The Optimal Enforcement of Insider Trading Regulations

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Regulating insider trading lessens the adverse selection problem facing market makers, enabling them to quote better prices. An optimal enforcement policy must balance these benefits against the costs of enforcement. Such a policy must specify (i) the conditions under which the regulator conducts an investigation, (ii) the penalty schedule imposed if an insider is caught, and (iii) a transaction tax to fund enforcement. We derive the policy that maximizes investors’ welfare. This policy entails investigations following large trading volumes or large price movements or both. Insiders caught making large trades are assessed the maximum penalty, but small trades are not penalized. Given this policy, insiders trade most aggressively on news with an intermediate price impact but refrain from trading on moderate or extreme news.

I. Introduction

There is a large literature in finance, economics, and law debating the costs and benefits of insider trading. Almost exclusively, this liter-
ature compares welfare in an economy with insider trading to welfare in the same economy without insider trading. The problem of enforcing insider trading regulations is ignored. In effect, this amounts to an assumption that enforcing a prohibition on insider trading is costless and therefore perfectly effective. This article deals directly with the problem of enforcing insider trading regulations. We model a market in which insider trading has social costs and we characterize the optimal enforcement of insider trading, accounting for the fact that enforcement is costly and there is a limit to the size of the penalty that can be imposed.

To be effective, insider trading regulations require monitoring and enforcement in a market setting. This kind of problem has two distinguishing features. First, regulators are guided by market data, which are a noisy signal about whether a crime has been committed. Regulators cannot directly observe how much insider trading has occurred. Instead, enforcement resources must be allocated on the basis of information such as trading volume and stock price changes. The second distinguishing feature is that the social cost and private benefit of the crime depend on the behavior of other market participants. The trades that market makers and other (noninsider) traders are willing to make, and hence market prices, depend on the extent to which insider trading regulations are enforced. Other financial market enforcement problems that have these features include front-running by brokers and financial market manipulations. The enforcement of antitrust laws also shares these features. For example, collusion and predatory pricing are not directly observed. Antitrust enforcement must be guided by data on prices, quantities sold, and the number of competitors (see, e.g., Besanko and Spulber 1989). Further, the cost and benefit of antitrust violations depend on the response of competitors and customers (see, e.g., Salant 1987). Two key questions to be addressed in such problems are, How should the regulator’s strategy for investigating for violations depend on the market data, and how high should the penalty be for detected violations?

We take as the objective of the enforcement policy the maximization of the expected utility of uninformed investors who trade to rebalance their portfolios. These traders are harmed by insider trading because of the adverse selection problem. That is, in quoting bid and ask prices, a market maker must take account of expected losses that he will incur when trading with better-informed insiders. To compensate for these losses, a market maker widens the bid-ask spread relative to a market with no insiders. In turn, this wider spread, a form of transaction cost, leads uninformed traders to trade
less, resulting in less efficient portfolio decisions. The role of an optimal enforcement policy in this setting is to lessen the adverse selection problem associated with insider trading, enabling market makers to quote more favorable prices.

An enforcement policy specifies the circumstances under which the regulator undertakes a costly investigation for insider trading, an insider trading penalty schedule, and a tax to be levied and used for the regulator’s budget. The regulator bases his investigation policy on the available information, in particular trading volume and a public release of information concerning share value. The latter information may have been known to the insider before its public release. If so, it is the source of the potential profit from insider trading. Trading volume comprises both an insider’s trade and the trades of the noninsiders, and without conducting a costly investigation, the regulator cannot disentangle the trades of the two. The insider, if investigated and found to have traded, may be required to pay a penalty, which can depend on the size of his trading profit.

We find the following results. Although we allow for a random investigation policy, we show that a simple nonrandom threshold policy is optimal. That is, the regulator investigates for insider trading if and only if trading volume exceeds some threshold, where the threshold depends on the information released concerning share value. This policy is the most efficient deterrent to insider trading because investigation efforts concentrate on trading volumes that provide the greatest evidence that the insider traded.

This result differs from the results of the costly state verification literature (see Townsend 1979; Mookherjee and Png 1989). There, an agent buys insurance to protect himself from adverse realizations of a random endowment. After privately observing this endowment, the agent makes a report to the insurer. The insurer can either pay the agent on the basis of the report or undertake a costly audit to check the veracity of the agent’s report. It is shown that the optimal contract calls for the insurer to do random audits, with the probability depending on the agent’s report. The randomization economizes on auditing costs. With insider trading, the optimal investigation policy is nonrandom conditional on total trading volume. Note, however, that it is still random from the perspective of the insider because trading by noninsiders is random. Our result is similar to results obtained in the principal-agent literature. For instance, Dye (1986) analyzes a principal-agent model in which the principal can expend resources to audit the agent and observe his effort. The contract specifies the probability of an audit as a function of output, a random variable conditional on effort. He shows that the optimal contract calls for a nonrandom audit policy conditional on output:
audit if and only if output is below some threshold. As in the analysis here, an audit takes place for the outputs that are most probable if shirking did occur.

We also show that the optimal penalty schedule has the following form. If the insider is caught trading more than a critical level, then the insider must pay the maximum feasible penalty. But if the insider is caught trading less than the critical level, then the insider pays no penalty. In effect, it is optimal to legalize a limited amount of insider trading. The intuition is as follows. Consider an enforcement policy that consists of a penalty schedule that imposes the maximum penalty for any level of insider trading and a threshold investigation policy as discussed above. Given this enforcement policy, suppose that the insider would trade $x > 0$ shares and have an expected profit net of penalties equal to $\pi$. Now, holding the investigation policy fixed, lower the penalty for some trade $x' < x$ so that if the insider trades $x'$ shares his expected profit net of penalties is still $\pi$. This change is preferred by the regulator. Net losses to the insider have not changed, but expected investigation costs are lower: if the insider trades less, the trading volume is less likely to exceed the threshold, and so the regulator is less likely to investigate. This reasoning applies as long as the penalty can be lowered, and it implies that there be a range of insider trading for which the insider pays no penalty. Moreover, the insider finds it optimal to make the largest trade in this range, and so in equilibrium, no penalty is imposed.

That an optimal penalty schedule may increase with the level of the crime was first discussed by Stigler (1970), who argued that a benefit of small penalties for small crimes is that they induce people not to commit large crimes. Among the articles that elaborate on Stigler’s argument are Shavell (1991) and Mookherjee and Png (1992, 1994). These articles consider two cases: one in which the probability of apprehension can be made to depend on the severity of the offense (e.g., how much money a thief stole) and one in which the probability of apprehension is independent of the severity of the offense (e.g., police monitor highway traffic and identify moderate speeders with the same likelihood as egregious speeders). In these models, it is optimal to have the expected penalty increase with the severity of the offense. In the first case, this is optimally accomplished by setting the penalty to the maximum possible and increasing the probability of apprehension with the severity of the offense. In the second case, the penalty itself must increase with the severity of the offense. In our analysis, with the optimal enforcement policy both the probability of apprehension and the penalty increase with the severity of the offense. As in the other models, this policy induces the insider to trade less than he would if the maximum feasible pen-
alty was always imposed. The difference, however, is in the benefit associated with inducing a less severe offense. As discussed above, in our analysis, the benefit of inducing the insider to trade less is that it lowers investigation costs. The net loss to the insider is actually the same as in the case in which the maximum feasible penalty is always imposed.

The solution to the optimal enforcement problem entails prices, an investigation threshold, and insider trades with the following features. Traders face better prices when investigation costs are low, the probability that the insider is informed is low, and the maximum feasible penalty is high. Also, we show that the investigation threshold is lower when extreme information is released. For instance, trading volumes that trigger an investigation following a takeover announcement will not trigger an investigation following a more modest event such as the announcement of earnings that are slightly higher than analysts’ expectations. This implies that the insider refrains from trading when he has very extreme news but trades on more modest news. This result is in contrast to models in which insider trading is not regulated, for example, Kyle (1985). There, the more extreme the insider’s news, the bigger the insider’s trade. Another result is that the investigation threshold is lower the more likely it is that the insider knew the information before its release. For example, trading volumes that trigger an investigation following the announcement of merger negotiations, an event an insider is likely to have known about, will not trigger an investigation following the announcement of a hostile takeover, an event an insider is less likely to have known about. This implies that the insider refrains from trading on news that he is very likely to have known in advance but trades on news that he is less likely to have known.

In practice, the enforcement process often begins with the stock exchange. Exchanges are self-regulatory organizations that monitor trading and report unusual activity to the Securities and Exchange Commission (SEC). For instance, at the New York Stock Exchange (NYSE), the Market Surveillance Division includes a group called Stock Watch. This group monitors trading patterns looking for abnormal movements in volumes and prices. If an abnormal pattern is observed and cannot be easily explained, it is referred to NYSE investigators. Suppose that there is abnormally high trading volume in the morning and a takeover announcement in the afternoon. If the trading cannot be attributed to some other event, then NYSE investigators broaden the investigation beyond this single day, establishing a period of review covering the sequence of events leading up to the takeover announcement. They compare the set of individu-
als who traded to a set of individuals who may have had access to nonpublic information regarding the takeover. This includes individuals who work at the acquiring and target firms as well as the investment banks and law firms. If the NYSE investigators uncover suspicious trades, then the information is passed along to the SEC. Our analysis focuses on investigations that resemble this type of exchange referral. In particular, investigations are triggered by volume and public news regarding a stock’s value. According to Meulbroek (1992), 31 percent of SEC investigations come from exchange referrals, 41 percent come from public complaints (such as tips from employees or ex-spouses), and the remaining 28 percent come from various other sources including broker referrals and other SEC cases.

Insider trading cases prosecuted by the SEC are civil cases. Hence only monetary penalties are involved. Alternatively, the SEC may pass a case to the Department of Justice for criminal prosecution, in which case incarceration becomes a possibility. Our analysis considers only monetary penalties for insider trading.

To our knowledge, this is the first article to analyze the optimal enforcement of insider trading regulations. As noted above, most of the existing literature compares the performance of economies with insider trading to that of economies without insider trading. For example, Ausubel (1990), Bhattacharya and Spiegel (1991), Leland (1992), and Bernhardt, Hollifield, and Hughson (1995) analyze the effect of insider trading on the liquidity of a market and the benefits derived from trading. Fishman and Hagerty (1992), Leland (1992), Khanna, Slezak, and Bradley (1994), and Bernhardt et al. (1995) analyze the effect of insider trading on the informational efficiency of stock prices and consequent efficiency of production decisions. Manne (1966), Dye (1984), Easterbrook (1985), and Fischer (1992) analyze the effect of insider trading on managerial contracting.

In those articles, it is assumed that enforcing insider trading regulations is costless. Shin (1996) assumes that enforcement is costless but analyzes a model in which a limited amount of insider trading

\[1\] For example, consider the case of Hasbro Inc. Average daily trading volume for Hasbro is about 255,000 shares (on the American Stock Exchange). On January 23, 1996, 684,000 shares traded. In addition, after 3 weeks of no trading volume in Hasbro call options, 145 contracts traded on January 22 and 166 contracts traded on January 23 (on the Pacific Stock Exchange). On January 24, Mattel Inc. publicly announced an offer to acquire Hasbro. While the increase in the trading volume could be the result of informed parties’ trading ahead of the takeover bid announcement, it could also simply be a random fluctuation. Ultimately, the exchanges and the SEC must determine whether to pursue investigations. See the Wall Street Journal, January 26, 1996.
is socially beneficial. In his analysis, the regulator’s investigation policy is assumed to be independent of the other variables in the analysis and the form of the penalty schedule is exogenous. Spiegel and Subrahmanyam (1995) discuss the deterrence of insider trading when convictions are based on circumstantial evidence, for example, price moves and volume. They focus on mistakenly penalizing noninsiders. We do not address this issue. In our analysis, enforcement may err by not penalizing a guilty individual but not the reverse.

The article is organized as follows. Section II describes the model. The analysis and an example are presented in Section III. Section IV concludes the article with a discussion of possible extensions to the analysis.

II. The Model

Shares of a risky asset are traded. The value of a share is a random variable denoted by $\theta$. There is a single trading date before the realization of $\theta$. The modeling of the trading process follows Admati and Pfleiderer (1989) and Easley and O’Hara (1992). A risk-neutral market maker posts a bid price, $b$, at which he will accept all sell orders and an ask price, $a$, at which he will accept all buy orders. We assume that market making is competitive, and so the market maker is willing to quote a bid (ask) price as long as his expected profit on the bid (ask) side of the market is at least zero.

There are $2n$ risk-averse traders who are uninformed regarding $\theta$ and trade for portfolio reasons. Uninformed traders are endowed with cash and shares and have a concave utility function over final wealth. Assume that traders $\{1, \ldots, n\}$ are endowed with long share positions and traders $\{n + 1, \ldots, 2n\}$ are endowed with short share positions. Uninformed traders privately observe their own endowment. Given quoted prices, uninformed traders choose their optimal trades. With bid and ask prices such that $b \leq E[\theta] \leq a$, uninformed traders who are endowed with long share positions will choose to either sell shares at the bid or not trade and uninformed traders who are endowed with short share positions will choose to either buy shares at the ask or not trade. Assume that the endowments of traders with short share positions are independent of the endowments of traders with long share positions. Hence for a given bid and ask, the total number of shares bought by uninformed traders is independent of the total number of shares sold by uninformed traders.

Additionally, there is one risk-neutral insider. With some probabil-
ity the insider privately observes the realization of $\theta$ and with some probability the insider observes nothing. Let the random variable $\phi \in \mathbb{R} \cup \{\emptyset\}$ denote the insider’s information. If the insider is informed, $\phi = \theta$; if the insider is uninformed, $\phi = \emptyset$. We let $q(\theta) = \Pr(\phi = \theta | \theta)$ and assume $E[\theta | \phi = \emptyset] = E[\theta]$. Only the insider knows whether he is informed. Given quoted prices, the insider chooses a trade to maximize his expected profit net of any penalties associated with insider trading.

A regulator enforces the insider trading regulations. Once trading is completed, the regulator observes three variables: the total number of shares purchased from the market maker, the total number of shares sold to the market maker, and the share payoff, $\theta$. The regulator does not, however, observe individual traders’ orders. For a cost $c$, the regulator can conduct an investigation. As a result of this investigation, the regulator observes the number of shares that the insider traded, if any.

The market maker pays a tax per share traded, $t$, to the regulator, which is used to cover the cost of investigating insider trading. This tax is passed through to traders via the bid and ask prices quoted by the market maker. The regulator’s expected expenditures on investigations net of taxes and penalties collected must be no higher than $K$, the regulator’s funding from other sources. If $K = 0$, the regulator’s activity is supported solely by insider trading penalties and market maker taxes; if $K > 0$, the regulator can run a deficit in regulating this market; and if $K < 0$, the regulator must run a surplus in regulating this market.

Given that the number of shares bought by uninformed traders is independent of the number of shares sold by uninformed traders and given that the market maker must earn a zero expected profit on both sides of the market, we can focus separately on the bid and ask sides of the market. We present the analysis for the bid side of the market. The analysis for the ask side of the market is identical.

Consider the optimal regulation of the bid side of the market. Prior to trade, the regulator determines the optimal penalty schedule and investigation policy. The regulator’s objective is to maximize

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2 Note that we allow the probability $q(\theta)$ that the insider is informed to vary with the realization of $\theta$ (e.g., the insider could more likely be informed about large price moves). In addition, it is not necessary for $q$ to depend only on $\theta$. We could easily allow an additional random variable $v$ that specifies the “type” of information (e.g., earnings announcement vs. takeover offer) and let $q$ be a function of $(\theta, v)$. We avoid this generalization for notational convenience.

3 In practice, the transaction tax collected by the SEC from the NYSE is $0.03 per $1,000 traded.
the expected utility of the uninformed traders. This objective reflects the fact that there are no social gains, just costs, associated with insider trading in our model. The insider has no risk sharing or liquidity reasons for trading, and his trading causes a distortion of prices and portfolio choices by the uninformed. With a bid of $b$, let $V(b)$ denote the weighted sum of the ex ante indirect utility functions of the uninformed traders who are endowed with long share positions. That is, $V(b) = \sum_{i=1}^n \lambda_i V_i(b)$ for some positive constants $\lambda_i$, and

$$V_i(b) = E[\max_y E[U_i(e_i + yb + (z_i - y)\theta) | e_i, z_i]],$$

where $e_i$ and $z_i$ are trader $i$’s endowments of cash and shares, respectively, and $y$ is the number of shares sold.

An investigation policy, $G(w, \theta)$, specifies the probability of an investigation as a function of the total number of shares sold, $w$, and the share payoff, $\theta$. If the insider sells $x$ shares and the uninformed traders sell a total of $Y$ shares, then $w = x + Y$. If caught, the insider pays a penalty denoted $P(x, \theta)$. We assume that the insider can be forced to give up all of his trading profit, $x(b - \theta)$, plus pay an additional monetary fine. The maximum total penalty that can be imposed is $M[x(b - \theta)] \geq x(b - \theta)$. This maximum could correspond to a resource constraint of the insider (e.g., $M = x(b - \theta) + m$, where $m$ is the insider’s initial wealth) or a legal constraint such as a treble damages provision (i.e., $M = 3x(b - \theta)$). We make the technical assumption that $M[x(b - \theta)]/x$ is nonincreasing in $x$. We also assume that the insider cannot be rewarded by the regulator. Hence, $0 \leq P(x, \theta) \leq M[x(b - \theta)]$. Finally, $P(0, \theta) = 0$: if the insider does not trade, no penalty can be levied. Note that in contrast to the usual enforcement literature, here the maximum feasible penalty varies with the insider’s profit. Typical analyses assume that the maximum feasible penalty is a constant, independent of the size of the offender’s gain (effectively assuming that the offender’s gain cannot be recovered).

III. Optimal Enforcement

In this section we solve the regulator’s problem. After we state the optimization problem, propositions 1–4 present results that simplify

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4 In fact, we could also include the insider’s utility in the regulator’s objective. This would not alter the form of the optimal policy derived in the paper.
A. The Regulator’s Problem

The total volume of trade observed by the regulator is the aggregate of informed and uninformed trading. Each uninformed trader \( i \) chooses to trade

\[
y_i(e_i, z_i, b) = \text{argmax}_y E[U_i(e_i + yb + (z_i - y)\theta)|e_i, z_i].
\]

We define the random variable \( Y = \sum_{i=1}^n y_i(e_i, z_i, b) \), the total number of shares sold by the uninformed. Note that the distribution of \( Y \) depends on \( b \), though we suppress this dependence for notational convenience.

The risk-neutral insider trades to maximize expected profits, inclusive of penalties. The expected profit given the information \( f \) and a sale of \( x \) shares can be written as

\[
\pi(x, \phi, G, P, b) = E[x(b - \theta) - G(x + Y, \theta)P(x, \theta)|\phi].
\]

A feasible strategy for the insider is a nonnegative random variable \( X \) that is independent of \( (\theta, Y) \) given \( f \).

Therefore, we can write the regulation problem as follows:

\[
\max_{(G, P, X, b, t)} V(b) \quad (P1)
\]

subject to

\[
\text{(IC)} \quad X \in \text{argmax}_{x \geq 0} \pi(x, \phi, G, P, b),
\]

\[
\text{(MM)} \quad E[(\theta - b - t)(X + Y)] = 0,
\]

\[
\text{(Reg)} \quad E[G(X + Y, \theta)[c - P(X, \theta)] - t(X + Y)] \leq K.
\]

The incentive-compatibility constraint for the insider (IC) requires that \( X \) is an optimal trade for the insider given the investigation policy and penalty schedule. The market maker’s participation constraint (MM) requires that his expected profit, net of the tax, must be zero. The regulator’s budget constraint (Reg) requires that the expected investigation cost less revenues from taxes and penalties must be no more than \( K \).

Proposition 1 shows that \( (P1) \) can be simplified. The regulator’s objective is equivalent to maximizing the bid, and the market maker’s and regulator’s constraints can be combined. Combining the constraints relies on the assumption that the market maker is risk-neutral.
Proposition 1. The optimization problem (P1) is equivalent to the following:

$$\max_{(G, P, X, b)} (G, P, X, b)$$  \hspace{1cm} (P2)

subject to

- (IC) $X \in \arg\max_{x \geq 0} \pi(x, \phi, G, P, b)$,

- (CB) $E[cG(X + Y, \theta) + \pi(X, \phi, G, P, b) - (\theta - b)Y] \leq K$.

Also, the insider’s strategy $X$ can be taken to be a deterministic function of $\phi$.

Proof. The uninformed traders’ utility is clearly increasing in $b$. Thus maximizing $V(b)$ is equivalent to maximizing $b$. The combined budget constraint (CB) is equal to the difference of the regulator’s and the market maker’s constraints, from the definition of $\pi$. Thus anything feasible under (P1) is also feasible under (P2). Finally, a feasible policy under (P2) is also feasible under (P1) when the tax $t = E[(\theta - b)(X + Y)]/E[X + Y]$ is used.

To see that the insider’s strategy can be taken to be a deterministic function of $\phi$, note that if the incentive-compatibility constraint does not determine $X$ uniquely, we can select $X$ from among the solutions to the incentive-compatibility constraint to be the smallest trade that minimizes $E[G(x + Y, \theta)]$. This only weakens the combined budget constraint. Q.E.D.

The combined budget constraint (CB) reflects the fact that, from the regulator’s perspective, the costs associated with insider trading are the expected investigation costs $E[cG]$ and the insider’s expected profit $E[\pi]$. These costs must be covered by the market maker’s profit from trading with the uninformed traders $E[(\theta - b)Y]$ and the regulator’s budget $K$. The tax no longer appears because it is a transfer from the market maker to the regulator.

The structure of our analysis is as follows. For any bid $b$, one can compare alternative enforcement policies by comparing the total cost (investigation costs plus insider profits) of implementing them. In particular, we make the following definition.

Definition. For a given bid $b$, we say that the policy $(G, P, X)$ is incentive compatible if $X \in \arg\max_{x \geq 0} \pi(x, \phi, G, P, b)$. We say that the policy $(G^*, P^*, X^*)$ is cheaper than $(G, P, X)$ if $(G^*, P^*, X^*)$ is incentive compatible and

$$E[cG^*(X^* + Y, \theta) + \pi(X^*, \phi, G^*, P^*, b)]$$

$$\leq E[cG(X + Y, \theta) + \pi(X, \phi, G, P, b)].$$
It is clear that when one is solving (P2), any bid $b$ that can be attained with a given enforcement policy and budget can also be attained with a cheaper policy (if there is one). Therefore, in solving (P2) we can restrict attention to enforcement policies that are cheaper than all others.

We now use this idea to characterize the form of the insider’s trading strategy, the penalty schedule, and the investigation policy. Notably, proposition 3 shows that we can restrict attention to penalty functions of a simple form and proposition 4 shows that we can restrict attention to nonrandom investigation policies. From these results, propositions 5 and 6 characterize the optimal solution to the regulator’s problem.

First we show that the bid side of the market does not trigger an investigation if the realization of $q$ exceeds $b$.

**Proposition 2.** For any bid $b$ and any incentive-compatible policy $(G, P, X)$, define

$$ G^*(w, \theta) = \begin{cases} 0 & \text{if } \theta \geq b \\ G(w, \theta) & \text{if } \theta < b. \end{cases} $$

Then $(G^*, P, X)$ is cheaper than $(G, P, X)$.

**Proof.** Given the lower bound on $P$, $\pi(x, \phi, G, P, b) \leq E[x(b - \theta)|\phi] = x(b - E[\theta|\phi])$. Thus the insider earns zero profits for $E[\theta|\phi] \leq b$, for any investigation policy. Therefore, setting $G^*(w, \theta) = 0$ when $\theta \geq b$ reduces investigation costs without altering the insider’s profits. Q.E.D.

Our next result, proposition 3, shows that we can restrict attention to penalty functions of the following form. For trades above a threshold, the insider pays the maximum penalty $M[x(b - \theta)]$. For trades below this threshold, the insider gives up a multiple $a(q)$ of his trading profit. Note that $a(q)$ may be less than one. That is, the regulator may allow the insider to keep some of the trading profit.

To see the intuition for this result, consider an arbitrary penalty schedule $P$ and suppose that $X(\theta)$ is the insider’s optimal trade under $P$ given information $\theta = \Theta$. Now consider two penalty schedules as alternatives to $P$. The first imposes the penalty $\alpha'x(b - \theta)$ for trades less than or equal to $X(\theta)$ and imposes the maximum feasible penalty, $M[x(b - \theta)]$, for trades greater than $X(\theta)$, where $\alpha'$ is chosen so that $\alpha'X(\theta)(b - \theta) = P(X(\theta), \theta).$ That is, the penalty is not changed at $X(\theta)$ (see fig. 1). This change must leave the insider at least as well off as before because the insider could choose the same trade and face the same penalty as before. Next consider the penalty $\min\{\alpha''x(b - \theta), M[x(b - \theta)]\}$, where $\alpha''$ is chosen so that $\alpha''x(b - \theta) \geq P(x, \theta)$ for all $x$ (see fig. 1). This change must leave the insider
no better off than under \( P \) because now for any trade, the insider faces a penalty that is greater than or equal to \( P(x, \theta) \). Since the penalty schedule defined by \( \alpha' \) makes the insider better off and the schedule defined by \( \alpha'' \) leaves him worse off, there must be an intermediate schedule defined by some \( \alpha^* \in [\alpha', \alpha''] \) that leaves his expected profit unchanged. This schedule imposes the penalty \( \alpha^* x(b - \theta) \) for trades less than or equal to \( X(\theta) \) and imposes the maximum feasible penalty, \( M[x(b - \theta)] \), for trades greater than \( X(\theta) \). Since we have increased the penalty and hence lowered the profits for trades larger than \( X(\theta) \), the insider’s optimal trade under this new schedule must be some \( x^* \leq X(\theta) \). For this \( x^* \), the incentive-compatibility constraint is satisfied by definition. To see that the penalty schedule defined by \( \alpha^* \) leads to a cheaper enforcement policy, note that since \( x^* \leq X(\theta) \), the insider has lower trading profits if he is not investigated. If he is investigated, he forfeits a higher fraction of his profits (\( \alpha^* \geq \alpha' \)). Hence, since his expected profits \( \pi \) are unchanged, he must be investigated with lower probability. Thus expected investigation costs are lower whereas expected insider profits are unchanged, and so it is a cheaper enforcement policy. Finally, note that nothing is changed if we now impose the maximum penalty for trades larger than \( x^* \). Thus, for a given \( \theta \), we have replaced the arbitrary penalty schedule \( P \) with a simpler schedule that can be characterized by two parameters, a penalty rate \( \alpha^* \) and the threshold \( x^* \), where the penalty becomes the maximum possible.

![Diagram](image-url)
Proposition 3. For any bid $b$ and any incentive-compatible policy $(G, P, X)$, there exists a cheaper policy $(G^*, P^*, X^*)$ such that, for some function $\alpha^*: \mathbb{R} \to [0, \infty)$,

$$P^*(x, \theta) = \begin{cases} \alpha^*(\theta)x(b - \theta) & \text{if } 0 < x \leq X^*(\theta) \\ M[x(b - \theta)] & \text{if } x > X^*(\theta). \end{cases}$$

(1)

Proof. See the Appendix.

Given the form of the penalty function, we now show that if the distribution of the total number of shares sold, $w$, conditional on the insider’s trade, $X$, satisfies the monotone likelihood ratio property (MLRP), then attention can be restricted to nonrandom investigation policies for which the regulator investigates if and only if the number of shares sold exceeds a threshold. Since $w = X + Y$, the MLRP in this setting corresponds to the following restriction on the distribution of $Y$.

Definition. We say that the MLRP holds for $b$ if, given $b$, $Y$ has a continuous and log-concave density function.

If the MLRP holds, then the density $f$ of $Y$ is such that $f(w - x)/f(w)$ is increasing in $w$ if $x \geq 0$. That is, a large number of shares sold is stronger evidence that the insider is selling than a small number of shares sold. In this case the nonrandom investigation policy discussed will concentrate investigations over trading volumes for which insider trading is most likely and will be at least as good as, and generally better than, any other policy.

The proof of this result relies on a technical lemma. This lemma shows that if a nonrandom investigation policy leaves unchanged the insider’s probability of being investigated relative to some arbitrary policy for a given trade, then for all smaller trades the insider faces a lower probability of investigation and for all larger trades the probability of investigation increases.

Define $G_w$ as the threshold investigation policy in which an investigation is conducted if and only if the number of shares sold exceeds $w$. That is, $G_w(X + Y, \theta) = 1[X + Y \geq w]$.

Lemma. Suppose that the MLRP holds for $b$ and $Y$ has support $[y_0, y_1]$. If, for some $w \in [x + y_0, x + y_1]$,

$$E[G_w(x + Y, \theta) | \theta] = E[G(x + Y, \theta) | \theta],$$

then if $x' \leq (\geq) x$,

$$E[G_w(x' + Y, \theta) | \theta] \leq (\geq) E[G(x' + Y, \theta) | \theta].$$

The intuition for why we can restrict attention to threshold investigation policies is as follows. First, begin with an arbitrary investigation policy $G$, and suppose that we modify it to a threshold investiga-
Fig. 2.—Simplifying the investigation policy

The investigation policy, $G_w(0)$, where $w(x)$ denotes the threshold that leaves the probability of investigation unchanged given an insider trade of $x$. By the lemma, $G_w(0)$ implies an increased probability of investigation for any trade above zero. Thus the insider’s profit given $G_w(0)$ must be lower than the profit given $G$ for any positive trade (see fig. 2).

Alternatively, suppose that we modify $G$ to a policy $G_w(X(\theta))$ that leaves the probability of investigation unchanged, given an insider trade of $X(\theta)$, the optimal trade under the original investigation policy $G$. By the lemma, $G_w(X(\theta))$ implies higher profits for trades below $X(\theta)$. This potentially increases the insider’s maximal profit (see fig. 2).

Therefore, there exists a trade $x^*$ between zero and $X(\theta)$ such that the policy $G_w(x^*)$ keeps the insider’s maximal profit unchanged. Under this new policy, the insider’s optimal trade $x^*$ is less than $X(\theta)$ since the probability of an investigation has been increased for trades larger than $x^*$. This means that there must be a lower probability of investigation for the insider’s profits to remain unchanged. Also, if the insider is uninformed and therefore trades zero, we know by the lemma that the probability of investigation is lower. Thus this change must reduce expected investigation costs and therefore results in a cheaper enforcement policy. Thus, for a given $\theta$, we have replaced the arbitrary investigation policy $G$ with a nonrandom investigation policy that can be characterized by a single parameter, the investigation threshold.

**Proposition 4.** If the MLRP holds for $b$, then for any incentive-compatible policy $(G, P, X)$, there exists a cheaper policy $(G^*, P^*, X^*)$ such that $P^*$ satisfies (1) and for some function $W: \mathbb{R} \to [0, \infty]$,
We can now characterize the optimal enforcement policy in more detail. Proposition 5 establishes that the optimal penalty schedule imposes the maximum penalty if the insider’s trade exceeds some threshold and allows the insider to keep his trading profit if his trade is below the threshold; that is, $a^*(w, G) = 0$.

Let $PM$ denote the penalty function that imposes the maximum penalty for any trade by the insider; that is, $PM(x, q) = M[x(b - \theta)]$. Then define

$$
\pi^*(\phi, G, b) = \max_{x \geq 0} \pi(x, \phi, G, P_M, b),
$$

which is the insider’s maximal expected profit when facing the harshest possible penalty schedule. Next define

$$
x^*(\phi, G, b) = \begin{cases} 
0 & \text{if } E[\theta] \geq b \\
\frac{\pi^*(\phi, G, b)}{b - \phi} & \text{if } \phi = \theta < b,
\end{cases}
$$

which is the trade that delivers a gross trading profit of $\pi^*$.

Then we have the following proposition.

**Proposition 5.** If the MLRP holds for $b$, then for any incentive-compatible policy $(G, P, X)$, there exists a cheaper policy $(G^*, P^*, X^*)$ such that $X^*(\phi) = x^*(\phi, G^*, b)$,

$$
P^*(x, \theta) = \begin{cases} 
0 & \text{if } 0 < x \leq X^*(\theta) \\
M[x(b - \theta)] & \text{if } x > X^*(\theta),
\end{cases}
$$

$$
G^*(w, \theta) = \begin{cases} 
0 & \text{if } w < W(\theta) \\
1 & \text{if } w \geq W(\theta),
\end{cases}
$$

for some function $W$. Given these strategies, the insider’s profit is given by $\pi^*(\phi, G^*, b)$.

**Proof.** Given any incentive-compatible policy $(G, P, X)$, we can first apply proposition 2 to show the result for $E[\theta] \geq b$. Next apply proposition 4 to assume that $G$ has the desired form. Incentive compatibility implies that $\pi(X(\phi), \phi, G, P, b) \geq \pi^*(\phi, G, b)$. Now define $X^*$ and $P^*$ according to the statement of the proposition. Then $\pi(X^*(\phi), \phi, G, P^*, b) = \pi^*(\phi, G, b)$, so the insider’s profits have been reduced. Also, since $\pi(X(\phi), \phi, G, P, b) \leq X(\phi)(b - \phi)$, we
must have $X^*(\phi) \leq X(\phi)$. But given the form of $G$, this implies that $E[G(X^* + Y, \Theta)] \leq E[G(X + Y, \Theta)]$, so that investigation costs are reduced. Hence total costs have been reduced. It remains to check the incentive-compatibility constraint. First note that for $x \leq X^*(\phi)$, $\pi(x, \phi, G, P^*, b) = x(b - \phi)$. Thus the insider does not prefer any smaller trade. Next note that, for $x > X^*(\phi)$, the maximum penalty is imposed, so that by the definition of $\pi^*$, $\pi(x, \phi, G, P^*, b) \leq \pi^*(\phi, G, b)$. Thus the insider does not prefer any larger trade, and the incentive-compatibility constraint holds. Q.E.D.

Note that the optimal enforcement policy has the feature that the insider’s trade is $x^*$ and he pays no penalty. His profit, $x^*(b - \theta) = \pi^*$, is the same as the profit he would receive had the penalty schedule been $p_0$ and he traded optimally. Thus the gain from employing the optimal penalty schedule in proposition 5 does not come from reducing the insider’s profit. Instead it comes from the reduction in expected investigation costs that results from the smaller insider trade.

Proposition 5 characterizes the insider’s optimal trade, the optimal penalty schedule, and the form of the optimal investigation policy. To completely characterize the solution we need to determine the optimal investigation threshold, $W(\Theta)$, and the optimal bid, $b$. With the simple form for the optimal penalty function, we can rewrite the regulator’s problem in a simpler fashion: maximize the bid subject to a single constraint.

**Proposition 6.** The optimization problem (P2) is equivalent to

$$\max_{(W, b)} b \quad \text{(P3)}$$

subject to

$$E[cG^*(x^*(\phi, G^*, b) + Y, \Theta) + \pi^*(\phi, G^*, b) - (\Theta - b)Y] \leq K, \quad (4)$$

where $G^*$ is defined as in (3).

**Proof.** Follows immediately from proposition 5. Q.E.D.

In general, there is no closed-form solution for the optimal bid, $b$, the insider’s optimal trade, $X^*(\Theta)$, and the optimal investigation threshold, $W(\Theta)$. We do, however, have the following comparative statics results.

**Proposition 7.** The optimal bid $b$ increases as the investigation cost $c$ decreases, the probability the insider is informed $q(\Theta)$ decreases, or the maximum feasible penalty $M[x(b - \Theta)]$ increases.

**Proof.** We simply need to show that the constraint (4) is relaxed with any of these changes, since then (by continuity of this constraint in $b$) $b$ can be increased. First, a decrease in $c$ decreases investigation costs and so relaxes (4). If $q(\Theta)$ decreases, the insider has lower aver-
age profits, relaxing (4). Also, the probability of investigating decreases, since investigating is less likely when the insider is absent. This decreases investigation costs and again relaxes (4). Finally, an increase in \( M(x(b - \theta)) \) reduces \( \pi^*(\phi, G^*, b) \), which relaxes (4).

Q.E.D.

To illustrate additional properties of the solution we present a numerical example in subsection B. We show that a number of the properties illustrated in the example hold more generally in subsection C below.

B. An Illustration of the Optimal Enforcement Policy

We continue to focus on the bid side of the market, recognizing that the analysis for the ask side is the same. There is one uninformed trader, endowed with no cash but with \( z \) shares of stock; \( z \) has a truncated exponential distribution on the support \([z_l, z_u]\), with density \( f(z) \sim e^{-z/s} \). The parameter \( s \) represents the relative likelihood of a large versus a small exposure for the uninformed; in particular, a larger \( s \) leads to a larger \( z \) in the sense of first-order stochastic dominance.\(^5\) Assume \( z_l = 25,000 \) and \( z_u = 75,000 \). If the uninformed trader sells \( y \) shares at the bid, \( b \), then his final wealth is \( \omega = (z - y)\theta + by \) and his utility is \( U(\omega) = -e^{-\omega} \). Assume that \( \theta \) is normally distributed with a mean of 100 and a standard deviation of 10. Therefore, given a bid of \( b \) and an endowment of \( z \), the uninformed trader’s optimal trade is \( y = z - [(100 - b)/200] \). The investigation cost is 25,000 and the maximum feasible penalty equals the trading profit plus a fine of 30,000, that is, \( M(x(b - \theta)) = x(b - \theta) + 30,000 \). The regulator’s budget is given by \( K = 0 \), so the transaction tax must cover the expected investigation cost.

We consider the following three examples, which vary the probability that the insider is informed and the extent of uninformed trading.

Case A. \( s = 10^3 \) and \( q(\theta) = 0.20 \).

Case B. \( s = 10^9 \) and \( q(\theta) = 0.25 \) (\( q \) up).

Case C. \( s = 20,000 \) and \( q(\theta) = 0.20 \) (\( Y \) down).

Table 1 summarizes the solution for each of these cases.

Figure 3 presents the optimal investigation threshold, the volume that induces the regulator to investigate. Note that the threshold is lower, and hence enforcement efforts are more aggressive, the more extreme the realization of \( \theta \). Thus modest trading volumes will trigger an investigation following a significant announcement such as

\(^5\) For example, as \( s \to \infty \), the distribution is approximately uniform; for \( s \to 0 \), the distribution becomes concentrated at \( z_u \).
TABLE 1

<table>
<thead>
<tr>
<th>Case</th>
<th>Bid b</th>
<th>E[Y]</th>
<th>17.2%</th>
<th>4,300</th>
<th>.085</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>99.897</td>
<td>50,000</td>
<td>17.2%</td>
<td>4,300</td>
<td>.085</td>
</tr>
<tr>
<td>B</td>
<td>99.894</td>
<td>50,000</td>
<td>19.5%</td>
<td>1,730</td>
<td>.097</td>
</tr>
<tr>
<td>C</td>
<td>99.886</td>
<td>40,530</td>
<td>17.4%</td>
<td>1,220</td>
<td>.107</td>
</tr>
</tbody>
</table>

Fig. 3.—Optimal investigation threshold W(θ)

A bankruptcy filing or a takeover offer, whereas an announcement that earnings are slightly below (or above) analysts’ expectations would lead to an investigation only if excessive volume were observed.

Also, the threshold is lower if the probability that the insider is informed is higher (case A compared to case B). The more likely the insider has prior information, the more aggressive enforcement efforts are. For example, consider the announcement of merger negotiations to the announcement of a hostile takeover bid, and suppose that both events lead to the same stock price change and are preceded by the same trading volume. If it is more likely that the insider had prior knowledge of the merger negotiations, then this announcement might trigger an investigation whereas the announcement of the hostile takeover might not.

Finally, consider how the distribution of uninformed demand af-
flects the investigation threshold by comparing case A and case C. Uninformed trading is lower in case C than in case A (in the sense of first-order stochastic dominance), so that high volume is more suspicious in case C than in case A. We see from figure 3 that this leads to a lower investigation threshold for case C.

Figure 4 presents the insider’s equilibrium trade. For extreme realizations of $q$, the insider does not trade. The potential profit from trading is high, but the low investigation threshold for such realizations deters the insider. For realizations of $q$ that are close to $E[q]$, the potential profit from trading is low. Hence even with a high investigation threshold the threat of paying the maximum penalty deters the insider from trading much. The insider trades the most and has the highest profit for intermediate realizations of $q$. By contrast, in typical models of unrestricted insider trading, an insider’s trades and profits are higher when his information is more extreme. Here, the insider would prefer not to observe news that is too surprising.

Figure 4 shows that the insider’s trade is lower when the probability that the insider is informed is higher (case A vs. case B). Thus the insider trades more aggressively on information that he is less likely to have known.

Figure 4 also shows the effect of a lower uninformed volume on the insider’s trade (case A vs. case C). A lower level of uninformed trading has opposing effects on the insider. First, for a given investigation threshold, less uninformed trading implies a lower probability that an investigation is triggered. Second, as shown in figure 3,
less uninformed trading leads the regulator to lower the investigation threshold, making an investigation more likely. Third, less uninformed trading leads to a lower bid (due in large part to a higher transaction tax per share to support the regulator’s budget), decreasing the gains to insider trading. Figure 4 shows that for this example the net result is a decrease in insider trading, except for realizations of \( \theta \) close to \( E[\theta] \).

C. Additional Properties of the Optimal Policy

We now establish several additional properties that must hold for an optimal policy. We do so for an environment in which the maximum feasible penalty equals the insider’s trading profit plus a fixed amount, as in the example of the preceding subsection.

Assumption A. \( M = x(b - \theta) + m \), and \( m > c \).

For a given bid \( b \), let \( [y_l(b), y_h(b)] \) denote the support of the distribution of the total number of shares sold by uninformed traders. If the regulator sets an investigation threshold of \( y_l(b) \) and imposes the maximum feasible penalty, then the insider would not find it profitable to trade at all since an investigation would always occur. Since \( m > 0 \), however, a higher threshold may also deter insider trading. Define \( w_l(\theta, b) \) to be the highest threshold such that an insider who observes \( \theta \) and faces a bid of \( b \) would choose not to trade. That is, \( w_l(\theta, b) = \max\{w: \pi^*(\theta, G_w, b) = 0\} \). Clearly, \( w_l(\theta, b) \in [y_l(b), y_h(b)] \).

The threshold \( w_l \), which fully deters insider trading, serves as an obvious lower bound on an optimal investigation threshold \( W^*(\theta) \) since any lower threshold would only increase investigation costs. The next result shows that this threshold must be lowered when the potential profit per share, \( b - \theta \), is higher.

Proposition 8. \( w_l(\theta, b) \) is decreasing in \( b - \theta \). As \( b - \theta \to \infty \), \( w_l(\theta, b) \to y_l(b) \). As \( b - \theta \to 0 \), \( w_l(\theta, b) \to y_h(b) \).

Proof. See the Appendix.

We now state some properties of the optimal investigation threshold \( W^*(\theta) \) and the profit of the insider. In particular, we show that the insider’s expected profit is near zero when the probability that he is informed is very low or very high.

Proposition 9. (i) The optimal threshold can be bounded as follows: \( w_l(\theta, b) \leq W^* \leq y_h(b) \). (ii) If the insider is informed with certainty, \( q(\theta) = 1 \), full deterrence is optimal; that is, \( W^* = w_l(\theta, b) \) and \( \pi^* = 0 \). (iii) As the probability that the insider is informed approaches zero, \( q(\theta) \to 0 \), the investigation threshold approaches its upper bound and the insider’s ex ante expected profit approaches zero; that is, \( W^* \to y_h(b) \) and \( q(\theta) \pi^* \to 0 \).
Proof. See the Appendix.

The next result relates the optimal investigation threshold and the insider’s trade and expected profit to the realization of \( \theta \). We show that the feature illustrated in figure 4, that the insider’s trade (and profit) is near zero when the realization of \( \theta \) is very low or very close to the bid, holds more generally.

**Proposition 10.** (i) As the realization of \( \theta \) approaches the bid, the optimal threshold approaches its upper bound and the insider’s trade and expected profit approach zero. That is, as \( b - \theta \to 0 \), \( W^* \to y_{\theta}(b), x^* \to 0 \), and \( \pi^* \to 0 \). (ii) Suppose that \( \lim_{\theta \to -\infty} q(\theta) > 0 \); that is, the probability that the insider is informed is bounded away from zero for very low realizations of \( \theta \). Then as the realization of \( \theta \) becomes very low, the optimal threshold approaches full deterrence and the insider’s trade and expected profit approach zero. That is, as \( b - \theta \to \infty \), \( W^* \to y_{\theta}(b), x^* \to 0 \), and \( \pi^* \to 0 \).

Proof. See the Appendix.

Note that proposition 10 implies that the insider will trade most aggressively (and the regulator will be most tolerant) for news that has an intermediate effect on the stock price.

**IV. Concluding Remarks**

There are a number of possible extensions to the enforcement problem analyzed here. For example, we assumed that the regulator can commit to an investigation policy. What happens, however, if such a commitment is not possible? A striking feature of the regulatory solution is that the optimal penalty for the equilibrium level of insider trading is zero. If an investigation takes place, a cost is incurred but no penalty is collected. Given this, perhaps the regulator would have an incentive not to investigate even if the policy dictates otherwise.

Solving a no-commitment case requires the specification of an ex post objective for the regulator. A natural case to consider is one in which the regulator investigates for insider trading if the expected penalty to be collected in an investigation exceeds the cost of an investigation, that is, if

\[
E[P(X, \theta) | w, \theta] \geq c.
\]  

(5)

In the absence of commitment, the optimal regulatory policy must satisfy the added constraint that \( G(w, \theta) = 1 \) if (5) holds and \( G(w, \theta) = 0 \) otherwise. Given the MLRP, this constraint immediately delivers a threshold investigation policy like that derived in proposition 4. The optimal penalty schedule will be very different though. The
penalty for the equilibrium trade must be at least as high as the investigation cost, for otherwise there will be no investigation.

In the model, insider trading lowers welfare by distorting prices and inducing inefficient portfolio decisions by investors. If the insider were an employee of the firm, additional effects emerge. For example, the opportunity to earn insider trading profits would reduce the salary necessary to meet the employee’s reservation wage. Thus some of the costs of insider trading for the firm would be recouped through lower wages. This is a potential benefit associated with insider trading. If this were factored into the regulator’s objective, it would presumably reduce the optimal level of enforcement. However, even in this setting our characterization of the optimal investigation policy and penalty schedule would continue to hold, since these policies were designed to lower investigation costs without changing insider trading profits. In fact, by the same argument we can extend our characterization to cases with any arbitrary dependence of the regulator’s objective on the insider’s profit.

We assumed in our model that the insider’s information is precisely \( q \). One could assume instead that the insider observes a noisy signal of \( q \). For instance, the insider might know that earnings are below expectations but does not know the exact effect this will have on the stock price. Because of risk neutrality, this will not affect our results in any way as long as the regulator knows ex post the information that the insider might have had (e.g., the regulator sees the earnings announcement that was known to the insider). For the most part, this amounts to simply reinterpreting \( q \) as the expected stock price given the insider’s information.

The situation is different, however, if the regulator cannot determine ex post what information the insider might have had. In this case, the insider’s trade is not perfectly predictable conditional on the regulator’s information. In effect, there are multiple “types” of insiders since the insider now has private information ex post. As in Mookherjee and Png (1994), a model in which the offender has a privately observed type, this may lead to a penalty schedule that increases gradually as a function of the insider’s trade.

How do the results change if there is more than one insider? In that case, the probability that an insider is detected depends on the other insiders’ trades. This leads to a trading game between insiders. Since insiders do not internalize the cost their trades impose on

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6 On the other hand, if there are agency problems within the firm, the ability of the employee to trade shares might make it more costly for the firm to provide incentives. See, e.g., Fischer (1992), Admati, Pfleiderer, and Zechner (1994), and DeMarzo and Bizer (1995). If the regulator were to take this into account too, the net effect could be either a higher or a lower level of enforcement.
other insiders (by raising the probability of an investigation), for any enforcement policy we might expect more insider trading in aggregate. While this is bad for the adverse selection problem, it may make insider trading easier to detect. Thus the net effect on welfare is ambiguous.

Finally, one could consider the effect of different trading processes. We assumed that there is no limit on the trade size that a market maker would accept. It may, however, be optimal for the market maker to post a maximum trade size (under the assumption that traders cannot split their orders). While this may constrain uninformed traders, it may enable the market maker to quote better prices since he stands to lose less to the insider. Note that for a given bid, varying the maximum trade size will vary the distribution of uninformed trading, \( Y \). But as long as the MLRP holds for \( b \), our results on the structure of the optimal enforcement policy will not change. The least-cost method for sustaining any given bid and ask prices and maximum trade sizes will entail the same form of investigation policy and penalty schedule that was identified here.

We assumed that the market maker quotes a bid and an ask prior to observing the order flow. Alternatively, as in Kyle (1985), market makers could observe the order flow and then compete on prices. Since the order flow provides information about the level of insider trading \( X(\theta) \), it provides information about \( \theta \). In the current solution of our model, however, \( X(\theta) \) is not monotone in \( \theta \). In fact, insider trading tends to zero for extreme values of \( \theta \). Because of this, the market makers’ equilibrium price schedule might not be monotone in the order flow, a conclusion very different from that of standard models of insider trading.

Appendix

*Proof of Proposition 3*

First consider the case \( \phi = \theta < b \). Fix some \( \phi = \theta < b \). Since we hold \( \phi, \theta, G \), and \( b \) fixed for the remainder of the proof, we suppress these arguments and write \( \pi(x, P) \) for \( \pi(x, \phi, G, P, b) \). Also define \( P_{a', x} \) for constants \( a \) and \( x' \) to represent the penalty schedule such that, for all \( x \), if \( \theta' \neq \theta \), \( P_{a', x}(x, \theta') = P(x, \theta') \); otherwise

\[
P_{a', x}(x, \theta) = \begin{cases} 
\min \{ax(b - \theta), M[x(b - \theta)] \} & \text{if } 0 < x \leq x' \\
M[x(b - \theta)] & \text{if } x > x'.
\end{cases}
\]

The function \( P_{a', x} \) is a modification of \( P \) at a particular value of \( \theta \). First we find a penalty of the form above that does not change the insider’s expected profits. Define \( a' = P(X(\phi), \theta) / [X(\phi)(b - \theta)] \). Note that \( \pi(X(\phi),...
Recall that \( \alpha^* \geq \alpha' \) and gross trading profits are lower, \( x^*(b - \theta) \leq X(\phi)(b - \theta) \). Thus, for this equality to hold, it must be that \( E[G(x^* + Y, \theta)\{\theta]\] \leq E[G(X(\phi) + Y, \theta)\{\theta]\]. Hence investigation costs are lower, and the new policy is cheaper.

Finally, consider \( \phi \) such that \( E[\theta|\phi] \geq b \). From proposition 2, if \( E[\theta|\phi] \geq b \), we can define \( G^* \) such that the regulator does not investigate. Hence, for such \( \phi \) we can also modify \( P^* \) to satisfy (1) without consequence. Q.E.D.

**Proof of Lemma**

It is enough to show that \((x' - x)E[G_\text{r}(x' + Y, \theta) - G(x' + Y, \theta)|\theta] \geq 0\). Since \( \theta \) is constant throughout the proof, we suppress it as an argument of
G. Let \( f \) be the density of \( Y \), and assume that \( f \) is log concave. Then we have

\[
(x' - x)E[G_w(x' + Y, \theta) - G(x' + Y, \theta)|\theta]
\]

\[
= (x' - x) \int [G_w(x' + y) - G(x' + y)]f(y)\,dy
\]

\[
= (x' - x) \int [G_w(z) - G(z)]f(z - x')\,dz
\]

\[
= \int_{B} [G_w(z) - G(z)]L(z, x, x')f(z - x)\,dz
\]

\[
+ \int_{A \setminus B} (x' - x)[G_w(z) - G(z)]f(z - x')\,dz,
\]

where \( A = [x' + y_0, x' + y_1] \), \( B = [x + y_0, x + y_1] \), and \( L(z, x, x') = (x' - x)[f(z - x')/f(z - x)] \).

Since \( f \) is log concave, \( f(z - x')/f(z - x) \) is decreasing (increasing) in \( z \) if \( x' \leq (\geq) x \). Therefore, \( L(z, x, x') \) is increasing in \( z \). Next, note that, for \( z < w \), \( G_w(z) = 0 \leq G(z) \) and, for \( z \geq w \), \( G_w(z) = 1 \geq G(z) \). Therefore, \([G_w(z) - G(z)][L(z, x, x') - L(w, x, x')] \geq 0 \), or

\[
[G_w(z) - G(z)]L(z, x, x') \geq [G_w(z) - G(z)]L(w, x, x').
\]

Thus we have

\[
\int_{B} [G_w(z) - G(z)]L(z, x, x')f(z - x)\,dz
\]

\[
\geq \int_{B} [G_w(z) - G(z)]L(w, x, x')f(z - x)\,dz
\]

\[
= L(w, x, x') \int_{B} [G_w(z) - G(z)]f(z - x)\,dz
\]

\[
= 0,
\]

where the last equality follows from the hypothesis of the lemma.

Next note that, for \( z \in A \setminus B \), if \( x' > x \), then \( z > x + y_1 \geq w \); if \( x' < x \), then \( z < x + y_0 \leq w \). Hence, on \( A \setminus B \), \((x' - x)[G_w(z) - G(z)] \geq 0 \). Therefore,

\[
\int_{A \setminus B} (x' - x)[G_w(z) - G(z)]f(z - x')\,dz \geq 0,
\]

which proves the lemma. Q.E.D.

**Proof of Proposition 4**

First apply proposition 2, so that \( G \) and \( P \) can be presumed to have the desired form for \( E[\theta|\theta] \in b \). Now consider the case \( \phi = \theta < b \). Using proposition 3, we can also assume without loss of generality that \( P \) satisfies (1).

Fix some \( \phi = \theta < b \). For \( w' \), \( w \in [0, \infty] \), let \( G_{w'}(w, \theta) = 1 [w \geq w'] \) (i.e., the indicator function on the event \( w \geq w' \)) and \( G_{w'}(w, \theta') = G(w, \theta') \) for all \( \theta' \neq \theta \). Note that with the policy \( G_w \), if the insider learns \( \phi = \theta \) and trades \( x \), investigation occurs with probability \( \Pr(Y \geq w' - x) \).
Let \([\gamma_0, \gamma_1]\) be the support of \(Y\). Then for any \(x\), we define \(w(x) \in [x + \gamma_0, x + \gamma_1]\) by

\[
E[G_{w(x)}(x + Y, \Theta) | \Theta] = \Pr[Y \geq w(x) - x] = E[G(x + Y, \Theta) | \Theta].
\]

Such a \(w(x)\) exists since, by the MLRP, \(\Pr(Y \geq \gamma)\) is continuous and strictly decreasing in \(y\). Also, because \(Y\) is continuously distributed, \(E[G(x + Y, \Theta) | \Theta]\) is continuous in \(x\), which implies that \(w(x)\) is continuous. If the insider trades \(x\), then the probability of an investigation is the same with \(G_{w(x)}\) as with \(G\).

We now show that we can modify the investigation schedule for a fixed \(\phi = \theta\) to have the desired form. The strategy of the proof is to find a 0-1 investigation schedule that gives the insider the same expected profits.

Since \(w(x), \phi, \theta, P, \) and \(b\) remain fixed, we suppress them and write \(\pi(x, G)\) for \(\pi(x, \phi, \theta, G, P, b)\).

By the definition of \(w(x), \pi(X(\phi), G_{w(X(\phi))}) = \pi(X(\phi), G)\). Therefore,

\[
\max_{x \in X(\phi)} \pi(x, G_{w(X(\phi))}) \leq \pi(X(\phi), G).
\]

(see fig. 2). Next note that, by the lemma, \(E[G_{w(x)}(x + Y, \Theta) | \Theta] \geq E[G(x + Y, \Theta) | \Theta]\) for any \(x > 0\). Thus, for any trade, the insider faces a higher probability of being investigated with \(G_{w(x)}\) than with \(G\). This decreases the insider’s profits, and since \(X(\phi)\) is optimal given \(G\), we have

\[
\max_{x \in X(\phi)} \pi(x, G_{w(x)}) \leq \pi(X(\phi), G).
\]

The continuity of \(w(\cdot)\) and the distribution of \(Y\) imply that \(\pi(x, G_{w(x)})\) is continuous in \(x\). We can therefore find \(x^w \in [0, X(\phi)]\) such that

\[
\max_{x \in X(\phi)} \pi(x, G_{w(x)}) = \pi(X(\phi), G).
\]

(A2)

Let \(x^*\) solve (A2) and let \(X^*(\phi) = x^*\). We claim that \((G_{w(x^*)}, P, X^*)\) is cheaper than \((G, P, X)\).

To see this, note that since \(x^w \leq X(\phi)\), for all \(x \geq X(\phi)\), we have from the lemma that \(E[G_{w(x^*)}(x + Y, \Theta) | \Theta] \geq E[G(x + Y, \Theta) | \Theta]\). This reduces the profits associated with trades above \(X(\phi)\). Combined with (A2), this implies that

\[
x^* \in \arg\max_x \pi(x, G_{w(x^*)}),
\]

and the incentive-compatibility constraint is satisfied.

To see that it is cheaper, since \(\pi(x^*, G_{w(x^*)}) = \pi(X(\phi), G)\), we need only to check that investigation costs have not risen. Since \(P\) satisfies (1) and \(x^* \leq X(\phi)\), we know that \(\alpha^* = P(X(\phi), \Theta)/[X(\phi)(b - \theta)] = P(x^*, \Theta)/[x^*(b - \theta)]\). Thus \(\pi(x^*, G_{w(x^*)}) = \pi(X(\phi), P)\) can be rewritten as

\[
x^*(b - \theta)[1 - \alpha^*E[G_{w(x^*)}(x^* + Y, \Theta) | \Theta]] = X(\phi)(b - \theta)[1 - \alpha^*E[G(X(\phi) + Y, \Theta) | \Theta]].
\]

Since gross trading profits are lower, \(x^*(b - \theta) \leq X(\phi)(b - \theta)\), for this equality to hold, it must be that \(E[G_{w(x^*)}(x^* + Y, \Theta) | \Theta] \leq E[G(X(\phi) + Y, \Theta) | \Theta]\).
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Investigation costs are lower for the case \( \phi = \theta \). We also need to check the effect on the investigation probability when the insider is uninformed, \( \phi = \emptyset \). In this case \( X(\phi) = 0 \), and note that from the lemma, since \( 0 \leq x^* \), \( E[G_{aw}(Y, \theta) \mid \emptyset] \leq E[G(Y, \theta) \mid \theta] \). Thus total investigation costs decrease, and \((G_{aw}, P, X^*)\) is cheaper than \((G, P, X)\).

Finally, note that since \( x^* \neq X(\phi) \), \((P, X^*)\) no longer satisfies (1). The proof is therefore complete by another application of proposition 3 to adjust \( P \) to the desired form. Q.E.D.

**Proof of Proposition 8**

Obviously, \( \pi^*(\theta, G_w, b) \) is decreasing in \( \theta \) and increasing in \( w \). Hence \( w_i \) is increasing in \( \theta \).

Suppose \( w > y_i + 2\epsilon \). Then considering \( x = \epsilon \), we have

\[
\pi^* \geq \epsilon(b - \theta)F(y_i + \epsilon) - m[1 - F(y_i + \epsilon)].
\]

Hence \( \pi^* > 0 \) if

\[
\theta < \theta^* = b - \frac{m[1 - F(y_i + \epsilon)]}{\epsilon F(y_i + \epsilon)}.
\]

Thus, for any \( \epsilon, \theta < \theta^* \), implies \( w_i < y_i + 2\epsilon \).

For \( w \leq y_i \), \( \pi^* \leq y_i(b - \theta)F(w) - m[1 - F(w)] \). Therefore, \( y_i(b - \theta)F(w) \geq m[1 - F(w)] \), Thus, as \( \theta \to b, F(w) \to 1, \) and \( w_i \to y_i \). Q.E.D.

**Proof of Proposition 9**

i) First, \( W^* \geq w_i \) is obvious. Note that the optimization problem (P3) implies that \( W^*(\theta) \) solves

\[
\min_w H(w) = q(\theta) \pi^*(\theta, G_w, b)
\]

\[
+ cq(\theta)[1 - F(w - x^*(\theta, G_w, b))] + c[1 - q(\theta)][1 - F(w)],
\]

which implies the first-order condition

\[
\frac{\partial H}{\partial w} = q(\theta) \left[ \frac{\partial \pi^*}{\partial w} + cf(w - x^*) \left( 1 - \frac{\partial x^*}{\partial w} \right) \right] - c[1 - q(\theta)]f(w) = 0 \quad (A3)
\]

for \( w = W^*(\theta) \). Fix \( b \) and \( \theta \) and suppose that \( W^*(\theta) = w > y_i \). If we define

\[
x_u \in \arg\max_{x \geq 0} x(b - \theta)F(w - x) - m[1 - F(w - x)], \quad (A4)
\]

then \( \pi^* = x_u(b - \theta)F(w - x_u) - m[1 - F(w - x_u)] > 0 \). By the envelope theorem,

\[
\frac{\partial \pi^*}{\partial w} = [x_u(b - \theta) + m] f(w - x_u) = (b - \theta) F(w - x_u), \quad (A5)
\]
where the second equality follows from the first-order condition for $x_w$. Since $x^* = \pi^*/(b - \theta)$, this implies that

$$\frac{\partial x^*}{\partial w} = F(w - x_w).$$

(A6)

Finally, note that $\pi^* > 0$ also implies that $x_w(b - \theta) + m > m/F(w - x_w)$.

Since $w > y$, $f(w) = 0$ and

$$\frac{\partial H}{\partial w} = q(\theta)\{[x_w(b - \theta) + m]f(w - x_w) - cf(w - x^*)[1 - F(w - x_w)]\}
\begin{align*}
> q(\theta) & \left\{\frac{mf(w - x_w)}{F(w - x_w)} - cf(w - x^*)[1 - F(w - x_w)]\right\} \\
> q(\theta) & \left\{\frac{mf(w - x^*)}{F(w - x^*)} - cf(w - x^*)[1 - F(w - x_w)]\right\} \\
> q(\theta) & f(w - x^*)(m - c) > 0,
\end{align*}

where we use the fact that $x_w > x^*$ and $f(w)/F(w)$ is decreasing in $w$ by the MLRP. This contradicts (A3), and hence we must have $W^*(0) \leq y$.

ii) Suppose $q(\theta) = 1$. It is obvious that $W^* \geq w$. Consider a threshold $w > w^*$. Then $\pi^* > 0$ and $\partial H/\partial w = [x_w(b - \theta) + m]f(w - x_w) - cf(w - x^*)[1 - F(w - x_w)]$, which is positive as in the proof above. Hence $W^* = w$.

iii) Note that $\partial H/\partial w \leq q(\theta)(b - \theta) - c[1 - q(\theta)]f(w)$. Integrating this expression implies

$$H(y) - H(w) \leq q(\theta)(b - \theta)(y - w) - c[1 - q(\theta)][1 - F(w)]
\begin{align*}
& \leq q(\theta)(b - \theta)(y - w) - c[1 - q(\theta)][1 - F(w)].
\end{align*}

Since $W^*$ minimizes $H$, $1 - F(W^*) \leq q(\theta)(b - \theta)(y - w)/[c[1 - q(\theta)]].$

Thus, as $q(\theta) \to 0$, $W^* \to y$. Also, since $\pi^* \leq y(b - \theta)$, $q(\theta)\pi^* \to 0$ as well. Q.E.D.

Proof of Proposition 10

i) First, the previous results that $w_i \to y_i$ as $\Theta \to b$ and $W^* \leq y_i$ imply $W^* \to y_i$. Next, define $x_w$ as in (A4). If $W^* = w$, the first-order condition for (A4) implies that if $x_w > 0$, then

$$x_w = \frac{F(w - x_w)}{f(w - x_w)} - \frac{m}{b - \theta}.
$$

By the MLRP, $F(w - x_w)/f(w - x_w)$ is increasing in $w$ and decreasing in $x_w$. Hence, if

$$\frac{F(y - \epsilon)}{f(y - \epsilon)} - \frac{m}{b - \theta} \leq 0,$
then $x_\omega < \epsilon$. But for any $\epsilon > 0$, \[F(y_e - \epsilon) / f(y_e - \epsilon) - m/(b - \theta) < 0\] for $\theta$ sufficiently close to $b$. Therefore, $x_\omega \to 0$ as $\theta \to b$. The result follows since $\pi^* \leq x_\omega (b - \theta)$ and $x^* \leq x_\omega$.

ii) Suppose $W^* > y_\omega + 2\epsilon$. Then $\pi^* \geq \epsilon (b - \theta) F(y_e + \epsilon) - m [1 - F(y_e + \epsilon)]$. But since the regulator can prevent insider trade by always investigating ($w = y_\omega$) and paying $\epsilon$ for sure, the optimality of $W^*$ implies $\pi^* \leq c / q$. Therefore, for any $\epsilon$, if

$$b - \theta > \frac{c / q(\theta)}{\epsilon F(y_e + \epsilon)},$$

then $W^* \leq y_\omega + 2\epsilon$. Therefore, $W^* \to y_\omega$ as $\theta \to -\infty$.

Note that the definition of $x_\omega$ implies $x_\omega < W^* - y_\omega$ (otherwise the insider is surely investigated and pays $m$). Thus $x^* \leq x_\omega \to 0$.

Suppose $\pi^* > 0$. Then the first-order condition (A3) for $W^*$ together with (A5) and (A6) implies that, for $w = W^*$,

$$(b - \theta) F(w - x_\omega) = c f(w - x^*) [1 - F(w - x_\omega)] + c f(w) \left[ \frac{1 - q(\theta)}{q(\theta)} \right].$$

By the MLRP, the density $f$ is bounded and hence so is $(b - \theta) F(w - x_\omega)$. This implies that $\pi^* \leq x_\omega (b - \theta) F(w - x_\omega) \to 0$. Q.E.D.

References

- Easterbrook, Frank H. “Insider Trading as an Agency Problem.” In Princi-


