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# Optimal Incentive Contracts When Agents Can Save, Borrow, and Default[1]

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The standard Principal–Agent (PA) model assumes that the principal can control the agent's consumption profile. In an intertemporal setting, however, Rogerson (1985, *Econometrica* **53**, 69–76) shows that given the optimal PA contract, the agent has an unmet precautionary demand for savings. Thus the standard PA model is invalid if the agent has access to credit markets. In this paper we generalize the standard PA model to allow for saving and borrowing by the agent. We show that the impact of such access critically depends upon the treatment of default. If default is not permitted, efficiency is strictly reduced by the introduction of credit markets, and the equilibrium level of borrowing or saving is indeterminate in the model. If default is allowed, however, the optimal contract depends upon the level of bankruptcy protection in the economy, which is described by a minimum level of wage income. We show that there is an optimal intermediate range of bankruptcy protection. Within this range, allowing default increases efficiency in the economy relative to the case of no default. Also, the model predicts specific levels of consumer debt, interest rates, and default rates as functions of the level of bankruptcy protection level. *Journal of Economic Literature* Classification Numbers: D80, G21, G28, J30. 

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## 1. INTRODUCTION

As an empirical matter, most workers have the opportunity to adjust the timing of their consumption relative to the timing of their income. Even in the most primitive setting, workers can store nonperishable output. In industrialized market economies with well-developed financial markets, individuals have access to competitive markets for both saving and borrowing. In contrast with this observation, models of agency typically assume that by setting the wage, the principal (or firm) exercises complete control over the agent's (worker's) consumption profile. In this paper we extend the standard principal–agent model to allow for access to credit markets. In particular, we give the agent unrestricted access to saving, riskless borrowing, *and also* risky borrowing incorporating the possibility of default.

The starting point for our analysis is the well-known result, due to Rogerson (1985a), that in an intertemporal setting the optimal principal–agent (PA) contract derived without access to financial markets leaves the agent with a precautionary demand for saving. Riskless saving benefits a risk-averse agent by providing partial insurance against future wage uncertainty. The problem, of course, is that further insurance reduces overall efficiency because the agent, facing weaker incentives, works less. Thus, the solution to the standard PA model is *infeasible* in an environment with unrestricted access to saving. Given the importance of the PA model in modern economics, it therefore seems critical to extend the basic model to incorporate access to credit markets.

Alternatively, one could argue that the PA model is still valid if accompanied by restrictions on saving, as with the interlinking of credit and labor contracts in sharecropping (see Braverman and Stiglitz (1982)). Such restrictions are not commonly observed in modern economies and in any case would be difficult and costly to enforce. In addition, in order to determine the value of imposing such restrictions, one first needs a model of optimal contracts in their absence. We develop such a model and explore some of the consequences for contract design.

While extending the standard principal–agent model to include saving is straightforward, adding debt raises several interesting issues. Given a stochastic future wage, allowing the agent to borrow introduces the possibility of bankruptcy. Thus, the model must specify how assets are allocated in default. Moreover, since the probability of default depends indirectly on the agent's incentives, lenders may also have an interest in restricting the agent's access to credit markets. Rather than make a single specific assumption regarding these issues, we develop a model that is general enough to consider a variety of alternative scenarios. Doing so allows us to address a number of important questions, including: What effect does the introduction of saving and riskless borrowing have on the principal-agent model when bankruptcy is precluded? What is the effect of risky debt with bankruptcy possibilities? Do there exist bankruptcy protection levels such that permitting bankruptcy enhances efficiency? What effect does bankruptcy protection have on labor contracts, interest rates, default rates, and debt levels?

First, we avoid the complication of default and introduce saving and riskless borrowing into the model. In this case, the agent's maximum total debt is limited by the agent's minimum future wage. The agent's saving/borrowing decision adds a new incentive compatibility constraint into the standard PA model. Moreover, this constraint is binding, and the agent's equilibrium utility is decreased relative to the standard model. Though the opportunity to save has an impact on equilibrium, no saving or borrowing need be observed in equilibrium since any riskless transfers can be offered directly through the labor contract.

Next, we introduce risky debt and the possibility of default to the model. We assume that there is a minimal level $\lambda$ of wage income which is protected in bankruptcy. For $\lambda = 0$, there is no bankruptcy protection, and the agent defaults only when it is infeasible to repay the loan. For $\lambda > 0$, absolute priority is violated and the agent can retain some wages in default. We are interested in the effect of different levels of bankruptcy protection on equilibrium outcomes.

As mentioned earlier, in this setting lenders are also concerned with the agent's incentives and the impact that further borrowing or lending might have on these incentives. We consider two alternative environments. First, we analyze the case of *exclusive* debt, in which the lender can renegotiate the loan terms if the agent borrows or saves in the future. We also consider the case of *non-exclusive* debt, as in Bizer and DeMarzo (1992), in which lenders have no ability to monitor or restrict future borrowing or saving by the agent. In both settings, we introduce a methodological technique that allow us to solve for the optimal contract in certain cases without solving explicitly the implied extensive form games, which are analytically intractable in general.

Our main results are as follows. We find that in general there exist bankruptcy protection levels for which the introduction of risky debt increases efficiency relative to the case of allowing only riskless transfers. In particular, it is important that bankruptcy protection is neither too strong nor too weak, but lies within an intermediate range. Within this range however, the exact level of bankruptcy protection is unimportant for efficiency. While this is true for both exclusive and non-exclusive debt, the efficient range for the bankruptcy protection level is reduced in the case of non-exclusive debt.

Unlike the case of riskless debt, we find that equlibrium wage contracts and banking activity are uniquely determined in this setting. Wage contracts differ from those predicted by the standard principal-agent model in that they are back-loaded; that is, the agent's first period wage is reduced and the average second-period wage is higher than in the PA model. Given such a back-loaded wage, the agent borrows in equilibrium in order to smooth his consumption. This borrowing creates a "debt overhang" (similar to Myers (1977)) which eliminates the agent's incentive to save and self-insure.[2]

[2] That is, enough of the benefits of saving would be captured by the debt holders in bankruptcy to make it unattractive for the agent.

Our model also generates predictions regarding the level of interest rates, default rates, and consumer debt as a function of the level of bankruptcy protection. We show that while marginal interest rates and default rates are naturally increasing in the level of bankruptcy protection, debt levels may either increase or decrease.

Related papers by Fudenberg *et al.* (1990), and Allen (1985) embed banking in a moral hazard problem and investigate whether there exist advantages to long-term versus short-term contracts that cannot be explained by intertemporal consumption smoothing. These models assume that there is no possibility of default and hence all loans are at the risk-free rate. As we demonstrate, however, in the absence of default the labor contracts are Pareto inferior to those derived with risky debt. In a paper analyzing the implications of moral hazard for Ricardian equivalence, Rotemberg (1987) demonstrates that for specific parameter values, borrowing with bankruptcy can offset government saving and maintain work incentives. His paper does not, however, explore the full equilibrium relationship between the firm, the worker, and the financial sector. Haltiwanger and Waldman (1986) introduce alternative capital market assumptions, including risky debt, into a labor contracting problem, but in their paper the agent's productivity is exogenously determined, so moral hazard is not at issue. In the related context of insurance, Arnott and Stiglitz (1991) analyze how the market for insurance may be adversely affected by the agent's ability to obtain supplementary insurance from family and friends.

More generally, this paper is related to the common agency literature introduced by Bernheim and Whinston (1986). That literature explores a situation in which several principals attempt to influence the action of a single agent. In this paper, the firm that employs the worker, as well as the banks that extend loans to the worker, are all concerned about the worker's effort decision and its impact on their profits. Thus, they can be intrepreted as multiple principals for the common worker.[3] In our setting, however, the contracts are entered into sequentially rather than simultaneously. In that sense it is more closely related to the models of sequential contracting in Bizer and DeMarzo (1992) and DeMarzo and Bizer (1994).

Finally, there is a separate literature in incomplete markets in which the beneficial role of debt with default is considered. Zame (1993) and Dubey *et al.* (1988) show that when markets are incomplete, requiring agents to fulfill all obligations can severely limit contingent contracting. Allowing default increases the set of feasible trades and can enhance welfare.

The rest of the paper is organized as follows. In Section 2 we introduce a standard two-period version of the principal–agent model and show that at its solution, the agent has an unmet desire to save at the market interest rate. Section 3 then extends the model to allow for banking (borrowing and saving) by the agent that cannot be controlled by the principal. Since this adds additional incentive constraints, welfare is weakly reduced by allowing banking. Section 4 considers the case in which only riskless borrowing is allowed, and shows that the agent is strictly worse

[3] Our setting differs from the standard model of common agency in that the bank only cares about the worker's effort indirectly due to its effect on wages.

off than in the standard PA model. Sections 5 and 6 then show that efficiency can be improved by allowing risky borrowing coupled with bankruptcy protection. Section 5 considers the case of exclusive debt in which the agent borrows from at most one lender, and Section 6 allows unrestricted borrowing from many lenders. Finally, Section 7 concludes the paper, summarizing the results and suggesting areas for further research.

## 2. THE STANDARD PRINCIPAL–AGENT PROBLEM

To highlight the implications of access to credit markets, we first formulate the model in the standard principal–agent setting. A profit-maximizing, risk-neutral principal employs a risk-averse worker to undertake a project that requires the worker's labor, but worker effort is unobservable. Therefore, a contract must be designed that balances the objective of providing work incentives against the objective of efficiently assigning risk. We assume that there is competition among principals (or that the agent chooses the contract), so that the observed contract is the one most preferred by the agent that yields non-negative profits for the principal. (This is not critical; see footnote 6.)

To simplify the intertemporal issues presented, we restrict attention to a two-period model. Upon accepting a contract, the agent works in the first period, earns a wage, and consumes. In the second period, the agent works, and profits of the project are realized. The agent is then paid a wage and consumes. The payoff of the project is given by a bounded random variable $X$, whose distribution is affected by the agent's effort. Since the principal only observes the payoff $X$, and only in the second period, the labor contract takes the form of the sharing rule $S = (s_1, s_2(\cdot))$. That is, the contract consists of a payment $s_1$ by the firm to the worker in the first period, and a second-period wage schedule $s_2(X)$ that is contingent on the project outcome.[4] The worker's preferences are represented by an additively separable Von Neumann–Morgenstern utility function,

$$U(c_1, c_2, e) = u_1(c_1) + \beta E[u_2(c_2) \mid e] - V(e),$$

where $c_t \geq 0$ is consumption expenditures in period $t$ and $e \geq 0$ represents the total effort expended. Note that we can interpret $u_t$ as an indirect utility function over period $t$ income; we assume that $u_t$ is increasing and strictly differentiably concave. Also, we can interpret effort $e$ as occurring soley in period one, or solely in period two, or spread out across the periods; the precise timing is not important. $V(e)$ represents the disutility of effort, where $V$ is increasing and strictly differentiably convex, with $V'(0) = 0$. Finally, the expectation is conditional upon effort since the distribution $F(x, e)$ of the project outcome depends upon $e$. In general, we

[4] The model can be generalized to the case in which the project produces output in both periods so that the first period wage is also stochastic. All of our results still apply state by state according to the first period outcome.

think of higher effort leading to higher output, though we make no such explicit assumption at this point. The distribution $F$ may be either continuous, discrete, or mixed, but we assume the support is independent of effort. Let $f(x, e)$ be either the density or the probability mass of outcome $x$ given effort $e$ as appropriate. In addition, we assume that $f_e(x, e)$ exists and is continuous in $e$. Letting $w$ be the agent's initial wealth, if the agent accepts the contract $S$, consumption is given by[5]

$$c_1 = w + s_1, \qquad c_2 = s_2.$$

Note that all of the agent's initial wealth is consumed in period one, since at this point we have not introduced an explicit saving technology. Of course, the agent can save indirectly through the labor contract, by setting $s_1 < 0$. The consequences of alternative saving options are the heart of this paper, and they will be introduced in Section 3.

Letting $r$ represent the competitive interest rate, the standard principal–agent contracting problem, which we denote by (PA), is given by:

$$\begin{array}{lll}
\text{(PA)} & \max_{S,e} & U(c_1, c_2, e) \\
& \text{subject to} & s_1 \leq \frac{1}{1+r} E[X - s_2(X) \mid e], \\
& & e \in \arg\max_{\hat{e}} U(c_1, c_2, \hat{e}), \\
& & c_1 = w + s_1, \quad c_2 = s_2.
\end{array}$$

The first constraint is the principal's participation constraint, since the risk-neutral firm should earn non-negative profits.[6] The second constraint ensures incentive compatibility for the agent's effort choice. As is well known, the solution to this problem involves an optimal tradeoff between work incentives and risk-sharing. We denote the optimal allocation and effort resulting from (PA) by $c^* = (c_1^*, c_2^*)$ and $e^*$, respectively. To facilitate later comparisons, we assume that this solution is unique (up to changes on sets of measure zero) and interior (i.e., that $c^* > 0$ and $e^* > 0$).

A similar formulation of a multiperiod principal–agent problem has been considered by Rogerson (1985a). In that article, Rogerson suggests that the optimal principal–agent contract is feasible only if the agent's access to credit is severely curtailed. In particular, Rogerson shows that at the solution to (PA), the agent has an unmet demand to save some of his first-period wage for consumption in the second period. We confirm this result in the context of our model:

PROPOSITION 1 (Rogerson, 1985a). *At the optimal solution to* (*PA*), *the agent wishes to save at the interest rate* $r$.

[5] Here and throughout the paper, we write $s_2$ to represent the random variable given by $s_2(X)$.

[6] We have written the problem to maximize the agent's utility, although we could also have written it to maximize the principal's profits subject to a participation constraint for the agent. This would not alter our results.

*Proof.* At a solution to (PA), the optimal sharing rule must satisfy the following first-order condition (see Rogerson, 1985a, Proposition 1):

$$\frac{1}{u_1'(c_1^*)} = \frac{1}{1+r} E \frac{1}{\beta u_2'(c_2^*)}. \tag{1}$$

We can interpret the LHS of (1) as the marginal cost to the principal of increasing the agent's first-period utility, and the RHS as the marginal cost of increasing the agent's second-period utility through a uniform increase of the second-period payment. Since uniform changes to $s_2$ do not change incentives, these marginal costs must be equated at the solution.

Since $e^* > 0$, the incentive compatibility constraint implies that the second-period wage (and hence consumption) must not be constant across states. Therefore, the strict form of Jensen's inequality requires

$$u_1'(c_1^*) < (1+r) E \beta u_2'(c_2^*). \tag{2}$$

Hence $(1+r)$ exceeds the agent's marginal rate of substitution $u_1'/E[\beta u_2']$, indicating that the agent would like to save at the market interest rate $r$. ■

The intuition for this result is clear. Since the agent's utility function is concave, the marginal impact of an additional dollar of income is higher at lower levels of consumption. Therefore, it is easier for the firm to provide work incentives for the agent if second-period consumption is lower overall. Thus, the need to provide such incentives distorts second-period consumption downward, leaving the agent with a "precautionary" demand for savings.

An implication of this result, however, is that in many standard economic environments, the second-best allocation may not be achievable. That is, since labor contracts do not in practice impose constraints on savings, the agent should be able to save at the competitive, risk-free rate $r$. The theorem implies, however, that any optimal (PA) consumption profile is not consistent with the agent's ability to save. Therefore, we have

COROLLARY. *If the agent can save at the risk-free rate r, the solution to* (*PA*) (*i.e., the second-best allocation*) *cannot be attained.*

Since incorporating the desired additional saving directly into the labor contract (by reducing $s_1$ and increasing $s_2$) is not in the firm's interest, it must be the case that such saving alters work incentives and reduces firm profits. Thus, if the agent can save after the labor contract is signed, he or she can impose an externality on the firm, as saving alters the incentive scheme of the original contract and acts as a form of insurance for the agent against the second-period outcome. Of course, this externality is imposed only if the firm insists on offering the (PA) contract. If instead the firm knows that the agent has access to a bank, then it will take this into account when designing the labor contract. In the absence of restrictions on savings, then, the standard principal–agent model is an inadequate model of labor contracts.

In the following section, we extend the (PA) model to incorporate banking and to explore the implications of banking for consumption, production, and the form of optimal contracts. The major result is that bankruptcy protection can enhance efficiency. Before demonstrating this formally, it is useful to sketch the intuition here. As shown above, agency costs are minimized by pushing the agent's marginal rate of substitution so that (2) holds. However, if only riskless saving and riskless borrowing is permitted, then at the agent's optimal saving/borrowing decision, the agent's intertemporal Euler condition is

$$u_1'(c_1^*) \geq (1+r)E\beta u_2'(c_2^*),$$

where the inequality may be strict since the agent may be constrained on the amount of borrowing permitted (in order for it to remain riskless). This contradicts (2). On the other hand, if risky borrowing is permitted, then the agent's intertemporal Euler condition is

$$u_1'(c_1^*) = (1+r)E[\beta u_2'(c_2^*) \mid \text{no default}],$$

since in the presence of a debt overhang, saving (or borrowing less) only increases consumption in those states in which the agent is not bankrupt. Since those are states in which consumption is high and marginal utility is low, this implies that (2) holds. That is, risky borrowing makes it possible to "push" the agent's marginal rate of substitution in the direction that reduces the agency problem. However, risky borrowing will not enhance efficiency if it requires the risk-averse agent to suffer very low consumption in the future. Hence, the combination of risky borrowing with adequate bankruptcy protection is optimal.

## 3. THE PA MODEL WITH BANKING

We now extend the standard principal–agent model to incorporate the possibility of saving and borrowing by the agent. In this section we construct a general model that allows the agent to enter into an arbitrary number of savings or loan agreements with different, competitive banks.

In particular, we suppose the following sequence of events: first, the worker and the firm sign a labor contract $S$ as described in (PA). After this contract is signed, however, the agent may approach a bank and secure a loan agreement $(l, d)$. The amount $l$ represents the loan received by the agent in the first period, with $l < 0$ corresponding to saving. The agent then promises to repay the amount $d$ in the second period (again, $d < 0$ corresponds to withdrawal of savings). In fact, the agent may open several accounts with different banks; we index banks by $j$ and let $(l_j, d_j)$ be the agreement with bank $j$.

Finally, after securing a set of loan agreements with the banks, the agent works on the project and chooses a particular level of effort. In the second period, the outcome of the project $X$ is realized. The firm then pays the agent according to the labor contract $S$, and the agent, in turn, repays the banks according to his or her loan obligations.

Several assumptions are embedded in this formulation. First, we allow the agent to procure any loan contract that is acceptable to the bank. That is, we do not allow the firm to restrict the agent's access to banking, either explicitly or implicitly via the labor contract. This corresponds to our goal of exploring the impact of unrestricted banking on labor contracts. Second, we consider only standard debt contracts and do not permit the repayment $d$ to depend explicitly on the project outcome.[7] In essence, we assume that only the firm and the worker write contracts on the project outcome; courts will not enforce general state-contingent contracts written by banks. This is consistent with the types of contracts that are observed in practice.[8]

Given some set of loan contracts, we must determine the agent's consumption. Let $(l, d) = \sum_j (l_j, d_j)$. Then the agent's consumption in the first period is given by

$$c_1 = w + s_1 + l. \tag{3}$$

If default were not possible, then consumption in the second period would simply be $c_2 = s_2 - d$ and all loans would be risk-free. In reality, of course, default is possible and is observed. In fact, full repayment is not feasible if $d > s_2$. Thus loans may be risky. We incorporate this feature into our model of banking by assuming that a bankruptcy law exists allowing the agent to declare bankruptcy and avoid repayment of the loan. We assume that in bankruptcy, the agent's assets are seized, together with any wages in excess of some minimum "protected" level, denoted by $\lambda$. For $\lambda > 0$, this is interpreted as a bankruptcy protection law,[9] while for $\lambda = 0$, this restriction simply embodies the feasibility constraint on repayment. Values of $\lambda > 0$ might correspond, for example, to the fact that many states protect debtors from extreme hardship by assuring them of a minimal level of wages.

[7] Note, however, that though debt contracts are not explicitly state-dependent, this does not imply that the actual repayment will be independent of the project outcome once we allow for default.

[8] Presumably, this is true since it is very costly for banks and courts to enforce more general state-contingent contracts. One could try to model this along the lines of the costly-state-verification literature. While debt has an advantage in lowering verification costs, it is not clear that standard debt would be optimal in this setting. We do not pursue this optimal security design problem here, but instead restrict ourselves to the empirically relevant case of standard debt contracts.

[9] This roughly corresponds to the procedure in Chapter 7 bankruptcy.

Given the bankruptcy protection level $\lambda$, an agent with contract $S$ and aggregate debts $d$ has second-period consumption given by

$$c_2(d, S) = \max[s_2 - d, \min(s_2, \lambda)], \tag{4}$$

since the agent has the option of either repaying the debt or declaring bankruptcy and keeping at most $\lambda$.[10] Of course, the difference between the agent's wage $s_2$ and actual consumption $c_2$ represents the amount repaid to the banks. Thus, given $S$ and $d$, the actual aggregate repayment made to the banks is given by

$$s_2 - c_2 = \min[d, \max(s_2 - \lambda, 0)]. \tag{5}$$

Clearly, unless $d \leq \max(s_2 - \lambda, 0)$ with probability one, the repayments made by the agent depend upon the realization of the project outcome $x$. In this case, at least one bank must hold a risky loan.

Without further assumptions it is difficult to characterize the precise set of loan contracts which are available to the agent. Given the equilibrium strategy of the banks, however, there will be *some* set of aggregate loan contracts the agent can obtain by visiting an arbitrary number of banks. We adopt the notation

$$L(S) \equiv \left\{(l, d) : (l, d) = \sum (l_j, d_j) \text{ available in equilibrium given wages } S\right\}.$$

For now, we do not further characterize the set $L(S)$. We will characterize this set throughout the remainder of the paper under a variety of restrictions.

Thus, after signing the labor contract $S$, the agent can then approach the banks and, via some combination of contracts, obtain any aggregate loan in $L$. After signing these loans, the agent then works on the project and chooses an optimal level of effort. Obviously, in equilibrium the agent will choose the loans and effort to solve the following optimization problem:

$$\begin{array}{lll} B(S, L): & \max_{l,d,e} & U(c_1, c_2, e), \\ & \text{subject to} & (l, d) \in L(S), \\ & & c_1 = w + s_1 + l, \\ & & c_2 = \max[s_2 - d, \min(s_2, \lambda)]. \end{array}$$

That is, an equilibrium in the banking subgame should be a solution to $B(S, L)$.

[10] Alternatively, we could model bankruptcy protection as a guarantee against losing more than some fraction of income (for example, federal statutes require forfeiture of at most 25% of income). Then (4) would become $c_2 = \max[s_2 - d, \delta s_2]$. This would not substantially alter our main results. Another implicit assumption in this formulation is that the wage payment $s_2$ is nonnegative. Relaxing this assumption and accounting for feasibility implies $c_2 = \max[s_2 - d, \min(s_2, \lambda), 0]$. Our results are also robust to this generalization, though we omit it for simplicity.

Note that effort is chosen by the agent after the aggregate loan contract is determined. Hence at a solution to B($S$, $L$), effort satisfies

$$e \in \arg\max_{\hat{e}} U(c_1, c_2, \hat{e}). \tag{6}$$

Because of the time separability of utility, optimal effort only depends on the distribution of $c_2$. We can write this as a function $e(c_2(d, S))$, where we assume an arbitrary (measurable) selection if the optimum is not unique.

We assume that the financial sector is competitive, is risk neutral, and faces the competitive interest rate $r$.[11] In equilibrium, banks will only accept loan contracts on which they expect to earn non-negative profits. Because this is true for each individual loan contract, it must be true for the aggregate loan contract as well. Therefore, at the equilibrium solution $(l, d, e)$ to the banking subgame $B(S, L)$, we must have

$$l \leq \frac{1}{1+r} E[\min[d, \max(s_2 - \lambda, 0)] \mid e]. \tag{7}$$

That is, the expected discounted value of the total repayment should exceed the aggregate loan extended to the agent.

Having described the banking subgame, we can now describe the optimal contracting problem faced by the firm when the agent cannot be prevented from engaging in outside saving or borrowing:

$$\text{(PAB)} \qquad \begin{array}{ll} \max_{S,l,d,e} & U(c_1, c_2, e), \\ \text{subject to} & s_1 \leq \frac{1}{1+r} E[X - s_2(X) \mid e], \\ & (l, d, e) \in \arg\max B(S, L), \\ & c_1 = w + s_1 + l, \\ & c_2 = \max[s_2 - d, \min(s_2, \lambda)]. \end{array}$$

Note that the objective function, and the firm's participation constraint are identical to those of the standard principal–agent problem (PA). Also, since $B(S, L)$ implies (6), the incentive compatibility constraint on effort is also equivalent. The key difference between (PAB) and (PA) results from the additional incentive compatibility constraint for the loan contract; under (PAB) the agent must also be at an optimum with regard to his or her banking opportunities. The fact that (PAB) embodies additional constraints suggests the following obvious result:

PROPOSITION 2. *Suppose* $(S, l, d, c, e)$ *is feasible under* (*PAB*). *Then* $(\hat{S}, c, e)$ *is feasible under* (*PA*), *where* $\hat{s}_1 = s_1 + l$ *and* $\hat{s}_2 = c_2$. *The agent is thus weakly better off under* (*PA*) *than under* (*PAB*).

[11] Note we are assuming that financial intermediaries have no advantage relative to the firm in extending credit to the agent. See, however, the remarks in Section 7.

*Proof.* The nonnegative profit constraint for the firm in (PAB) implies $s_1 \leq \frac{1}{1+r} E[X - s_2 \mid e]$. Also, from (7), we have $l \leq \frac{1}{1+r} E[s_2 - c_2 \mid e]$. Together, these two conditions imply that

$$s_1 + l \leq \frac{1}{1+r} E[X - c_2 \mid e] = \frac{1}{1+r} E[X - \hat{s}_2 \mid e].$$

Therefore, $\hat{S}$ entails nonnegative profits for the firm under (PA). Finally, since consumption is identical, the optimality of effort in (PA) follows from the optimality of effort in (PAB). ■

To determine the precise relationship between (PA) and (PAB), further analysis of the banking sector is required, which we address in the following sections.

## 4. THE PAB MODEL WITHOUT DEFAULT

In this section we consider the impact of allowing the agent to save or to engage in riskless borrowing. The agent is not permitted to borrow to an extent that would entail default. This is represented by the following set of aggregate loan contracts:[12]

$$L^0(S) = \{(l, d) : d = (1 + r)l, \text{ and } d \leq 0 \text{ or } d \leq s_2 - \lambda \text{ almost surely}\}.$$

We then denote by (PAB$^0$) the principal–agent problem in which the banking subgame is represented by $B(S, L^0)$. We have the following immediate result, which is somewhat anticipated by the results of Section 2.

PROPOSITION 3. *For any bankruptcy protection level $\lambda$, the agent is strictly better off under* (*PA*) *than under* (*PAB*$^0$). *The labor contract and amount of saving/borrowing are indeterminate under* (*PAB*$^0$). *In particular, no saving or borrowing need be observed in equilibrium.*

*Proof.* At a solution to (PAB$^0$), the agent must not prefer to increase savings (or reduce debt). This implies the (FOC)

$$u_1'(c_1) \geq (1 + r)\beta E[u_2'(c_2) \mid e].$$

This contradicts (2), which must hold at a solution to (PA). Hence, by Proposition 2, (PAB$^0$) is inferior to (PA). Also, if $(c_1, c_2)$ is a solution to (PAB$^0$), it can be implemented by the labor contract $s_1 = c_1 - w$ and $s_2 = c_2$, in which case the agent does not borrow or lend. In addition, $s_1 = c_1 - w + k/(1 + r)$ and $s_2 = c_2 - k$ is also an optimal labor contract for any $0 \leq k \leq c_2$ almost surely, and for $k < 0$ if $c_2 \geq \lambda$ almost surely. ■

[12] Note that $d \leq 0$ corresponds to savings, while $d \leq s_2 - \lambda$ almost surely implies that the debt is repaid with probability one and there is no default. Hence these contractual restrictions are equivalent to the restriction to riskless borrowing and saving.

## 5. THE PAB MODEL WITH EXCLUSIVE DEBT

In this section we relax the assumption that loan contracts must be risk-free and allow the agent to borrow against the risky portion of his or her future wage. Of course, banks rationally anticipate the possibility of default in this case and set the loan rate accordingly. The difficulty is that the probability of default now depends upon the agent's wage, which depends upon effort. Thus, a bank holding a risky loan is to some extent concerned about the agent's incentives. Because these incentives are affected by borrowing/saving activity, a bank holding a risky loan is potentially affected by the agent's future banking decisions.

We first consider an environment in which banks can monitor the agent's future banking activity. In particular, we assume that the bank has the option to renegotiate its debt contract with the agent in the event that the agent borrows or lends elsewhere. We think of this as an "exclusivity" convenant associated with the debt contract.[13]

The first step in our analysis is to determine the set of aggregate loan contracts that would be available to the agent in this environment. A useful benchmark is to compare this set with the set of loan contracts that would be accepted if the agent could contract with only a single bank. We have the following result:

PROPOSITION 4. *Suppose loan contracts are exclusive. Then the set of aggregate loan contracts $L^x(S)$ available to an agent with wages $S$ satisfies*

$$L^x(S) \supset L^1(S) \equiv \left[(l, d) : l \leq \frac{1}{1+r} E[\min(d, \max[s_2 - \lambda, 0]) \mid e(c_2(d, S))]\right].$$

*Proof.* Suppose the agent requests $(l, d) \in L^1(S)$ from a bank. By definition, if the agent engages in no further banking, the bank earns non-negative profits with such a contract. On the other hand, suppose the agent does engage in further banking. If $(l, d)$ is a risk-free contract, the bank's profits are unaffected and so still nonnegative. If $(l, d)$ is a risky loan, the bank's profits may be affected by further banking, but since the first bank has the option to reset the terms, its profits will again be nonnegative. Hence $(l, d)$ would be accepted by the bank, and so $L^1(S)$ is available in equilibrium. ■

The set $L^1(S)$ is not a complete characterization of $L^x(S)$, however. This is because there may be loan contracts the agent could request which would not be profitable if the agent did no further borrowing or lending, but which would be profitable given the agent's optimal level of additional banking after receiving the first contract. Since the first bank can rationally forecast the agent's future banking decisions, such an initial loan contract would be accepted.

[13] As a practical matter, such covenants are not commonplace with standard consumer loans, though they may be more common for small business loans. In any case, we will drop the "exclusivity" assumption in the next section, and so this case can be viewed as an alternative benchmark.

The entire set of loans $L^x(S)$ is therefore difficult to characterize. Fortunately, it is not necessary for our analysis, as shown below:

PROPOSITION 5. *The solutions to $B(S, L^x)$ and $B(S, L^1)$ coincide.*

*Proof.* Let $(l, d, e)$ solve $B(S, L^x)$. Then by (7), $(l, d) \in L^1(S)$, and so $(l, d, e)$ solves $B(S, L^1)$. ■

Thus, the PA problem with exclusive banking, which we denote by (PAB$^x$), is equivalent to (PAB) with the banking subgame $B(S, L^1)$.

Recall from Proposition 3 that with riskless borrowing and saving, no banking activity need be observed in equilibrium. The following result demonstrates that the opposite is true with exclusive borrowing: banking is necessary in equilibrium if the bankruptcy protection level lies within the range of consumption.

PROPOSITION 6. *Suppose $s_2 \leq \lambda$ and $s_2 > \lambda$ both occur with positive probability. Then $d \neq 0$ at any solution to $B(S, L^x)$.*

*Proof.* If $d = 0$ solves $B(S, L^x)$, then the agent must have no incentive to save or borrow given $S$. Note that for $d \leq 0$, $(d/(1+r), d) \in L^1(S)$; i.e., the agent can save at the risk-free rate. He or she will choose not to save only if the first-order condition

$$u_1'(c_1)\frac{1}{1+r} \geq \beta E[u_2'(c_2) \mid e] \tag{8}$$

is satisfied, where $c_2 = s_2$ and $c_1 = w + s_1$.

Next define $l^1(d) = \frac{1}{1+r} E[\min[d, \max(s_2 - \lambda, 0)] \mid e(c_2(d, S))]$. Then at $d = 0$ the marginal loan terms available to the agent are given by

$$\partial/\partial d\, l^1(d)|_{d=0+} = \Pr(s_2 > \lambda \mid e)/(1+r), \tag{9}$$

since the bank is only repaid when the agent has more than $\lambda$. This implies the first-order condition for the agent to choose not to borrow

$$u_1'(c_1)\frac{1}{1+r}\Pr(c_2 > \lambda \mid e) \leq \beta E[u_2'(c_2)1[c_2 > \lambda] \mid e],$$

where $1[\,\cdot\,]$ is the indicator function.

Since $s_2 = c_2 > \lambda$ with positive probability, this can be rearranged to yield

$$u_1'(c_1)\frac{1}{1+r} \leq \beta E[u_2'(c_2) \mid c_2 > \lambda, e]. \tag{10}$$

Therefore, combining the first-order necessary conditions (8) and (10) for $d = 0$

to be optimal implies that

$$E[u_2'(c_2) \mid c_2 > \lambda, e] \geq E[u_2'(c_2) \mid e].$$

But since $u$ is strictly concave, this can only hold if $s_2 = c_2 > \lambda$ with probability one. ■

The intuition for this result is that bankruptcy produces a "kink" at $d = 0$. Suppose the agent is indifferent toward or does not wish to engage in riskless saving. This implies the agent must be indifferent toward or prefer riskless borrowing. But if the agent does borrow, default will occur when $s_2 \leq \lambda$. Thus borrowing is risky and allows the agent to lower consumption in the highest states only. Since the agent weakly prefers riskless borrowing, risk aversion implies the agent will strictly prefer risky borrowing. Hence the agent must either borrow or save at solution to the banking subgame. Thus, unlike the case of Section 4, banking activity will be observed in equilibrium.

### 5.1. *Efficiency of (PAB$^x$)*

From Proposition 2, we know that (PAB$^x$) is weakly inferior to the standard problem (PA). Moreover, we observed in Proposition 3 that the availability of riskless borrowing and saving leads to a strict decrease in welfare. In this section we introduced the possibility of exclusive risky debt, which allows for an even richer set of contracts available to the agent; that is, $L^x(S) \supset L^0(S)$. Thus, it would seem intuitive that the availability of exclusive borrowing would lead to even further incentive constraints in the PA problem, and an even further decline in welfare. However, this intuition fails to take into account the optimal response of the labor contract to the availability of risky debt. In fact, we demonstrate in this section that under some circumstances, (PAB$^x$) may lead to equivalent welfare to (PA), albeit with a very different form of labor contract. We also show that efficiency depends critically on the level of bankruptcy protection.

Recall that $c_2^*$ is the equilibrium consumption at the solution to the standard PA problem. Our first result demonstrates that if the bankruptcy protection level is either below or above the range of $c_2^*$, (PAB$^x$) is inferior to (PA):

PROPOSITION 7. *Suppose $\lambda < c_2^*$ or $\lambda > c_2^*$ with probability one. Then* (*PAB$^x$*) *is strictly inferior to* (*PA*).

*Proof.* By Proposition 2, (PAB$^x$) is weakly inferior to (PA). Since (PA) has a unique solution, (PAB$^x$) is strictly inferior to (PA) unless $c_2^*$ can be implemented under (PAB$^x$). Suppose $\lambda < c_2^*$ with probability one. If $c_2^*$ is part of a solution to (PAB$^x$), the agent must default with probability zero at this solution. But since the agent has access to risk free saving, this cannot be a solution to the banking subgame by Proposition 1, and so cannot solve (PAB$^x$). Next suppose $\lambda > c_2^*$ with probability one. Then the agent has an aggregate loan of at most $l = 0$ since any debt is almost surely not repaid at all. Hence, the agent would be again be able to save at the risk free rate, so this also cannot be a solution in the banking subgame. ■

The previous result leaves open the possibility that for intermediate levels of bankruptcy protection—that is, if $c_2^* \geq \lambda$ and $c_2^* \leq \lambda$ both occur with positive probability—(PAB$^x$) might not be inefficient relative to (PA). The intuition for this is the following: the inefficiency with banking stems from the agent's ability to save and self-insure. In an equilibrium with risky debt, however, the agent cannot engage in riskless saving. A marginal dollar saved (or used to reduce the debt) will increase the agent's consumption in the highest states, but not in the states for which $c_2 < \lambda$ and the agent is in default.[14] Thus, if the labor contract is structured to induce the agent to borrow, it may block the agent's ability to self-insure. We now explore this possibility.

We begin by examining the necessary form of an efficient labor contract. This labor contract, $S^*$, together with a loan contract, $(l^*, d^*)$, must implement the (PA) allocation $c^*$. If $\Pr(c_2^* = \lambda \mid e^*) = 0$ (which holds generically), these labor and loan contracts must have the following form:[15]

$$\begin{aligned} s_2^* &= \begin{cases} c_2^* & \text{if } c_2^* \leq \lambda \\ c_2^* + d^* & \text{if } c_2^* > \lambda \end{cases}, \\ s_1^* &= \frac{1}{1+r} E[X - s_2^* \mid e^*], \\ l^* &= \frac{1}{1+r} E[s_2^* - c_2^* \mid e^*] = \frac{1}{1+r} E[d^* 1[c_2^* > \lambda] \mid e^*]. \end{aligned} \tag{11}$$

Thus, (PAB$^x$) is second-best efficient if and only if there exists a $d^* > 0$ such that $(l^*, d^*)$ is an optimal loan in the banking subgame $B(S^*, L^x)$. To provide some intuition regarding the appropriate choice of $d^*$, consider the following. The reason for the inefficiency of (PAB) relative to (PA) is that when the agent saves or borrows with the bank, the bank does not adjust its terms to reflect the externality imposed on the firm. This externality stems from the incentive effect of saving or borrowing. If for some choice of $d^*$ this externality disappeared, efficiency could be restored.

How can we measure this externality? Note first that the impact of a change in the agent's effort on the total expected cash flows available to both the firm and the bank is given by

$$\frac{\partial}{\partial e} E[X - c_2^* \mid e^*]. \tag{12}$$

Next note that given the contract above, the bank receives $d^*$ if and only if $c_2^* > \lambda$.

[14] This intuition is related to the "debt overhang" problem in corporate finance (Myers (1977)). Though saving is valuable to the agent, enough of the gains go to the debt holders to eliminate its appeal.

[15] Of course, the labor contract could be modified on a set of events of measure zero. We ignore this indeterminacy, though strictly speaking the statements hold "almost everywhere."

Thus, the impact of a change in the agent's effort on the total expected cash flows available to the bank alone is given by

$$\frac{\partial}{\partial e} E[d^* 1[c_2^* > \lambda] \mid e^*] = d^* \frac{\partial}{\partial e} \Pr(c_2^* > \lambda \mid e^*), \tag{13}$$

where $1[\,\cdot\,]$ is the indicator function.

Hence, the externality the bank can impose on the firm through the incentive effect is given by the difference between (12) and (13). This suggests that $d^*$ should be chosen to equalize the externality terms (12) and (13); that is, the level of debt supporting the second-best should be given by

$$d^* = \frac{\partial}{\partial e} E[X - c_2^* \mid e^*] \bigg/ \frac{\partial}{\partial e} \Pr(c_2^* > \lambda \mid e^*). \tag{14}$$

The following theorem substantiates this claim:

PROPOSITION 8. *Let* $(S^*, l^*, d^*)$ *be given by* (11) *and* (14). *Then if* $d^* > 0$, *the loan contract* $(l^*, d^*)$ *satisfies the first-order conditions of* $B(S^*, L^x)$. *If the first-order conditions are sufficient, then* ($PAB^x$) *is equivalent to* (PA).

*Proof.* We compare the loan terms offered by the bank to the terms of a "hypothetical" loan by the firm under (PA). Since the firm would offer a loan at terms maintaining zero profits, and since $c^*$ is the optimal such consumption profile, the agent would be unwilling to borrow (or save) at the terms offered by the firm. Thus, the theorem is proved by showing that the bank would offer identical terms.

Recall that $B(S, L^x)$ is equivalent to $B(S, L^1)$. The set of loans available in $L^1(S)$ can be represented by the loan schedule

$$l(\hat{d}; S^*) = \frac{1}{1+r} E[s_2^* - \hat{c}_2 \mid \hat{e}],$$

where $\hat{c}_2 = c_2(\hat{d}, S^*)$ and $\hat{e} = e(c_2(\hat{d}, S^*))$. The loan schedule offered by the firm, however, is defined by the condition of zero expected profits for the aggregate payments made to the agent (i.e., wages plus loan):

$$l_f(\hat{d}; S^*) = \frac{1}{1+r} E[X - \hat{c}_2 \mid \hat{e}] - s_1^*.$$

Also, from the definition of $S^*$,

$$s_1^* = \frac{1}{1+r} E[X - s_2^* \mid e^*].$$

Combining these three conditions yields

$$l_f(\hat{d}; S^*) - l(\hat{d}; S^*) = \frac{1}{1+r}(E[X - s_2^* \mid \hat{e}] - E[X - s_2^* \mid e^*]). \quad (15)$$

Hence, the difference between the loan schedule offered by the firm and that offered by the bank is given by the change in the profits of the labor contract resulting from the effort change induced by the debt. Differentiating this expression, we can compare the marginal interest rate offered by the firm and the bank at $d^*$:

$$l_f'(d^*; S^*) - l'(d^*; S^*) = \frac{1}{1+r}\frac{\partial}{\partial e}E[X - s_2^* \mid e^*]\frac{\partial}{\partial d}e(c_2(d^*, S^*)).$$

Further, since $s_2^* = c_2^* + d^* 1[c_2^* > \lambda]$,

$$\frac{\partial}{\partial e}E[X - s_2^* \mid e^*] = \frac{\partial}{\partial e}(E[X - c_2^* \mid e^*] - d^* \Pr(c_2^* > \lambda \mid e^*)) = 0, \quad (16)$$

from the definition of $d^*$. Therefore, $l_f' = l'$ and the first-order conditions of the banking subgame are satisfied. If the first-order characterization of $B(S^*, L^x)$ is sufficient, the labor contract $S^*$ will indeed implement the second-best allocation, and so is optimal for (PAB$^x$) and equivalent to (PA). ■

To complete the analysis of the general case then, it remains to demonstrate conditions implying $d^*$ is positive and is globally optimal in $B(S^*, L^x)$. First, we demonstrate that standard assumptions guarantee $d^* > 0$.

DEFINITION. The distribution function $F$ satisfies MLRC if $f_e(x, e)/f(x, e)$ is increasing in $x$ for all $e$. It satisfies CDFC if $F_{ee}(x, e) \geq 0$ for every $x$ and $e$.

These are precisely the assumptions used by Grossman and Hart (1983) and Rogerson (1985b) to guarantee the validity of the first-order approach for analyzing (PA). These assumptions provide a sufficient condition for $d^* > 0$, though they are not necessary, as the example of the next section demonstrates.

PROPOSITION 9. *Suppose the distribution $F$ satisfies MLRC and CDFC. Then the level of debt $d^*$ defined in* (14) *is positive.*

*Proof.* By our assumption on $F$, the first-order approach to the principal–agent problem is justified. Using this approach, $\frac{\partial}{\partial e}E[X - c_2^* \mid e^*] > 0$; that is, the principal would always like the agent to work harder at the solution to (PA). Furthermore, the MLRC implies that $c_2^*(X)$ is nondecreasing in $X$ (see Holmstrom (1979)), so that $c_2^* > \lambda$ if and only if $X > k$ for some $k$. Thus, since $\Pr(c_2^* > \lambda \mid e^*) \in (0, 1)$, and since MLRC implies first-order stochastic dominance,

$$\frac{\partial}{\partial e}\Pr(c_2^* > \lambda \mid e^*) = \frac{\partial}{\partial e}[1 - F(k, e)] = -F_e(k, e) > 0.$$

Therefore, the level of debt $d^*$ specified in (14) is indeed positive. ■

Finally, we would like to establish similar conditions guaranteeing the sufficiency of the first-order characterization for $B(S^*, L^x)$. Unfortunately, in the general case, the loan schedule is typically not concave, so that verifying the sufficiency of the first-order conditions is not tractable. For completeness, we present a condition which will guarantee that $S^*$ solves (PAB$^x$). The condition implies that firm profits do not decrease after the agent contracts with the bank, so that the bank cannot exploit this externality. We remark that this condition is much stronger than required and certainly not necessary, as the example of the following section will show.

PROPOSITION 10. *Suppose that for the labor contract $S^*$ described above,*

$$e^* \in \arg\min_{\hat{e}} E[X - s_2^* \mid \hat{e}].$$

*Then the loan contract* $(l^*, d^*)$ *solves* $B(S^*, L^x)$, *and* (*PAB*$^x$) *is equivalent to* (*PA*).

*Proof.* From Eq. (15), the above condition implies

$$l_f(\hat{d}; S^*) - l(\hat{d}; S^*) \geq 0.$$

Therefore, the loan terms offered by the bank are always worse than those that would be offered by the firm, so that the given loan must be optimal. ∎

Again, the conditions of Proposition 9 and Proposition 10 are restrictive and much stronger than necessary. We demonstrate their usefulness, however, by showing that they apply in the case in which the project has only two outcomes.

PROPOSITION 11. *Suppose the project has only two possible outcomes $x_g > x_b$, and let effort be normalized so that $e = \Pr(X = x_g)$. Then* (*PAB*$^x$) *is equivalent to* (*PA*) *if $c_2^*(x_g) > \lambda > c_2^*(x_b)$.*

*Proof.* The conditions of Proposition 9 are trivially satisfied, so that

$$d^* = (\partial/\partial e E[X - c_2^* \mid e^*])/(\partial/\partial e \Pr(c_2^* > \lambda \mid e^*)) = \Delta x - \Delta c_2^* > 0,$$

where $\Delta x = x_g - x_b$ and $\Delta c_2 = c_2^*(x_g) - c_2^*(x_b)$. But given this choice of $d^*$, from (11) we have $\Delta s_2^* = \Delta x$. But then $E[X - s_2^* \mid e]$ is independent of $e$, and Proposition 10 holds. ∎

Again, we stress that these conditions are much stronger than necessary. In general, we expect that $d^* > 0$ and optimal for $B(S^*, L^x)$ even if these conditions are not satisfied. The following section provides such an example.

### 5.2. *An Example*

Consider the economy described as follows:

$$X \in [0, \infty), \text{ with distribution } F(x, e) = 1 - \exp[-x/e],$$
$$u_1(c) = u_2(c) = 2\sqrt{c}, \quad V(e) = \beta\delta e^2, \quad \beta = 1/(1+r) = 0.95,$$
$$w = 0.475, \quad \delta = 1.$$

First, we solve the standard problem (PA) for this economy. From the incentive compatibility constraint and the first-order condition for (PA) we have

$$c_2^*(x) = \left[\sqrt{c_1^*} + \delta e^*(x - e^*)\right]^2.$$

Also, the zero profit condition for the firm implies that

$$c_1^* = \frac{1}{2+r}[w(1+r) + e^* - \delta^2(e^*)^4].$$

Finally, we determine $e^*$ to maximize the agent's utility. Under this parameter specification, $e^* = 0.4591$. This implies that $c_1^* = 0.4456$ and $c_2^*(x) = (0.4568 + 0.4591x)^2$.

Since $c_2^*(X) \geq 0.4568^2 = 0.2086$, we must choose a bankruptcy protection level $\lambda > 0.2086$ to implement the second-best. Let $\lambda = 0.25$. Then $c_2^* > \lambda$ if and only if $x > 0.0942 \equiv x_\lambda$. Further calculation (using the FOC for $e^*$) yields

$$\frac{\partial}{\partial e} E[X - c_2^* \mid e^*] = 2\delta^2 e^{*3},$$
$$\frac{\partial}{\partial e} \Pr(c_2^* > \lambda \mid e^*) = (x_\lambda / e^{*2}) \exp[-x_\lambda / e^*];$$

we can then calculate $d^* = 0.5318$. Further calculation yields $l^* = 0.4108$, and $s^*$ is given by (11).

Figure 1 depicts effort as a function of debt together with the loan schedule offered by the bank in $B(S^*, L^1)$. In addition, Fig. 1 shows the agent's indifference curve through $(l^*, d^*)$, demonstrating that $d^*$ is indeed optimal in $B(S^*, L^1)$. Hence, (PAB) is second-best efficient for this example, even though the economy does not satisfy the assumptions of Proposition 9 or Proposition 10.

Finally, we examine the comparative statics associated with different levels of bankruptcy protection for this economy. Figure 2 demonstrates the impact of varying the bankruptcy protection level $\lambda$ in the range of $c_2^*$ on the equilibrium level of debt and the probability of bankruptcy.

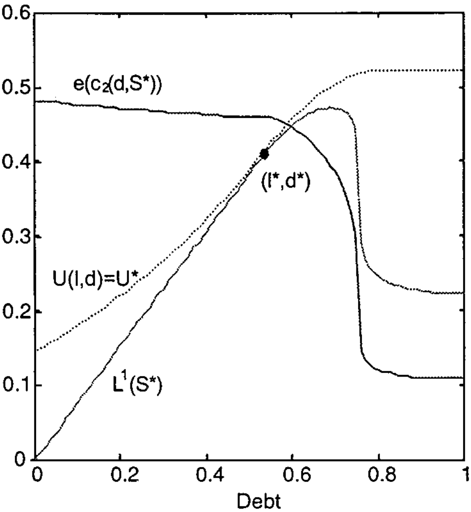


**FIG. 1.** Effort, loan schedule, and indifference curve for the example.

Note that the probability of default increases with the level of bankruptcy protection. Thus, quoted interest rates also increase with $\lambda$. Interestingly, the amount of debt is non-monotonic in the level of bankruptcy protection. Also, this shows that in the context of this model, variations in consumer debt and default associated with changes in bankruptcy protection do not necessarily have implications for efficiency.

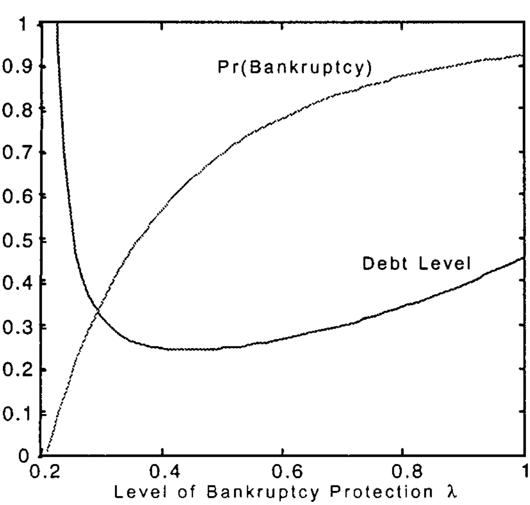


**FIG. 2.** Equilibrium debt and default probability given $\lambda$.

## 6. THE PAB MODEL WITH NONEXCLUSIVE BANKING

An important criticism of the PAB model with exclusive debt is that it assumes that banks have the ability to monitor the agent's future banking activity. In this section, we relax this assumption and assume instead that banks can only observe the agent's prior credit history. Thus, loan contracts can only be made contingent on prior, but not future, loan contracts signed by the agent. This is typically the case for most consumer loans, which are made based on the individual's credit history but do not restrict future borrowing. In this environment the bank must set its loan terms based on rational expectations regarding the incentive effects of any future banking activity. Bizer and DeMarzo (1992) study such an environment in the absence of a labor contracting problem and demonstrate that non exclusive or "sequential" banking introduces an inefficiency in equilibrium loan contracts by effectively introducing yet another incentive constraint related to the agent's ability to do further saving or borrowing with a new bank. Here we study the impact of nonexclusive loan contracts on equilibrium labor contracts.

Formally modeling this environment requires the construction of an infinite horizon game to represent the banking subgame.[16] The game (adapted from Bizer and DeMarzo (1992)) can be described as follows. There is an infinite number of banks $t = 1, \ldots, \infty$. Period one of the previous model is now divided into an infinite number of subperiods, such that in subperiod $t$, the agent makes a take it or leave it offer $(l_t, d_t)$ to bank $t$. The bank observes the agent's labor contract $S$, and all prior loan commitments, and can either accept or reject the offer. After the bank announces its decision, the agent can either cease banking activities, or proceed to bank $t + 1$ and request another loan contract. This continues until the agent decides to stop with the current aggregate loan contract. The agent then consumes the first-period income and works on the project. When the second-period outcome is realized, all contracts are settled and the agent consumes the second-period income. Given a sequence of accepted loan contracts $((l_1, d_1), \ldots, (l_T, d_T))$ obtained by the agent, the agent's consumption profile is as before

$$c_1 = w + s_1 + l,\; c_2 = \max[s_2 - d, \min(s_2, \lambda)],$$

where $(l, d) = \sum_t (l_t, d_t)$, the aggregate loan contract obtained by the agent. We also need to describe the payments to the banks. Suppose a bank agrees to the loan $(l_t, d_t)$. At maturity, the debt $d_t$ will be paid from the agent's "excess wages" (wages above the protected level), plus any other assets/savings held by the agent, and less the amount owed to prior creditors. That is, we assume that there is a strict prioritization of loans in the order they are signed.[17] Thus, the payment to bank $t$

[16] An infinite horizon is necessary since there is no "last period" in which the agent is guaranteed not to do further borrowing or saving. If such a last period existed, we would effectively be back in the exclusivity environment of Section 5.

[17] Note that unlike the exclusivity provision of the previous section, prioritization can be enforced in bankruptcy and does not require monitoring of further banking activity.

in the second period is given by

$$\min\left(d_t, \left[(s_2 - \lambda)^+ + \sum_{i=1}^{T}(-d_i)^+ - \sum_{i=1}^{t-1} d_i^+\right]^+\right),$$

where the notation $x^+ \equiv \max(x, 0)$.

Solving for a subgame-perfect equilibrium of this game is difficult in general, though the reader is referred to Bizer and DeMarzo (1992) and DeMarzo and Bizer (1994) for an explicit game-theoretic treatment. Rather than solve the game explicitly in this paper, we instead derive some sufficient conditions for establishing an equilibrium. We then show that, because of the endogeneity of the labor contract, these conditions are adequate to make conclusions regarding efficiency in this model.

We begin by defining

$$L^*(S) = \left\{ \sum (l_j, d_j) \text{ available in equilibrium given wages } S \text{ and nonexclusive banking} \right\}.$$

Given a wage contract $S$, the banking subgame with nonexclusive debt is described by the problem $B(S, L^*)$. We write (PAB*) to denote the PAB problem in this setting.

Recall that in our solution to the exclusive banking problem, (PAB$^x$), we were able to reduce the problem $B(S, L^x)$ to the simpler problem $B(S, L^1)$ in which the agent has access to the loans that would be offered if there were only a single bank. We next show that it is possible to use a similar reduction to derive a sufficient condition for a solution to $B(S, L^*)$.

First we define the set of loans that a new bank would offer an agent with labor contract $S$ and a preexisting aggregate loan $(l, d)$, if the agent could not borrow or save further:

$$L^1(S, l, d) \equiv \left\{(l', d') : l' \leq \frac{1}{1+r} E[\min[d', ((s_2 - \lambda)^+ - d)^+] \mid e(c_2(d + d', S))]\right\}.$$

Essentially, $L^1(S, l, d)$ is what is available to the agent if after signing a set of nonexclusive contracts, the agent can then sign an exclusive contract. Of course, we are not actually allowing exclusive contracts in this setting. But since the terms under an exclusive contract are more favorable to the agent then under a nonexclusive contract (there is no threat of an externality), if the agent would not enter into a new exclusive contract then the agent would also not enter into a new nonexclusive contract. As shown below, this yields a convenient way of identifying if, given a preexisting aggregate loan $(l, d)$, the agent would choose to stop banking.

PROPOSITION 12. *Suppose* $(l, d, e)$ *solves* $B(S, L^1)$ *and that the problem*

$$B(S, l, d, L^1): \quad \begin{array}{ll} \max_{l', d', e'} & U(c_1, c_2, e'), \\ \text{subject to} & (l', d') \in L^1(S, l, d), \\ & c_1 = w + s_1 + l + l', \\ & c_2 = \max[s_2 - d - d', \min(s_2, \lambda)]. \end{array}$$

*is solved with* $(l', d') = 0$. *Then* $(l, d, e)$ *solves* $B(S, L^*)$.

*Proof.* First suppose the agent already holds the aggregate loan $(l, d)$, and let $(l', d')$ be the additional aggregate loan the agent would obtain in equilibrium. Since in equilibrium the banks that provide these new loans all earn nonnegative profits, it must be the case that $(l', d') \in L^1(S, l, d)$. But then by hypothesis we can take $(l', d') = 0$. Thus, given $(l, d)$, the agent does no further borrowing or saving in equilibrium.

Next suppose the agent initially requests the loan $(l, d)$ from the first bank. If the bank accepts this loan, the agent does no further borrowing or saving. Since $(l, d) \in L^1(S)$, the bank therefore earns nonnegative profits. Hence $(l, d)$ is accepted by the first bank, and so $(l, d) \in L^*(S)$.

Finally, since in any equilibrium with nonexclusive debt all banks earn nonnegative profits, the loan contract that solves $B(S, L^*)$ must be in $L^1(S)$. Therefore, the solution to $B(S, L^*)$ is weakly inferior to the solution to $B(S, L^1)$. Since by hypothesis $(l, d)$ is optimal in $B(S, L^1)$, and since $(l, d) \in L^*(S)$, it must also be optimal in $B(S, L^*)$. ■

Thus, if given a particular loan contract the agent does not want to engage in further banking under a new exclusive contract, then the agent will also not wish to engage in further banking under a new nonexclusive contract. This methodological trick is extremely useful since $L^1$ is much more tractable than $L^*$ (in fact, a general characterization of $L^*$ is unlikely to exist!).

In the previous section we demonstrated that under certain conditions, (PAB$^*$) may in fact be equivalent to (PA). We next investigate whether this might also be true of (PAB$^*$). Based on the previous result, if at the solution to (PAB$^*$) the agent does not want to contract with a new exclusive bank, then this is also a solution to (PAB$^*$):

PROPOSITION 13. *Suppose* (*PAB*$^x$) *is equivalent to* (*PA*), *and let* $(S^*, l^*, d^*, e^*)$ *solve* (*PAB*$^x$). *If* $B(S^*, l^*, d^*, L^1)$ *is solved with* $(l', d') = 0$, *then* $(S^*, l^*, d^*, e^*)$ *also solves* (*PAB*$^*$) *and thus* (*PAB*$^*$) *is equivalent to* (*PA*).

*Proof.* By Proposition 12, $(l, d, e)$ solves $B(S, L^*)$. The result then follows from Proposition 2. ■

Recall from the results of Section 5 that subject to technical conditions, (PAB$^x$) is equivalent to (PA) if the bankruptcy protection level $\lambda$ is within the range of the optimal consumption profile $c_2^*$. We now consider under what conditions $B(S^*, l^*, d^*, L^1)$ is solved with $(l', d') = 0$.

First consider additional saving. If the agent does save with a new bank at interest rate $r$, what effect will this have on consumption? Recall that given $(l^*, d^*, S^*)$, the agent is going bankrupt in those states with $c_2^* \leq \lambda$. In these states, then, this additional saving will be seized by the first bank. Therefore, additional saving only increases consumption in those states with $c_2^* > \lambda$. This leads to the first-order condition for the agent not to save further

$$u_1'(c_1^*) \geq \beta(1+r)E[u_2'(c_2^*)1[c_2^* > \lambda]], \tag{17}$$

where $1[\,\cdot\,]$ is the indicator function.

Next consider additional borrowing. If the agent borrows a marginal dollar from a new bank, the bank will only receive payment in those states in which the agent's wage exceeds the bankruptcy protection level by more than the debt $d^*$ due the first bank. That is, the second bank is repaid only in those states for which $c_2^* > \lambda$. Thus the new bank will charge a marginal interest rate of $\rho$ on new debt, where $\rho$ solves[18]

$$1+\rho = (1+r)/\Pr(c_2^* > \lambda).$$

Because the new loan only reduces the agent's consumption in the same states $c_2^* > \lambda$, the first-order condition for the agent not to borrow further is given by

$$u_1'(c_1^*) \leq \beta(1+\rho)E[u_2'(c_2^*)1[c_2^* > \lambda]] = \beta(1+r)E[u_2'(c_2^*) \mid c_2^* > \lambda]. \tag{18}$$

Note that (17) and (18) can be viewed as restrictions on the parameter $\lambda$. The question is whether these restrictions can be jointly satisfied. This is resolved below:

PROPOSITION 14. *Suppose the second-best consumption profile $c_2^*$ is continuously distributed with support $[\underline{c}, \overline{c}]$. Then there exists a nonempty range $[\underline{\lambda}, \overline{\lambda}] \subset (\underline{c}, \overline{c})$ such that for $\lambda$ in this range, the first-order conditions* (17) *and* (18) *hold.*

*Proof.* For $\lambda = \overline{c}$, (17) obviously holds. If $\lambda = \underline{c}$, however, Eq. (2) shows that (17) fails (if the agent defaults with probability zero, there is an incentive to save). Since the RHS of (17) is decreasing in $\lambda$, this implies there is a $\underline{\lambda} > \underline{c}$ such that (17) holds if and only if $\lambda \geq \underline{\lambda}$.

Next note that for $\lambda = \underline{c}$, by Eq. (2) we find that (18) holds. The RHS of (18) is decreasing in $\lambda$, however, and Eq. (1) shows that (18) fails for $\lambda \to \overline{c}$. Therefore, there is a $\overline{\lambda} < \overline{c}$ such that (18) holds if and only if $\lambda \leq \overline{\lambda}$.

Finally, we need to show that $\underline{\lambda} < \overline{\lambda}$. For this, note that at $\overline{\lambda}$, (18) holds with equality. But then (17) holds strictly since the RHS of (17) equals the RHS of (18) times $\Pr(c_2^* > \lambda)$. ■

[18] This is the same calculation as in Eq. (9).

Therefore, there is a range of λ for which the first-order conditions of B($S^*$, $l^*$, $d^*$, $L^1$) are satisfied at 0. Thus, assuming these first-order conditions are sufficient, Proposition 13 implies that (PAB*) is second-best efficient. Thus, the effect of nonexclusive banking relative to exclusive banking is to reduce the range of efficient λ from $[\underline{c}, \overline{c}]$ to $[\underline{\lambda}, \overline{\lambda}]$.

We conclude this section by demonstrating this result for the numerical example of Section 5. Evaluating conditions (17) and (18), we find the relevant range for λ in this example is

$$\lambda \in [0.2192, 0.2513].$$

Suppose $\lambda = 0.23$. In this case the equilibrium debt $d^* = 0.9145$. We plot in Fig. 3a the agent's indifference curve through ($l^*$, $d^*$) as well as the loan schedules $L^1(S^*)$ and $(l^*, d^*) + L^1(S^*, l^*, d^*)$. For a closer look, in Fig. 3b we plot the difference between the indifference curve and each loan schedule in a neighborhood of $d^*$. Note that both the local and global conditions are satisfied.

The figure also illustrates another feature of the nonexclusive banking case. Note, for example, that the curve $(l^*, d^*) + L^1(S^*, l^*, d^*)$ is below $L^1(S^*)$ for $d < d^*$. This is so since $d < d^*$ implies the agent is saving at the new bank. However, some of this saving will be claimed by the old bank when the agent defaults. Thus, the agent is better off borrowing less initially (and getting a less risky and hence lower interest rate loan) than borrowing a lot (at a higher interest rate) and then saving at a new bank at the risk-free rate. This gives further intuition as to why saving is no longer attractive to the agent in equilibrium, overturning the Rogerson result.

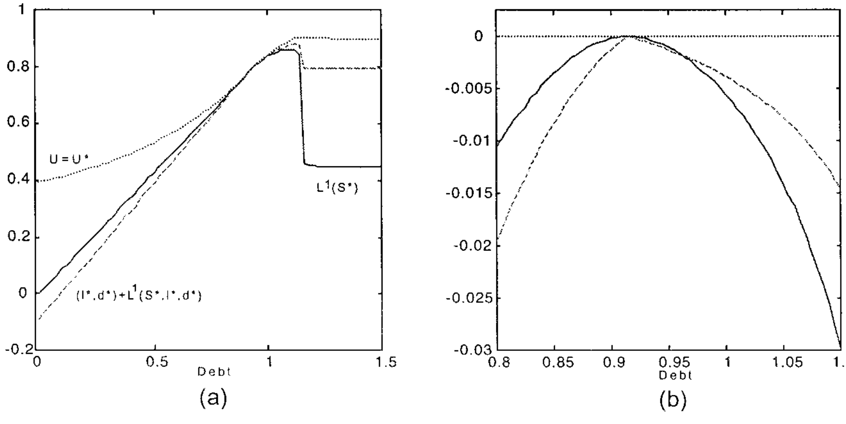


**FIG. 3.** (a) Indifference curve and loan schedules in equilibrium; (b) Indifference curve minus loan schedules near equilibrium debt.

## 7. CONCLUSIONS

Because modern economies generally permit financial contracting, and in any event the simplest financial arrangements (such as saving) would be difficult to prohibit, we extend the Principal-Agent Model to include financial contracting. Introducing financial markets into the standard principal–agent contracting problem significantly modifies previous conclusions regarding the structure of optimal labor contracts, efficiency, and the demand for banking services. In this section we review our results and explore their implications.

Our central result addresses the role of bankruptcy protection and standard debt contracts in improving efficiency. In particular, if bankruptcy protection is not available, or if it is too lenient, access to financial markets strictly increases agency costs. Within a broad intermediate range, however, bankruptcy protection restores efficiency in the presence of competitive financial markets. We summarize this result below:

*Result 1.* If bankruptcy protection $\lambda$ is too low or too high, then (PAB) is necessarily inefficient relative to (PA). If $\lambda$ is in an intermediate range $[\underline{\lambda}, \bar{\lambda}]$, however, then (PAB$^*$) is generally (subject to technical conditions) equivalent to (PA) in terms of efficiency.

The intuition for the result is that the seizure of assets which occurs in bankruptcy is a potential substitute for the restrictions on savings required in the standard principal–agent setting. For it to be effective, however, it must be the case that agents are willing to risk bankruptcy in equilibrium. This would not be true if bankruptcy is too severe; hence the need for bankruptcy protection.

We remark that although our analysis has emphasized the cases in which efficiency is restored via bankruptcy protection, this has been for reasons of tractability. More generally, our results suggest that the level of bankruptcy protection has important efficiency consequences in this setting even in cases in which second-best efficiency is not obtained.

When (PAB$^*$) is second-best efficient, our analysis has strong implications for optimal contract design. First, we note the effect on the saving decision of the agent:

*Result 2.* Under the labor contract derived in (PA), if the agent were given access to a bank, the agent would choose to save. When labor contracts take into account the possibility of banking as in (PAB$^*$), the contract induces the agent to borrow in equilibrium.

Two additional results follow indirectly from our analysis. First, we note that the optimal labor contract is altered by the presence of external credit markets:

*Result 3.* In the optimal (PAB$^*$) labor contract, wage payments are delayed relative to the standard (PA) contract. In addition, the (PAB$^*$) wage is more sensitive to the agent's performance than that under (PA).

This result has a further implication. Thus far we have assumed that firms and banks have the same discount rate. Suppose in fact that financial intermediaries are more efficient at extending credit than firms. Because under (PAB*) the wage payments are delayed relative to (PA), the (PAB*) outcome would be more efficient in this case. This may justify the observed trend in modern economies away from restrictive “company store” arrangements towards unrestricted access to banking with potential bankruptcy.

Finally, we note the following comparative statics result regarding changes in the level of bankruptcy protection:

*Result 4.* Increasing bankruptcy protection $\lambda$ in the range supporting second-best efficiency may either increase or decrease the equilibrium level of consumer debt. The frequency of bankruptcy and observed interest rates, however, are both increasing in the level of bankruptcy protection.

This follows from the fact that the agent defaults whenever $c_2^* < \lambda$. Recall, however, that though default rates, indebtedness, and interest rates are affected by the level of bankruptcy protection, there is no real effect on the economy in terms of final production, consumption and utility. This is because incentive contracts adjust in response to the change in bankruptcy protection.

Finally, we remark that we have concentrated our attention in this paper to the case in which the agent has access to saving and standard debt contracts. We have not considered the introduction of more general state-contingent contracts in the capital markets. As an empirical matter, this corresponds with the range of contracts typically observed; we do not generally find private markets for unemployment insurance, for instance. Presumably, part of the explanation for this observation are the large verification costs necessary with more complex state-contingent contracts. One possible extension of the analysis of this paper would be to consider an optimal contract design in this setting.

There are settings where consideration of more general contracts is important. For instance, if the agent is the manager running a publicly traded firm, the agent’s incentives could be affected by buying or selling shares in the firm. An analysis of some of the issues that arise in that setting can be found in Admati *et al.* (1994), DeMarzo and Bizer (1994), and Garvey (1997).

Unfortunately, consideration of arbitrary contingent contracts in the presence of bankruptcy protection would be tremendously challenging in this framework. In fact, even the extension of the banking subgame of this paper to the case of sequential contingent contracting with default is a difficult theoretical problem which has not been solved in general in the literature. (See, however, Kahn and Mookherjee (1998) for analysis of a special case.) In any case, given the prevalence of standard debt contracts in practice, the analysis of this paper seems a sensible place to begin. And while our precise results would not necessarily survive in the presence of alternative contracts, we believe our general conclusion—that outside contracting and bankruptcy protection laws have important consequences for the optimal PA contract—to be quite robust.

In conclusion, our analysis demonstrates that the introduction of external credit markets need not disrupt efficiency in agency environments. This is likely to be the case if credit markets are sufficiently sophisticated to allow for risky borrowing with an adequate level of bankruptcy protection. Though efficiency need not be reduced, observed incentive contracts will differ in this environment. Initial wages are reduced, and future wages have larger performance bonuses. Also, agents borrow against these future wages in equilibrium and risk default with positive probability.

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