A Theory of Hard and Soft Information *

Jeremy Bertomeu† and Iván Marinovic ‡

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Abstract

We study optimal disclosure via two competing communication channels; hard information whose value has been verified and soft disclosures such as forecasts, unaudited statements and press releases. We show that certain soft disclosures may contain as much information as hard disclosures, and we establish that: (a) exclusive reliance on soft disclosures tends to convey bad news, (b) credibility is greater when unfavorable information is reported and (c) misreporting is more likely when soft information is issued jointly with hard information. We also show that a soft report that is seemingly unbiased in expectation need not indicate truthful reporting. We demonstrate that mandatory disclosure of hard information reduces the transmission of soft information, and that the aggregation of hard with soft information will turn all information soft.

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JEL Classification: D72, D82, D83, G20.

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†Baruch College. Email: jeremy.bertomeu@gmail.com
‡Stanford University. Email: imvial@stanford.edu
1 Introduction

The coexistence of hard and soft information is a fundamental characteristic of the measurement process. A disclosure can be soft, in the form of a measure that “can easily be pushed in one direction or another” (Ijiri 1973, p. 36), or hard, having been subjected to a verification after which “it is difficult to disagree” (ibid). For example, firms have asset classes ranging from tangible assets to traded securities which are subject to a formal verification procedure. Forward-looking assets are more difficult to objectively verify and are typically regarded as being soft. For example, the value of many intangibles (e.g., goodwill, patents, and brands) may require unverifiable estimates of future risks.

The literature on strategic communication is extensive but, to our knowledge, most prior theories have focused on two broad families of models; either models in which disclosed information is hard (Jovanovic 1982; Dye 1985) or misreporting is costly (Dye 1988), with no other channel for credible soft communication, or models in which the strategically reported information is soft (Crawford and Sobel 1982) and may coexist with an exogenous hard signal (Stocken 2000; Lundholm 2003) or may become hard with a fixed probability (Sansing 1992; Marinovic 2013). Within both families of models, strategic communication concerns either hard information or soft information. Our primary contribution is to combine these two families by developing a theory of competing channels of communication in which the manager chooses how to report soft information and decides whether to issue additional hard, but costly, verified disclosure. Put differently, we evaluate the strategic release of hard information in a framework where the manager might also credibly communicate some soft information in the absence of verification.

The following are three key elements of our research design.

1. The operation of a firm involves both verifiable and unverifiable as-
sets, but the nature of the unverifiable assets prevents an objective external validation of the true value of that asset. For instance, firms report their order backlog and material contracts (Rajgopal, Shevlin, and Venkatachalam 2003; Li 2013), which are technically verifiable facts, but also supplement these with revenue and earnings forecasts which are not directly verifiable.

2. Management has some discretion over whether to issue hard information. A highly institutionalized example of this is the case of the automobile industry which publishes weekly production numbers, given voluntarily, via its trade journal, Ward’s Automotive Reports (Bertomeu, Evans, Feng, and Tseng (2013)). From the revenue side, Regulation S-K requires a disclosure of the order backlog in its regulatory filings (Item 101(c) VIII), an item that seems prevalent in manufacturing (Rajgopal, Shevlin, and Venkatachalam 2003), but such disclosures are often accelerated and issued as press releases or updates.

3. There is uncertainty about the managerial propensity to manipulate soft information. This assumption stems from from the cheap talk literature and is motivated from the fact that many types of soft reports are informative (Kirk and Vincent 2014), but they are neither perfectly truthful nor fully devoid of credibility (Rogers and Stocken (2005)). We exploit this assumption in our model to examine soft information as an imperfect substitute the strategic release of hard information.

Our model has several natural applications in accounting, as the dis-

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1 An article in the Dow Jones Institutional News (January 28th 2015) titled “Boeing Posts Stronger-Than-Expected Results – 2nd Update” notes that “Boeing is working through a record $430 billion order backlog for commercial jets. That comes as some industry executives and investors have voiced concern about a potential bubble in jetliner demand.” Its main competitor, Airbus, similarly issues regular updates on its backlog.
tinction between hard and soft information can manifest itself at different levels of the financial reporting process.

First, firms can decide to become private, which can lift requirements for various costly regulatory filings or mandated audits (i.e., go dark, see Bushee and Leuz 2005); many regulations have changed the conditions under which firms are subject to mandatory filings (Lennox and Pittman 2011). Our theory also provides insights regarding “unaudited” financial statements, now available in data sets covering private firms (see, e.g., Minnis (2011)).

Second, mandatory audits must be conducted by a registered auditor and, in the US, according to generally accepted auditing standards; however, audit quality varies as a function of the resources committed to the audit (Becker, DeFond, Jiambalvo, and Subramanyam 1998), implying the differential hardness of financial statements. Some audit opinions may be qualified, reflecting a failure to obtain enough hard evidence to substantiate certain account balances.

Third, at the firm level, there is significant cross-sectional variation in earnings quality (Dechow, Ge, and Schrand 2010), as defined by the mapping from economic fundamentals to verifiable accounting variables. Firms with low earnings quality may rely more on managing “soft” accruals. Fourth, firms use many other voluntary channels to certify assets, ranging from rating agencies to investment banks (Kraft 2013). Certain public disclosures, made as part of management discussions and analy-

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2 The Public Company Accounting Oversight Board defines the objective of the audit to “obtain reasonable assurance about whether the financial statements are free of material misstatement.” (see AU Section 110), but not complete certainty regarding the absence of misstatement. This implies that the degree of verification of items that are hard to verify can vary across audits. In certain cases, auditors specifically indicate a disclaimer and/or qualification, over items that could not be appropriately tested. Consistent with this idea, empirical studies document that capitalized goodwill is valued at a discount relative to other tangible assets (Lys, Vincent, and Yehuda 2012) and financial instruments classified at level 3 (“marked to model”) are valued at a discount relative to level 1 (Kolev 2009).
ses, press releases or management forecasts, can be supported by hard verified facts, or disclosed to the market informally as a soft unverifiable disclosure.

We describe next the primary insights gained from our analysis. The market jointly learns about the firm’s final cash flow and the manager’s propensity to be untruthful. An unfavorable soft report would not have been issued by an untruthful manager and, therefore, is perceived as fully credible so that the market impounds the information as if it were hard. This prediction adds to cheap talk models where unfavorable reports are more informative but never fully credible (Crawford and Sobel 1982; Morgan and Stocken 2003). The model is consistent with the empirical observation that unfavorable reports tend to be more credible (see, e.g., Williams 1996; Rogers and Stocken 2005). We also predict that markets should not react further when a verified signal is released (ex-post) if a preceding soft report was unfavorable.3

The second main insight is that a manager who is more likely to mis-report is more willing to verify and release hard information, even though issuing hard information reduces her ability to manipulate. To explain this key property of our model, we reiterate that not all information can be made hard. Hence, what managers lose in terms of discretion to over-report the verifiable information, they can gain in credibility for the remaining soft disclosure.

Untruthful managers will tend to issue higher soft reports, naturally facing stronger market skepticism. We demonstrate that untruthful managers are always more willing to issue hard information, relative to truthful managers. We thus predict that situations in which managers release

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3 This property may allow for a strict test of the theory. In cheap talk models, such as Crawford and Sobel (1982) and Morgan and Stocken (2003), the signal is never fully credible so that the market will always learn (to some extent) from an ex-post verified signal. In our model, if the soft signal is unfavorable, any subsequent hard report of this information should have been perfectly anticipated.
more hard information are also more likely to feature aggressive soft reports and have a greater likelihood of issuing overstatements.

We then apply our theory to two current debates about standard-setting, where soft information is believed to be an important concern. We consider the consequence of reducing the amount of discretion in the reporting of any verifiable information. The mandatory disclosure of hard information has the unintended consequence of reducing information about the soft, unverifiable components of firm value. In other words, there is a trade-off between the quality of hard versus soft information. Regulation cannot increase the social provision of one without reducing the other. Next, we examine the consequences of aggregating hard and soft information into financial statements. For example, fair-value accounting aggregates both hard information, e.g., prices obtained from liquid markets, and soft information, e.g., shallow markets with estimates of varying quality. We establish that this process makes investors as uncertain about the firm value as they would be if the entire value of the firm were soft, unverified and subject to manipulation.

Related Literature. Our study develops a model of communication with multidimensional information. It nests the cheap talk models of Sansing (1992), da Silva Pinheiro (2013) and Marinovic (2013) with the truthful disclosure of hard information in Milgrom (1981) and Jovanovic (1982). The distinctive feature of our approach is that both soft and hard communication are strategic choices, so we may compare the channels of communication on which the manager chooses to rely.

A prior literature on strategic communication considered the role of

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4 A strict empirical test of this proposition (which rules out alternative explanations) goes beyond the scope of our study. However, some of the descriptive evidence is consistent with our prediction. For example, IPOs are often viewed as institutional mechanisms that require significant releases of hard information (part of it is due to increased regulatory monitoring and part of it is because of additional work by auditors and investment bankers). Teoh, Wong, and Kao (1998) find that many IPO firms feature abnormally large accruals, followed by poor performance.
public hard information in making soft communication credible. An early example of this approach, in the context of a revelation mechanism is provided in Gigler and Hemmer (1998). In their model, which is extended by Gigler and Jiang (2011) and Sabac and Tian (2012), a hard signal can serve as a contracting variable to discipline truth-telling in the reporting of soft information. In cheap talk settings, such pre-commitment to a mechanism is not feasible or practical; however, a hard signal, which comes after the release of soft information, can help convey soft information (Stocken 2000; Lundholm 2003) because it provides the means to ex-post respond to the true information. The monitoring role of the hard information is achieved because the realization of hard signals is correlated to the soft realization and facilitates truth-telling. To avoid repeating the insights already reported in the literature, we abstract away from these aspects in our baseline model by assuming that the hard signal, when reported, is not correlated to the soft signal.

Within the applied cheap talk literature, several papers have used assumptions that are similar to ours. We share with this literature the assumption that information is soft and communication need not be truthful (see, e.g., Newman and Sansing 1993; Gigler 1994; Fischer and Stocken 2001, 2010; Che, Dessein, and Kartik 2013). Our model specifically relates to the subfamily of cheap talk models that include payoff uncertainty, where the willingness or ability to misreport is not publicly known. In Morgan and Stocken (2003), the audience is uncertain about the actual degree of preference misalignment, that is, how much the sender cares about a stock price increase versus the accuracy of the report, a trade-off that could apply not only to analysts but also to management. In our model, we assume a different kind of payoff uncertainty: the sender is sometimes constrained to tell the truth, but the receiver is uncertain about

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5 Other recent studies involve soft communication followed by an exogenous hard signal, see, e.g., Corona and Randhawa (2010), Ramakrishnan and Wen (2014) and Julien and Park (2014).
the sender’s actual reporting discretion. The strategic communication of a soft report by constrained senders was introduced by Sansing (1992) and Benabou and Laroque (1992), and recently extended by Chen (2011), Marinov (2013) and da Silva Pinheiro (2013). Our model builds on this setting to derive the content of soft information. However, our main objective concerns the strategic release of hard information as an alternative channel of communication.

2 The Model

2.1 Assumptions and equilibrium definition

Let \( \tilde{\pi} \) denote the firm value and assume that it depends on two components: a verifiable asset \( \tilde{h} \) and an unverifiable asset \( \tilde{s} \). Specifically, the verifiable asset might be a physical asset (e.g., equipment, inventories, property) or a security traded in a liquid market, and the unverifiable asset might be an intangible whose value is firm-specific and forward-looking.

\[ \tilde{\pi} = \tilde{h} + \tilde{s} \]

For \( x \in \{h, s, \pi\} \), \( \tilde{x} \) has density \( f_x(.) \), log-concave distribution \( F_x(.) \), support over \( \mathbb{R} \), finite first moment \( m_x \) and thin left tail, i.e., \( \lim_{y \to -\infty} \frac{d}{dy} \ln f_x(y) = \infty \).

The difference between the two types of assets is that \( \tilde{h} \) can be objectively verified by an external party and reported for a cost \( c > 0 \). In our model, if a measure has been verified, there is no further room for disagreement about the true underlying value of the asset. Hence, we refer the assumption of a thin left tail is satisfied by most commonly used distributions. Logconcavity is used here for expositional purposes and used to show the existence of a unique solution to the equations characterizing the equilibrium. However, even if logconcavity does not hold, it can be shown that there is a unique perfect sequential equilibrium in our model (the solution to the equations with the smallest probability of verification).
to the output of the verification process as hard information.

A2. The manager has the option to provide hard information, denoted by an indicator variable \( d \in \{0, 1\} \), in which case \( \tilde{h} \) becomes public and a cost \( c > 0 \) is incurred.

[Ijiri (1975)] defines soft information as a measure that is subjective and therefore can be manipulated. We operationalize this definition in terms of a report that a manager may alter. We denote the soft report \( r \) and, as in [Sansing (1992)] and [Benabou and Laroque (1992)], assume that the report is subject to discretion represented by a binary random variable \( \tilde{\tau} \in \{0, 1\} \).

If \( \tilde{\tau} = 0 \), the manager is unconstrained and has complete discretion over \( r \) and if \( \tilde{\tau} = 1 \), the manager is constrained and cannot over-report.

A3. The manager issues a report \( r \in \mathbb{R} \) where, if \( \tilde{\tau} = 1 \) (constrained manager), \( r \) must satisfy \( r \leq ds + (1 - d)(h + s) \). Reporting discretion \( \tilde{\tau} \) is independent from \( (\tilde{h}, \tilde{s}) \) and such that \( \Pr(\tilde{\tau} = 1) = \gamma \). Only the manager knows the realization of \( \tilde{\tau} \).

Assumption A3 states that some managers are averse to over-claiming the true value of their assets (e.g., a person who downplays his true value might not be seen as unethical or a manager who understates for no personal gain is unlikely to face a criminal prosecution). This assumption closely follows [Sansing (1992)] who interprets \( \tilde{\tau} = 1 \) as a setting in which the external environment will reflect the information. In our setting, we

\[\text{\footnotesize{\textsuperscript{7}}Our model has a pure strategy equilibrium, where all managers with soft information above a certain level (conditional on whether they verify or not) issue the same maximal report. What makes this pooling outcome possible is that the constrained manager under-reports because any higher report would have no credibility. Hence, the model can also be solved with the restriction that constrained managers should report their exact information (cannot under-report). Under this restriction, any equilibrium will feature mixing by the unconstrained manager. Importantly, the implied market prices and expected utilities, for each type of manager, are equivalent to the solution given here given}}\]
extend this approach to a setting with partial verification.\footnote{If \( \gamma = 1 \), all managers are truthful; therefore, the soft communication is fully informative and no hard information will ever be issued. If \( \gamma = 0 \), no soft information is credible, implying that the threshold for releasing hard information will coincide with Jovanovic (1982).}

The manager maximizes the trading value of the firm, which we denote as \( P(I) \) where \( I = (d, dh, r) \in \mathcal{I} \) is the information set of outside investors. If \( h \) is verified, we set \( d = 1 \) so that investors observe \( I = (1, h, r) \). If \( h \) is not verified, we set \( d = 0 \) so that investors observe \( I = (0, 0, r) \) and only know the soft report \( r \).

\textbf{A4.} The firm is priced at \( P(I) = \mathbb{E}[\tilde{\pi}|I] - dc \), where \( I \) indicates all public information (soft and hard information, if any). The manager chooses his hard and soft reports to maximize this market price.

A Nash equilibrium consists of reporting strategies and prices such that (a) reporting strategies maximize prices and (b) prices form according to the Bayes rule, whenever possible.

Communication games rarely have a unique equilibrium, and our model is no exception. To address the equilibrium multiplicity, we impose several refinements that are plausible and practical for our setting. We restrict attention to price functions that are non-decreasing and, as in Sansing (1992) and Marinovic (2013), equilibria such that (c) the unconstrained manager does not condition his report on payoff irrelevant information.\footnote{That is, the reporting strategy of unconstrained managers may only depend on information they choose to verify. This restriction rules out sunspot equilibria because any information reported as soft does not directly enter the unconstrained manager’s problem. Note that payoff irrelevance would have no bite in the standard cheap talk setting of Crawford and Sobel (1982) because the true state is part of the utility function of the sender.} As in these prior studies, this criterion reduces the large multiplicity of equilibria from the cheap talk component of our model. Because our model
borrows from the persuasion literature, which is a subclass of signaling games, we use (d) the perfect sequential equilibrium (“PSE”) of Grossman and Perry (1986), a fairly standard signaling refinement. With a slight abuse of language, we refer to a Nash equilibrium satisfying these refinements as a PSE.

**Definition 1** A Perfect Sequential Equilibrium \( \Gamma \) (“PSE”) consists of a reporting strategy \( S = (D(\cdot), R(\cdot)) \) and a nondecreasing price function \( P(\cdot) \), such that:

(a) at any information set \( \omega = (h, s, \tau) \) of the manager, \( (D(\omega), R(\omega)) \in \arg\max_{d,r} P(d, dh, r) \) s.t. \( \tau r \leq \tau(ds + (1 - d)\pi) \),

(b) at any public information set \( I = (d, dh, r) \) such that, whenever possible, the price system is obtained from Bayes rule, \( P(I) = \mathbb{E}(\tilde{\pi}|I) - cd \),

(c) for \( (\omega, \omega') \) such that \( \tau = \tau' = 0 \) and \( D(\omega)h = D(\omega')h' \), then \( R(\omega) = R(\omega') \),

(d) \( \not\exists (r_0, h_0, d_0, p_0, \Lambda_0) \) such that \( p_0 \leq \mathbb{E}(\tilde{\pi}|\omega \in \Lambda_0) - cd_0 \) and \( \Lambda_0 \) is a non-empty subset of that type space that contains all types \( \omega \) that can send \( (d_0, dh_0, r_0) \) and are better-off with \( p_0 \) over \( P(D(\omega), D(\omega)h, R(\omega)) \).

In essence, the PSE selects the equilibrium in which managers achieve their maximal credible price. This refinement selects a market price function that has a few intuitive properties. First, there is a maximal market price after a soft report which is the market price that is achieved by an unconstrained manager. Second, for any market price that is not maximal, the market believes that the manager is constrained and accepts the report as truthful. That is, in the absence of verification, if investors observe a report \( r \) that does not maximize price, they interpret this report as truthful and set the price at \( P(0,0,r) = r \). Otherwise, they set the price at \( P(1,h,r) = h - c + r \). This implies that the market price has the following
structure (which we show formally in the Appendix).

\[ P(0,0,r) = \min(r, \lambda_0), \quad (1) \]
\[ P(1,h,r) = h - c + \min(r,z), \quad (2) \]

where \( \lambda_0 \) is the maximal market price if hard information is not issued and \( z \) is the maximal valuation of the unverifiable component incremental if hard information is issued.

2.2 Discussion of the assumptions

In this section, we discuss our working assumptions A1-A3 and provide some complementary background and interpretation.

Assumption A1 states that the marginal contribution per unit of verifiable information to firm value is not a function of the unverifiable information \( s \). This exclusion is not critical to our analysis but focuses our discussion on informational channels only: under our specification, if \( s \) were publicly known, it would have no effect on the manager’s decision to verify. The incremental effect of a non-separability is very intuitive in our model. For example, if the assets are complements (e.g., a high-quality luxury item with a strong brand), the firm’s willingness to verify will be greater conditional on high soft information. Vice-versa, if the assets are substitutes (e.g., a low-quality non-durable hyped with advertising), the firm’s willingness to verify will be lower, conditional on low soft information. These cases are formally discussed in Section 4.3.

Assumptions A2 and A3 formalize the two possible channels of communication in our model; a perfect but costly verification versus a communication that relies on managerial credibility. The assumption that some managers are constrained in their ability to over-report plays a critical role in our analysis, and has various plausible interpretations, such as the possibility that the information derives from the external environment (Sans-
ing (1992)), an ethical behavioral type who is averse to lying (Benabou and Laroque (1992)) or the strength of internal control systems (Marinovic (2013)).

Lastly, we use two refinements to select a unique, intuitively compelling equilibrium in our setting. Payoff irrelevance speaks to the cheap talk component of our model and restricts the attention to equilibria, in which the manager conditions his strategy on variables that directly affect his utility function. This means that a manager will not condition his report on an asset value unless it is verified. Although we view payoff-irrelevance as appealing (especially in market settings with fewer obvious means of coordination), it is not innocuous; for example, Chakraborty and Harbaugh (2010) show that some information transmission is feasible, using payoff-relevant messages. The perfect sequential equilibrium of Grossman and Perry (1986) is primarily a signaling refinement that rules out overly pessimistic beliefs as a function of the choice to issue hard information (to this effect, it plays no role in the limit of the model with \( c = \infty \) in which there is only cheap talk). As do many other forward-induction arguments, it relies on the idea that the manager (or sender) is a first-mover and, therefore, can “offer” an off-equilibrium report and price that should be rationalized, if possible, by prospective investors. Implicitly, this allows the manager to coordinate expectations on equilibria that maximize the market price.

3 Equilibrium

The analysis is organized as follows. We derive the manager’s optimal disclosure of hard information \( D(\omega) \) and the manager’s reporting behavior

10 The existence of ethical concerns has received interest in the recent literature; see, e.g., Ederer and Fehr (2007), Evans et al. (2001) and Huddart and Oul (2012). In a recent study, Fischer and Huddart (2008) provide a model of ethics and social norms, and discuss the impact of ethical motives in the design of organizations.
Consider first the unconstrained manager’s choice whether to issue hard information. This manager compares the payoff from issuing hard information, which yields \( h - c + z \), to the payoff from issuing soft information only, which yields \( \lambda_0 \). Hence, the unconstrained manager issues hard information if and only if

\[
h \geq \lambda_0 - z + c.
\]

By assumption, the unconstrained manager is not tied by the realization of the unverifiable component \( s \): his decision to verify is only a function of the verifiable component \( h \). Hereafter, we refer to \( k \equiv \lambda_0 - z + c \) as the verification threshold.

The problem facing a constrained manager is different. He chooses to verify \( h \) if and only if

\[
h - c + \min(s, z) \geq \min(\lambda_0, h + s).
\]

This inequality can be reformulated as consisting of two conditions: (a) \( h + s - c \geq \lambda_0 \) and (b) \( h \geq k \). Condition (a) states that the unconstrained manager is willing to verify \( h \) provided that the firm value net of cost, \( h + s - c \), is higher than the maximal price that the manager could attain without verification \( \lambda_0 \). The constrained manager does not verify \( h \), unless the firm value \( h + s \) is relatively large, that is, above the maximal price. Condition (a) would be sufficient if all information were truthfully reported after verification and led to a price \( h + s - c \). However, in our model, some residual unverifiable information \( s \) implies an upper bound for the market price of \( s, \min(s, z) \). When the maximal price \( z \) is attained,
the constrained manager then faces the same verification problem as if she were unconstrained, implying that verification condition (b) is also required for the unconstrained manager. This is summarized in the following lemma.

**Lemma 1** The unconstrained manager issues hard information if and only if \( h \geq k \) while the constrained manager issues hard information if and only if \( h + s \geq \lambda_0 + c \) and \( h \geq k \).

The key consequence of Lemma 1 is that the unconstrained manager will verify \( h \) more often than the constrained one. Hence, a manager with greater ability to manipulate is more likely to issue hard information. The unconstrained manager not only (voluntarily) constrains her reporting strategy but also reveals herself more likely to be over-reporting, about \( s \), by issuing hard information.

Why would an unconstrained manager issue hard information when a constrained manager would not? Soft communication can be fully credible, even if hard information is absent. To be precise, a soft report \( r \) that does not attain the maximal price \( \lambda_0 \) must have been issued by a constrained manager. Therefore, a constrained manager with low total firm value does not need to incur a costly verification to reveal truthfully her information and attain a price \( h + s \). In comparison, the unconstrained manager always chooses to attain the maximal price, even if the firm value is low, and thus never issues a fully credible soft disclosure. For an unconstrained manager, costly verification is the only means to overcome the market’s skepticism about the residual soft information.

The next step is to characterize how managers report their soft information. We focus on the case after \( h \) is verified. Unconstrained managers issue a report that attains the maximal price \( h + c - z \), denoting this report as \( R(1, h, z) = r_z \). In equilibrium, one can show that \( r_z \) is also the report issued by constrained managers with \( s \geq r_z \) (and who issue hard informa-
tion \( h \geq k \) because their reporting constraint does not bind, and they can mimic the choice of an unconstrained manager.

The market price after \( h \) and \( r_z \) can then be factored into a hard and a soft component:

\[
P(1, h, r_z) = h - c + \mathbb{E}[\tilde{s}|D(h, \tilde{s}, \tilde{\tau}) = 1, R(h, \tilde{s}, \tilde{\tau}) = r_z]. \tag{4}
\]

In Equation (4), the first term is the pricing of the hard disclosure minus the verification cost, for which there is no remaining uncertainty, and the second term is the pricing of the soft information. To price the second term, observe that a constrained manager who can report \( r_z \) faces the same problem as an unconstrained manager and verifies under the same circumstances. Hence, the conditioning event in Equation (4) can be restated as either the manager is unconstrained or the manager is constrained and can report \( r_z \), i.e., \( \tilde{s} \geq r_z \). The market price then simplifies to

\[
P(1, h, r_z) = h - c + \mathbb{E}[\tilde{s}|\tilde{\tau} = 0 \cup \{\tilde{\tau} = 1, \tilde{s} \geq r_z\}]. \tag{5}
\]

This market price function yields a key implication of our model regarding the equilibrium report \( r_z \). Recall that the PSE selects the equilibrium that maximizes the market price. Hence, if there is a continuum of credible pairs \((r_z, z)\), the PSE selects the pair maximizing \( P(1, h, r_z) = h - c + z \). For any expectation \( z \) a constrained manager with \( s < z \) (resp., \( s > z \)) decreases (resp., increases) this expectation. In turn, this implies that the equilibrium report that maximizes the price is \( r_z = z \). The same argument can be applied to the case in which \( h \) is not verified; hence, absent a verification, both unconstrained managers and constrained managers whose firm value is \( h + s \geq \lambda_0 \) issue a soft report equal to \( \lambda_0 \).

Lemma 2 If \( h \) is verified, unconstrained managers and constrained managers with \( s \geq z \) issue a soft report \( z \). If \( h \) is not verified, unconstrained managers and
constrained managers with $h + s \geq \lambda_0$ issue a soft report $\lambda_0$. Hence, the soft report made by unconstrained managers is, in equilibrium, not discounted by the market.

Although Lemma 2 states that managers, on average, report the market price, it does not mean that all managers report truthfully. To the contrary, the report $z$ aggregates the biases of both constrained and unconstrained managers. However because the biases offset one another, the equilibrium report does not trigger any market skepticism. While unconstrained managers over-report (in expectation), constrained managers, with $s \geq z$, always under-report down to $r = z$. Reports above the maximum price $z$ are not credible and are not made in equilibrium.

Our model thus establishes that the absence of a market discount after a report need not imply truth-telling across all managers. This property contrasts with costly signaling models, in which reports are generally discounted (Ewert and Wagenhofer 2005; Laux and Stocken 2012).

Next, we elaborate on the economic intuition that supports the absence of a discount after the report $r_z$ is made. We know that only the high report $r_z$ is indicative of unconstrained managers; hence, any other lower report $r < r_z$ is priced as being truthfully made. If the price $z$ were to discount the report $r_z$ and remain below $r_z$, it would be feasible for an unconstrained manager to deviate from the report $r_z$ (the report discounted to $z$) to a report $r \in (z, r_z)$, which is not discounted and priced at $r > z$. In summary, because an unconstrained manager can simply alter her reporting strategy to attain any undiscounted report made by unconstrained managers, the report that she chooses to issue cannot be discounted.

We can then write the prices $z$ and $\lambda_0$ using Bayes’ rule as follows:
\[
\begin{align*}
z &= \mathbb{E}[\bar{s}|\bar{\tau} = 0 \cup \{\bar{\tau} = 1, \bar{s} \geq z\}], \quad (6) \\
\lambda_0 &= \mathbb{E}[\bar{\pi}|D(\bar{\omega}) = 0, R(\bar{\omega}) = \lambda_0] \\
&= \mathbb{E}[\bar{\pi}|\Omega], \quad (7)
\end{align*}
\]

where \( \Omega \equiv \{\bar{\tau} = 0, \bar{h} < k\} \cup \{\bar{\tau} = 1, \bar{\pi} \in [\lambda_0, \lambda_0 + c]\} \cup \{\bar{\tau} = 1, \bar{\pi} \geq \lambda_0 + c, \bar{h} < k\}. \) The last equality follows immediately from expanding the conditions under which \( h \) is not verified, as given by Lemma 1. Any maximal prices \((z, \lambda_0)\) that satisfy (6)-(7) sustain a PSE. We demonstrate in the next Proposition that a unique PSE exists.

**Proposition 1** There exists a unique PSE and it is given as follows:

1. The market price \( P(d, dh, r) \) equals (i.a) \( P(0, 0, r) = \min(r, \lambda_0) \) or (i.b) \( P(1, h, r) = h - c + \min(r, z) \), where \((z, \lambda_0)\) is defined by Equations (6)-(7).
2. The hard report \( h \) is issued, i.e., \( D(h, s, \tau) = 1 \), if and only if \( h \geq \lambda_0 - z + c + \tau \max(0, z - s) \).
3. The constrained (resp., unconstrained) manager reports (iii.a) \( R(h, s, 1) = \min(s, z) \) (resp., \( R(h, s, 0) = z \)) when issuing hard information, or (iii.b) \( R(h, s, 1) = \min(h + s, \lambda_0) \) (resp., \( R(h, s, 0) = \lambda_0 \)) when issuing only soft information.

The set of results presented in Proposition 1 are driven by what we believe to be two simple economic forces. The first force is that soft information is fully revealing after a low report is observed, because it indicates that the manager has forfeited an alternative report that would have an increased price; however, for the same reason, high soft reports that maximize price are not credible and create a demand for verification that is a function of the degree of soft communication. The second force is that the value of verification is the greatest in managers who bear the greatest market skepticism. Because skepticism is always the greatest following higher soft reports made by unconstrained managers, the unconstrained manager that are the most willing to verify.

Building and expanding on these intuitions, we summarize the primary highlights of the Proposition. First, soft information is always partially informative; in fact, parts (i.a) and (i.b) reveal that soft information is as informative as hard information, provided the information that is disclosed is relatively unfavorable. The mechanism of this form of communication is that a report that does not appear to maximize the price is interpreted by the market as having been issued by a constrained manager. Figure 1 depicts the
constrained manager’s reporting strategy as a function of \( s \), for \( h > k \). The manager issues hard information only if \( s \) is such that the firm overall value is so large that it would not be credible in the absence of verification.

Second, the presence of only soft disclosures tends to indicate that the manager has received relatively unfavorable information from all sources. Put differently, the act of releasing more hard information is good news about not only the hard assets but also the unverifiable assets. This effect encourages the unconstrained manager to issue hard information more frequently, as illustrated in Figure 2.

Third, a high soft report need not be biased; in fact, we show that the market responds to a soft report by considering the report as true in expectation. The intuition for this result is that the PSE selects the greatest credible prices, \( \lambda_0 \) and \( z \). This price is necessarily formed by taking expectations over all unconstrained managers as well as constrained managers who cannot over-report and thus whose observed signal is above the report. Hence, the price-maximizing report exactly matches the price and is unbiased. Of practical interest, the model provides an economic rationale for why environments with little outside verification need not exhibit large biases in expectation.\footnote{This property is different from standard cheap talk models, whose primary research question concerns the quantity of information transmission, but language is essentially}

\[ \text{Figure 1: Reporting strategy of constrained manager (for a given } h \text{).} \]
Fourth, the region in which the hard information is issued is wider if the manager is unconstrained, implying that the posterior probability of being constrained is reduced after hard information is released. Consistent with our first result, the constrained manager uses an alternative mode of communication that does not require any verification when the firm’s value is low. This implies that a constrained manager tends to be less willing to report hard information; the unconstrained manager, on the other hand, anticipates issuing a high soft report and, therefore, obtains the greatest benefit from verification.

Two limit cases provide additional intuition about the determination of the reporting strategies, as well as linkages to the prior literature on hard information. In the model, the parameter $\gamma$ is a measure of the fraction of managers that are constrained and, therefore, captures the market trust in a soft report. In the limit case where $\gamma \to 0$, nearly all managers are assumed to be unconstrained, and our model converges to classic costly disclosure, in which hard information is issued if and only if $\tilde{h} \geq k$, the unique solution to $k - c = E[\tilde{h}|\tilde{h} \leq k]$. That they become (nearly) uninformative does not mean, however, that soft reports will be random or set at very high levels. As we have seen earlier, conditional on issuing hard information, the report is equal to the expected cash flow; hence, in the limit, nearly all (unconstrained) managers report the expected cash flow, i.e., $z = E[s]$ if no hard information is issued and $\lambda_0 = E[s] + E[\tilde{h}|\tilde{h} \leq k]$. In this context, a market defined by the action that it induces. By contrast, in our model, messages have a literal meaning defined as what a truthful manager would have reported; we can then analyze biases as the deviation from literal meaning.

Figure 2: Equilibrium release of hard information in the $(h,s)$ space.
with low levels of trust will manifest through soft reports that are imprecise and highly concentrated around the mean.

The other limit case occurs when \( \gamma \to 1 \), which means that most managers are likely not to over-report. The market then will be able to interpret a soft report as true, even if this soft report is very favorable, implying that \( z, \lambda_0 \to \infty \). This situation naturally implies that most (constrained) managers will be such that \( \pi \leq \lambda_0 \) and will find no benefit in issuing hard information. It is intuitive that a high degree of trust completely removes the need for hard information. The model will feature unraveling because a constrained manager will report the true cash flow and be perfectly identified, with a probability that converges to one. This is perhaps balanced by the observation that a vanishing fraction of remaining unconstrained managers will issue very aggressive reports \( \lambda_0 \) or \( z \), so over-reporting will be infrequent but very large.

We describe next comparative statics for the model that apply even when these limit cases are not reached.

**Corollary 1** The following comparative statics hold:

(i) The soft report \( z \) increases in the propensity to be constrained \( \gamma \) (the effect on \( \lambda_0 \) is ambiguous).

(ii) The soft report \( \lambda_0 \) conditional on no hard information increases in the cost of the hard report \( c \) (further, \( z \) does not depend on \( c \)).

Whenever the sole residual source of uncertainty is the unverifiable component \( s \), an increase in the degree of market trust \( \gamma \) will allow more favorable soft information to be credibly transmitted to investors. The highest price \( z \) will then increase and more firms, owned by constrained managers with \( s < z \), will be priced at their fundamental value. Fewer unconstrained managers, on the other hand, will report more aggressively since they must issue a soft report equal to \( z \) to achieve the highest price.

In the absence of hard information, when the uncertainty concerns the entire firm value \( \pi \), the effect of \( \gamma \) is ambiguous. We know from the limit cases that the soft report will change from \( \lambda_0 = \mathbb{E}[\pi|h \geq k] \) to \( \lambda_0 \to \infty \) as \( \gamma \) changes from \( \gamma = 0 \) to \( \gamma \to 1 \). However, the change need not be monotonic for interior values of \( \gamma \). An increase in \( \gamma \) increases the price \( z \), potentially raising managers’ willingness to issue hard information. This tends to reduce expectations about the verifiable component and reduce \( \lambda_0 \). Moreover, as noted earlier, the increase in \( \gamma \) raises expectations about the unverifiable component since more managers are likely to be constrained, which increases \( \lambda_0 \).

Figure 3 offers a numerical illustration when the verified and unverified information are normally distributed. Increasing \( \gamma \) then results in higher prices and a lower likelihood
of verification. Similarly, a higher cost of verification \( c \) results in higher soft information prices and a lower likelihood of verification.

Another determinant of optimal reporting behavior is the cost of issuing the hard report \( c \). When \( z \) is interpreted as the market price that follows a soft report on \( \tilde{h} + \tilde{s} \), the fact that \( \lambda_0 \) should increase if \( h \) alone can be reported at higher cost is intuitive. Of somewhat greater interest is that \( \lambda_0 \) is both the price and the equilibrium report of unconstrained managers. These managers report more aggressively when it is more costly to release hard information. That is, when it is difficult to send hard information, more favorable realizations of \( \tilde{h} \) are disclosed as soft information. This tends to increase the maximum price when only soft information is reported because, according to assumption A3, constrained managers cannot over-report. Unconstrained managers then benefit from the higher maximum price by over-reporting to a greater extent. Similarly, since all reports below \( \lambda_0 \) are not discounted and perfectly reveal \( h + s \), the amount of communication through soft reports increases when hard information is less widespread. Put differently, the presence of hard information, which serves as an alternative communication channel, provides some discipline to the soft reporting channel and reduces a constrained manager’s willingness to overstate.

Figure 3: The effect of \( \gamma \) and \( c \), with \( \tilde{h}, \tilde{s} \sim N(0, 1) \). We assume \( c = 1 \) (left panel) and \( \theta = 1 \) (right panel)
4 Applications

4.1 Mandatory disclosure

In the first application, we examine the economic consequences of mandatory disclosure, which is defined as the requirement that the verifiable asset is subject to verification. This environment of mandatory disclosure is formally captured by constraining \( d = 1 \) in part (a) of Definition \[ \text{Definition 1}. \]

Note that a rationale for imposing mandatory disclosure in such settings is that, first, unconstrained managers are more likely to exploit the optionality of issuing hard information and, second, because they issue relatively more hard information, low-quality reports could be issued after hard information is released.

We formally discuss these arguments. Because the equilibrium has the same general form as the baseline model, let us now denote \( z^m \) as the report made by an unconstrained manager about \( \tilde{s} \). Restating some of the results obtained in the baseline model, we know that (a) all constrained managers with a soft signal \( \tilde{s} \) greater than \( z^m \) will issue the report \( z^m \), (b) the market price conditional on reporting \( r = z^m \) is equal to \( z^m \) where

\[
\begin{align*}
  z^m &= E[\tilde{s} | t = 0 \cup \{ t = 1, \tilde{s} \geq z^m \}].
\end{align*}
\]

It thus follows that in the model with mandatory disclosure, the pricing of the soft information is the same as in the model without mandatory disclosure, i.e., \( z^m = z \). Put differently, if a firm reports \( h \) as hard information in an environment without mandatory disclosure, it achieves exactly the same surplus in an environment with mandatory disclosure. Therefore, mandatory disclosure does not improve the quality of the soft information whenever hard information is voluntarily issued.

The effect of mandatory disclosure can therefore be limited to its effect on firms that did not report \( h \) as hard information in the absence of regulation. Such firms had voluntarily opted not to issue \( h \) as hard information, and because of revealed preferences, cannot be better off when this option is removed. This implies the following result regarding the desirability of mandatory disclosure.

**Corollary 2** The environment with no mandatory disclosure requirement Pareto-dominates, i.e., is weakly preferred regardless of \((h, s, \tau)\) (strictly for some realizations), the environment with mandatory disclosure.

This observation is intuitive. As in many signaling games, the voluntary disclosure process provides excessive incentive to separate and then induces the over-provision of hard information. A mandatory disclosure requirement worsens this problem by forcing
all firms to incur the separation cost, even those that opted not to separate. Put differently, mandatory disclosure has no effect on firms that already incurred the cost in the absence of regulation but removes the option to issue unverified information from firms that would have been better off doing so. Consequently, in this model, imposing mandatory disclosure is weakly detrimental to all firms.

We show next that mandating hard disclosures for one component of the information does not imply a Blackwell improvement to the information received.

**Corollary 3** The environment with mandatory disclosure does not dominate (in the sense of Blackwell) the environment with no mandatory disclosure.

Mandatory disclosure does increase the information available about the verifiable component $\tilde{h}$, but it does not necessarily increase the information available about the unverifiable component $\tilde{s}$. The reason is straightforward: when given the opportunity to voluntarily withhold hard information, the act of withholding indirectly reveals information about the manager’s constraint $\tilde{\tau}$ and, hence, about the unverifiable component $\tilde{s}$. This informational channel is broken as soon as all hard disclosures become mandatory.

### 4.2 Aggregation

We have assumed, thus far, that investors observe the presence of hard information and can perfectly disentangle hard from soft information. Nonetheless, aggregation presents a fundamental constraint on a firm’s reporting system; for example, as noted by Beyer (2012), “even though the company likely owns many different machines, buildings and properties, it reports just a single number on its balance sheet which reflects the aggregate value of all its property, plant and equipment.” Similarly, certain environments might not be conducive to reporting hard and soft information separately, or to indicating that a formal verification has been properly conducted. We investigate the effect of aggregation constraints on the production of hard information.

Suppose that investors only observe an aggregate report $I = \{y, d\}$ about the firm value $\pi$ where

$$y \equiv d(h + r) + (1 - d)r,$$

but do not see the individual components of the report. In other words, investors observe the total amount reported and whether or not the value of $h$ was verified (but they do not observe the value of $h$).

As previously, the PSE takes the form of a price function

$$P(I) = \min(y, z_d) - cd$$

23
where \( z_d \) is both the report made by an unconstrained manager choosing to verify (if \( d = 1 \)) or not to verify (if \( d = 0 \)), as well as the market expectation after this report is observed.

**Corollary 4** When investors only observe the aggregate report \( y \), neither the constrained nor the unconstrained manager verify the component \( \tilde{h} \).

In Corollary 4, we demonstrate that the potential benefits of verification are fully dissipated by the process of aggregation in the presence of soft information. Certainly, the verification induces some confidence about part of the firm value but, because the market does not know which part of the aggregate report has been verified, an unconstrained manager can simply inflate the unverifiable component to attain any level of aggregate report. De facto, the process of aggregation makes all the information soft.\(^{12}\)

This effect suggests that the mandatory inclusion of unverifiable information in the financial statements may damage the information quality of aggregate numbers in financial reporting. This is clearly the case when the verification is voluntary, since managers then choose to verify. Next, we show that this is also true in the case of a mandatory verification, i.e., when setting \( d = 1 \).

**Corollary 5** When investors only observe the aggregate number \( y \), mandatory verification does not affect price. Hence, the reporting strategy with aggregation is the same as if \( \tilde{\pi} \) is entirely unverifiable (i.e., \( d = 0 \)).

Mandatory verification is entirely ineffective as a means of resolving investor uncertainty when soft information is aggregated with hard information. Investors obtain exactly the same information about total value from the manager’s report, as they would when a hard signal \( h \) is not available to the manager. Given the cost of hard information, this result suggests that mandating hard information is not desirable to managers, and it does not increase the amount of information available in the market.

### 4.3 Interactions between verifiable and unverifiable information

We have emphasized an environment in which, in the absence of informational asymmetries, the pricing of \( \hat{h} \) and \( \hat{s} \) would be entirely independent. We generalize this approach and consider settings in which there are some interactions between the verifiable and

\(^{12}\) This result relies in part on our assumption of unbounded support.
unverifiable components of the firm; \( v(h, s) \equiv E(\pi(h, s)|h, s) \) represents the firm value, where the differentiable function \( \pi(., .) \) satisfies \( \partial \pi(h, s)/\partial h > 0 \) and \( \partial \pi(h, s)/\partial s > 0 \).

For instance, as in Langberg and Sivaramakrishnan (2008), we could assume that \( \pi = h \cdot s \) represents a situation in which a single asset generates cash flows \( h \in \mathbb{R}_+ \) only if an event whose unknown probability \( s \in [0, 1] \) occurs. Here, \( h \) could be the nominal/face value of the asset, and \( 1 - s \) the probability that the asset defaults. In a credit sales application, the firm could disclose the nominal value of its receivables \( h \) but might not be able to disclose the probability of bad debts except as a soft disclosure. Because we allow the function \( v(h, s) \) to be non-separable, this formulation can also capture settings in which \( \tilde{h} \) and \( \tilde{s} \) are correlated.

The structure of the equilibrium is similar to that of the baseline model. It is characterized by a number \( \lambda_0 \) and a function \( z(h) \), where \( \lambda_0 \) is the maximum price of a firm issuing only soft information, and \( z(h) \) is the maximum price of the soft report, given that a hard signal \( h \) has been verified. Since the value of \( h \) may affect the distribution of \( s \), the reporting strategy of the unconstrained manager may depend on the value of \( h \).

The unconstrained manager issues hard information when \( h \geq k \), where the threshold \( k \) is now implicitly defined by

\[
v(k, z(k)) - c = \lambda_0. \tag{10}
\]

As in the baseline model, the constrained manager issues hard information less frequently, i.e., hard information is released if and only if \( h \geq k \) and \( v(h, s) \geq \lambda_0 + c \). Defining the set of all \((h, s)\) satisfying these conditions as \( M_0 \), there exists an equilibrium characterized by the following equations (see Appendix for the proof):\(^{13}\)

\[
\lambda_0 = E[v(\tilde{h}, \tilde{s})|\{\tilde{\tau} = 0, \tilde{h} \leq k\} \cup \{\tilde{\tau} = 1, (\tilde{h}, \tilde{s}) \notin M_0, v(\tilde{h}, \tilde{s}) \geq \lambda_0\}], \tag{11}
\]

\[
v(h, z(h)) = E[v(h, s)|\{\tilde{\tau} = 0\} \cup \{\tilde{\tau} = 1, s \geq z(h)\}]. \tag{12}
\]

Figure 4 illustrates the optimal decision to issue hard information. In this example, we set \( v(h, s) \) to satisfy \( \partial^2 v(h, s)/\partial h \partial s > 0 \) so that the veri-

\(^{13}\) The existence of a solution in the case of an unbounded support is shown using the same techniques as in the baseline, and it also requires lower tails to be sufficiently thin. The uniqueness of a solution to these equations is non-trivial; nevertheless, it can be shown that \( z(h) \) is unique (provided logconcavity is assumed) and that if there are multiple solutions for \( \lambda_0 \), the highest \( \lambda_0 \) is Pareto-dominant (and, therefore, would be selected by the PSE criterion).
fiable and unverifiable components are complements. Then a lower realized soft signal will also reduce the impact of the verifiable information on firm value. The certification threshold for the constrained manager will be convex as managers with lower soft information become less marginally willing to incur the verification cost.

5 Empirical applications

We discuss a few empirical applications of our theory when it is applied to particular contexts with soft information.

First, we predict that the quality (or information content) of a soft communication is related to both the type of news being disclosed – whether positive or negative – as well as the truthfulness of the firm. Management forecasts will be more precise, i.e., have lower forecast error, if they concern negative news. This prediction is consistent with results in the cheap talk literature (see, e.g., Crawford and Sobel 1982; Morgan and Stocken [Morgan and Stocken])
but differs from costly misreporting models, in which the expected error is constant or increasing in the report being made (Einhorn and Ziv 2012). Relative to the extent cheap talk literature, our models makes the stronger prediction that forecasts are perfectly credible when they are unfavorable, implying that unfavorable forecast errors are likely to be pure noise, instead of related to a conflict of interest.

Second, we predict that the determinants of the manager’s truthfulness, if they are not perfectly observable by the market, explain managers’ aggressiveness in making soft reports. This implies that factors, such as the litigation environment, internal controls, incentives, and insider trading, which are difficult to directly observe, are drivers of forecast errors (Hutton and Stocken 2009; Rogers and Stocken 2005). It should be noted that this prediction applies to information that is not public or, to be more practical, harder to access or difficult to process by the market (e.g., estimated from a model). Our stylized model also predicts that factors that are easily observable should not drive forecast errors, which imply that most common capital market variables should not correlate to forecast errors. This prediction is consistent with the absence of a clear correlation of many firm characteristics with forecast biases.

Third, we characterize the relationship between disclosure over competing channels. Our working hypothesis is that there is some managerial discretion regarding the level of hardness in financial reports. Proxies for the hardness of the information released may involve metrics of accounting quality (e.g., accruals), audit quality or, on a case-by-case basis, whether the firm chooses to be listed on a US exchange, uses the services of a rating agency or, in certain countries, decides to hire an independent auditor (Lennox and Pittman 2011). In our model, factors that are indicative of hardness in financial statements (e.g., accounting quality measures, high book to market, auditor characteristics, etc.) might correlate to aggressiveness in soft reporting (e.g., aggressive management forecasts).
Fourth, we predict that firms choose to “stay dark,” or becoming private, thus relying on soft information only when they have more unfavorable information. We suggest that misreporting tends to be less frequent in publicly traded (regulated) firms. The theory thus implies that private firms may self-select to release higher-quality information. To our knowledge, this prediction is yet to be tested. However such tests might be conducted in the future using newly available databases on private firms (Minnis 2011).

Fifth, we offer predictions about the consequences of recent regulations that change how certain types of soft information are being disclosed. We predict that the aggregation of both hard and soft information tends to make all information soft; for example, Level 3 fair value assets (i.e., marked to model) are evaluated based on a combination of hard (audited) inputs as well as soft management forecasts and models. We predict that these items will have low levels of informativeness (Kolev 2009). We also predict that the move toward an expanded use of fair value, requiring more comprehensive mandatory disclosure than exists at present will reduce the quality and quantity of soft disclosures, in the sense that there will be fewer voluntary disclosures (e.g., forecasts, press releases), or fewer informative management statements and discussions.

6 Conclusion

In this study, our objective was to develop a theory that re-connects two distinct subset of the literature: models of strategic communication of hard information and models of strategic soft communication. This problem was motivated by the fact that the institutional accounting process attempts to make information hard (Ijiri 1975). Nevertheless, significant amount of information are released through alternative soft channels that are not verified via external means. These sources of information are not
fully understood, even though they appear to account for a significant fraction of all publicly available information. Indeed, this fact pervades the foundations of modern capital market research. The classical study of Ball and Brown (1968) found that most of the information appears to be received by markets prior to the release of financial statements.

In this paper we postulate that certain forms of communication are possible within these channels as long as markets trust that some (but not necessarily all) managers do not strategically over-report. We compare the form of soft communication to the release of verified hard information and, in doing so, attempt to provide a framework for these two main forms of communication. This is the first step toward the proper understanding of soft information. Although we have focused on a simple reporting problem, there is likely more to learn from applying the framework to other settings, which may range from contracting or capital structure, to the design of market regulations.

Appendix

Proof of price functions (1)-(2). Using the price conjectures in (1)-(2), Lemmas 1 and 2 are immediate and shown in text. Therefore, we now prove now that this price conjecture must be used in any PSE.

For further use, we define the maximal market prices $\lambda_0 = \max_r P(0, 0, r)$ and $z$ such that $h - c + z = \max_r P(1, h, r)$. We then denote $r_\lambda$ (resp., $r_z$) as the soft report made by the unconstrained manager without verification (resp., with verification). Because the proofs when $h$ is not verified are identical, we only provide the proofs when $h$ is verified.  

Step 1. The market price must satisfy $P(1, h, r) = h - c + r$ if $P(1, h, r) < h - c + z$ and $P(0, 0, r) = r$ if $P(0, 0, r) < \lambda_0$.

14 To save space, we omit here several straightforward implications of PSE. First, the maximal price $z$ is not a function of $h$. Second, the pricing function described in (1)-(2) can also be used off-equilibrium to solve for the optimal verification strategies.
Suppose \( r \) is such that \( P(1, h, r) < h - c + z \) with \( P(1, h, r) < h - c + r \). Note that an unconstrained manager issues a report that attains \( h - c + z \) so the market must expect \( r \) to have been issued by constrained managers only, implying that \( P(1, h, r) > h - c + r \). For this to hold, some managers with \( \hat{s} > r \) issue \( r \). Now, let us define \( \Lambda_0 \) as the set of all managers with \( s \geq \hat{s} \) and who are better-off with price \( h - c + \hat{s} \) over their equilibrium payoff. Since constrained managers cannot over-report, we know that \( E[\pi | \tilde{\omega} \in \Lambda_0] \geq h - c + \hat{s} \). Next, the choice of \( p_0 = h - c + \hat{s} \), \( h_0 = h \), \( d_0 = 1 \) and \( r_0 = \hat{s} \) contradicts PSE.

**Step 2.** Conditional on verifying \( h \), unconstrained and constrained managers with \( s \geq z \) report \( r_z = z \). Conditional on not verifying \( h \), unconstrained and constrained managers with \( h + s \geq \lambda_0 \) report \( r_\lambda = \lambda_0 \).

We know that the market price conditional on issuing the soft report \( r_z \) is given by

\[
z = E[s | \tilde{\tau} = 0 \cup L],
\]

where \( L \) is the set of constrained managers with \( s \geq r_z \) and choosing to report \( r_z \).

In a PSE, it must be that \( (r_z, L) \) maximize \( z = E[s | \tilde{\tau} = 0 \cup L] \). Otherwise, we could find \( (r'_z, L') \) and construct \( \Lambda_0 \) that includes unconstrained and constrained types in \( L' \) verifying \( h \), and set \( p_0 = h - c + E[s | \tilde{\tau} = 0 \cup L'] \), \( d_0 = 1 \), \( h_0 = h \) and \( r_0 = r' \), which would contradict a PSE. Recall that the expectation is equal to \( z \) so including any \( s \) below (above) \( z \) decreases (increases) the expectation. Hence, this expectation is maximal if any unconstrained manager with \( s \geq z \) is in \( L \) whereas \( s < z \) is not in \( L \). This implies a choice of \( (r_z, L) \) equal to \( r_z = z \) and \( L = (z, +\infty) \).

**Step 3.** The price function has the form given in equations (1)-(2).

Using steps 1 and 2, we know that \( P(1, h, r) = h - c + r \) if \( r < z \) and \( P(1, h, r) = z \) if \( r \geq z \). Therefore \( P(1, h, r) = h - c + \min(r, z) \).

**Proof of Proposition 1.** We already know that a unique \( z \) exists and, if \( \lambda_0 \) exists and is unique, Lemmas 1-3 establish the Proposition. It remains to
be shown that \( \lambda_0 \) as defined by equation (2), has a unique solution. In this proof it is convenient to use the notation \( \theta = \gamma / (1 - \gamma) \).

We first establish existence of a solution. The conditional expectation in equation (2) can be explicitly written in integral form as follows:

\[
\theta \int_{-\infty}^{\infty} \int_{\lambda_0 + c - h}^{\infty} (\pi - \lambda_0) dF + \theta \int_{-\infty}^{\infty} \int_{\lambda_0 + c - h}^{\infty} (\pi - \lambda_0) dF + \int_{-\infty}^{\infty} (\pi - \lambda_0) dF = 0 \tag{A-1}
\]

where \( dF = f_s(s) ds f_h(h) dh \) and \( \pi = h + s \). By a suitable change of variables (i.e., \( x = h - \lambda_0 \) and \( y = s + x \)), Equation (A-1) can be rewritten as:

\[
\theta \int_{-\infty}^{\infty} l(x) f_h(x + \lambda_0) dx + \int_{-\infty}^{c-z} v(x) f_h(x + \lambda_0) dx = 0 \tag{A-2}
\]

where

\[
v(x) = \theta \int_{c}^{\infty} s f_s(s - x) ds + x, \quad l(x) = \theta \int_{0}^{c} y f_s(y - x) dy > 0.
\]

Defining the function \( \varphi(h) \equiv l(h) + v(h) \), Equation (A-2) can be rewritten as:

\[
\int_{c-z}^{\infty} l(h) f_h(h + \lambda_0) dh + \int_{-\infty}^{c-z} \varphi(h) f_h(h + \lambda_0) dh = 0. \tag{A-3}
\]

This establishes the lemma. ■

The following lemma is required.

**Lemma A.3** \( v(h) \) is increasing in \( h \). Furthermore, there exists \( h^+ \) such that \( v(h^+) = 0 \), and \( h^+ < \min(c - z, 0) \).

**Proof.** To show that \( v'(h) > 0 \) notice that

\[
v(h) = \theta \int_{c}^{\infty} s f_s(s - h) ds + h = \theta \int_{c-h}^{\infty} (s + h) f_s(s) ds + h.
\]
hence \( v' (h) = \theta c f_s (c - h) ds + \theta \int_{c-h}^{\infty} f_s (s) ds + 1 > 0 \). Now observe that \( v(0) = \theta \int_c^{\infty} f_s (s) ds > 0 \). Also,

\[
v(c - z) = \theta \int_c^{\infty} s f_s (s - c + z) ds + c - z \]
\[
= \theta \int_{-c}^{\infty} (s - z) f_s (s) ds + c - z = c [\theta [1 - F_s (z)] + 1] > 0.
\]

This establishes the lemma.

**Lemma A.4** Let \( f (\cdot) \) be a density that satisfies strict log-concavity. Then for any \( h_1 < h_2 \in (-\infty, \infty) \) and \( a > 0 \), 

\[
f(h_2 + a) f(h_1 - a) < \frac{f(h_2)}{f(h_1)}.
\]

**Proof.** This follows from the log-concavity of \( f \) because log-concavity implies the monotone likelihood ratio property (see e.g., Milgrom (1981)). The fact that \( \lim_{\lambda_0 \to -\infty} \zeta (\lambda_0) = -\infty \) is immediate. On the other hand, \( \lim_{\lambda_0 \to -\infty} \zeta (\lambda_0) = 0 \), so we still need to show that \( \zeta (\lambda_0) > 0 \) if \( \lambda_0 \) is low enough.\( ^{15} \)

We decompose \( \zeta (\lambda_0) \) as the sum of three terms:

\[
\zeta (\lambda_0) = Q_1 (\lambda_0) + Q_2 (\lambda_0) - Q_3 (\lambda_0)
\]

(A-4)

where

\[
Q_1 (\lambda_0) = \theta \int_{-\infty}^{\infty} r (x) f_h (x + \lambda_0) dx; \quad Q_2 (\lambda_0) = \theta \int_{h_0}^{\infty} v (h) f_h (h + \lambda_0) dh,
\]
\[
Q_3 (\lambda_0) = \theta \int_{h_0}^{\infty} -v (h) f_h (h + \lambda_0) dh.
\]

such that \( Q_1, Q_2, Q_3 \) are all positive and they all tend to zero as \( \lambda_0 \to -\infty \). We shall show however that:

\[
\lim_{\lambda_0 \to -\infty} \frac{Q_1 (\lambda_0)}{Q_3 (\lambda_0)} = \infty.
\]

\( ^{15} \) This is always true when the support of the random variables under consideration is bounded below because the cost is positive, but the unbounded case requires us to use the assumption of thin tails.
When $\lambda_0$ is negative, logconcavity implies that

$$Q_1(\lambda_0) = \theta \int_{-\infty}^{\infty} r(x) f_h(x + \lambda_0) \, dx \geq \theta \int_{0}^{\infty} r(x) \frac{f_h(x + \lambda_0)}{f(x)} f(x) \, dx$$

$$\geq \theta \int_{0}^{\infty} r(x) \frac{f_h(0 + \lambda_0)}{f(0)} f(x) \, dx = \frac{f_h(\lambda_0)}{f_h(0)} Q(0).$$

where $Q(0) = \int_{0}^{\infty} r(x) f(x) \, dx$. Similarly, when $\lambda_0$ is negative, log-concavity implies that

$$Q_3(\lambda_0) = \int_{-\infty}^{h^+} -v(h) f_h(h + \lambda_0) \, dh = \int_{-\infty}^{h^+} -v(h) \frac{f_h(h + \lambda_0)}{f_h(h)} f_h(h) \, dh$$

$$\leq \frac{f_h(h^+ + \lambda_0)}{f_h(h^+)} \int_{-\infty}^{h^+} -v(h) f_h(h) \, dh = Q_3(0) \frac{f_h(h^+ + \lambda_0)}{f_h(h^+)}.$$

Hence for $\lambda_0 < 0$, we must have

$$\frac{Q_1(\lambda_0)}{Q_3(\lambda_0)} \geq \frac{Q(0) \frac{f_h(h^+)}{f_0(h^+)} \frac{f_h(\lambda_0)}{f_h(0)}}{Q_3(0) \frac{f_h(h^+ + \lambda_0)}{f_h(h^+)} \frac{f_h(\lambda_0)}{f_h(0)}}.$$

Taking the limit as $\lambda_0 \to -\infty$ yields

$$\lim_{\lambda_0 \to -\infty} \frac{Q_1(\lambda_0)}{Q_3(\lambda_0)} \geq \lim_{\lambda_0 \to -\infty} \frac{Q(0) \frac{f_h(h^+)}{f_0(h^+)} \frac{f_h(\lambda_0)}{f_h(0)}}{Q_3(0) \frac{f_h(h^+ + \lambda_0)}{f_h(h^+)} \frac{f_h(\lambda_0)}{f_h(0)}}.$$

Finally, by the assumption that $\lim_{\lambda_0 \to -\infty} \frac{\partial \ln f_h(\lambda_0)}{\partial \lambda_0} = \infty$, we obtain that $\lim_{\lambda_0 \to -\infty} \frac{f_h(\lambda_0)}{f_0(\lambda_0 + h^+)} = \infty$ given that $h^+ < 0$. Hence

$$\lim_{\lambda_0 \to -\infty} \frac{Q_1(\lambda_0)}{Q_3(\lambda_0)} = \infty.$$

Therefore, the left-hand side of $(A - 4)$ must be positive when $\lambda_0$ is sufficiently low. According to the intermediate value theorem there must be a $\lambda_0$ that satisfies equation $\zeta(\lambda_0) = 0$. ■
Next we establish the uniqueness of $\lambda_0$. The proof is by contradiction and relies heavily on logconcavity. Before proving uniqueness we establish a series of lemmas.

**Lemma A.5** For any $p \in (-\infty, \infty)$, $a < b \in (-\infty, \infty)$, define

$$Q(p) = \int_{a}^{b} g(h) f(h + p) \, dh,$$

where the function $g(x)$ satisfies $g(x) \geq 0$ for all $x \in [a, b]$. If $Q(p) > 0$ for all $p$, then for any $p_1 < p_2$,

$$\frac{f(p_2 + b)}{f(p_1 + b)} \leq \frac{Q(p_2)}{Q(p_1)} \leq \frac{f(p_2 + a)}{f(p_1 + a)}.$$

**Proof.** We can write $Q(p_2)$ as:

$$Q(p_2) = \int_{a}^{b} g(h) f(h + p_1) f(h + p_2) \, dh. \quad (A-5)$$

According to Lemma A.4, for all $x \in [a, b]$ the following inequalities hold

$$\frac{f(b + p_2)}{f(b + p_1)} \leq \frac{f(x + p_2)}{f(x + p_1)} \leq \frac{f(a + p_2)}{f(a + p_1)}.$$

Substituting these inequalities into equation (A-5) yields

$$\frac{f(b + p_2)}{f(b + p_1)} Q(p_1) \leq Q(p_2) \leq \frac{f(a + p_2)}{f(a + p_1)} Q(p_1).$$

This completes the proof. ■

To use the properties of logconcavity, we need to determine whether and where the function $\varphi(h)$ is non-negative. We do this in the next lemma.

**Lemma A.6** $\varphi(h) \geq 0$ if and only if $h \geq h^-$ where $\varphi(h^-) = 0$. Furthermore $h^- < h^+ < \min(0, c - z)$, where $\nu(h^+) = 0$.  

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Proof. From Lemma (4.3) \( v(h) \) single crosses the horizontal axis from below at \( h^+ < \min(0, c - z) \). Hence,

\[
\varphi(h) = v(h) + r(h) > 0
\]

for all \( h \geq h^+ \). We next show that \( \varphi \) single crosses the horizontal axis at a point \( h^- < h^+ \). Notice that

\[
\varphi(h) = v(h) + r(h) = h + \theta \int_0^\infty s f_s(s - h) \, ds = h + \theta \int_{-h}^\infty (x + h) f_s(x) \, dx,
\]

where the last equality follows from defining \( x \equiv s - h \) and changing variables. It is routine to verify that \( \varphi'(h) > 0 \), and that \( \lim_{h \to -\infty} \varphi(h) = -\infty \) and that \( \lim_{h \to -\infty} \varphi(h) = \infty \). We can partition the integration range into three parts and write Equation (4.3) as

\[
\int_{-\infty}^\infty r(h) f_h(h + \lambda_0) \, dh + \int_{h^-}^{c-z} \varphi(h) f_h(h + p) \, dh - \int_{-\infty}^{h^-} (-\varphi(h)) f_h(h + p) \, dh = 0.
\]

This allows us to use the properties of logconcavity to show that the existence of multiple solutions leads to a contradiction. Suppose there are two prices \( \lambda_0' > \lambda_0 \) that solve this equation. Applying Lemma (4.3) to each term implies that

\[
\frac{Q_1(\lambda_0')}{Q_1(\lambda_0)} \leq \frac{f_h(\lambda_0' + c - z)}{f_h(\lambda_0 + c - z)} < \frac{f_h(\lambda_0' + h^-)}{f_h(\lambda_0 + h^-)}
\]

where the second inequality follows from the fact that \( h^- < c - z \) and the monotone likelihood ratio. Similarly as in Lemma (4.3) we conclude that

\[
\frac{Q_2(\lambda_0')}{Q_2(\lambda_0)} \leq \frac{f_h(\lambda_0' + h^-)}{f_h(\lambda_0 + h^-)} \quad \text{and} \quad \frac{Q_3(\lambda_0')}{Q_3(\lambda_0)} \geq \frac{f_h(\lambda_0' - h^-)}{f_h(\lambda_0 + h^-)}.
\]
Then,

$$Q_1(\lambda_0') + Q_2(\lambda_0') + Q_3(\lambda_0') < [Q_1(\lambda_0) + Q_2(\lambda_0) + Q_3(\lambda_0)] \frac{f_h(\lambda_0')}{f_h(\lambda_0 + h)},$$

which is a contradiction.

Proof of Corollary 1. Given its uniqueness, we only need to check the derivative of the left-hand side of equation (A−3) with respect to $c$ and apply the implicit function theorem. Differentiating the left-hand side of (A−3) with respect to $c$ yields:

$$(r(c - z) + \varphi(c - z)) f_h(c - z + p) > 0,$$

where the inequality follows from the fact that $r(\cdot) > 0$ and $\varphi(c - z) > 0$ (see Lemma (A.6)). Because the left-hand side of (A−3) crosses the horizontal axis only once and from above, then by implicit function theorem we must have:

$$\frac{\partial \lambda_0}{\partial c} > 0.$$

The effect of $\theta$ on $z$ is straightforward.

Proof of Corollary 2. Notice that under mandatory disclosure, a constrained manager induces a price equal to $\min(s, z) + h - c$ and the unconstrained manager induces a price equal to $z + h - c$, where $z$ is given by equation (6). In a voluntary disclosure regime, the managers would still have access to these prices, but they would also have the option of inducing the prices without verification. In particular, the constrained manager would have the option of inducing a price $\min(\pi, \lambda_0)$ and the constrained manager would have the option of inducing $\lambda_0$. This option implies that all manager types are better off in the voluntary disclosure regime.

Proof of Corollary 3. In the model without mandatory disclosure, for all $s \geq z$ but $\pi < \lambda_0$, the constrained manager receives a fair valuation from the market, since investors face no uncertainty about the value of $\pi$. In
contrast, with mandatory disclosure, any report above \( z \) is discounted by the market, because the reports of constrained and unconstrained managers are necessarily pooled. Without mandatory disclosure, investors’ posterior distribution of \( \pi \) is degenerate when \( \{ \tau = 1, s > z, \pi \leq \lambda_0 \} \), but it is not degenerate with the mandatory disclosure of \( h \). In other words, there are states of nature under which the market is ex-post less (more) uncertain under the voluntary (mandatory) disclosure regime. This finding shows that mandatory disclosure does not lead to a Blackwell improvement.

\[ \text{Proof of Corollary 5.} \]  
Assume the constrained manager reports truthfully. Suppose, aiming for a contradiction, that there are two markets: market \( A \) where \( h \) is verified and market \( B \) where the information about \( \pi \) is released in a soft form. Assume that the maximum prices in these two markets differ. For example, \( p_A - c > p_B \). Let \( A \subseteq R \) be the set of reports that induce \( p_A - c \) in equilibrium. Then the unconstrained manager will always issue verified reports that lie in \( A \). We assume her reports will not depend on private information because this information is unverifiable. That is, regardless of the true value of \( (h, s) \) the unconstrained manager can always issue a report in \( A \). The constrained manager, on the other hand, will verify \( h \) and issue a report in \( A \) if and only if \( \pi \in A \). By construction, any report \( y \in A^c \) will be fully credible, even in the absence of verification, because such reports can only be issued by constrained managers. In other words, \( P(0, 0, y) = y \) for all \( y \in A^c \). It follows that \( \sup A^c = p_A - c \). This means that for any \( \pi \geq p_A - c \) the constrained manager will verify his reports; hence, \( (p_A - c) \in A \). This implies that in equilibrium

\[
p_A = E[\tilde{\pi}|R(\tilde{\omega}) = p_A - c, D(\tilde{\omega}) = 1] \\
= t(p_A - c) + (1 - t) E[\tilde{\pi}|R(\tilde{\omega}) = p_A - c, D(\tilde{\omega}) = 1, \tilde{\tau} = 0].
\]

where \( t = E[\tilde{\tau}|R(\tilde{\omega}) = p_A - c, D(\tilde{\omega}) = 1] \). This equality holds if \( E[\tilde{\pi}|R(\tilde{\omega}) = \]
\[ p_A - c, D(\tilde{\omega}) = 1, \tilde{\tau} = 0 > p_A - c, \] which implies on the one hand that the unconstrained manager report depends on unverifiable information. Furthermore, on average, the unconstrained manager’s report is lower than the true value of \( \pi \).

**Proof of existence for general case.** The existence of a solution to Equations (11)-(12) follows because for any \( h, a, z \) exists which solves

\[
v(h, z) - \frac{(1 - \gamma) \mathbb{E}[v(h, \tilde{s})|h] + \gamma \int_z^\infty v(h, s) f_s(s|h) \, ds}{(1 - \gamma) + \gamma \mathbb{P}(s > z|h)} = 0.
\]

Notice indeed that the left hand side of the above equation is negative (positive) when \( z(h) \to -\infty \) (when \( z(h) \to \infty \)). In fact, under mild technical conditions (which include bounded support), we have

\[
\lim_{z(h) \to -\infty} \frac{(1 - \gamma) \mathbb{E}[v(h, \tilde{s})|h] + \gamma \int_z^\infty v(h, s) f_s(s|h) \, ds}{(1 - \gamma) + \gamma \mathbb{P}(s > z|h)} = \mathbb{E}[v(h, \tilde{s})|h] > \lim_{z \to -\infty} v(h, z) = -\infty
\]

and

\[
\lim_{z \to \infty} \frac{(1 - \gamma) \mathbb{E}[v(h, \tilde{s})|h] + \gamma \int_z^\infty v(h, s) f_s(s|h) \, ds}{(1 - \gamma) + \gamma \mathbb{P}(s > z|h)} = \mathbb{E}[v(h, \tilde{s})|h] < \lim_{z \to \infty} v(h, z) = \infty.
\]

On the other hand, given \( z(h) \) and \( k \), the value of \( \lambda_0 \), if it exists, is defined by the following equation

\[
\lambda_0 - \mathbb{E}[\tilde{\varphi}|\Omega] = 0
\]

where

\[
\Omega \equiv (\tilde{\tau} = 0, \tilde{h} \leq k) \cup (\tilde{\tau} = 1, \tilde{h} \geq k, \tilde{\varphi} \in [\lambda_0, \lambda_0 + c]) \cup (\tau = 1, \tilde{h} \leq k, \tilde{\varphi} \geq \lambda_0),
\]

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and $k$ is defined by Equation (10). To establish the existence of $\lambda_0$, notice that one can set $\lambda_0$ to be small enough so that $\lambda_0 \rightarrow \inf(\text{supp} F_h)$. In this case, again under mild technical conditions, the left hand side of the above equation would be negative because as $k$ converges to the infimum of the support of the distribution $F_h$, we have

$$ E[\tilde{v}|\Omega] \rightarrow E[\tilde{v}|\tilde{v} \in [\lambda_0, \lambda_0 + c]] > \lambda_0 $$

On the other hand, if $\lambda_0 \rightarrow \infty$ the left hand side must be positive because

$$ \lim_{\lambda_0 \rightarrow \infty} E[\tilde{v}|\Omega] = E[\tilde{v}] $$

The existence of $\lambda_0$ would then follow from the intermediate value Theorem, given the continuity of the function $E[\tilde{v}|\Omega]$ with respect to $\lambda_0$. The continuity of the above function, in turn, is guaranteed in virtue of the continuity of $v(h, s)$ and the continuity of the probability densities of both $\tilde{h}$ and $\tilde{s}$. ■
References


