the incentive to smooth consumption over the entire consumption path. My analysis shows that many popular utility functions in finance and economics satisfy the condition needed for no-trade results.

The introduction of multiperiod consumption and intertemporal substitution bridges information-based trading and consumption-based asset pricing. Consumption-based asset pricing models can be used to analyze information-based trading, and the concern for rare disaster events alone can generate around 15% – 35% of total consumption as the trading volume.

The no-trade equilibrium outcome is robust to whether information is publicly or privately observed and whether the equilibrium concept is market competitive equilibrium or game theoretic equilibrium concept. If information-based trading exists in public information scenarios, then in the corresponding private information case, in the the Perfect Bayesian equilibrium traders may reveal their private information strategically, leading to a different equilibrium outcome.
A Appendix A

Proof of Proposition 1

Proof. Suppose there’s a feasible and mutually acceptable trade $z$ at $\omega'$ starts at time $t$, then for every $i$ and every $\omega \in R_t(\omega')$

$$E_i[U^i(e^i + z^i)|F_t^i] \geq E_i[U^i(e^i)|F_t^i].$$

(19)

Suppose trader $j$ strictly prefer to trade at $\omega$. Consider the trade $z^* = 1_{R_t(\omega')}z$, where $1_{R_t(\omega')} = 1$ if and only if $\omega \in R(\omega')$. It is a feasible trade since $z$ is feasible. Viewing $z^*$ ex ante,

$$E_i[U^i(e^i + z^{*,i})] = E_i[E_i[U^i(e^i + z^i1_{R_t(\omega')})|F_t^i]]$$

$$= E_i[1_{R_t(\omega')}E_i[U^i(e^i + z^i)|F_t^i]] + E_i[1_{R_t(\omega')}E_i[U^i(e^i)|F_t^i]]$$

$$\geq E_i[U^i(e^i)]$$

(20)

where $R_t^c$ denotes the complement of $R_t$. This inequality is strict for agent $j$. Now consider the ex-ante trade $z^{**} = E[z^*|\theta]$. Because agents’ beliefs are concordant, the $\theta$-trade $z^{**}$ is feasible and ex ante is strictly Pareto-superior to the null trade. Contrary to the assumption about the initial allocation.

If traders are strictly concave in $C^i_1$, and $z$ is not null, then $0.5z^{**}$ is a Pareto-improving $\theta$-trade, a contradiction. □

Proof of Proposition 2

Proof. Suppose there’s a feasible and mutually acceptable trade $z$ at $\omega'$ starts at time $t$, then for every $i$ and every $\omega \in R_t(\omega')$

$$E_i[U^i(e^i + z^i)|F_t^i] \geq E_i[U^i(e^i)|F_t^i].$$

(21)
Suppose trader $j$ strictly prefer to trade at $\omega$. Consider the trade $z^* = 1_{R_t(\omega')}z$, where $1_{R_t(\omega')} = 1$ if and only if $\omega \in R(\omega')$. It is a feasible trade since $z$ is feasible. Viewing $z^*$ ex ante,

$$E_i[U^j(e^i + z^{*i})] = E_i[E_i[U^j(e^i + z_{1_{R_t(\omega')}})|F_i^t]]$$

$$= E_i[1_{R_t(\omega')}E_i[U^j(e^i + z^i)|F_i^t]] + E_i[1_{R_t(\omega')}E_i[U^j(e^i)|F_i^t]]$$

$$\geq E_i[U^j(e^i)]$$  \hspace{1cm} (22)

This inequality is strict for agent $k$. Now consider an one time trade $z^{**} = \sum_t E(z^*_k|\theta_t)$. Because agents' beliefs are dynamically concordant, $z^{**}$ is feasible. Because agents are risk-aversion-dominating

$$E_i[U^j(e^i + z^{**i})] = E_i[U^j(\sum_{i=1}^{t-1} e^i_j + \sum_{i}^{T} E[e^i_k + z^{*i}_k|\theta_k])$$

$$\geq E_i[U^j(\sum_{i=1}^{T} (e^i_k + z^{*i}_k))]$$

$$\geq E_i[U^j(e^i)]$$  \hspace{1cm} (23)

the trade ex ante is strictly Pareto-superior to the null trade, contrary to the fact that the initial allocation is Pareto-optimal relative to $\theta_t$-trade. If traders are strictly concave in $C^i_1$, and $z$ is not null, then $0.5z^{**}$ is a Pareto-improving $\theta_t$-trade, a contradiction. \hfill \Box

**Proof of Proposition 3**

Proof. For $k=1$

$$U(C^i_1) = F(c^i_1, E[\mu_1(U(\{c^i_2, \ldots \})])[F^t_1])$$

$$\leq F(E(c^i_k|\theta_k), E[\mu_k(U(\{E(c^i_k|\theta_k), \ldots \})])[F^t_k])]$$

$$= U(\{E_k(c^i_k|\theta_1), \ldots \})$$  \hspace{1cm} (24)

For any $k > 1$, consider $U(C^i_{k-1}) = F(c^i_{k-1}, E[\mu_k(U(C^i_k))]|F^t_k])$. Then:

$$U(C^i_{k-1}) = F(c^i_{k-1}, E[\mu_{k-1}(U(\{c^i_k, \ldots \})]|F^t_k])$$

$$\leq F(c^i_{k-1}, E[\mu_{k-1}(U(\{E(c^i_k|\theta_k), \ldots \})]|F^t_k])$$

$$= U(\{c^i_{k-1}, E(c^i_k|\theta_k), c^i_{k+1}, \ldots \})$$  \hspace{1cm} (25)
It follows that

\[ U(C^i_{k-2}) = F(c^i_{k-2}, E[\mu_k(U(C^i_{k-1}))|\mathcal{F}^i_{k-1}]) \]

\[ \leq F(c^i_{k-2}, E[\mu_k(U({c^i_{k-1}, E(c^j_k|\theta_k), c^i_{k+1}, \ldots}))|\mathcal{F}^i_{k-1}]) \]

\[ = U({c^i_{k-2}, c^i_{k-1}, E(c^i_k|\theta_k), c^i_{k+1}, \ldots}) \]

(26)

By iterated induction, \( U(C^i_1) \leq U({c^i_1, \ldots, E_k(c^i_k|\theta_k), \ldots}) \).
B Appendix B

Proof of Lemma 1

Given the consumption plan \( \{\pi^i_{jk}\} \). At any time \( t \geq 1 \), given the endowment evolution equation 6, if \( \theta_t = \theta^j \) and \( x_t = k \), then agent \( i \)’s utility function \( U^i(e_t, \theta_t, x_t) \) can be rewritten as:

\[
U^i(e_t, j, L) = \{(\pi^i_{jL}e_t)^{\rho_i} + \beta [P_{LH}U^i_{LH}(g(1-b)e_t, \theta^L, H)^{1-\gamma_i} + P_{LL}U^i_{LL}(g(1-b)e_t, \theta^L, L)^{1-\gamma_i}]^{\frac{\rho_i}{1-\gamma_i}} \}^{\frac{1}{\rho_i}}
\]

and

\[
U^i(e_t, j, H) = \{(\pi^i_{jH}e_t)^{\rho_i} + \beta [P_{HH}U^i_{HH}(ge_t, \theta^H, H)^{1-\gamma_i} + P_{HL}U^i_{HL}(ge_t, \theta^H, L)^{1-\gamma_i}]^{\frac{\rho_i}{1-\gamma_i}} \}^{\frac{1}{\rho_i}}
\]

Since both agent utility functions the endowment evolution equation is homogeneous of degree 1, then:

\[
U^i(e_t, j, L) = e_t U^i_{jL}
\]

\[
= e_t \{(\pi^i_{jL})^{\rho_i} + \beta [P_{LH}U^i_{LH}^{1-\gamma_i} + P_{LL}U^i_{LL}^{1-\gamma_i}]^{\frac{\rho_i}{1-\gamma_i}} \}^{\frac{1}{\rho_i}}
\]

and

\[
U^i(e_t, j, H) = e_t U^i_{jH}
\]

\[
= e_t \{(\pi^i_{jH})^{\rho_i} + \beta g^{\rho_i} [P_{HH}U^i_{HH}^{1-\gamma_i} + P_{HL}U^i_{HL}^{1-\gamma_i}]^{\frac{\rho_i}{1-\gamma_i}} \}^{\frac{1}{\rho_i}}
\]

where \( U^i_{jk} \) is agent \( i \)’s time \( t \) utility when \( e_t = 1, \theta_t = \theta^j \). Without loss of generality, I will focus on \( U^i_{jk} \). For convenience, denote:

\[
F_L \equiv P_{LH}U^i_{LH}^{1-\gamma_i} + P_{LL}U^i_{LL}^{1-\gamma_i};
\]

\[
F_H \equiv P_{HH}U^i_{HH}^{1-\gamma_i} + P_{HL}U^i_{HL}^{1-\gamma_i}.
\]

Let \( q_{jk} \) be the price the security that pays 1 unit of consumption tomorrow given the pay-off
relevant state is $\theta^j$ today and $\theta^k$ tomorrow. At any time $t \geq 1$:

$$q_{jk} \frac{\partial U_{i}^j}{\partial c_l} = \frac{\partial U_{i}^k}{\partial c_{l+1}} \quad i \in \{A, B\} \quad (33)$$

If $k = H$, then

$$\frac{\partial U_{i}^j}{\partial c_{l+1}} = U_{i}^j \frac{1 - \rho_i}{\pi^j_H \rho_i} \quad (34)$$

$$\frac{\partial U_{i}^j}{\partial c_{l+1}} = U_{i}^j \frac{1 - \rho_i}{\pi^j_H \rho_i} \beta g \rho_i \frac{1 - \gamma_i}{\pi^j_H \rho_i} \quad (35)$$

$$\equiv U_{i}^j \frac{1 - \rho_i}{\pi^j_H \rho_i} D_{i}^j \quad (36)$$

where $D_{i}^j$ is independent of current state $\theta^j$. Thus

$$\frac{D_{i}^A}{\pi^A_{jH} \rho_A^{-1}} = q_{jH} = \frac{D_{i}^B}{\pi^B_{jH} \rho_B^{-1}} \quad (36)$$

then

$$\frac{q_{HH}}{q_{LH}} = (\frac{\pi^A_{LH}}{\pi^A_{H} \rho_A^{-1}})^{\rho_B} = (\frac{\pi^B_{LH}}{\pi^B_{H} \rho_B^{-1}})^{\rho_A} = (\frac{1 - \pi^A_{LH}}{1 - \pi^A_{H}})^{\rho_B} \quad (37)$$

This equation holds if and only if $\pi^i_{HH} = \pi^i_{LH}$, $i \in \{A, B\}$. Similarly, one can prove that $\pi^i_{HL} = \pi^i_{LL}$, $i \in \{A, B\}$.

**Solving the Stationary Recursive Competitive Equilibrium**

Given Lemma 1, agent $i$’s utility becomes:

$$U_{L}^i = \{(\pi^A_L)^{\rho_i} + \beta g (1 - b)^{\rho_i} [P_{LH} U_{H}^{1 - \gamma_i} + P_{LL} U_{L}^{1 - \gamma_i} \frac{\rho_i}{\rho_A - 1}] \} \frac{1}{\pi^A_L} \quad (38)$$

$$U_{H}^i = \{(\pi^B_H)^{\rho_i} + \beta g (1 - b)^{\rho_i} [P_{HH} U_{H}^{1 - \gamma_i} + P_{HL} U_{L}^{1 - \gamma_i} \frac{\rho_i}{\rho_B - 1}] \} \frac{1}{\pi^B_H} \quad (39)$$

F.O.C suggests that

$$q_{H} (\pi^i_{H})^{\rho_i - 1} = D_{i}^H \quad (40)$$

$$q_{L} (\pi^i_{L})^{\rho_i - 1} = D_{i}^L \quad (41)$$

I now provide an outline of the algorithm:
1. Guess that in the equilibrium, for agent A, $\pi^A_H$ equals some value $a$.

2. Given $\pi^A_H$, solve corresponding $\pi^A_L$ and $\{U^i_k\}$ by F.O.C condition 40 and definition of $\{U^i_k\}$ via value function iteration.

3. Given the solution $\pi^A_L$, solve corresponding $\pi^A_H$ and $\{U^i_k\}$ by F.O.C condition 41 and definition of $\{U^i_k\}$ via value function iteration.

4. Iterate on Steps 2 and 3 until convergence.

5. $q_H$ and $q_L$ can be obtained from F.O.C conditions.

Given the numerical solution to the allocation $\pi^i_k$ and utility functions $\{U^i_k\}$, there may exist more than one trade that deliver the equilibrium allocation $\{\pi^i_k\}$. Here I propose one trade $z^*$ such that agents trade $|\pi^i_H - \pi^i_L|$ current consumption if and only if they observe signal $H$. At each time $t \geq 1$, let $M_k$ be agent A’s holding of the security that pays 1 unit of consumption at time $t$ if $\theta_t = \theta^k$. At time 0, after the ex ante trade, agent A’s position is:

1. $M_L = \pi^A_L$;
2. $M_H = \pi^A_L + (\pi^i_H - \pi^i_L)q_H$;

In every period, agent A’s trading strategy is to sell $(\pi^i_H - \pi^i_L)q_H$ unit of security that pays 1 unit of consumption tomorrow and purchase $(\pi^i_H - \pi^i_L)$ unit of current consumption if and only if she observes signal $H$.

To compute the equity price (the claim on total current and future endowment), let $W_k$ be the price of the equity when $e_t = 1$ and the signal is $k$, then:

$$W_H = 1 + \sum_{k=H,L} P_{Hk}q_kgW_k; \quad (42)$$

$$W_L = 1 + \sum_{k=H,L} P_{Lk}q_kg(1-b)W_k. \quad (43)$$
References


