Sequential Banking

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We study environments in which agents may borrow sequentially from more than one lender. Although debt is prioritized, additional lending imposes an externality on prior debt because, with moral hazard, the probability of repayment of prior loans decreases. Equilibrium interest rates are higher than they would be if borrowers could commit to borrow from at most one bank. Even though the loan terms are less favorable than they would be under commitment, the indebtedness of borrowers is greater. Further, additional lending causes the probability of default to increase. The results apply to markets for consumer, corporate, and international debt.

I. Introduction

Credit markets often consist of independent, competing lenders. We study resource allocation under “sequential banking,” meaning that a borrower may apply for and obtain a sequence of loans, each from a different lender (bank). Sequential banking introduces a time-consistency element into credit markets. Although a borrower would be better off if it were possible to commit to obtaining only one loan, the borrower has an incentive to approach additional banks for further borrowing. In the subgame-perfect equilibrium, interest rates charged on loans are higher than when the borrower can commit to

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obtaining only one loan. Although interest rates are higher, bor-
rowing is also greater, and the probability of default is greater as well.

Sequential banking matters when there is moral hazard, that is,
when unobservable decisions of borrowers affect the return to a loan.
As borrowing increases, the borrower may take actions that lower the
probability of repayment of earlier loans. If a new loan is issued by
an independent creditor, the terms of this loan will not reflect the
resulting devaluation of existing debt. This contrasts with a one-bank
environment, in which all effects on prior credit are internalized by
the sole creditor.¹

Fama and Miller (1972) have shown that if debt is prioritized
(meaning that existing loans retain seniority over new ones) and capi-
tal markets are perfect, then existing creditors are unaffected by sub-
sequent borrowing. Their argument is that prioritization eliminates
the externality arising from the straightforward dilution of earlier
loans by new loans.² Our analysis introduces an externality through
moral hazard and shows how this influences loan contracts even when
all debt is fully prioritized.

The externality we study might in principle be eliminated by
clauses making loan terms contingent on future borrowing from
other sources. As an empirical matter, however, many loan contracts
do not contain such clauses. In a sample of firms involved in lever-
aged buy-outs, Asquith and Wizman (1990) report that less than 16
percent of corporate bonds contain clauses restricting total debt.
Smith and Warner (1979) argue that such clauses are not widely used
because they introduce other inefficiencies. For instance, debt cove-
nants might include clauses that permit future borrowing only with
the approval of existing creditors. This would give veto power to
creditors and allow them to extract some of the surplus from new
investments, thereby reducing the borrower’s incentives to pursue
new projects. Alternatively, loan contracts could specify ex ante the
precise circumstances under which further borrowing would be al-
lowed. Implementing a fully state-contingent contract would be prob-
lematic, however, because of costs of determining whether a state has
arisen under which the contract allows further borrowing.

A simpler approach is to disallow further borrowing but make debt
callable. The borrower could then refinance the loan should further

¹ Several authors study environments in which the signing of one contract does not
preclude access to further contracts. Rothschild and Stiglitz (1976), Jaynes (1978), and
Arnott and Stiglitz (1989) show that Nash equilibria need not exist. Kahn and Mook-
herjee (1989) and Lacker and Weinberg (1990) invoke coalitional solution concepts to
define equilibrium. In contrast, our sequential structure implies the existence of a
unique subgame-perfect Nash equilibrium.

² For this reason, debt contracts often contain covenants explicitly prohibiting fur-
ther debt of equal or greater priority.
borrowing become desirable. This solves the sequential banking problem only if the call price equals the fair market value of the debt in the absence of further borrowing. The contract could either specify fair market value ex ante (which is equivalent in complexity to a fully state-contingent contract) or base the repayment on the current market price of the firm's debt. The current market price of the firm's debt need not correspond to fair market value, however, because of the firm's legal obligation to repurchase these claims before making new investments. In particular, existing debt holders can bid up the price of the debt and thereby extract some of the surplus from these investments.³

Rather than model the costs of such clauses explicitly, we simply treat future borrowing as unobservable, and we examine the costs associated with sequential banking.

II. One-Bank Equilibrium

Consider an economy in which a single borrower can obtain at most a single loan contract from a competitive banking sector. The borrower lives for two periods and may undertake a project that yields a known first-period income of \( x_1 \) and a stochastic income in the second period of \( X_2 \). Following Holmstrom (1979), assume that the distribution of second-period income is affected by the amount of work effort \( e \in \mathcal{E} \) applied by the borrower but that this effort is unobservable. The borrower may wish to save or borrow in the first period to smooth consumption.⁴

The economy contains an infinite number of competitive, risk-neutral, profit-maximizing banks. The borrower may enter into a loan contract with one of these banks. The loan contract is a pair \((l, d)\) specifying the loan amount \( l \) provided by the bank in the first period together with the debt amount \( d \) that the borrower must repay in the second period. Saving is a negative loan amount followed by a negative repayment.

After receiving first-period income \( (x_1) \), the borrower makes a loan-and-repayment proposal of \((l, d)\) to a bank in the first stage of the game. That bank responds by accepting or rejecting the proposal. If the offer is rejected, however, the borrower may then approach another bank with a loan-and-repayment proposal in the second stage, and so on, until an offer is accepted in stage \( t \). On reaching

³ An additional cost of restrictive clauses might result from monitoring to guard against forms of borrowing, such as trade credit, that are not readily observable.
⁴ More generally, one can model a borrower as an employee of a firm and examine the equilibrium relationship among the firm, the worker, and a bank (see Bizer and DeMarzo 1990).
agreement, the borrower may not seek additional loan contracts. The borrower with loan $l$ consumes $c_1 = x_1 + l$ in the first period and works on the project, expending effort $e$. In the second period, income of $x_2$ is realized, and the borrower must repay the debt amount $d$. If the borrower is able to repay, consumption is $c_2 = x_2 - d$. If the borrower is unable to repay and defaults on the loan, the bank receives $x_2$ and the borrower consumes zero.

Because payoffs in the game depend only on the final loan contract $(l, d)$, rejected offers do not matter, so the outcome of the game is $(l, d, t, e, x_2)$. (An outcome in which no loan request is accepted is $l = d = t = 0$.) Given such an outcome, the payoff to the borrower is the borrower's utility, $U(c_1, c_2, e)$, where $c_1 = x_1 + l$ and $c_2 = (x_2 - d)^+$ if a loan is accepted and $c_i = x_i$ otherwise. All banks that refuse a proposal receive a payoff of zero, whereas a bank that accepts a loan $(l, d)$ has a net payoff of $-(1 + r)l + \min(d, x_2)$, where $r$ is the competitive, risk-free interest rate faced by the bank.

A strategy for a borrower is a sequence of loan requests, together with associated effort choices. A strategy for the bank is a loan policy, which is a set of loan proposals it is willing to accept from the borrower, contingent on the current history of prior requests.

In equilibrium, the policy of each bank should maximize profits given the strategy of the borrower, and the strategy of the borrower should maximize utility given the banks' policies. This is a Nash equilibrium. To rule out equilibria in which banks threaten the borrower with unfavorable terms off the equilibrium path, we restrict attention to subgame-perfect equilibria; that is, we require all players to maximize their payoffs after any contingency. We use the term "one-bank equilibrium" (OBE) to mean such a subgame-perfect equilibrium.

The distribution of second-period income is fixed once the borrower's work effort is determined. The expected profit of the bank given a loan contract $(l, d)$ if the borrower takes effort $e$ is then

$$\pi(l, d, e) = E[\min(d, X_2) - l(1 + r)|e].$$

Standard assumptions imply that $\pi(l, d, e)$ is continuous, which we assume throughout. Similarly, the expected utility for the borrower given the loan contract and work effort is

$$U(l, d, e) = E[U(x_1 + l, (X_2 - d)^+, e)|e],$$

which we also assume to be continuous.

In any subgame-perfect equilibrium, banks accept only loan proposals that earn nonnegative expected profits. Profits on a loan contract depend on the effort of the borrower, which is unknown to the bank when the contract is signed. From subgame perfection, however, the borrower always chooses effort optimally given the loan
contract. For simplicity, we assume that, given any loan contract, the borrower's optimal effort is unique, though this can be generalized. That is, $e(l, d) = \arg\max_{e'} U(l, d, e')$ is taken to be a continuous function. Thus $\pi$ and $U$ can be expressed as functions of $(l, d)$ alone by defining $\pi(l, d) = \pi(l, d, e(l, d))$, and similarly for $U(l, d)$. We prove the following theorem (all proofs are in the Appendix).

**Theorem 1.** The loan contract $(\hat{l}, \hat{d})$ is an OBE outcome if and only if $(\hat{l}, \hat{d})$ maximizes $U(l, d)$ subject to $\pi(l, d) = 0$.

The theorem allows us to characterize the OBE by considering the loan contract that is optimal for the borrower within the set of zero-profit loan contracts. A useful way to view an OBE is then to consider the loan amount that yields zero profits for a given debt level. This zero-profit loan schedule is

$$l(d) = \max \{ l \mid \pi(l, d) = 0 \}$$

$$= \max \left\{ l \mid l = \frac{1}{1 + r} E[\min(d, X_2) | e = e(l, d)] \right\}.$$ 

Theorem 1 then says that an OBE is the point on the loan schedule that maximizes the borrower's utility.6

### III. One-Bank Equilibrium: The Two-State Case

We now simplify the model in order to provide intuition useful for understanding later results. In particular, we assume that, with probability $p$, a good outcome occurs ($x_2 = G$), and with probability $1 - p$, a bad outcome occurs ($x_2 = B < G$). Suppose also that the probability $p$ rises with the borrower's work effort; then effort may be parameterized by $p$. We model the borrower's utility from expected consumption and effort as

$$u(c_1) + \beta E[u(c_2) | p] - v(p),$$

where $u$ is strictly differentiable concave and increasing with $u(0) = 0$. We assume that $v$, which represents the disutility of effort required to achieve a given $p$, is strictly differentiably convex. Finally, we as-

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5 Under the assumptions that $\mathcal{E}$ is compact and the project payoff is bounded, an OBE necessarily exists but may not be unique.

6 The equilibrium conditions we derive are robust to alternative institutional arrangements. For example, one could allow the borrower to make a take-it-or-leave-it offer to only one bank, or the borrower could announce a debt amount and the banks could offer loans. The latter is equivalent to corporate debt for which the face value is determined but the price is set in the market.

7 As is common in models of moral hazard, we suppress the exogenously given probabilities over states of nature.
sume that \( v'(0) = 0 \) and \( v'(1) > u(G) \), which guarantees that optimal effort induces a probability of success between zero and one.

By the strict convexity of \( v \), the optimal effort choice of the borrower is unique and is characterized by the first-order condition

\[
v'(p) = \beta[u(c_G) - u(c_B)],
\]

where \( c_G = (G - d)^+ \) and \( c_B = (B - d)^+ \) represent consumption in the good and bad state, respectively. Note that the effort decision is affected by the borrower's loan contract only through the level of debt \( d \). In fact, if we write \( p(d) \) to indicate the success probability given the borrower's debt commitment, then

\[
p'(d) > 0 \quad \text{if } d < B,
\]

\[
p'(d) < 0 \quad \text{if } B < d < G.
\]

Effort increases with debt if there is no probability of bankruptcy. On the other hand, if the borrower has borrowed sufficiently that bankruptcy occurs when the bad outcome is realized, then increases in indebtedness decrease effort.

The equilibrium loan schedule in the two-state case is then

\[
l(d) = \frac{1}{1 + r} E[\min(d, X_2) | p = p(d)] = \frac{1}{1 + r} \{ p(d) \min(d, G) + [1 - p(d)] \min(d, B) \}.
\]

In the region \( d \leq B \), the loan is risk-free, so that \( l(d) = d/(1 + r) \). In the region \( B < d < G \), however, debt is risky because bankruptcy occurs in the bad state. For such debt levels the slope of the loan schedule satisfies

\[
l'(d) = \frac{1}{1 + r} p(d) + \frac{1}{1 + r} p'(d)(d - B).
\]

One may interpret \( l'(d) \) as the marginal discount rate applied to loans. Equation (2) says that this marginal discount rate is composed of an actuarial term capturing the discounted expectation that the marginal dollar is repaid and a second term embodying the incentive effect of debt. Recall that \( p'(d) < 0 \) for \( B < d < G \), so the second term is negative. That is, the incentive effect raises the marginal interest rate on additional borrowing above the actuarially fair rate on current debt. The reason is that as additional debt is acquired, effort declines. The marginal interest rate charged on an additional dollar of debt must take into account not only the probability that the marginal
dollar of debt is repaid, but also the implicit devaluation of inframarginal (prior) debt that arises as the reduction in effort lowers the probability that inframarginal debt is repaid.

From theorem 1, equilibrium occurs at the point on the loan schedule that maximizes the borrower's utility. If it occurs in the bankruptcy region, it must satisfy the first-order condition

\[ u'(x_1 + l(d))l'(d) = Bp(d)u'(G - d). \]  

Because of moral hazard, the loan schedule \( l(d) \) always has an interior maximum; that is, there is a critical level of debt \( d^m \in [B, G] \) such that an increase in debt beyond \( d^m \) actually decreases the present value of repayments (see fig. 1 below). Thus an increase in debt need not lead to an increase in the loan amount the bank is willing to provide even though there is a positive probability of repayment. The region to the right of \( d^m \) may be thought of as the inefficient segment of the loan schedule because a point in this region would be dominated by a point on the upward-sloping portion of the loan schedule that involves the same present consumption for the borrower but higher future consumption. Obviously, an OBE cannot occur in this inefficient region.

IV. Incentives for Further Borrowing

We now relax the assumption of commitment to a single bank so that the worker may obtain loans from a new bank after an initial loan has been granted. To see whether commitment is a binding constraint, we calculate the terms that a new bank would offer on an additional dollar of debt, and we check the worker's demand for an additional loan at these terms. We assume that there is always a bank in the economy with which the borrower has no preexisting debt.

If the worker has acquired a sufficiently small amount of debt from previous banks so that there is no possibility of bankruptcy, then all existing debt is riskless. In this case, an additional dollar of debt is priced at the same riskless rate as previous debt; that is, the marginal interest rate is simply \( r \). Suppose instead that the worker has acquired an amount \( d \) of existing debt such that there is a nonzero probability of bankruptcy on existing debt; that is, \( d > B \). Now if the new bank offers an additional dollar of debt, the new bank would be paid back only in the good state; in the bad state, prior claims would exhaust the worker's income. Provided that the new bank lends an amount \( \Delta d \leq G - d \), the expected present value of the new bank's claim would then be \( [1/(1 + r)]p(d + \Delta d)\Delta d \). At the margin, the new bank would therefore be willing to lend at rate \( [1/(1 + r)]p(d) \) per unit of
new debt. These terms are better than those offered by the bank with which the borrower has existing indebtedness because the new bank’s terms do not take into account the effective devaluation of existing debt caused by new borrowing. Hence the borrower may have an incentive to seek further credit from a new bank. That is, if we make the following assumption, then the borrower risks bankruptcy in the OBE, so that theorem 2 follows.

Assumption A. Current income \( x_1 \) is sufficiently small that \( U(l(d), d) \) is increasing for \( d \leq B \).

Theorem 2. In the OBE, \( d > B \) and the borrower would prefer to borrow further from a new bank.

Thus the commitment assumption underlying the OBE is binding. If any bank were to accept a loan proposal that gave the borrower the OBE level of debt, the borrower would have no demand for additional funds from that particular bank. However, the borrower would have a demand to borrow from a new bank. Intuitively, the reason is that new banks do not pay for the externality they impose on prior banks, and as a result their loan terms are more favorable.

V. Sequential Banking Equilibrium

We now dispense fully with the commitment assumption and assume that the borrower may request and obtain loans repeatedly, each from a new bank. Specifically, we permit the borrower to request additional loans until satisfied with the loan portfolio. At that point, the borrower quits the banking phase, consumes first-period income plus borrowing, and expends effort on the project. When the second-period income is realized, the borrower repays debts and consumes any remaining wealth.

We allow each bank to observe the borrower’s entire credit history when the borrower applies for a loan.\(^8\) At each stage \( t \) of the banking phase, the borrower goes to bank \( t \) and requests a loan contract \((l_t, d_t)\). Bank \( t \) accepts or rejects the loan request. We denote the bank’s action by \( a_t \in \{0, 1\} \), with \( a_t = 1 \) indicating acceptance.

Let \( h_t = (l_t, d_t, a_t) \) denote the outcome of each round of banking, so \( h^t = (h_1, \ldots, h_t) \) is the borrower’s entire credit history after visiting \( t \) banks. Let \( H^t \) denote the set of possible histories at stage \( t + 1 \).

After requesting loans, the borrower chooses effort given the current history \( h^T \). The project outcome is then realized, and debts are repaid. An outcome of the game is thus \((h^T, e, x_2) \in H^T \times \mathcal{E} \times \mathcal{R}\).

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\(^8\) In a different context, Pauly (1974) examines the more extreme assumption that firms cannot observe existing contracts. Kletzer (1984) applies a similar model to debt and demonstrates that Nash equilibria do not exist in general.
After $(h^T, e)$, the total loan and debt commitment of the borrower are

$$L(h^T) = \sum_{t=1}^{T} a_t l_t, \quad D(h^T) = \sum_{t=1}^{T} a_t d_t.$$ 

The payoff to each bank is the expected profit of its loan, under the assumption that banks can borrow funds at the risk-free rate $r$. Debt is fully prioritized, so each loan contract is repaid only after all earlier commitments are satisfied. Because $D(h^{t-1})$ is the borrower’s total debt commitment prior to a contract signed in stage $t$, debt $d_t$ is repaid from remaining income $[x_2 - D(h^{t-1})]^+$. Thus bank $t$ receives

$$\pi_t(h^T, e) \equiv -(1 + r) l_t + E \left[ \min(d_t, [X_2 - D(h^{t-1})]^+) \right] | e]$$

if it accepts the loan, and zero otherwise. The payoff of the borrower is $U(L(h^T), D(h^T), e)$. If the borrower never stops searching for a loan, utility is zero and all banks receive zero. A strategy for the borrower is a function $s(h^t) \in \mathbb{R}^2 \cup \{q\}$ that indicates the borrower’s loan request given a credit history and a function $e(h^t) \in \mathcal{E}$ indicating the borrower’s effort after obtaining the loans in $h^t$. In particular, $s(h^t) = q$ means that the borrower quits searching for new loans after the history $h^t$, and $s(h^t) = (l, d)$ indicates that the borrower visits bank $t + 1$ and requests the loan $(l, d)$.

A strategy for each bank $t$ is a loan policy $a_t: H^{t-1} \times \mathbb{R}^2 \rightarrow \{0, 1\}$, where $a_t(h^{t-1}, l, d) = 1$ if and only if the bank is willing to accept loan request $(l, d)$ from a borrower with credit history $h^{t-1}$. We suppress the subscript $t$ since it is implicit given the history; that is, $a(h^t, l, d) = a_{t+1}(h^t, l, d)$. Finally, $\sigma = (s, e, a)$ denotes the strategy profile of both the borrower and the banks. A strategy profile $\sigma = (s, e, a)$ is a “sequential banking equilibrium” (SBE) if it is a subgame-perfect equilibrium of the sequential banking game.

We now characterize the equilibrium strategies. Given a strategy profile $\sigma = (s, e, a)$, it is possible to compute the outcome of the game. In particular, define $\rho(\sigma; h^t) \equiv h^T$, where $h^T$ is recursively defined by

$$h_t = (s(h^{t-1}), a(h^{t-1}, s(h^{t-1}))) \quad \text{for } t = \tau + 1, \ldots, T,$$

and $T$ is such that $s(h^T) = q$. Thus $\rho(\sigma; h^t)$ is the final credit history obtained conditional on the history $h^\tau$. For convenience, abbreviate $L(\rho(\sigma; h^t))$ as $L(\sigma; h^\tau)$, which is the ultimate loan acquired by the borrower if play is according to $\sigma$ and after $h^\tau$ is reached. A similar interpretation applies to $D(\sigma; h^\tau)$ and $e(\sigma; h^\tau)$. As in the OBE case, any loan policies adopted by the banks induce an implicit loan schedule faced by the borrower. For the SBE game, given policies $a$, the
aggregate available loan schedule is

\[ l^a(d; h') = \sup \{ l \mid l = L(s', e, a; h'), \]
\[ d = D((s', e, a); h'), \text{ for some strategy } s'. \]

That is, \( l^a(d; h') \) is the best total loan the borrower can receive by carrying total debt \( d \), given credit history \( h' \) and banks' policy \( a \).

With these definitions, we can characterize an SBE.

**Theorem 3.** A strategy profile \( a = (s, e, a) \) is an SBE if and only if

1. \( e(h') \in \argmax_{e \in E} U(L(h'), D(h'), e) \),
2. \( a(h', l, d) = 1 \) (respectively \( 0 \)) if

\[ l < (>) \frac{1}{1 + r} E[\min(d, [X_2 - D(h')]^+)|e(\sigma; h', (l, d, 1))|, \]

and

3. \( L(\sigma; h'), D(\sigma; h'), \) and \( e(\sigma; h') \) solve

\[ \max_{l, d, e} U(l, d, e) \]

subject to \( l = l^a(d; h'), \quad e \in E. \)

Note that condition 2 implies that banks do not accept loans leading to negative expected profits. An immediate corollary follows.

**Corollary 4.** In any SBE, banks earn nonnegative expected profits. Thus if \( (l, d) \) is the aggregate loan contract resulting from some SBE, \( \pi(l, d) \geq 0 \).

Corollary 4 says that the set of aggregate loans available in equilibrium is not expanded by the opportunity to bank sequentially. The aggregate SBE loan is thus also feasible in the OBE environment, so that the borrower is always at least weakly better off in any OBE than in any SBE. In addition, because any bank would accept a loan leading to nonnegative expected profits, corollary 4 suggests that even in the sequential banking environment, it is unnecessary for the borrower to visit more than one bank. This is established in the following theorem.

**Theorem 5.** Suppose that strategy profile \( \sigma = (s, e, a) \) is an SBE. Then there exists another SBE, \( \hat{\sigma} \), with the same equilibrium aggregate loan and effort, in which the borrower contracts with at most one bank.

VI. Sequential Banking Equilibrium: The Two-State Case

We now characterize an SBE under the assumption that the project has only two possible outcomes, \( G \) and \( B \), as in Section III.
Suppose first that banks are naive and assume that the borrower will borrow no further. Under naivete, the first bank will accept any loan along the original loan schedule from the OBE game. The borrower could choose the OBE loan, point $O$ in figure 1. Note that the chord from the bankruptcy point ($d = B$) in the loan schedule through point $O$ has a slope equal to the inverse of the average interest rate, $[1/(1 + r)]p(d_0)$. At this point, however, suppose that a second naive bank accepts loans along a zero expected profit loan schedule that again assumes that the borrower would borrow no further. From Section IV, this new bank would accept a marginal loan at the rate $[1/(1 + r)]p(d_0)$. Thus the second bank’s loan schedule rises from point $O$ with an initial slope equal to the slope of the chord and then tails off because the second bank must worry about devaluation of its own preexisting debt. Suppose that the borrower requests point $M$. But now a third naive bank, whose loan schedule has an initial slope equal to the slope of the chord from $O$ to $M$, would accept point $N$, and so forth.

The naivete of each bank in this scenario is clear: each bank earns losses as soon as the borrower borrows further and moves to a point above the bank’s zero-profit loan schedule. Each bank earns losses because it calculates expected profits believing that the borrower does not borrow further, but even if the borrower promises not to do so,
the borrower has a clear incentive to renege and approach additional banks.

There is, however, a locus of loan proposals that would be credible in the sense that if the borrower ever obtained a loan from this set, the borrower would seek no additional funds.

**Lemma 6.** Define the no further borrowing (NFB) schedule so that
\[ I = \text{NFB}(d) \text{ if and only if } u'(x_1 + I) = (1 + r)u'(G - d). \] (4)

If the borrower has a current aggregate loan \((l, d)\), with \(d \geq B\) and \(l \geq \text{NFB}(d)\), then any additional loan involving nonnegative profits for the banks makes the borrower worse off.

Thus condition (4) identifies the set of loans that the borrower could request and credibly promise to refrain from further borrowing.

Suppose that the borrower were to request a loan contract beyond the NFB schedule. Since the borrower has no incentive to seek additional loans, the bank can evaluate this request as though it were in an OBE (commitment) environment. Hence, it should accept any loan request that is both below the OBE loan schedule and beyond the NFB schedule. This means that the borrower should be able to receive any loan in this region. The following lemma identifies the best possible such loan.

**Lemma 7.** Let \(d^*\) be the smallest \(d\) such that \(\text{NFB}(d) = l(d)\), and let \(l^* = l(d^*)\) and \(e^* = e(l^*, d^*)\). Then \((l^*, d^*, e^*)\) solves
\[
\max_{l, d, e} U(l, d, e)
\text{subject to } \text{NFB}(d) \leq l \leq l(d).
\]

That is, of the set of loans beyond the NFB schedule and below \(l(d)\), the best possible loan for the borrower occurs at the highest possible intersection of these two curves. As we have argued, the borrower should be able to get this loan from the bank since the bank need not fear further borrowing by the borrower. Hence, in any equilibrium of the sequential banking game, the borrower should be no worse off than with this loan since it should always be available. This is verified in the following theorem.

**Theorem 8.** If \((l, d, e)\) is the aggregate loan and effort that result from some SBE, then \(U(l, d, e) \geq U(l^*, d^*, e^*)\). Furthermore, \(d \leq d^*\).

This theorem restricts the set of possible SBE outcomes. We can uniquely identify the equilibrium if we place an additional assumption on the borrower's preferences.

**Assumption B.** In the region \(d \in [B, d^*]\), the borrower's indifference curves are convex in \(l\) and \(d\).
In the absence of moral hazard, the borrower’s indifference curves would be convex in \( l \) and \( d \). With moral hazard, this property is no longer guaranteed, but a sufficient condition for the convexity of indifference curves is a restriction on the marginal utility of consumption in the good state, as in the following lemma.

**Lemma 9.** If for \( d \leq d^* \) the borrower’s expected marginal utility in the good state, \( pu_G'(G - d) \), is increasing in \( d \), then assumption \( B \) is satisfied.

We now show that, with this convexity assumption, the loan \((l^*, d^*)\) identified earlier is the only possible equilibrium outcome. Consider an arbitrary outcome below the NFB line, for example, point \( X \) in figure 2. This point cannot be an equilibrium since the borrower could approach a new bank and request an additional loan to arrive at point \( Y \), the intersection of the new bank’s loan schedule and the NFB line. As shown above, the new bank should accept this request because the borrower has no incentive for further borrowing from the point \( Y \). Thus the bank need not fear a devaluation of its debt and hence will not earn negative expected profits. Moreover, the borrower has an incentive to request a loan to point \( Y \). To see this, recall that the chord from \( X \) to \( Y \) has a slope corresponding to the actuarially fair interest rate given a total loan of \( Y \). Moreover, the definition of the NFB line implies that the borrower’s indifference curve at \( Y \) is tangent to this chord since the actuarial interest rate is
the rate charged by a subsequent new bank. Finally, our assumption of convexity of the indifference curve implies that this indifference curve through $Y$ must lie above the chord and hence above point $X$.

In this way, points below the NFB line cannot be equilibria: the borrower has the desire and the ability to move to the NFB. Any equilibrium, then, must lie on or above the NFB. Also, since banks earn nonnegative profits, an equilibrium must also lie on or below the original zero-profit loan schedule, $l(d)$. But given these two constraints, lemma 7 and theorem 8 imply that any SBE must then have aggregate outcome $(l^*, d^*)$.

This equilibrium is supported by the following strategies: Starting from any point $X$ below the NFB, the borrower requests a loan to point $Y$ on the NFB, as in figure 2. The banks, on the other hand, will accept only those loans on or below the chord connecting the bankruptcy point to $(l^*, d^*)$. (We refer to this chord as the linear loan schedule.)

To see this, suppose that the borrower requests any loan strictly above this chord. If this loan were accepted, the borrower would then borrow from a new bank to the intersection of the new zero-profit loan schedule and the NFB. This intersection will occur above the aggregate zero-profit loan schedule, so that the original loan must earn negative expected profits. Hence banks should not accept loan requests above the chord.

Next, suppose that the borrower requests a loan on the chord, for example, point $Z$ in figure 2. Again, with such a loan, the borrower would get a new loan to the NFB. It is important to note in this case that the intersection of the NFB and the new bank’s loan schedule is precisely the point $(l^*, d^*)$. Hence, the first bank does not earn losses and so should accept the point $Z$.

Thus the linear loan schedule is optimal given the borrower’s strategy. Since this same argument can be made on any subgame, all banks should offer such a linear loan schedule. But if all banks offer this linear loan schedule, the best available loan for the borrower is the intersection with the NFB.

The following theorem makes this result precise.

**Theorem 10.** The unique SBE outcome involving bankruptcy is $(l^*, d^*, e^*)$. Furthermore, the equilibrium loan schedule $l^*(d)$ implies a constant interest rate of $[1/(1 + r)]p(d^*)$ for $d \in [B, d^*]$.

The linear loan schedule arises because all banks anticipate future borrowing by the borrower until indebtedness reaches $d^*$. Thus the loan terms must be determined by the probability of bankruptcy induced by this final portfolio. This is in sharp contrast to the OBE, where the loan schedule was nonlinear throughout the range of debt involving bankruptcy. Further, we can make the following compari-
son of indebtedness and interest rates between the environments of banking with commitment to a single bank and sequential banking.

**Theorem 11.** The level of debt in the SBE outcome exceeds that in the OBE.\(^9\)

**Theorem 12.** The equilibrium loan schedule under SBE implies higher average interest rates than under OBE for every loan; that is, \(l(d) \leq l^*(d)\) for all \(d\).

Thus the externality created by sequential banking is sufficient to induce greater indebtedness than in the case of commitment to a single bank. Though the OBE is no longer supported because of the potential of a new bank to offer highly attractive loan terms, in the SBE no such attractive offers are made. Interest rates are uniformly higher in the SBE, yet the borrower chooses a higher level of indebtedness than in the commitment case.

In contrast to the OBE, it is interesting to note that the SBE loan may occur on the downward-sloping, and thus inefficient, portion of the zero-profit schedule \(l(d)\). Therefore, though indebtedness is higher in an SBE than in an OBE, the actual loan amount received may decrease.

**VII. Modifications and Extensions**

The SBE is robust to modifications of the game form that correspond to a variety of institutional arrangements. For example, the negotiation process between the bank and the borrower can be modified to accommodate various market structures without affecting the equilibrium. Borrowers could request \(l\) and banks offer an interest rate (as in consumer and privately held corporate debt), or borrowers could announce a face value \(d\) and lenders respond with offers of \(l\) (as in publicly held corporate debt).

Indeed, under the latter scenario of borrowers committing to a face value \(d\) at each stage, the SBE obtains even if the number of banks is finite. Fundamentally, the SBE results from the availability of additional loans at terms that do not reflect the devaluation of prior debt. With precommitment to \(d\) on announcement, however, the devaluation of prior debt occurs immediately. Because this devaluation is now sunk, all banks, including those with preexisting debt, are willing to offer identical terms. Thus sequential banking can lead to inefficiencies even if there are only two banks.

More generally, the SBE analysis can be applied to other situations in which agency costs lead to externalities across contracts. Indeed,

\(^9\) Note, however, that if \(x_1\) is large, so that assumption A is not satisfied, then the inefficiency of sequential banking may be so severe that the borrower chooses \(d < B\).
the proof of theorem 10 depends only on the fact that the NFB locus is characterized by the first-order conditions and that indifference curves are convex. Subject to these conditions, the model can be extended to a multiple-state framework. One difference, however, is that a multiple-state version of our model entails an equilibrium in which higher interest rates will be charged on debt of lower priority. The reason is that lower-priority debt is repaid in fewer states, a consideration that does not arise in the two-state framework.

Additionally, there are many other applications in which these basic conditions hold. For example, sequential banking considerations apply to the classic “debt overhang” problem identified by Myers (1977), in which increases in risky debt may reduce the incentives of equity holders to undertake new profitable investments. The resulting underinvestment thus imposes an externality on all existing debt holders. The trade-off between the tax advantages and agency costs of debt can lead to convex indifference curves for risk-neutral equity holders, and thus the results of Section V extend to this model of corporate debt.

Another possible extension of sequential banking is the problem of asset substitution, which occurs if managers can choose between projects of varying riskiness. In this case, increased debt levels might exacerbate management’s preference for risky assets and thus reduce the value of all debt. This creates an externality similar to that considered here.

VIII. Conclusion

If debt contracts do not prevent further debt acquisition by borrowers, then equilibrium is affected by the opportunity to bank sequentially. Interest rates and indebtedness are higher than if borrowers could restrict themselves to borrow from only one bank. Equilibrium might even occur in the inefficient region of the loan schedule, where a reduction in the amount to be repaid would increase present consumption. The crux of the problem is that additional lending imposes an externality on prior lending because increased indebtedness might reduce the likelihood that prior debt is repaid. In an environment in which lenders recognize this externality, equilibrium results in reduced welfare of borrowers. Models of covenant protection should trade off the costs of restrictive covenants with the costs of sequential banking that we describe.

The environment we examine has a direct bearing on markets for individual, corporate, and international debt. In each of these markets, borrowers can obtain funds from multiple, independent lenders. For instance, in a study of consumer credit, Domowitz and Sartain...
(1991) find that growth in unsolicited, preapproved credit cards is positively associated with the probability of filing for personal bankruptcy.

In addition, subordinated corporate debt has been used by firms with preexisting debt from other sources to increase their indebtedness. Asquith and Wizman (1990) find evidence that bonds with covenants restricting the extent of total funded debt tend to maintain or even gain value in leveraged buy-outs. In contrast, corporate bonds with weaker covenants enforcing priority alone suffered a devaluation from the substantial increase in junior debt associated with leveraged buy-outs. This evidence is consistent with our theoretical prediction that further borrowing imposes an externality on existing debt.

Finally, developing countries have historically had access to independent lenders and have more recently experienced difficulty meeting their repayment obligations. Many commentators, including Froot (1989) and Krugman (1989), have argued that third-world borrowers have inefficiently high levels of debt. Indeed, the debate over third-world debt reduction through forgiveness hinges, as Krugman states, on “arguments that countries are on the wrong side of the debt relief Laffer curve” (p. 265). Our model demonstrates that an optimizing borrower could accept inefficiently large amounts of debt in equilibrium. If so, a policy implication is that attempts to reduce third-world debt through loan forgiveness might be undone by additional debtor borrowing; forgiveness would simply represent a lump-sum transfer from existing lenders to borrowers.

Appendix

Proof of theorem 1.—For sufficiency, suppose \((l, d, e) \in \text{argmax}_{l', d', e'} U(l', d', e')\) subject to \(r(l, d) = 0\). The following strategies are an OBE: the agent requests \((l, d)\) from any bank and chooses effort optimally; banks accept any loan with \(r(l, d) \geq 0\).

For necessity, suppose that \((l, d, e)\) is an OBE outcome. Clearly, \(r(l, d) = 0\) since if \(r(l, d) < 0\), the bank should reject the loan; for \(r(l, d) > 0\), the agent would prefer to borrow more. Suppose that there is some \((l, d, \delta)\) such that \(U(l, d, \delta) > U(l, d, e)\) and \(r(l, d) = 0\). By subgame perfection, \(e = e(l, d)\) so that \((l, d) \neq (l, d)\). Then there is some \(e > 0\) such that \(U(l - e, d, e(l - e, d)) > U(l, d, e)\) and \(r(l - e, d) > 0\). But then \((l, d, e)\) cannot be an OBE.

In a sample of 12 such transactions, they found this devaluation to be on the order of 10 percent.

If inefficiently high debt were to arise merely because of an adverse ex post shock, the market might still provide incentives to recontract to an efficient equilibrium. That such recontracting has not been observed is sometimes attributed to a public-goods problem among lenders. In contrast, recontracting would not occur in our model even if all debt had been issued by a single lender.
because the agent could instead ask for and obtain \((\hat{d} - \epsilon, \hat{d})\) from the first bank. Q.E.D.

Proof of theorem 2.—The first-order condition for the worker’s optimal debt choice implies \(u'(c_i)l'(d) = \beta p(d)u'(c_G)\), where \(c_G\) is second-period consumption in the good state. Also, \(l'(d) > \frac{1}{1+\frac{1}{1+r}}p(d)\) for \(d > B\) so that
\[
u'(c_i)\frac{1}{1 + \frac{1}{1+r}}p(d) > \beta p(d)u'(c_G).
\]
Since a new bank is willing to offer a marginal loan rate of \(\frac{1}{1+\frac{1}{1+r}}p(d)\) on new debt, this inequality implies that utility would be increased by further borrowing. Q.E.D.

Proof of theorem 3.—Straightforward from subgame perfection. Q.E.D.

Proof of theorem 5.—Let \(h^T = \rho(\sigma), l = L(h^T),\) and \(d = D(h^T).\) Then define a new strategy \(\delta\) equivalent to \(s\) except that \(\delta(l, 1) = q\); thus the agent requests the total loan from the first bank then quits if accepted.

Also, define a new bank policy \(d\) to coincide with \(a\) except that \(d((l, 1, d'), 1, d) = a((h^T, 1), 1', d').\) That is, the first bank accepts the total loan request, and if the agent then makes further requests, banks adopt the same policy they use after \(h^T\) in the original equilibrium. It will then be in the agent’s best interest to quit after the first loan.

Clearly, \(\sigma = (\delta, e, \hat{d})\) yields the same aggregate loan, but from one bank. Since effort is the same, this bank earns nonnegative profits. Then \(\sigma\) is an SBE since the play along any subgame is identical to an equivalent subgame under \(\sigma.\) Q.E.D.

Proof of lemma 6.—Suppose that the agent has obtained an aggregate loan \((l, d)\) such that \(d \geq B\) and \(l \geq NFB(d).\) Suppose that the agent obtains an additional loan \((l, d)\) from new banks. If these new banks earn nonnegative profits, then \(l' < \frac{1}{1+\frac{1}{1+r}}p(d + d)d\), so that the agent’s utility is bounded above by
\[
f(d) = u\left(x_1 + l + \frac{1}{1 + \frac{1}{1+r}}p\hat{d} + \beta p\hat{u}(G - d - \hat{d}) - v(\hat{p}).\right)
\]
Since \(f(0) = U(l, d),\) the lemma is proved by showing \(f'(d) < 0\) for all \(d > 0.\) This follows, however, since \(p' < 0\) and \(l \geq NFB(d).\) Q.E.D.

Proof of lemma 7.—Since the NFB is downward sloping, \(l(d) \geq l \geq NFB(d)\) implies \(d \geq d^*\) and so involves further borrowing. Hence by lemma 6, the agent is worse off at any such point. Q.E.D.

Proof of theorem 8.—From lemma 6, the agent will do no further borrowing given a loan beyond the NFB locus. Thus any bank should be willing to accept a loan \((l, d)\) such that \(l < l(d)\) and \(l > NFB(d).\) Hence, the agent could get arbitrarily close to \(U(l^*, d^*, e^*)\) by requesting such a loan. An optimal strategy must therefore yield utility greater than or equal to \(U(l^*, d^*, e^*).\) Finally, this implies \(d < d^*\) since, from lemma 6, borrowing beyond \(d^*\) must involve either negative bank profits or lower utility for the agent. Q.E.D.

Proof of lemma 9.—Under the given assumption, \(U(l, d, p)\) is concave in \((l, d),\) so that (by the envelope theorem) the indifference curves are convex. Q.E.D.

Proof of theorem 10.—First we show that no equilibrium outcome can occur in the bankruptcy region with aggregate loan contract \((l, d)\) such that \(l < NFB(d)\) and
\[
l > \frac{1}{1 + \frac{1}{1+r}}\{p(d^*)d + [1 - p(d^*)]B\}.
\]
Consider any history resulting in such a loan portfolio. Consider an additional loan \((l, d)\) such that \(l + \hat{l} = \text{NFB}(d + \hat{d})\) and \(\hat{l} = [1/(1 + r)]p(d + d)d\). The definition of NFB implies

\[
u'(x_1 + l + \hat{l}) \frac{1}{1 + r} p(d + \hat{d}) = \beta p(d + \hat{d}) u_2'(G - d),
\]

so that at this new loan, the agent's indifference curve is tangent to the line connecting \((l, d)\) and \((l + \hat{l}, d + \hat{d})\), which has slope \([1/(1 + r)]p(d + \hat{d})\). By convexity of the agent's indifference curve in this region, this implies \(U(l, \hat{d}, p(d)) < U(l + \hat{l}, d + \hat{d}, p(d + d))\). Hence, there exist additional loans arbitrarily close to \((l, \hat{d})\) that would make the agent better off and would be accepted by the next bank. Therefore, the current history cannot be an equilibrium outcome.

Any equilibrium outcome must have aggregate loan \((l, d)\) such that \(l \geq \text{NFB}(d)\), or

\[
l \leq \frac{1}{1 + r} \{p(d*) + [1 - p(d*)]B\}.
\]

However, among such loans, the unique optimal loan involving nonnegative profits for the banks is given by \((l*, d*)\). Hence, this must be the unique SBE, if one exists.

Next, we demonstrate strategies supporting this equilibrium. Let \((l, d)\) be the current total aggregate loan contracted by the agent. Define

\[
l(\hat{d}; d) = \frac{1}{1 + r} \{p(d + \hat{d}) \min(\hat{d}, (G - d)^+) + [1 - p(d + \hat{d})] \min(\hat{d}, (B - d)^+)\},
\]

the loan schedule corresponding to zero expected profits for the new bank given that the agent does no further borrowing. Also define \(d^*(l, d)\) as the smallest additional debt such that \(l*(1, d) = l(d^*, d) \geq \text{NFB}(d + d*)\). Note that \(l^*(l, d) > 0\) if and only if \(l < \text{NFB}(d)\).

Finally, define \(d'(l, d)\) by

\[
d'(l, d) = \underset{\hat{d} \leq 0}{\text{argmax}} \max_{\hat{p}} u\left(x_1 + l + \frac{1}{1 + r} \hat{d}\right) + \beta \hat{p}u(G - d - \hat{d}) - v(\hat{p}),
\]

so that \(l'(l, d) = [1/(1 + r)]d'(l, d)\) is the optimal amount for the agent to save given current position \((l, d)\). Note that \(l'(l, d) < 0\) if and only if \(l > \text{NFB}(d)\). The agent's strategy is then simply

\[
s(l, d) = \begin{cases} 
\text{request } (l^*(l, d), d^*(l, d)) & \text{if } l^*(l, d) > 0 \\
\text{request } (l'(l, d), d'(l, d)) & \text{if } l'(l, d) < 0 \\
\text{quit} & \text{otherwise.}
\end{cases}
\]

As the uniqueness argument above demonstrated, it is easy to check that this implies that the optimal bank strategy is to accept a loan proposal \((l, d)\) from an agent with prior debt \((l, d)\) with \(d \geq B\) and such that \(l + l \leq \text{NFB}(d + d)\) if and only if

\[
\hat{l} \leq \frac{1}{1 + r} p(d + d^*(l, d)) \hat{d}.
\]

This is precisely the linear loan schedule identified by the theorem.
To verify that the agent’s strategy is an optimal response to the banks’ strategy, consider \((l, d)\) such that \(l \geq \text{NFB}(d)\). Suppose that the bank is prepared to accept a request \((l, d)\) \((d > 0)\). The agent’s utility must be lower after obtaining \((l, d)\) than with only \((l, d)\). To see this, recognize that the bank accepts \((l, d)\) expecting the agent to quit immediately or to save first and then quit. In the first case, \((l, d)\) must yield nonnegative profits for the bank, so the claim is proved by lemma 6. In the second case, the total additional loan \((l + l', d + d')\) involves nonnegative profits for both banks and hence again makes the agent worse off; but by the definition of \((l', d')\), \((l, d)\) must be even worse. Hence, beyond the NFB locus, any loan that is accepted by the banks lowers the agent’s utility. Thus the agent never borrows beyond NFB. Moreover, if currently below the NFB locus, the agent should request loans only at or below it. Yet given the linear loan schedule, the best such loan is exactly \((l^*, d^*)\). Q.E.D.

Proof of theorem 11.—\(U(l_1, d_1) \geq U(l^*, d^*)\) implies \(d_1 \leq d^*\) exactly as in the second part of the proof of theorem 8. Further, a comparison of the first-order conditions for each verifies \(d_1 \neq d^*\). Q.E.D.

Proof of theorem 12.—Immediate from theorem 11 and the definition of the linear loan schedule. Q.E.D.

References


