The Leverage Ratchet Effect

Anat R. Admati
Peter M. DeMarzo
Martin F. Hellwig
Paul Pfleiderer[*]

July 31, 2013

This version: December 31, 2015

Abstract

This paper explores the dynamics of corporate leverage when funding decisions are made in the interests of shareholders. In the absence of prior commitments or regulations, shareholder-creditor conflicts give rise to a *leverage ratchet effect*, which induces shareholders to resist reductions while favoring increases in leverage even when total-value maximization calls for the opposite. Unlike inefficiencies based on asymmetric information, the leverage ratchet effect applies to all forms of leverage reduction, including earnings retentions and rights offerings.

The leverage ratchet effect is present even in the absence of frictions other than the inability to write complete contracts. The effect creates an agency cost of debt that lowers the value of the leveraged firm. Standard frictions magnify the impact of the effect. In a dynamic context, since leverage becomes effectively irreversible, firms may limit leverage initially but then ratchet it up in response to shocks. The resulting leverage dynamics lead to history-dependent capital structure that cannot be explained by simple tradeoff considerations.

Leverage can be adjusted in many ways. For example, leverage reductions can be achieved by issuing equity to either buy back debt or purchase new assets, or by selling assets to buy back debt. We study shareholders' preferences over different ways to adjust leverage. A benchmark result gives conditions for shareholder indifference, but generally, shareholders have clear rankings over the alternatives. For example, shareholders prefer reducing leverage by selling assets, even at distressed prices, if by doing so they can benefit at the expense of senior creditors.

1. Introduction

In the standard tradeoff theories of capital structure, firm leverage is determined by trading off the benefits of using debt against bankruptcy costs and agency costs of debt. The agency costs stem from conflicts of interest between borrowers and creditors that create distortions in investment decisions such as underinvestment or excessive risk taking. In this article we examine another agency cost of debt that arises from conflicts of interest in decisions regarding leverage adjustments over time. In the absence of prior commitments, these conflicts of interest induce leverage patterns that can be very different from those suggested by pure tradeoff theories. Analyses that overlook the effect and its interactions with other frictions cannot explain the richness of leverage patterns and dynamics that we observe. Our results apply generally, but they are particularly relevant to banking, where leverage levels are high and creditors are too weak to prevent shareholders and managers from pursuing their objectives at the expense of creditors and others.

Funding decisions made by shareholders of a leveraged firm can be quite different from those that maximize the total value of the firm. Specifically, if shareholders are unable to commit to all future actions, a *leverage ratchet effect* arises, which creates a tendency for leverage to increase but not decrease. Moreover, there is a feedback loop between the agency distortions on the two sides of the balance sheet, where the agency costs due to the leverage ratchet effect are made worse by the investment distortions associated with agency conflicts. For example, higher leverage intensifies shareholders' desire to choose excessively risky investments, and at the same time shareholders' ability to shift risk intensifies the ratchet effect.

To see the leverage ratchet effect in a simple setting, consider a firm on the verge of costly financial distress. As long as it is solvent, the firm can reduce its leverage by issuing new shares, and thus lower the costs of the distress, yet such actions are rarely seen. This resistance of shareholders and managers to issuing new shares is sometimes explained by reference to asymmetric information along the lines of Myers and Majluf (1984).[1] The Myers-Majluf argument, however, does not explain resistance to earnings retention or rights offerings, neither of which is subject to market undervaluation of new shares.[2] The uniform resistance we see to all

---

[1] See for example Bolton and Freixas (2006) and Kashyap et al. (2010, 2011). In the policy debate, many other flawed claims are made, as discussed in Admati et al (2013), Admati and Hellwig (2013a,b,c 2014).

[2] Indeed, Myers and Majluf (1984) emphasize that, with the information asymmetries they consider, raising funds by retaining earnings should be preferred to new borrowing. Further, when leverage reductions are imposed by regulation, adverse selection becomes irrelevant. The expected "dilution costs" (losses due to undervaluation) for the shareholders when the firm has above-average return prospects should be offset by the expected gains when the firm's prospects turn out to be especially poor In other words, removing discretion also mitigates any negative signal associated with recapitalizations (see Admati et al (2013, Section 6) and Kashyap, Hansen and Stein (2011, p. 10)). Of course, managers might want to protest increased equity requirements in an attempt to show that *their* firm is undervalued in the market. However, relative to the average price, some firms are actually overvalued by investors, and their informed managers may well know it.

these forms of leverage reduction, especially in banking, is much better explained by the agency conflicts that create the leverage ratchet effects we analyze in this paper.

As noted in Black and Scholes (1973), shareholder resistance to leverage reduction occurs in the frictionless markets setting assumed in Modigliani-Miller (1958). If the shareholders issue new shares to buy back some of a firm's risky debt, the debt becomes less risky and more valuable. There is a wealth transfer from shareholders to debt holders.[3] Conversely, if shareholders can raise new debt of equal priority in order fund a payout for themselves, wealth is transferred in the other direction. Thus, shareholder resistance to leverage reduction and incentives for increased leverage are present even in a simple setting with no frictions other than the inability to write complete contracts.

Of course, in the Modigliani-Miller context there are no costs associated with these leverage changes. Our results go further than the observation by Black and Scholes (1973) by showing that shareholders' resistance to leverage reductions persists even when the leverage reduction would enhance firm value by alleviating frictions such as bankruptcy or agency costs. Indeed, shareholders resist leverage reduction *no matter how large the potential gain to the total value of the firm*. We also show that shareholders have incentives to increase leverage even when new debt must be junior and there is thus no mechanical dilution of existing creditors. These incentives can arise even when, due to other frictions, the new debt would destroy firm value.

These effects create an important additional agency cost of debt. First, in response to negative shocks that shift the tradeoffs regarding leverage due to frictions, shareholders may avoid reducing leverage even if this would raise total firm value. Second, anticipations of shareholders' future actions may affect initial contracting in ways that make borrowing more expensive and reduce firm value.

Our analysis of the leverage ratchet effect raises doubts about the ability of the traditional tradeoff theory of capital structure to fully explain capital structure. The effect implies that, absent strict (and costly) covenants to constrain it effectively, leverage begets more leverage and can become addictive. Because past leverage decisions distort future leverage choices, capital structure becomes history-dependent. Explaining capital structures requires a dynamic approach but, as we show, even in a relatively simple setting, the dynamics of leverage can be very complex and the standard comparative statics from tradeoff theory may no longer apply.

---

[3] We assume throughout our analysis that creditors are small and dispersed so that conflicts of interest cannot be dealt with by collective bargaining. Therefore, the price at which debt is repurchased must reflect the value that can be obtained by retaining a marginal amount of debt. As a result, the post-buyback price of debt, and the buyback price, will typically be higher than the price without a buyback. The resulting increase in the value of debt comes largely at the expense of shareholders. The importance of the difference between collective bargaining and unilateral actions of the debtor is also stressed by Strebulaev and Whited (2012). However, they do not consider buybacks in markets, but study callable debt, where the call option requires a repayment of the amount that was originally borrowed, plus a premium.

For example, at any level of tax rates, the level of leverage that is implied by the leverage ratchet depends on what tax rates were in the past. Firms generally ratchet up their leverage over time, with the pace of such changes driven both by economic shocks, the tax benefits of debt, and the sensitivity of the debt price to total leverage. Shareholders will seek to increase leverage in response to shocks that increase the benefits of debt, but will resist leverage reductions even when lower leverage would increase the total value of the firm. Indeed, we show that shareholders may actually seek to increase leverage even when tax rates are reduced, in complete contrast to tradeoff theory predictions.[4] However, observed leverage is not necessarily higher than tradeoff theory would predict. Because the initial cost of debt reflects creditors' expectations about the risks of further debt increases, firms initially may be under-leveraged relative to the predictions of tradeoff theory. Only over time, and depending on shocks to asset values and other considerations, will there be a tendency for firms to become over-leveraged.

Shareholders often have many ways available to adjust leverage. For example, when required to reduce leverage, shareholders can (i) sell assets and use the proceeds to reduce debt levels; (ii) issue new equity to buy back debt; or (iii) issue new equity to buy additional assets. We demonstrate an important equivalence result, showing that shareholders are indifferent among all ways leverage can be changed if assets are homogeneous, there is one class of debt outstanding, and sales or purchases of assets do not, by themselves, generate or destroy value for shareholders.

We show that shareholders will have strong preferences concerning leverage adjustments, however, whenever the conditions of the equivalence result don't hold. For example, if there are multiple classes of debt (rather than just a single class) and shareholders must reduce leverage, they prefer to sell assets and buy back the most junior debt. Intuitively, asset sales impose some of the cost of the deleveraging on the remaining senior debt holders, whose claims will in the end be backed by fewer assets. These distributive effects may dominate even if asset sales, possibly occurring at fire-sale prices, are inefficient and reduce the value of the firm.

The paper is organized as follows. Section 2 presents the basic model and preliminary results on shareholder resistance to leverage reductions. Section 3 considers how agency conflicts interact and potentially create feedback effects and how the leverage ratchet is intensified by investment-related conflicts. Section 4 develops a dynamic equilibrium model of leverage, and demonstrates that firms will limit leverage initially but "ratchet up" indefinitely in response to shocks. In Section 5 we consider alternative ways for a firm to adjust leverage other than pure recapitalization. Section 6 provides concluding remarks and discusses the application of our analysis to banking and the role of capital regulation.

---

[4] Evidence of such an asymmetric response in the context of tax rate changes is provided in recent work by Heider and Ljungqvist (2015).

Related Literature

As mentioned above, the observation that in the absence of frictions shareholders cannot gain by giving up their default option was first made by Black and Scholes (1973). Leland (1994) pointed to a similar resistance (which he terms "surprising") in the context of a specific model in which debt is homogeneous and presumed fixed, bankruptcy costs are proportional, and only small marginal changes of *pari passu* debt are permitted.

The fact that creditors can gain when debt is repurchased by a borrower in the open market has been noted in the literature concerning market-based solutions to the sovereign debt crisis of the 1980s. See, for example, Frenkel et al. (1989) and Bulow and Rogoff (1990). The theory developed in that literature was confirmed in the Bolivian debt buyback of 1988 and more recently in the Greek debt buyback of 2012.[5]

Shareholders' resistance to leverage reduction due to conflicts of interests with creditors is related to the underinvestment problem created by debt overhang noted by Myers (1977). As in Myers (1977), shareholders resist leverage reductions because the creditors obtain benefits at shareholder expense. Indeed, the underinvestment problem arises in Myers (1977) because it is assumed that new investments must be funded by equity (or junior debt). Importantly, unlike the underinvestment problem in Myers (1977), which only occurs when the net present value of the project is not large enough for the shareholders' share of the benefits to cover its cost, shareholders' resistance to leverage reductions persists *no matter how much the leverage reduction would increase the total value of the firm*. Worse, other agency costs of debt intensify shareholder resistance to leverage reduction even though the alleviation of these agency frictions increase the benefit of leverage reduction to total firm value. This intensity of the conflict of interest is due to the fact that shareholders do not generally capture *any* of the benefits of reduced leverage; the benefits accrue entirely to creditors and are not shared like the value of an investment.

In the literature on dynamic capital structure, it is common to explore shareholders' decisions with respect to payouts and default without allowing changes in the capital structure prior to default. Models that allow capital structure adjustments often assume that it is prohibitively costly to reduce leverage in distress, or that debt can only be recalled at par or at a premium. In addition, these models generally assume that new debt cannot be issued unless all existing debt is retired first.[6] The assumption that firms are required to repurchase all existing debt before making any changes in their debt level is clearly inconsistent with actual practice. Moreover, this assumption effectively rules out any leverage ratchet effect a priori. Our analysis assumes instead that new debt can be issued and existing debt can be repurchased any time at competitive market

---

[5] By contrast, van Wijnbergen (1991) showed that, in the 1990 buyback of Mexican debt under the Brady plan, which involved collective bargaining, creditors had to agree to terms under which they neither gained nor lost from the buyback.

[6] See e.g., Fischer, Heinkel, Zechner (1989), Leland (1998), Goldstein, Ju, and Leland (2001), Strebulaev (2007), and Titman and Tsyplakov (2007). Hennessey and Whited (2005) allow only one-period riskless debt.

prices.[7] In contrast to much of the literature on dynamic capital structure, we do not assume that there is an exogenous transactions cost to issuing equity.

Dangl and Zechner (2007) analyze the dynamics of leverage and the choice between long- and short-term debt. They observe that with long-term debt shareholders do not have incentives to reduce leverage when the firm has poor performance. Short-term debt requires the firm pay off all its debt frequently (at par) and effectively causes the firm to start afresh with new tradeoffs a-la Modigliani and Miller each time this debt matures. The frequent issuance of short-term debt entails higher transactions costs, which must be traded off against any benefits created. An important assumption in Dangl and Zechner (2007) is that there are covenants in place that prohibit the issuance of any new debt that would increase the total face value of debt outstanding, which again rules out the ratchet effect that we explore in this paper.

Another related paper is Brunnermeier and Oehmke (2013), which shows that shareholder incentives potentially lead to a "maturity rat race," since shortening the maturity structure of a firm's liabilities dilutes the longer-term creditors. The key assumption in Brunnermeier and Oehmke (2013) is that, although the firm can commit to a total amount of debt, it cannot commit to a particular maturity structure of that debt. They observe that this inability to commit is especially applicable to financial institutions. Our paper is similarly based on borrower-creditor conflict of interest and on the difficulty of making binding commitments. Unlike Brunnermeier and Oehmke (2013), however, our focus is on the firm's leverage choices rather than on the maturity structure of a fixed amount of debt.

Bizer and DeMarzo (1992) demonstrate that in the presence of agency costs of debt, lack of commitment leads borrowers to choose excessive leverage. Their setting is focused on the case of a risk-averse borrower or sovereign, rather than a corporation, but the desire to increase leverage once existing leverage is in place mirrors our finding regarding shareholders' resistance to leverage reductions. We also use solution methods developed in Bizer and DeMarzo (1994) for our analysis of the dynamic equilibrium.

Leverage ratchet effects are particularly strong for highly leveraged firms, such as banks. If their debts are explicitly or implicitly guaranteed, creditors have fewer (or no) incentives to put in place debt covenants that might mitigate the effect. Leverage choices can therefore become extremely inefficient. In discussing governance in banking, Becht et al (2011) also note that creditor governance or "discipline" is particularly weak for banks, which can lead to excessive leverage and risk.

As discussed in the concluding remarks, our paper is particularly applicable to banking, and also has policy implications. The results in this paper complement our earlier paper (Admati et al. 2013), in which we argued that banks choose socially inefficient levels of leverage because they do not internalize the significant negative external effect of their distress and default. If banks'

---

[7] Some papers make the assumption that new debt can be issued *pari passu* with existing debt, which can help overcome the underinvestment problem identified in Myers (1977). Unless existing creditors benefit from additional investments, issuing such debt can reduce the value of existing debt and violate the creditors' seniority. In the spirit of Myers (1977), we assume for most of our analysis that violating the seniority of existing creditors is not possible.

funding costs are lower when they have high leverage, the reason is debt subsidies (e.g. taxes and government guarantees) provided at taxpayers' expense. In the current paper we show that high leverage is likely to be *privately* costly when viewed from the combined perspectives of all creditors and shareholders. Because high leverage in banking creates systemic risk that increases the fragility of the system and the likelihood of a financial crisis, high leverage is also *socially inefficient*. Guarantees meant to provide stability have the harmful effect of feeding the addiction to leverage implied by the leverage ratchet effect by removing creditors' incentives to restrict leverage through covenants or high interest rates. The results of this paper therefore strengthen our conclusion that, in the context of banking, effective capital regulation (i.e., regulation of leverage levels) is essential. Our analysis of shareholder preferences over various modes of leverage reduction also sheds light on how banks react to regulations that are framed in terms of regulatory ratios.

2. Debt Overhang and Resistance to Recapitalization

In this and the following section, we develop a simple but general reduced-form, static tradeoff theory model in order to highlight the logic and demonstrate the robustness of the leverage ratchet effect. Subsequently, in Section 4, we embed this static model in a dynamic setting and examine the leverage ratchet's dynamic consequences.

Consider a firm that has made an investment in risky assets and has funded itself partly with debt. We begin with a simple tradeoff model of capital structure based on taxes and net default costs, which we generalize later as we consider additional frictions. For our basic argument, we make the following assumptions:

Firm Investment: The firm has existing real investments given by A. Investment returns are realized in the future ("date 2"), with final value $\tilde{x}A$.

Firm Liabilities: The firm is funded by equity, together with a debt claim with total face value D that is due at date 2 when the asset returns are realized. If $\tilde{x}A \geq D$, debt claims will be honored in full. If $\tilde{x}A < D$, the firm will be forced to default.

We begin by considering three standard frictions that may affect the payouts of the firm's securities at date 2. These are taxes, bankruptcy costs, and third party (government) subsidies.

Taxes: We assume the firm may incur a corporate tax liability if asset returns exceed required debt payments. The tax benefits are determined by the firm's total debt outstanding and are given by $t(\tilde{x}, A, D) \in [0, \tilde{x}A - D]$ when $\tilde{x}A > D$. We

7

assume that no tax is paid when $\tilde{x}A \leq D$. Finally, we assume that the total tax liability is weakly decreasing in D, i.e. $t_D(\tilde{x}, A, D) \leq 0$.[8]

Default costs (net of subsidies):

If $\tilde{x}A < D$, the firm is unable to fulfill its obligation to debt holders and must default unless it receives a subsidy from the government or some other third party.[9] Let $n(\tilde{x}A, D)$ be the *net* default costs for the firm, which is the difference between the bankruptcy cost and any third party subsidy. In the event that $\tilde{x}A > D$, there are no subsidies and no bankruptcy costs and thus $n(\tilde{x}A, D) = 0$. If $\tilde{x}A < D$, we assume that $\tilde{x}A - n(\tilde{x}A, D) \in [0, D]$. Note that the net default costs could be negative if the subsidy exceeds the bankruptcy cost – which means that the firm's debt holders will receive *more* than $\tilde{x}A$ – but we assume that, at best, subsidies bring the available funds up to the amount D that is needed to avoid default.

With these assumptions, the payoffs on the firm's debt and its equity are as follows:

	If $\tilde{x}A < D$	If $\tilde{x}A \geq D$
Payoff to Shareholders	0	$\tilde{x}A - t(\tilde{x}, A, D) - D$
Payoff to Debt Holders	$\tilde{x}A - n(\tilde{x}A, D)$	D

Pricing at Date 1: All securities are traded in perfect Walrasian markets. We normalize the risk-free interest rate to zero and set prices of securities at date 1 equal to their expected payoff with respect to the risk-neutral distribution F of the return $\tilde{x}$ on the firms' assets.[10] The distribution F has full support on $[0, \infty)$, and we assume the firm takes F as given and independent of its leverage choice.[11] Throughout the paper, when we refer to probabilities, it is with respect to this risk-neutral measure.

Given our assumptions about payouts and pricing, it follows that at date 1 the values of the firm's debt and its equity are:

---

[8] Note that there may be other direct effects of leverage on cash flows that can be included in the function t. For example, the firm may have to pay higher wages as in Berk, Stanton, and Zechner (2010), or have reduced costs due to disciplining benefits as argued by Jensen (1986). The key assumption is that on the margin, the combined direct consequences of leverage on free cash flows embodied by t are weakly increasing in D.

[9] We allow for the possibility of subsidies because of their importance in the recent financial crisis.

[10] Setting the interest rate to zero is without loss of generality (we can alternatively interpret prices as future values). The existence of a risk-neutral distribution (or pricing kernel) F is implied by the absence of arbitrage.

[11] That is, we assume the firm acts as a price-taker with respect to this pricing kernel. Thus, as is standard in the corporate finance literature, we are ignoring any general equilibrium consequences of the individual firm's security choices.

$$\text{Total value of debt} = V^D(D, A)$$

$$= \int_{D/A}^{\infty} D \, \mathrm{d}F(x) + \int_{0}^{D/A} \left(xA - n(xA, D) \right) \mathrm{d}F(x) \tag{1}$$

and

$$\text{Value of equity} = V^E(D, A) = \int_{D/A}^{\infty} \left(xA - t(x, A, D) - D \right) \mathrm{d}F(x). \tag{2}$$

The total value of the firm is therefore $V^F(D, A) \equiv V^E(D, A) + V^D(D, A)$.

Now suppose the firm considers reducing leverage by buying back a portion of its outstanding debt. For now, we hold fixed the assets of the firm, and assume the cash used for the buyback is raised through a rights offering to existing shareholders, or a market offering of equity or other equity-like securities (such as preferred shares). Alternatively, the firm may use cash on hand that it would have alternatively paid out as a dividend. If the firm plans to buy back debt with a nominal claim equal to Δ, then upon the announcement of the recapitalization, the value of the firm will change from $V^F(D, A)$ to $V^F(D - \Delta, A)$.

If the firm is inefficiently over-leveraged, then this change in firm value will be positive. It is natural to expect that some fraction of this gain will be captured by the firm's debt holders, diminishing the benefit to equity holders. Intuition suggests, however, that equity holders should still gain if the benefit of the recapitalization is sufficiently large. On the contrary, we will show that, independent of the source of funds or the potential benefit to total firm value, the fraction of any gain captured by the firm's debt holders *always* exceeds 100%, deterring equity holders from ever *voluntarily* recapitalizing.[12]

To understand this result, consider first the pricing of the firm's debt. Equation (1) above implies that, without the buyback, the date 1 market price of debt per unit of nominal face value is equal to:

$$q(D, A) = \frac{V^D(D, A)}{D} = 1 - \underbrace{F(D/A)}_{\text{Probability of Default}} \left(1 - \underbrace{E\left[\left. \frac{\tilde{x}A - n(\tilde{x}A, D)}{D} \right| \tilde{x}A < D \right]}_{\text{Expected Recovery Rate}} \right). \tag{3}$$

That is, the debt price decreases with the probability of default but increases with the expected recovery rate.

Next observe that if the firm wants to buy back debt in the open market, it cannot do so at the price given in (3). The repurchase price must be such that debt holders are at the margin indifferent between selling debt and holding on to it.[13] The buyback price of the debt must

---

[12] We take up whether a possibly forced recapitalization is feasible and consistent with limited liability in Section 5.

[13] For extensive discussions of this holdout effect, see Frenkel et al. (1989) and Bulow and Rogoff (1990).

therefore be equal to the market price $q(D-\Delta,A)$ that prevails at the post-buyback debt level. Thus, upon announcement of the debt buyback, the value of the firm's debt will change to

$$V^D(D-\Delta)+\Delta q(D-\Delta,A),$$

and, because they must pay the cost of the buyback, the value of equity will change to

$$V^E(D-\Delta)-\Delta q(D-\Delta,A).$$

We assume that that the firm's managers and shareholders assess such a buyback only on the basis of how it affects shareholders' wealth.[14] Therefore, the buyback will be undertaken only if the market value of the firm's equity would increase with the buyback:

$$V^E(D-\Delta,A)-\Delta q(D-\Delta,A)>V^E(D,A). \tag{4}$$

The following proposition shows that, no matter how large the gain to total firm value is, equity holders are always harmed by a recapitalization.

Proposition 1 (Shareholder resistance to Recapitalization): *Equity holders are strictly worse off issuing securities to recapitalize the firm and reduce its outstanding debt. Losses to equity holders arise from the loss of their default option, the reduction in dilution of existing debt, and higher taxes. The loss to equity holders is mitigated by bankruptcy costs, and increased by the presence of taxes or default subsidies.*

Proof: Using (2) and (4), we can write the gain to shareholders from changing from debt D to $D-\Delta$ as

$$\begin{aligned}
G(D,D-\Delta) &\equiv V^E(D-\Delta,A)-V^E(D,A)-\Delta\times q(D-\Delta,A)\\
&= \int_{(D-\Delta)/A}^{D/A}(xA-D)\,\mathrm{d}F(x)\\
&\quad + \Delta\times\left(1-F\left(\frac{D-\Delta}{A}\right)-q(D-\Delta,A)\right)\\
&\quad + \int_{D/A}^{\infty}t(x,A,D)\,\mathrm{d}F(x)-\int_{(D-\Delta)/A}^{\infty}t(x,A,D-\Delta)\,\mathrm{d}F(x)
\end{aligned} \tag{5}$$

The first term in (5) captures the *default option effect*: the loss of equity's default option given final asset values between $D-\Delta$ and D. This term is strictly negative given our assumption that F has full support.

---

[14] We do not consider the governance issues associated with the decision to issue shares or make a rights offering. Under U.S. law, a rights offering can be made without shareholder approval, though it may still fail if investors do not find it in their interest to acquire the new shares. In most other countries, rights offerings must be approved by shareholder meetings.

The second term in (5) captures the *dilution effect*: the portion of the debt repurchase price that compensates debt holders for their share of any recovery in default. From (3), we can see that this term is negative and decreases with the (ex post) expected recovery rate of the debt:

$$\Delta \times \left(1 - F\left(\frac{D-\Delta}{A} \right) - q(D-\Delta, A) \right) =$$

$$-\Delta \times \underbrace{F\left(\frac{D-\Delta}{A} \right)}_{\text{Probability of Default}} \times \underbrace{E\left[\left. \frac{\tilde{x}A - n(\tilde{x}A, D-\Delta)}{D-\Delta} \right| \tilde{x}A < D-\Delta \right]}_{\text{Expected Recovery Rate}} \leq 0. \tag{6}$$

Note that this term would be zero at the margin if the debt repurchased were strictly junior to any remaining debt, as in that case the expected recovery rate would be zero for the marginal dollar of junior debt. When the debt repurchased has equal priority with the remaining debt, as assumed thus far, and the remaining debt has a positive recovery rate, this term is negative. Note also that default subsidies will raise, and bankruptcy costs will lower, this cost.

Finally, the third term in (5) is the *tax effect*: it is negative because taxes are non-increasing in D.

Thus, combining these three effects, we see that

$$V^E\left(D-\Delta, A \right) - V^E\left(D, A \right) - \Delta \times q(D-\Delta, A) < 0 \tag{7}$$

and hence shareholders always lose from a recapitalization. ∎

Proposition 1 restates and generalizes observations that have been made elsewhere in the literature. Black and Scholes (1973) note that in a setting with perfect markets shareholders lose from repurchasing debt. Equation (5) shows that this result is due to the loss of the default option and the expectation of a positive recovery rate on the debt. Leland (1994) demonstrates a similar result (which he finds "surprising") in the context of a continuous-time tradeoff model with linear taxes and a particular model of default costs, without providing intuition for the result. To the best of our knowledge, the full generality of Proposition 1 has not been clearly articulated nor fully appreciated in the capital structure literature.[15]

We have assumed thus far that the firm has only a single class of debt outstanding. If the firm has several classes of debt, shareholders will naturally find it most attractive to buy back the cheapest class first, which will be the most junior class outstanding. Note that the only difference between these classes of debt is their expected recovery rates. Because the expected recovery rate is always non-negative, however, the above logic still applies and we have the following immediate generalization:

---

[15] Indeed, resistance to recapitalization is often justified by appealing to transactions costs or lemons costs associated with equity issues (see e.g., Bolton and Freixas (2006)). Of course, such explanations do not explain the failure of recapitalizations via rights offerings or when firms have cash available to pay out as dividends, whereas Proposition 1 immediately applies.

Proposition 2 (Shareholder Resistance to buying back *any* debt class): *Shareholders are strictly worse off issuing equity to repurchase any class of outstanding debt. The loss increases with the seniority of the debt purchased.*

Note that shareholders' resistance to a recapitalization does not depend on the tax benefits of debt. More strikingly, note that shareholders will resist a recapitalization no matter how large the potential gain to firm value. All of the benefits produced by a debt buyback – which in our model thus far come from reduced bankruptcy costs – accrue to existing debt holders. Shareholders – who must buy back the debt at a market price that reflects the reduced likelihood of default – are unable to appropriate these gains and so will resist a recapitalization.

The observation that shareholders resist a recapitalization even when it would raise the value of the firm stands in contrast to the standard tradeoff theory of capital structure, where firms are presumed to choose their debt levels so as to maximize total firm value given the countervailing frictions of tax benefits and distress and other costs associated with leverage. In fact, shareholder and firm value maximization coincide only when capital structure decisions are taken *ex ante*, before any debt has been issued. Once existing debt is in place, shareholder-creditor conflicts emerge, with important consequences for how both the asset *and* liability side of the firm's balance sheet will be managed going forward.[16]

Indeed, the consequences of debt overhang for recapitalization are stronger than those for equity-financed investment as described in Myers (1977). When a firm must issue equity to undertake a valuable project, the loss to the shareholders due to the wealth transfer to risky debt holders brought about by the reduction in leverage can be more than offset by the positive net present value (NPV) of the project, a portion of which the shareholders capture. Thus, if the NPV of the project is large enough, Myers's underinvestment problem disappears, and the outcome is efficient. By contrast, when a debt buyback would increase the total firm value, debt overhang always results in a loss of efficiency. Again, no matter how large the gain in firm value, shareholders will always resist the recapitalization.

Matters would be different if there were collective bargaining about the price of debt in the buyback.[17] For example, if debt contracts had collective action clauses, the firm's management, acting on behalf of shareholders, could negotiate a buyback agreement with debt holder representatives. In such negotiations, and with the no-buyback outcome as a default option, debt holders would end up sharing their gains from the buyback with the shareholders. This sharing of gains cannot be achieved in a market buyback. And even in a negotiation, if debt holders are

---

[16] This point is central to the literature on dynamic theory of capital structure; see for example Strebulaev and Whited (2012). However, despite its name, this literature is more concerned with the dynamics of default and investment decisions for a given capital structure than with the evolution of capital structure through new issues and repurchases of debt and equity. Moreover, leverage changes are often restricted exogenously; e.g. Bhamra et al. (2010, p. 1499) state "In common with the literature, we assume that refinancings are leverage increasing transactions since empirical evidence demonstrates that reducing leverage in distress is much costlier."

[17] In a different setting the impact of collective bargaining on debt dynamics is also noted by Strebulaev and Whited (2012).

dispersed, holdouts could be likely. In other words, at terms for which shareholders would not resist a recapitalization, we would expect (at least some) debt holders to resist, precluding a purely voluntary leverage reduction.[18]

The difference between a buyback through collective bargaining and a buyback through the market is due to the fact that the buyback through the market itself raises the market price. Payoffs per unit of debt improve both because the default probability goes down and because, in the event of default, the available asset value (net of bankruptcy costs) is split among fewer claimants. With higher payoffs, the market price of the debt must increase. An exception to this rule occurs only if the buyback has no effect on the default probability and, in the event of default, debt holders do not get anything.[19,20]

Our results above establish the resistance of shareholders to pure recapitalizations for all equity-based sources of funding. In Section 5 we will consider leverage reduction modes that involve either asset sales or the acquisition of new assets using equity funding. In that context, we also discuss the compatibility of leverage reduction with limited liability of existing shareholders.

3. Agency and Leverage Ratchet

So far we have focused on the tradeoff between the tax benefits of leverage and bankruptcy costs. Bankruptcy costs alone, however, are not the only detrimental consequence of leverage for firm value. In this section we consider additional agency conflicts that result from debt overhang, and find an important feedback: Agency costs increase both the benefit of, and resistance to, recapitalizations. Moreover, existing leverage distorts future leverage decisions to further exacerbate debt overhang, leading to a leverage ratchet effect.

3.1. Investment Distortions Increase Resistance

It is well known that the presence of leverage in the firm creates debt-equity conflicts related to investment. In particular, leverage may induce equity holders to increase the risk of the firm's assets via asset substitution (Jensen and Meckling, 1976), or to fail to undertake new

---

[18] Note that in general it is not sufficient to negotiate solely with creditors whose debt will be repurchased – that is, it is not enough just to overcome the holdout problem – remaining creditors need to share some of their gains as well.

[19] A recent example of this effect can be seen in the buyback of Greek debt in 2012, when about one half of the debt was repurchased. The market price in August 2012, before the news about the buyback plans transpired, was about 17c per euro of debt, whereas the buyback was transacted in different batches between 30c and 40c per euro. This outcome mirrored the Bolivian experience of 1988, which was documented and analyzed by Bulow and Rogoff (1990). In contrast, in the Mexican debt buyback in 1990, collective bargaining seems to have been used to prevent the private creditors from obtaining any windfalls; see van Wijnbergen (1991).

[20] Our analysis raises potential concerns about the practice of using mark-to-market accounting for a firm's liabilities. Mark-to-market accounting is usually based on the principle that reported valuations should reflect the scope the firm has for its actions in markets, e.g. in selling assets or buying back debt. If the market price of the firm's debt is based on the expectation that there will not be a buyback, this price is not relevant for a buyback. A mark-to-market valuation that is based on this price does not actually reflect the scope for a buyback that the firm has. As a monopsonist or a monopolist in the markets for its own securities, the firm should not take the prices of its securities as given and independent of its own actions. Market prices of the firm's securities depend on its actions, especially actions related to changes in capital structure.

investment opportunities (Myers, 1977). Indeed, these costs are often presumed to be even more significant than bankruptcy costs in the determination of optimal leverage from the perspective of tradeoff theory.

In this section we generalize our analysis to allow for both asset substitution and underinvestment. These agency frictions raise the cost of leverage for total firm value, and thus increase the potential benefit from a recapitalization. Yet we will show that despite this benefit, future debt-equity conflicts only *increase* shareholder resistance to any recapitalization.

To see the intuition for this result, consider first the case of asset substitution. Suppose the distribution of asset returns may be affected by actions taken by shareholders (or managers acting on behalf of shareholders). We denote these actions by θ, and the resulting asset returns by $\tilde{x}_\theta$, which has distribution $F(x \mid \theta)$. In this setting, it is natural to extend our notation and define the value of equity as follows:

$$
\begin{aligned}
V^E(D, A) &\equiv \max_\theta V^E(D, A, \theta) \\
&= \max_\theta \int_{D/A}^{\infty} \left(xA - t(x, A, D) - D \right) \, \mathrm{d}F(x \mid \theta).
\end{aligned}
\tag{8}
$$

We assume in (8) that the actions θ are taken to maximize the value of equity. Let θ^* be the action choice at the target level of debt, $D - \Delta$, i.e.,

$$
\theta^* \equiv \arg\max_\theta V^E(D - \Delta, A, \theta).
\tag{9}
$$

To see that asset substitution increases shareholder resistance to a recapitalization, note that

$$
\begin{aligned}
V^E(D - \Delta, A) - V^E(D, A) &= V^E(D - \Delta, A, \theta^*) - \max_\theta V^E(D, A, \theta) \\
&\leq V^E(D - \Delta, A, \theta^*) - V^E(D, A, \theta^*).
\end{aligned}
\tag{10}
$$

Thus, the increase in the value of equity post-recapitalization is even smaller now than in the setting without agency costs (that is, with θ fixed at θ^*, the level of risk that shareholders would choose given lower leverage).

As the above argument reveals, the result that agency costs increase shareholders' resistance to recapitalization follows directly from the shareholders optimization problem. Thus, we can apply the same argument to demonstrate that *any* shareholder discretion over future firm investment will lead to a similar result.

For example, suppose that in addition to determining asset risk θ, management (on behalf of shareholders) has the opportunity to invest in additional assets a by raising capital k from shareholders (or reducing planned equity payouts). Moreover, suppose these decisions will be made at a later date and conditional on some future information z that is relevant to both asset returns and the profitability of the investment opportunity. Specifically, if we let $k(a, z)$ be the

cost of making investment a given information z, then the equity value function *conditional* on the investment policy functions $a(z)$ and $\theta(z)$ can be written as

$$V^E(D,A,\theta,a) \equiv$$

$$E_z \left[\int_{D/(A+a(z))}^{\infty} \big(x(A+a(z)) - t\big(x, A+a(z), D\big) - D \big)\, dF\big(x\big|z, \theta(z)\big) - k(a(z), z) \right] \quad (11)$$

where the expectation is over possible information states *z*. Equity holders choose the policies (θ, a) to maximize (11) given outstanding debt *D*.

In this case, in addition to asset substitution, leverage may lead to future underinvestment due to the traditional debt overhang problem identified by Myers (1977). The next result demonstrates that, once again, the possibility of future underinvestment and risk shifting, while detrimental to total firm value, will only increase the cost to shareholders from a current recapitalization.

Proposition 3 (Agency Costs): *Although shareholder-creditor conflicts regarding investment may raise the benefits of a leverage-reducing recapitalization for total firm value, they also raise the costs of a recapitalization for shareholders (relative to a setting in which investments were fixed at the optimal policy given lower leverage).*

Proof: See Appendix.∎

The intuition for Proposition 3 follows the same logic as (10): Agency costs mitigate the decline in the value of equity as leverage increases, as shareholders take actions that transfer wealth from creditors.[21] But this effect implies that equity holders also gain less from a leverage reduction, and they must pay more for the debt in anticipation that such wealth transfers will be diminished. Thus, even though agency costs raise the cost of leverage, they impede shareholders' incentive to reduce it.

3.2. Leverage Distortions: The Ratchet Effect

The standard tradeoff theory of capital structure posits that firms choose debt in order to maximize total firm value given the countervailing frictions of tax benefits versus distress and agency costs associated with leverage. Our prior results suggest, however, that once leverage is already in place, debt overhang creates a powerful dynamic that will distort shareholder incentives. In particular, we show that not only will the shareholders not choose to reduce leverage, they will *always* prefer to *increase* leverage if they have the opportunity to do so, even if this additional leverage further reduces firm value. In other words, leverage begets additional leverage, creating a leverage ratchet effect.

---

[21] Essentially, we follow the proof of Proposition 1 state-by-state conditional on *z*.

The observation that shareholders can gain by issuing new debt that has equal (or higher) priority to its existing debt is well known, and results from the fact that the new debt will dilute the claims of existing creditors. Fama and Miller (1972) argue that seniority provisions, requiring any new debt to be junior to existing creditors, prevent such dilution.

In the presence of agency costs, however, strict priority rules are insufficient to fully insulate senior creditors from the consequences of future debt issues. As demonstrated by Bizer and DeMarzo (1992), sequential borrowing with *junior* debt can still be detrimental to more senior claims because of its influence on future firm actions, such as risk-shifting, underinvestment, or earlier default. In other words, by increasing future agency costs, new junior debt can harm existing senior creditors. [22] Equity holders do not internalize this harm, distorting their decision to engage in additional borrowing, as illustrated in Figure 1. As we show below, this feedback effect creates an additional agency cost of debt: *existing leverage distorts future leverage decisions of the firm*.
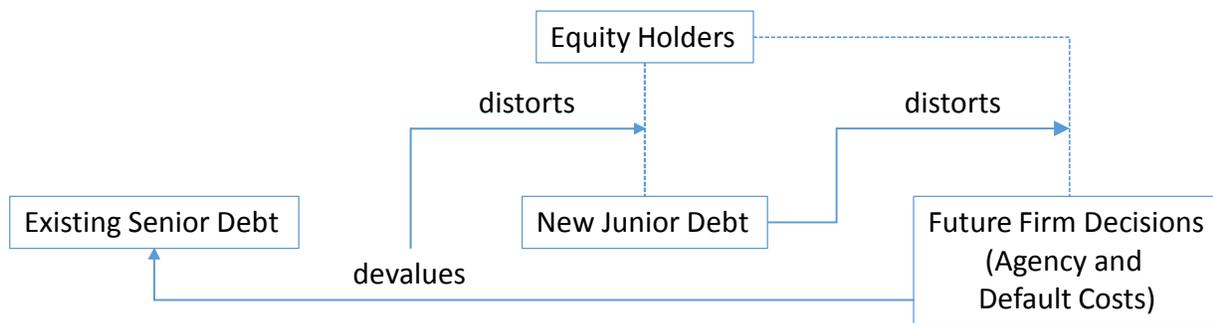

Figure 1: Sequential Borrowing Agency Distortions
Even when new debt must be junior, it distorts future incentives in a way that harms senior creditors. This loss to senior creditors is not internalized by shareholders, distorting incentives with regard to the decision to increase leverage.

To demonstrate the leverage ratchet effect, consider our setting with taxes, default costs, and asset substitution, and suppose that any existing debt is protected so that any new debt issued is junior to all other outstanding debt claims (and hence there is no direct dilution of existing creditors). Nonetheless, we show that under broad conditions, a leveraged firm will *always* find it optimal to increase its leverage. To formalize this result, let $G(D,D')$ be the gain to shareholders when a firm with existing debt D increases its debt to $D' \geq D$, by issuing new junior debt with face value $D' - D$:

$$G(D,D') = V^E(D',A) - V^E(D,A) + (D'-D)q^J(D,D',A) \tag{12}$$

where $q^J(D,D',A)$ is the price at which the new junior debt is sold. Then we have the following key result:

---

[22] Brunnermeier and Oehmke (2013) show that a similar effect arises if shareholders can issue new debt with a shorter maturity than existing debt, as its earlier maturity gives it effective seniority.

Proposition 4 (Leverage Ratchet Effect): *Given initial debt D, suppose the firm has the opportunity to adjust its debt on a one-time basis. Then, the optimal choice $D^*(D) = \arg\max_{D'} G(D, D')$ satisfies:*

- *If the firm has no initial debt, then the amount of debt D*(0) that maximizes shareholders' gain also maximizes the total value of the firm.*
- *If the firm has outstanding debt $D > 0$, shareholders never gain by reducing leverage. Moreover, if the probability of default is continuous at D and the marginal expected tax benefit of debt is positive, it is always optimal for shareholders to increase leverage by issuing new debt even if this new debt must be junior to existing claims, and even if it reduces total firm value; that is $D^*(D) > \max(D, D^*(0))$.*

Proof: See Appendix. ■

The first statement in Proposition 4 is obvious – absent pre-existing debt, shareholders internalize any costs to creditors via the price they will receive for the new debt, and hence will choose leverage to maximize total firm value. This observation is the basis for the standard optimality prediction of the tradeoff theory.

The second statement in Proposition 4, however, makes clear that when there is a marginal tax benefit from debt, this prediction must fail if the firm makes new leverage decisions once existing debt is in place. It implies that even if the firm is already excessively leveraged (relative to the tradeoff theory optimum), equity holders will still be tempted to increase leverage further.

While the proof of this result in full generality is complicated by technicalities related to differentiability and continuity, the intuition is straightforward. All of standard *costs* associated with leverage in the tradeoff theory – default costs, investment-related agency costs, and even distortions of future leverage choices – are decisions which are *optimized* by equity holders (e.g., equity holders optimally determine when to exercise their put option to default). Thus, by a standard envelope argument, these costs have no first order impact on the cost of new debt to shareholders, and thus the only first order effect is the incremental tax benefit. Intuitively, from (8), letting θ be the optimal level of future investment decisions by shareholders given debt D,

$$\tfrac{\partial}{\partial D}V^E(D, A) = -\left(1 - F\left(D/A|\theta\right)\right) - \int_{D/A}^{\infty} t_D\left(x, A, D\right) dF\left(x|\theta\right)$$

$$= -q^J(D, D, A) - \int_{D/A}^{\infty} t_D\left(x, A, D\right) dF\left(x|\theta\right)$$

(13)

and thus from (12),

$$G_2(D, D) = -\int_{D/A}^{\infty} t_D\left(x, A, D\right) dF\left(x|\theta\right) > 0.$$

(14)

That is, the marginal gain from new incremental debt is equal to the associated incremental tax shield.[23] Thus, independent of the amount of debt already in place, shareholders always have a positive incentive to increase debt further until its interest tax shield is fully exploited.

We illustrate the result of Proposition 4 in Figure 2, which shows G and V^E for different levels of initial debt D.
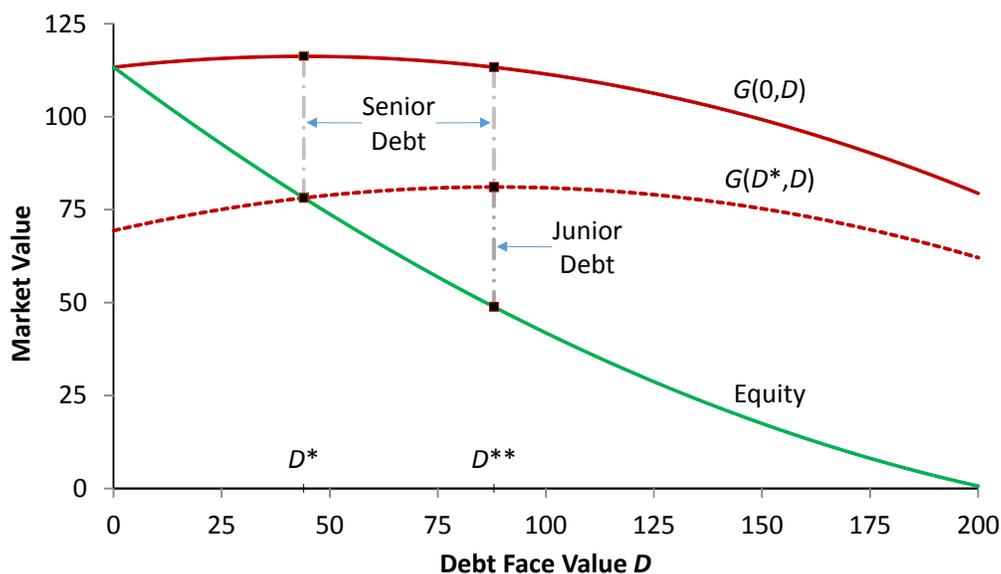

Figure 2: The Leverage Ratchet Effect
Although a new issue of junior debt reduces firm value, it decreases the value of senior debt by even more, leading to a gain for shareholders. At the margin, shareholders benefit from a higher interest tax shield, whereas default and agency costs are second order.

If the firm initially has no debt, equity holders would prefer debt D^* that maximizes total enterprise value (equity plus debt). Once debt D^* is in place however, equity holders face the potential gain $G(D^*, D)$, and so would ideally choose D^{**} if given a one-time opportunity to issue new junior debt. Note that total enterprise value is lower at D^{**}, but equity holders still gain because the value of the senior debt declines. Senior creditors lose because for them, the increased agency and default costs that arise with greater total leverage have a first-order impact on their payoff. (We will describe the specific numerical parameters used to generate Figure 2 in Section 4.1.)

---

[23] We note in passing that this observation provides some justification for the standard practice in capital budgeting valuation to consider only incremental tax benefits associated with new debt while ignoring the impact of bankruptcy and agency costs. Once existing debt is in place this approach correctly captures the marginal impact to the firm's shareholders.

4. Leverage Ratchet in Dynamic Equilibrium

Our results thus far demonstrate that under shareholder control, leverage is "irreversible" once put in place and moreover creates a desire for even more leverage. Our analysis, however, has been restricted to one-time static adjustment of the firm's debt level. In a dynamic context, creditors will include the cost of such future distortions in the price they are willing to pay for the debt upfront. Moreover, this price adjustment will naturally affect the firm's optimal initial leverage choice. In this section we develop a simple and tractable dynamic model to explore and highlight the key consequences of the leverage ratchet effect.

We begin in Section 4.1 by developing a simple stationary dynamic model of a leveraged firm, in which the firm earns taxable income at a fixed rate until the random arrival of an exit event. Because interest on debt is tax deductible, the firm has a tax benefit from leverage, but we assume debt distorts the payoff at exit either due to default or agency costs. We then illustrate the leverage ratchet effect in the context of this model and consider the equilibrium debt choice when the firm has repeated opportunities to issue debt in Section 4.2. In Section 4.3 we allow for shocks to the income and value of the firm and show how these shocks will lead leverage to increase over time. Finally, in Section 4.4 we discuss the potential role of countervailing forces such as repayment of maturing debt, asset growth, and debt covenants in mitigating the leverage ratchet.

4.1. A Simple Dynamic Tradeoff Model

Suppose the firm generates earnings before interest and taxes at a constant rate y until the arrival at random time τ of a liquidation event. (This setting could correspond, for example, to an investment in a fixed income security or portfolio.) The interest rate r, liquidation arrival rate λ, and tax rate t are constant, and the firm issues debt with fixed face value D and constant coupon rate c such that $cD \leq y$. The value of the firm at liquidation is given by Y_θ, which is independent of τ, and where the parameter θ reflects investment or strategy choices that are chosen to maximize shareholder value. In that case, the value of the firm's equity is given by[24]

$$V^E(D) = \max_\theta E\left[\int_0^\tau e^{-rs}(y - cD)(1-t)ds + e^{-r\tau}(Y_\theta - D)^+\right]$$
$$= \frac{r}{r+\lambda}\frac{(y-cD)(1-t)}{r} + \frac{\lambda}{r+\lambda}\max_\theta E[(Y_\theta - D)^+].$$

(15)

For $D \geq 0$, we define the function

$$\phi(D) \equiv \max_\theta E[(Y_\theta - D)^+]$$

(16)

to represent the expected payoff to equity in liquidation, and note that by standard arguments ϕ must be nonnegative, strictly decreasing (if $\phi > 0$) and weakly convex in the debt face value D,

---

[24] We use the notation $x^+ = \max(x,0)$, and the fact that given arrival rate λ, $E[e^{-r\tau}] = \lambda/(r+\lambda)$.

and $\phi'(0) \geq -1$. For technical convenience we assume Y_θ is continuously distributed and ϕ is twice differentiable. Finally, we write $\theta(D)$ to represent the argmax in (16).

We assume that in the event of default, debtholders are unable to recover any of the asset value and receive a payoff of zero. This assumption simplifies but is not necessary for our results (and we will develop shortly an equivalent setting without bankruptcy costs). Given a fixed face value, the total value of debt is therefore

$$V^D(D) = E\left[\int_0^\tau e^{-rs} cD\,ds + e^{-r\tau} D\mathbf{1}[Y_{\theta(D)} \geq D]\right] = \frac{r}{r+\lambda}\frac{cD}{r} + \frac{\lambda}{r+\lambda} D \Pr(Y_{\theta(D)} \geq D). \quad (17)$$

From (16) and the fact that Y_θ is continuously distributed,

$$\phi'(D) = -\Pr(Y_{\theta(D)} \geq D) = -\Pr(Y_{\theta(D)} > D). \quad (18)$$

Hence the debt has a price per dollar of face value of

$$q(D) = \frac{r}{r+\lambda}\left(\frac{c}{r}\right) - \frac{\lambda}{r+\lambda}\phi'(D). \quad (19)$$

Note that, by our assumption of zero recovery value, this price applies to both junior and senior debt.

We assume the firm exhausts the tax benefits of debt if the coupons cD exceed the cash flows y, or equivalently if $D > \bar{D} \equiv y/c$. The total value of the firm is then

$$V^F(D) = V^E(D) + V^D(D) = \frac{y(1-t) + t\min(y, cD) + \lambda(\phi(D) - D\phi'(D))}{r+\lambda}. \quad (20)$$

The convexity of ϕ implies that $\phi(D) - D\phi'(D)$ declines with D, and thus (20) captures the standard tradeoff between tax savings and bankruptcy concerns. Solving for the level of debt which maximizes firm value, we see that an interior solution D^* must satisfy the first-order condition

$$D^* = \frac{tc}{\lambda\phi''(D^*)}, \quad (21)$$

which makes the tradeoff explicit.

Bankruptcy Costs or Agency Costs?

Our formulation of bankruptcy concerns includes both moral hazard (in the choice of θ) and bankruptcy costs (given a zero recovery rate). We note, however, that the same function ϕ, which indicates the expected payoff to shareholders in liquidation, can also be derived from a pure agency model or a pure bankruptcy cost model, described as follows:

1. *Pure Agency Costs:* With probability θ, the firm has a successful exit with value $g(\theta)$, and is worthless otherwise. That is, $Y_\theta \in \{0, g(\theta)\}$ and $\Pr(Y_\theta = g(\theta)) = \theta$. In this case,
$$\phi^A(D) = \max_{\theta \in [0,1]} \theta(g(\theta) - D)^+.$$

2. *Pure Bankruptcy Costs:* The liquidation value is independent of θ; that is, $Y_\theta = Y$. However, in the event of default, debtholders recover nothing. In this case,
$$\phi^B(D) = E[(Y - D)^+].$$

The following result demonstrates that we can construct the corresponding exit value g, or distribution for Y, to match any payoff function ϕ:

Lemma (Equivalence of Agency and Bankruptcy Cost Models): *Given any ϕ from (16), there exists an exit value g in the pure agency model, and a distribution for Y in the pure bankruptcy cost model, such that $\phi = \phi^A = \phi^B$.*

Proof: See Appendix. ∎

The following example will be used throughout this section to illustrate the dynamic issues created by the leverage ratchet effect.

Example 1: Consider a pure bankruptcy cost model in which Y is uniformly distributed on $[0, \overline{Y}]$. Then

$$\phi(D) = \frac{(\overline{Y} - D)^2}{2\overline{Y}}. \tag{22}$$

This same payoff function arises from the pure agency model with exit value $g(\theta) = \overline{Y}\left(1 - \tfrac{1}{2}\theta\right)$, so that higher leverage induces shareholders to choose riskier projects (lower θ). In either case, using (21), the debt level that maximizes total firm value is given by

$$D^* = \min\left(\frac{tc}{\lambda}Y, \overline{D}\right).$$

Time Inconsistency

Now suppose that the firm starts out with the value-maximizing debt level. Will it stay there? In the absence of binding covenants, the answer is no: Once debt is in place, re-optimization by shareholders leads to an additional increase in leverage.

To see this effect, given initial debt $D \leq \bar{D}$, we can calculate the gain to equity holders from a permanent change to debt level $D + d \leq \bar{D}$ as follows:[25]

$$G(D, D+d) = V^E(D+d) - V^E(D) + dp(D+d)$$

$$= \underbrace{\frac{r}{r+\lambda} \frac{tcd}{r}}_{\text{Incremental Tax Shield}} - \underbrace{\frac{\lambda}{r+\lambda}\left(\phi(D) - \left(\phi(D+d) - d\phi'(D+d)\right)\right)}_{\text{Incremental Agency/Bankruptcy Costs}}. \qquad (23)$$

The second term on the right-hand side of (23), which is non-negative by the convexity of ϕ, represents the incremental agency or bankruptcy costs borne by shareholders (via the debt price). As in Proposition 4, this term is of the second order at $d = 0$, and we have the following analog to (14):

$$G_2(D, D) = \begin{cases} \dfrac{tc}{r+\lambda} & \text{if} \quad D < \bar{D} \\ 0 & \text{if} \quad D > \bar{D} \end{cases}. \qquad (24)$$

In other words, the gain from a marginal dollar of new debt is simply equal to the value of its incremental tax shield. The optimal quantity of *new* debt can be found be setting $G_2(D, D+d) = 0$, which implies $d = \min\left(d^*, \bar{D} - D\right)$ where

$$d^* = \frac{tc}{\lambda\phi''(D+d^*)}. \qquad (25)$$

As long as the upper bound for the tax shield has not been reached, the optimal amount of new debt is strictly positive.

Thus, the simple tradeoff level of debt that is obtained by maximizing the value of the firm initially is *not* time consistent.[26] Unless this level itself exhausts the tax shield, shareholders find it desirable to raise debt further. Like the value-maximizing amount of initial debt, the optimal incremental debt increases with the tax shield and decreases with the likelihood of liquidation and the intensity of the agency or bankruptcy costs (measured by ϕ'').

With the quadratic specification of ϕ in Example 1,

$$d^* = \frac{tc}{\lambda}\bar{Y}. \qquad (26)$$

Note that, if $d^* < \bar{D}$, then this quantity is the same as the firm-value maximizing debt level D^* in Example 1. Therefore, if the firm already has optimal leverage D^*, shareholders would choose to double the firm's leverage if given the opportunity. For example, in Figure 2 of Section 3.2 (which

---

[25] Recall that because recovery rates are zero, this calculation applies independent of the priority of the new debt. In other words, the incentives we are identifying apply even if existing debtholders can enforce seniority with respect to any new debt.

[26] For an early discussion of time inconsistency in repeated lending decisions, see Hellwig (1977).

is illustrated using parameters $r = c = 5\%$, $y = 10$, $t = 40\%$, $\lambda = 10\%$, and $\bar{Y} = 220$) we have $D^* = d^* = 44$ and $D^{**} = 88$. Indeed, until the tax shield is exhausted, shareholders always want to issue new debt with face value d^*, regardless of how much debt they already have. This finding is illustrated by the dashed straight line (parallel to the 45º line) in Figure 3 below, which shows shareholders' desired total leverage $D + d^*(D)$ for any value of D (using the same parameters as above).

4.2. Stable Leverage without Commitment

We have shown that if shareholders are granted a one-time opportunity to issue debt, they will *always* have an incentive to issue new debt up to the point that the tax shield is exhausted. While the incentive to increase leverage is obvious if new debt has equal or higher priority, it persists even when all new debt must be junior to any existing claims so that there is no dilution. Thus, if shareholders are unconstrained in their ability to issue junior debt, and have this opportunity repeatedly, their initial choice of a debt level will not persist. The simple tradeoff approach, determining leverage by value maximization, will not correspond to an equilibrium of the dynamic model.

How will leverage be determined in a dynamically consistent setting? In addressing this question, one must take account of the fact that creditors will recognize the leverage ratchet effect and price the debt appropriately in anticipation of future leverage changes. To analyze this possibility, we consider the following dynamic game. At each time *s*, the firm has existing debt D_s. It then announces a new quantity of junior debt to issue (or repurchase). The price q_s of the incremental debt is set competitively in the market; in equilibrium, of course, this price will reflect creditors' anticipation of the firm's future leverage choices given the new debt level. While this new debt must be junior to any existing debt, it is otherwise unconstrained. Absent commitment, what will be the equilibrium leverage of the firm?

To determine the equilibrium leverage choice, consider first the case in which $D_s \geq \bar{D}$. In that case, the firm has exhausted the debt tax shield, and has no incentive to issue additional debt. From our earlier results in Section 2, equity holders will also not choose to repurchase debt. Thus, debt level $\bar{D}$ or higher is "stable" – once it is attained, equity holders will not benefit from any change.

Next, suppose D_s is such that equity holders will gain by adjusting debt to $\bar{D}$. That is suppose $G(D_s, \bar{D}) > 0$. Then it is clear that D_s cannot be a stable outcome, as equity holders would gain by issuing new debt until $\bar{D}$ is reached. Buyers of the new debt would be willing to pay $q(\bar{D})$ since they know the firm will not change debt from that level once it is attained.

Finally, suppose the current debt level D_s is the maximal debt level below $\bar{D}$ such that $G(D_s, \bar{D}) \leq 0$. Then equity holders could not gain by issuing any new debt, since if they did, by

the previous logic creditors would anticipate that they would keep issuing until the debt had face value $\bar{D}$. But at the price $q(\bar{D})$, issuing new debt would not be profitable for shareholders.

Repeating this logic, we obtain the following construction for an equilibrium. There exists a set of stable debt levels, $D^0 > D^1 > \ldots$, defined recursively by $D^0 = \bar{D}$ and

$$D^{n+1} = \max\{D < D^n : G(D, D^n) \le 0\}. \tag{27}$$

Note that given our assumptions, G is continuous, so (27) implies $G(D^{n+1}, D^n) = 0$; that is, equity holders are just indifferent between sequential stable debt levels. We then have the following equilibrium, in which the firm's leverage "ratchets up" to the next stable debt level:

Proposition 5 (Stable Leverage Equilibrium): *The following strategies represent a subgame perfect dynamic leverage equilibrium: Given any debt level D, shareholders immediately increase leverage to the next highest stable leverage level D^n, defined as*

$$D^+(D) \equiv \begin{cases} \min\{D^n : D^n \ge D\} & \text{if} \quad D < \bar{D} \\ D & \text{if} \quad D \ge \bar{D} \end{cases}.$$

The price of debt is given by $q(D^+(D))$.

Proof: See Appendix. ∎ [27]

We illustrate this dynamic equilibrium using the setting of Example 1 above:

Example 1 continued: Let ϕ be as in Example 1 with $\bar{Y} \ge \bar{D}$. Then for $D \le \bar{D}$, $G(D - \hat{d}, D) = 0$ implies

$$\hat{d} = 2tc\lambda^{-1}\bar{Y} = 2d^*. \tag{28}$$

Therefore, $D^n = \bar{D} - n\hat{d}$. We illustrate this equilibrium in Figure 3 (for parameters $t = 40\%$, $r = c = 5\%$, $\lambda = 10\%$, $y = 10$, and $\bar{Y} = 220$, as given earlier). Note that the tax shield is exhausted with debt level $\bar{D} = y/c = 200$.

From (26), the debt level D^* that maximizes $G(D, D^*)$ is $D^*(D) = D + 44$. From (28), the debt level $\hat{D}$ that makes equity holder indifferent so that $G(D, \hat{D}) = 0$ is $\hat{D}(D) = D + 88$.[28] The stable debt levels are therefore $D^0 = \bar{D} = 200$, $D^1 = 200 - 88 = 112$, and $D^2 = 112 - 88 = 24$.

---

[27] The equilibrium we describe is the unique subgame perfect equilibrium given pure strategies in *D*. Other equilibria are possible, however, if we allow mixing or non-Markov strategies (e.g. mixing between D^n and D^{n-1}, or moving from one to the other after some period of time, would lead to an increase in all subsequent stable points). That said, if debt quantities are discrete (e.g. there is a minimum increment to the face value of $1), then the equilibrium we describe is generically unique (since then $G(D^n, D^{n-1}) < 0$).

[28] $\hat{D}(D) = D + 88$ up to the point $D = D^1$. For $D > D^1$, because there is no tax shield above D^0, $\hat{D}(D) = D + \sqrt{(D^0 - D) \times 88}$.

Figure 3 plots D, $D^*(D)$, $\hat{D}(D)$ and finally the stable equilibrium $D^+(D)$ which is a step function showing the jump to the next stable point.

   With these parameters, given no initial debt, the firm would choose $D^+(0) = 24$, which is lower than the firm value maximizing level $D^*(0) = 44$. Note that if cash flows were lower so that $y = 8$, however, then $D^0 = \bar{D} = 160$, and $D^+(0) = 72$. In that case, the firm would choose higher leverage than the firm value maximizing level (which is still 44). ∎
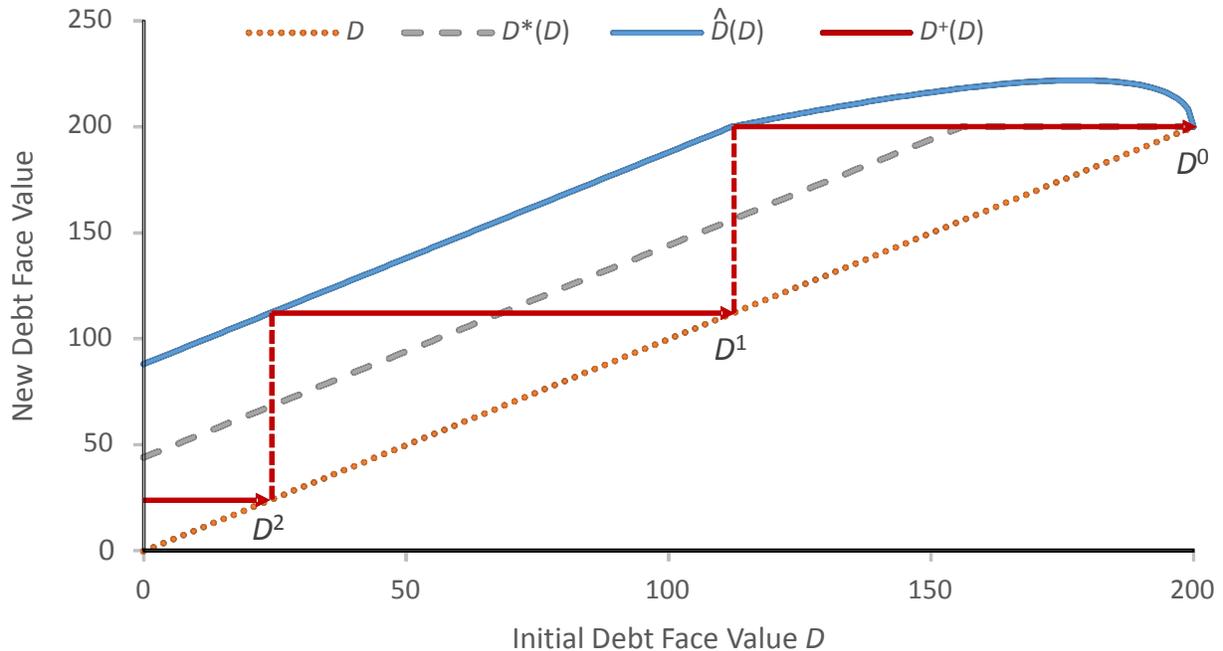

Figure 3: Stable Leverage Equilibrium

   The example shows that, because of the leverage ratchet effect, the equilibrium level of firm leverage can be quite different from anything predicted by the static tradeoff approach. Whereas the tradeoff approach predicts a debt level of 44, the lowest equilibrium debt level with the ratchet effect is 24; however, once debt is at 24, a small perturbation might lead shareholders to raise the level all the way to 112. And while the first level is almost 50% lower than that predicted by the tradeoff theory, the second is more than 100% higher. More importantly, the mechanisms determining potential equilibrium leverage are quite different from the simple optimization presumed by the tradeoff approach.

   The equilibrium described in Proposition 5 is unnaturally stark. In particular, the result that the firm makes a one-time adjustment to its debt and then debt remains stable from that point onward depends on our assumption that all of the parameters of the firm are constant over time. More realistically, we should expect the firms' cash flows, likelihood of and value in liquidation, as well as macro factors such as interest rates and tax rates, to be time varying. In that case,

permanently stable debt levels as described in Proposition 5 cannot be expected, as we demonstrate next.

4.3. Shocks and Leverage Ratchet Dynamics

Thus far, we have allowed the firm to adjust its leverage in an environment without shocks. In that case, once a stable leverage level is attained, there is no reason for the firm to adjust leverage further. However, suppose the firm is now subjected to shocks to its cash flows, tax rate, or the intensity of bankruptcy or agency costs. How will the firm's leverage respond to these shocks? And how will debt be priced in anticipation of these shocks and the firm's reaction to them?

Intuitively, starting from any stable debt level prior to the shock, it is unlikely that this debt level will remain stable after the shock. Thus, as in our prior analysis, we would expect the firm to increase leverage to the next stable debt level given the new parameters. If shocks are repeated, then after each shock we will see leverage ratchet upward until the point that the tax shield has been exhausted or the firm defaults.

We formalize this by extending our prior model to allow for the Poisson arrival of a regime shift. We index regimes by $j \in \{1, \ldots, J\}$. In regime j, tax rates, interest rates, debt coupons, and cash flows are given by (t_j, r_j, c_j, y_j), and there is a random arrival of a liquidation event with arrival intensity λ_{j0} and payoff ϕ_j. In addition, there is an independent random arrival with intensity λ_{jk} of a shock that moves to regime k. The debt is priced and leverage decisions are made in anticipation of these potential shocks. We apply the same logic as in Proposition 5 to establish the following:

Proposition 6 (Leverage Ratchet Dynamics): *There exists a subgame perfect equilibrium of the following form: For each regime j there will be a set of stable debt levels*
$$D_j^0 = \bar{D}_j \equiv y_j / c_j > D_j^1 > \ldots > D_j^n > \ldots.$$
Upon entering regime j with current debt D, the firm will increase leverage to the next stable level
$$D^{j+}(D) = \min\{D_j^n : D_j^n \geq D\} \cup \{D : D \geq D_j^0\}.$$

Proof: See Appendix. ∎

We show in the proof of Proposition 6 how to construct the stable points for each regime. As before, starting from D_j^n, we find the next lower debt level such that equity holders would just be indifferent between not issuing additional debt, and issuing up to the point D_j^n. The equity value function and debt price are calculated given this issuance policy. Because debt always ratchets upward, we can solve for these values by induction on the level of debt D.

Over time, regime shocks will cause the equilibrium debt level to increase monotonically as the firm ratchets up to the next stable point with each transition to a new regime. As long as the

stable points for each regime do not coincide, then Proposition 6 implies that the debt level of the firm will continue to ratchet up over time. Indeed we have the following immediate result:

Corollary (Limit Values): *Suppose the regimes are recurrent (i.e., starting from any regime there is a positive probability of ultimately transitioning to any other regime). Then starting from debt level D, the debt level will increase over time until the next universal stable point in the set* $\cap_j \{D_j^n\}$ *or, if no such point exists, to* $\bar{D} \equiv \max_j \bar{D}_j$.

Example 2: Consider a setting as in Example 1 with two regimes that differ in terms of the firm's cash flow stream with $y_1 = 10 > y_2 = 8$. Both regimes share the same (t, r, c, λ, ϕ) as in Example 1 and Figure 3. Then we can show that the stable points for each regime are distinct (until the point that tax shield becomes exhausted) and will alternate in magnitude, with the distance between them shrinking with the frequency of the regime shifts.

Figure 4 shows the stable points for each regime when the arrival intensity of a regime shift is $\lambda_{12} = \lambda_{21} = 20\%$ (first panel) or 100% (second panel), corresponding to an average regime length of 5 years or 1 year. In the first panel, starting with no leverage, the firm's initial debt choice is 19 (if cash flows are low) or 0 (if cash flows are high). Then, with each subsequent change in the level of the firm's cash flow, debt will "ratchet up" to the next stable point shown in the figure. After at most 3 shocks, debt will exceed the first best ($D^*(0) = 44$), and after 8 shocks debt will reach its maximal level of 174; at that level of debt the tax shield is exhausted for the low cash flow firm and this level is a common stable point. In the second panel, with an average regime length of only one year, the distances between adjacent stable points are smaller, and so are the steps by which debt ratchets up. In that case it will take up to 9 shocks for debt to exceed $D^*(0) = 44$, and up to 24 shocks to reach the maximal debt level of 181. If regime changes, i.e. the shocks to extraneous data determining the desirability of leverage, arrive continuously, the ratcheting up of debt will become continuous as well (except for a possible jump prior to the final stable point).[29] ∎

This example confirms the intuition that when there are fluctuations in factors that affect the costs or benefits of leverage, the leverage ratchet effect will induce shareholders to repeatedly "ratchet up" the leverage of the firm. (Similar results are obtained, for example, if the tax rates or default rates fluctuate over time.) Naturally, in anticipation of future ratchets, the equity holders limit the amount of additional leverage they are willing to take on today. Whereas the firm will be initially under-leveraged relative to the value-maximizing debt choice, it will ultimately have excessive leverage, with leverage at any point determined by its cash flow history.

---

[29] See DeMarzo and He (2015) for the development of the model in a general continuous-time framework, as well as a methodology for characterizing equilibrium.
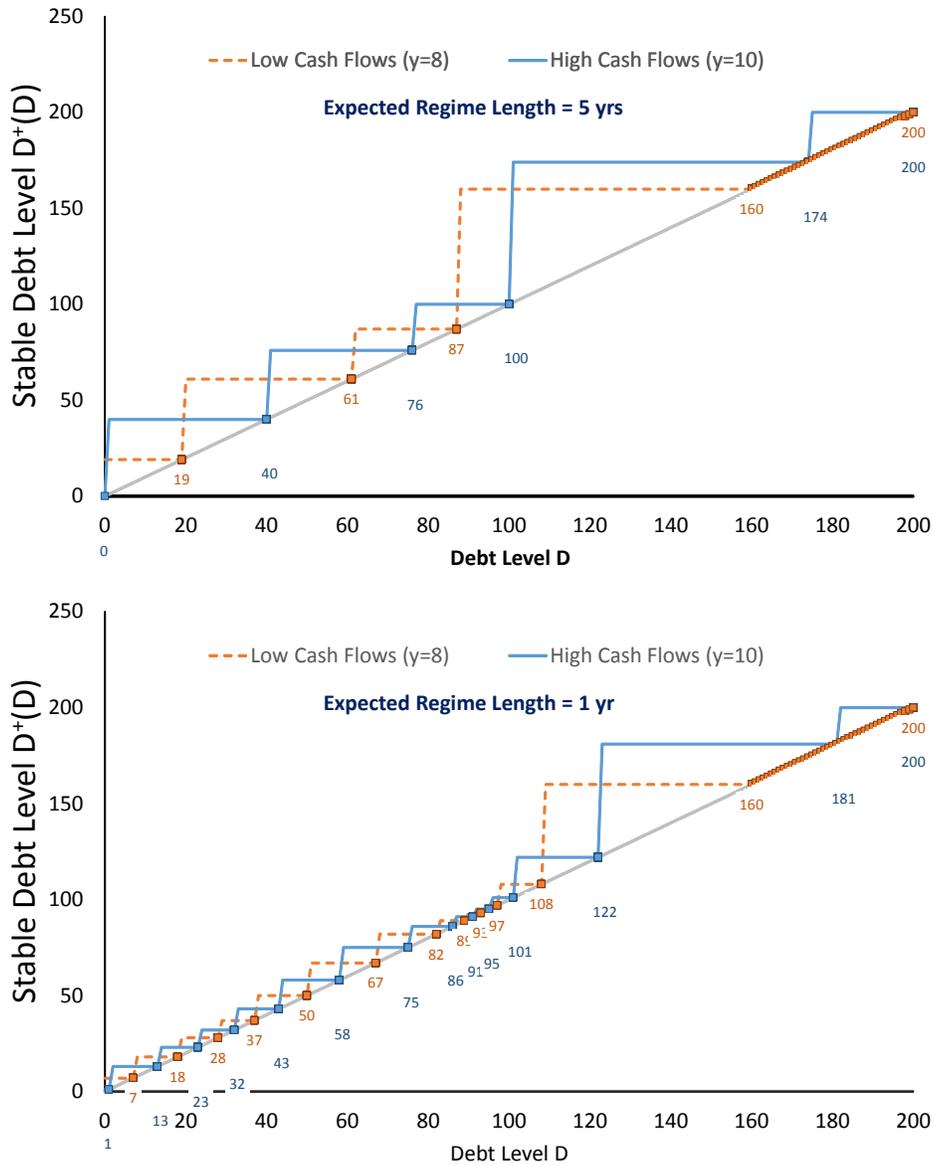
Figure 4: Leverage Ratchet Dynamics with Regime Shocks

4.4. Countervailing Forces: Covenants, Maturity, and Growth

Our analysis has shown that shareholders of a corporation resist leverage reductions and instead may increase leverage even when these choices reduce the total value of the firm. As the preceding example starkly illustrates, absent countervailing forces, the resulting leverage ratchet effect may lead initial leverage to be low — because the cost of debt reflects the anticipated future inefficiencies due to agency conflicts — but with a gradual transition to ever higher leverage as debt is increased in response to exogenous shocks.

Of course, in reality there are countervailing forces that may prevent leverage from increasing or cause it to decline. First, debt covenants may place some restriction on funding choices by shareholders. Second, finite debt maturity provides an ex ante commitment to retire at least some existing debt. Third, asset growth may raise the value of equity and thereby reduce leverage. We discuss each of these in turn.

Creditors who understand that they may be harmed by the subsequent behavior of shareholders or managers can put in place covenants to try to prevent such behavior. For example, covenants may impose caps or other restrictions on equity payouts or future debt issuance.[30] In reality, even with covenants in place, shareholders retain some discretion, because covenants are necessarily incomplete and do not cover all possible states of the world, they are costly to enforce, and they may be renegotiated. Leaving discretion for shareholders may be useful for allowing them the flexibility to respond to opportunities that may benefit all stakeholders. However, discretion leaves room for debt overhang, risk shifting and leverage ratchet effects. The scope that shareholders have for exploiting their flexibility is especially large if there are many creditors and free-rider problems prevent creditors from being able to respond to abuses by shareholders of their flexibility.

Issuing debt with a finite maturity is equivalent to an ex ante commitment by shareholders to repurchase debt at face value and thus reduce leverage at a pre-specified time. In the context of our example in Section 4.3, for example, if some portion of the firm's debt were to mature, leverage would fall and the ratchet would "restart" from a lower debt level. If all of the firm's debt were to mature at the same time, shareholders and managers would again reevaluate leverage from the position of an unleveraged firm. If maturities are very short, the resulting constraints on the leverage ratchet effect might be very tight.

The tradeoff, however, is that relying on short-maturity borrowing, or concentrating debt maturity, exposes the firm to rollover risk or to the risk of a "debt run" which can provoke a costly liquidation of assets or outright default.[31] This risk is particularly high if the firm's assets are illiquid, their value is uncertain, and debt maturities are short. In this case, incomplete information about asset values may induce creditors to run upon the slightest piece of bad news, driving the firm into default and insolvency because the assets can only be liquidated at large discounts, if at all.

Asset growth provides an alternative mechanism by which leverage might decline. If asset values rise, the value of equity grows, which counteracts the ratchet effect. Asset growth can occur simply because the market value of the firm's existing assets increases. Growth may also arise

---

[30] Note, however, that the most common restriction whereby any new debt must be junior to existing debt is insufficient to prevent the costs associated with the leverage ratchet effect. Because of default costs and agency costs, even the issuance of junior debt can harm existing creditors.

[31] See, e.g. He and Xiong (2012) and He and Milbradt (2014). Diamond and He (2014) also show that short maturity may exacerbate debt overhang. Importantly, if short-term debt can be renewed at prices that do not reflect the firm's risk, then it cannot help alleviate the inefficiency of excessive leverage. A particular example is the case where the debt is explicitly or implicitly guaranteed, like bank deposits. With guarantees, runs may be prevented but the ratchet effect is actually reinforced and exacerbated. We discuss the critical relevance of our model to banking in the concluding remarks.

endogenously if new investment opportunities are sufficiently profitable so that shareholders and managers want to exploit them even if they must put up additional equity. (Here, of course, the underinvestment effect of Myers (1977) works in the opposite direction.)

Changes in asset values, however, are not always beneficial. Indeed, the leverage ratchet effect is likely to be most costly in the aftermath of a decline in asset values. In related work, DeMarzo and He (2015) develop a continuous-time model similar to Leland (1998), with finite debt maturity and (potentially endogenous) investment in assets whose cash flows evolve as geometric Brownian motion. They demonstrate that absent commitment, the leverage ratchet effect leads to continuous debt issuance by the firm, at a speed determined by the ratio of the tax benefits to the convexity of the equity value function (analogous to (25)). The countervailing effects of debt maturity and asset growth lead to a mean-reverting leverage policy in which the equilibrium debt level is strongly history dependent and proportional to a weighted-aggregate of the firm's past earnings.[32]

Fundamentally, our results in this paper suggest that naïve tests of "tradeoff theory" are likely to fail empirically. Observed levels of leverage will depend on historical choices and their interaction with both covenants and other frictions, with adjustments to leverage driven by the asymmetric preferences of shareholders.

The leverage ratchet effect also has clear implications for leverage dynamics. It suggests that firms may have asymmetric responses to shocks that impact leverage choices, such as changes in tax rates. Increases in the value of the debt tax shield should induce increases in leverage, but reductions in the value of the tax shield would not cause a similar fall in leverage. Such asymmetry has been documented empirically by Heider and Ljungqvist (2015).

5. Alternative Ways to Adjust Leverage

In the paper thus far, we have considered only pure recapitalizations as a means to change leverage. In a pure recapitalization the (real) assets of the firm are held fixed. But a pure recapitalization is not the only method available to change leverage. For example, when shareholders must reduce leverage, either due to covenants or to regulations, leverage can be also be reduced through adjustments to the scale of the firm's assets through either of the following transactions:

- **Asset Sales**: The firm sells assets and uses the proceeds to repurchase debt, thus lowering leverage without issuing new equity.

---

[32] Specifically, given earnings Y_t, D_t is proportional to $\left(\int_0^t \delta^s Y_{t-s}^{1/\gamma} ds \right)^\gamma$. DeMarzo and He (2015) also show that while the price of debt is lower in anticipation of new issues, the equilibrium value of equity is the same as it would be if the firm committed not to issue new debt – shareholders dissipate the tax benefits of new leverage via increased agency or default costs.

- **Asset Expansion:** The firm issues equity and uses the proceeds to buy additional assets, thus reducing leverage without repurchasing debt.[33]

One important question is whether our finding of shareholder resistance to leverage reduction through a pure recapitalization also applies to these alternative ways to reduce leverage, and what method shareholders would prefer if they have a choice.

The three different alternatives to reduce leverage are illustrated in Figure 5.
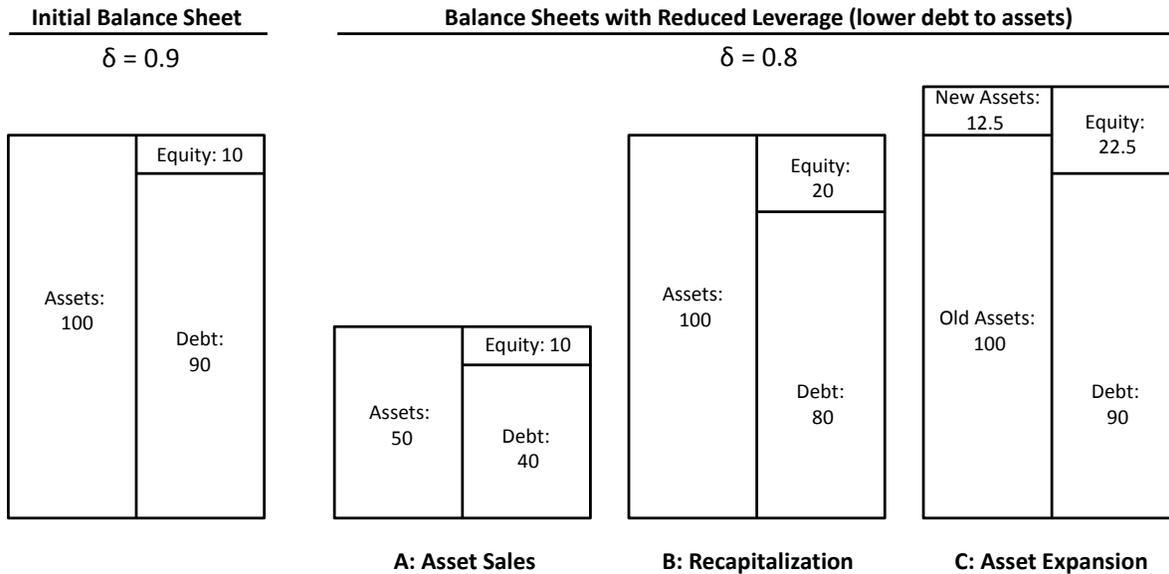

Figure 5: Alternative Responses to Increased Equity Requirements

In this figure, we assume that the ratio of debt to assets must be reduced from 90% to 80%.[34] This can be achieved by selling half of the firm's assets (asset sales), by issuing equity equal to 10% of the firm's assets and using the proceeds to buy back debt (pure recapitalization), or by issuing equity equal to 12.5% of the firm's assets and using the proceeds to invest in new assets (asset expansion). The figure exhibits shows how the firm's balance sheet changes under each of these alternatives.

Given these alternatives, it is not obvious which outcome we should expect if shareholders are forced by covenants or regulations to reduce leverage but allowed to choose their preferred method. We begin by identifying conditions under which all modes of leverage reduction are equally undesirable to shareholders, i.e., shareholders are indifferent among them. We then relax these conditions to see how the preferred mode of leverage reduction depends on the types of assets and securities involved in the transactions and the associated frictions. We will also discuss, given

---

[33] Asset expansion was the subject of the original analysis of debt overhang in Myers (1977). Myers shows that because existing debt holders capture some of the benefit of the new investment via reduced credit risk, shareholders may refuse to undertake a new positive NPV investment project.

[34] Here we are measuring debt in terms of its face or book value, as would be the case in typical covenants.

the desirability of leverage increases according to the leverage ratchet effect, shareholders' preferences over different modes of increasing leverage when allowed by covenants or regulations.

For simplicity, as in Sections 2 and 3, we evaluate shareholders' preferences in this section as though the leverage adjustment is a one-time change. This assumption may be justified in the context of a binding covenant or regulation – shareholders' must comply with a given leverage requirement and then have no flexibility to increase leverage further. Extending the analysis to consider a dynamic equilibrium as we did in Section 4 is certainly possible using the methods we developed there, but is beyond the scope of this paper. As we will see, when leverage adjustments include both asset and securities transactions, and especially with multiple debt classes, the number of possible transactions is very large.

5.1 An Equivalence Result

The alternative approaches to reducing leverage introduced above result in different sizes (assets levels) for the firm. Let D_0 be the current face value of debt and A_0 be the current level of assets for the firm, so that $\delta_H = D_0 / A_0$ is its current debt-asset ratio. Suppose that firm is required to reduce its debt-asset ratio to a lower level $\delta_L < \delta_H$. If the firm can choose any combination of debt and assets (D_1, A_1) satisfying this debt-asset ratio – i.e., such that $D_1 = \delta_L A_1$ – which combination will shareholders prefer?

If $A_1 \neq A_0$, then assets will be either sold or purchased as part of the leverage reduction. We assume first that the assets are perfectly homogeneous, so that each unit of the assets today will generate a payoff of $\tilde{x}$ in the future. (We comment on the more general case of asset heterogeneity in Section 5.3.) We also assume that frictions related to taxes and net bankruptcy costs are homogenous with firm size. Specifically, for all (A, D) and $\delta = D / A$,

$$t(x, A, D) = t(x, 1, \delta) A \quad \text{and} \quad n(xA, D) = n(x, \delta) A. \tag{29}$$

In addition, we assume that if agency costs exist, they are also homogeneous with respect to firm size. In particular,

$$\theta^* \equiv \arg\max_\theta V^E(D, A, \theta) = \arg\max_\theta V^E(\delta, 1, \theta) \tag{30}$$

for all (A, D).[35]

Using the expressions for the value of debt and equity in Section 2, we see that when the assets and all frictions are homogeneous, the total value of the firm (equity plus debt) is proportional to its asset holdings and is given by:

---

[35] For simplicity, we do not explicitly model agency costs due to the Myers (1977) underinvestment problem. As long as new investment opportunities scale with firm size, however, the same results would apply.

$$V^F(A,D) = \int_\delta^\infty xA - t(x,A,D) \, dF\left(x,\theta^*\right) + \int_0^\delta xA - n(xA,D) \, dF\left(x,\theta^*\right)$$

$$= \left[\int_0^\infty x \, dF\left(x,\theta^*\right) - \int_\delta^\infty t(x,1,\delta) \, dF\left(x,\theta^*\right) - \int_0^\delta n(x,\delta) \, dF\left(x,\theta^*\right) \right] A \qquad (31)$$

$$\equiv v(\delta)A.$$

The homogeneity of the firm's assets also implies that the average price of the firm's debt, which we denote by $q(\delta)$, depends only on the leverage ratio $\delta = D/A$:

$$q(\delta) = q\left(\frac{D}{A}\right) = \frac{V^D(D,A)}{D}$$

$$= \int_{D/A}^\infty dF\left(x,\theta^*\right) + \int_0^{D/A} \frac{xA - n(xA,D)}{D} \, dF\left(x,\theta^*\right) \qquad (32)$$

$$= \int_\delta^\infty dF\left(x,\theta^*\right) + \frac{1}{\delta} \int_0^\delta (x - n(x,\delta)) \, dF\left(x,\theta^*\right).$$

Recall from Section 2 that if the firm has a single class of debt outstanding, it will be forced to pay the price $q(\delta_L)$ to repurchase its outstanding debt in the market as this price is the value of the debt to a bondholder who refuses to tender. Thus, to reduce its debt level to $D_1 \leq D_0$, the firm must spend $q(\delta_L) \times (D_0 - D_1)$ on debt repurchases.

Assume that the firm is a price taker in the asset market and the price at which the firm can buy or sell assets is p. It follows that to move from initial balance sheet positions (D_0, A_0) to the new balance sheet positions (D_1, A_1), the value of equity the firm must issue is:

$$\text{New Value of Equity Issued} = N = p \times (A_1 - A_0) + q(\delta_L) \times (D_0 - D_1). \qquad (33)$$

The total change in the firm's equity value from the transaction is given by:

$$\text{Change in Total Equity Value} = \nabla V^E = V^E(D_1, A_1) - V^E(D_0, A_0). \qquad (34)$$

We can therefore determine the effect of the leverage change on existing shareholders by subtracting (33) from (34). Specifically, the gain or loss for existing shareholders is given by $\nabla V^E - N$.

We are now in a position to evaluate the effect on existing shareholders from alternative methods of reducing leverage. Recall that in a pure recapitalization, there is no change to the firm's assets $(A_1 = A_0)$. With pure asset sales, all reductions in debt are financed by asset sales, so that $N = 0$. In a pure asset expansion, no debt is repurchased so that $D_1 = D_0$.

We can ask whether shareholder losses differ across these or other intermediate scenarios. As one would expect, the answer depends, among other things, on the relation between the price at which the assets can be bought and sold and the value the firm obtains by holding the asset on its balance sheet. Recall from (13) that

$$v\left(\delta_L\right) = \int_0^\infty x\,\mathrm{d}F\left(x,\theta^*\right) - \int_{\delta_L}^\infty t\left(x,1,\delta_L\right)\mathrm{d}F\left(x,\theta^*\right) - \int_0^{\delta_L} n(x,\delta_L)\,\mathrm{d}F\left(x,\theta^*\right), \qquad (35)$$

is the expected payoff of the assets to the firm net of taxes and of (net) default costs. When $p = v(\delta_L)$ and the final debt-asset ratio is equal to δ_L, buying or selling assets does not affect the value of equity. From the perspective of shareholders, the net present value (NPV) of asset sales and purchases is zero. If $p < v(\delta_L)$, the NPV of asset purchases is positive, and if $p > v(\delta_L)$, value is gained by selling assets.[36] The NPV of asset sales and purchases depends on the leverage ratio because this ratio affects taxes and other frictions incurred by the firm.

We begin with the following benchmark result, which assumes the asset price $p = v(\delta_L)$. (The firm's behavior at other values of the asset price will be considered in Section 5.3.) In this case, given homogenous assets and liabilities, shareholder losses are the same for all forms of leverage reductions:

Proposition 7 (An Equivalence Result): *Assume that $p = v(\delta_L)$, there is only one class of debt, assets and frictions are homogeneous, and the firm faces no transactions costs in buying or selling assets or the securities it issues. Then shareholders find pure recapitalization, asset sales, and asset expansion equally undesirable methods to reduce leverage. Specifically, starting from the initial position $\left(D_0, A_0\right)$, with $\delta_H = D_0 / A_0$, shareholder losses are equal to*

$$\left(q\left(\delta_L\right) - q\left(\delta_H\right)\right)D_0 + \left(v\left(\delta_H\right) - v\left(\delta_L\right)\right)A_0 > 0 \qquad (36)$$

for all $\left(D_1, A_1\right)$ with $\delta_L < \delta_H$ and $D_1 = \delta_L A_1$.

Proof: See Appendix. ∎

As an immediate corollary, this proposition implies that, under the given conditions, shareholders will resist leverage reductions through asset sales or asset expansion just as they resist leverage reductions through pure recapitalization. In other words, the leverage ratchet effect applies regardless of the mode of leverage adjustment.

At first sight, the result is perhaps surprising, but the intuition is straightforward. If asset and security sales or purchases have zero NPV, they cannot change the total value of the firm.

---

[36] Note that *holding the leverage ratio δ fixed*, the value of equity is proportional to total firm value, so that shareholder incentives coincide with maximizing firm value.

Because debt holders gain from the decline in leverage, the shareholders must lose an equal amount. The gain for debt holders is determined by the change in the average price of the debt, which depends only on the change in the firm's leverage ratio. All of this is captured in the first term in (36). The second term represents changes in the value of existing assets due to changes in tax benefits, bankruptcy costs or subsidies resulting from the reduction in leverage.

While the expression for shareholder losses is natural, it is not obvious (by inspection) that its sign should be positive. As remarked in Section 2, the standard intuition is that the sign will depend on the relative magnitude of the debt gains (captured by the change in q) and the efficiency gains (captured by the change in v). But Proposition 2 guarantees that the former will always dominate, even when the leverage reduction increases firm value ($v(\delta_L) > v(\delta_H)$).

If the reduction in leverage is mandated by regulation or covenants, one question that arises is whether the reduction can be achieved without violating the limited liability of shareholders. For the move from (D_0, A_0) to (D_1, A_1) to be compatible with limited liability of existing shareholders, we must have $N \leq V^E(D_1, A_1)$; that is, the amount raised via new equity cannot exceed the market value of the firm's equity after the change – as this value is the maximum value of the claim that can be given to new investors. The following result shows that, under the assumptions of Proposition 7, the validity of this condition is independent of whether the reduction of leverage occurs through asset sales, pure recapitalization or asset expansion.

Proposition 8 (Limited Liability): *Under the assumptions of Proposition 7, a move from (D_0, A_0) to (D_1, A_1) with $D_1 = \delta_L A_1$ is compatible with limited liability of existing shareholders if and only if $\delta_H \leq v(\delta_L)/q(\delta_L)$.*

Proof: Compatibility with limited liability of shareholders requires that

$$V^E(A_1, D_1) = v(\delta_L) A_1 - q(\delta_L) D_1 \geq N = p(A_1 - A_0) + q(\delta_L)(D_0 - D_1),$$

which is equivalent to $v(\delta_L)(A_0 + A_1 - A_0) \geq p(A_1 - A_0) + q(\delta_L) D_0$, or,

$$v(\delta_L) \geq q(\delta_L)\delta_H + (p - v(\delta_L))\left(\frac{A_1 - A_0}{A_0}\right) \tag{37}$$

which leads to the condition $v(\delta_L) \geq q(\delta_L)\delta_H$ when $p = v(\delta_L)$. For a pure asset sale, we also need to check that the firm can deleverage to new level δ_L without needing to raise new equity; that is, there exists $A_1 \in [0, A_0]$ such that $p \times (A_0 - A_1) = q(\delta_L)(D_0 - \delta_L A_1)$. Solving for A_1 we have

$$A_1 = \frac{(p - q(\delta_L)\delta_H)}{(p - q(\delta_L)\delta_L)} A_0, \tag{38}$$

35

which is in the range $[0, A_0]$ if and only if $p = v(\delta_L) \geq q(\delta_L)\delta_H$. ∎

Because $q(\delta_L) \leq 1$, a sufficient condition for the leverage reduction to be feasible is $pA_0 \geq D_0$, which is the conventional condition for assessing the firm to be solvent. Under this condition, a leverage reduction can always be achieved via an asset sale from Eq. (38), and can be achieved under limited liability for any method if, as we have assumed here, $v(\delta_L) = p$.

We have focused our attention throughout this section on leverage reductions. The same arguments can be applied, however, to analyze alternative modes to *increase* leverage. For leverage increases, the different modes corresponding to the above discussion are: (i) asset sales: selling assets and using the proceeds to repurchase equity (or pay dividends), (ii) recapitalizations: issuing debt and using the proceeds to repurchase equity (or pay dividends), and (iii) asset expansions: issuing debt and using the proceeds to purchase new assets. The following result is immediate:

Proposition 9 (Equivalence for Leverage Increases): *Under the same conditions as* Proposition *7, shareholders are also indifferent between different forms of increasing leverage from* $\delta_L = D_1 / A_1$ *to* $\delta_H = D_0 / A_0 > \delta_L$, *and shareholder gains are given by*

$$\big(q(\delta_L) - q(\delta_H)\big)D_1 + \big(v(\delta_H) - v(\delta_L)\big)A_1 > 0. \tag{39}$$

Note that, while the losses and gains in (36) and (39) appear symmetric, they differ in magnitude due to the difference in the base level of debt and assets. Thus, for example, holding assets fixed, if the firm were to increase debt from D_1 to D_0 and then subsequently must reduce it back to D_1, the net payoff to shareholders would be

$$\big(q(\delta_L) - q(\delta_H)\big)\big(D_1 - D_0\big) < 0, \tag{40}$$

which is negative since $q(\delta_L) > q(\delta_H)$ and $D_1 < D_0$.[37]

5.2 Multiple Classes of Debt

In many settings, the conditions under which Proposition 7 holds are violated, and shareholders have a preference for one mode of leverage reduction over the others. In this section we show that an important case in which shareholders may have strong preferences about how to reduce leverage is when not all debt in the firm's capital structure has the same priority.

We continue to assume that the asset payoffs and the costs and benefits associated with frictions are perfectly homogenous with firm size. But now suppose the firm has multiple classes

---

[37] We are assuming in (40) that the second transaction is unanticipated at the time of the first, otherwise the debt price at the time of the first transaction would differ. Equation (40) also holds when asset levels change as long as the asset price p is the same for both transactions.

of *existing* debt with different levels of priority. In this case, if $D_1 < D_0$, it is optimal for the firm to repurchase the most junior debt first, as it will be the least expensive. The price at which junior debt can be repurchased depends on the precise capital structure of the firm (as well as any default costs or subsidies). Without going into the details of this dependence, we note that the price q^J at which junior debt can be repurchased should satisfy

$$\Pr(x \geq \delta_L \mid \theta_L^*) \leq q_L^J < q(\delta_L). \tag{41}$$

where, as above, $\theta_L^* = \arg\max_\theta V^E(\delta_L, 1, \theta)$. The lower bound in (41) reflects the fact that the price of the junior debt should be no less than the probability that the firm does not default, since in that case it will be repaid. The strict inequality for the upper bound follows as long as seniority "matters" in the sense that there exist some states of the world in which junior debt holders have lower recovery rates in default than more senior creditors.

The fact that junior debt is cheaper to repurchase breaks the indifference obtained in Proposition 7. Now, shareholders will be better off the more junior debt that is repurchased. In particular, we have the following important result:

Proposition 10 (Multiple Classes of Existing Debt): *Assume that the firm must reduce leverage from δ_H to δ_L, that $p = \nu(\delta_L)$ and that (41) holds. Then,*

  i. *If the firm can repurchase junior debt, shareholders find asset sales preferable to a pure recapitalization, which in turn is preferable to an asset expansion.*

  ii. *In the case of asset expansion, the ability to purchase junior debt makes no difference since no debt is repurchased.*

  iii. *In the case of a pure recapitalization the shareholders lose less with the ability to repurchase junior debt than they lose when there is only one debt class, but they still lose.*

  iv. *In the case of asset sales, shareholders may actually gain if the reduction in leverage is sufficiently small.*

Proof: See Appendix. ∎

To understand this result, note that the only change relative to Proposition 7 is the price at which the debt is repurchased. There, we assumed debt was reduced from D_0 to D_1 at price $q(\delta_L)$. If instead the debt is repurchased at price $q_L^J < q(\delta_L)$, the net cost to shareholders will be reduced by

$$(q(\delta_L) - q_L^J)(D_0 - D_1). \tag{42}$$

If leverage is reduced via an asset expansion, there is no change in the level of debt ($D_1 = D_0$), and thus the cost of the leverage reduction to shareholders is unchanged from Proposition 7. But with a recapitalization, the amount of debt is reduced, and so (42) is strictly positive and the cost to

shareholders is lower. (We know from Proposition 2, however, that despite this benefit a recapitalization is still costly to shareholders.)

If the firm chooses to reduce leverage using an asset sale, the final debt level will be lower than in a recapitalization (see Figure 5). As a result, the savings in (42) is even larger, making an asset sale the least costly method for shareholders to reduce leverage.[38] Indeed, as Part (iv) states, it is possible that shareholders actually gain from leverage reduction achieved through an asset sale if the debt is sufficiently junior and the required reduction in leverage is not too large.

Shareholders' preference to deleverage via asset sales can be viewed as another manifestation of the classic underinvestment problem associated with debt overhang identified by Myers (1977). Here we assumed asset sales are zero NPV ($p = v(\delta_L)$), but shareholders would prefer asset sales even if sold at a price somewhat below their fair market value; in other words, shareholders can gain from a "fire sale" even though it reduces total firm value. Importantly, however, our results demonstrate that this preference arises only if shareholders have the option to repurchase *junior* debt. If covenants or regulations require that senior debt be retired first, the preferences in Proposition 10 would be reversed.

More generally, we can consider a full range of transactions in which the firm buys or sells assets and issues or repurchases securities. Letting a be the increase in assets (at price p), and d_i be the increase in debt class i (at price q^i), both of which could be negative if assets are sold or debt is bought back, the gain to equity holders as a result of such a transaction is given by

$$\underbrace{V^E\left(A + a, D + \sum_i d_i\right) - V^E(A, D)}_{\text{change in equity value}} \underbrace{- pa + \sum_i q^i d_i}_{\text{transaction proceeds}}.$$

There are obviously many possible combinations of transactions, and shareholders' preferences among them will depend both on the impact on the redistribution of payoffs among existing and new claimholders and various potential frictions. The following result provides a partial ranking based on transactions involving exchanges of assets or equity for a single class of debt assuming that asset sale and purchases do not by themselves create value.

Proposition 11 (Transaction Ranking): *Assume* $p = v(\delta)$, *and assets and frictions are homogenous. Consider transactions in which a leveraged firm exchanges assets or equity for debt, where the debt traded is either homogenous (i.e. there is a single class or all classes participate pro rata), or is strictly junior or strictly senior to all other debt. Then, for given small change in leverage* δ, *shareholder preferences across such transactions are given by* Figure 6.

Proof: See Appendix. ∎

---

[38] While this intuition is correct, the proof is complicated by the fact that with multiple classes of debt, the average price at which the junior debt is repurchased may in general be higher in the event of an asset sale. Despite this potential price difference, we show that the net cost to shareholders is always reduced relative to a recapitalization.

Proposition 10   Indifference (Prop. 7)

Leverage Decreasing $\delta+\varepsilon \to \delta$	Sell/Buy:	A/D_J	>	E/D_J	>	E/D A/D E/A	>	E/D_S	>	A/D_S
	Perfect Market:	$+/-$		$-$		$-$		$-$		$-$

Leverage Increasing $\delta-\varepsilon \to \delta$	Sell/Buy:	D_S/A	>	D_S/E	>	D/E D/A A/E	>	D_J/E	>	D_J/A
	Perfect Market:	$+$		$+$		$+$		0		$-$

Indifference (Prop. 9)   Classic Debt Overhang (Myers 1977)

Figure 6: Shareholder Preferences for Alternative Transactions

Figure 6 shows the relative ranking of shareholder payoffs for a marginal increase or decrease in leverage. Also shown is the sign of the shareholder payoff absent taxes or other frictions; taxes will lower the payoff for leverage decreasing transactions and raise it for leverage increasing ones. Highlighted in the figure are the indifference results of Proposition 7 and the Proposition 9 for leverage increases, as well as the results for junior debt in Proposition 10.

With perfect markets, shareholders are indifferent to increasing leverage by issuing junior debt in exchange for equity. Shareholders always lose, however, in the reverse transaction (selling equity and purchasing junior debt). The reason for the asymmetry is the holdup problem shareholders face when repurchasing debt – the price for all claims is the value of the *last* dollar repurchased. A similar asymmetry holds when exchanging assets and junior debt.

We also highlight in Figure 6 the classic result of Myers (1977) that debt overhang implies a cost for shareholders when funding asset purchases with junior securities (equity or junior debt). Note that, absent taxes, this cost is the same whether it is leverage increasing or leverage decreasing. Alternatively, if asset purchases can be financed with senior or pari passu debt, shareholders will gain from the transaction, which may lead to overinvestment.

Figure 6 reveals a rich menu of alternative transactions with quite different consequences for shareholders even when all asset sales or purchases are zero NPV and there are no other frictions such as taxes or distress costs. Which transaction shareholders ultimately choose will depend on the constraints imposed by regulations and covenants. Considering these preferences together with other frictions in a fully dynamic context without perfect commitment is likely to lead to even more complex capital structure and investment dynamics than those we explored in the simple model in Section 4.

5.3 Other Considerations That Affect Shareholder Preferences

Divergence of Internal and External Asset Values

Proposition 7 concerns the case in which $p = v(\delta_L)$, which means that no value is directly gained or lost by buying or selling assets. What can we say about shareholder preferences at other prices? In this analysis, we begin by taking the asset price p as parametrically given, without considering whether it is consistent with market equilibrium. This corresponds to the standard approach of analyzing the behavior of price-taking agents by considering their demand and supply choices at *any* parametrically given prices. We will introduce equilibrium considerations once we discuss the parametric analysis.

If $p > v(\delta_L)$, the market price of assets exceeds the value of those assets when held by the firm. If $p < v(\delta_L)$, the firm can increase shareholder value by purchasing assets at the market price and holding them. The change in shareholder value is:

$$\nabla V^E - N = -\big(q(\delta_L) - q(\delta_H)\big)D_0 - \big(v(\delta_H) - v(\delta_L)\big)A_0 - \eta\big(A_1 - A_0\big), \qquad (43)$$

where $\eta = p - v(\delta_L)$. The third term shows that shareholders will prefer reducing leverage through asset sales when $\eta > 0$. Moreover, a pure asset sale may be feasible even if a pure recapitalization (or asset expansion) is not.[39] When $\eta < 0$, on the other hand, shareholders prefer asset purchases, and asset purchases may be feasible when a pure recapitalization is not.[40]

Taking the asset price as given is justified if the individual firm or bank can be thought of as a price taker operating in a large market. However, when we consider what occurs when there is a policy change that affects a large number of firms, e.g., an increase in bank capital requirements, we must recognize that the price-taking assumptions may no longer be justified. Even though an individual firm acting alone may be justified in taking the market price of assets as given, when all firms change their behavior in response to changes in regulatory requirements, it can be expected that the equilibrium market price will change.

For example, in the case of banking regulation, assume that the initial capital requirements correspond to the debt-asset ratio δ_H and that, for this debt-asset ratio, the equilibrium asset price is equal to $p_0 = v(\delta_H)$, the price at which banks with the debt-asset ratio δ_H are just indifferent about their asset holdings. Now suppose capital requirements are tightened, so that leverage must fall to δ_L, and that, because of a reduction in tax benefits and subsidies net of bankruptcy costs, we have $v(\delta_L) < v(\delta_H)$. If the price would remain equal to $v(\delta_H)$, all banks would want to

---

[39] Specifically, a pure asset sale only requires $p \geq q(\delta_L)\delta_H$ whereas a recap requires $v(\delta_L) \geq q(\delta_L)\delta_H$.

[40] An asset expansion is feasible iff $v(\delta_L) \geq q(\delta_L) \times \delta_H + \eta \times \left(\dfrac{\delta_H}{\delta_L} - 1\right)$.

respond to the new requirement by selling assets to buy back debt. Unless there are third parties willing to hold assets at this price, the asset price $p = v(\delta_H)$ will no longer clear the market. The new equilibrium price must fall to $p = v(\delta_L)$, as we are assuming in Proposition 7, unless there are third parties willing to hold *all* the assets at a higher price. Furthermore, while a bank might initially appear solvent with $p_0 A_0 \geq D_0$, if it is the case for some banks that $p_1 A_0 < D_0$, these banks may only be revealed to be insolvent through their inability to recapitalize and satisfy the new requirements.

Throughout our discussion, we have assumed that the constraints on leverage involve a debt-asset ratio D / A, which is fixed without regard to current market prices. In practice, covenants or capital requirements are often based (at least to some extent) on current values, imposing an upper bound on a ratio such as $q(\delta_L) D / p_1 A$ or $D / p_1 A$. The first corresponds to a ratio based solely on market values, the second corresponds to a case where assets are marked to market but debt levels are measured at the face value of liabilities. All of our results continue to apply with either of these measures (as they simply represent rescaling of the target leverage ratio). Note that if either $q(\delta_L) D / p_1 A$ or $D / p_1 A$ must be equal to δ_L, then, because $q(\delta_L) > q(\delta_H)$ and $p_1 < p_0 = v(\delta_H)$, the deleveraging effect is larger than it would be if D / A were required to equal δ_L. That is, when the leverage ratio is based on market values, rather than quantities, the effect of deleveraging is exacerbated.

Heterogeneous Assets

Proposition 7 treats the firms' assets as though they are homogeneous, with each asset unit having return of x so that the total return on all assets is simply xA. In reality, assets are heterogeneous, with differing risk and return. Nevertheless, the results of Proposition 7 continue to apply even when assets are heterogeneous as long as any asset sales or purchases correspond to a "representative portfolio" and so have the same risk and return as the average asset in the firm.

Of course, given the option, shareholders will generally have preferences with respect to which assets to sell or purchase. If a firm deleverages through asset sales, shareholders prefer to sell relatively safe assets. In contrast, they will prefer to purchase relatively risky assets if the firm expands. This preference is just another manifestation of the asset substitution agency problem that we have discussed above.

As a concrete example, suppose the firm holds a mix of risky assets and safe assets. In particular, suppose it holds quantity A_r of risky assets with return $\tilde{x}_r$ and A_s of safe assets with a riskless return. Note that we can normalize quantities so that each "unit" of assets (risky or safe) has market price p. Thus the firm has total assets $A = A_r + A_s$ with aggregate return $\tilde{x}$ given by

$$\tilde{x} \equiv \frac{\tilde{x}_r A_r + p A_s}{A}.$$

Suppose the firm considers reducing leverage by selling safe assets and using the proceeds to buyback debt. At the conclusion of the asset sale, the firm's leverage ratio must equal δ_L and the value of the assets sold must equal the value of the debt repurchased:

$$pa_s = q(\delta_L)(D_0 - D_1) = q(\delta_L)(D_0 - \delta_L(A - a_s)).$$

We then have the following immediate corollary to Proposition 7, showing the equivalence of "selective" asset sales and asset substitution:

Corollary (Asset Sales and Asset Substitution): *Reducing leverage via the sale of safe assets is equivalent, in terms of shareholder payoff, to recapitalizing the firm (to the same leverage ratio) and simultaneously selling safe assets and purchasing risky ones.*

Proof: Suppose the firm first exchanges its holdings a_s of safe assets for risky ones at the market price p, and then sells those risky assets to reduce leverage through an asset sale. This transaction clearly has the same shareholder payoff as simply selling the safe assets directly. But since the firm's assets are homogenous after the asset exchange, by Proposition 7 this has the same shareholder payoff as an asset exchange followed by a pure recapitalization. ∎

Note that the equivalence between asset sales and asset substitution ignores potential transactions costs. Once these are considered, asset sales are likely to be strictly preferred, as it avoids both the need to purchase risky assets and to issue equity. We discuss additional impacts of transactions costs in the next subsection below.

In the context of capital regulation for banks, an attempt is made under Basel II and Basel III to address the problems created by asset substitution and risk shifting. This is done by assigning risk weights to assets and formulating capital requirements in terms of the size of the risk-weighted asset base. If the risk weighting system worked perfectly and completely removed the ability of bank managers and shareholders to engage in asset substitution and risk shifting when assets are sold or purchased, asset heterogeneity would not necessarily undermine the equivalence result given in Proposition 7. In particular, if risk weighting effectively means that the value of debt depends only on leverage as measured by the risk weighting system, so that $q(\delta_L)$ will be the same no matter what the mode of leverage reduction, then the conditions for Proposition 7 to hold are potentially restored even with heterogeneous assets.

In practice, risk weighting falls short of removing the ability of banks to increase risk and engage in asset substation. Indeed, the regulations often involve transparently inappropriate risk weights, e.g., a zero risk weight for sovereign debt or for highly rated securities even when they clearly carry some potentially significant risks. Making matters worse is the fact that in practice the implementation of the risk weighting system relies in part on the banks' own internal risk models and is therefore highly manipulable. When risk weights are imperfect, the same logic as the preceding result implies that banks will have an incentive to reduce leverage by selling assets that are safer than their risk weight implies, and holding on to assets that are riskier than their assigned weights. Again, such selective sales are another mode of asset substitution that banks

may engage in when capital regulations only impose capital ratios rather than specifying the mechanism by which they should be achieved.

Transactions Costs

Proposition 7 is based on the assumption that the firm faces no transactions costs in changing the scale of its assets or in issuing and retiring securities. Not surprisingly the introduction of transactions costs can lead to one alternative being preferred over the others, since the three ways of changing leverage that we consider involve different pairs of transactions as shown below:

	The firm purchases:	The firm sells:
Asset Sales	Debt	Assets
Recapitalization	Debt	Equity
Asset Expansion	Assets	Equity

Asset expansion will be the preferred alternative if the transactions costs involved in repurchasing debt are particularly large relative to the other transactions, but this is unlikely to be the case. The transactions costs involved in equity issuance and asset sales are likely to be more important. If equity issuance costs are large relative to those in asset transactions, then asset sales, since they involve no equity transactions, will be the preferred alternative. If the transactions costs involved in selling assets are particularly large compared to equity issuance costs (e.g., the firm faces extreme "firesale conditions" in liquidating assets), then recapitalization or asset expansion will be preferred. Without making specific assumptions about the magnitude of the various transactions costs, little more can be said about what approach will be most advantageous for shareholders.

Asymmetric Information

A key component of transactions costs in settings such as the ones we are considering is due to the possibility that the firm's managers have private information about the firm's assets and growth opportunities that both existing "outside" shareholders and others in the market do not have. Managers will want to sell assets that the market is overvaluing and similarly will want to issue equity if they perceive the market is overpricing the firm's shares. The possibility that managers will make strategic choices based on their private information can account for a significant part of the bid/ask spread for transactions involving the firm's assets and securities. Information asymmetries can be particularly important in asset sales and equity issuance and this explains why transactions costs for these are likely to be larger than those associated with debt buybacks.

Asymmetric information factors that would affect the valuation of the firm's assets in the asset sales approach clearly also give rise to asymmetric information issues affecting the market valuation of the firm's equity when the firm issues equity directly (as opposed to a rights offering)

to recapitalize or expand its assets.[41] It is clear that if there is asymmetric information about the value of the assets in place, there must be asymmetric information about the value of the firm's equity. If a recapitalization must be done through a new share issuance as opposed to a rights offering, it is not immediately obvious whether it will be more expensive for the firm's shareholders to sell assets and deleverage or sell equity and recapitalize.

In some circumstances asymmetric information about asset values makes the shareholders indifferent between deleveraging and recapitalizing. Assume that the market undervalues the firm's assets in the following sense: while managers know that the realized value on the firm's assets will be $\tilde{x}(1+\omega)A$ for $\omega > 0$, the market assumes that the realized value of the assets will only be $\tilde{x}A$. Essentially this means that for each asset unit that the market perceives, the firm effectively has $1+\omega$ units and this difference is perceived by the firm's managers who make decisions about how the firm should change its leverage.

Proposition 12 (Equivalence with Asymmetric Information): *Assume that there is only one class of debt, the firm has homogeneous assets and faces no transactions costs in buying or selling assets or the securities it issues other than that implied by the market's undervaluation of its assets and the firm must decrease its leverage (as measured by outsiders) from δ_H to δ_L. Then for all $\omega \geq 0$, assets sales and pure recapitalization through a common share offering are equally undesirable for shareholders based on the managers' information.*

Proof: See Appendix. ∎

Asymmetric information imposes costs on the current shareholders in both the asset sale and pure recapitalization cases because the firm is selling assets or equity at prices below their values. Although a greater dollar amount of assets must be sold in the asset sales approach than the dollar amount of equity that needs to be issued to effect a recapitalization, the underpricing of equity is larger in percentage terms because of leverage, and this is just sufficient to make the loss due to underpricing the same.[42]

---

[41] Note that in Myers and Majluf (1984) the asymmetric information relates to the value of assets in place as well as the value of the new investment opportunity. The key assumption in their analysis is that the firm can only raise equity through an offering of common shares and not, for example, through a rights offering. With symmetric information, as in Proposition 7, the mode of issuance does not matter. With asymmetric information, it does make a difference. In a sale of new shares to the market, the market's assessment of the firm directly impacts the amount of money raised by the firm. In a rights offering, if it succeeds, the market's assessment of the firm does not affect the amount of money raised by the firm, but only the value of shares and therefore the value of the rights. The attitude of existing shareholders to a rights offering then depends on whether they are short-term investors, who are interested only in the current share price, or long-term investors à la Myers-Majluf, who care about share prices in the future, when the market will have learned about the underlying values.

[42] One might wonder why the results we obtain for asymmetric information differ from those presented earlier where we assumed $p - v(\delta) = \eta$. There, because there is symmetric information about the value of the assets held by the firm, there is uniform agreement that the firm should either be selling assets if $\eta > 0$ and buying assets if $\eta < 0$. Whether the firm should grow or shrink is unambiguous, and the preferred mode of leverage reduction depends only on the *amount* of assets sold or bought. With asymmetric information, when equity is issued, the price is based on the market's perception of the *total* value of the assets and any losses are due the market's undervaluation of that total.

If the firm's assets are heterogeneous, the situation involving asymmetric information becomes more complex. Transactions costs due to asymmetric information are likely to be lowest on the least risky assets. As discussed above, asset substitution considerations indicate that the shareholders will want to sell low-risk assets when deleveraging, but will want to buy high-risk assets in the asset expansion approach to reducing leverage. This means that transactions costs concerns and asset substitution will tend push shareholders toward the deleveraging alternative. With deleveraging, incentives associated with asset substitution and transactions cost minimization are aligned. This is not the case with asset expansion.

Note, however, that deleveraging is not always the preferred alternative from a transactions costs perspective. If most assets are hard to value by outsiders and managers can pick the assets they sell, then the adverse selection effects can be greater with asset sales than they are when equity is sold. This is because equity represents a claim on a portfolio of assets rather than an adversely selected subset. The transactions costs associated with issuing equity can be lower than those involved in selling hard to value assets. This could tip the balance in favor of recapitalization.

It should also be noted that one way to reduce leverage that involves almost no transactions costs due to asymmetric information is for the firm to retain earnings and build equity "internally." Adverse selection costs can also be eliminated by raising equity through a rights offering. Shareholders resistance to these ways of reducing leverage is entirely due to conflicts of interest related to debt overhang.

6. Concluding Remarks

In this paper we analyzed an agency cost of debt that stems from the inefficient capital structure choices likely to be made by leveraged firms. This agency cost is due to conflicts of interests shareholders have with creditors, which bias shareholders against reducing and toward increasing leverage. We refer to this bias as the *leverage ratchet effect* and show that it can have a profound impact on leverage choices, both initially and in the dynamics of leverage.

Because of the leverage ratchet effect, firms that start out with little debt may increase their leverage over time and end up with inefficiently high debt levels. If creditors anticipate this development, they will charge relatively high interest *ex ante*. The leverage ratchet effect thus increases the *ex-ante* cost of debt funding. These higher *ex-ante* costs of debt funding in turn may lead firms to choose low levels of leverage, at least initially, in order to avoid the negative effects of subsequent distortions, including from excessive leverage. The leverage ratchet effect may

---

As discussed above, the same amount of assets is effectively sold at undervalued prices when equity is issued as when assets are sold directly. Whereas earlier the losses or gains were based on the amount of assets sold, with asymmetric information the losses are based on the market's valuation of all the assets. It does not matter whether the assets are directly sold or indirectly sold through issuance of equity — the loss is the same. A long-term investor, who is patient enough to wait until the market has learned the correct value for the assets, would however take a different view if equity were raised through a rights offering; see the preceding footnote. Such an investor would prefer the rights offering to a sale of assets.

therefore explain the so-called zero-leverage puzzle, since with no leverage the effect does not exist at all.[43] At the same time, the effect can explain why highly indebted firms tend to remain highly indebted and may even increase their leverage if permitted to do so. It also explains why firms in financial distress are unlikely to reduce their indebtedness by retaining earnings or raising additional equity while they can still do so, i.e., before they become insolvent.

Like the other agency conflicts associated with debt, the leverage ratchet effect reflects a commitment problem. With complete contracting and the ability to fully commit to all subsequent funding and investment choices, the leverage ratchet effect would not exist. In most cases in practice participants will find it impossible or inefficient to eliminate the leverage ratchet effect by prior contracting. Although debt covenants can be designed to counter the effect, e.g., by forbidding all borrowing until the current debt is retired, such restrictive covenants are costly if they unduly reduce the flexibility of the firm in subsequent decisions. Covenants may also be ineffective or inefficient because their enforcement involves collective-action problems on the side of creditors. If creditors can act unilaterally, inefficient outcomes can occur when one creditor intervenes at the expense of others; if creditors must act collectively, they may be too disorganized to have any effect.

Our analysis suggests that the traditional contract-theoretic approach to studying capital structure in terms of tradeoffs between total costs and benefits associated with different capital structures must be modified to take account of imperfect commitment to subsequent funding choices. When this modification is included, the explanatory power of the traditional tradeoffs theories is reduced. A fully dynamic model must capture the interdependence among commitment problems, shareholder biases toward leverage increases, and the *ex-ante* expectations that take into account the distorted incentives.

The impact of the leverage ratchet effect is likely to differ across firms and industries. It is likely to be particularly severe for firms whose multiple creditors find it difficult to impose and enforce debt covenants. A key example involves banks, which typically operate with extremely high levels of leverage, traditionally in the form of bank deposits. Depositors and other creditors of banks tend to be dispersed, so they are not in a good position to deal with the collective-action problems involved in monitoring banks and enforcing contractual covenants. Incentives for these creditors of banks to use *ex ante* contracting as well as *ex post* monitoring to protect their interests are further weakened by the existence of explicit and implicit government guarantees.

Creditors' passivity and the presence of debt guarantees encourage more risk taking and reinforce the leverage ratchet effect. If leverage is high and debt is guaranteed, the intense conflict of interest between shareholders and creditors with respect to leverage and risk are transformed to an intense conflict between manager and shareholders and the provider of the debt guarantees or the broader public. None of the countervailing forces discussed in Section 4.4 that reduce the impact of the leverage ratchet effect have an impact in this case. Leverage levels can and would become dangerously high unless controlled effectively by regulations. As long as depositors

---

[43] See, for example, Strebulaev and Yang (2013).

maintain balances and short-term creditors continue to refinance the debt, even insolvent banks can persist before a potential run or intervention by regulators.

Much of the theoretical literature on banking has sought to explain the observed high leverage of banks as a result of optimal contracting. One line of argument focuses on the idea that short-term debt "disciplines" bank managers. Another focuses on the desire of investors to have assets that are "liquid." As we have explained in some detail elsewhere, both lines of argument have serious conceptual and empirical weaknesses that render them inadequate for explaining the observed choices of banks' or for guiding policy.[44]

Our analysis suggests that high leverage in banking and the growth of this leverage over the past century may well reflect the leverage ratchet effect, combined with the expansion of the safety net of banking. Indeed, equity levels in banking have consistently declined over the last 150 years, just as safety nets from central banks, deposit insurance and governments have expanded, enabling banks to obtain debt funding under easier and more favorable terms than those they would face without guarantees.[45] Changes in capital regulation, such as the use of complex and manipulable risk weights, have allowed leverage to increase further in recent years. (See our discussion of leverage adjustment with heterogeneous assets in Section 5.3.) The increased use of derivatives and the opacity of many markets has made actual leverage and risk exposures have become almost impossible to measure.

Our model suggests that banks will resist regulations that mandate leverage reduction and will seek to increase leverage whenever they have the opportunity to do so, in line with much of what we observe. The fact that banks often seek to make payouts to shareholders, thus maintaining or increasing their leverage, is also consistent with our model and contrary to the "pecking order" theory of capital structure according to which retained earnings are the most preferred source of funding for any corporation.

The significant expansion of short-term bank borrowing in the early 2000s is also consistent with our analysis and with the "maturity rat race" discussed in Brunnermeier and Oehmke (2013). Repo borrowing, which played a major role in this expansion, involves a sale of collateral and a commitment to repurchase it at a later time. It provides a way to issue new debt ahead of any incumbent debt by essentially allowing the repo lender to jump the queue of claimants (including depositors) in default. Since the repo collateral is not available to repay other creditors,

---

[44] On models claiming that debt and fragility discipline, see Admati et al. (2013, Section 5) and Admati and Hellwig (2013b). On models that focus on liquidity provision by banks, see Admati et al. (2013, Section 7) and Admati and Hellwig (2013a, Chapter 10, 2014). Pfleiderer (2014) discusses some of the same issues, and more generally the misuse of theoretical models in economics and finance.

[45] See Berger et al. (1995) and Admati and Hellwig (2013a, Chapters 2, 9 and 10). Note that deposits represent unsecured debt, which implies that assets purchased with deposit funding can be used as collateral, making additional borrowing relatively easy and often free of covenants and restrictions. In addition, deposit insurance is often implicit and thus free, as are implicit guarantees based on expectations of central bank and government support. Even when banks pay for deposit insurance, the fee cannot reflect the highly time-varying risk to the insurer. On subsidies associated with expectations of support, see Admati (2014).

adding debt of this type lowers the value of existing claims. This is in line with our results in Section 5.[46]

Because of the leverage ratchet effect, the funding patterns that we see in the real world cannot be presumed to be efficient even in the absence of externalities affecting third parties. Expectations of excessive deadweight costs of default and inefficient investments induced by high leverage raise the *ex-ante* cost of borrowing and may make shareholders shy away from borrowing altogether, even if this means losing some otherwise profitable investment opportunities.

With banks, the inefficiencies created by the ratchet effect are even greater because high leverage exacerbates systemic risks from bank failures and makes the financial system more fragile, which harms the broader economy, including most small investors. These observations suggest that there is a critical role for regulation to correct the distortions and control excessive leverage, particularly if creditor discipline is missing due to creditors being dispersed and protected by guarantees. In a sense, regulations help correct the distortions due to lack of commitment. This conclusion reinforces the discussion of Admati et al (2013), and Admati and Hellwig (2013a,b, 2014), where we argue that much higher equity requirements than those currently enforced would be highly beneficial and entail minimal, if any social cost. Our analysis also reinforces the view that policymakers should re-examine the corporate tax code with a view to eliminating the discrimination in favor of debt relative to equity, which is highly distortive.

Our results in Section 5 concerning the choice of leverage-adjustment methods that involve both assets and securities transactions are highly relevant for the design of capital regulation, particularly with respect to the use of regulatory ratios and other tools available to control banks' actions as asset values evolve. If regulation forces a bank to reach a specific capital ratio, shareholders have incentives to respond in ways that minimize their costs and keep the value of their equity claim as high as possible. They can do this, for example, by selling relatively safe assets and buying back junior debt, a prediction in line with the behavior of European banks in fall of 2011 when authorities forced them to increase their core equity.[47]

If policymakers are concerned that deleveraging by banks in response to increased capital requirements would have harmful effects, they should set targets for *absolute* amounts of equity rather than ratios, at least during a transition period. They should also use their authority to manage banks' actions during the transition. An obvious measure is to ban dividends and other payouts to shareholders, as well as cash bonuses to managers, until the desired equity levels are reached. To accelerate leverage reduction, banks whose shares are publicly traded might be required to raise new equity. Raising new equity is always possible if a bank is solvent; indeed, inability to raise new equity can be taken as evidence that the bank may well be insolvent. Since capital regulations often focus on accounting measures that banks have strong incentives to manipulate, and since

---

[46] In contrast to our analysis, which emphasizes the borrower's desire to expand leverage, Brunnermeier and Oehmke (2013) assume that total debt is fixed and show that the lack of commitment induces a "maturity rat race".

[47] In particular, they repurchased so-called *hybrid* debt, which before Basel III, had to some extent been counted as "capital", namely the so-called "Tier 2 Capital". The scope for counting such hybrid debt as bank capital was much reduced under Basel III. On the use of hybrids instead of equity, see Admati et al (2013, Section 8).

insured or secured creditors do not impose the usual debt burden, banks can persist in insolvency or near insolvency for extended periods if regulators show forbearance.[48]

More generally, our analysis shows that capital structure outcomes depend critically on the degree to which firms can commit to subsequent choices and on the amount of flexibility shareholders retain in determining leverage levels. The borrower-creditor conflict that generates the leverage ratchet effect, and contracting frictions that prevent full commitment, imply that the leverage levels we observe are highly history-dependent. We have also shown that in a dynamic context with imperfect commitments, leverage levels can be sensitive to small changes in the costs and benefits of debt financing. Models that overlook the critical effect of contracting and commitment issues are unlikely to explain the richness of observed leverage patterns, including the great variety of capital structures across firms and industries.

---

[48] See Admati and Hellwig (2013a, Chapter 11) for a discussion of how capital regulation can be made more effective than currently implemented.

Appendix: Remaining Proofs

Proof of Proposition 3: Note that the expectation in (11) is with respect to the information z . Then, using the same argument as in Proposition 1, holding the policy functions fixed,

$$
V^E(D-\Delta,A,\theta,a)-V^E(D,A,\theta,a)=
$$

$$
E_z\left[\Delta\times\left(1-F\left(\frac{D-\Delta}{A+a(z)}\Big|z,\theta(z)\right)\right)+\int_{(D-\Delta)/(A+a(z))}^{D/(A+a(z))}\left(x(A+a(z))-D\right)\mathrm{d}F\left(x|z,\theta(z)\right)\right.
$$

$$
-\int_{(D-\Delta)/(A+a(z))}^{\infty}t\left(x,A+a(z),D-\Delta\right)\mathrm{d}F\left(x|z,\theta(z)\right)
$$

$$
\left.+\int_{D/(A+a(z))}^{\infty}t\left(x,A+a(z),D\right)\mathrm{d}F\left(x|z,\theta(z)\right)\right]
$$

(44)

$$
<E_z\left[\Delta\times\left(1-F\left(\frac{D-\Delta}{A+a(z)}\Big|z,\theta(z)\right)\right)\right]
$$

$$
=\Delta\times\Pr\left[x_{\theta(z)}\left(A+a(z)\right)>D-\Delta\right]
$$

As in Proposition 1, the inequality follows because shareholders forfeit their default option for final asset values between $D-\Delta$ and D , and have a higher expected tax burden. The last equality states that the increase in the value of equity per dollar of debt repurchased is less than the ex-ante probability of no default at the lower level of leverage.

The proof then follows using exactly the same argument as in (10) above. Let θ^* and a^* be the optimal risk and investment policy functions for equity holders given debt $D-\Delta$:

$$
V^E\left(D-\Delta,A\right)=\max_{\theta,a}V^E\left(D-\Delta,A,\theta,a\right)=V^E\left(D-\Delta,A,\theta^*,a^*\right)
$$

(45)

Then,

$$
\begin{aligned}
V^E\left(D-\Delta,A\right)-V^E\left(D,A\right)&=V^E\left(D-\Delta,A,\theta^*,a^*\right)-\max_{\theta,a}V^E\left(D,A,\theta,a\right)\\
&\le V^E\left(D-\Delta,A,\theta^*,a^*\right)-V^E\left(D,A,\theta^*,a^*\right)\\
&<\Delta\times\Pr\left[x_{\theta^*(z)}\left(A+a^*(z)\right)>D-\Delta\right]\\
&\le\Delta\times q^D\left(D-\Delta,A\right)
\end{aligned}
$$

(46)

The first inequality follows since we have fixed the investment policy functions at a level that may not be optimal with higher leverage (due to agency costs), the second follows from above, and the third follows since the repurchase price of the debt will be at least the no default probability (and will be strictly higher if the debt has a non-zero recovery rate in any default states). ∎

Proof of Proposition 4: Note that $q^J(0,D,A)=q(D,A)$, and therefore $Dq^J(0,D,A)=V^D(D,A).$ That is, proceeds from issuing debt are equal to the total value of the firm's debt. Hence,

$$
\begin{aligned}
G(0,D)&=V^E(D,A)-V^E(0,A)+Dq^J(0,D,A)\\
&=V^E(D,A)+V^D(D,A)-V^E(0,A)
\end{aligned}
$$

(47)

Thus, D maximizes $G(0,D)$ if and only if it maximizes total firm value.

For the second result, note that our earlier results already establish that shareholders lose if the firm reduces debt $(D' < D)$ regardless of the seniority of the debt that is repurchased. Therefore, it is enough to establish that the marginal benefit of an increase in leverage from its current level is positive. Specifically, we need to show the right-hand derivative of G at $D' = D$,

$$\left.\frac{\partial G(D, D')}{\partial D'}\right|_{D'=D_-} = \left.\frac{\partial \left(V^E(D', A) + (D' - D)q^J(D, D', A)\right)}{\partial D'}\right|_{D'=D_-}, \tag{48}$$

is positive. Let θ^* be the optimal risk choice with debt level D. From the definition of V^E, and using the fact that holding the risk choice fixed at θ^* only reduces the gain to equity holders, we have

$$\left.\frac{\partial V^E(D', A)}{\partial D'}\right|_{D'=D_+} \geq \left.\frac{\partial}{\partial D'}\left[\int_{D'/A}^{\infty} \left(xA - t(x, A, D') - D'\right) dF\left(x|\theta^*\right)\right]\right|_{D'=D_+}$$

$$= -\int_{D_+/A}^{\infty} dF\left(x|\theta^*\right) - \int_{D_+/A}^{\infty} t_D(x, A, D) dF\left(x|\theta^*\right)$$

$$> -\int_{D_+/A}^{\infty} dF\left(x|\theta^*\right) = -\Pr\left(xA > D|\theta^*\right)$$

where the final inequality follows from the assumption that tax benefits are positive.

Next, for $D' \geq D$, define $\pi(D')$ to be the proceeds raised from the new debt:

$$\pi(D') \equiv (D' - D)q^J(D, D', A) = (D' - D)\int_{D'/A}^{\infty} dF\left(x|\theta'\right) + \int_{0}^{D'/A} \left(xA - n(\tilde{x}A, D') - D\right)^+ dF\left(x|\theta'\right).$$

Then let $\hat{\pi}(D') \equiv (D' - D)\int_{D'/A}^{\infty} dF\left(x|\theta'\right)$. Because $\pi(D) = \hat{\pi}(D) = 0$ and $\pi(D') \geq \hat{\pi}(D')$ for $D' > D$, we have

$$\pi'(D_+) \geq \hat{\pi}'(D_+) = \lim_{\varepsilon \downarrow 0} \frac{\hat{\pi}(D + \varepsilon) - \hat{\pi}(D)}{\varepsilon} = \lim_{D' \downarrow D} \int_{D'/A}^{\infty} dF\left(x|\theta'\right) = \lim_{D' \downarrow D} \Pr\left(xA > D'|\theta'\right).$$

That is, the marginal price per dollar of junior debt is at least the probability of no default (and could be higher in the presence of default subsidies). Thus we have shown

$$\left.\frac{\partial G(D, D')}{\partial D'}\right|_{D'=D_-} > \lim_{D' \downarrow D} \Pr\left(xA > D'|\theta'\right) - \Pr\left(xA > D|\theta^*\right) = 0 \tag{49}$$

where for the final equality we use the fact that the probability of default is continuous at D. ∎

Proof of Lemma: For the pure bankruptcy cost model, define the distribution for Y such that

$$\Pr(Y \leq D) = 1 + \phi'(D) = 1 - \Pr(Y_{\theta(D)} > D) = \Pr(Y_{\theta(D)} \leq D).$$

Then we have

$$\phi^{B'}(D) = -\Pr(Y > D) = -\Pr(Y_{\theta(D)} \leq D) = \phi'(D).$$

Because the derivatives match and they share the same limit, we have $\phi^B = \phi$.

For the pure moral hazard model, define

$$\theta(D) = -\phi'(D).$$

Next define g as

$$g(\theta(D)) = D + \frac{\phi(D)}{\theta(D)}$$

on the range of $\theta(D)$ and zero elsewhere. Then we can rewrite the shareholders' optimization as

$$\phi^A(D) = \max_\theta \ \theta(g(\theta) - D)^+ = \max_{\hat{D}} \ \theta(\hat{D})(g(\theta(\hat{D})) - D)^+$$

Now

$$\begin{aligned} \theta(\hat{D})(g(\theta(\hat{D})) - D) &= \theta(\hat{D})(\hat{D} - D) + \phi(\hat{D}) \\ &= \phi(\hat{D}) + \phi'(\hat{D})(D - \hat{D}) \\ &\leq \phi(D) \end{aligned}$$

where the last inequality follows from the convexity of ϕ. Hence we have $\phi^A = \phi$. ∎

Proof of Proposition 5: To verify an equilibrium, note first that given the equilibrium leverage strategy, the debt pricing is rational for creditors since the firm is expected to maintain leverage permanently at $D^n = D^+(D)$. Next note that if $D < D^n = D^+(D)$, then $G(D, D^n) > 0$ from (27). Thus, shareholders gain from increasing debt to D^n. Moreover, it is suboptimal to delay this increase in debt, as it would delay earning the gain $G(D, D^n)$.

Finally, we need to establish that shareholders would not prefer some alternative debt choice or sequence of choices. From the prior argument, it is sufficient to consider only changes to some other stable point $D^m \neq D^n$. Note that for $D^m < D$, $G(D, D^m) < 0$ since shareholders both lose tax benefits and bear (via the debt price) incremental agency or bankruptcy costs when buying back debt. For $D \leq D^n < D^m$, note that because $p(D^m) \leq p(D^n)$,

$$G(D, D^m) \leq G(D, D^n) + G(D^n, D^m) \leq G(D, D^n),$$

where the last inequality follows since $G(D^n, D^{n-1}) \leq 0$ by (27). ∎

Proof of Proposition 6: Let $V_j^E(D)$ and $p_j(D)$ be the payoff to equity and the price of debt if the firm has stable debt D until the next shock arrives, and let λ_j be the total arrival rate $\sum_k \lambda_{jk}$. If the firm enters regime k with debt D, let $D^{k+}(D)$ denote the next stable debt level in regime k. Then upon entering regime k, the value of equity and the debt price will be

$$\hat{V}_k^E(D) = V_k^E(D^{k+}(D)) + (D^{k+}(D) - D)p_k(D^{k+}(D)), \text{ and (50)}$$

$$\hat{p}_k(D) = p_k(D^{k+}(D)),$$

where

$$V_j^E(D) = \left(\frac{(y_j - c_j D) - t_j (y_j - c_j D)^+ + \lambda_{j0} \phi_j(D) + \sum_{k>0} \lambda_{jk} \hat{V}_k^E(D)}{r_j + \lambda_j} \right)^+, \text{ and} \tag{51}$$

$$p_j(D) = \frac{c_j - \lambda_{j0} \phi_j{}'(D) + \sum_{k>0} \lambda_{jk} \hat{p}_k(D)}{r_j + \lambda_j} \times 1\left[V_j^E(D) > 0 \right].$$

Note that in (51), we account for equity's option to default when $D > \overline{D}_j$. By the identical logic as **Proposition 5**, the stable points are defined by

$$D_j^{n+1} = \max\{D < D_j^n : G_j(D, D_j^n) \leq 0\} \tag{52}$$

where

$$G_j(D, D') \equiv V_j^E(D') - V_j^E(D) + p_j(D')(D' - D). \tag{53}$$

Note that we can calculate the equilibrium value function and stable points via backward induction on the debt level D, beginning from debt level $\overline{D} \equiv \max_j \overline{D}_j$. Once $D = \overline{D}$, there will be no further increases in debt, and the system (50) and (51) can be solved using standard methods. ∎

Proof of Proposition 7: After the change, the total value of equity will be:

$$V^E(A_1, D_1) = v(\delta_L) A_1 - q(\delta_L) D_1. \tag{54}$$

Therefore,

$$\nabla V^E = \left(v(\delta_L) A_1 - q(\delta_L) D_1 \right) - \left(v(\delta_H) A_0 - q(\delta_H) D_0 \right). \tag{55}$$

Thus, the total change in value for existing shareholders is

$$\begin{aligned}
\nabla V^E - N &= \left(v(\delta_L) A_1 - q(\delta_L) D_1 \right) - \left(v(\delta_H) A_0 - q(\delta_H) D_0 \right) \\
&\quad - p(A_1 - A_0) - q(\delta_L)(D_0 - D_1) \\
&= \left(v(\delta_L) - p \right) A_1 - \left(q(\delta_L) - q(\delta_H) \right) D_0 - \left(v(\delta_H) - p \right) A_0 \\
&= -\left(q(\delta_L) - q(\delta_H) \right) D_0 - \left(v(\delta_H) - v(\delta_L) \right) A_0.
\end{aligned} \tag{56}$$

Since this does not depend on either A_1 or D_1, it is the same for all changes that lead to a given reduction in the leverage ratio. We know already from our results in Section 2 that shareholder losses are positive for a pure recapitalization, so they must be positive for any method, proving the result. ∎

Proof of Proposition 10: As before we have

$$\nabla V^E = \left(v(\delta_L) A_1 - q(\delta_L) D_1 \right) - \left(v(\delta_H) A_0 - q(\delta_H) D_0 \right),$$

but given the lower cost q_L^J of repurchasing the junior debt, the total value of equity issued is

$$N^J = p \times (A_1 - A_0) + q_L^J \times (D_0 - D_1) = N - \left(q(\delta_L) - q_L^J \right)(D_0 - D_1),$$

and therefore the change in value for existing shareholders is:

$$\nabla V^E - N^J = \nabla V^E - N + \left(q(\delta_L) - q_L^J \right)\left(D_0 - D_1 \right)$$
$$= -\left(q(\delta_L) - q(\delta_H) \right) D_0 - \left(v(\delta_H) - v(\delta_L) \right) A_0 + \left(q(\delta_L) - q_L^J \right)\left(D_0 - D_1 \right). \tag{57}$$

For a pure asset expansion, we have $D_0 = D_1$, and thus the loss to shareholders in (57) is identical to that in the case of a single debt class. However, this loss is reduced with a recapitalization or asset sale, since then $\left(q(\delta_L) - q_L^J \right)\left(D_0 - D_1 \right) > 0$.

But while shareholders' losses are smaller in a recapitalization, we know from Proposition 2 that shareholders still lose even if they can repurchase the junior debt at the minimal price in (41).

We have established (ii) and (iii), and the second statement in (i). To complete (i), we must show that asset sales are preferable to a recapitalization. Let D_{1R} be the debt remaining after a recapitalization and D_{1A} be the debt remaining after an asset sale, and note that $D_0 > D_{1R} > D_{1A}$. Let q_{LR}^J be the average repurchase price of the junior debt $(D_0 - D_{1R})$ in a recapitalization. Therefore, relative to an asset expansion, shareholders gain

$$\left(q(\delta_L) - q_{LR}^J \right)\left(D_0 - D_{1R} \right)$$

from a recapitalization. For the asset sale, let q_{LA}^{J1} be the average repurchase price of the most junior $(D_0 - D_{1R})$ of debt, and q_{LA}^{J2} be the average repurchase price of the remaining $(D_{1R} - D_{1A})$ of "mezzanine" debt. Then the gain relative to an asset expansion for an asset sale can be written as

$$\left(q(\delta_L) - q_{LA}^{J1} \right)\left(D_0 - D_{1R} \right) + \left(q(\delta_L) - q_{LA}^{J2} \right)\left(D_{1R} - D_{1A} \right).$$

Comparing the two, we see that the gain for an asset sale is larger because $q_{LA}^{J2} \le q(\delta_L)$ (mezzanine debt has weakly lower priority to the remaining senior debt) and $q_{LA}^{J1} \le q_{LR}^J$ (since the most junior tranche has even lower priority relative to the remaining senior debt in an asset sale), and moreover at least one of the inequalities must be strict given at least two classes of debt.

Finally, to show (iv) – that shareholders *may* gain with asset sales if they can repurchase junior debt – it is sufficient to consider the case in which there are no frictions. In that case, given assets A and debt $D(A)$, the value of equity is

$$V^E = \int_{D(A)/A}^{\infty} \left(xA - D(A) \right) dF(x)$$

With perfect markets, assets can be sold for $p = E[x]$, with the proceeds used to purchase debt with face value p / q^J. Therefore, $D'(A) = p / q^J$ and so,

$$\frac{d}{dA} V^E = \int_{\delta}^{\infty} \left(x - D' \right) dF(x) = \int_{\delta}^{\infty} x \, dF(x) - D' \int_{\delta}^{\infty} dF(x) = \left(E[x \mid x > \delta] - \frac{E[x]}{q^J} \right) \Pr(x > \delta)$$

Equity holders gain from asset sales if the above expression is negative, or equivalently,

$$q^J E[x \mid x > \delta] < E[x]. \tag{58}$$

Now, the value of the junior debt can be written

$$q^J = \int_{\delta}^{\infty} dF(x) + \alpha \int_0^{\delta} x / \delta \, dF(x)$$

where $\alpha \in [0,1]$ is the expected recovery rate of the junior debt relative to average recovery rate of the firm's debt (which is strictly positive given our assumption that F has full support). If the debt is fully prioritized so that all debt repurchased is junior to any debt retained, then $\alpha = 0$, whereas if the debt is all of one class, then $\alpha = 1$ and $q^J = q$. Substituting this value for q^J in (58) we get

$$\left(\int_\delta^\infty dF(x) + \alpha \int_0^\delta x/\delta\, dF(x)\right) E[x \mid x > \delta]$$

$$= \int_\delta^\infty x\, dF(x) + \alpha \int_0^\delta x\, dF(x) \frac{E[x \mid x > \delta]}{\delta} < \int_0^\infty x\, dF(x) \tag{59}$$

Simplifying, we see that (59) holds and shareholders can gain from an asset sale if the debt repurchased is sufficiently junior so that its relative recovery rate satisfies $\alpha < \delta/E[x \mid x > \delta]$. ■

Proof of Proposition 11: For the case of leverage reductions, the only case left to prove is when debt repurchased must be senior, in which case the total value of equity issued is

$$N^S = p \times (A_1 - A_0) + q_L^J \times (D_0 - D_1) = N - \left(q(\delta_L) - q_L^S\right)(D_0 - D_1),$$

where $q_L^S > q(\delta_L)$ is the price of senior debt. The change in value for existing shareholders is:

$$\nabla V^E - N^S = \nabla V^E - N + \left(q(\delta_L) - q_L^J\right)(D_0 - D_1)$$

$$= -\left(q(\delta_L) - q(\delta_H)\right)D_0 - \left(v(\delta_H) - v(\delta_L)\right)A_0 + \left(q(\delta_L) - q_L^S\right)(D_0 - D_1). \tag{60}$$

For a recapitalization or asset sale, $D_0 > D_1$, and so relative to an asset expansion (in which debt is unchanged), shareholder losses increase by

$$\left(q_L^S - q(\delta_L)\right)(D_0 - D_1) > 0.$$

By a similar argument as in the proof of Proposition 10, because the amount of debt repurchased is larger in an asset sale, the total loss from an asset sale exceeds that from a recapitalization.

The ranking for leverage increases follows by a symmetric argument, with signs reversed. When selling junior debt rather than repurchasing it, however, note that the firm will receive the average value of the new debt issued rather than repurchasing it at the marginal value of the last dollar repurchased. As a result, with perfect markets, selling strictly junior debt and repurchasing equity has zero value to shareholders, while the reverse transaction (selling equity and purchasing junior debt) entails a strict loss. ■

Proof of Proposition 12: Let $q(\delta)$ is the market value of a unit of debt (face value is equal to 1) when the perceived leverage is δ and let $e(\delta)\, pA$ be the *total* market value of equity when the market value of assets is equal pA and the (perceived) leverage is δ.

In a recapitalization the firm must issue equity sufficient to buy back Δ_D units of debt so that

$$\frac{D - \Delta_D}{A} = \delta_L, \text{ or } D - \Delta_D = \delta_L A \tag{61}$$

The true value of current equity holders' claim after recapitalization will be:

$$\left(1 - \frac{q(\delta_L)\Delta_D}{pA - q(\delta_L)(D - \Delta_D)}\right) e\left(\frac{\delta_L}{1+\omega}\right) p(1+\omega) A \tag{62}$$

The *total* value of equity after the leverage reduction (from the perspective of the informed insiders) is $e\left(\delta^{True}\right)p\left(1+\omega\right)A$ where $\delta^{True}=\delta_L/\left(1+\omega\right)$ and $p\left(1+\omega\right)A$ is the managers' assessment of the value of the assets. Note that true leverage as perceived by the managers is less than the market perceived leverage since the market is undervaluing the assets. The fraction of the total equity claim retained by current shareholders is based on the amount that must be raised through issuing equity to buy back the debt, i.e., $q\left(\delta_L\right)\Delta_D$, and the market's valuation of equity after the recapitalization, i.e., $pA-q\left(\delta_L\right)\left(D-\Delta_D\right)$.

Substituting (61) into (62), we have

$$\left(1-\frac{q\left(\delta_L\right)\Delta_D}{pA-q\left(\delta_L\right)\left(D-\Delta_D\right)}\right)e\left(\frac{\delta_L}{1+\omega}\right)p\left(1+\omega\right)A=\left(\frac{pA-q\left(\delta_L\right)D}{pA-q\left(\delta_L\right)\left(D-\Delta_D\right)}\right)e\left(\frac{\delta_L}{1+\omega}\right)p\left(1+\omega\right)A$$
$$=\left(\frac{pA-q\left(\delta_L\right)D}{pA-q\left(\delta_L\right)pA\delta_1}\right)e\left(\frac{\delta_L}{1+\omega}\right)p\left(1+\omega\right)A=\left(\frac{pA-q\left(\delta_L\right)D}{1-q\left(\delta_L\right)\delta_L}\right)e\left(\frac{\delta_L}{1+\omega}\right)\left(1+\omega\right)$$

(63)

In reducing leverage through assets sales the amount of debt bought back must solve:

$$\frac{D-\Delta_D}{pA-q\left(\delta_L\right)\Delta_D}=\delta_L\quad\text{or}\quad\Delta_D=\frac{D-pA\delta_L}{1-q\left(\delta_L\right)\delta_L}$$

(64)

Since $A-q\left(\delta_L\right)\Delta_D/p$ will be the new level of assets after the deleveraging is completed, the value of the equity claim after the asset sales is:

$$e\left(\frac{\delta_L}{1+\omega}\right)p\left(1+\omega\right)\left(A-\frac{q\left(\delta_L\right)\Delta_D}{p}\right)$$

(65)

Using (64), we find that the new level of assets will be:

$$\left(A-\frac{q\left(\delta_L\right)}{p}\left(\frac{D-pA\delta_L}{1-q\left(\delta_L\right)\delta_L}\right)\right)=\left(\frac{pA-pAq\left(\delta_L\right)\delta_L-q\left(\delta_L\right)D+pAq\left(\delta_L\right)\delta_L}{p-pq\left(\delta_L\right)\delta_L}\right)$$
$$=\left(\frac{pA-q\left(\delta_L\right)D}{p-pq\left(\delta_L\right)\delta_L}\right)$$

(66)

This means that (65) becomes

$$e\left(\frac{\delta_L}{1+\omega}\right)\left(1+\omega\right)\left(\frac{pA-q\left(\delta_L\right)D}{1-q\left(\delta_L\right)\delta_L}\right)$$

(67)

Since this is precisely equal to (63), the shareholders are indifferent between recapitalization and asset sales.

References

1) Admati, Anat R. (2014), Admati, Anat R. (2014), "Examining the GAO Report on Expectations of Government Support for Bank Holding Companies," Testimony for hearing of Senate Committee on Banking, Housing and Urban Affairs Subcommittee on Financial Institutions and Consumer Protection, July 31.

2) Admati, Anat R., Peter M., DeMarzo, Martin F. Hellwig and Paul Pfleiderer (2013), "Fallacies, Irrelevant Facts, and Myths in the Discussion of Capital Regulation: Why Bank Equity is Not Socially Expensive" [Note: This is a new version that replaces the 2011 paper with a similar title.]

3) Admati, Anat R., and Martin F. Hellwig (2013a), *The Bankers' New Clothes: What's Wrong with Banking and What to Do about It*, Princeton University Press. (Excerpts and links available at http://bankersnewclothes.com/)

4) Admati, Anat R., and Martin F. Hellwig (2013b), "Does Debt Discipline Bankers? An Academic Myth about Bank Indebtedness," Working paper.

5) Admati, Anat R., and Martin F. Hellwig (2014), "The Parade of Bankers' New Clothes Continues: 23 Flawed Claims Debunked," Working paper.

6) Becht, Marco, Patrick Bolton, and Ailsa Roell (2011), "Why bank governance is different?" *Oxford Review of Economic Policy* 27:437–463.

7) Berger, Allen N., Richard J. Herring, and Giorgio P. Szegö (1995), "The Role of Capital in Financial Institution," *Journal of Banking and Finance,* 19:93-430

8) Berk, Jonathan, Richard Stanton and Josef Zechner (2010), "Human Capital, Bankruptcy, and Capital Structure, *Journal of Finance* 65:891-926.

9) Bhamra, Harjoat S., Lars-Alexander Kuehn, and Ilya A. Strebulaev (2010), "The Aggregate Dynamics of Capital Structure and Macroeconomic Risk," *Review of Financial Studies* 23:4187-4241.

10) Bizer, David S. and Peter M. DeMarzo (1992), "Sequential Banking," *Journal of Political Economy* 100:41-61.

11) Bizer, David S. and Peter M. DeMarzo (1994), "Sequential Trade and the Coase Conjecture: A General Model of Durable Goods Monopoly with Applications to Finance." Working paper.

12) Black, Fischer and Myron S. Scholes (1973), "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy* 81:637-654.

13) Bolton, Patrick and Xavier Freixas (2006), "Corporate Finance and the Monetary Transmission Mechanism," *Review of Financial Studies* 19:829-870.

14) Bolton, Patrick and Martin Oehmke (2014), "Should Derivatives Be Privileged in Bankruptcy?" *Journal of Finance*, forthcoming.

15) Brunnermeier, Markus, and Martin Oehmke (2013), "The Maturity Rat Race," *Journal of Finance*, 68:483-521.

16) Bulow, Jeremy and Kenneth Rogoff (1990), "Cleaning up Third-World Debt Without Getting Taken to the Cleaners," *Journal of Economic Perspectives* 4:31-42.

17) Dang, Thomas and Josef Zechner (2007), "Debt Maturity and the Dynamics of Leverage," Working paper, University of Vienna.

18) Diamond, Douglas W. and Zhiguo He (2014), "A Theory of Debt Maturity: The Long and Short of Debt Overhang," *Journal of Finance* 69:719-762.

19) Fama. E. F. and Merton H. Miller (1972), *The Theory of Finance*, New York: Holt, Rhinehart and Winston.

20) Fischer, Edwin O., Robert Heinkel, and Josef Zechner (1989), "Optimal Dynamic Capital Structure Choice: Theory and Tests," *Journal of Finance* 44:19-40.

21) Frenkel, Jacob A., Michael P. Dooley, and Peter Wickham (eds.) (1989), *Analytical Issues in Debt*, International Monetary Fund.

22) Goldstein, Robert, Nengjiu Ju, and Hayne E. Leland (2001), "An EBIT-based Model of Dynamic Capital Structure," *Journal of Business* 74:483-512.

23) He, Zhiguo and Wei Xiong (2012), "Debt Financing in Asset Markets," *American Economic Review*, Papers and Proceedings, 88-94.

24) Heider, Florian and Alexander Ljungqvist (2015), "As Certain as Debt and Taxes: Estimating the Tax Sensitivity of Leverage from State Tax Change," *Journal of Financial Economics*, forthcoming.

25) Hellwig, Martin F. (1977), "A Model of Borrowing and Lending with Bankruptcy," *Econometrica* 45:1879-1906.

26) Hennessy, Christopher A. and Toni M. Whited (2005), "Debt Dynamics," *Journal of Finance* 60:1129-1165.

27) Jensen, Michael C. (1986), "Agency Cost Of Free Cash Flow, Corporate Finance, and Takeovers" *American Economic Review* 76:323-329.

28) Jensen, Michael C. and William H. Meckling (1976), "Theory of the Firm, Managerial Behavior, Agency Costs and Ownership Structure," *Journal of Financial Economics* 3:305-360.

29) Kashyap, Anil, K., Jeremy C. Stein, and Samuel Hanson (2010), "An Analysis of the Impact of "Substantially Heightened" Capital Requirements on Large Financial Institutions," Working Paper, University of Chicago and Harvard University.

30) Kashyap, Anil, K., Samuel Hanson and Jeremy C. Stein, (2011), "A Macroprudential Approach to Financial Regulation" *Journal of Economic Perspectives*, 3-28.

31) Leland, Hayne E. (1994), "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure," *Journal of Finance* 49:1213-1252.

32) Leland, Hayne E. (1998), "Agency Costs, Risk Management, and Capital Structure," *Journal of Finance* 53:1213-1243.

33) Modigliani, Franco, and Merton H. Miller (1958), "The Cost of Capital, Corporation Finance, and the Theory of Investment," *American Economic Review* 48:261-297.

34) Myers, Stewart C. (1977), "Determinants of Corporate Borrowing," *Journal of Financial Economics* 5:147-175.

35) Myers, Stewart C. and Nicholas S. Majluf (1984), "Corporate Financing and Investment Decisions when Firms Have Information that Investors Do Not Have," *Journal of Financial Economics* 13:187-222.

36) Pfleiderer, Paul (2014), "Chameleons: The Misuse of Theoretical Models in Finance and Economics," Working paper.

37) Strebulaev, Ilya (2007), "Do Tests of Capital Structure Mean What They Say?" *Journal of Finance* 62:1747-1787.

38) Strebulaev, Ilya and Baozhong Wang (2013), "The Mystery of Zero-Leverage Firms," *Journal of Financial Economics* 109:1-23.

39) Strebulaev, Ilya and Toni M. Whited (2012), "Dynamic Models and Structural Estimation in Corporate Finance," *Foundations and Trends in Finance* 6:1-163.

40) Titman, Sheridan and Sergey Tsyplakov (2007), "A Dynamic Model of Optimal Capital Structure," *Review of Finance* 11:401-451.

41) Van Wijnbergen, Sweder (1991), Mexico and the Brady Plan (with comments by Mervyn King and Richard Portes), *Economic Policy* 12:13 – 56.