Dynamic Coalitions*

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July 3, 2014

Abstract

We present a theory of dynamic coalitions for a legislative bargaining game in which policies continue in effect in the absence of new legislation. We characterize Markov perfect equilibria with dynamic coalitions, which are decisive sets of legislators whose members strictly prefer preserving the coalition to having it end. Dynamic coalitions satisfy internal stability and exclusion risk conditions. They can be minimal winning or surplus and can award positive allocations to non-coalition members. Policies supported can be efficient or inefficient. Vested interests can support policies that no legislator would propose if forming a new coalition. If uncertainty is associated with policy implementation, a continuum of policies are supported. These equilibria have the same allocation in every period when the coalition persists. Dynamic coalitions also exist in which members tolerate a degree of implementation uncertainty, resulting in policies that can change without the coalition dissolving. We compare to experimental results. JEL Classification: C73, D72

Keywords: Coalitions, Dynamic Bargaining, Policy Stability

*The authors wish to thank Daniel Diermeier, Hülya Eraslan, Salvatore Nunniari, Thomas Palfrey, Jan Zápal, and seminar participants at Stanford University, Johns Hopkins University, the University of Iowa, the Canadian Economic Theory Conference, the Priorat Workshop in Theoretical Political Science, the NBER Political Economy Public Finance Workshop, and the University of California Los Angeles for helpful comments and suggestions.

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1 Introduction

Many economic decisions involve dynamic considerations. Most laws and government programs are continuing and remain in effect in the absence of new legislation. Rules promulgated by regulatory commissions also remain in effect until modified or rescinded. Social security, welfare, and other redistributive programs are typically continuing, and distributions are governed by formulas that are changed only infrequently. Tax rates also continue in the absence of new legislation. At the state level, legislatures establish continuing policies including Medicaid eligibility and benefits, and regulatory commissions establish prices, rules governing lifeline and other cross-subsidization programs, and environmental and energy-efficiency policies. These programs have the property that the policy adopted or in place in the current period becomes the status quo for the next period. Policy choice thus can be viewed as a dynamic legislative bargaining game with an endogenous status quo in which a legislature has the opportunity to choose the policy in every period and agenda-setting control can change over time. Despite their dynamic nature and the opportunities for change, many policies are stable and are supported by coalitions that persists from one period to the next. Government formation in parliamentary systems also involves dynamic considerations and policies, and often government coalitions and their policies persist in both interelection and election periods. In a dynamic setting the risk faced by political actors in changing policy or disrupting a stable coalition is that a new coalition formation round could exclude some current coalition members.

We study endogenous dynamic coalitions and the policies they support. We consider simple strategies, which we call basic strategies, that have the feature that legislators propose the status quo if it is supported by the dynamic coalition (i.e., the status quo is in the supported set), and if the status quo is not in the supported set, legislators choose a policy in the supported set that is most advantageous to them, and randomize over coalition partners. For status quo policies not in the supported set there is uncertainty over which coalition will form in the future. This provides an incentive for current coalition members to maintain the status quo – to avoid the risk of being excluded from a future coalition. The coalition is dynamic because in equilibrium its members strictly prefer that the coalition and its policy continue rather than have the coalition end.

Despite the dynamic nature of the game, there are two straightforward necessary and sufficient conditions for the existence of Markov perfect equilibria with dynamic coalitions: (i) internal stability of the supported set of policies, and (ii) exclusion risk, which formalizes the preferences of the members of the dynamic coalition to preserve the coalition rather than risk exclusion when new coalitions are formed. This exclusion risk is not player-specific but rather is collective in that all coalition members bear the risk when any one of them breaks the tacit agreement. The punishment for defection thus
is collective. The risk of exclusion is present in any dynamic voting game with a less than unanimity collective choice rule. The approach taken in the paper is applicable to a broad class of dynamic political economy problems with a collective choice rule and players who face a risk of exclusion from future coalitions.

Studies of distributive politics typically find that benefits are allocated to more than a minimal majority of legislative districts even though a minimal winning coalition could do strictly better.\(^1\) In parliamentary systems governments often include more parties than required for a majority in parliament, and minority governments are observed in a number of political systems.\(^2\) The theory presented here shows that dynamic coalitions may be minimal winning or surplus and may allocate benefits to all legislators. We show that dynamic coalitions may include senior and junior members, where the latter have strictly lower allocations than do senior members. Equilibria also exist in which one legislator has a strictly greater allocation than do others, which can be interpreted as a minority government supported by other legislators with a continuing interest in preserving the government.

Distributive programs, such as those providing pork, are often viewed as inefficient, and dynamic coalitions can support inefficient policies. One source of inefficiency is due to a special class of dynamic coalitions we call \textit{vested interests}. These vested interests support a policy that no legislator would propose in a new coalition formation round. That policy is supported because a decisive set of legislators fears the risk of exclusion and hence is willing to support the status quo policy even if it is Pareto inefficient. This also provides an explanation for gridlock. That is, the status quo policy persists even though no legislature would propose that policy if the status quo were outside the gridlock set. When inefficient policies are supported, dynamic coalitions result in political failure in the sense of Besley and Coate (1998).

Many policies have a degree of uncertainty associated with their implementation, and that uncertainty affects not only the current policy but also the status quo in the next period. The uncertainty could be due to exogenous factors or to endogenous factors associated with delegation to an administrative agency or regulatory commission or to choices made by those affected by the policy. The basic model is extended to include implementation uncertainty, and a class of specific-policy coalition equilibria is characterized in which a dynamic coalition persists as long as uncertainty is not realized and ends when it is realized. The dynamic coalition, while it persists, implements the same policy in each period, and the originator of the dynamic coalition shares the gains from proposal power with the coalition partner but not necessarily equally.

A specific-policy coalition ends when implementation uncertainty changes the policy, but a coalition

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\(^1\) See Primo and Snyder (2008), Stein and Bickers (1994), and Weingast (1994).

\(^2\) See Ansolabehere, Snyder, Strauss and Ting (2005), Laver and Shepsle (1996), and Strom (1990) for theory and evidence on government formation.
could tolerate some change due to uncertainty. Coalition equilibria exist that support a set of tolerated policies where the coalition persists if the policy remains in the set and ends if it is outside the set. A tolerant coalition thus is more valuable to its members than is the corresponding specific-policy coalition. The originator of a tolerant coalition shares proposal power with the coalition partner, and because of the realized implementation uncertainty, the coalition partner could in some periods have a larger allocation than the originator. Tolerant coalitions provide an explanation for coalition governments that survive small shocks but fail in crises.

The dynamic legislative bargaining game considered is an extension of the sequential legislative bargaining game introduced by Baron and Ferejohn (1989). In that game the stationary equilibrium outcome is reached with the first proposal, and the decisive set supporting the bargain is minimal winning. The proposer captures what otherwise would be the allocation of those legislators excluded from the decisive set and does not share the gains with other members of the decisive set. In the dynamic game we show that a dynamic coalition forms in one-step and in the absence of implementation uncertainty persists thereafter. For simple majority rule, however, the sequential equilibrium policy cannot be supported by a coalition equilibrium because the internal stability condition is not satisfied; i.e., an out legislator can propose a replacement policy that is attractive to a decisive set of legislators. Proposal power thus is mitigated within a dynamic coalition. With supermajority rule, however, the sequential legislative bargaining equilibrium can be supported.

This paper adds to a recent literature in dynamic political economy examining models with an endogenous status quo. Anesi and Seidmann (2013) consider a dynamic legislative bargaining model similar to ours, where their equilibrium proposals are based on the identities of legislators and must provide a punishment for at least one other legislator. The coalition equilibria we characterize are identity free and hence are simpler on that dimension. Anesi and Seidmann (2013) show that almost any outcome is possible as a simple solution in their dynamic legislative bargaining model, whereas we find that most policies cannot be supported by coalition equilibria. In the following sections we discuss the relation between their model and results and ours.

Kalandrakis (2004) was the first to characterize Markov perfect equilibria in a dynamic legislative bargaining model, and in the equilibrium a rotating dictator captures all the surplus in a period. For a finite set of alternatives Battaglini and Palfrey (2012) also present an equilibrium that rotates among minimal winning coalitions, and they conduct laboratory experiments that implement their dynamic game. In our equilibria policies are stable and are reached in no more than one step. Stability in one step is also present in Anesi (2010), Acemoglu, Egorov and Sonin (2013), and Diermeier, Egorov and Sonin (2013), who characterize Markov perfect equilibria and relate them to the solution concept of stable sets. To do this, they use a finite policy space and a discount factor approximately equal to one.
Deirmeier, Egorov, and Sonin, for example, use von Neumann and Morgenstern internal and external stability conditions to characterize the set of supported policies. Anesi (2010), however, shows that external stability is not a necessary condition for a policy to be supported as a stationary Markov perfect equilibrium. Our exclusion risk condition is the necessary condition needed for the existence of Markov perfect equilibria in basic strategies. The relation between their approach and ours is discussed in more detail in Section 5.3.

Bowen and Zahran (2012) and Richter (2014) identify equilibria that exhibit compromise where more than a minimal majority of legislators receive an allocation. Bowen and Zahran find compromise in a Markov perfect equilibrium with risk-averse legislators, and Battaglini and Palfrey (2012) find compromise in a quantal response equilibrium with risk-averse players. When inefficient policies are possible, Richter (2014) identifies Markov perfect equilibria (MPE) in which all legislators share the benefits equally. The challenge in this line of research is to identify equilibria with intuitive properties that can be easily applied in a variety of settings. We provide such a characterization and use it to study dynamic coalitions and the policies they support.


Cooperation or policy moderation in a dynamic policy-making environment has been studied in Dixit, Grossman and Gul (2000), Lagunoff (2001), and Acemoglu, Golosov and Tsyvinski (2011). In contrast to these papers we consider Markov perfect equilibria in a game with a purely distributive policy, where there is no natural incentive to form coalitions or induce policy moderation. Besley and Coate (1998), Battaglini and Coate (2007), Battaglini and Coate (2008), Acemoglu, Egorov and Sonin (2012), and Baron, Diermeier and Fong (2012) show that dynamic incentives can lead to inefficiency. In our distributive policy setting we show that dynamic coalitions may support Pareto dominated policies because the members of the coalition prefer that the current policy continue rather than risk exclusion from a new coalition with a different policy.

Duggan and Kalandrakis (2012) provide a general existence result for dynamic legislative bargaining games with an endogenous status quo when there is uncertainty over legislators’ preferences.\footnote{Duggan (2012) also proves a general existence result for MPE in noisy stochastic games that requires norm-continuity of state transition probabilities. Norm-continuity is violated with voting, however.}
In the environment considered here, legislators’ preferences are fixed so the results of Duggan and Kalandrakis (2012) do not apply. We establish existence by construction. Ray and Vohra (2013) embed a bargaining model in a framework using deterministic agenda setting and voting protocols that accommodate cooperative game solution concepts. Acemoglu, Egorov, and Sonin and Deirmeier, Egorov, and Sonin use these protocols in their models. The random recognition rule used for selection of a proposer in our model is not in the class of protocols considered by Ray and Vohra.

The basic model is introduced in the next section, and Section 3 presents the basic strategies used in the coalition equilibria. Section 4 presents necessary and sufficient conditions that characterize all coalition equilibria for the basic model and Section 5 presents some important special cases of dynamic coalitions. Section 6 introduces implementation uncertainty, and specific-policy coalition equilibria are characterized in Section 7. Section 8 considers coalitions that tolerate a degree of implementation uncertainty. Section 9 compares the coalition equilibria to the results of the Battaglini and Palfrey experiments, and conclusions are provided in the final section.

2 The Basic Model

The model represents a political process with an endogenous status quo where in each period legislators can adopt a new policy or leave the status quo in place. Legislators in this model could also be thought of as party leaders in a parliamentary system forming a government or governing once in office, factions or blocks of legislators with aligned preferences, or members of a commission bargaining each period over the allocation of a budget or a regulatory policy. In each period \( t = 1, 2, \ldots \), legislator \( i \in \{1, \ldots, n\} \) is recognized with probability \( p = \frac{1}{n} \) to propose a policy, which is then voted against the status quo policy from the previous period according to an \( m \)-majority rule, where \([\frac{n+1}{2}] \leq m \leq n-1\).

The winner becomes the policy in place in the current period and the status quo for the next period. In each period legislators allocate a dollar, possibly with waste, so the feasible set of policies in each period is \( X \subseteq \{ x \in \mathbb{R}^n, \sum_{i=1}^{n} x_i \leq 1 \} \). A proposal by a legislator in period \( t \) is a policy \( y^t \in X \), and the status quo policy at the beginning of period \( t \) is denoted \( q^{t-1} \in X \). The agenda on which legislators vote is \( \{q^{t-1}, y^t\} \), and the implemented policy in period \( t \) is denoted by \( x^t \) and \( q^t = x^t \). Legislator \( i \) derives utility \( u(x^t_i) \) from the allocation it receives in period \( t \), where \( u(\cdot) \) is increasing. Legislators maximize the expectation of the discounted, infinite stream of utilities \( \sum_{t=1}^{\infty} \delta^{t-1} u(x^t_i) \), where \( \delta \in [0, 1) \) is the discount factor. An extension in which discount factors and selection probabilities can differ among legislators is presented in Appendix A. A generalization of the results for the basic model in Section 4 are given for the extended model.

A history of the game includes all proposals made, the identity of the proposer, votes cast and
policies implemented. A stationary Markov perfect equilibrium is a subgame perfect equilibrium in which strategies depend only on the payoff-relevant history, which at the proposal stage is the status quo $q^{t-1}$, and does not depend on calendar time. A stationary Markov strategy for legislator $i$ is a pair of functions $(\sigma_i, \omega_i)$, where $\sigma_i : X \rightarrow X$ is a proposal strategy and $\omega_i : X \times X \rightarrow \{0, 1\}$ is a voting strategy.\(^5\) Legislator $i$’s proposal strategy $\sigma_i(q^{t-1}) = y^t$ selects a proposal $y^t$ conditional on the status quo. Legislator $i$’s voting strategy $\omega_i(q^{t-1}, y^t)$ assigns a vote conditional on the proposal and the status quo, where $\omega_i(q^{t-1}, y^t) = 1$ denotes a vote for the proposal. The proposal is approved if and only if $\sum_{i=1}^n \omega_i(q^{t-1}, y^t) \geq m$. The status quo $q^t$ in period $t + 1$ is then

\[
q^t = \begin{cases} 
q^{t-1} & \text{if } \sum_{i=1}^n \omega_i(q^{t-1}, y^t) < m \\
y^t & \text{if } \sum_{i=1}^n \omega_i(q^{t-1}, y^t) \geq m.
\end{cases}
\]

The state thus evolves as proposals are made and votes are cast.

Letting $\sigma$ and $\omega$ denote a profile of strategies, the continuation value $v_i(\sigma, \omega|q^{t-1})$ for $i$ depends on $t$ only through the state and is defined by

\[
v_i(\sigma, \omega|q^{t-1}) = E^t[u(q^t_i) + \delta v_i(\sigma, \omega|q^t)],
\]

where $E^t$ denotes expectation with respect to the selection of the proposer and any uncertainty affecting payoffs and transitions.

A perfect equilibrium requires that in every subgame for each $t$ every legislator’s dynamic payoff is optimal given the equilibrium strategies of the other legislators. That is, a stationary Markov strategy profile $(\sigma^*, \omega^*)$ is a perfect equilibrium if and only if

\[
v_i(\sigma^*, \omega^*|q^{t-1}) \geq v_i((\sigma^*_{-i}, \hat{\sigma_i}), (\omega^*_{-i}, \hat{\omega_i})|q^{t-1}), \text{ for all } (\hat{\sigma_i}, \hat{\omega_i}), i = 1, \ldots, n, \text{ and all } q^{t-1} \in X,
\]

where the strategies $(\hat{\sigma}_i, \hat{\omega}_i)$ may depend on any history of actions and states. We henceforth refer to a stationary Markov perfect equilibrium simply as an equilibrium.

The focus in this paper is on dynamic coalitions of legislators and the policies they support. We formally define a dynamic coalition.

**Definition 1.** A **dynamic coalition** is a decisive set of legislators who strictly prefer to continue the status quo policy from one period to the next rather than pursue a new policy not supported by any coalition.

In the next section we introduce a class of **basic strategies**, and stationary Markov perfect equilibria using basic strategies are referred to as **coalition equilibria**. In Section 4 necessary and sufficient conditions for the existence of a coalition equilibrium are presented, and the policies and dynamic coalitions that arise in coalition equilibria are characterized.

\(^5\)We abuse notation slightly by writing proposal strategies as pure strategies. The coalition equilibria we characterize involve mixing, but the mixing among pure strategies is simple, so for clarity we use pure strategy notation.
3 Basic Strategies and Coalition Equilibria

This section introduces a class of strategies that are simple, result in one-step equilibria, and have proposals in a subset of X. The strategies are symmetric and hence do not favor any particular legislator in coalition formation. Equilibria employing basic strategies are coalition equilibria.

We focus on supporting a symmetric set \( Z \subseteq X \) of policies in equilibrium, and we call \( Z \) the supported set. Let \( S \subseteq X \) be a set of policies, and let \( Z(S) \) be the operator that returns the set of all permutations of the policies in \( S \). The set \( Z = Z(S) \) thus is symmetric and provides the same opportunities to each legislator, so no legislator is advantaged. As an example, let \( n = 3 \) and \( S = \{ (\frac{1}{2}, \frac{1}{2}, 0) \} \). Then \( Z(S) = \{ (\frac{1}{2}, \frac{1}{2}, 0), (\frac{1}{2}, 0, \frac{1}{2}), (0, \frac{1}{2}, \frac{1}{2}) \} \), and \( Z = Z(S) \) is symmetric.

Let \( Z_i \subseteq Z \) be the set of policies that are most advantageous for legislator \( i \), and denote the maximum allocation in \( Z \) as \( z_{\text{max}} \). Then \( Z_i = \{ z \in Z : z_i = z_{\text{max}} \} \) is the set of policies in which \( i \) receives the largest allocation. In the example \( Z_1 = \{ (\frac{1}{2}, \frac{1}{2}, 0), (\frac{1}{2}, 0, \frac{1}{2}) \} \). In equilibrium, proposing policies in \( Z_i \) is equivalent to choosing coalition partners among whom the proposer is indifferent. To ensure basic strategies are well-defined, we assume \( Z \) is compact and is such that \( Z_i \) is finite.

**Definition 2.** A basic strategy profile \( (\sigma, \omega) \) such that for all \( i = 1, \ldots, n \) and for some \( Z \):

(i) Legislators propose the status quo if it is in the supported set \( Z \) and otherwise randomize over favorable policies in \( Z \) to form a new coalition

\[
\sigma_i(q^{t-1}) = \begin{cases} 
q^{t-1} & \text{if } q^{t-1} \in Z \\
\{ z \in Z_i \text{ with probability } \frac{1}{|Z_i|} \} & \text{if } q^{t-1} \notin Z,
\end{cases}
\]

(ii) Voting strategies are stage undominated and legislator \( i \) votes for the status quo when indifferent between the status quo and the proposal

\[
\omega_i(q^{t-1}, y') = \begin{cases} 
1 & \text{if } E^t[u(y'_i) + \delta v_i(\sigma, \omega|y')] > E^t[u(q^{t-1}_i) + \delta v_i(\sigma, \omega|q^{t-1})] \\
0 & \text{if } E^t[u(y'_i) + \delta v_i(\sigma, \omega|y')] \leq E^t[u(q^{t-1}_i) + \delta v_i(\sigma, \omega|q^{t-1})],
\end{cases}
\]

A basic strategy incorporates an indifference rule in (ii) under which a legislator votes for the status quo when indifferent between it and a proposal.

With basic strategies the set \( Z \) includes the policies implemented once a dynamic coalition is in place. Once the status quo is in the set \( Z \), legislators continue to propose policies in \( Z \) even if they are not members of the dynamic coalition because no policy outside of \( Z \) receives a majority of votes.\(^6\) If there is a deviation from the equilibrium path so that the status quo is not in \( Z \), the tacit coalition agreement is broken and the proposer \( j \) selected in the next period randomizes over the policies in \( Z_j \).

\(^6\)Proposing \( q^{t-1} \) if \( q^{t-1} \in Z \) allows the continuation values to be easily calculated.
A legislator thus faces a risk of exclusion, which serves as a collective punishment for all legislators in the dynamic coalition. The indifference rule requires that legislators vote for the status quo when indifferent between it and a proposal, which assures the stability of the coalition.

We define a coalition equilibrium as a stationary Markov perfect equilibrium in which basic strategies are employed.

**Definition 3.** The strategy profile \((\sigma, \omega)\) is a coalition equilibrium if it is a Markov perfect equilibrium using basic strategies.

To analyze dynamic coalitions and the policies they support, we ask which \(Z\) are supported with coalition equilibria. We provide an example in the next section and characterize coalition equilibria more generally in the following sections.

### 3.1 A Coalition Equilibrium Example

This example illustrates a coalition equilibrium, demonstrates the two necessary conditions for existence, and identifies the members of the dynamic coalition. Let \(n = 3, m = 2,\) and \(u(x_i) = x_i,\) and consider the set \(S = \{(\frac{1}{2}, \frac{1}{2}, 0)\}\) from which the supported set \(Z\) is generated. In a coalition equilibrium the status quo \(q_{t-1} \in Z\) continues in place, so the continuation value for legislators \(i\) and \(j\) receiving \(\frac{1}{2}\) in each period under the status quo is \(\frac{1}{2(1-\delta)}\), and 0 for the other legislator \(k.\) Naturally, legislators receiving \(\frac{1}{2}\) under the status quo constitute the dynamic coalition. In Section 4.2 we formalize the identification of the members of a dynamic coalition. For \(q_{t-1} \notin Z\) the proposer \(i\) is selected with probability \(\frac{1}{3}\) and randomizes over the policies in \(Z_i,\) which in the conjectured equilibrium are approved. In expectation a legislator \(i\) obtains \(\frac{2}{3}\left(\frac{1}{2(1-\delta)}\right) = \frac{1}{3(1-\delta)}\) for all \(i.\) We show next that basic strategies constitute a Markov perfect equilibrium.

Suppose the status quo is in \(Z.\) Without loss of generality consider \(q_{t-1} = (\frac{1}{2}, \frac{1}{2}, 0)\) so that legislators 1 and 2 constitute the dynamic coalition, and assume that legislator 3 is selected as the proposer. Suppose legislator 3 proposes to coalition member 2 that the two form a replacement coalition with policy \(y^t = (0, \frac{1}{2}, \frac{1}{2}).\) That is, a deviation to another policy in \(Z\) is proposed. If 2 votes for the proposal, legislators 2 and 3 constitute a replacement coalition with continuation value \(\frac{1}{2(1-\delta)}\). Legislator 2 is indifferent between voting for the status quo and the proposal, however, and according to the indifference rule in basic strategies, along with legislator 1 votes against the proposal sustaining the present coalition \(\{1, 2\}\) and \(q_{t-1}.\) Since legislators 1 and 2 vote against any other proposal in \(Z,\) we say \(Z\) satisfies internal stability.

Next suppose legislator 3 offers more than \(\frac{1}{2}\) to legislator 2. That is, a deviation outside of the set \(Z\) is proposed. Suppose 2 votes for the proposal, so the tacit coalition agreement is broken. The dynamic coalition of 1 and 2 thus ends with a new coalition formation round commencing in the next
period, giving a continuation value of $\frac{1}{1(1-\delta)}$. The most attractive proposal that can be offered to 2 is $(0, 1, 0)$, which gives legislator 2 the highest dynamic payoff for any proposal not in $Z$. Legislator 2 rejects the proposal if and only if

$$\frac{1}{2} + \delta \frac{1}{2(1-\delta)} \geq 1 + \delta \frac{1}{3(1-\delta)},$$

which is satisfied if and only if $\delta \geq \frac{3}{4}$. Since $(0, 1, 0)$ is the most attractive proposal, there is no proposal by 3 (nor by 1 or 2) that can break the coalition for $\delta \geq \frac{3}{4}$. As long as the continuation payoff for a proposal outside $Z$ is lower than the continuation payoff to coalition members from continuing a policy in $Z$, the coalition is sustained. The continuation payoff is lower because of the possibility of being excluded from the coalition formed in the next period, i.e., exclusion risk.

If $q_{t-1} \notin Z$, the selected legislator $i$ is to propose a policy in $Z_i$. To show that the proposer has no incentive to propose a policy not in $Z_i$, consider the most attractive status quo $(1, 0, 0)$ for legislator 1. Legislator 1 could propose the status quo yielding a payoff of 1 in the current period, but as shown in (1), the proposer (weakly) prefers the equilibrium proposal $y_i^t = \left(\frac{1}{2}, \frac{1}{2}, 0\right)$ or $y_i^t = \left(\frac{1}{2}, 0, \frac{1}{2}\right)$ for $\delta \geq \frac{3}{4}$. But, for $\delta = \frac{3}{4}$ legislator 1 is indifferent between the status quo and a proposal in $Z_i$, and under the indifference rule votes for the status quo. The discount factor thus must be strictly greater than $\frac{3}{4}$. The set $Z$ thus is supported by a coalition equilibrium with dynamic coalitions if and only if $\delta > \frac{3}{4}$. The coalition equilibrium is attained in one step because legislators propose in $Z_i \subset Z$, and the dynamic coalition sustains that policy thereafter.

The indifference rule used in basic strategies specifies that a legislator votes for the status quo when indifferent. The role of the indifference rule is to preclude a coalition member from deviating to an apparently equivalent proposal and replacement coalition by a non-coalition legislator. That is, if $q_{t-1} = \left(\frac{1}{2}, \frac{1}{2}, 0\right)$ and legislator 3 proposes $y_i^t = \left(0, \frac{1}{2}, \frac{1}{2}\right)$, legislator 2 is indifferent between the status quo and the proposal. If legislators vote for the proposal when indifferent, however, legislator 2 would break the coalition with legislator 1. The indifference rule of voting against the proposal when indifferent thus is needed to support coalition equilibria in basic strategies in the dynamic legislative bargaining game.

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7 The indifference rule used here is thus fundamentally different from the indifference rule used in sequential legislative bargaining. In the latter a legislator votes for the proposal when indifferent between it and the alternative so as to avoid an open set of accepted proposals, whereas here the indifference rule provides stability within the set of supported policies.

8 The equilibria established by Kalandrakis, Bowen and Zahran, Duggan and Kalandrakis, and others specify that a legislator who is indifferent between the status quo and the proposal votes for the proposal. Battaglini and Palfrey assume that a legislator votes for the proposal with positive probability. Anesi and Seidmann use an indifference rule that is conditional on the status quo and the proposal. For example, consider the equilibrium policy $(c, c, w)$ with $0 < w < 1 - 2c$, which can be a simple solution in their terminology. If $q^0 = (0, 1, 0)$ and legislator 1 proposes $(c, c, w)$, legislator 2 is assumed to vote for the proposal if $\delta$ is such that legislator 2 is indifferent. When $q^1 = (c, c, w)$ and legislator 3 proposes $(c, w, c)$, legislator 1 is assumed to vote for the status quo when indifferent between the two coalitions each of which yields $c$ in every period.
4 Dynamic Coalitions in the Basic Model

4.1 Continuation Values

For any symmetric set \( Z \) the continuation values in a coalition equilibrium are straightforward to characterize as illustrated by the example. Basic strategies call for all legislators to propose the status quo if it is in \( Z \), so any policy in \( Z \) is an absorbing state. For any status quo not in \( Z \), legislator \( i \), if selected, proposes with equal probability all elements of the set \( Z_i \), each of which is assumed to be approved in the conjectured coalition equilibrium. Symmetry implies that \(|Z_i| = |Z_j|\), so the probabilities of receiving particular allocations are identical across legislators. This gives Lemma 1, the proof of which is straightforward and hence is omitted. To simplify the notation, let \( v_i(q^{t-1}) \) denote \( v_i(\sigma,\omega|q^{t-1}) \).

**Lemma 1.** In the basic model if \((\sigma,\omega)\) is a coalition equilibrium with supported set \( Z \):

(i) The continuation value for player \( i \) for \( q^{t-1} \in Z \) is \( v_i(q^{t-1}) = u(q^{t-1}) = \frac{u(q^{t-1})}{1-\delta} \).

(ii) The continuation value for player \( i \) for \( q^{t-1} \notin Z \) is \( v_i(q^{t-1}) = v^* = \frac{\bar{u}}{1-\delta} \), where

\[
\bar{u} = \frac{1}{n|Z_j|} \sum_{z \in Z_j} \sum_{i=1}^{n} u(z_i).
\]

With a symmetric set \( Z \) the continuation value for any legislator is a constant \( v^* \) for all \( q^{t-1} \notin Z \), where \( v^* \) is the discounted average utility \( \bar{u} \) available in the proposals made in \( Z_i \). This feature of the basic strategies simplifies the characterization of coalition equilibria, because every possible deviation from the set \( Z \) is met by the same response (random formation of a new coalition in the next period). All such deviations thus result in the same continuation value \( v^* \) for all legislators, which as illustrated in the example, simplifies the analysis of possible deviations. It also means that a proposer cannot sweeten a proposal without a new coalition formation round beginning in the next period.

4.2 A Class of Simple Coalition Equilibria

The following proposition identifies a class of coalition equilibria that includes the example in the previous section. The proof of Proposition 1 is presented as a special case of the extended model with heterogeneous selection probabilities and discount factors to demonstrate the robustness of coalition equilibria. All proofs for Section 4 are presented in Appendix A.

**Proposition 1.** In the basic model \( Z \) is supported by a coalition equilibrium if \( \delta > \delta^* \equiv \frac{u(1) - u(z_{max})}{u(1) - \bar{u}} \), and

(a) \( z_i = z_{max} \) for at least \( m \) legislators, for all \( z \in Z \).
(b) \( u(z_j) < u(z_{\text{max}}) \) for some \( j \) and some \( z \in Z \).

Part (a) of Proposition 1 states that at least a minimal majority of legislators receives allocations equal to the maximum allocation in \( Z \). This condition ensures that policies in \( Z \) satisfy internal stability, since no group of \( m \) or more legislators can be made strictly better off with another policy in \( Z \). Part (b) states that at least one other allocation must give strictly lower utility than \( z_{\text{max}} \). This condition assures that there is exclusion risk. Each legislator receiving \( z_{\text{max}} \) fears being excluded from the next coalition formed and receiving a payoff strictly lower than \( u(z_{\text{max}}) \).

To illustrate the argument underlying the proof of Proposition 1, note that with basic strategies any \( z \in Z \) is an absorbing state, so any deviation to a \( z' \in Z \) from the coalition equilibrium supporting \( Z \) involves a comparison between \( z_i \) and \( z'_i \) for legislator \( i \). Since at least a majority have \( z_i = z_{\text{max}} \), under the indifference rule no deviation in \( Z \) receives a majority vote. For a deviation to a policy \( z'' \notin Z \) legislator \( i \) has a dynamic payoff no greater than \( u(1) + \delta v^* \). On the equilibrium path \( i \) receives \( v_i(z) = \frac{u(z_{\text{max}})}{1-\delta} \), which is greater than the dynamic payoff from any policy not in \( Z \) for a sufficiently high discount factor \( \delta \in (\delta, 1) \), where the bound \( \delta \) corresponds to the supported set \( Z \) satisfying conditions (a) and (b). Such a \( \delta \) exists if and only if \( u(z_i) > \pi \) for a minimal-winning coalition, or if and only if the utility for a minimal-winning coalition is at least as great as the average utility available in the policies proposed if the coalition ends. The bound \( \delta \) is strictly less than one because by (b), \( Z \) contains at least one element that gives a strictly lower utility than \( z_{\text{max}} \).

The following Corollary identifies properties that a dynamic coalition formed in the coalition equilibria identified in Proposition 1 can have.

**Corollary 1.** Policies supported by the dynamic coalitions in Proposition 1 have the following properties:

(a) Dynamic coalitions can support both efficient and inefficient policies.

(b) Dynamic coalitions can support positive allocations to legislators outside the dynamic coalition.

(c) Dynamic coalitions can have surplus members; that is, the size of the dynamic coalition can be strictly larger than minimal winning.

Part (a) states that dynamic coalitions can support inefficient policies. That is, a policy with waste can be sustained in a coalition equilibrium, representing a political failure in the sense of Besley and Anesi and Seidmann use a condition analogous to (b), requiring that for each player at least one proposal provides a low payoff.

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9 The basic strategy in Definition 2(i) has a proposer propose the status quo if it is in the supported set \( Z \), but if \( Z \) satisfies the conditions of Proposition 1, the basic proposal strategy can be sharpened to propose \( q^{t-1} \) only if it is in \( Z_i \). That is, a policy \( q^{t-1} \in Z_i \) gives \( i \) and at least \( m - 1 \) others the maximum payoff, so at least \( m \) legislators vote for \( q^{t-1} \). The continuation values proposing \( q^{t-1} \in Z_i \) are the same as with basic strategies.

10 Anesi and Seidmann use a condition analogous to (b), requiring that for each player at least one proposal provides a low payoff.
Coate (1998). An inefficient policy is sustained because the coalition members fear being excluded from a future coalition and receiving a lower continuation payoff. While there are short term payoffs that may be better for a coalition member, the coalition member fears what may happen subsequent to receiving the short-term payoff. Thus the dynamics sustaining the coalition are similar to the dynamics described in Acemoglu, Egorov and Sonin (2012), who also find that equilibria can support inefficient policies.

Part (b) states that dynamic coalitions can support policies in which non-coalition members receive positive allocations. This is observed in practice as noted in the introduction, where benefits are distributed to more than a bare majority. Such policies are supported as an equilibrium because coalition members prefer to maintain the status quo rather than risk exclusion.

Part (c) states that a dynamic coalition can be strictly larger than minimal winning, as in parliamentary governments that have a surplus of parties. This again results because the members of the dynamic coalition fear the risk of exclusion in a new coalition formation round.

Coalition equilibria have stable policies, providing perfect risk smoothing over time. For risk averse legislators this allows coalition equilibria to exist for lower discount factors compared to risk neutral preferences. Proposition 1 identifies the bound on the discount factor corresponding to a set $Z$ supported by a coalition equilibria, and the following Corollary presents the bound for a class of particularly simple coalition equilibria in which all non-coalition members receive no allocation.

**Corollary 2.** For all $\delta > \delta_*$ there exists a coalition equilibrium in which $s \in [m, n - 1]$ legislators receive $z_{\text{max}}$ and $n - s$ legislators receive 0. The bound $\delta_*$ is

$$\delta_* = \frac{u(1) - u(z_{\text{max}})}{u(1) - \left[ \frac{s}{n} u(z_{\text{max}}) + \frac{n-s}{n} u(0) \right]},$$

which is strictly increasing in $s$.

For $u(x)$ concave enough, $\delta_*$ can be arbitrarily small. That is, normalize $u(\cdot)$ so that $u(1) = 1$ and $u(0) = 0$, and note that as $u(\cdot)$ becomes more concave, $u(z_{\text{max}})$ approaches 1 and $\delta_*$ approaches 0. The theory of coalition equilibria thus can be applied to a large class of political settings, even where risk-averse political actors place very low value on the future. The discount factor in a particular application depends on the length of a period. In a legislative application the time between proposals can be relatively short in which cases discount factors are high, whereas if a period corresponds to an interelection period, the discount factor could be relatively low. As Corollary 2 indicates, a coalition equilibrium can exist if parties or their leaders are risk averse.
4.3 Existence and Characterization of Coalition Equilibria

Proposition 1 identifies a class of coalition equilibria, and the following proposition provides necessary and sufficient conditions for the existence of a coalition equilibrium supporting a set $Z$. Let $M$ denote a set of $m$ legislators, and let $\mathcal{M}$ denote the collection of all such $M$. Let $W(z)$ denote the set of policies in $Z$ that are strictly preferred by an $m$-majority of legislators; i.e., $W(z) = \{z' \in Z | \exists M \in \mathcal{M} \ni u(z'_i) > u(z_i), i \in M\}$.\footnote{Strict preference is used because of the indifference rule.} Then, if $W(z) = \emptyset$, there is no policy $z' \in Z$ that defeats $z$. To break a dynamic coalition, a deviation proposal must attract the vote of the legislator with the $m$th largest allocation for any $z \in Z$, and let the smallest of these be denoted by $z_{min}^m$.

**Proposition 2.** In the basic model there exists a $\delta \leq \frac{u(1) - u(z_{min}^m)}{u(1) - \bar{u}} < 1$ such that a coalition equilibrium supporting the set $Z$ exists if and only if $\delta > \bar{\delta}$ and:

(a) No policy in $Z$ can be defeated by another policy in $Z$ in a pairwise comparison; i.e., $W(z) = \emptyset$ for all $z \in Z$.

(b) At least a majority $M$ of legislators strictly prefers the coalition to continue rather than dissolve; i.e., $v_i(z) > v^*$ for $m$ legislators, for all $z \in Z$, or equivalently, $u(z_{min}^m) > \bar{u}$.

Proposition 2 gives two intuitive conditions for a coalition equilibrium supporting a set $Z$. Condition (a) ensures that no deviation from an allocation in $Z$ to another allocation in $Z$ is (strictly) attractive for a minimum-winning coalition. This internal stability condition precludes replacement coalitions. Condition (b) is the exclusion risk condition that ensures that a decisive set of legislators prefer to continue the coalition rather than face a new coalition formation round and the risk of exclusion. Legislators then have no incentive to deviate from a policy in $Z$ to a policy not in $Z$. More formally, Condition (b) states that the utility of the $m$th largest allocation in $z$ must be strictly larger than the average utility in $Z$, for all $z$. This condition is different from von Neumann Morgenstern external stability, which requires that no policy not in $Z$ can defeat a policy in $Z$.\footnote{Condition (b) is thus different from the external stability conditions used by Anesi (2010) and Diermeier, Egorov and Sonin (2013). This difference is considered in more detail in Section 5.3.} From Condition (b) it is immediate that the lower bound on the discount factor is strictly less than 1.

Conditions (a) and (b) only involve comparisons of single-period payoffs. Because these comparisons are single-period, legislators’ voting and proposal behavior is indistinguishable from behavior in a single-period model in which legislators receive utility $\bar{u}$ if a coalition ends.

A generalization of Proposition 2 is presented in Appendix A for an extension of the model in which legislators have different discount factors $\delta_i$ and different selection probabilities $p_i$. In contrast
to sequential legislative bargaining, the continuation values of the legislators are irrelevant to the
selection of coalition partners even though when the discount factors are unequal the continuation
values are ordered by $p_i > p_j$, i.e., $v_i(q^{t-1}) > v_j(q^{t-1})$ for $q^{t-1} \notin Z$. All that matters to proposers
when $q^{t-1} \notin Z$ is that they receive the largest payoff $z_{\text{max}}$ in the next period, since a proposal in $Z_i$
is an absorbing state.

The following two corollaries provide examples of policies that satisfy Condition (a) but fail Con-
dition (b) in Proposition 2, and hence are not supported by dynamic coalitions.

**Corollary 3.** Dynamic coalitions cannot support a dictator outcome $(1,0,\ldots,0)$ or any policy such
that fewer than $m$ legislators receive positive allocations.

It is straightforward to see that condition (b) is violated in the case of the dictator outcome. It is
natural to expect dynamic coalitions not to support the dictator outcome since less than a minimal
majority receives positive benefits.\textsuperscript{13}

**Corollary 4.** If $u(x_i) = x_i$ and all policies in $S$ are efficient, no dynamic coalition is universal (with
all legislators receiving equal allocations). If $S$ is a singleton, no coalition equilibrium can support the
universal coalition for any $u(\cdot)$.

Although $S = \{(\frac{1}{n},\ldots,\frac{1}{n})\}$, that is, the universal policy, is not supported by a coalition equilibrium
when all the policies in $S$ are efficient, if an inefficient policy is included in $S$, for example $S = \{(\frac{1}{n},\ldots,\frac{1}{n}),\frac{1}{n},0,\ldots,0\}$ with $m$ legislators receiving $\frac{1}{n}$ in the second policy, then $Z(S)$ is
supported by a coalition equilibrium. The inclusion of the policy $(\frac{1}{n},\ldots,\frac{1}{n},0,\ldots,0)$ in $S$ provides
the exclusion risk, so there is a threat of collective punishment for coalition members.\textsuperscript{14} Universal
ccoalitions thus can form when a threat is present.

### 4.4 Identifying Dynamic Coalitions

For each policy in the supported set $Z$ there exists a dynamic coalition. Lemma 2 guides us to
identifying members of the dynamic coalition with a simple condition.

**Lemma 2.** Let $(\sigma,\omega)$ be a coalition equilibrium with supported set $Z$. Legislator $i$ is a member of the
dynamic coalition corresponding to $z \in Z$ for some $\delta > \delta_0$ if and only if $u(z_i) > \bar{u}$.

By Lemma 2 a legislator is a member of the dynamic coalition corresponding to a $z \in Z$ if its
single-period payoff with $z$ is strictly larger than the average payoff available in $Z_i$. For the equilibrium

\textsuperscript{13}This is also a feature of the equilibria characterized by Anesi and Seidmann.

\textsuperscript{14}This is similar to Richter (2014), who shows that equal division can be supported by allowing some of the dollar to
be wasted.
given in Proposition 1 (and the previous three player example), all legislators with allocation $z_{\text{max}}$ are members of the dynamic coalition because their utility is maximal and a new coalition formation round poses the risk that they receive the lower payoff in part (b) (or worse). Thus, if the dynamic coalition ends, its members are collectively punished by the risk of exclusion from the next dynamic coalition formed.

We illustrate the identification of the members of a dynamic coalition using Lemma 2 and demonstrate that they need not all have the maximum allocation in $Z$. Consider $n = 3$, $m = 2$, $u(x_i) = x_i$, $S = \{(c, c, w)\}$, and $c > w$. Suppose the status quo is $z = (c, c, w)$, so the continuation value when the coalition dissolves is $\nu^* = \frac{2c+w}{3(1-\delta)}$ and the average payoff $\bar{u}$ in the coalition is $\frac{2c+w}{3}$. Then, $c > \bar{u}$ for $i = 1, 2$, and $w < \bar{u}$. Legislator 3 benefits when the coalition ends and hence is not in the dynamic coalition, whereas legislators 1 and 2 are punished if the coalition ends. In a three-member legislature when $S$ is a singleton, the dynamic coalition is always minimal-winning.

As another illustration consider $n = 5$, $m = 3$, $u(x_i) = x_i$, $S = \{(c, c, c, a, b)\}$, with $c > a > b$. If the status quo is $(c, c, c, a, b)$, legislators 1, 2, and 3 strictly prefer that the coalition continue rather than dissolve, whereas legislator 4 strictly prefers that it continue if and only if $a > \frac{1}{3}(3c + b)$. If the policy is efficient, legislator 4 strictly prefers that the coalition continue if and only if $a > \frac{1}{3}$. Consequently, if $a$ is sufficiently high, the dynamic coalition has four members (a surplus coalition, where legislator 4 can be thought of as a “junior” member). If $S = \{(c, a, a, a, 0)\}$, $m = 3$, and $c > a > c$, the party receiving $c$ can be thought of as a single-party minority government supported by three other parties. Minority governments thus provide policies sufficiently beneficial to a set of other parties to maintain their support.

Alternatively, suppose $S = \{(c, c, a, a, 0)\}$, $m = 4$, and $c > a > \frac{2}{3}c$. If the status quo is $(c, c, a, a, 0)$ then legislators (parties) 1 and 2 can be thought of as a minority government supported by parties 3 and 4. If $S = \{(c, a, a, a, 0)\}$, $m = 3$, and $c > a > \frac{2}{3}$, the party receiving $c$ can be thought of as a single-party minority government supported by three other parties. Minority governments thus provide policies sufficiently beneficial to a set of other parties to maintain their support.

5 Special Cases of Dynamic Coalitions

5.1 Vested Interests

Proposition 2 does not require the maximum allocation in each policy in $S$ to be the same. This suggests that coalition equilibria can support a policy that is not proposed from status quos other than itself. These policies can be thought of as supported by interests vested in the initial status quo. Vested interests are defined as:
**Definition 4.** A dynamic coalition has *vested interests* if its members support a policy that would never be selected in a new coalition formation round.

To identify this feature of coalition equilibria, note that the set $Z$ supported by a coalition equilibrium includes the policies in $\bigcup_i Z_i$ that are proposed by legislators when the status quo is not in $Z$. The set $Z$ also can include policies that are supported by dynamic coalitions but would never be proposed by any legislator when forming a new coalition. Let the latter set of policies be denoted by $Q^0$. That is $Q^0 = Z \setminus (\bigcup_i Z_i)$. The set $Q^0$ can be empty as when $S = \{(\frac{1}{2}, \frac{1}{2}, 0)\}$ or more generally for $|S| = 1$. For $|S| > 1$ the set $Q^0$ can be nonempty. For example, if $u(x_i) = x_i$, $m = 3$, and $S = \{z', z''\}$ where $z' = (a, b, b, c, c)$ and $z'' = (b, b, b, d, d)$, $a > b > d > c$, and $\frac{1}{2}(a+2c) < b$, legislators only propose a permutation of $z'$ and never propose a permutation of $z''$ when a coalition is being formed. If the initial status quo is $z''$, however, legislators propose $z''$, and it persists thereafter. Hence $Q^0 = Z(\{z''\})$. The policies in $Q^0$ are supported by a coalition equilibrium with vested interests.

Vested interests thus preserve the initial status quo. This can be interpreted as a form of gridlock in dynamic legislative bargaining in the sense that every legislator prefers and has the opportunity to propose a different policy yet all proposals are defeated by the vested interests. This is driven by the indifference rule. No majority strictly prefers an alternative policy to $z$, so the will of the majority is not thwarted. To illustrate this notion of gridlock, suppose that a parliamentary system of government with policy $z$ falls because of an event such as a scandal and a new government formation opportunity commences. If the policy of the fallen government remains in place, the interests vested in the fallen government can prevail resulting in no change in policy. If the policy is subject to a shock, however, as in Sections 6-8, a different coalition can emerge with a policy $z' \notin Q^0$.

A variety of policies can be sustained by equilibria when there are vested interests. For example, if $c = 0$, $z'$ is minimal winning, yet vested interests prevail. If $d = b = \frac{1}{5}$, $z''$ is the universal policy. A universal policy thus can be supported by a coalition equilibrium in two ways. First, it can be an element of $Z_i$ and hence be proposed when a coalition is being formed as illustrated in the previous section, and second, it can be supported by vested interests even though it would never be proposed when a coalition is being formed. The latter requires $z'$ in the example to be inefficient, and if $z'$ is efficient, the universal policy $z''$ cannot be supported by vested interests.

Vested interests can also support Pareto dominated policies, since those interests can prefer such a policy to avoid the risk of exclusion in a new coalition formation opportunity in the next period.

**Corollary 5.** There always exists a policy in the supported set that strictly improves the payoff of a member of the group of vested interests. Furthermore, such a policy can Pareto dominate the policy supported by the vested interests.
For example, for $n = 5$, $m = 3$, and $u(x_i) = x_i$, consider $S = \{z', z''\}$, where $z' = (a, b, b, b, c)$, $z'' = (b, b, b, b, 0)$, $a > b \geq c \geq 0$, and $b > \frac{a+c}{2}$. The policy $z'$ Pareto dominates $z''$, but $W(z'') = \emptyset$, and $Z = Z(S)$ can be supported by a coalition equilibrium. Vested interests thus can result in a political failure.

5.2 The Sequential Bargaining Outcome

Coalition equilibria are symmetric, the ex ante values of the game are the same for all legislators, and all legislators have an equal probability of being in a coalition, which are also the predictions of sequential legislative bargaining theory. In sequential bargaining the proposer captures all the benefits of proposal power; that is, the equilibrium policies with $\delta = 1$ are of the form $(1 - \frac{m-1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}, 0 \ldots, 0)$, with $m - 1$ legislators receiving $\frac{1}{n}$ and $n - m$ legislators receiving 0. If $u(x_i) = x_i$, the sequential bargaining outcome cannot be supported because $\Pi = \frac{1}{n}$ and $z_{\text{min}}^{m} \leq \frac{1}{n}$, so a minimal-winning coalition is not punished when the coalition dissolves (i.e., the exclusion risk condition is violated). If $|S| \geq 2$, and $S$ includes policies that lower the average allocation, thereby providing a risk of exclusion, internal stability is also violated for simple majority rule. A supermajority rule is necessary for this purpose, and the following corollary gives necessary conditions for coalition equilibria to support the sequential bargaining outcome.

Corollary 6. If $u(x_i) = x_i$, a dynamic coalition supports the sequential bargaining outcome only if $|S| \geq 2$, $m > \frac{n+1}{2}$, and $\frac{1}{n} \geq z_{\text{min}}^m > \Pi$.

5.3 A Three-Member Legislature

For $n = 3$ Proposition 1 indicates that any policy of the form $(c, c, w)$ with $c > w$ is supportable by a coalition equilibrium. Conditions $(a)$ and $(b)$ in Proposition 1 are also necessary when $u(\cdot)$ is strictly increasing.

Proposition 3. If $n = 3$, $m = 2$, and $u(\cdot)$ is strictly increasing, a coalition equilibrium supports the set $Z$ if and only if $\delta > \delta \equiv \frac{u(1) - u(c)}{u(1) - w}$, $z = (c, c, w^*)$ for all $z \in Z$, and $c > w^*$ for some $z \in Z$.

These policies all have equal sharing within the dynamic coalition as illustrated in Figure 1. Thus in a three-member legislature a dynamic coalition does not give the proposer any advantage over its coalition partner.

The proposition follows from the internal stability condition $(a)$ in Proposition 2, and more generally for $n$ odd and $m = \frac{n+1}{2}$, all members of a minimal winning dynamic coalition have the same allocation. The reason is that if not, at least one member of the coalition can strictly gain from another member’s allocation, and there are $\frac{n-1}{2}$ legislators outside the dynamic coalition whose payoff can also be strictly improved by giving them a coalition member’s share.
Figure 1: Supported Policies $S = \{(c, c, 1 - 2c)\}$

The special case of a three-member legislature provides a simple comparison of policies supported by dynamic coalitions and the von Neuman and Morgenstern stable outcomes. Restricting attention to the case of efficient policies and linear utility we have the following corollary.

**Corollary 7.** If $n = 3$, $m = 2$, $u(x) = x$ and all policies in $S$ are efficient, that is all policies are of the form $(c, c, 1 - 2c)$, for any $\delta \in \left(\frac{3}{4}, 1\right)$ allocations supported by a dynamic coalition satisfy $c > 1 - \frac{2}{3}\delta$.

Figure 2 illustrates the supported policies in Corollary 7 as a function of the discount factor. The lower diagonal border is open as is the vertical border at $\delta = 1$. The upper bound represents the example presented in Section 3.1. The higher the discount factor the larger is the set of policies that are supported by a coalition equilibrium because the more patient legislators are the greater is the set of short-run temptations a legislator can resist.

Figure 2: Supported policies as a function of $\delta$

Anesi (2010), Acemoglu, Egorov and Sonin (2012), and Diermeier, Egorov and Sonin (2013) consider models for the case in which $\delta \approx 1$, which allows them to compare the Markov perfect equilibria
in a bargaining game to stable sets.\footnote{These models rely on a finite policy space, and Diermeier, Egorov and Sonin (2013) find that although some players have vetoes and those players also have monopoly proposal rights, the non-veto players effectively form a blocking group that prevents the veto players from fully exploiting them. Nunnari (2014) considers a dynamic legislative bargaining model with a veto player and shows that the veto player fully exploits the non-veto players. Similarly, Kalandrakis (2010) shows that a player with monopoly proposal rights can fully exploit the other players using a mixed proposal strategy. Nunnari (2014) and Kalandrakis (2010) assume a continuous policy space. Kalandrakis (2010) also argues that for any fixed $\delta < 1$ the pure strategy equilibrium in Diermeier and Fong (2011), which is similar to that in Diermeier, Egorov and Sonin (2013), does not exist as the finite grid on the policy space becomes finer. Kalandrakis (2010) notes that whether these equilibria exist with a continuous policy space is an open question.} Anesi (2010) and Diermeier, Egorov and Sonin (2013) focus on policies that satisfy internal and external stability conditions consistent with the von Neumann Morgenstern stable set $\Sigma$, which for $n = 3$ is $\Sigma = \{(\frac{1}{2}, \frac{1}{2}, 0), (\frac{1}{2}, 0, \frac{1}{2}), (0, \frac{1}{2}, \frac{1}{2})\}$, whereas Acemoglu, Egorov and Sonin (2012) develop a related dynamic stability concept.\footnote{The assumption that $\delta \approx 1$ corresponds to choices that can be changed immediately as Acemoglu, Egorov and Sonin (2012) note.}

For a model with veto players who are also monopoly proposers Diermeier, Egorov and Sonin (2013) show that a policy is in $\Sigma$ if and only if it is supported by a MPE in which veto players propose only policies they strictly prefer.\footnote{The restriction of MPE to proposals that strictly improve a veto player’s payoff is substantive as it rules at MPE with cycles.} For a dynamic legislative bargaining model as in the present paper Anesi shows that every policy in $\Sigma$ can be supported by a MPE but the converse is not true. The difference between the two results is that, as in this paper, Anesi uses the indifference rule of voting for the status quo when indifferent and Diermeier, Egorov, and Sonin use the indifference rule of voting for the proposal when indifferent. Thus, Diermeier, Egorov and Sonin (2013) find that internal and external stability conditions are both sufficient and necessary for a policy to be supported by a MPE, Anesi finds that external stability is not necessary, so a set larger than $\Sigma$ is supported for $\delta \approx 1$. In the theory presented here for $\delta \approx 1$ all policies of the form in Corollary 7 with $c \in (\frac{1}{3}, \frac{1}{2}]$ are supported. The exclusion risk condition in Proposition 2 can be thought of as the replacement for the external stability condition for a dynamic legislative bargaining model.

The internal stability and exclusion risk conditions in Proposition 2 are necessary and sufficient for a coalition equilibrium to exist and allow a set of policies larger than the stable set to be supported and for discount factors as low as $\frac{3}{4}$ for a linear utility function. Also, the limit as $\delta \to 1$ of the supported set is not the stable set but a larger set. Moreover, if the stable set $\Sigma \notin Z$, it is defeated by all policies in $Z$. That is, the incentives in a dynamic game are fundamentally different from those in the static game.

Proposition 3 provides a benchmark for the case of implementation uncertainty beginning in Section 6. If efficient policies with non-coalition members receiving a zero allocation are focal, this provides a unique benchmark $S = \{(\frac{1}{2}, 0, 0)\}$ for comparison. Furthermore, $\left(\frac{1}{2}, \frac{1}{2}, 0\right)$ is supportable by a coalition equilibrium only when $S = \{(\frac{1}{2}, 1, 0)\}$. In contrast to these equilibria in which the gains from proposal power are shared equally within the coalition, with implementation uncertainty dynamic coalitions
with unbalanced allocations are possible, as considered in Sections 5-7. That is, the internal stability condition can be weakened when there is implementation uncertainty.

6 Implementation Uncertainty

Uncertainty can be associated with the implementation of a policy, and that uncertainty can affect not only the payoff in the current period but also the status quo for the following period. Since the uncertainty affects the policy that is implemented, the status quo can move away from the coalition policy in which case the coalition could dissolve and a new coalition form as in Section 7 or the coalition members could tolerate the new policy as in Section 8. Dynamic coalitions can survive the possibility of implementation uncertainty and can support unbalanced allocations among the coalition members. The analysis is presented for \( n = 3, \ m = 2, \ u(x_i) = x_i \) and efficient policies, which is the case studied by Kalandrakis (2004) and Battaglini and Palfrey. Implementation uncertainty could take many forms and the model used simplifies the analysis of the incentive constraints and facilitates comparative statics analysis of the set of policies supported by dynamic coalitions.

Uncertainty resulting from implementation represents the observation that policy does not always work as intended. The implementation of legislation is typically delegated to administrative agencies or regulatory commissions that develop the details for the application of the policy. A degree of uncertainty can be associated with that delegation, and legislators take that uncertainty into account in choosing a policy. The uncertainty could also be associated with the response to the enacted policy by those affected, and the realization of that uncertainty can affect the status quo and the strategies of legislators in the future. As an extreme example, with the support of the American Association of Retired Persons Congress overwhelmingly enacted the Medicare Catastrophic Coverage Act of 1988 which provided generous benefits for catastrophic care under Medicare and financed the benefits through increases in Medicare premiums. Before the change could be fully implemented, Medicare recipients began protesting the forthcoming premium increases, and facing uncertainty about the impact of the Act, Congress quickly repealed the Act it had passed in the previous session.

Implementation uncertainty could depend on whether the legislature retains the current policy or chooses a new policy. When the legislature retains the current policy, implementation uncertainty is assumed to be present with probability \( \eta \), and when the legislature chooses a new policy, the

\[18\] In their MLQRE Battaglini and Palfrey assume that players use behavioral strategies that place positive probability on every available action (on a grid). That probability is proportional to the continuation value, and as that proportion increases the limit points correspond to MPE. This uncertainty affects strategies and hence payoffs but does not affect state transitions other than through the strategies. Duggan and Kalandrakis (2012) show the existence of stationary MPE in pure strategies for a class of dynamic games that accommodate uncertainty in the current period payoffs and in the transitions from one state to another, but they do not provide an equilibrium construction. They consider stationary legislative equilibria which have the property that legislators vote for the proposal when indifferent and show that these equilibria have smooth continuation values. Since we assume legislators vote for the status quo when indifferent and our assumed uncertainty is different from theirs, these results do not apply to the equilibria we characterize.
corresponding probability is $\gamma < 1$. With the complementary probabilities there is no uncertainty and hence the policy implemented equals that adopted by the legislature. When implementation uncertainty is realized, its magnitude is represented by a continuous, mean zero, random shock that is publicly observable. The former specification allows comparative statics analysis in terms of a single parameter, and the latter specification means that the probability is zero that the shocked policy equals the policy adopted by the legislature. A legislator cannot receive more that 1 or less than 0, so the shocked allocation may be truncated, in which case the truncated amount is assumed to be reallocated among the legislators to ensure the policy remains in the feasible set. Details of the assumed implementation uncertainty are given in Appendix B, and we present the substance here.

Let $Z(c) \equiv Z(S(c))$, where $S(c) = \{(1 - c, c, 0)\}$ and $c \leq \frac{1}{2}$, so $z_{\text{max}} = 1 - c$.

**Assumption 1 (Substance).** Suppose $q^{t-1} \in Z(c)$ and $y^t \in Z(c)$. If $y^t = q^{t-1}$, with probability $1 - \eta$, $q^t = q^{t-1}$ and with probability $\eta$, a shock $\tilde{\theta}^t$ distorts the policy as follows:

$$q_i^t = \begin{cases} 
1 - c + \theta^t & \text{if } y_i^t = 1 - c \\
 c - \theta^t & \text{if } y_i^t = c, \\
 0 & \text{if } y_i^t = 0.
\end{cases}$$

where $\tilde{\theta}^t$ is distributed uniformly on $[-\hat{\theta}, \hat{\theta}]$ and $\theta^t$ is the realization of $\tilde{\theta}^t$. If $y^t \neq q^{t-1}$ is approved, with probability $1 - \gamma$, $q^t = y^t$, with probability $\gamma$ a shock $\tilde{\varepsilon}^t$ distorts the policy as above but with with the realization $\varepsilon^t$ replacing $\theta^t$, where $\tilde{\varepsilon}^t$ is distributed uniformly on $[-\varepsilon, \varepsilon]$. If $q^{t-1} \notin Z(c)$ or $y^t \notin Z(c)$ is approved, allocations are similarly affected.

Implementation uncertainty could be greater when a new policy is adopted than when the current policy is continued, and the following assumption formalizes this.

**Assumption 2.** Implementation of a new policy $y^t \neq q^{t-1}$ has a higher probability of a shock than implementation of the current (status quo) policy, i.e., $1 > \gamma \geq \eta$, and a stochastically larger shock, i.e., $\varepsilon \geq \hat{\theta}$.

### 7 Specific-Policy Equilibria with Implementation Uncertainty

This section shows by construction the existence of a set of coalition equilibria that support policies with unbalanced allocations to the members of a dynamic coalition; i.e., in $Z(c)$. Since in these equilibria $S(c)$ consists of a single policy, the policy is specific to that $c$. The originator of a specific-policy coalition has proposal power and may not share the gains equally with the coalition partner. These coalitions persist with probability $1 - \eta$ and dissolve with probability $\eta$ when implementation uncertainty is realized. A new coalition then forms in the next period. The following bound on
the shocks to the policy facilitates the exposition by simplifying the expressions for the continuation values.

**Assumption 3.** \( \varepsilon \leq \frac{1}{3} \).

Basic strategies are used in establishing the existence of a *specific-policy equilibrium* supporting \( Z(c) \).

**Proposition 4.** With implementation uncertainty given in Assumptions 1-3, there exists a \( c^+ \leq \frac{1}{2} \), such that for all \( \delta > \delta^o \equiv \frac{3-\frac{3}{2}n^o}{4-\gamma-3\eta-\frac{3}{2}(1-\eta)n^o} \), all \( c \in [c^+, \frac{1}{2}] \) and not too much uncertainty; i.e., \( (\gamma, \eta) \in R(\theta) \equiv \{(\gamma, \eta)|1 - \gamma - 3\eta(1 - \frac{n^o}{2}) > 0\} \), a coalition equilibrium exists supporting \( Z(c) \).

Proposition 4 identifies a class of specific-policy equilibria that are indexed by the allocation \( c \) to the coalition partner, where the originator of the coalition receives the larger allocation \( 1 - c \). By Proposition 3 the only \( c \) supportable in the absence of uncertainty, i.e., when \( \gamma = \eta = 0 \), is \( c = \frac{1}{2} \), but with implementation uncertainty strictly unbalanced allocations as illustrated in Figure 3 are supported.

![Figure 3: The supported sets \( Z(c) \), \( c \in [c^+, \frac{1}{2}] \)](image)

Proposition 4 is proven in four steps. First, the continuation values corresponding to basic strategies are derived. Second, bounds on \( c \) are identified such that legislators have no incentive to deviate from basic strategies. Third, we derive \( \delta^o \) such that for all \( \delta > \delta^o \) there is a non-empty set of \( c \) satisfying these bounds. Finally, we establish restrictions on the implementation uncertainty such that \( \delta^o \) is strictly less than one, completing the proof. These results are stated and proven in Appendix B, and the intuition is developed in this section.

The originator of the coalition receives an allocation of \( 1 - c \geq \frac{1}{2} \) every period in which the coalition continues, and the coalition partner receives \( c > \frac{1}{3} \). The continuation value of the originator is thus greater than that of the partner, and a higher probability \( \eta \) of implementation uncertainty...
decreases the continuation value to the coalition members and increases the continuation value to
the legislator not in the coalition, so greater uncertainty means that sustaining a coalition is less
likely. The continuation values are increasing in δ and independent of γ. For a status quo not in the
supported set \( Z(c) \) the continuation value is \( \hat{v} = \frac{1}{3(1-\delta)} \) because of the random selection rule and the
proposer’s randomization among potential partners. It is the difference between the dynamic payoffs
when in the coalition persists and \( \hat{v} \) when the coalition dissolves that provides the incentive for the
partner to accept the lower payoff.

When \( c \) is not too small, the coalition partner has an incentive both to accept the coalition proposal
and maintain it once the dynamic coalition has formed. We demonstrate that all supportable \( c \) are
strictly greater than \( \frac{1}{3} \).

The allocation \( c \) must be such that the coalition partner receiving \( c \) in the status quo has no
incentive to propose (or accept) an allocation in which he receives \( 1-c \), and this requires that \( \eta < \gamma \)
for \( c < \frac{1}{2} \). Greater uncertainty from changing the policy allows the allocations to the dynamic coalition
members to differ.

The allocation \( c \) also must be such that the coalition partner being offered \( c \) accepts the coalition
proposal for status quos not in \( Z(c) \). Remaining at a status quo outside \( Z(c) \) has a continuation
value of \( \hat{v} \), and for \( c \) sufficiently high the coalition partner is willing to accept the greater uncertainty
associated with a a new policy.

We show that for a sufficiently high discount factor an equilibrium with \( c \leq \frac{1}{2} \) exists and that
the lower bound on the discount factor is strictly less than one. This requires that implementation
uncertainty is not too great. For example, consider a status quo policy \((0, 1, 0)\) for which legislator 1
makes the proposal \((1-c, c, 0)\). The status quo is very attractive for legislator 2, and with \( \gamma \) high it is
very likely that the coalition allocation \( c \) is never realized. Furthermore, with \( \eta \) sufficiently high once
the coalition has formed it dissolves with high probability so accepting the proposal is not attractive.
So for \( \gamma \) and \( \eta \) sufficiently high legislator 2 could reject the proposal regardless of the discount factor.
We establish that \( \delta^o < 1 \) if \((\gamma, \eta) \in R(\theta)\), which completes the proof of Proposition 4.

**Corollary 8.** The set of policies supported by specific-policy coalition equilibria is strictly increasing
in \( \delta \) for \( \delta \in (\delta^o, 1) \).

The set of policies supported by a specific-policy equilibrium is strictly decreasing in the discount
factor, since the coalition partner has a stronger incentive to propose \( 1-c \) for itself the more important
is the future. In a specific-policy coalition equilibrium with \( c < \frac{1}{2} \) the originator of the coalition does
not share the gain from proposal power equally with the coalition partner. The lower bound \( c^+ \) is
greater than \( \frac{1}{3} \), however, so the coalition partner receives more in each period than in sequential
legislative bargaining because of its opportunity to reverse roles and propose the coalition originator’s policy.

If the probability of implementation uncertainty for both the current and a new policy are equal ($\gamma = \eta$) and $\eta$ is not too large, equal division within the coalition is the only policy supported by a specific-policy coalition equilibrium.

**Corollary 9.** For $\gamma = \eta$, the set of allocations $c$ that can be supported as a coalition equilibrium is the singleton $\{\frac{1}{2}\}$ for $\eta < \frac{1}{5^2} [4 - 2(4 - \frac{3}{2} \theta)^{\frac{1}{2}}].$

The equal division property in Proposition 3 is thus robust to implementation uncertainty provided that the probability that uncertainty is realized is the same when a new policy is adopted as when the policy remains the same and that probability is not too high.

### 8 Tolerant Coalitions

Specific-policy dynamic coalitions dissolve if implementation uncertainty is realized, since the shock moves the implemented policy away from the coalition policy. A dynamic coalition could, however, tolerate some change in policy due to implementation uncertainty. This section identifies tolerant coalition equilibria in which coalition members accept a degree of variation in the coalition policy, i.e., the coalition persists if the policy remains in a tolerated set of policies and dissolves if it is outside the set. The coalition thus withstands small but not large shocks.

We show that basic strategies support policies in a set $\bigcup_{c=\xi}^{1/2} Z(c)$ for some $\xi \leq \frac{1}{2}$. Letting $\zeta \equiv [\xi, 1 - \xi]$, the coalition persists when the realization $\theta^t$ of the implementation uncertainty satisfies $c - \theta^t \in \zeta$. In this case $z_{\text{max}} = 1 - \xi$, so if a status quo policy is not in $\bigcup_{c=\xi}^{1/2} Z(c)$, with basic strategies the proposer randomizes over policies that give him $1 - \xi$. If the status quo is in $\bigcup_{c=\xi}^{1/2} Z(c)$ for any $c \in \zeta$, the status quo is proposed. The following assumption assures that the implementation uncertainty is sufficiently great that the coalition can dissolve for all $c \in \zeta$ from either a very high or a very low realization of $\theta^t$. This simplifies the expressions for the continuation values and facilitates the comparison between tolerant coalition and specific-policy coalition equilibria.

**Assumption 4.** $1 - 2\xi \leq \theta \leq \xi \leq \eta.$

The bound $\xi$ of the set $\zeta$ is obtained from the incentive constraints for legislators to propose and accept policies corresponding to $\zeta$. The incentive constraints identify a set of bounds $\xi \in [\xi^+, \frac{1}{2}]$ on the tolerated allocations, where $\xi^+$ is the analogue of $c^+$ in Proposition 4 for a specific-policy equilibrium. Analogues of the bounds obtained from the incentive constraints for specific-policy equilibria are obtained, but additional incentive constraints are present. Appendix C provides the characterization, and this section develops the intuition underlying the equilibrium.
The bounds are derived from incentive constraints corresponding to deviations at the boundaries of $\zeta$ and $X$, and an additional deviation must be considered with tolerant equilibria. The status quo could be a policy $(1 - \alpha, \alpha, 0)$ where $\alpha \in (c, 1)$, which has the property that the current period allocation $1 - \alpha + \theta^t$ is less likely to be truncated at 1 than $1 - c + \theta^t$ and also has a higher probability that the allocation will be in $\zeta$ than a status quo $(1, 0, 0)$. A characterization of the $\alpha$ that maximizes the dynamic payoff and of the corresponding bound is provided in Appendix C. In addition, starting from a status quo not in $\bigcup_{c=x}^{\frac{1}{2}} Z(c)$, the proposing legislator may propose $(1 - \alpha, \alpha, 0)$ rather than the equilibrium proposal in $\bigcup_{c=x}^{\frac{1}{2}} Z(c)$. To ensure no such deviation is attractive requires an upper bound is characterized in Appendix C.

The incentive constraints for an equilibrium are satisfied if the upper bound is at least one-half and where the lower bound $c^+$, which is the maximum of the four lower bounds obtained from the incentive constraints, is less than one-half. Three of these bounds are less than one-half for $\delta > \delta^o$ given in Proposition 4, and the fourth is shown in Appendix C to less than one-half. The lower bound $c^+$ can be greater than the lower bound for a specific policy equilibrium because with a tolerant coalition equilibrium a coalition member receiving $c$ in the status quo has a stronger temptation to propose $1 - c$ because the probability of preserving the coalition is higher than in the corresponding specific-policy proposal.

An equilibrium supporting $\bigcup_{c=x}^{\frac{1}{2}} Z(c), c \in [c^+, \frac{1}{2}]$, is referred to as a tolerant coalition equilibrium. Coalition members in period $t + 1$ propose the status quo when the allocation in period $t$ is in $\zeta$. If the realized implementation uncertainty $\theta^t$ is such that $c - \theta^t$ is not in $\zeta$, the coalition dissolves and the legislator $i$ selected in the next period proposes a policy in $\bigcup_{c=x}^{\frac{1}{2}} Z(c)$ that yields $1 - c$ to $i$. Tolerant coalitions thus form immediately with the composition of the coalition determined by the selection of a proposer and the random selection of a coalition partner. When a tolerant coalition forms, the originator of the coalition receives a strictly greater allocation than the coalition partner, but following a tolerated realization of the implementation uncertainty, the allocation to the other coalition member and the corresponding continuation value can be larger. When a tolerant coalition forms with $c = c^+$, the probability that it dissolves due to implementation uncertainty is $\frac{1}{2} \eta$, since the distribution of $\tilde{\theta}^t$ is symmetric about 0. If the coalition persists beyond one period, the probability that it dissolves in the next period is smaller, since $c \geq c^+$.

As with specific-policy equilibria, restrictions are required so that the implementation uncertainty is not so great that the set of tolerated allocations is empty. Identifying the set $R^T(\xi; \bar{\theta})$ of allowable implementation uncertainty and a bound on the discount factor is complex and presented in Appendix C. Section 8.1 establishes existence of tolerant coalition equilibria for $\gamma > \eta = 0$, and by continuity tolerant coalition equilibria exist for at least small $\eta$. 

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The continuation values corresponding to a tolerant coalition equilibrium are given in Appendix C and the continuation values for a specific-policy coalition corresponding to \( c^+ \) can be compared to the continuation values for a tolerant coalition with \( c = c^+ \).

**Proposition 5.** Consider a \( c = c^+ < \frac{1}{2} \) such that both a specific-policy coalition equilibrium and a tolerant coalition equilibrium exist. If \( \eta > (\leq) 0 \), the continuation values for specific-policy coalition members are strictly less than (equal to) the continuation values for tolerant coalition members.

With \( \eta = 0 \) in a specific-policy coalition equilibrium, once formed the coalition continues with probability one, as does a tolerant coalition. The continuation values on the equilibrium path thus are the same in the two equilibria. For \( \eta > 0 \), implementation uncertainty can be realized on the equilibrium path in which case the specific-policy coalition dissolves, whereas the probability is positive that the allocation remains in \( \zeta \) and the tolerant coalition continues. A tolerant coalition thus is more valuable to its members than is the corresponding specific-policy coalition when \( \eta > 0 \).

For \( \gamma = \eta \) and \( \xi = \theta \) the unique tolerant coalition policy has equal allocations for the coalition members, so the coalition is no more tolerant than the specific-policy coalition in Proposition 4. This is formalized in the following corollary.

**Corollary 10.** For \( \gamma = \eta \) and \( \xi = \theta \), coalition members receive equal allocations, and a tolerant coalition is no more tolerant than the specific-policy coalition.

### 8.1 Implementation Uncertainty Only When Policy Changes

To provide a further characterization of tolerant coalition equilibria, consider the case in which there is implementation uncertainty (\( \gamma > 0 \)) associated with a change in policy but no implementation uncertainty (\( \eta = 0 \)) when the policy is not changed. This allows a complete characterization of the set of policies supported by a tolerant coalition equilibrium and the comparative statics properties on the bound on that set. When \( \eta = 0 \) once the coalition policy is on the equilibrium path it remains there, so a tolerant coalition is no more valuable than a specific-policy coalition, yet the equilibria are not the same. As shown in Proposition 8, the set of policies supported by a tolerant coalition equilibrium can be strictly smaller than the set of policies in Proposition 4 supported by a specific-policy equilibria. The proofs of Propositions 7-9 are presented in Appendix C.

The following proposition states that for sufficiently high discount factors a tolerant coalition equilibrium exists with the equilibrium policy of the most tolerant coalition identified by partner’s incentive to propose or accept the larger coalition allocation \( 1 - c \).

**Proposition 6.** With implementation uncertainty given in Assumptions 1 and 4 and for \( \eta = 0 \), there exists a \( \delta^c \) satisfying \( \delta^c = \frac{3}{1 - \gamma} < \delta^c \) < 1 such that for all \( \delta > \delta^c \), a coalition equilibrium exists supporting \( \bigcup_{\xi=\xi} Z(c) \) for all \( \xi \in [c^{**}, \frac{1}{2}] \).
The policy proposed by the originator of the coalition is of the form \((1 - c^*, c^*, 0)\) for \(\delta > \delta^\zeta\). The following proposition characterizes the policy.

**Proposition 7.** For \(\delta > \delta^\zeta\) and \(\gamma > \eta = 0\), the allocation \(c^*\) to the coalition partner is (i) strictly less than \(\frac{1}{2}\), (ii) strictly greater than \(\frac{1}{3}\), (iii) strictly decreasing in \(\gamma\), and (iv) strictly decreasing in \(\delta\).

The originator of a tolerant coalition thus receives a strictly larger share in the first period of the coalition than does the coalition partner, but the originator’s share is less than in sequential legislative bargaining theory. The proposer shares the gain from proposal power with the coalition partner, but the implementation uncertainty allows the originator to take the larger share. The allocation to the originator is greater the more important is the future and hence the more valuable is the dynamic coalition to the coalition partner. As with specific-policy coalition equilibria, the set of policies supported by the most tolerant coalition with \(c^*\) is increasing in the discount factor. Similarly, the coalition is more valuable the greater is the probability \(\gamma\) of implementation uncertainty when the policy is changed.

Specific-policy equilibria, however, can support a larger divergence between the allocation to the coalition originator and the partner than in the most tolerant coalition equilibrium.

**Proposition 8.** For \(1 > \gamma > \eta = 0\) and \(\delta > \delta^\zeta\), the set \([c^*, \frac{1}{2}]\) of coalition partner allocations supported by the class of specific-policy coalition equilibria strictly contains the set \([c^*, \frac{1}{2}]\) of coalition partner allocations supported by a tolerant coalition equilibrium.

The intuition underlying Proposition 8 is as follows. When \(\eta = 0\) the value of a specific-policy coalition corresponding to \(c\) equals the value of a tolerant coalition corresponding to \(c\), since once on the equilibrium path the policy does not change provided no coalition member deviates from the equilibrium strategies.\(^{20}\) A deviation from the tolerant coalition equilibrium strategies, however, is not as costly to a coalition member as is a deviation from the specific-policy equilibrium strategies because the former deviation could still result in an allocation in the set \(\zeta\) as a result of the realization of \(\tilde{e}_t\), whereas with a specific-policy coalition the coalition dissolves with probability one whenever the policy is shocked. The incentive constraint is thus tighter for a tolerant coalition equilibrium than for a specific-policy coalition equilibrium, and \(c^* > c^*\).

When \(\eta > 0\), the probability that a tolerant coalition persists is higher than the probability that a specific-policy coalition persists, since the shocked allocation can remain in the set \(\zeta\). The higher probability means that the continuation value for a tolerant coalition is higher than that in Proposition 5. This effect is in the opposite direction of the effect characterized in Proposition 8, and the bound \(c^*\) can be lower than \(c^*\) for a specific-policy coalition if \(\frac{\eta(1 - \delta(1 - \gamma))}{\gamma(1 - \delta(1 - \eta))} > \frac{\theta}{2}\), which requires \(\theta < \frac{\varepsilon}{2}\) when \(\eta = 0\) the continuation values are the same for specific-policy and tolerant coalition equilibria.
A tolerant coalition equilibrium thus could have a greater difference between the allocations of the coalition members than in a specific-policy coalition.

9 Evidence from the Battaglini-Palfrey Experiments

Battaglini and Palfrey conducted two experiments related to the basic model with policies restricted to be efficient. In the first, referred to as the no-Condorcet winner (NCW) experiment, the policy space consisted of four policies, and the second, referred to as the continuous experiment, was an approximation to the efficient boundary of $X$.

The NCW experiment consisted of 20 matches with 70 randomly assigned committees of 3 players each. Each match continued to the next round (period) with probability 0.75, so $\delta = \frac{3}{4}$, and matches lasted between 1 and 10 rounds for a total of 291 rounds. In every round each player chose a provisional proposal, and one proposal was selected randomly from those proposals. In each round each committee had 60 units of experiment currency to allocate by majority rule, and the available allocations were $S = \{(30, 30, 0), (20, 20, 20)\}$ and its permutations. A unique coalition equilibrium exists for all $\delta \in [0, 1)$ for $Z = \mathcal{Z}((30, 30, 0))$. The universal policy $(20, 20, 20)$ can be supported as a subgame perfect equilibrium for $\delta > \frac{3}{4}$ with player-specific punishments implemented by the other two players. Three-player coalitions were present in 5 of the 70 committees, and all but one resulted from an initial status quo of $(20, 20, 20)$. That is, players maintain the status quo if the allocation is of the form $(30, 30, 0)$ and otherwise propose keeping 30 and allocating 30 randomly to one of the other players.

Dynamic coalitions are present in 74.2% of the rounds, however, and if the rounds with a universal initial status quo are excluded, 78.5% of the rounds have coalitions that persist from one round to the next. Table 4 in Battaglini and Palfrey gives the frequency with which a particular coalition persist from one round to the next. The frequencies for coalitions $\{1, 2\}$, $\{1, 3\}$, and $\{2, 3\}$ are 0.77, 0.79, and 0.75, respectively.

A stronger test for the presence of a dynamic coalition is for the players favored by the status quo to vote against any proposal that differs from the status quo. The votes have been examined for those players who were indifferent between a proposal and the status quo when the two policies differed. In those rounds player one voted for the status quo in 23 of 35 rounds, player two in 27 of 53 rounds, and player three in 19 of 30 rounds. In the aggregate, the players voted for the status quo with probability 0.585. For those rounds in which a coalition was present, the corresponding numbers are 20 of 27, 24 of 45, and 16 of 23, respectively, so in the aggregate the players in a coalition voted for the status quo with probability 0.632. These data provide a degree of support for the indifference rule of voting for the status quo when indifferent.

We show in Appendix A how these player-specific punishments can be constructed.
Table 1 presents data on the duration of dynamic coalitions. The 70 committees experienced 78
dynamic coalitions. Forty-seven coalitions lasted only 1 or two rounds, and 18 of those ended because
the match ended. A dynamic coalition cannot be present in the first round if the initial status quo
is (20,20,20), so the probability that a coalition could form in the first round is bounded above by
three-quarters. The longest lasting coalitions were one of 9 rounds and 3 of 8 rounds. For 2 of the
coalitions with 8 rounds the game ended after the 8th round. Twenty-two coalitions lasted throughout
the match. The longest-lived coalition was formed in the second round and continued for 9 rounds
until the match ended.

The continuous allocation space experiment was implemented with a discrete grid with 60 available
units allocable in increments of one, a common discount factor of $\delta = \frac{5}{6}$, and equal recognition
probabilities. The experiment consisted of 10 matches in which 12 participants were randomly assigned
to three-person committees. The prediction from Proposition 3 is that dynamic coalitions support
policies with equal allocations among the coalition members and a lower, possibly 0, allocation to the
third player.

Table 2 presents the experiment results for majoritarian, universal, and dictatorial policies, where a
universal policy is defined as each player receiving at least 15 units, a dictatorial policy has one player
receiving at least 50, and for the policies that are neither universal nor dictatorial are majoritarian with
$M_{ij}$ denoting a policy in which $i$ and $j$ receive at least as much as $k$. The numbers in parentheses
in Table 2 are the number of policies in each category. Participants played a universal policy with
frequency 0.370, a majoritarian policy with frequency 0.536, and a dictatorial policy with frequency
0.093.

The probability of transitioning to the same majoritarian policy set is 0.522 for $M_{12}$ and 0.582 for
$M_{13}$. The transition probability for $M_{23}$ is only 0.313, and the probability of transitioning from $M_{23}$
to the universal allocation set is 0.281. With the exception of $M_{23}$, the probabilities are substantially
higher than with random transitions. Since not all initial status quos were in supportable sets, some
first-period transitions should be random according to the theory. Table 3 thus reports the transition
probabilities for rounds after the first. Overall, the transition probabilities for majoritarian coalitions
are slightly higher than in Table 2 for $M_{12}$ and $M_{13}$, and slightly lower for $M_{23}$. In Tables 2 and 3 the
transition probabilities for universal policies are 0.848 and 0.875, respectively, indicating the presence
of a dynamic coalition supported by a subgame perfect equilibrium with player-specific punishments.
Overall, the transition probabilities provide modest support for the presence of dynamic coalitions.
Battaglini and Palfrey show that a logistic quantal response equilibrium in which players’ best response

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22 A programming error resulted in the loss of the initial status quo in 4 of the 10 matches, so the data discussed here
include only the 6 matches for which the initial status quo is known.
23 These definitions are slightly different from those used by Battaglini and Palfrey.
functions are subject to random shocks fits the experiment data when players are sufficiently risk averse.

Several experiments implementing the sequential legislative bargaining game introduced by Baron and Ferejohn find behavior that only weakly supports the theory. These experiments do not allow communication among participants. Agranov and Tergiman (2012) show that allowing participants in a legislative bargaining experiment to communicate results in behavior that more strongly supports the theory. The cheap talk communication allowed in the experiment takes place after a proposer has been selected, and any player can send a message to any subset of other players. Participants in the experiment used the communication opportunity to learn about the reservation values of other participants and to induce the proposer to include them in the majority.

As in the experiment by Agranov and Tergiman, communication among experiment participants could increase the frequency with which dynamic coalitions are formed, extend their duration, and select among multiple equilibria. The authors and Salvatore Nunnari are conducting such an experiment.

10 Conclusions

Public policymaking is a dynamic process in which the opportunity to set the agenda gives legislators temporary power that can be used to their advantage. Distributive policy in particular could be prone to opportunistic behavior, and shifting agenda-setters could lead to policy instability. Yet most policies exhibit a measure of stability. This paper shows that dynamic coalitions can form in one step beginning from any status quo and once formed support policies that are stable.

The originator of a dynamic coalition has agenda-setting or proposal power as in sequential bargaining theory, but in contrast to that theory the originator of a dynamic coalition shares proposal power more equally with the other members of the coalition. Sharing is required to satisfy dynamic incentive constraints resulting from the opportunity of coalition members to propose or accept alternative policies and to vote against the coalition policy when the status quo is favorable. In the basic model for a three member legislature the dynamic incentives require the originator to share proposal power equally with the coalition partner.

Uncertainty can be associated with the implementation of policies, and that uncertainty can be greater when the policy changes than when it remains unchanged. Specific-policy coalition equilibria exist in the presence of implementation uncertainty, and in the policies supported the originator of the coalition can receive more than the coalition partner. The coalition partner accepts the smaller allocation rather than break the coalition and face increased uncertainty when a new policy is chosen.

Coalitions in specific-policy equilibria dissolve when implementation uncertainty is realized, but a coalition could tolerate a degree of uncertainty and continue to the next period. A tolerant coalition continues when the implemented policy remains in a set of tolerated policies, but if the shock is large as in a crisis, the coalition dissolves. A tolerant coalition thus can persist over time, and policies have a degree of stability, provided that the realized implementation uncertainty is not too large.

Coalition equilibria have particularly simple strategies with legislators proposing the status quo if it is in the supported set of policies and otherwise proposing a new coalition with coalition partners selected randomly. When legislators are risk averse, coalition equilibria seem natural, since they provide perfect risk smoothing over time. These equilibria could arise in the laboratory with the strategies of legislators coordinated through straightforward communication between the coalition originator and potential coalition partners.

The theory of dynamic coalitions can be applied to understand government formation in a multiparty parliamentary democracy. The theory predicts that governments can be formed even though party leaders are politically impatient. The theory explains that surplus and minority governments in addition to minimal-winning governments can form, and a consensus government can form if there is a threat. If there is uncertainty in implementing promised benefits, governments can survive small shocks. However, large shocks, which can be thought of as crises, can lead to the dissolution of a government. The theory thus provides an explanation for why and when governments fail. The theory also provides an explanation for failed government formation attempts when there are interests vested in the initial status quo that prevent a new, potentially Pareto improving, government to form.

The theory of dynamic coalitions can be extended in a number of directions. In the pure distribution game considered here, the preferences of legislators are directly opposing yet stable coalitions can form. With a policy space in which preferences are partially aligned, dynamic coalitions should also be present although their characteristics could differ. In particular, the extent to which proposal power is shared would depend on the preference alignment, as would coalition size and the set of policies supported. For example, in every period the legislature could allocate a budget between a public good and a distributive policy with legislators having quasi-linear preferences. The model could be extended to incorporate a richer model of government spending in which investment, debt and tax policies are chosen. In this setting, the focus of the analysis could be the role of dynamic coalitions in limiting the inefficiency resulting from the inability of government to commit to long-term policies. Another extension would be to enrich the model of politics by incorporating features of political institutions such as bicameralism, and committees with continuing agenda control. This might include incorporating periodic elections with the legislative bargaining model representing policy-making during inter-election periods. Elections could dislodge coalitions and create incentives for particular coali-
tions to form. The theory of dynamic coalitions could provide a foundation for a theory of endogenous political parties that arise through legislative bargaining. A number of these extensions could be taken to the laboratory.

Appendix A

Dynamic Coalitions in the Basic Model

We first present a more general extended model and prove Proposition 1e, the analogue to Proposition 1 in the basic model. The basic model is a special case of the extended model, so the proof of Proposition 1 is immediate.

The extended model

The extended model is the same as the basic model, but we allow heterogenous discount factors \( \delta_i \) and heterogenous proposal probabilities \( p_i < 1 \) for legislator \( i \).

Continuation values in the extended model are accordingly

\[
v_i(q^{t-1}) = E_t[u(q^t) + \delta_i v_i(\sigma, \omega|q^t)].
\]

All other definitions including the definition of basic strategies in the extended model are analogous to those in the basic model.

Lemma 1e gives continuation values for the extended model.

Lemma 1e. In the extended model, if \((\sigma, \omega)\) is a coalition equilibrium supporting a symmetric set \( Z \):

(a) The continuation value for legislator \( i \) for \( q^{t-1} \in Z \) is

\[
v_i(q^{t-1}) = \frac{u(q^{t-1})}{1 - \delta_i}.
\]

(b) The continuation value for legislator \( i \) for \( q^{t-1} \not\in Z \) is

\[
v_i(q^{t-1}) = v_i^* \equiv \frac{p_i u(z_{\text{max}}) + (1 - p_i)\overline{\pi}}{1 - \delta_i}.
\]

where

\[
\overline{\pi} \equiv \frac{1}{|Z_j|} \sum_{z \in Z_j} u(z_i), \text{ for } j \neq i.
\]

Let \( p_{\text{max}} \equiv \max\{p_1, \ldots, p_n\} \), and \( \delta_{\text{min}} \equiv \min\{\delta_1, \ldots, \delta_n\} \).

Proposition 1e. In the extended model a symmetric set \( Z \) is supported by a coalition equilibrium if \( \delta_{\text{min}} > \tilde{\delta} \equiv \frac{u(1) - u(z_{\text{max}})}{u(1) - p_{\text{max}} u(z_{\text{max}}) - (1 - p_{\text{max}}) \overline{\pi}} \), and

(a) \( z_i = z_{\text{max}} \) for at least \( m \) legislators, for all \( z \in Z \).

(b) \( u(z_j) < u(z_{\text{max}}) \) for some \( j \) and some \( z \in Z \).
Proof. The proof proceeds by checking incentives to deviate from basic strategies, assuming (a) and (b) in Proposition 1e are true and \( \delta_{\text{min}} > \delta \).

First consider \( q^{t-1} \in Z \). Suppose that \( i \) is the proposer, and consider the votes of other legislators. The equilibrium proposal is the same as the status quo, so legislators vote for the status quo under the indifference rule.

Consider \( i \)'s incentives to make a deviation proposal. The best potential deviation inside \( Z \) gives \( i \) a dynamic payoff of \( \frac{u(z_{\text{max}})}{1-\delta_i} \), and with the status quo \( i \)'s dynamic payoff is \( \frac{u(q_i^{t-1})}{1-\delta_i} \). If \( u(q_i^{t-1}) < u(z_{\text{max}}) \), legislator \( i \) has an incentive to deviate to an allocation in \( Z \) unless a minimal-winning coalition will reject the proposal. A minimal-winning coalition will reject the proposal if (a) holds, because at least one member of the minimal-winning coalition can only be kept indifferent. Under the indifference rule that member rejects the deviation proposal.

The best potential deviation outside \( Z \) gives 1 to legislator \( i \) and nothing to the other legislators. If such a proposal passed, the continuation value for \( i \) would be \( v_i^* \). If legislator \( i \) has the allocation \( z_{\text{max}} \) under the status quo, legislator \( i \) has no incentive to deviate if

\[
\begin{align*}
\frac{u(1) + \delta_i v_i^*}{1-\delta_i} &\leq \frac{u(z_{\text{max}})}{1-\delta_i} \\
\iff \frac{u(1) + \delta_i p_i u(z_{\text{max}}) + (1-p_i)\overline{\pi}}{1-\delta_i} &\leq \frac{u(z_{\text{max}})}{1-\delta_i} \\
\iff \frac{u(1) - u(z_{\text{max}})}{u(1) - p_i u(z_{\text{max}}) - (1-p_i)\overline{\pi}} &\leq \delta_i,
\end{align*}
\]

(A.2)

The left hand side of (A.2) is increasing in \( p_i \), so for \( \delta_{\text{min}} > \delta = \frac{u(1) - u(z_{\text{max}})}{u(1) - p_{\text{max}} u(z_{\text{max}}) - (1-p_{\text{max}})\overline{\pi}} \) this holds for all \( i \). Furthermore, if legislator \( i \) does not receive \( z_{\text{max}} \) under the status quo, at least a minimal-winning coalition does, so for \( \delta_{\text{min}} > \delta \) a minimal-winning coalition rejects a policy outside of \( Z \). By (b) \( \overline{\pi} < u(z_{\text{max}}) \), and since \( p_{\text{max}} < 1 \), we have \( \delta < 1 \).

Second, consider \( q^{t-1} \notin Z \). Consider a legislator’s incentives to accept a proposal \( y^t \in Z \). A minimal-winning coalition obtains a dynamic payoff of \( \frac{u(z_{\text{max}})}{1-\delta_i} \) under the equilibrium strategies. The highest status quo payoff available gives a coalition parter an allocation of 1 and continuation payoff \( v_i^* \). Legislators will reject the proposal if indifferent, hence legislators must have a strict incentive to accept the proposal. This is true if (A.2) holds strictly, or if \( \delta_{\text{min}} > \delta \).

Consider a legislator’s incentives to propose a policy \( y^t \in Z \) that gives him an allocation \( z_{\text{max}} \). Suppose \( q_i^{t-1} = 1 \), then legislator \( i \) will prefer the equilibrium proposal to remaining at the status quo if \( \delta_i > \delta \). This is true for all \( i \) if \( \delta_{\text{min}} > \delta \). Legislators do not have an incentive to propose any other policy in \( Z \), since the dynamic payoff for any other allocation in \( Z \) is strictly less than the dynamic payoff from the equilibrium proposal \( \frac{u(z_{\text{max}})}{1-\delta_i} \).

Consider a proposal \( y^t \notin Z \). Any proposer \( i \) receives at most 1 when \( q_i^t = 1 \) and as above for \( \delta_i > \delta \) prefers a proposal in \( Z_i \) that yields the dynamic payoff \( \frac{u(z_{\text{max}})}{1-\delta_i} \). A majority prefers the proposal
to the status quo, so \( y^t \) is approved.

**Proof of Proposition 1**

The proof of Proposition 1 follows from Proposition 1e. It is straightforward to verify that for \( p_i = \frac{1}{n} \) and \( \delta_i = \delta \) for all \( i \) the lower bound on the discount factor simplifies to \( \delta \equiv \frac{u(1) - u(z_{\text{max}})}{u(1) - u(z_{\text{min}})} \).

**Proof of Proposition 2**

**Sufficiency:** The proof of sufficiency proceeds by checking incentives to deviate from basic strategies assuming \((a)\) and \((b)\) are true and \( \delta > \delta^* \).

First consider \( q^{t-1} \in Z \). If \( i \) is the proposer, consider the votes of other legislators. With a proposal \( y^t = q^{t-1} \), legislators are indifferent and vote for \( q^{t-1} \) under the indifference rule. Consider legislator \( i \)'s incentives to make a deviation proposal. The best potential deviation inside \( Z \) gives legislator \( i \) a dynamic payoff of \( u(z_{\text{max}}) \), and with the status quo legislator \( i \)'s dynamic payoff is \( \frac{u(q^{t-1})}{1-\delta} \). If \( u(q^{t-1}) < u(z_{\text{max}}) \), legislator \( i \) has an incentive to deviate to an allocation in \( Z \) unless a minimal-winning coalition will reject the proposal. If \( W(z) = \emptyset \) for all \( z \in Z \), there is no deviation in \( Z \) that can defeat \( q^{t-1} \). Legislator \( i \) therefore has no incentive to propose this deviation.

To deviate to a policy \( z' \notin Z \), legislator \( i \) must find an \( m \)-member coalition to accept \( z' \). A deviation proposal must include the legislator with the \( m \)-th largest allocation for any \( z \in Z \) and the smallest of these is \( z_{\text{min}}^m \). This deviation is rejected by the \( m \)-member coalition if

\[
\delta(z') + \delta v^* \leq \frac{u(z_{\text{min}}^m)}{1-\delta}
\]

\[
\Leftrightarrow (1-\delta)u(z') + \delta \pi \leq u(z_{\text{min}}^m)
\]

\[
\Leftrightarrow \delta \geq \frac{\delta(z')}{\delta(z')} = \frac{u(z') - u(z_{\text{min}}^m)}{u(z') - u(z_{\text{min}}^m)}.
\]

Since \( \delta(z') \) is increasing in \( z' \), \( \delta \geq \delta(1) = \frac{u(1) - u(z_{\text{min}}^m)}{u(1) - u(z_{\text{min}}^m)} \) is sufficient to ensure no such deviation is possible.

Second, consider \( q^{t-1} \notin Z \). Repeating the argument above, some majority prefers \( z \in Z \) to any \( z' \notin Z \) if \( \delta > \delta(1) \), so the equilibrium proposal is accepted. The proposer also prefers any \( z \in Z_i \) to any \( z' \notin Z \). Legislator \( i \) does not have an incentive to propose any policy in \( Z \setminus Z_i \), since the dynamic payoff is strictly less than the dynamic payoff \( \frac{u(z_{\text{max}})}{1-\delta} \) from the equilibrium proposal.

**Necessity:** We show necessity by way of contradiction. Suppose \( Z \) can be supported by a coalition equilibrium, and suppose \( W(z') = \emptyset \) for some \( z' \in Z \). Then there exists a \( z'' \in Z \) such that \( m \) legislators strictly prefer \( z'' \) to \( z' \). Consequently, a majority prefers to vote for a deviation to \( z'' \), so \( z' \) cannot be supported as a coalition equilibrium. Condition \((a)\) thus is necessary. Next, suppose that \( Z \) is supported by a coalition equilibrium and Condition \((b)\) is not satisfied. Suppose that there
is a \( z' \in Z \) such that \( v^* \geq v_i(z') \) for some \( i \) in each \( M \in M \). Then a minimal-winning coalition will not accept the proposal \( z' \in Z \), since \( u(q_{i}^{t-1}) + \delta v^* < v_i(z') \) does not hold for a minimal-winning coalition. Hence, \( v_i(z') > v^* \) for \( i \) in \( M \) is necessary, and \( v_i(z') > v^* \Leftrightarrow u(z_{m}^{\text{min}}) > \bar{u} \). Finally we show that a lower bound on the discount factor, \( \delta \leq \frac{u(1) - u(z_{m}^{\text{min}})}{u(1) - \bar{u}} \) is necessary. Consider a status quo policy \( z \in Z \) such that every legislator has strictly positive utility. Order the elements of \( z \) such that \( z_1 \geq \ldots \geq z_n > 0 \) and suppose \( z_m = z_{m}^{\text{min}} \). Suppose legislator \( n \) proposes a deviation policy \( z' \not\in Z \). The most profitable deviation gives legislators \( i = m, \ldots, n-1 \) some allocation \( z'_i \) where \( u(z'_i) > u(z_i) \), and legislator \( n \) keeps the allocation \( z'_n = 1 - \sum_{i=m}^{n-1} z'_i \) for itself. Suppose \( u(z'_n) > u(z_n) \), then for \( \delta \) close to zero, such a proposal is voted for, but if \( \delta \geq \frac{u(z'_i) - u(z_{m}^{\text{min}})}{u(z'_i) - u(z_{m}^{\text{min}})} \) legislator \( m \) will reject the proposal. By a similar argument if the status quo is \( z' \not\in Z \) legislator \( m \) must have a strict incentive to accept the proposal \( z \in Z \), hence it is necessary for \( \delta > \frac{u(z'_i)}{u(z'_i) - u(z_{m}^{\text{min}})} \), where \( z' \not\in Z \) gives the \( m \)-member coalition strictly higher utility than \( z \in Z \). The most binding constraint \( \delta \) is thus determined by the elements of \( Z \). Since \( \frac{u(z'_i)}{u(z'_i) - u(z_{m}^{\text{min}})} \) is increasing in \( z'_i \) we have \( \delta \leq \frac{u(1) - u(z_{m}^{\text{min}})}{u(1) - \bar{u}} \). ■

**Dynamic Coalitions in the Extended Model**

**Proposition 2e.** In the extended model, for each \( i \), there exists a \( \delta_i \leq \frac{u(1) - u(z_{m}^{\text{min}})}{u(1) - p_i u(z_{\text{max}}) -(1-p_i)\bar{u}} \) such that a coalition equilibrium supporting the set \( Z \) exists if and only if \( \delta_i > \delta_i \) for all \( i \) and:

(a) No policy in \( Z \) can be defeated by another policy in \( Z \); i.e., \( W(z) = \emptyset \) for all \( z \in Z \).

(b) At least a majority \( M \) of legislators is punished if the coalition dissolves; i.e., \( v_i(z) > v_i^* \) for all \( i \in M \), for all \( z \in Z \), or equivalently \( u(z_{m}^{\text{min}}) > p_i u(z_{\text{max}}) + (1 - p_i)\bar{u} \) for all \( i \).

The proof is analogous to the proof of Proposition 2.

**Proof of Lemma 2**

By Definition 1 a legislator is a member of a dynamic coalition if it strictly prefers to maintain the status quo than support a policy that leads to a random coalition formation opportunity. With basic strategies a random coalition is formed if a policy not in \( Z \) is selected. Thus legislator \( i \) is a member of the dynamic coalition for some status quo \( q^{t-1} \in Z \) if and only if

\[
\frac{u(q_{i}^{t-1})}{1 - \delta} > u(y_i) + \delta v^*.
\]

This is true for all policies \( y \not\in Z \) and is true for some \( \delta \) if and only if \( u(q_{i}^{t-1}) > \bar{u} \).

**Proof of Proposition 3**

Proposition 1 shows part (a) and (b) are sufficient for an equilibrium to exist. We show below that they are necessary. We begin with the following general result.
Lemma 3. If \( u(\cdot) \) is strictly increasing, a set \( Z \) satisfies Condition (a) of Proposition 2, i.e., \( W(z) = 0 \), only if there are \( m \) or fewer distinct allocations in \( z \), for all \( z \in Z \).

Proof. By way of contradiction suppose \( Z \) is supportable as a coalition equilibrium, and there are more than \( m \) distinct payoffs in \( z \) for some \( z \in Z \). Consider the status quo \( q^{t-1} = z \) giving dynamic payoff \( u(z_i) \) to legislator \( i \). Since there are more than \( m \) distinct allocations in \( z \) and \( u(\cdot) \) is strictly increasing, there exists a proposal \( q' \in Z \) that is a permutation of \( z \) such that \( u(q'_i) > u(z_i) \) for at least \( m \) legislators. That is, consider that \( z \) is ordered such that \( z_1 \geq \ldots \geq z_n \). Next consider an alternate policy \( z' \) such that \( z'_i = z_{i+1} \) for \( i \neq 1 \), and \( z_1 = z_n \). Since \( z' \) is a permutation of \( z \) and \( Z \) is symmetric, \( z' \in Z \) and gives dynamic payoff \( u(z'_i) \) to legislator \( i \). Since there are at least \( m+1 \) distinct payoffs, there are at least \( m \) legislators that receive a strictly higher dynamic payoff under \( z' \), hence a minimal-winning coalition will vote for the deviation \( z' \), contradicting that \( Z \) is supportable as a coalition equilibrium.

To illustrate why this is necessary, consider the example \( n = 5 \), \( m = 3 \), \( u(x_i) = x_i \) and \( z = (a, a, b, c, d) \in Z \) with \( a > b > c > d \). If \( q^{t-1} = z \), legislator 3 receiving \( b \) under the status quo can propose the policy \( z' = (d, a, a, b, c) \in Z \). Since all elements of \( Z \) are absorbing states, legislators 3, 4 and 5 are strictly better off and vote for \( z' \). Hence, \( Z \) cannot be supported by a coalition equilibrium.

Note that Lemma 3 means that many policies cannot be supported as coalition equilibria.

By Lemma 3 a necessary condition for Proposition 2 part (a) to be satisfied is that there are \( m = 2 \) or fewer distinct allocations in each \( z \in Z \), implying \( z_i = z_j \) for some \( i, j \), for each \( z \in Z \).

Next, for any two policies \( z, z' \in Z \), if \( z_i = z_j \), and \( z'_j = z'_k \) for any \( i, j, k \), it must be that \( z_i = z_j = z'_j = z'_k \). Suppose not, then assume \( z_i = z_j > z'_j = z'_k \). Since both policies are in \( Z \), both are absorbing states, and if \( q^{t-1} = z' \) legislators \( j \) and \( k \) would vote to change the policy to a permutation of \( z \) in which they receive the allocations \( z_i \) and \( z_j \). This violates Condition (a) of Proposition 2.

The arguments above show that it is necessary for each policy in \( S \) to have two identical allocations which we will denote by \( c \). So \( S = \{(c, c, w), (y, c, c)\} \), for example. This also implies \( z_m = z_2 = c \) for all \( z \in Z \). We show (a) in Proposition 1 is necessary, that is, \( z_{\max} = c \) for all \( z \in Z \). Suppose \( |S| = 1 \) and \( z_{\max} > c \), then \( \pi > u(c) = u(z_m) \) which is a contradiction of Condition (b). Clearly, \( z_{\max} \neq c \), since that violates the definition of \( z_{\max} \). Next suppose \( |S| > 1 \). If the maximum allocation for any \( z \in Z \) is strictly larger than \( c \), then Proposition 2 Condition (b) is again violated by the previous argument. Suppose \( z_i = c \) for some \( z \in Z \) and \( z'_j > c \) for some \( z' \in Z \). Then \( z_{\max} \geq z'_j > c \). Furthermore, any element of \( Z \) with its maximum allocation \( z''_m = c \) is not in \( Z_j \) for any \( j \) (since \( Z_j \) gives \( j \) the highest possible allocation in \( Z \)). Then \( \pi \geq u(c) = u(z_m) \) which violates Proposition 2 Condition (b). So for each \( z \in Z \), we must have \( z_2 = c = z_{\max} \). If Proposition 1 Condition (b) fails,
then $z = (c, c, c) \forall z \in Z$, and this violates Proposition 2 Condition (b) that $\pi < u(c)$. ■

**Universal Efficient Allocations and Player-Specific Punishments**

The universal allocation can be obtained with strategies that are conditioned on information in addition to the status quo. For example, denote $\bar{x} = (\frac{1}{n}, \ldots, \frac{1}{n})$ as the universal allocation, and consider proposal strategies where a legislator proposes $\bar{x}$ for any status quo as long as no deviation has occurred and assume that the proposals $\bar{x}$ are approved under stage-undominated voting strategies. If legislator $i$ deviates and proposes $y^t \neq \bar{x}$, legislators $j \neq i$ punish $i$ by playing the coalition equilibrium in which they form a dynamic coalition and each takes $\frac{1}{n-1}$ leaving 0 for the deviator. If legislator $i$ becomes the proposer after the deviation, legislator $i$ proposes the status quo. Voting strategies are stage undominated with legislators voting for the proposal if indifferent between the proposal and the status quo and the equilibrium is not in the coalition equilibrium punishment phase. This punishment is player-specific rather than collective, and does not use Markov strategies as used in the equilibrium characterized in Proposition 2.\textsuperscript{25} Note that legislators $j \neq i$ strictly prefer to punish legislator $i$, because their dynamic payoff is $u(\frac{1}{n})$ rather than $u(\frac{1}{n+1})$ if they return to the equilibrium path.

The best deviation for $i$ gives 1 to $i$ and zero to all other legislators. The continuation value $v^*_i$ for $i$ then satisfies

$$v^*_i = \frac{1}{n} \left[ u(1) + \delta v^*_i \right] + \frac{n-1}{n} \left[ u(0) + \delta \frac{u(0)}{1-\delta} \right],$$

which yields

$$v^*_i = \frac{1}{n-\delta} \left[ u(1) + (n-1) \frac{u(0)}{1-\delta} \right].$$

Legislator $i$ has no incentive to deviate if

$$\frac{u(\frac{1}{n})}{1-\delta} \geq u(1) + \delta v^*_i \quad \Leftrightarrow \quad \delta \geq \delta^* = \frac{n(u(1) - u(\frac{1}{n}))}{nu(1) - (n-1)u(0) - u(\frac{1}{n}).}$$

Player-specific punishments using a coalition equilibrium in which the deviator receives 0 thus can be used to generate a larger class of equilibria than can be attained with Markov strategies. Whether player-specific strategies are used in political settings is unclear, but in the United States Congress there is little if any evidence that deviators are individually punished.\textsuperscript{26}

\textsuperscript{25}That is, Markov strategies are conditioned on the status quo, and if $q^{t-1} \notin Z$, it is known that some player has deviated from the equilibrium path, but which player deviated is not identified. A player-specific punishment is conditioned on more of the history of play than just the status quo policy. In the particular player-specific punishment discussed here, the status quo policy and the identity of the deviator are needed to specify the strategies.

\textsuperscript{26}The case in which Senator James Buckley of New York was supposedly punished for attempting to strip pork from an appropriations bill is often cited as an example of a player-specific punishment, but he was never punished. See Krehbiel (1991) (Ch. 2).
Battaglini-Palfrey experiment results

Table 1: Duration of Coalitions

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NB: 70 committees
Table 2: Transition Probabilities – Continuous Experiment (All Rounds)

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<th>Outcome</th>
<th>$q^t$</th>
<th>M12</th>
<th>M13</th>
<th>M23</th>
<th>U</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Status quo $q^{t-1}$</td>
<td>M12 (46)</td>
<td>0.522</td>
<td>0.130</td>
<td>0.217</td>
<td>0.087</td>
<td>0.043</td>
</tr>
<tr>
<td></td>
<td>M13 (55)</td>
<td>0.145</td>
<td>0.582</td>
<td>0.073</td>
<td>0.091</td>
<td>0.109</td>
</tr>
<tr>
<td></td>
<td>M23 (32)</td>
<td>0.063</td>
<td>0.250</td>
<td>0.313</td>
<td>0.281</td>
<td>0.094</td>
</tr>
<tr>
<td></td>
<td>U (92)</td>
<td>0.087</td>
<td>0.054</td>
<td>0.011</td>
<td>0.848</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>D (23)</td>
<td>0.217</td>
<td>0.130</td>
<td>0.087</td>
<td>0.043</td>
<td>0.522</td>
</tr>
</tbody>
</table>

Table 3: Transition Probabilities – Continuous Experiment (Rounds after First)

<table>
<thead>
<tr>
<th>Outcome</th>
<th>$q^t$</th>
<th>M12</th>
<th>M13</th>
<th>M23</th>
<th>U</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Status quo $q^{t-1}$</td>
<td>M12 (42)</td>
<td>0.524</td>
<td>0.119</td>
<td>0.214</td>
<td>0.095</td>
<td>0.048</td>
</tr>
<tr>
<td></td>
<td>M13 (47)</td>
<td>0.128</td>
<td>0.596</td>
<td>0.085</td>
<td>0.064</td>
<td>0.128</td>
</tr>
<tr>
<td></td>
<td>M23 (24)</td>
<td>0.083</td>
<td>0.333</td>
<td>0.292</td>
<td>0.208</td>
<td>0.083</td>
</tr>
<tr>
<td></td>
<td>U (88)</td>
<td>0.080</td>
<td>0.034</td>
<td>0.011</td>
<td>0.875</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>D (23)</td>
<td>0.217</td>
<td>0.130</td>
<td>0.087</td>
<td>0.043</td>
<td>0.522</td>
</tr>
</tbody>
</table>
Appendix B - For online publication

Implementation uncertainty

The maintained assumption on implementation uncertainty ensures that when uncertainty is realized the policy remains in the feasible set $X$. Assumption 1 gives details of the specification of implementation uncertainty.

**Assumption 1.** If a proposal $y^t = q^{t-1}$ is adopted, with probability $1 - \eta$ the policy implemented equals the proposal, and with probability $1 > \eta \geq 0$ the policy is distorted by a uniformly distributed shock $\theta^t$ with mean zero and support $[-\theta, \theta]$. (i) For $y^t \in Z(c)$ if the realization $\theta^t$ is such that $c - \theta^t \geq 0$, the legislators in the coalition receive $1 - c + \theta^t$ and $c - \theta^t$. If $c - \theta^t < 0$ for legislator $\ell$, $\ell$ receives $0$ and the other coalition member $\ell'$ receives $1$. (ii) For a proposal $y^t = q^{t-1} \notin Z(c)$, if a legislator $\ell$ receives $1$ in $y^t$ and $1 + \theta^t \geq 1$, $\ell$ receives $1$. If $1 + \theta^t < 1$, $\ell$ receives $1 + \theta^t$ and one other legislator selected at random receives $-\theta^t$. If only two legislators receive positive allocations in $y^t = (1 - x_\ell, x_\ell, 0)$, where $0 < x_\ell \leq \frac{1}{2}$, they receive $1 - x_\ell + \theta^t$ and $x_\ell - \theta^t$, respectively, if $x_\ell - \theta^t \geq 0$. If $x_\ell - \theta^t \leq 0$, $\ell$ receives $1$ and $\ell'$ receives $0$. If all three legislators receive positive allocations in $y^t$, the allocations with the shock are $x_\ell + \alpha_\ell \theta^t$, $\ell = i, j, k$, where $|\alpha_\ell| \leq 1$, $\ell = i, j, k$, and $\alpha_i + \alpha_j + \alpha_k = 0$. If $x_\ell + \alpha_\ell \theta^t \leq 0$ for some $\ell$, $\ell'$ receives $0$ and $-\alpha_\ell \theta^t$ is allocated randomly among the other legislators.

If a proposal $y^t \neq q^{t-1}$ is adopted, with probability $1 - \gamma$ the policy implemented equals the proposal, and with probability $1 > \gamma \geq 0$ the policy is distorted from the proposal by a uniformly distributed shock $\varepsilon^t$ with support $[-\varepsilon, \varepsilon]$. (iii) For $y^t \in Z(c)$ the allocations are as in (i) with the realization $\varepsilon^t$ replacing $\theta^t$. (iv) For $y^t \notin Z(c)$ the allocations are as in (ii) with the realization $\varepsilon^t$ replacing $\theta^t$.

Specific-policy equilibrium policies

For notational convenience define $z_{ij}(c) \in Z(c)$ as the policy that allocates $1 - c$ to legislator $i$, $c$ to legislator $j$, and $0$ to legislator $k$, so legislator $i$ receives the highest allocation and legislator $j$ is the coalition partner.

**Lemma 4.** With implementation uncertainty given in Assumptions 1-3, if $(\sigma, \omega)$ is a coalition equilibrium for a set $Z(c)$ with $c > \frac{1}{3}$:

(i) The continuation value for legislator $i$ for $q^{t-1} \in Z(c)$ is

$$v_i(q^{t-1}) = \frac{3(1 - \delta)q_i^{t-1} + \eta\delta}{3(1 - \delta)(1 - \delta(1 - \eta))}, \quad q_i^{t-1} \in \{1 - c, c, 0\}. \quad (B.1)$$

(ii) The continuation value for legislator $i$ for $q^{t-1} \notin Z(c)$ is

$$v_i(q^{t-1}) = \hat{v} = \frac{1}{3(1 - \delta)}.$$

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Proof. Let \( v_i(y^t), i = 1, 2, 3 \), denote the continuation value when the proposal is \( y^t \), conditional on no shock being realized. Let \( v_i(y^{t^t}) \) denote the continuation value when \( y^t \) is implemented conditional on the shock \( \varepsilon^t \) being realized, and \( v_i(y^{\theta^t}) \) denote the continuation value when \( y^t \) is implemented conditional on the shock \( \theta^t \) being realized.

The continuation value \( \hat{v}_1 \) when \( q^{t-1} \not\in Z(c) \) is given by

\[
\hat{v}_1 = (1 - \gamma) \left[ \frac{1}{3} \left( \frac{1}{2} (1 - c + \delta v_1(z_{12}(c))) \right) + \frac{1}{2} (1 - c + \delta v_1(z_{13}(c))) \right] + \frac{1}{3} \left( \frac{1}{2} (c + \delta v_1(z_{21}(c))) + \frac{1}{2} \delta v_1(z_{23}(c)) \right) + \frac{1}{3} \left( \frac{1}{2} (c + \delta v_1(z_{31}(c))) + \frac{1}{2} \delta v_1(z_{32}(c)) \right) + \gamma \left[ \frac{1}{3} \left( 1 - c + E^t \varepsilon^t + \delta v_1(z_{12}^t) \right) + \frac{1}{2} (1 - c + E^t \varepsilon^t + \delta v_1(z_{13}^t)) \right] + \frac{1}{3} \left( \frac{1}{2} (c - E^t \varepsilon^t + \delta v_1(z_{21}^t)) + \frac{1}{2} \delta v_1(z_{23}^t) \right) + \frac{1}{3} \left( \frac{1}{2} (c - E^t \varepsilon^t + \delta v_1(z_{31}^t)) + \frac{1}{2} \delta v_1(z_{32}^t) \right),
\]

and the continuation values \( \hat{v}_2 \) and \( \hat{v}_3 \) for the other legislators are analogous. By symmetry \( \hat{v}_i = \hat{v}, i = 1, 2, 3. \)

Given \( q^{t-1} \not\in Z(c) \), the policy \( z_{ij}^t \) resulting from a proposal \( y^t \neq q^{t-1} \) is not in \( Z(c) \) with probability one, so the continuation values \( v_i(z_{ij}^t) = \hat{v}, i = 1, 2, 3, i \neq j. \) For \( q^{t-1} = z_{ij}(c) \) the allocation \( z_{ij}^t \in Z(c) \) with probability zero, so the continuation value \( v_i(z_{ij}^t) = \hat{v}, i = 1, 2, 3, i \neq j. \) For \( q^{t-1} \in Z(c) \) the dynamic payoffs \( v_i(z_{12}(c)), i = 1, 2, 3, \) are given by

\[
v_1(z_{12}(c)) = (1 - \eta) [1 - c + \delta v_1(z_{12}(c))] + \eta [1 - c + E^t \hat{\theta}^t + \delta \hat{v}]
\]

\[
v_2(z_{12}(c)) = (1 - \eta) [c + \delta v_2(z_{12}(c))] + \eta [c + E^t \hat{\theta}^t + \delta \hat{v}]
\]

\[
v_3(z_{12}(c)) = (1 - \eta) \delta v_3(z_{12}(c)) + \eta \delta \hat{v}.
\]

Continuation values for the other policies in \( Z(c) \) are defined analogously. Solving (B.2)-(B.5) and the analogous conditions simultaneously yields the continuation values in Lemma 4. \( \blacksquare \)

Lemma 5. (i) No legislator has an incentive to deviate from the basic strategies if and only if\(^{27}\)

\[
\begin{align*}
c^* & \leq c \leq \frac{1}{2}, \text{ and} \\
\bar{c}^* & < c < 1 - c^*,
\end{align*}
\]

\(^{27}\)Lemma 5 indicates that Assumption 3 can be weakened to \( \varepsilon \leq \min\{c^*, \bar{c}^*\} \) for \( \delta > \delta^o. \)
where
\[
\begin{align*}
c^* &= \frac{3 - 2\delta(\gamma - \eta)}{3(2 - \delta(\gamma - \eta))}, \\
c^o &= \frac{3 - \delta(2 + \gamma - 3\eta) - \frac{1}{3}\eta\theta(1 - \delta(1 - \eta))}{3(1 - \delta(\gamma - \eta))},
\end{align*}
\]

(ii) \(c^* > \frac{1}{3}\).

Proof. The proof of part (i) proceeds by checking incentives to deviate from basic strategies given the continuation values in Lemma 4.

Suppose \(q^{t-1} \in Z(c)\), so \(q^{t-1} = z_{ij}(c)\) for some \(i\) and \(j\). Consider the incentives of legislators to vote for the equilibrium proposal. With basic strategies the proposal \(z_{ij}(c) \in Z\) is the same as the status quo, so legislators vote for the status quo.

Consider \(i\)'s incentive to propose a deviation. Proposing \(z_{ji}(c)\) or \(z_{ki}(c)\) with \(j\) and \(k\) voting for the proposal changes the status quo if approved, and since \(c \leq \frac{1}{2}\) and \(\eta \leq \gamma\), \(z_{ij}(c)\) is preferred by \(i\).

Formally, the incentive constraint is
\[
(1 - \eta)[1 - c + \delta v_i(z_{ij}(c))] + \eta[1 - c + E^t \tilde{\theta}^t + \delta \hat{v}] \geq (1 - \gamma)[c + \delta v_i(z_{ji}(c))] + \gamma[c - E^t \tilde{\varepsilon}^t + \delta \hat{v}],
\]
which can be verified using \(\eta \leq \gamma\).

Proposing \(z_{ik}(c)\) results in a change in the status quo if approved, so \(i\) prefers to propose \(z_{ij}(c)\), since by stochastic dominance
\[
(1 - \eta)[1 - c + \delta v_i(z_{ij}(c))] + \eta[1 - c + E^t \tilde{\theta}^t + \delta \hat{v}] \geq (1 - \gamma)[1 - c + \delta v_i(z_{ik}(c))] + \gamma[1 - c + E^t \tilde{\varepsilon}^t + \delta \hat{v}].
\]

Proposing \(z_{jk}(c)\) or \(z_{kj}(c)\) changes the status quo if approved, and \(i\) prefers to propose \(z_{ij}(c)\), since
\[
(1 - \eta)[1 - c + \delta v_i(z_{ij}(c))] + \eta[1 - c + E^t \tilde{\theta}^t + \delta \hat{v}] \geq (1 - \gamma)\delta v_i(z_{jk}(c)) + \gamma \delta \hat{v}.
\]

The best proposal deviation for \(i\) outside the set \(Z(c)\) gives 1 to \(i\) with \(i\) and \(k\) voting for the proposal. Proposer \(i\) prefers not to deviate if and only if
\[
(1 - \eta)[1 - c + \delta v_i(z_{ij}(c))] + \eta[1 - c + E^t \tilde{\theta}^t + \delta \hat{v}] \geq 1 - \gamma \tilde{\theta}^t + \delta \hat{v},
\]
where \(1 - \gamma \tilde{\theta}^t = 1 - \gamma \left[\int_{-\varepsilon}^{0}(1 + \varepsilon^t)\frac{d\varepsilon^t}{\varepsilon^t} + \int_{0}^{\varepsilon}(\varepsilon^t)\frac{d\varepsilon^t}{\varepsilon^t}\right]\) is \(i\)'s expected truncated payoff in period \(t\). Then legislator \(i\) does not deviate if
\[
c \leq c^u = z^2(1 - \delta(1 - \eta)) + \frac{2\delta(1 - \eta)}{3}.
\]

Consider \(j\)'s incentives to propose a deviation. Proposals \(z_{ki}(c)\) or \(z_{ik}(c)\) give \(j\) an allocation of 0, and \(j\) has no incentive to propose these over \(z_{ij}(c)\), since
\[
(1 - \eta)[c + \delta v_j(z_{ij}(c))] + \eta[c + E^t \tilde{\theta}^t + \delta \hat{v}] \geq (1 - \gamma)[\delta v_j(z_{ki}(c))] + \gamma[E^t \tilde{\varepsilon}^t + \delta \hat{v}].
\]
If \(j\) proposes \(z_{kj}(c)\), \(j\) and \(i\) vote against it as above.
If \( j \) proposes \( z_{ij}(c) \) or \( z_{jk}(c) \), \( j \) receives \( 1 - c \) in expectation in the current period, and with probability \( 1 - \gamma \) the continuation value is \( v_j(z_{ij}(c)) = v_j(z_{ij}(c)) \) and with probability \( \gamma \) the continuation value is \( \hat{v} \). Legislator \( j \) has no incentive to deviate if and only if
\[
(1 - \eta)[c + \delta v_j(z_{ij}(c))] + \eta[c - E^t \hat{\theta} + \delta \hat{v}] \geq (1 - \gamma)[1 - c + \delta v_j(z_{jk}(c))] + \gamma[1 - c + E^t \hat{\theta} + \delta \hat{v}]
\]
\[
\Leftrightarrow c \geq \frac{3 - 2\delta(1 - \eta)}{3(1 - \gamma)} = c^*. \quad (B.7)
\]
If \( j \) proposes \( y^t \notin Z(c) \), the best proposal gives 1 to \( j \) (\( j \) and \( k \) vote for it), and the expected truncated payoff is \( 1 - \gamma \frac{\hat{v}}{\eta} + \delta \hat{v} \). Legislator \( j \) prefers the equilibrium proposal if and only if
\[
(1 - \eta)[c + \delta v_j(z_{ij}(c))] + \eta[c - E^t \hat{\theta} + \delta \hat{v}] \geq 1 - \gamma \frac{\hat{v}}{\eta} + \delta \hat{v}
\]
\[
\Leftrightarrow c \geq c^* = \frac{3 - 2\delta(1 - \eta)}{3(1 - \gamma)}. \quad (B.8)
\]
Note that \( c^* + c^n = 1 \).

Consider \( k \)'s incentive to propose a deviation. As shown above if \( c \in [\max\{c^*, c^\ell\}, c^n] \), \( i \) and \( j \) prefer \( z_{ij}(c) \) to any other allocation. Hence, any proposal by \( k \) different from \( z_{ij}(c) \) will be rejected. Legislator \( k \)'s payoff is the same if he proposes \( z_{ij}(c) \) and it is accepted, or proposes another allocation that is rejected, hence legislator \( k \) has no incentive to deviate from the equilibrium strategies.

**Suppose** \( q^{t-1} \notin Z(c) \). Consider \( j \)'s incentive to vote for the equilibrium proposal \( z_{ij}(c) \). The best status quo for \( j \), gives 1 to \( j \). Legislator \( j \) along with \( i \) vote for \( z_{ij}(c) \) rather than the status quo if and only if
\[
1 - \eta \frac{\theta}{\eta} + \delta \frac{1}{(1 - \eta)} < (1 - \gamma)[c + \delta v_j(z_{ij}(c))] + \gamma[c - E^t \hat{\theta} + \delta \hat{v}],
\]
where
\[
1 - \eta \frac{\theta}{\eta} = 1 - \eta + \eta \left[ \int_{-\theta}^{0} (1 + \theta^t) \frac{d\omega}{2\pi} + \int_{0}^{\theta} \frac{d\omega}{2\pi} \right]
\]
is the expected truncated payoff in period \( t \). Legislator \( j \) accepts the proposal if
\[
c > \frac{3 - 2\delta(2 + \gamma - 3\eta) - \frac{1}{3(1 - \gamma)} \delta \theta(1 - \delta(1 - \eta))}{\eta \delta(1 - \eta)} = c^n.
\]
Since \( \eta \frac{\theta}{\eta} \leq \gamma \xi \), \( c^n \geq c^\ell \), so \( c^\ell \) is not binding.

Consider \( i \)'s incentive to make a proposal other than \( z_{ij}(c) \). Using the continuation values in (B.1), \( z_{ij}(c) \) gives \( i \) the highest payoff among proposals in \( Z(c) \), so there is no incentive to make any other proposal in \( Z(c) \).

If \( q^{t-1} \) gives 1 to \( i \), \( i \) strictly prefers \( z_{ij}(c) \) to the status quo if and only if
\[
1 - \eta \frac{\theta}{\eta} + \delta \frac{1}{(1 - \eta)} < (1 - \gamma)[1 - c + \delta v_i(z_{ij}(c))] + \gamma[1 - c + E^t \hat{\theta} + \delta \hat{v}]
\]
\[
\Leftrightarrow c < \hat{c}_1 = \frac{2\delta(1 - \eta) + \frac{3}{2}\eta \theta(1 - \delta(1 - \eta))}{3(1 - \delta(1 - \eta))}.
\]
Since \( \eta \frac{\theta}{\eta} \leq \gamma \xi \), \( c^n \geq \hat{c}_1 \), so \( c^n \) is not binding. Note that \( \hat{c}_1 = 1 - c^n \).

If \( i \) does not receive 1 in \( q^{t-1} \), \( i \) prefers a proposal \( z_{ij}(c) \) to a proposal that gives 1 to \( i \) if and only
If

\[ 1 - \gamma + \delta \frac{1}{4(1-\gamma)} \leq (1 - \gamma)(1 - c + \delta \nu_i(z_{ij}(c)) + \gamma[1 - c + E^t \xi + \delta \nu], \]

\[ \Leftrightarrow c \leq \hat{c}_2 \equiv \frac{2\delta(1-\gamma) + \frac{3}{2} \eta \nu(1 - \delta - \eta)}{\delta(1 - \delta - \eta)}. \]

Note that \( \hat{c}_1 \leq \hat{c}_2 \), since \( \eta \leq \gamma \), so \( \hat{c}_2 \) is not binding.

To prove part (iii) it is straightforward to show that \( c^* \) is decreasing in \( \gamma \) and increasing in \( \eta \).

Evaluating \( c^* \) at \( \gamma = 1 \) and \( \eta = 0 \) yields \( c^* = \frac{3 - 2\delta}{3(2 - \delta)} \), which implies that \( c^* > \frac{1}{3} \), for all \( \gamma \in [0, 1) \) and \( \eta \in [0, 1) \). Consequently, if \((1 - c, c, 0)\) is a coalition equilibrium proposal, \( c - \xi > 0 \), and hence \( c - \theta > 0 \). Also, \( 1 - c - \xi < 1 \), so \( 1 - c + \theta < 1 \). Then \( \xi \leq \frac{1}{3} \) is sufficient for the coalition allocations \( c \) and \( 1 - c \) to be in \([0, 1]\).

**Lemma 6.** For \( \delta > \delta^o \) there exists a \( c \) such that \( c^o < c < 1 - c^o \) and \( c^* \leq c \leq \frac{1}{2} \). That is, \( c^o < \frac{1}{2} \) and \( c^* \leq \frac{1}{2} \), when \( \delta > \delta^o \).

**Proof.** The lower bound \( c^* \leq \frac{1}{2} \) for all \( \gamma \geq \eta, \delta > 0 \). The lower (upper) bound \( c^o \) \((1 - c^o)\) is strictly less (greater) than \( \frac{1}{2} \) for \( \delta > \delta^o \) as

\[
\delta^o = \frac{3 - 2\eta \theta}{4 - \gamma - 3\eta \theta (1 - \eta)}. \]

**Lemma 7.** For \((\gamma, \eta) \in R(\theta) \equiv \{(\gamma, \eta)|1 - \gamma - 3\eta(1 - \frac{\eta \theta}{2}) > 0\}, \delta^o \in (0, 1).

**Proof.** It is straightforward to show that \( \delta^o < 1 \Leftrightarrow (\gamma, \eta) \in R(\theta) \).

**Corollary 11.** For \( \gamma \leq \frac{2}{3}, c^* = c^o \) for \( \delta \geq \delta^+ \) and \( c^* = c^o \) for \( \delta^o < \delta < \delta^+ \), where

\[
\delta^+ \equiv \frac{(4(1 - \eta) + \frac{2}{3} \eta \theta(2 + \gamma - 3\eta))}{2(\gamma - \eta)(2 - \gamma - \eta - \frac{3}{4} \eta \theta(1 - \eta))} \quad \text{(B.9)}
\]

\[
-\sqrt{(4(1 - \eta) + \frac{2}{3} \eta \theta(2 + \gamma - 3\eta))^2 - 4(3 - \frac{2}{3} \eta \theta)^2(\gamma - \eta)(2 - \gamma - \eta - \frac{3}{4} \eta \theta(1 - \eta))}}.
\]

**Proof.** The difference \( c^* - c^o \) is increasing in \( \delta \) for \( \gamma \leq \frac{2}{3} \). To show this, differentiation yields

\[
\frac{\partial(c^* - c^o)}{\partial \delta} = \frac{\gamma - \eta}{3(2 - \delta - (\gamma - \eta))^2} + \frac{(2 - \frac{2}{3} \eta \theta(1 - \delta - (\gamma - \eta))^2}{3(2 - \delta - (\gamma - \eta))^2}.
\]

If \( \gamma = \eta \), the first line of (B.10) is positive. If \( \gamma \neq \eta \), the second line is positive if \( 2 - 3\gamma - \eta(1 - \frac{2}{3} \theta(1 - \gamma)) > 0 \), which is the case for \( \gamma \leq \frac{2}{3} \). The greater lower bound is then \( c^* \) if and only if \( \delta \geq \delta^+ \), where \( \delta^+ \) in (B.9) is obtained by equating \( c^o \) and \( c^* \) in (B.6).

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Discussion: When the discount factor is high ($\delta \geq \delta^+$), the binding incentive constraint is for the coalition partner to stay on the equilibrium path; i.e., to accept the allocation $c$ and not propose a policy in $Z(c)$ that would yield $1-c$. When $\delta \in (\delta^o, \delta^+)$, the binding incentive constraint is for the potential coalition partner to accept the coalition originator’s proposal for any status quo. The binding incentive constraints are associated with the coalition member who receives the lower allocation.

The following lemma characterizes $c^+$ in terms of the probability $\gamma$ of implementation uncertainty with $c^+ = c^*$ for low $\gamma$ and $c^+ = c^o$ for higher $\gamma$.

**Lemma 8.** $c^+ = c^*$ for $\gamma \leq \gamma^c$ and $c^+ = c^o$ for $\gamma > \gamma^c$, where

$$\gamma^c \equiv 1 + \frac{1}{85} \left[ 3\eta \theta (1 - \delta (1 - \eta)) - [(8 - 3\eta \theta)(1 - \delta (1 - \eta)) (16 + (8 - 3\eta \theta)(1 - \delta (1 - \eta)))^{\frac{1}{2}} \right].$$

(B.11)

**Proof.** The bound $c^*$ is decreasing in $\gamma$, and $c^o$ is increasing in $\gamma$, so the difference $c^* - c^o$ is decreasing in $\gamma$. The greater lower bound is then $c^*$ if and only if $\gamma \geq \gamma^c$, where $\gamma^c$ in (B.11) is obtained by equating $c^o$ and $c^*$ in (B.6). ■

**Proof of Corollary 9**

Substituting $\gamma = \eta$ into $c^*$ given in (B.6) yields $c^* = \frac{1}{2}$. The condition $\eta < \frac{1}{32} \left[ 4 - 2 (4 - \frac{3}{2} \theta)^{\frac{1}{2}} \right]$ implies $(\eta, \eta) \in R(\theta)$, so $\delta^o < 1$ by Lemma 7 and hence $c^o < \frac{1}{2}$. ■
Appendix C - For online publication

Tolerant coalition equilibrium

The analogues $c^{**}$ of $c^*$ and $c^{oo}$ of $c^o$ in (B.6) are

\[
c^{**} = c^* - \frac{\delta \left( (\gamma - \eta) - (1 - \delta(1 - \eta)) \left( \gamma - \eta \right) \right) \nu(c^{**})}{2 - \delta(\gamma - \eta)},
\]

\[
c^{oo} = c^o - \frac{\delta \left( (1 - \gamma) \eta + \gamma(1 - \delta(1 - \eta)) \frac{\eta}{1 - \delta(\gamma - \eta)} \right) \nu(c^{oo})}{1 - \delta(\gamma - \eta)},
\]

where

\[
\nu(c) = \frac{1 - 2c}{6[2\delta(1 - \delta(1 - \eta)) - \delta(1 - 2c)]}.
\]

Lemma 9. With implementation uncertainty given in Assumptions 1, 2, and 4, if $(\sigma, \omega)$ is a coalition equilibrium for some set $\bigcup_{c \in \zeta} Z(c)$:

(i) The continuation value $v_i(q^{-1})$ for legislator $i$ for $q^{-1} \in \bigcup_{c \in \zeta} Z(c)$ is

\[
v_i(q^{-1}) = \frac{3(1 - \delta)q_i^{-1} + \eta \delta + 3(1 - \delta)\eta \delta \nu(c)}{3(1 - \delta)(1 - \delta(1 - \eta))}, \quad q_i^{-1} \in \{1 - c, c, 0\},
\]

(ii) the continuation value $v_i(q^{-1})$ for legislator $i$ for $q^{-1} \notin \bigcup_{c \in \zeta} Z(c)$ is

\[
v_i(q^{-1}) = \hat{v} \equiv \frac{1}{3(1 - \delta)}.
\]

(iii) $\nu(c) \geq 0$ and $\nu(c) > 0$ if $c < \frac{1}{2}$.

Proof. For $c \in \zeta$ the dynamic payoffs are given by

\[
\bar{v}_i(z_{ij}(c)) = (1 - \eta) [1 - c + \delta \bar{v}_i(z_{ij}(c))] + \eta [1 - c + E^t \bar{\theta}^t + \delta \bar{v}_i(z_{ij}^t)]
\]

\[
\bar{v}_j(z_{ij}(c)) = (1 - \eta) [c + \delta \bar{v}_j(z_{ij}(c))] + \eta [c - E^t \bar{\theta}^t + \delta \bar{v}_j(z_{ij}^t)]
\]

\[
\bar{v}_k(z_{ij}(c)) = (1 - \eta) \delta \bar{v}_k(z_{ij}(c)) + \eta \delta \bar{v}_k(z_{ij}^t),
\]

where

\[
\bar{v}_t(z_{ij}^t) = \int_{c-}\theta^{c-1} \bar{v}_t(z_{ij}(c - \theta^t)) \frac{1}{\theta^t} d\theta^t + \int_{c-}^{\epsilon} \bar{v}_t(z_{ij}(c - \theta^t)) \frac{1}{\theta^t} d\theta^t
\]

\[
+ \int_{\epsilon}^{\theta} \bar{v}_t(z_{ij}(c - \theta^t)) \frac{1}{\theta^t} d\theta^t.
\]

In the first and third integrals in (C.8), $c - \theta^t \notin \zeta$, so $\bar{v}_t(z_{ij}(c - \theta^t)) = \bar{v}_t(z_{ij}(c'))$ for some $c' \notin \zeta$, where $\bar{v}_t(z_{ij}(c'))$ is not a function of $c$ or $\theta^t$ according to the equilibrium strategies.

In the second integral in (C.8) $c - \theta^t \in \zeta$. Conjecture that for $c \in \zeta$, $\bar{v}_i(z_{ij}(c))$ is linear in $1 - c$, $\bar{v}_j(z_{ij}(c))$ is linear in $c$ and that these are given by $\bar{v}_i(z_{ij}(c)) = a_i + b_i(1 - c)$ and $\bar{v}_j(z_{ij}(c)) = a_j + b_j c$. 
Then \( \bar{v}_i(z_{ij}(c - \theta^t)) = a_i + b_i(1 - c + \theta^t) \), and \( \bar{v}_j(z_{ij}(c - \theta^t)) = a_j + b_j(c - \theta^t) \). Conjecture that \( \bar{v}_k(z_{ij}(c)) \) is constant in \( c \). Then for \( \ell = i, j \)

\[
\bar{v}_\ell(z_{ij}^\theta) = \frac{\bar{v}_\ell(z_{ij}(c))(2c - 1 + 2\delta)}{2\eta} + \frac{(1 - 2\delta)(2a + b\ell)}{4\eta} \tag{C.9}
\]

\[
\bar{v}_k(z_{ij}^\theta) = \frac{\bar{v}_k(z_{ij}(c))(2c - 1 + 2\delta)}{2\eta} + \frac{(1 - 2\delta)a_k}{2\eta} \tag{C.10}
\]

Substituting into (C.5)-(C.7) and matching coefficients gives

\[
a_i = a_j = \frac{\delta \eta}{2(1 - \sigma(1 - \eta))} \left[ \frac{\delta \eta}{2(1 - \sigma(1 - \eta))} - \delta \eta(1 - 2\eta) \right] + a_k
\]

\[
a_k = \frac{\delta \eta a_k}{2(1 - \sigma(1 - \eta))} \left[ \frac{\delta \eta}{2(1 - \sigma(1 - \eta))} - \delta \eta(1 - 2\eta) \right]
\]

\[
b_i = b_j = \frac{1}{1 - \delta(1 - \eta)}.
\]

Substituting the coefficients and simplifying (C.9)-(C.10) gives

\[
\bar{v}_\ell(z_{ij}^\theta) = \frac{(1 - 2\delta)(1 - 2\bar{v}_\ell(z_{ij}(c'))(1 - \delta))}{2(1 - \sigma(1 - \eta))} + \bar{v}_\ell(z_{ij}(c')) \tag{C.11}
\]

\[
\bar{v}_k(z_{ij}^\theta) = \frac{(1 - 2\delta)\bar{v}_k(z_{ij}(c'))(1 - \delta)}{2(1 - \sigma(1 - \eta))} + \bar{v}_k(z_{ij}(c')) \tag{C.12}
\]

Simplifying (C.5)-(C.7) gives

\[
\bar{v}_i(z_{ij}(c)) = \frac{\frac{1}{2} - c}{1 - \sigma(1 - \eta)} + \beta_i \tag{C.13}
\]

\[
\bar{v}_j(z_{ij}(c)) = \frac{\frac{1}{2} - c}{1 - \sigma(1 - \eta)} + \beta_j \tag{C.14}
\]

\[
\bar{v}_k(z_{ij}(c)) = \beta_k \tag{C.15}
\]

where for \( \ell = i, j \),

\[
\beta_\ell = \frac{\eta \delta \bar{v}_\ell(z_{ij}(c'))}{1 - \sigma(1 - \eta)} + \frac{\eta \delta(1 - 2\delta)(1 - 2\delta)\bar{v}_\ell(z_{ij}(c'))}{2(1 - \sigma(1 - \eta))} \quad \text{and}
\]

\[
\beta_k = \frac{\eta \delta \bar{v}_k(z_{ij}(c'))}{1 - \sigma(1 - \eta)} + \frac{\eta \delta(1 - 2\delta)(1 - \delta)\bar{v}_k(z_{ij}(c'))}{2(1 - \sigma(1 - \eta))}.
\]

For \( q^{t-1} \neq z_{ij}(c) \) for all \( c \in \zeta \), the continuation value \( \bar{v}_\ell(q^{t-1}) \) is, using the equilibrium strategies,

\[
\bar{v}_\ell(q^{t-1}) = (1 - \gamma) \left[ \frac{1}{2} \left( c + \delta \bar{v}_\ell(z_{ij}(c)) + \frac{1}{2} \left( 1 - c + \delta \bar{v}_\ell(z_{ij}(c)) \right) \right) \right]
\]

\[
+ \frac{1}{2} \left[ \frac{1}{2} \left( c + \delta \bar{v}_\ell(z_{ij}(c)) + \frac{1}{2} \delta \bar{v}_\ell(z_{jk}(c)) \right) \right]
\]

\[
+ \gamma \left[ \frac{1}{2} \left( 1 - c + E' \xi^t + \delta \bar{v}_\ell(z_{ij}^t) \right) + \frac{1}{2} \left( 1 - c + E' \xi^t + \delta \bar{v}_\ell(c_{ij}^t) \right) \right]
\]

\[
+ \frac{1}{2} \left[ \frac{1}{2} \left( c - E' \xi^t + \delta \bar{v}_\ell(c_{ij}^t) \right) \right]
\]

\[
+ \frac{1}{2} \left[ \frac{1}{2} \left( c - E' \xi^t + \delta \bar{v}_\ell(c_{jk}^t) \right) \right], \tag{C.16}
\]
where \( \bar{v}_\ell(z_{ij}(c)), \ell = i, j, k, \) are given by (C.13)-(C.15) and
\[
\bar{v}_\ell(z_{ij}^t) = \int_{c-\varepsilon}^{c+\varepsilon-1} \bar{v}_\ell(z_{ij}(c - \varepsilon^t)) \frac{1}{2\varepsilon} \, d\varepsilon^t + \int_{c+\varepsilon-1}^{c+\varepsilon} \bar{v}_\ell(z_{ij}(c - \varepsilon^t)) \frac{1}{2\varepsilon} \, d\varepsilon^t + \int_{c-\varepsilon}^{c-\varepsilon} \bar{v}_\ell(z_{ij}(c - \varepsilon^t)) \frac{1}{2\varepsilon} \, d\varepsilon^t.
\] (C.17)

In the first and third integrals in (C.17), \( c - \varepsilon^t \not\in \zeta, \) so \( \bar{v}_\ell(z_{ij}(c - \varepsilon^t)) = \bar{v}_\ell(q^{t-1}). \) In the second integral in (C.17) \( c - \varepsilon^t \in \zeta, \) so \( \bar{v}_\ell(z_{ij}(c - \varepsilon^t)) \) is given by (C.13)-(C.15). Then substituting from (C.13)-(C.15) and simplifying gives
\[
\bar{v}_i(z_{ij}^t) = \left(1 - \frac{2\eta}{2\varepsilon}\right) \frac{\theta(1 - 2(1 - \delta)\bar{v}_i(q^{t-1}))}{\beta(1 - \delta(1 - \eta)) - \delta\eta(1 - 2\varepsilon)} + \bar{v}_i(q^{t-1})
\] (C.18)
\[
\bar{v}_j(z_{ij}^t) = \left(1 - \frac{2\eta}{2\varepsilon}\right) \frac{\theta(1 - 2(1 - \delta)\bar{v}_j(q^{t-1}))}{\beta(1 - \delta(1 - \eta)) - \delta\eta(1 - 2\varepsilon)} + \bar{v}_j(q^{t-1})
\] (C.19)
\[
\bar{v}_k(z_{ij}^t) = -\left(1 - \frac{2\eta}{2\varepsilon}\right) \frac{\theta(1 - 2(1 - \delta)\bar{v}_k(q^{t-1}))}{\beta(1 - \delta(1 - \eta)) - \delta\eta(1 - 2\varepsilon)} + \bar{v}_k(q^{t-1}).
\] (C.20)

By symmetry \( \bar{v}_i(q^{t-1}) = \bar{v}_j(q^{t-1}) = \bar{v}_k(q^{t-1}) = \bar{v}_\ell(q^{t-1}). \) Substituting (C.18)-(C.20) into (C.16) and solving gives
\[
\bar{v}_\ell(q^{t-1}) = \hat{v} = \frac{1}{3(1-\delta)}, \ell = i, j, k.
\] (C.21)

This proves part (ii) of the lemma.

To prove part (i), by part (ii) \( \hat{v} = \frac{1}{3(1-\delta)} \) is the continuation payoff for any allocation such that \( c \not\in \zeta, \) hence \( \bar{v}_\ell(z_{ij}(c')) = \hat{v} = \frac{1}{3(1-\delta)}, \) for \( c' \not\in \zeta. \) Substituting \( \bar{v}_\ell(z_{ij}(c')) = \frac{1}{3(1-\delta)} \) into (C.13)-(C.15) yields (C.4).

To prove part (iii), first note that the numerator of (C.3) is nonnegative, since \( \zeta \leq \frac{1}{2}. \) Using \( \theta \geq 1 - 2\varepsilon \) from Assumption 4 the denominator of (C.3) yields
\[
2\theta(1 - \delta(1 - \eta)) - \delta\eta(1 - 2\varepsilon) \geq \theta(2 - \delta(2 - \eta)) > 0,
\]
so \( \nu(\zeta) \geq 0. \) If \( \zeta < \frac{1}{2}, \) the numerator is strictly positive. \[\blacksquare\]

The following lemma identifies policies that can be supported by tolerant dynamic coalitions.

**Lemma 10.** With implementation uncertainty given in Assumptions 1, 2, and 4 basic strategies are a coalition equilibrium supporting \( \bigcup_{\zeta \subset \zeta} Z(c) \) for all \( \zeta \in [\zeta^+, \frac{1}{2}], \) if \( \delta_2 \geq \frac{1}{2}, \) and
\[
\zeta^+ = \max\{c^{*}, c^{oo}, c^{oo}, c^{\ell}\}.
\] (C.22)

**Proof.** The proof proceeds by checking incentives to deviate from basic strategies supporting \( \bigcup_{\zeta \subset \zeta} Z(c). \) The following lemma identifies the continuation values when a coalition dissolves, which is required to check incentives to deviate.
Lemma 11. If \( q^{t-1} = z_{ij}(c) \) for \( c \in \zeta \), the continuation value \( \bar{v}_t(z_{ij}^\ell) \) for \( c - \bar{\varepsilon} < \bar{\varepsilon} \) when \( y^t \neq q^{t-1} \) is proposed is given by

\[
\bar{v}_i(z_{ij}^\ell) = \bar{v}_j(z_{ij}^\ell) = \frac{1}{3(1-\delta)} + \frac{\theta}{2} \nu(\bar{\varepsilon}) \tag{C.23}
\]

\[
\bar{v}_k(z_{ij}^\ell) = \frac{1}{3(1-\delta)} - 2\frac{\theta}{2} \nu(\bar{\varepsilon}). \tag{C.24}
\]

If \( q^{t-1} = z_{ij}(c) \) for \( c \in \zeta \), the continuation value \( \bar{v}_t(z_{ij}^\ell) \) for \( c - \theta < \bar{\varepsilon} \) when \( y^t = z_{ij}(c) \) is proposed is given by

\[
\bar{v}_i(z_{ij}^\ell) = \bar{v}_j(z_{ij}^\ell) = \frac{1}{3(1-\delta)} + \nu(\bar{\varepsilon}) \tag{C.25}
\]

\[
\bar{v}_k(z_{ij}^\ell) = \frac{1}{3(1-\delta)} - 2\nu(\bar{\varepsilon}). \tag{C.26}
\]

Proof. The first part follows from substituting \( \bar{v}_t(q^{t-1}) = \frac{1}{3(1-\delta)} \) into (C.18)–(C.20). The second part follows from substituting \( \bar{v}_t(z_{ij}(c')) = \frac{1}{3(1-\delta)} \) into (C.11) and (C.12). \( \blacksquare \)

The continuation values in (C.23)-(C.26) are constant in \( c \) because of Assumption 4 and the uniformly distributed shocks. The continuation values \( \bar{v}_t(z_{ij}^\ell) \) and \( \bar{v}_t(z_{ij}^\ell) \), \( \ell = i, j \), are greater than \( \frac{1}{3(1-\delta)} \) because a proposal \( z_{ij}(c), c \notin \zeta \) could result in a tolerant coalition allocation, whereas it equals the specific-policy allocation with probability 0. Note that \( \bar{v}_t(z_{ij}^\ell) \geq \bar{v}_t(z_{ij}^\ell) \), \( \ell = i, j \), and \( \bar{v}_k(z_{ij}^\ell) \leq \bar{v}_k(z_{ij}^\ell) \), since \( \bar{\varepsilon} \geq \theta \). If \( \theta = \bar{\varepsilon} \), \( \bar{v}_t(z_{ij}^\ell) = \bar{v}_t(z_{ij}^\ell) \), \( \ell = i, j, k \).

We show in the next lemma that the optimal proposal for the originator of a coalition gives the proposer 1 – \( \bar{\varepsilon} \).

Lemma 12. For \( y^t \neq q^{t-1} \notin \bigcup_{c=\bar{\varepsilon}}^{\hat{c}} Z(c) \), the optimal proposal by the originator \( i \) of a tolerant coalition is \( z_{it}(\bar{\varepsilon}) \), \( \ell = j, k \).

Proof. Legislator \( i \) proposes \( z_{ij}(c) \in \bigcup_{c=\bar{\varepsilon}}^{\hat{c}} Z(c) \), which yields an expected dynamic payoff \( EU_i(c) \) given by

\[
EU_i(c) = (1 - \gamma)[1 - c + \delta \bar{v}_i(z_{ij}(c))] + \gamma[1 - c + E^t \bar{v}_i(z_{ij}^\ell)], \tag{C.27}
\]

where \( \bar{v}_i(z_{ij}(c)) \) is given in (C.4) and \( \bar{v}_i(z_{ij}^\ell) \) is given in (C.23). From Lemma 11 \( \bar{v}_i(z_{ij}^\ell) \) does not depend on \( c \), so differentiating (C.27) yields

\[
\frac{dEU_i(c)}{dc} = -1 - \frac{\delta(1-\gamma)}{1-a(1-\eta)} < 0.
\]

Consequently, \( i \) prefers the lowest \( c \in \zeta \), so \( c = \bar{\varepsilon} \) is optimal. \( \blacksquare \)

We now check incentives to deviate from basic strategies.

Suppose \( q^{t-1} \notin \bigcup_{c=\bar{\varepsilon}}^{\hat{c}} Z(c) \). Then \( q^{t-1} = z_{ij}(c) \) for some \( i \) and \( j \) and some \( c \in \zeta \). Consider the incentives of legislators to accept the equilibrium proposal. With basic strategies the proposal is the same as the status quo, so legislators vote for the status quo.
Consider $i$’s and $j$’s incentives to propose a deviation. The lowest allocation for $i$ or $j$ when the status quo is in $\bigcup_{c \in Z} Z(\epsilon)$ is $\underline{c}$, so consider $\epsilon^{-1} = z_{ij}(\underline{c})$ and $j$’s incentives. Since $i$’s payoff is strictly higher, $i$ has no incentive to deviate if $j$ does not have an incentive to deviate.

From Lemma 12, the best deviation proposal for $j$ in $\bigcup_{c \in Z} Z(\epsilon)$ gives $j$ the allocation $1 - \underline{c}$. Legislator $j$ will not propose this if

$$(1 - \eta)[\underline{c} + \delta \hat{v}_j(z_{ij}(\underline{c}))] + \eta[\underline{c} - E^{\hat{\nu}} + \delta \hat{v}_j(z_{ij}^*)] \geq (1 - \gamma)[1 - \epsilon - \delta \hat{v}_j(z_{ij}(\underline{c}))] + \gamma[1 - \underline{c} + E^{z'} + \delta \hat{v}_j(z_{ij}^*)]$$

$$\iff \underline{c} \geq c^*.$$  \hfill (C.28)

If $j$ proposes $y' \notin \bigcup_{c \in Z} Z(\epsilon)$, the best proposal such that the realized policy is not in $\bigcup_{c \in Z} Z(\epsilon)$ gives 1 to $j$. Legislator $j$ has no incentive to deviate if

$$(1 - \eta)[\underline{c} + \delta \hat{v}_j(z_{ij}(\underline{c}))] + \eta[\underline{c} - E^{\hat{\nu}} + \delta \hat{v}_j(z_{ij}^*)] \geq 1 - \gamma \hat{\nu} + \delta \frac{1}{3(1 - \delta)}$$

$$\iff \underline{c} \geq c^\ell.$$  

Legislator $j$ may propose $y' \notin \bigcup_{c \in Z} Z(\epsilon)$ such that the realized policy has some probability of being in $\bigcup_{c \in Z} Z(\epsilon)$. Consider a policy that awards $1 - a^1$ to legislator $j$, where $0 \leq a^1 \leq \underline{c}$. Legislator $j$ has no incentive to deviate if

$$(1 - \eta)[\underline{c} + \delta \hat{v}_j(z_{ij}(\underline{c}))] + \eta[\underline{c} - E^{\hat{\nu}} + \delta \hat{v}_j(z_{ij}^*)] \geq (1 - \gamma)[1 - a^1 + \delta \hat{v}] + \gamma E^{(1 - a^1 + \epsilon') + \delta \hat{v}_j(y')},$$  \hfill (C.29)

where $\hat{v}_j(y')$ is the continuation payoff from the realized policy. This realized policy may be in $\bigcup_{c \in Z} Z(\epsilon)$ or not. If the realized allocation can be less than $\underline{c}$, then by Assumption 1

$$E^{(1 - a^1 + \epsilon') + \delta \hat{v}_j(y')} = (1 - \gamma)[1 - a^1 + \delta \hat{v}] + \gamma E^{(1 - a^1 + \epsilon') + \delta \hat{v}_j(y')}$$

$$= (1 - \gamma)(1 - a^1) + \gamma E^{(1 - a^1 + \epsilon')}
+ \delta \left[(1 - \gamma)\hat{v} + \int_{a^1 - \epsilon} a^1 + \epsilon + \delta \hat{v}_j(z_{ij}^* (1 - \epsilon')) d\epsilon' \right]$

$$= 1 - a^1 - \gamma \frac{(1 - \epsilon')^2}{\epsilon} + \delta \left[\hat{v}(1 - \gamma + \frac{\epsilon}{\epsilon}(2 \epsilon + \epsilon - 1)) + \gamma \frac{3(1 - \delta) \frac{1 + \delta \eta + 3(1 - \delta) \nu(\epsilon)}{3(1 - \delta)(3(1 - \eta))} (1 - 2 \epsilon) \right].$$

The most attractive deviation maximizes this expected payoff with respect to $a^1$. This expected dynamic payoff is decreasing in $a^1$, so the optimal $a^1 = 0$. This is the case described above, hence there is no incentive to deviate if $\epsilon \geq c^\ell$.

If the realized allocation cannot be less than $\underline{c}$, i.e., $1 - \epsilon - \epsilon > a^1 > \epsilon - \epsilon$

$$E^{(1 - a^1 + \epsilon') + \delta \hat{v}_j(y')} = 1 - a^1 + \int_{a^1 - \epsilon} a^1 (\epsilon' + \delta \hat{v}_j(z_{ij}((1 - a^1)) d\epsilon' + \int_{a^1 - \epsilon} a^1 (\epsilon' + \delta \hat{v}) d\epsilon' + \int_{a^1 - \epsilon} a^1 (\epsilon' + \delta \hat{v}) d\epsilon$$  \hfill (C.30)

The right side of (C.29) is quadratic and strictly concave in $a^1$ with a maximum at $\hat{a}^1$ given by

$$\hat{a}^1 = \frac{-2(-\epsilon - \epsilon) + 3(1 - \delta) \nu(\epsilon)}{\gamma(1 + \delta \eta)},$$  \hfill (C.31)
which must satisfy the constraints $1 - \xi - \zeta > \hat{a}^1 > \xi - \zeta$. Let the corresponding lower bound be \( \hat{c}^{\ell} = c^{\ell}(\hat{a}^1) \), so \( j \) (and also \( i \)) vote for \( z_{ij}(\xi) \) if \( \xi \geq \hat{c}^{\ell} \).

Consider \( k \)'s incentive to propose a deviation. If \( \xi \in [\max\{\xi^*, c^{\ell}, \hat{c}^{\ell}\}, \frac{1}{\mathcal{Z}}], \) \( i \) and \( j \) prefer \( z_{ij}(c) \) for all \( c \in [\xi; 1 - \xi] \) to any new allocation. Hence, any proposal by \( k \) other than the status quo will be defeated. Hence legislator \( k \) has no incentive to deviate from the equilibrium strategies.

**Suppose** \( \mathbf{q}^{t-1} \notin \bigcup_{c \in \mathcal{Z}} \mathcal{Z}(c) \). Consider \( j \)'s incentive to vote for the equilibrium proposal \( z_{ij}(c) \). The best status quo for \( j \) such that the realized policy from the status quo is not in \( \bigcup_{c \in \mathcal{Z}} \mathcal{Z}(c) \) gives 1 to \( j \). Legislator \( j \) votes for \( z_{ij}(\xi) \) rather than the status quo, and \( i \) also votes for \( z_{ij}(c) \), if and only if

\[
(1 - \gamma)[\xi + \delta \bar{v}_j(z_{ij}(\xi))] + \gamma[\xi - E^t \bar{\varepsilon}^t + \delta \bar{v}_j(z_{ij}^t)] > 1 - \eta^\theta + \delta \frac{1}{3(1 - \gamma)} \Rightarrow \xi > \hat{c}^{oo}.
\]

The status quo \( \mathbf{q}^{t-1} \notin \bigcup_{c \in \mathcal{Z}} \mathcal{Z}(c) \) may be such that the policy when implementation uncertainty is realized is in \( \bigcup_{c \in \mathcal{Z}} \mathcal{Z}(c) \) with some probability. Consider that the status quo gives 1 - \( a^2 \) to legislator \( j \). Legislator \( j \) votes for \( z_{ij}(\xi) \) if and only if

\[
(1 - \gamma)[\xi + \delta \bar{v}_j(z_{ij}(\xi))] + \gamma[\xi - E^t \bar{\varepsilon}^t + \delta \bar{v}_j(z_{ij}^t)] > (1 - \eta)[1 - a^2 + \delta \bar{v}] + \eta E^t[1 - a^2 + \tilde{\theta} + \delta \bar{v}_j(y^\theta)].
\]

The right side of the incentive constraint is quadratic and strictly concave in \( a^2 \) with a maximum at \( \hat{a}^2 \) given by

\[
\hat{a}^2 = \frac{-(2\theta - \eta\theta)(1 - \delta(1 - \eta)) + \delta\eta(\bar{\varepsilon}^t - \theta + \eta\delta v(\xi))}{\eta(1 + \delta\eta)}.
\]

The optimal \( \hat{a}^2 \) must satisfy the constraints analogous to those for \( a^1 \). Let the corresponding lower bound be \( \hat{c}^{oo} \equiv c^{oo}(\hat{a}^2) \), so legislator \( j \) (and also \( i \)) votes for \( z_{ij}(\xi) \) if \( \xi \geq \hat{c}^{oo} \).

Consider \( i \)'s incentive to propose \( y^j \neq z_{ij}(\xi) \). By Lemma 9 \( z_{ij}(\xi) \) gives \( i \) the highest dynamic payoff among proposals in \( \bigcup_{c \in \mathcal{Z}} \mathcal{Z}(c) \), so there is no incentive to make any other proposal in \( \bigcup_{c \in \mathcal{Z}} \mathcal{Z}(c) \). Since legislator \( j \) has no incentive to deviate when receiving 1 or 1 - \( \hat{a}^2 \) and legislator \( i \)'s allocation is at least as great as \( j \)'s, legislator \( i \) has no incentive to deviate to receive 1 or 1 - \( \hat{a}^2 \).

Consider the case in which \( i \) does not receive 1 in \( \mathbf{q}^{t-1} \). Legislator \( i \) prefers a proposal \( z_{ij}(\xi) \) to a proposal that gives 1 to \( i \) if and only if

\[
(1 - \gamma)[1 - \xi + \delta \bar{v}_i(z_{ij}(\xi))] + \gamma[1 - \xi + E^t \bar{\varepsilon}^t + \delta \bar{v}_i(z_{ij}^t)] \geq 1 - \gamma^\theta + \delta \frac{1}{3(1 - \gamma)}.
\]

Note that the right side of the inequality is less than \( 1 - \eta^\theta + \delta \frac{1}{3(1 - \gamma)} \) since \( \eta^\theta \leq \gamma \xi \), so \( \xi < 1 - c^{oo} \) is sufficient for this to be satisfied.

Legislator \( i \) may also propose \( y^t \notin \bigcup_{c \in \mathcal{Z}} \mathcal{Z}(c) \) such that the realized proposal is in \( \bigcup_{c \in \mathcal{Z}} \mathcal{Z}(c) \) with positive probability. From the previous analysis the optimal proposal has \( \hat{a}^1 \). Legislator \( i \) prefers
If and only if \( z_{ij}(\xi) \) \( \iff \hat{c}_2 \geq \xi \) if and only if\(^{28}\)
\[
(1 - \gamma)[1 - \xi + \delta \bar{v}_i(z_{ij}(c))] + \gamma[1 - \xi + E^t \bar{v}_t + \delta \bar{v}_i(z^t_{ij})] \geq (1 - \gamma)[1 - \hat{a}^1 + \delta \bar{v}_i] + \gamma E^t[1 - \hat{a}^1 + \bar{v}_t + \delta \bar{v}_j(y^t)]
\]
(C.32)

For this to be a feasible upper bound, we require \( \hat{c}_2 \geq \frac{1}{2} \), which is shown in Lemma 15.

Note that \( \hat{c}_2 \geq \frac{1}{2} \) implies that \( c^{**} \geq \hat{c}^{\ell \ell} \) so \( \hat{c}^{\ell \ell} \) is not binding. The right side of (C.32) is the same as the right side of (C.29), and the left side of (C.32) is the same as the (C.28). In addition the left side of (C.28) and (C.29) are the same, so (C.32) implies the constraint giving \( c^{**} \) is tighter than the constraint giving \( \hat{c}^{\ell \ell} \), so \( \hat{c}^{\ell \ell} \) is not binding.

The bounds \( c^{oo} \) and \( c^{\ell \ell} \) are less than their counterparts for specific-policy equilibria, and the following lemma gives sufficient conditions for \( c^{oo} < \frac{1}{2} \), \( c^{\ell \ell} < \frac{1}{2} \), and \( c^{**} \leq \frac{1}{2} \).

**Lemma 13.** For \( \delta > \delta^o \) the bounds satisfy \( c^{**} \leq \frac{1}{2} \), \( c^{oo} < \frac{1}{2} \), and \( c^{\ell \ell} < \frac{1}{2} \).

**Proof.** To show that \( c^{**} \leq \frac{1}{2} \), first totally differentiate (C.1) to show that \( c^{**} \) is increasing in \( \theta \). Since \( c^{**} \) is increasing in \( \theta \), the maximum is at \( \theta = \bar{\xi} \), in which case
\[
c^{**} = c^* + \frac{\delta(\gamma - \eta)(1 - \delta)\nu(c^{**})}{2(\gamma - \eta)} = \frac{3 - \delta(\gamma - \eta)(2 - 3(1 - \delta)\nu(c^{**}))}{3(2 - \delta(\gamma - \eta))}.
\]
Then,
\[
c^{**} < \frac{1}{2} \iff 0 < \delta(\gamma - \eta)(1 - \delta)\nu(c^{**}).
\]
Substituting for \( \nu(c^{**}) \) and rearranging yields
\[
1 > (1 - \delta)^{2\bar{\theta}} \left( 1 - \delta(1 - \eta) - \delta \eta(1 - 2c^{**}) \right)\]
\[
\iff 2\bar{\theta}(1 - \delta(1 - \eta)) - \delta \eta(1 - 2c^{**}) > (1 - \delta)(1 - 2c^{**})
\]
\[
\iff 2\bar{\theta}(1 - \delta(1 - \eta)) - (1 - 2c^{**})(1 - \delta(1 - \eta)) > 0
\]
\[
\iff 2\bar{\theta} > 1 - 2c^{**}.
\]
The last line holds since by Assumption 4 we have \( 1 - 2c^{**} \leq \bar{\theta} < 2\bar{\theta} \).

For \( \delta > \delta^o \), \( c^o < \frac{1}{2} \), and since \( c^{oo} \leq c^o \), \( c^{oo} < \frac{1}{2} \). Note that \( \hat{c}^{\ell \ell} \leq \hat{c} \leq c^o \) so \( \hat{c}^{\ell \ell} < \frac{1}{2} \) for \( \delta > \delta^o \). \( \blacksquare \)

---

\(^{28}\)Legislator \( j \) also votes for \( z_{ij}(c) \) if \( \xi \leq 1 - \hat{c}^{oo} \).
Proof of Proposition 5

The difference between the continuation values in (C.4) and (B.1) for the coalition originator $i$ receiving $1 - c$ under the status quo $q^{t-1}$ is

$$
\bar{v}_i(q^{t-1}) - v_i(q^{t-1}) = \frac{\delta \nu(c)}{1 - \delta (1 - \eta)},
$$

which is positive for $\eta > 0$ and $\xi < \frac{1}{2}$. If $\eta = 0$, the continuation values are the same. The same argument establishes the result for the coalition partner receiving $c$. ■

Tolerant Coalition Equilibria for $\gamma > \eta = 0$

The most tolerant coalition has $c^+$ in (C.22) equal to $c^{**}$, $c^{oo}$, $\hat{c}^{oo}$ or $c^{ell}$. With $\eta = 0$, $\hat{c}^{oo} = c^{oo}$, $c^{ell} = c^e$ and solving (C.1) and (C.2) for $c^{**}$ and $c^{oo}$ yields

$$
c^{**} = \frac{3 - \delta \gamma (2 - \frac{1}{\xi})}{3(2 - \delta \gamma (1 - \frac{1}{\xi}))},
$$

$$
c^{oo} = \frac{3 - 2\delta - \delta \gamma (1 + \frac{1}{\xi})}{3(1 - \delta \gamma (1 + \frac{1}{\xi}))}.
$$

Proof of Proposition 6

By Lemma 10 basic strategies constitute a coalition equilibrium if $c^+ = \max\{c^{**}, c^{oo}, \hat{c}^{oo}, c^{ell}\}$ and $\hat{c}_2 \geq \frac{1}{2}$. The proof of Proposition 6 proceeds first by establishing that $\hat{c}_2 \geq \frac{1}{2}$ for $\delta$ large enough. We then show for $\delta$ large enough the greatest lower bound on the coalition member’s allocation is $c^{**}$. By Lemma 13, $c^{**} \leq \frac{1}{2}$. Hence $\bigcup_{\xi = \frac{1}{2}} Z(c)$ is supported by a coalition equilibrium for all $\xi \in [c^{**}, \frac{1}{2}]$, which is non-empty since $c^{**} \leq \frac{1}{2} \leq \hat{c}_2$.

The following lemma gives the value for $\hat{a}^1$ when $\eta = 0$ and $\delta$ is large enough. The value of $\hat{a}^1$ is required to calculate $\hat{c}_2$.

Lemma 14. For $\eta = 0$ and $\delta \geq \hat{\delta} \equiv \frac{(1 - \xi)\gamma + 2\xi(1 - \gamma)}{2\xi(1 - \gamma) + \frac{3}{4} \gamma}$, we have $\hat{a}^1 = 1 - \xi - \xi$.

Proof. For $\eta = 0$ the optimal $\hat{a}^1$ in (14) is

$$
\hat{a}^1 = \frac{1}{\gamma} \left( -\xi(2 - \gamma)(1 - \delta) + \delta \gamma \left( \frac{2}{3} - \xi \right) \right),
$$

which is valid if and only if $1 - \xi - \xi > \hat{a}^1 > \xi - \xi$. Then,

$$
1 - \xi - \xi > \hat{a}^1 \quad \iff \quad 1 - \xi > -\frac{1}{\gamma} (2\xi(1 - \delta)(1 - \gamma)) + \frac{2}{3} \delta \quad \text{(C.33)}
$$

$$
\hat{a}_1 > \xi - \xi \quad \iff \quad \xi < -\frac{1}{\gamma} (2\xi(1 - \delta)) + \frac{2}{3} \delta.
$$

The condition in (C.33) is violated for $\delta \geq \hat{\delta}$ given by

$$
\hat{\delta} = \frac{(1 - \xi)\gamma + 2\xi(1 - \gamma)}{2\xi(1 - \gamma) + \frac{3}{4} \gamma},
$$

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which is strictly less than 1 for all $\xi > \frac{1}{4}$, which is required by the lower bound $c^{**}$. Consequently, for $\delta \geq \hat{\delta}$, $\hat{a}^1 = 1 - \xi - \xi$. That is, the best deviation proposal maximizes the probability that the allocation is in a tolerated interval $[\xi, 1 - \xi]$. \[\Box\]

The next lemma establishes for $\delta$ large enough $\hat{\delta}_2 \geq \frac{1}{2}$.

**Lemma 15.** For $\eta = 0$ we have $\hat{\delta}_2 \geq \frac{1}{2}$ for all $\delta > \max\{\hat{\delta}, \bar{\delta}\}$ where $\bar{\delta} \equiv \frac{\xi - \frac{1}{2}(\xi - \xi)}{v(1 - \gamma) + \xi - \frac{1}{2}(\xi - \xi)}$.

**Proof.** The incentive constraint in (C.32) that gives $\hat{\delta}_2$ evaluates to

$$1 - \frac{\delta}{1 - \delta} - \delta - \frac{\delta}{1 - \delta} (\frac{2}{3} - \xi) \geq 1 - a^1 + \delta \hat{v} - \gamma \frac{(\xi - a^1)^2}{4\xi} + \frac{\delta}{(1 - \delta)2\xi} ((a^1 - \xi + \xi) \left(\frac{2}{3} - \frac{1}{2}(a^1 + \xi + \xi)\right)), \tag{C.34}$$

where the right side is maximized at $\hat{a}^1$. For $\delta \geq \hat{\delta}$, $\hat{a}^1 = 1 - \xi - \xi$ from Lemma 14, so the right side of (C.34) simplifies to

$$1 - \frac{\xi}{1 - \xi} - \frac{\delta}{1 - \delta} (\frac{2}{3} - \xi) \geq \xi + \xi + \delta \hat{v} - \gamma \frac{(\xi + a^1 - 1)^2}{4\xi} + \frac{\delta}{(1 - \delta)2\xi} \left((\gamma - \xi + \xi)\right), \tag{C.35}$$

Then (C.35) is satisfied if the difference $\Delta$ between the two sides is positive, where

$$\Delta \equiv \left(1 - 2\xi - \xi\right) + \frac{\delta}{(1 - \delta)2\xi} \left(\frac{2}{3} - \xi\right) + \frac{\gamma}{2\xi} (\xi + 2\xi - 1)^2. \tag{C.36}$$

The expression for $\Delta$ is quadratic in $\xi$. The second derivative of $\Delta$ with respect to $\xi$ is $\frac{\partial^2 \Delta}{\partial \xi^2} = \frac{\gamma}{2\xi} > 0$ so $\Delta$ is convex in $\xi$. At $\xi = 1$ the derivative of $\Delta$ with respect to $\xi$ evaluates to $\frac{\partial \Delta}{\partial \xi} = -2 - \frac{\delta}{(1 - \xi)2\xi} + \gamma < 0$, so the derivative is negative for all feasible values of $\xi$. Then $\hat{\delta}_2$ is the value of $\xi$ such that $\Delta = 0$. By the implicit function theorem we can show that $\xi$ is increasing in $\delta$, so if $\hat{\delta}_2 = \frac{1}{2}$ for some $\delta$, then $\hat{\delta}_2 \geq \frac{1}{2}$ for all $\delta \geq \bar{\delta}$. By the implicit function theorem

$$\frac{\partial \xi}{\partial \delta} = -\frac{\partial \Delta}{\partial \xi} \cdot \frac{\partial \xi}{\partial \delta} \tag{C.37}$$

Differentiating $\Delta$ with respect to $\delta$ yields

$$\frac{d \Delta}{d \delta} = \frac{-\xi(1 - \gamma) - \gamma}{(1 - \delta)2\xi},$$

which is positive for $\xi < \frac{2}{3}$, and otherwise $\xi \geq \frac{2}{3} > \frac{1}{2}$. Consequently, if $\Delta|_{\xi=\frac{1}{2}} \geq 0$ for $\delta = \bar{\delta}$ then $\hat{\delta}_2 \geq \frac{1}{2}$ for all $\delta \geq \bar{\delta}$. The following characterizes $\delta$. Evaluating $\Delta|_{\xi=\frac{1}{2}}$ yields

$$\Delta|_{\xi=\frac{1}{2}} = -\xi(1 - \gamma) - \gamma + \frac{\delta}{6(1 - \delta)} + \frac{\gamma}{16\xi}. \tag{C.34}$$

The term $-\xi(1 - \gamma) - \frac{\gamma}{2} + \frac{\gamma}{16\xi}$ is nonnegative for $\xi \leq \xi^0(\gamma) \equiv \frac{\sqrt{2 - 2(1 - \gamma)} - 1}{4(1 - \gamma)}$, $\gamma \in (0, 1)$, in which case $\Delta|_{\xi=\frac{1}{2}} \geq 0$ for all $\delta \in [0, 1)$. For $\xi > \xi^0(\gamma)$, $\Delta|_{\xi=\frac{1}{2}} \geq 0$ if and only if

$$\delta \geq \bar{\delta}(\gamma, \xi) \equiv \frac{1}{1 + \frac{16\xi}{(1 - \gamma) + \frac{1}{2} + \frac{\gamma}{16\xi}}} < 1. \tag{C.35}$$

Consequently, for $\delta \geq \bar{\delta}(\gamma, \xi)$, $\Delta|_{\xi=\frac{1}{2}} \geq 0$, and $\hat{\delta}_2 \geq \frac{1}{2}$. The set $R_\gamma$ of $(\gamma, \xi)$ such that $\delta(\gamma, \xi) < 1$ is $R_\gamma = \{(\gamma, \xi) | \xi \geq \xi^0(\gamma), \gamma \in (0, 1)\}$, but since $\Delta|_{\xi=\frac{1}{2}} \geq 0$ for all $\delta$ when $\xi \leq \xi^0(\gamma)$, the bound $\delta(\gamma, \xi)$ in
(C.37) holds for all $(\gamma, \varepsilon) \in (0, 1) \times [1 - 2\varepsilon, \varepsilon]$.

The next lemma shows for $\delta$ large enough $c^{**} \geq \max\{c^{oo}, c^\ell\}$.

**Lemma 16.** For $\eta = 0$ there exists a $\delta^*$ and a $\delta^\ell$ such that for all $\delta \geq \delta^*$ we have $c^{**} \geq c^{oo}$, and for all $\delta \geq \delta^\ell$ we have $c^{**} \geq c^\ell$.

**Proof.** The difference between $c^{**}$ and $c^{oo}$ is

$$c^{**} - c^{oo} = \frac{3 - 2\delta \gamma + \frac{\delta \gamma}{\varepsilon}}{3(2 - \delta \gamma (1 + \frac{1}{6\varepsilon}))} - \frac{3 - \delta (2 + \gamma) - \frac{\delta \gamma}{1 + \frac{1}{6\varepsilon}}}{3(1 - \delta \gamma (1 + \frac{1}{6\varepsilon}))}.$$  \hspace{1cm} (C.38)

Evaluating (C.38) at $\delta = 0$ yields $(c^{**} - c^{oo})|_{\delta = 0} = -\frac{1}{2}$. Taking the limit as $\delta \to 1$ yields

$$\sup_{\delta \to 1} (c^{**} - c^{oo}) = \frac{(1 - \gamma)^2 + \frac{\gamma}{12\varepsilon}}{9(2 - \gamma (1 - \frac{1}{6\varepsilon}))(1 - \gamma (1 + \frac{1}{6\varepsilon}))} > 0.$$  

By the mean value theorem there exists one or more solutions to $c^{**} - c^{oo} = 0$ in $(0, 1)$. Let the largest of these be denoted by $\delta^*$.

For $\eta = 0$, $c^\ell = c^\ell$, so similarly, $c^{**} - c^\ell$ is positive as $\delta \to 1$ and negative for $\delta = 0$. Let $\delta^\ell \in (0, 1)$ denote the largest $\delta$ such that $c^{**} - c^\ell = 0$.

The difference $c^{**} - c^*$ is positive, since

$$c^{**} - c^* = \frac{3 - 2\delta \gamma + \frac{\delta \gamma}{\varepsilon}}{3(2 - \delta \gamma (1 + \frac{1}{6\varepsilon}))} - \frac{3 - 2\delta \gamma}{3(2 - \delta \gamma)} = \frac{\delta^2 \gamma}{36(2 - \delta \gamma (1 + \frac{1}{6\varepsilon}))} > 0.$$  \hspace{1cm} (C.39)

To determine the relation between $\delta^o$ and $\delta^*$, evaluate the difference $c^{**} - c^{oo}$ at $\delta = \delta^o = \frac{3}{4 - \gamma}$, which yields

$$(c^{**} - c^{oo})|_{\delta = \delta^o} = -\frac{2\gamma}{2(8 - \gamma (1 + \frac{1}{6\varepsilon}))} < 0.$$  

This implies that $\delta^* > \delta^o$. Since $\delta^* > \delta^o$, $c^* > c^\ell$ from the proof of Lemma 2, so $c^{**} > c^\ell$. ■

It is straightforward to show for $\eta = 0$, $\hat{\alpha}^2 = 0$, hence $\hat{c}^{oo} = c^{oo}$. Then, for $\delta > \delta^\ell \equiv \max\{\delta^*, \delta^\ell, \hat{\delta}, \overline{\delta}\} > \delta^o$, $c^{**}$ is the greatest lower bound. This completes the proof of Proposition 6. ■

**Proof of Proposition 7**

To prove (iii) and (iv), differentiate $c^{**}$ to obtain

$$\frac{dc^{**}}{d\delta} = \frac{\delta}{\gamma} \frac{dc^{**}}{d\gamma} = -\frac{\gamma}{3(2 - \delta \gamma (1 + \frac{1}{6\varepsilon}))^2} < 0.$$  

Properties (i) and (ii) are also straightforward to show. ■
Proof of Proposition 8

The difference $c^* - c^*$ is positive from (C.39), and the result follows from Corollary 11 and Proposition 6 with $\delta_+^{\bar{\epsilon}} \equiv \{\delta^-, \delta^+\}$. ■
References


