Majority Voting and Corporate Control: The Rule of the Dominant Shareholder

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This paper incorporates a model of corporate control into a general equilibrium framework for production economies with incomplete markets. The classical objective of value maximization is extended, but is indeterminate. Instead, firms are viewed as being subject to shareholder control via some decision mechanism. As long as this decision mechanism is responsive to a unanimous preference by shareholders, shareholder control is consistent with but stronger than value maximization. Next, the particular institution of majority voting by shareholders is examined. It is shown that for generic economies, a majority rule equilibrium for a firm implies that production is optimal for the largest, or dominant, shareholder. Finally, a more realistic control mechanism is considered in which majority voting by shareholders is constrained by a group of shareholders, or Board of Directors, who control the voting agenda. The result is that shareholders not on the Board have no influence on the equilibrium production choice of the firm.

1. INTRODUCTION

One striking characteristic of the modern public corporation is its widely distributed ownership. According to the neoclassical theory of the firm, such ownership simply reflects the efficient sharing of risk across a large number of individual shareholders. Though shareholders hold control rights to the firm in addition to their financial claim, these control rights have no value since there is no conflict regarding the production decision: profit maximization is unanimously supported by all shareholders.

The reality of the modern public corporation is much different from this neoclassical picture, however. One important source of conflict within the firm is that managers, who actively control the firm, may have different incentives than the shareholders. A large literature has developed exploring this source of conflict and the agency costs inherent in the relationship between owners and managers.

Another source of conflict, one which has received less theoretical attention but may be of significant practical importance, is disagreement between shareholders themselves. Though each shareholder may agree with "profit maximization" as an objective, they may not agree on the investment policies that will indeed maximize profits. In fact, the possibility of such disagreement is anticipated in the creation of the corporation itself, by installing the mechanism of majority voting to resolve such disputes. The widespread use of this mechanism to resolve shareholder disputes via "proxy battles", and the heterogeneity evident in the outcomes of these battles, substantiates the existence of this form of conflict.

The goal of this paper is to investigate both the source of this shareholder disagreement, and its resolution through the mechanism of majority voting. In particular, I characterize the relationship between the preferences of a firm's shareholders and its production objective.
The difficulty in investigating shareholder disagreement is of course that in standard Arrow-Debreu equilibrium models, disagreement does not occur. A natural source of disagreement emerges, however, if one relaxes a key assumption of the standard model: the completeness of markets. If markets are incomplete, "profit maximization" is no longer a well-defined objective for the firm, and shareholder disagreement may occur in equilibrium. Moreover, this source of disagreement seems quite natural and realistic. Investors typically have differing subjective assessments of investments due to the absence of markets. As a simple illustration, consider a car manufacturer's decision to invest in luxury or economy cars. Without futures markets for each type of car, individuals will in general disagree about future demand and market prices. Thus, there is no obvious objective criterion to determine how to invest—profit maximization becomes a subjective decision.

Other work concerning the objective of the firm with incomplete markets has primarily investigated the normative consequence of specific decision criteria. This originates with the work of Drèze (1974) who specifies an investment rule on efficiency grounds. Grossman and Hart (1979) propose a variant of the Drèze criterion for a multi-period model. A comparison and generalization of these approaches is provided by Milne and Starrett (1981). More recently, Duffie and Shafer (1986) have demonstrated that a form of value maximization by firms can lead to multiple, Pareto-ranked equilibria, and that shareholder disagreement is generic. Finally, Geanakoplos, Magill, Quinzii and Drèze (1987) have established that even the most promising decentralized decision procedure leads to generically inefficient allocations in stock market economies.

Surprisingly, although "one share-one vote" majority rule is perhaps the most common corporate control mechanism, little theoretical work has been done analysing its consequences. Gevers (1974) demonstrates that if the production decision is multi-dimensional, then there are economies which have no majority voting equilibrium. In fact, the results of Plott (1967) demonstrate that in pure voting games, generally no majority rule equilibrium exists unless preferences satisfy a very strict symmetry condition. Stock market economies, however, differ from standard voting environments in that votes can be traded and their distribution is typically uneven. This opens the possibility that in the case of corporate control, robust majority voting equilibria may exist. I show in this paper that although such equilibria sometimes exist, when they do they have the following strong characterization: production is optimal for the largest, or dominant, shareholder of the firm.

This result, while strong, is in the context of a model in which existence of an equilibrium is by no means guaranteed. In order to address this, I consider restrictions on the voting procedure that imply existence. In particular, I suppose that certain individuals have the power to determine which proposals are put before the shareholders. This "Agenda control" effectively gives these individuals veto power over the investment proposals. Drèze (1985) has considered majority voting in the firm when a Board of Directors has veto power and has shown that equilibria will exist in these models. This paper goes further by characterizing these equilibria. In fact, I show that (generically) equilibrium production is optimal for the firm's Board of Directors; that is, agenda control implies full control over the firm's investments.

1. Other means of guaranteeing existence have been proposed in the literature. Benninga and Muller (1979) make "spanning" assumptions, which Winter (1981) notes are quite restrictive and I show are non-generic. Sadanand and Williamson (1988) consider "direction restricted" majority voting, in which the production plan may only be modified along one dimension at a time. Equilibrium in that model depends upon the set of "basis" directions chosen. Also, it is not clear that such restrictions are commonly implemented.
2. PRODUCTION WITH INCOMPLETE MARKETS

The model consists of a two-period contingent-claim economy with financial assets denominated in a common numeraire (similar to that of Diamond (1967) or Drèze (1974)). For convenience, the numeraire commodity is the single consumption good in the economy. In the first period, the current state is known with certainty. The second period, however, is stochastic: the economy evolves into any of $S$ distinct states. Agents in the economy hold portfolios of the available assets as contingent claims on second period consumption.

A consumption plan in the economy is a column vector $x \in X = \mathbb{R}^{S+1}$, partitioned as $x_0 \in \mathbb{R}^+$, consumption in the first period, and $x_1 \in \mathbb{R}^S$, a contingent claim to second period consumption. That is, $x_{1s}$ denotes the quantity consumed in the second period if state $s \in S = \{1, \ldots, S\}$ occurs.

There are a finite number $h$ of individuals. Each agent $i \in H = \{1, \ldots, h\}$ has a utility function $u_i : X \to \mathbb{R}$ over consumption plans. Furthermore, each agent has an endowment $w_i \in X_i$ also partitioned as $w_{i0}$, the endowed first-period consumption, and $w_{i1}$, a stochastic second-period endowment.

Agents can trade $N$ available assets. Asset $j \in N = \{1, \ldots, N\}$ is described by its return vector $y^j \in \mathbb{R}^{S+1}$. Holding one share of asset $j$ entitles an agent to receive $y_{0j}$ in the first period, and $y_{1j}$ in the second period contingent upon state $s$. The $(S+1) \times N$ return matrix $Y$ is constructed from columns $y^j, j \in N$. The row vector $Y_{0j} \in \mathbb{R}^N$ and the $S \times N$ matrix $Y_1$ have similar interpretations.

Note that the assets above may be simple securities, representing a type of contingent contract agents can write and enforce. Alternatively, they may represent claims to productive activity; i.e. they may be shares of a firm. The set $J \subset N$ will be used to identify those assets which represent firm shares, and $Y^J$ is the matrix of columns of $Y$ corresponding to those firms. Thus the matrix $Y^J$ is regarded as the return matrix for firms once all production decisions have been made.

2.1. Agents' portfolio problem

Ideally, agents would like to trade arbitrary contingent claims to second-period consumption. They are constrained, however, to trade only the existing assets. Such trade takes place in a competitive market at the start of the first period. Once such trading has concluded, agents receive the payoffs to which they hold claim. Since there is a single consumption good, there is no need for a market for trade in the second period.

Each agent $i \in H$ has an initially allocated portfolio represented by the column vector $\tilde{\theta}^i \in \mathbb{R}^N$. The agent may trade this portfolio at competitive prices $\pi \in \mathbb{R}^N$ at the start of the first period, and so construct a new portfolio $\theta^i$ at a cost of $\pi(\tilde{\theta}^i - \theta^i)$. Finally, the agent receives the dividend stream $Y\theta^i$ corresponding to this new portfolio. Thus, each agent must solve the following portfolio problem:

\[
\begin{align*}
&\text{max}_{x, \theta} \quad u^i(x) \\
\text{subject to} \quad &x_0 \leq w_{0i} + \pi(\tilde{\theta}^i - \theta) + Y_{0i} \theta \\
&x_1 \leq w_{1i} + Y_{1i} \theta.
\end{align*}
\]

In order to characterize this problem in terms of marginal conditions, I make the following assumptions:
Assumption A. For each agent \( i \in H \), \((u^i, w^i, \theta^i)\) satisfy the following properties:

1. \( u^i \) is monotone and strictly differentiably concave on \( \mathbb{R}_{+}^{S} \),
2. \( \theta^i \geq 0 \), \( \theta_{N \setminus j} = 0 \),
3. \( w^i > 0 \),
4. If \( x > 0 \), and \( u^i(\tilde{x}) \geq u^i(x) \), then \( \tilde{x} > 0 \).

Assumption A1 is standard, and allows a first-order characterization. Assumption A2 corresponds to the interpretation of assets \( N \setminus j \) as zero net supply securities, whereas assets \( J \) represent shareholdings of firms. Finally, assumptions A3 and A4 guarantee that each agent’s optimal consumption plan is in the interior of the consumption set, thus eliminating tedious boundary conditions.

The monotonicity of the utility functions implies that the optimal consumption plan \( x^i \) of each agent is defined via the budget constraint once the portfolio choice \( \theta^i \) is known. Further, the differentiability and concavity of \( u^i \) allows each agent’s portfolio problem (P1) to be replaced with the first-order condition,

\[
D_{\theta} u^i(x^i) [Y_0 - \pi] + D_{x} u^i(x^i) Y_1 = 0.
\]

Now introduce the notation \( \nabla u^i = D_{\theta} u^i / D_{x} u^i \in \mathbb{R}_{+}^{S} \) to represent the marginal rate of substitution between first- and second-period consumption for an agent. Each component \( \nabla_s u^i \) may be thought of as the subjective “state price” at which agent \( i \) values output in state \( s \). Thus, the above condition may be rewritten,

\[
\pi = Y_0 + \nabla u^i(x^i) Y_1,
\]

so that current asset prices equal the discounted value of asset dividends, discounted according to agent \( i \)'s personal state prices.

This characterization motivates the definition of a particular subset \( Q \) of \( \mathbb{R}^S \):

\[
Q(\pi, Y) = \{ q \in \mathbb{R}_+^{S} : \pi = Y_0 + q Y_1 \}.
\]

That is, \( Q(\pi, Y) \) is the set of state prices which are consistent with current market prices. As Ross (1977) has shown, the existence of such state prices \( q \) guarantees that the securities market is “arbitrage-free”: there is no costless portfolio generating positive income in each state. Using this notation, the optimality condition for agent \( i \) can be rewritten \( \nabla u^i(x^i) \in Q(\pi, Y) \).

2.2. Production decision of the firm

The production choice for each firm is made at the conclusion of the first period. This decision specifies the level of a number of activities which determine output in the second period. By varying the level of these activities, the firm can alter its output across the possible states. Finally, different activity levels require different amounts of first-period investment.

Formally, each firm \( j \in J \) has access to a production technology involving a set of \( S_j \) different activities. At the end of the first period, the firm must choose a level for each activity, represented by \( v^j \in \mathbb{R}^{S} \). This requires some initial investment, given by the cost function \( C^j(v^j) \). In the second period, the output of the firm depends upon the activity levels chosen and the state of nature. This is given by the mapping \( A^j(v^j) \in \mathbb{R}^{S} \), where

2. Note that these state prices and utility functions imbed within them the agent’s subjective probability assessments of the various states.
$A_i(v^i)$ is the output in state $s$ given activities $v^i$. The firm has an initial endowment $t^j \in \mathbb{R}^{S+1}$, so that the production set of the firm can be described as

$$Y^j = \{ y \in \mathbb{R}^{S+1} : y_0 = t^j_0 - C_i(v), y_1 = t^j_1 + A_i(v), v \in \mathbb{R}^S \}.$$  

I make the following assumptions regarding the production technologies of the firms:

**Assumption B.** For each firm $j \in J$, $(C^j, A^j, t^j)$ satisfy the following properties:
1. $C^j$ is monotone and strictly differentially convex,
2. $A^j$ is monotone and differentiably concave, for all $s \in S$,
3. $C^j(0) = 0, A^j(0) = 0, t^j > 0$,
4. $D_v A^j(v)$ has full rank for all $v$,
5. $\|v_n\| \to \infty$ implies $\|A^j(v_n)\| \to \infty$.

In addition, there exists a $K$ such that
6. for any feasible $Y^j$, if $\sum_{i} \theta^j_i y^i + \sum_{i} w^i \geq 0$ then $\|Y^j\| < K$.

Assumptions B1 and B2 are quite standard, and correspond to convex costs and non-increasing returns for activities. Assumption B3 represents a "shutdown" option for each firm, and guarantees that firm shares have positive value. Assumption B4 states that the activities of the firm are "non-redundant", i.e. different marginal changes in activity levels imply different marginal output changes. This is useful since it implies that distinct proposals to change firm investment correspond to distinct output and profit streams. Finally, assumptions B5 and B6 are technical assumptions necessary for genericity results; together they imply that equilibrium production and activities can be bounded. Assumption B5 may be interpreted as a "productivity" assumption regarding investments—arbitrarily large investment leads to arbitrarily large output. Also, assumption B6 states that economy-wide feasibility of production plans rules out arbitrarily large production plans by individual firms, a natural restriction for an economy in which there are real resource constraints and production processes are not "perfectly reversible".

Given this production set, the issue of how a particular production choice $(y^j, v^j)$ is made is still unresolved. Consider first the case of effectively complete markets, in which any contingent claim can be constructed by an appropriate portfolio. In this case, there exist portfolios generating primitive "Arrow-securities" for each state. This is possible if and only if rank $(Y_i) = S$, which requires $N \geq S$. But then the set $Q(\pi, Y) = \{ q \}$, a singleton, and $q$ uniquely solves $\pi = Y_0 + q Y_1$. These state prices $q$ reflect the cost of constructing portfolios for each primitive security, and thus accurately represent the value of output in each state. Therefore, the profit maximization objective is naturally specified as

$$\max_{y \in Y^j} y_0 + q y_1.$$  

Unfortunately, unique state prices are not available when markets are incomplete. Indeed, the following assumption is maintained throughout the paper:

**Assumption C.** Markets are incomplete with $S > N$.

Under this assumption, the set $Q(\pi, Y)$ is a subset of $\mathbb{R}^S$ of at least dimension $S - N$, so that there exists a continuum of consistent state prices.

3. As a concrete example, suppose $S$ represents different weather conditions. Then $v^i$ might represent the amount of several different crops planted, with each variety having its own particular weather-sensitive yield characteristics.
A natural generalization of the standard Arrow-Debreu objective to the incomplete markets framework is to require firms to maximize the value of their output according to some consistent state prices. That is,

**Definition 1.** The production choice \((y^i, v^i)\) is **Value Maximizing** given \((\pi, Y)\) if it is a solution to

\[
\begin{align*}
\text{(P2)} & \quad \max_{y^i} y_0 + q^i y_1 \\
\text{where} & \quad y_0 = t_0^i - C^i(v) \\
& \quad y_1 = t_1^i + A^i(v)
\end{align*}
\]

for some consistent state prices \(q^i \in Q(\pi, Y)\).

Though this definition is a natural one, others are clearly possible. One alternative is to suppose that firms maximize their share price according to beliefs about how the market would value alternative production streams. Under the requirement that these beliefs never predict share prices permitting arbitrage, and the rational expectations assumption that these beliefs are supported in equilibrium, this objective can be shown to be equivalent to Definition 1 above (see Duffie and Shafer (1986) and DeMarzo (1988)).

Another plausible definition of the firms' objective is to suppose that firms simply compare their current production plan to alternatives which can be valued using market prices. That is, if second period output \(y^j\) is equivalent to the revenue \(Y^j, \theta^j\) generated by portfolio \(\theta^j\), that output must have a market value of \((\pi - Y^j)\theta^j\), the present net cost of the equivalent portfolio. Thus, firms' production choices should be value maximizing over all alternatives which can be priced in this fashion. It is easy to show that this objective is also equivalent to Definition 1 (see DeMarzo (1989)).

Finally, note that the value maximization problem (P2) can be simply characterized by the following necessary and sufficient first order condition:

\[
q^j D_v A^j(v^j) = D_v C^j(v^j), \quad \text{for some } q^j \in Q(\pi, Y).
\]

In words, the marginal cost of each activity must equal its marginal revenue, when discounted using state prices \(q^j\).

### 2.3. Equilibrium with value maximizing firms

The results of the previous section make it possible to endogenize the production decision of the firms and define an equilibrium in an economy with incomplete markets.

**Definition 2.** A **Value Maximizing Production Equilibrium (VMPE)** for the economy \(((u^i, w^i, \theta^j), (C^j, A^j, t^j), Y^{N/J})\) is a set of strategies, production plans, and asset prices \(((x^i, \theta^i), (y^j, v^j), \pi)\) such that

1. For all \(i \in H, (x^i, \theta^i)\) solves (P1) given \(\pi, Y, Y^{N/J}\).
2. For all \(j \in J, (y^j, v^j)\) solves (P2) given \(\pi, Y, Y^{N/J}\).
3. \(\sum_i x^i - w^i = Y \sum_i \bar{\theta}^i\).
4. \(\sum_i \theta^i = \sum_i \bar{\theta}^i\).

4. Because the set \(Q\) of allowable state prices depends on the production plans \(Y\), this requirement should be regarded as an equilibrium condition on firm beliefs.
The results of the previous sections enable the obvious characterization of any VMPE according to appropriate first order conditions, which are summarized here for convenience:

**Theorem 1.** A VMPE has the following characterization:
1. For all \( i \in H, \nabla u^i(x^i) \in Q(\pi, Y), \)
2. For all \( j \in J, q^jD_\sigma A^j(v^i) = D_\sigma C^j(v^i), \) for some \( q^j \in Q(\pi, Y), \)
3. \( \sum_i \theta^i = \sum_i \theta^i. \)

This characterization of the value maximization objective for the firm in terms of consistent state prices is quite useful. In DeMarzo (1988), I demonstrate that with such an objective, the Modigliani–Miller (MM) theorem regarding the irrelevance of corporate financial policy obtains. That paper also shows conditions for which the MM result extends to the shareholder control mechanisms to be considered later. Thus, if shareholder control conflicts exist in this model, they should be over production decisions alone, and financial policy can be ignored in this analysis.

As noted previously, the definition of VMPE corresponds to the usual Arrow–Debreu equilibrium for production economies when markets are complete. In that case, it must be that for all agents \( i \in H \) and firms \( j \in J, \)

\[
\nabla u^i(x^i) = q = q^j
\]

in equilibrium, so that agents and firms both value output in each state identically. This implies that all shareholders of the firm unanimously agree with the production choice made by the firm, production across firms is efficient, and the usual welfare results apply. Consequently, corporate control is not an issue in such models.

The situation is, of course, dramatically different when markets are incomplete. In general, agents’ state prices disagree with each other in equilibrium, due to differences in their endowments or in their subjective assessments of the likelihood of individual states. Hence, their evaluation of investment alternatives may also differ. Furthermore, shareholders’ evaluations are likely to differ from those of the firm, if the state prices \( q^j \) of the firm are arbitrarily chosen from the set \( Q. \) Because of this, VMPE allocations are in general inefficient, even in a constrained sense. In addition, the indeterminacy in the firms’ choice of state prices results in real indeterminacy of equilibrium allocations—in fact, in general there exists a continuum of Pareto-ranked equilibria (see Duffie and Shafer (1986)).

### 3. EQUILIBRIUM WITH SHAREHOLDER CONTROL

The definition of a VMPE in the previous section does not embody any relationship between the ownership of the firm and its investment policy. Indeed, when markets are incomplete the indeterminacy in firms’ state prices implies that production decisions are made irrespective of (and possibly contrary to) the preferences of their shareholders.

In actuality, modern firms are accountable to their shareholders. There typically exist mechanisms allowing shareholders to exercise control over the firm. A better notion of equilibrium, then, is one which captures this relationship by requiring that production plans be responsive to shareholder preferences. Thus, rather than modelling the firms as value maximizing with respect to arbitrary consistent state prices, condition 2 in the
definition of VMPE should be replaced by

2'. For all $j \in J$, the production plan $v^j$ is stable given the control mechanism; that is, there is no alternative plan which would be implemented by self-interested shareholders.

That is, equilibrium production $Y$ is such that, if shareholders choose optimal portfolios with the expectation that $Y$ is produced, their expectations are justified because these production plans will not be overturned given these shareholdings and preferences. To make this approach concrete, I next determine shareholder preferences over investment alternatives.

### 3.1. Shareholder production preferences

To establish the preferences of shareholders over production plans, it is necessary to specify how the agents are affected by a change in production. Given consumption $(x^i, \theta^i)$ for agent $i$ and an initial production plan $(y^j, v^j)$ for firm $j$, if the firm were to adopt some alternative plan $(\hat{y}^j, \hat{v}^j)$, the agent's new consumption would be $x^i + \theta^i(\hat{y}^j - y^j)$ in the absence of any portfolio changes. Of course, once the production change is announced, the shareholders may wish to change their shareholdings. But because portfolio rebalancing can only further increase utility (agents are not forced to trade), to show that agent $i$ is better off under the new production plan it is sufficient to show

$$u'(x^i + \theta^i(\hat{y}^j - y^j)) > u'(x^i).$$

A simple Taylor expansion of this condition leads to the following result:

**Theorem 2.** If the direction of deviation $dv^j$ for firm $j$'s activities satisfies

$$\theta^j[dv^j(\nabla u^i(x^i) A^j(v^j) - D_v C^j(v^j))] dv^j > 0,$$

for all $i \in B \subset H$, then for all sufficiently small $\alpha > 0$, if the firm were to adopt the new plan $v^j + \alpha dv^j$ the utility of every agent $i \in B$ will increase.

This condition can be interpreted quite naturally. Since agent $i$ values output in the second period according to state prices $\nabla u^i(x^i)$, the term $\nabla u^i A^j - D_v C^j$ represents the marginal "profit" associated with each activity when discounted by agent $i$. Thus, if agent $i$ holds a positive interest in the firm ($\theta^i > 0$), she would like the firm to increase those activities earning positive marginal profits and decrease those with negative profits. Short sellers of the firm would obviously like the firm to do the reverse.

The above proposition implies that as long as $\nabla u^i A^j - D_v C^j \neq 0$, shareholder $i$ has an incentive to modify the production of firm $j$. Recall from the definition of a VMPE that value maximizing firms choose production such that

$$q^j D_v A^j(v^j) - D_v C^j(v^j) = 0.$$

Since $\nabla u^i(x^i)$ need not equal $q^j$ if markets are incomplete, value maximization does not imply that shareholders are content with the firm's production plan.\(^7\)

5. This is equivalent to stating that shareholders have rational expectations regarding the decision mechanism of the firm.

6. While this condition is sufficient it may not be necessary. To determine necessity, one must specify agent's beliefs regarding the effect of the production change on asset prices. Under the assumption of competitive price perceptions (see Grossman and Hart (1979)), DeMarzo (1989) shows this condition is necessary for shareholder improvement as well, but this result is not needed for the analysis in this paper.

7. Note that smoothness of the production set is important here. If there were kinks in the production set, then at these kinks production might be optimal for an open set of state prices.
There is one circumstance, however, in which value maximization does imply unanimous shareholder support for a firm's production decision. Suppose that for any activity choice \( v' \), the output \( A'(v') \) is within the "span" of the existing securities; i.e. \( A'(v') = Y_1 \theta \) for some portfolio \( \theta \). In this case, the value of any alternative production plan can be uniquely determined using market information, so that all shareholders must agree with the value maximizing choice of the firm. This condition is referred to as a "spanning" condition, and its role in assuring shareholder unanimity was pointed out by Ekern and Wilson (1974) and Radner (1974). In the context of this paper, this spanning assumption implies that the marginal output \( DvA_i \) lies within the span of existing assets. Therefore, \( qDvA_i - DvC_i \) is independent of the choice of \( q \in Q(\pi, Y) \), and the value maximization condition of the firm also implies that the marginal profit associated with the firm's activities is zero when evaluated by every shareholder.

Although this spanning condition restores shareholder unanimity and eliminates the need to specify the control mechanism of the firm, it is an extremely strong assumption that is unlikely to be satisfied when markets are incomplete. Intuitively, the marginal output associated with any activity of the firm is unlikely to lie in the \( N \)-dimensional subspace of \( \mathbb{R}^S \) spanned by the securities. In fact, if \( N + S \leq S \), one might expect the securities payoffs \( Y_1 \) and the marginal output matrix \( DvA_i \) to be independent, so that security prices offer no information on how to discount activities. That is, the set

\[
\{qDvA_i(v') - DvC_i(v') | q \in Q(\pi, Y)\}
\]

is not a singleton (as with spanning) but equals \( \mathbb{R}^S \), implying no restriction whatsoever! Of course, if \( N + S > S \), the two cannot be independent and there is necessarily some relevant information in market prices, but again one might expect at least \( S - N \) dimensions of indeterminacy. In fact, I define this extreme alternative condition to spanning as follows:

**Definition 3.** The *No Spanning* condition holds given \((y', v')\) if for all firms \( j \in J \),

\[
\dim \{qDvA_i(v') - DvC_i(v') | q \in Q(\pi, Y)\} \geq \min [S, S - N].
\]

As I show in Theorem 7 of Section 4, this condition is satisfied for almost every economy in the class considered in this paper. Hence, shareholder disagreement over production decisions is expected in this model.

### 3.2. Shareholder control

It is now possible to reformulate the definition of an equilibrium to take account of the possibility of shareholder control. Because equilibrium production is likely to be sensitive to the actual control mechanism, in this section I propose a weak condition that should be satisfied by any reasonable mechanism.

A natural restriction on the corporate control mechanism is that only those shareholders holding a positive interest in the firm be given control rights. Because I have not imposed short sales restrictions, some shareholders of a firm may hold a negative interest in the firm. These shareholders have an interest in reducing the value of the firm. (Indeed, with free disposal, short-sellers would like the firm to produce zero output.) In reality, this problem is avoided since short-sellers are not given control rights, and the firm's managers are prohibited from short-selling. Thus, I define

\[ H^j = \{i \in H | \theta_i > 0\}, \]

as the set of agents who participate in the control of firm \( j \).
Another property to be expected of any reasonable control mechanism is that it be responsive to shareholders when there is unanimity. That is, if all shareholders in $H^j$ prefer an investment policy $\hat{\theta}^j$ to an existing plan $\nu^j$, it should be possible for the shareholders to exercise their control rights and implement $\hat{\theta}^j$. A control mechanism satisfying this property can be said to be Unanimity Responsive.

These two natural restrictions, though extremely weak, do have useful implications regarding the equilibria of the economy. In fact, together they are stronger than the requirement of value maximization by firms.

Theorem 3. Given the allocation $((x^i, \theta^i), (y^j, v^j))$, suppose the production plan of firm $j$ is stable with respect to some Unanimity Responsive control mechanism. Then the production plan solves (P2) for some $q^j$ in the convex hull of $\{\nabla u^i(x^i)| i \in H^j\}$.

Proof. Since the control mechanism is Unanimity Responsive, $v^j$ is stable implies that no $\nu^j$ exists which is preferred by all of the shareholders in $H^j$. From Proposition 2, this implies that there does not exist a $d\nu^j$ such that

$$\nabla u^i(x^i)D_v A^i(v^j) - D_v C^i(v^j) d\nu^j > 0, \forall i \in H^j.$$

By an alternative theorem, this is equivalent to the existence of non-negative weights $\lambda_i$ such that $\sum_{i \in H^j} \lambda_i = 1$ and

$$\sum_{i \in H^j} \lambda_i [\nabla u^i(x^i)D_v A^i(v^j) - D_v C^i(v^j)] = 0.$$

Thus, the result is established with $q^j = \sum_{i \in H^j} \lambda_i \nabla u^i(x^i)$.

Since the agents’ optimality condition implies $\nabla u^i(x^i) \in Q(\pi, Y)$ in equilibrium, the convex hull of these individual state prices is a subset of $Q$. Thus, shareholder control (via a unanimity responsive mechanism) is consistent with, and indeed stronger than, value maximization by firms when markets are incomplete.

4. CORPORATE CONTROL VIA MAJORITY VOTE

Now suppose that shareholders can influence the production plan of the firm if a majority (at least 50%) of the shareholders can agree on an alternative. This can be thought of as a shareholders’ meeting at which each share has one vote, and proposals are implemented if and only if they are majority preferred. In that case, we can define “stability” with respect to such a decision mechanism as follows:

Definition 4. Production choice $y^j$ by firm $j$ is Majority Stable in $H^j$ if there is no feasible alternative $\hat{y}^j$ which is preferred by at least one half of the shares in $H^j$.

Using the characterization of shareholder preferences provided by Proposition 2, it is possible to restate this definition in a more convenient form. First, introduce the notation,

$$P^i(d\nu^j) = \{i \in H^j : [\nabla u^i(x^i)D_v A^i(v^j) - D_v C^i(v^j)]d\nu^j > 0\},$$

8. In reality, of course, shareholders might not directly control production. One may view the control mechanism as a means for shareholders to install a manager who directs production on their behalf.


10. Note the tie-breaking rule used in this definition: only a weak majority is required to defeat the current plan. Since such ties are non-generic, the genericity results that follow are not sensitive to this assumption, and it is made to simplify the analysis.
Lemma 4. Production choice \((y^i, v^j)\) for firm \(j\) is Majority Stable in \(H^j\) only if for any changes \(dv^j\) to the activities of the firm,

\[
\sum_{i \in P^j(dv^j)} \theta_i^j < \frac{1}{2} \sum_{i \in H^j} \theta_i^j.
\]

Proof. By Proposition 2 there exists an alternative production plan \(v^j + \alpha dv^j\) which the members of \(P^j(dv^j)\) would prefer over \(v^j\). Hence, if these shareholders hold at least one half of the voting shares, this alternative would defeat \(v^j\) in a majority vote.

By adding the constraint that the production decision of each firm is Majority Stable, the majority voting mechanism defines a new notion of equilibrium. Since majority rule is obviously Unanimity Responsive, by the results of Section 3.2 we may define such equilibria as a subset of VMPE without loss of generality:

Definition 5. A Majority Rule Production Equilibrium (MRPE) is a VMPE such that for every firm \(j \in J\), production \((y^j, v^j)\) is Majority Stable in \(H^j\).

This definition implies “production-taking” behaviour on the part of shareholders. That is, each shareholder chooses a portfolio given beliefs about the production plan of each firm. In equilibrium, these beliefs are realized, since the production of each firm is not overturned by any majority coalition.

Unfortunately, it is difficult to characterize an MRPE in terms of simple first-order criteria. It is possible, however, to establish some important necessary conditions. First, define \(H^j(z)\) for any \(z \in \mathbb{R}^S\) to be the set of shareholders of firm \(j\) whose preference gradient regarding the firm’s activities are in direction \(\pm z\); that is,

\[
H^j(z) = \{i \in H^j : [\nabla u_i(x^i)D_eA^j(v^j) - D_eC^j(v^j)] = \lambda z, \lambda \in \mathbb{R}\}.
\]

Note that according to this definition, \(H^j(z)\) always contains \(H^j(0)\), the set of shareholders for whom \(\nabla u_iDA^j = DC^j\) (i.e. for whom the current activities of the firm are optimal).

I now give the following generalization of the Plott conditions for majority equilibria (Plott (1967)); see also Slutsky (1979) and Matthews (1980) for related results.

Theorem 5. Production \((y^j, v^j)\) for firm \(j\) is Majority Stable in \(H^j\) only if it is Majority Stable in \(H^j(z)\), for all \(z \in \mathbb{R}^S\).

Proof. See Theorem A2 in Appendix A. The proof shows by construction that if, for example, the direction \(+z\) contains at least half the votes in \(H^j(z)\), then these agents can form a majority coalition with the remaining shareholders and overturn the existing plan.

Using this result, it is possible to demonstrate that Majority Stability represents yet a further refinement of the set of equilibria:

Theorem 6. In any MRPE, for each firm \(j\), there exists some shareholder \(i \in H^j\) such that

\[
\nabla u_i(x^i)D_eA^j(v^j) - D_eC^j(v^j) = 0,
\]

so that production is optimal with respect to the state prices of shareholder \(i\).

Proof. Suppose the economy is at an MRPE. Then, from Theorem 5, for each firm \(j\), \((y^j, v^j)\) is Majority Stable in \(H^j(z)\), for any \(z \in \mathbb{R}^S\). But a deviation in direction \(dv^j = z\)
or in $dv^j = -z$ will gain half the votes in $H^j(z)$, unless $H^j(0) \neq \emptyset$. Thus, any $i \in H^j(0)$ establishes the result.\footnote{

11. Though this proof depends on the tie-breaking assumption, it is an easy matter to rule out ties for generic economies. Thus, the theorem still holds generically even if a strict majority is required to defeat the current plan. See the discussion of genericity below.

12. Though I do not address the issue of existence for this class of equilibria, generic existence can be easily established using the techniques of Duffie and Shafer (1986).

One way to interpret this result is to say that in a MRPE, the production plan of each firm looks as if it were chosen in the interests of some particular shareholder of the firm. It is useful to define the class of equilibria with this property separately:

**Definition 6.** A Designated Manager Production Equilibrium (DMPE) is a VMPE such that for each firm $j \in J$, there exists a manager $i(j) \in H^j$ such that production is optimal with respect to state prices $\nabla u^{i(j)}(x^{i(j)})$.

Note that for such DMPE, the firm can no longer choose $q_j$ arbitrarily from the positive-dimensional set $Q$. The firm's choice of $q_j$ is now determined by its manager. Since the indeterminacy of VMPE stemmed from the indeterminacy in the firm's choice of state prices, one might expect that such indeterminacy would not occur for DMPE. Intuitively, the model no longer has excess "degrees of freedom".

This intuition can be made precise by considering a generic analysis of the economy. A property of the economy is said to hold generically if it is true for an open, full measure set of endowments $((w^i), (t^i))$ and financial asset returns $Y^{N \times J}$. Generic properties can be thought of as occurring with probability one and as being robust to small perturbations of the economy. The following result establishes several useful generic properties of DMPE (and hence MRPE):

**Theorem 7.** DMPE are generically locally unique, satisfy No Spanning, and have full rank returns $Y_1$.\footnote{

Theorem C9 in DeMarzo (1990). The intuition is as follows. From Theorem 1, a DMPE implies that for every agent, $\pi = Y_0 + \nabla u^j(x^j) Y_1$. This system of $N$ equations determines the $N$ unknowns $\theta^j$ (which determine $x^j$ via the budget constraint). In addition, for every firm, $\nabla u^{i(j)}(x^{i(j)}) D_r A_r^j(v^j) = D_r C_r^j(v^j)$. This is a system of $S_j$ equations determining the $S_j$ unknowns $v^j$. Finally, the market-clearing condition, $\Sigma_i \theta^j = \Sigma_i \tilde{\theta}^j$, is a system of $N$ equations determining the $N$ unknowns $\pi$. Thus, the equilibrium conditions fully determine equilibrium consumption, production, and prices, so that generically the equilibrium is locally unique. Moreover, any additional restrictions, such as a drop in rank of the return matrix $Y_1$ or of the "augmented" matrix $[Y_1, D_r A_r^j]$ (if No Spanning is violated), would lead to an overdetermined system of equations that generically has no solution.

Thus we see that DMPE and MRPE allocations are (locally) determined. Also, the spanning conditions which guarantee unanimity cannot be expected to hold—in fact, the extreme opposite condition of No Spanning is generically satisfied.

To better understand MRPE, consider first firm for which $S_j = 1$. In this case, the production decision of the firm is a simple one dimensional choice: the amount to invest in its single production activity. It is clear that if the production choice of such a firm is Majority Stable, it must correspond to the preferences of the "median" shareholder with regard to this activity (see Black (1958)). This shareholder is therefore the one identified in Theorem 6 above.

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Similarly, suppose \( S - N = 1 \). Then the set \( Q(\pi, Y) \) is one-dimensional (generically) in a DMPE. Thus, there is only a single degree of freedom in the state prices of the individual agents, and preferences over firm production plans again vary over only a single dimension. The median voter characterization should therefore apply for every firm.

Such a characterization is not possible if \( S - N \geq 2 \) and \( S_j \geq 2 \). In this case, both the production decision and the set of consistent state prices \( Q \) are multi-dimensional. Thus, the median voter result does not apply to this environment unless shareholder preferences over production plans happen to be collinear. Otherwise, McKelvey (1976, 1979), Schofield (1978), Cohen (1979), and others have shown that majority voting is not transitive and can lead to cycling (as in the "Condorcet Paradox"). Moreover, this problem is so pervasive that in the standard one-person/one-vote environment, majority voting equilibria do not exist in general!

Before investigating the implications of these results for corporate governance, I first check to see if preferences generally "line up" in this model, in which case multi-dimensional cycling is avoided. The following result demonstrates, however, that such collinearity is non-generic:

**Theorem 8.** DMPE generically satisfy No Alignment of shareholder preference gradients; that is, for any \( j \in J \), if \( B \subset H \) with \( \#B = \min\{S_j, S - N\} \) and \( i(j) \in B \), then the preference gradients \( \{\nabla u^i DA^j - DC^j \mid i \in B\} \) are linearly independent.

**Proof.** See Theorem C9 in DeMarzo (1990). The intuition is identical to that of Theorem 7: the restriction that preferences are linearly dependent in equilibrium yields an overdetermined system of equations that cannot be satisfied generically.

Therefore, if \( S - N \geq 2 \) and \( S_j \geq 2 \), the decision problem faced by the firm and its shareholders is truly multi-dimensional, suggesting that existence of MRPE may be problematic. Unlike the standard voting literature, however, the distribution of votes across shareholders is endogenous in this model and, in general, unequal. The fact that individual shareholders may hold large numbers of votes might lead to majority-stability even though this would not occur under standard voting. For example, Figure 1 illustrates two cases in which majority-stability is possible (and robust to small changes in the parameters). Note, however, that neither of these would be an equilibrium under one person-one vote; rather, both examples achieve stability by virtue of the fact that production is optimal for a large blockholder of the firm.

The possibility of majority stability in this setting raises the question of how to characterize MRPE when they do exist—in particular, what is the relationship between equilibrium production and shareholder preferences? I now demonstrate that optimality for a large blockholder is in fact crucial for a robust MRPE:

**Theorem 9.** (Dominant Shareholder). Suppose \( S - N \geq 2 \) and \( S_j \geq 2 \). Then generically, if an MRPE exists the production of each firm \( j \) is optimal with respect to the state prices of its unique largest shareholder.

**Proof.** Consider the generic set of economies for which Theorem 8 holds. Since any MRPE is also a DMPE, the No Alignment property must also hold at any MRPE for these economies. Clearly, No Alignment implies that \( H^j(0) = \{i(j)\} \); i.e. the production of any firm \( j \) is optimal for a unique shareholder \( i(j) \in H^j \). Next, consider some other shareholder \( k \in H^j \). Since \( \min\{S_j, S - N\} \geq 2 \), No Alignment implies \( r \in H^j(\nabla u^k DA^j - DC^j) \) for any \( r \in H^j \setminus \{i(j), k\} \). Thus, \( H^j(\nabla u^k DA^j - DC^j) = \{i(j), k\} \). Then Theorem 5 requires that \((y^j, v^j)\) is Majority Stable in the set \( \{i(j), k\} \). But a proposal
in direction $\nabla u D A^i - D C^j$ fails to gain a majority in this set only if $\theta^i_j > \theta^j_k$. Thus, $i(j)$ must be the unique largest shareholder of firm $j$. 

The intuition for this result is that if the dominant shareholder were not satisfied with the firm’s production plan, she could find a majority coalition to support a proposal which would make her better off and the existing plan could not be majority stable. It does not imply, however, that a plan that is optimal for a firm’s dominant shareholder is necessarily a MRPE. This is only true if the dominant shareholder’s preferences are in some sense “modal” given equilibrium production and consumption. Simply put, optimality for the dominant shareholder is a (generically) necessary, but not sufficient, condition for a MRPE.

An alternative interpretation of Theorem 9 is the following: Suppose that many different distinct “types” of agents exist, with many or even a continuum of agents of each type. Each particular type is characterized by a commonality of preferences, endowments and expectations. In this case it is appropriate to describe economies by the characteristics of each type, rather than each individual agent. In this case then, if one considers a generic set of types for the economy, Theorem 9 states that in a MRPE production must be optimal for the largest (dominant) shareholder type. Further, because each individual shareholder is quite small (or even infinitesimal), portfolio changes by any single shareholder are unlikely to affect which type is dominant within the firm or the majority stability of the current plan. In this case, production is not influenced by individual portfolio choices and the assumption that shareholders are “production-takers” is truly justified.13

One implication of Theorem 9 is that in order to identify the set of MRPE for a generic economy, it is sufficient to examine the set of DMPE in which the designated

13. This interpretation might also be appropriate when considering the impact of small fixed costs associated with voting or production changes. With such frictions “almost identical” shareholders would not find it worthwhile to compete over small changes in the firm’s policy, and so could reasonably be regarded as belonging to a single shareholder type. The Theorem would then imply that the set of shareholders for whom production is “nearly” optimal should hold more votes than any other group of highly similar shareholders.
manager is the largest shareholder of the firm. If none of these DMPE is also majority stable, then no MRPE exists for this economy. Alternatively, if several such DMPE exist, there is the possibility of multiple MRPE.

As an illustration of this latter case, see again Figure 1. This depicts two MRPE associated with the production economy described in Appendix B. In this example, the firm faces a two parameter investment decision described by the vector \((v_1, v_2)\). In the first equilibrium, production is optimal for shareholder A who holds a majority (55%) stake in the firm. Though all other shareholders would prefer to shift investment from technology 2 to technology 1, A has full control of the firm. The share price in this equilibrium is 1.3. There is a second equilibrium, however, in which production is optimal for shareholder B, who is now dominant with 40% of the firm’s shares. Though shareholder B alone does not hold a majority of the firm’s shares, there is no majority coalition that can overturn this equilibrium. In this second equilibrium, the share price is 1.5, and the welfare of all shareholders except A is higher than in the first equilibrium. Besides illustrating the possibility of multiple MRPE, this example shows the importance of the endogeneity of shareholdings in determining the equilibrium.

5. AGENDA CONTROL IN VOTING PROCEDURES

The previous section investigated the implications of majority voting as a mechanism for shareholder control. The criterion of Majority Stability is quite strong, and so provides strong conclusions. It is well known, however, that majority voting equilibria often fail to exist. Though the endogeneity of shareholdings and the presence of large blockholders makes equilibrium more likely than in the standard legislative context, there are also examples of open sets of economies for which no such equilibrium exists, as was shown by Gevers (1974). The problem, again, can be traced to the intransitivity of Majority Voting in a multidimensional context, resulting in majority cycling.

Recent literature on legislative procedures and voting mechanisms (see, for example, Shepsle and Weingast (1981)) has pointed out that various structural aspects of the voting procedure may rule out such intransitivities and lead to stable outcomes. More realistic models of majority voting that take into account the determination of the voting agenda (i.e. the sequence of proposals voted on) no longer suffer from these problems of non-existence.

I study here the implications of a simple form of agenda determination for MRPE. Again suppose that firm shares are traded based on a proposed (or anticipated) production plan. After trading, some shareholder (or group of shareholders) with agenda control of the firm may make a proposal to change the production plan of the firm. This proposal is then voted on by the entire set of shareholders, and is implemented if a majority of the shares support it. Under such a mechanism, it is no longer necessary that a production plan be stable against all alternative plans, but only those that the agenda setter proposes.

In this setting, agenda control is equivalent to veto power over the set of considered proposals. Drèze (1985) considers such decision mechanisms, and shows that in the presence of short-sales restrictions, majority voting equilibria always exist in these models. Even with unlimited short-sales, generic existence could be proved as a natural extension of the results in Duffie and Shafer (1986) or Geanakoplos and Shafer (1990), though that is not the purpose here. Instead, I investigate whether the earlier characterization of MRPE is affected by this modification of the control mechanism.

14. See Drèze (1985) for an analysis of the existence of equilibria of this type.
5.1. Control by a board of directors

I consider the case in which the voting agenda of the firm is controlled by its board of directors. This is a somewhat more realistic description of the hierarchical control structure used by firms: shareholders are entitled to vote, but typically only on proposals made by the board. I assume that board members are either shareholders themselves, or act on behalf of certain shareholders. Further, I suppose that the board is governed by unanimity rule, so that each member of the board has veto power over any proposals.

Because the board of directors is a small group that meets directly, it is also natural to assume that the members of the board may bargain with each other over various proposals, and negotiate transfers (bribes?) between themselves to win acceptance of a proposal. Specifically, I allow them to propose transfer payments $t_j^i$, $i \in B^j$, that are feasible ($\sum_{i \in B^j} t_j^i = 0$) and conditional upon the acceptance of some alternative production plan. This leads to the following stability criterion for the firm's decision:

**Definition 7.** Production choice $y^j$ by firm $j$ is $B^j$-Constrained Majority Stable in $H^j$, where $B^j \subset H^j$, if there is no feasible alternative $\tilde{y}^j$ and feasible transfers $t_j^i$, $i \in B^j$, such that the proposal is preferred by all of the members of $B^j$ and by at least one half of the shares in $H^j$.

The following lemma demonstrates that, because of its ability to negotiate transfers, the Board effectively acts with unanimous preferences. Thus, it can be modelled as a single player:

**Lemma 10.** Production choice $(y^j, v^j)$ for firm $j$ is $B^j$-CMS in $H^j$ only if it is $b(j)$-CMS in $H^j = H^j \cup \{b(j)\} \setminus B^j$, where the pseudo-shareholder $b(j)$ has shareholdings and state prices defined by

$$
\theta_{j}^{b(j)} = \sum_{i \in B^j} \theta_{j}^i, \quad q^{b(j)} = \sum_{i \in B^j} \theta_{j}^i \nabla u'(x^i)/\sum_{i \in B^j} \theta_{j}^i.
$$

**Proof.** Suppose $y^j$ is not $b(j)$-CMS. Then there exists a production change $dy^j$ that is supported by $b(j)$ and gets half the votes in $H^j$. This implies $\theta^{b(j)}[dy^j_0 + q^{b(j)} dy^j_1] = T > 0$. If we define

$$
dt_j^i = \frac{T}{\# B^j} - \theta_{j}^i [dy^j_0 + \nabla u'(x^i) dy^j_1],
$$

it is clear that $\sum_{i \in B^j} dt_j^i = 0$, so that these transfers are feasible. Moreover, for each $i \in B^j$,

$$
\theta_{j}^i [dy^j_0 + \nabla u'(x^i) dy^j_1] + dt_j^i > 0,
$$

so that each board member is made better off by a change in direction $dy^j$ with transfers $dt_j^i$. But this implies that $y^j$ is not $B^j$-CMS. \\

This result implies that the preferences of the board can be viewed as those of single shareholder with state prices equal to the weighted average of the members state prices, where the weights are proportional to each members shareholdings. This criterion is

15. In practice, though explicit bribes may be illegal, implicit bribes via consulting fees, etc., are commonplace.

16. Note, however, that these transfers are not conditional on the state of the world. This is so since, in this model, all possible contingent trades should be represented in the asset return matrix.
similar to that proposed by Dreze (1974), with the restrictions that only board members preferences are considered, rather than the entire set of shareholders.

Given this result, I can now characterize (at least generically) the set of equilibria in which the production of each firm is majority stable with respect to proposals made by its board. I call such an equilibrium a **Board Constrained Majority Rule Production Equilibrium** (BC-MRPE).

**Theorem 11.** For generic economies with \( S \geq N + \sum_j S_j \), if for each firm \( j \in J \) with Board
\[ B_j, \]
both
\[ 1. \ S_j > 2, \text{ and,} \]
\[ 2. \ \theta_j^{b(j)} > \theta_j^i \text{ for all } i \in H^j \setminus B_j, \]
then in any BC-MRPE the production of each firm is optimal with respect to the state prices \( q^{b(j)} \) of its Board of Directors.

**Proof.** By Lemma 10, in a BC-MRPE the production of each firm must be \( b(j) \)-CMS in \( H^j \). Let \( z^j = q^{b(j)} DA^j - DC^j \). Then by Theorem A2 in Appendix A, production must be \( b(j) \)-CMS in \( H^j(z^j) \). If for some firm \( z^j \neq 0 \), then it must be the case that there exist shareholders with preference gradients in direction \(-z^j\), directly opposing \( b(j) \), whose cumulative shareholdings exceed \( \theta_j^{b(j)} \). Because the total shareholdings represented by the board are assumed to be larger than any other single shareholder, this implies that at least two shareholders must directly oppose the board. In Theorem C12 of DeMarzo (1990), however, it is shown that this type of joint opposition cannot occur generically if \( S_j > 2 \) (the required collinearity of preference gradients over-determines the equilibrium system). The technical condition \( S \geq N + \sum_j S_j \) is used to guarantee that Board members shareholdings (and hence \( q^{b(j)} \)) are uniquely defined. It is easily satisfied for economies in which firms have idiosyncratic technological risks, in which case \( S \) increases with the product \( \prod_j S_j \).

Thus, under the assumption that the Board, collectively, is the firm’s dominant shareholder, the theorem states that majority voting by shareholders generically plays no role: equilibrium production is optimal for each Board.

The intuition for this result is that if production were not optimal for the board, the board could find a proposal with majority support that made it better off. Moreover, one can show that the board can choose an appropriate sequence of proposals, each with majority support, such that the ultimate production plan adopted corresponds to the board’s optimum. This is illustrated in Figure 2. In this example, shareholder \( a \) has agenda control. Note that at each production proposal, \( a \) can form a coalition with one of the other shareholders to change the proposal to one that makes \( a \) better off. In fact, this continues until production is optimal for \( a \). The only way \( a \) would not be able to find such a coalition would be if both shareholders \( b \) and \( c \) directly opposed \( a \), but such collinearity of preferences is non-generic.

Of course, this sequence relies on the fact that shareholders consider each proposal myopically, and fail to anticipate the possibility of further proposals (both in their voting decision and their portfolio choices). There is also the assumption that conducting a vote is costless for the firm, and that the entire sequence can be executed before the firm’s investment decision must be made. This result is analogous to the cycling result of McKelvey (1976, 1979), who demonstrates that in a standard voting contest an agenda-setter can lead myopic voters to any outcome in the choice space.
6. CONCLUSION

In this paper I have developed a model of production economies with incomplete markets and investigated the issue of corporate control. The first sections of the paper demonstrated that unless an unrealistic “spanning” assumption is made, the standard profit maximization objective for the firm no longer makes sense, since output cannot be uniquely “priced” in the market. By allowing firms to conjecture some set of consistent state prices, however, the notion of value maximization was extended to incomplete markets. Most importantly, it was shown that any shareholder control mechanism that is responsive to a unanimous preference on the part of the shareholders will lead to production choices that are consistent with value maximization. This result may be useful in other contexts, since it implies that properties satisfied by equilibria in which firms are value-maximizing should also hold for any specific control mechanism.

Next I considered a very common type of control mechanism within the firm: majority vote by shareholders. Although results from the legislative voting literature lead one to expect that equilibria might not exist in general, the fact that the vote distribution is determined by endogenous and uneven shareholdings implies that MRPE can exist for some economies. Furthermore, a very strong characterization of such equilibria is possible. Although no “veto” players were in fact introduced, the equilibria which exist all look as if the largest shareholder has complete control over the firm.

Finally, the paper analysed the case in which a Board of Directors has agenda control within the firm. This restriction not only seems quite natural, but is important for existence of an equilibrium. Interestingly, if the Board can negotiate side payments amongst its members, it effectively acts with a single objective. Moreover, its control of the agenda implies that any majority rule equilibrium is optimal for the Board.

The results provided in this paper offer some justification to the notion that, in widely held firms, production decisions often seem to be at the complete discretion of the top management or Board of Directors, even though all shareholders appear to hold some voting power. This seems to suggest that firms may be quite unresponsive to the interests of a large number of their shareholders. Thus, it seems natural to consider another important type of control mechanism which operates by replacing these top managers and Board members: corporate takeovers. In fact, Grossman and Hart (1988) and Harris and Raviv (1988) have analysed the importance of one share–one vote majority mechan-

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FIGURE 2
A sequence of proposals by agenda setter a
isms in the context of corporate takeovers with complete markets. In their models, all shareholders agree with the firm's production decision, but conflicts arise due to private gains of control. It may be possible to use the analysis developed in this paper to investigate the takeover mechanism when shareholders disagree about production plans.

Another important extension of the results in this paper is to allow for a more general determination of the voting agenda. In practice, management, the board of directors, and shareholders each have the right to make proposals. One could consider the possibility of sequential voting on a series of such proposals. In this more complicated model, the possibility of trading shares (and thus voting rights) after the outcome of each vote must be considered, as well as the fact that shareholders are likely to act in a sophisticated manner and take into account the effect of the current vote on the ultimate equilibrium production choice.

APPENDIX

A. Major stability

In this section, I generalize the Plott (1967) conditions for a majority rule equilibrium. For related results see Slutsky (1979), Matthews (1980), and McKelvey and Schofield (1987).

Consider a majority voting game with a finite number of players \( i \in H \), each having \( v^i > 0 \) votes. Outcomes are chosen from the choice space \( Y \), an open subset of \( \mathbb{R}^k \). I include the possibility that one player has veto power over the proposals. There is a status quo outcome \( y \in Y \), and proposals to move to a new alternative \( y + dy, dy \in \mathbb{R}^k \) are considered. First define:

**Definition A.** The status-quo outcome \( y \in Y \) is \((i\text{-Constrained})\) Majority Stable in \( H \) if there is no alternative \( y + dy \in Y \) which is preferred by a set of players (including \( i \) and) holding at least one half the votes.

Players are assumed to have preferences at the status quo which can be represented by gradients \( \psi^i \in \mathbb{R}^k \). That is, agent \( i \) prefers \( y + dy \) to the status quo only if \( \psi^i dy > 0 \). Also, if \( \psi^i dy > 0 \), then for small enough \( \alpha > 0 \), agent \( i \) prefers \( y + \alpha dy \) to \( y \).

Let \( \sum_{i \in H} v^i = 1 \), and define \( P(dy) = \{ i \in H : \psi^i dy > 0 \} \) to be the set of players preferring movement in direction \( dy \). Also, let lowercase letters represent the number of votes in the corresponding sets, so that \( h = 1 \) and

\[
p(dy) = \sum_{i \in P(dy)} v^i.
\]

Thus we have the following obvious result:

**Lemma A1.** Given \((v^i, \psi^i)\), the status quo \( y \) is \((i\text{-Constrained})\) Majority Stable in \( H \) if and only if there does not exist \( dy \in \mathbb{R}^k \) such that \( p(dy) = \frac{1}{2} \) (and \( i \in P(dy) \)).

Finally, introduce the notation, for \( z \in \mathbb{R}^k \),

\[
H(z) = \{ i \in H : \psi^i = \lambda z, \lambda \in \mathbb{R} \}
\]

to represent the set of players whose preference gradients \( \psi^i \) are along the line \( z \). Note that \( i \in H(0) \) if and only if \( \psi^i = 0 \).

I now derive the following necessary condition for Majority Stability:

**Theorem A2.** Given \((v^i, \psi^i), i \in H\), the status quo \( y \) is Majority Stable in \( H \) only if it is Majority Stable in \( H(z) \), for all \( z \in \mathbb{R}^k \). Further, \( y \) is \((i\text{-Constrained})\) Majority Stable in \( H \) only if it is \((i\text{-Constrained})\) Majority Stable in \( H(\psi^i) \).

**Proof.** First consider the unconstrained case in which no player has veto power. Suppose \( y \) is defeated by \( \tilde{y} \) in \( H(z) \). This implies that the set

\[
H^+(z) = H(z) \cap P(\tilde{y} - y)
\]

contains at least half the votes in \( H(z) \); that is,

\[
h^+(z) \geq \frac{1}{2} h(z).
\]
First, choose $dy$ such that $zdy = 0$ and $\psi dy = 0$ if and only if $i \in H(z)$. Such a $dy$ always exists since the set $H$ is finite. It is now possible to partition the set $H$ as $H = P(dy) \cup P(-dy) \cup H(z)$. Therefore,

$$p(dy) + p(-dy) + h(z) = 1. \quad (2)$$

Next, let $dy_1 = -dy + \epsilon(y - y)$ for small enough $\epsilon > 0$ so that $P(dy_1) = P(-dy) \cup H^+(z)$. If $y$ is Majority Stable in $H$, then

$$p(dy_1) = p(-dy) + h^+(z) > \frac{1}{2}.$$

However, $(1)$, $(2)$, and $(3)$ together imply that

$$p(dy) + h^+(z) \geq \frac{1}{2}.$$

Thus, we can construct $dy_2 = dy + \epsilon(y - y)$ for small enough $\epsilon > 0$ so that $P(dy_2) = P(dy) \cup H^+(z)$ and $(4)$ implies

$$p(dy_2) \geq \frac{1}{2},$$

which, by Lemma A1, contradicts the stability of $y$ in $H$.

Finally, for the constrained case in which a single player $i$ has veto power, note that the logic of the above proof still holds in the case $z = \psi i$, since all winning coalitions constructed include $H^+(z)$, which contains $i$ by the initial hypothesis that $y$ defeats $y$ in $H(z)$.

Note that this result does not depend on the particular tie-breaking assumption used to define majority stability. If instead a strict majority were required to overturn the status quo, the above inequalities would switch from weak to strict and vice versa, but the result still holds.

**B. Multiple MRPE: an example**

In this section I describe an economy with equilibria depicted in Figure 1. There are three states in the second period, so that $S = 3$. There is a single firm whose shares represent the only asset. Thus $N = 1$ and markets are incomplete. Finally, there are four consumers, taken from the set $H = \{a, b, c, d\}$.

Consumers have Von Neumann–Morgenstern preferences given by

$$u'(x) = -e^{-x} - \frac{1}{3} \sum_{s=1}^{3} e^{-r'_{s}/r_{s}}.$$

Shares of the firm are initially distributed uniformly; that is, $\theta^i = 0.25$ for $i \in H$.

Finally, all agents have an endowment of one unit ($w_0 = 1$) in the first period. Second period endowments $w_1$ and preference parameters $r'$ are as follows:

- $w_1^a = (-0.3414, -2.005, -0.3031)$,
- $w_1^b = (4.321, 0.1513, -0.3177)$,
- $w_1^c = (18.59, -0.0587, -0.0944)$,
- $w_1^d = (39.12, 4.609, -0.1368)$,
- $r_1^a = (2.447, 0.0379, 6.263)$,
- $r_1^b = (0.0480, 2.058, 3.433)$,
- $r_1^c = (0.03816, 8.326, 5.771)$,
- $r_1^d = (0.0209, 0.0105, 19.71)$.

Next, we specify a two-dimensional production technology for the firm. The investment cost function is given by $C(v_1, v_2) = \frac{1}{2}(v_1^2 + v_2^2)$. The production technology is simply $A(v_1, v_2) = (v_1, v_2, 0)$. Finally, the firm has an initial endowment $t = (b, j, j, t)$. Given this specification, one can check that the following represent Majority Rule Production Equilibria:

**Equilibrium 1**

- **Production $p$**: $1.4$
- **Share price $\pi$**: $1.3$
- **Shareholdings ($\theta^i$)**: $(0.55, 0.25, 0.10, 0.10)$
- **Utility ($u'$)**: $(-9.46, -6.03, -4.63, -7.48)$

**Equilibrium 2**

- **Production $p$**: $4.1$
- **Share price $\pi$**: $1.5$
- **Shareholdings ($\theta^i$)**: $(0.25, 0.4, 0.20, 0.15)$
- **Utility ($u'$)**: $(-9.82, -5.82, -4.56, -7.43)$

Preference Gradients ($q'DA - DC$):

**Shareholder**

- **$a$**: $[0.0] [0.0]$ [0.0] [0.0]
- **$b$**: $[0.5] -0.35$ [0.0] [0.0]
- **$c$**: $[0.4] -0.38$ [0.0] [0.0]
- **$d$**: $[0.35] -0.25$ [0.01] [0.05]

Equilibrium investment and shareholder preference gradients are illustrated in Figure 1.
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