# Tailored Cheap Talk*

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March 2016

**Abstract**

We consider a cheap-talk game in which the persuader is able to collect information about the receiver's preferences in order to tailor communication and induce a favorable action. We find that the sender prefers not to learn the receiver's preferences with certainty, but to remain in a state of partial willful ignorance. The receiver prefers complete privacy except when information is necessary to induce communication from the sender. Surprisingly, joint welfare is always maximized by the sender's first-best level of information acquisition. The implications of our results are discussed in the contexts of online advertising, sales, dating and job search.

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# 1 Introduction

We consider a communication game in which a persuader tries to elicit a desired action from an agent by means of a compelling argument. In order to understand which arguments may indeed be compelling, the sender of the message can collect information about the receiver's preferences prior to the communication stage. This simple arrangement can lead to complex consequences because the receiver understands that the communication may have been appropriately tailored to appear persuasive.

Communication is a key feature of many matching markets: advertisers, salespeople, job candidates and romantic suitors put forth arguments to persuade their prospects of favorable match values. These claims, however, are often ex-ante unverifiable: whether an eatery serves the best hamburgers in town or an automobile is especially comfortable may require a visit to a restaurant or a dealership, and so the burden is often on the persuader to communicate in a way that merits a match in the eyes of the receiver.

In order to capture this phenomenon we build on the cheap-talk framework proposed by Crawford and Sobel (1982). The central distinction of our work is that we allow the sender to engage in endogenous information acquisition about the receiver's preferences and tailor communication accordingly. This assumption reflects the reality of multiple matching markets: Advertising platforms (including Facebook, Google, Microsoft and Yahoo) offer firms ways to tailor their advertisements in real time, according to users' search terms, demographics, cell phone usage/locations, browsing behaviors, device characteristics, etc.[1] Automobile salespeople and real

[1] See Google ad customizers (https://support.google.com/adwords/answer/6072565) and dynamic creatives (https://support.google.com/richmedia/answer/2691686), and Microsoft behavioral advertising (http://advertising.microsoft.com/en/behavioral-

estate agents tailor arguments to individual buyers based on elicited information in hopes of inducing test drives and bids for properties. Job seekers are able to gather information about potential employers and tailor resumes to their preferences. Similarly, individuals looking for romantic partners are approached by suitors who can collect information about their interests through online social platforms.

Relatedly, advances in data storage and processing have both led to widespread information acquisition as well as increased privacy concerns. For example, in January 2016 several consumer groups urged the Federal Communications Commission to establish higher privacy controls from internet service providers, citing in part the amounts of data collected for targeted advertising purposes.[2] Despite these efforts the debate around privacy is arguably far from settled and the merits of potential regulations remain hard to articulate. Our model sheds light on this debate by characterizing the effects of information acquisition and privacy on communication, trade and welfare.

In the examples above both the sender and the receiver exhibit matching preferences. The intuitive reason for this is that interactions between agents do not typically terminate immediately after a match takes place, but are often followed by post-match allocation stages. For example, a consumer who clicks on an online advertiser's link has to subsequently decide whether to buy a product at a posted price. Because of this, the advertiser prefers inducing clicks from consumers who are more likely to buy, and the

targeting) for examples. See also http://www.economist.com/news/special-report/21615871-everything-people-do-online-avidly-followed-advertisers-and-third-party for current tracking practices.

[2]See http://www.reuters.com/article/us-broadband-internet-privacy-idUSKCN0UY0CM.

consumers would also like to click on advertisements of firms carrying attractive offerings. Similarly, salespeople would like to attract high-potential buyers and both employers and employees negotiate over streams of future payoffs.

Our work is related to that of Bagwell and Ramey (1993), Gardete (2013) and Chakraborty and Harbaugh (2014) who consider informative communication as a way of matching products to buyers. Bagwell and Ramey (1993) show that informative communication may fail in the absence of costly signals in vertically-differentiated markets, and Gardete (2013) shows that if the incentives between firms and consumers are not well aligned then firms prefer to exaggerate their quality levels. Chakraborty and Harbaugh (2014) find that multidimensional communication helps sellers in the matching process. At the core of the credibility results is the initial insight by Farrell and Gibbons (1989) that senders may attain credibility because their messages are attractive to some audiences and simultaneously unattractive to others. Related to this idea, Chakraborty and Harbaugh (2010) show that by partitioning the message space - or by implicitly introducing comparative language - a sender can induce favorable responses from receivers in multiple contexts.

We extend this work by allowing the sender to engage in information acquisition about the receiver's preferences to tailor communication. In addition, we show that both the cost and the content of communication can be informative. While dissipative communication (e.g. Kihlstrom and Riordan, 1984; Milgrom and Roberts, 1986) is often considered as a substitute to the informative one (e.g. Bagwell and Ramey, 1993), in our analysis we find that these mechanisms can be complementary.

A related literature stream focuses on persuasion and disclosure. Ostrovsky and Schwarz (2010) consider the case of information disclosure between senders (schools) and receivers (employers) and find that senders may prefer to disclose only partial information in order to induce attractive receiver actions. Rayo and Segal (2010) document a similar finding in a setting with two-sided uncertainty. Kamenica and Gentzkow (2011) show that a sender can influence the receiver's action by affecting the signal realization process, while fully disclosing all informative outcomes. In addition to considering endogenous information acquisition, one of our key assumptions is that the sender lacks commitment power about the communication policy. This enables us to examine the tradeoff between information and credibility. The cheap-talk assumption relates to the fact that moderate misrepresentation is legal and sometimes expected in advertising contexts. Both the Federal Trade Commission in the United States and the Advertising Standards Authority in the United Kingdom allow advertisers to engage in 'puffery', i.e. reasonable exaggerations in advertising claims (see also Chakraborty and Harbaugh, 2010).

On the information acquisition front, Shen and Villas-Boas (2016) consider the case of a monopolist who uses first-period purchases to target advertising of another product at a latter stage, and de Cornière and de Nijs (2016) consider the case of a platform that may release consumer valuation information to bidders in an advertising auction. While these papers focus on modeling specific contexts, they do not address communication per se. Rather, advertising affects outcomes by increasing the likelihood that consumers are aware of the firm's offering.[3]

[3]Also, Roy (2000), Esteban, Gil, and Hernandez (2001), Iyer, Soberman, and Villas-Boas (2005), Deng and Mela (2016) and Tuchman, Nair, and Gardete (2016) consider

While different markets use different allocation rules (e.g. auctions in real estate, bargaining in automobile sales, etc), the common thread is that agents expect certain payoffs in case a match takes place. Rather than modeling the ultimate payoff-splitting rules we focus on the payoffs agents *expect* to earn if a match is produced. This assumption ensures that the results do not depend on subsequent value extraction activities that take advantage of the available information. While we abstract from the relationship between information acquisition and the value extraction ability of the sender, such an extension is straightforward in our model and is unlikely to have qualitative implications to our results. The first-order mechanism in our model is the fact that the sender benefits from information acquisition because he is able to increase the probability of trade by successfully persuading the receiver through compelling communication.[4]

The model considers the setting in which the sender features ex-ante 'transparent motives' (as defined by Chakraborty and Harbaugh, 2010) such that persuasion takes place despite the fact that the receiver understands the sender's preferred outcome clearly. Our main finding is that senders prefer to remain in a state of partial willful ignorance so as to preserve communication credibility. Moreover, we derive the first-best information level of the sender and show that it is attained by a communication policy that involves sampling from attractive messages at different rates. The extension to the case of costly communication reveals that both the cost and the content of the communication can be simultaneously informative

targeted advertising applications under monopoly and competition settings, but the mechanisms are different. In these cases advertising affects the receiver's payoffs directly, through complementarities or by generating awareness. Sahni, Wheeler, and Chintagunta (2016) consider the different mechanism of non-informative personalization.

[4]For economy of language we will often refer to a male sender and a female receiver.

for the receiver. We find that the receiver prefers complete privacy except when information is necessary to induce communication by the sender, in which case she prefers full disclosure. Surprisingly, joint welfare is always maximized by the sender's first-best level of information acquisition. We discuss the remaining results throughout the paper.

In the next section we discuss the main assumptions of the model. Section 3 presents the solution strategy and the main results. In Section 4 we consider the case of costly communication, in which case the sender's motives are not ex-ante transparent. Section 5 integrates the findings and provides a discussion of welfare implications. Section 6 concludes.

## 2 Preliminaries

We consider a matching model in which a sender and a receiver are independently located along a preference circle with uniform probability. The receiver's location is given by $\theta \sim U[0, 2\pi)$ and the sender's location is given by $q \sim U[0, 2\pi)$. In this market the receiver can take one of two actions, $a \in \{0, 1\}$. The goal of the sender is to induce action $a = 1$, which we refer to as a 'match'. In the online advertising context action $a = 1$ is typically a click on an advertisement.

If the receiver takes action $a = 1$ she earns utility $U^R = v^R - d(\theta, q)$ and the sender earns utility $U^S = v^S - d(\theta, q)$, where $v^R$ and $v^S$ are positive and $d(\theta, q)$ is a distance function. Action $a = 0$ yields the value of the outside option to both parties, with payoffs normalized to zero.

The preference structure captures the fact that in matching markets agents have preferences over the agents they may be matched to. For example, in automobile markets salespeople prefer attracting consumers who favor the brand/models being sold. In this case $U^R$ and $U^S$ capture the utility that the parties expect to earn in case the dealer is successful in inducing a consumer visit ($a = 1$) to the dealership.

The sender uses a message to communicate his own location and persuade the receiver of the merits of a match. We denote the sender's message as $m$, which lies in the circular domain $[0, 2\pi)$. The information acquisition level of the sender is given by parameter $\alpha \in [0, 1]$. Formally speaking, $\alpha$ is the probability that the sender is informed of the receiver's location $\theta$.

We assume that the cost of acquiring information about an additional receiver is equal to zero. This assumption allows us to focus the analysis on communication credibility concerns, as an alternative to the more straightforward case of costly information gathering. While in reality acquiring consumer data is costly, online advertisers typically obtain these services for free. In other cases information acquisition is costly on the margin but these costs tend to decrease over time. Finally, our results only require information acquisition to be cheap, but not necessarily free.

The final assumption is that the receiver does not know whether she has been identified by the sender, but observes the average level of information acquisition $\alpha$. Firms often communicate the types of data they gather about consumers. These efforts often deserve media attention and keep consumers aware of policy changes.[5] We assume that even if a given receiver is not aware of the particular data that the sender collects about her own preferences, she is aware of the average level of information acquisition.[6]

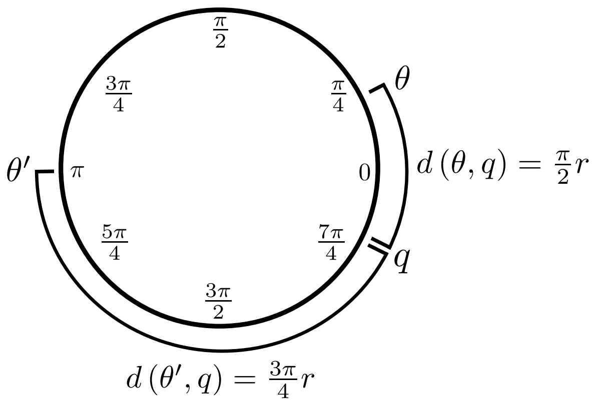
[5]See http://www.amazon.com/gp/help/customer/display.html?nodeId=468496 and https://privacy.google.com/data-we-collect.html for examples.

[6]In contrast, it is easy to show that the outcome of a model in which the receiver does not observe $\alpha$ is that the sender always prefers full information acquisition, and

# 3 Model and Solution Strategy

The distance function $d(\theta, q)$ represents the preference mismatch between the sender and the receiver and is given by $d(\theta, q) = r.\cos^{-1}(\cos(\theta - q))$. It is intuitively understood as the shortest angular distance between $\theta$ and $q$, multiplied by scalar $r > 0$. Figure 1 provides a representation. Parameter

Figure 1: Illustration of the Distance Function $d(\theta, q)$



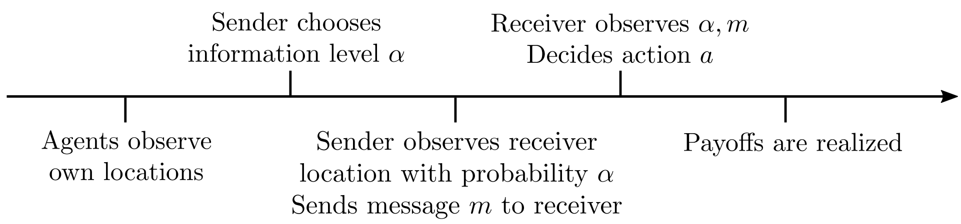

Note: The distance function $d(\theta, q)$ measures the shortest distance between locations $\theta$ and $q$, multiplied by $r$. Formally, it is given by $d(\theta, q) = r.\cos^{-1}(\cos(\theta - q))$.

$r$ captures the market differentiation level and simultaneously affects the match values of the sender and of the receiver by introducing a distance penalty. The parameter has a real world interpretation: in markets with large differentiation, which may be 'thin' and/or exhibit long tails, $r$ is large and parties expect to earn relatively low payoffs on average.

The timing of the game is given in Figure 2. First, both the sender and the receiver privately learn their respective locations. Second, the sender chooses the information level $\alpha \in [0, 1]$, and as a result learns the receiver's location with probability $\alpha$. The sender then decides on a message $m$. Upon

credibility trivially breaks down.

Figure 2: Timing




observing the information acquisition level and the message, the receiver finally decides her action and payoffs are realized.

We introduce the decision over the message after the one about information acquisition because in most markets senders are likely to have stable data collection policies whereas messages are personalized. Moreover, note that because location $q$ is not informative about the distance to the receiver's location, the order of the first two stages does not affect our analysis. Intuitively, no location $q$ offers the sender a specific differentiation that by itself would make him prefer different levels of $\alpha$.[7]

We look for perfect Bayesian equilibria such that at each information set agents maximize their utilities given their beliefs, and on-equilibrium path beliefs are given by Bayes rule. The latter requirement implies that the receiver's beliefs about the sender's location are consistent with the distribution induced by the communication policy. The receiver forms beliefs about the sender's type conditional on three pieces of information: her own type $\theta$, the message $m$ and the level of information $\alpha$. We denote these

[7]To be precise, while it is possible to construct equilibria where the sender's location can be used to affect information acquisition, this case is ruled out by a Markov perfect equilibrium restriction.

beliefs as $\widehat{f_{q|\theta,m,\alpha}}$, which should be considered fixed. Bayes rule implies

$$\widehat{f_{q|\theta,m,\alpha}} = \frac{f_{m^*|\theta,q,\alpha} \cdot f_{q|\theta,\alpha}}{f_{m^*|\theta,\alpha}} = \frac{f_{m^*|\theta,q,\alpha} \cdot f_q}{\int_0^{2\pi} f_{m^*|\theta,q,\alpha} \cdot f_q dq} \tag{1}$$

where $f_{m^*|\theta,q,\alpha}$ is the probability density function (p.d.f.) induced by the sender's optimal message policy $m^*$, and $f_{q|\theta,\alpha}$ is the p.d.f. of the sender's type conditional on the receiver's location $\theta$ and the information acquisition level $\alpha$. Because agents' locations are independent, it follows that $f_{q|\theta,\alpha} = f_{q|\alpha}$. Moreover, the problem of choosing $\alpha$ is invariant to the sender's location as we discuss above. Hence, $f_{q|\theta,\alpha} = f_{q|\alpha} = f_q$: because $q$ does not affect the choice of $\alpha$, knowledge of $\alpha$ does not affect the posterior knowledge of $q$.

We restrict our attention to intuitive outcomes in which the receiver is willing to match if and only if she receives an attractive message, i.e. we look for equilibria with the action policy

$$a^* = \begin{cases} 1, & m \in c_\theta \\ 0, & m \notin c_\theta \end{cases} \tag{2}$$

where $c_\theta \subseteq [0, 2\pi)$ is the set of persuasive messages.

We will later show that all messages fall on the equilibrium path and so no off-equilibrium path beliefs are required. We ignore the 'babbling' outcome, a feature present in cheap-talk games, in which the receiver ignores all statements of the sender, and the sender becomes indifferent across all potential messages, including mixing them in an uninformative fashion.

We focus on the cases in which i) the sender has ex-ante *transparent motives* and ii) communication is *decisive*. The first assumption means that the sender has a clearly preferred action, which is known to the receiver.[8] This assumption translates to restriction $v^S > \pi r$, i.e. the sender always prefer a match independently of the receiver's location. We consider the remaining cases in the next section.

The assumption that communication is decisive means that we focus on cases in which communication has the ability to induce a match. In contrast, there would be no point in communicating to a receiver who already expects a very high match value. This assumption translates into $v^R - E_q\left(d\left(\theta, q\right) \middle| \theta\right) = v^R - \frac{\pi r}{2} < 0$, i.e. the receiver prefers the outside option if no information about the sender's location is available.

## 3.1 Communication Stage

We start by solving the communication problem, at which stage the information acquisition level $\alpha^*$ has already been decided. Immediately before sending the message, the sender can be in one of two states: with probability $\alpha$ he has information about the receiver's location, and with probability $1 - \alpha$ he has not. When the sender knows the location of the receiver his utility is given by

$$U^S_{Info} = \max_m \mathbf{1}\left(m \in c_\theta\right) \left(v^S - d\left(\theta, q\right)\right) \tag{3}$$

In this case, the sender is indifferent across messages in $c_\theta$ because all of them are successful in inducing the preferred action. The optimal messaging strategy may involve mixing, a result that is common in the cheap-talk literature. We allow mixing because it may be optimal for the sender to

[8] This is in contrast with the work by Crawford and Sobel (1982), in which the receiver is uncertain about the sender's preferred action.

sample from different messages, potentially at different rates.

Consider now the case in which the sender does not know the location of the receiver. In order to solve this case we require a parametrization assumption about region $c_\theta$. We assume $c_\theta = \left\{m : d(\theta, m) \leq \overline{\Delta}\right\}$, which means that the receiver is willing to match as long she receives a message near her location. This parametrization has an intuitive appeal, as we will now describe. An uninformed sender solves communication problem

$$\begin{aligned} U^S_{NoInfo} &= \max_m E_\theta \left[ \mathbf{1}(m \in c_\theta) \left( v^S - d(\theta, q) \right) \middle| q \right] \\ &= \max_m \frac{1}{2\pi} \int_{d(\theta,m) \leq \overline{\Delta}} v^S - d(\theta, q)\, d\theta \\ &= \max_m \frac{1}{2\pi} \int_{m-\frac{\overline{\Delta}}{r}}^{m+\frac{\overline{\Delta}}{r}} v^S - d(\theta, q)\, d\theta \end{aligned} \tag{4}$$

i.e. the sender chooses the message that maximizes his expected utility. Differentiating under the integral sign yields the first-order condition

$$d\left(m^* + \frac{\overline{\Delta}}{r}, q\right) = d\left(m^* - \frac{\overline{\Delta}}{r}, q\right) \tag{5}$$

The optimal message is equidistant above and below the sender's location $q$, a condition that is satisfied by $m^* = q + k\pi$, $k = .., -2, -1, 0, +1, +2, ..$, and setting $k = 0$ minimizes the expected distance to a receiver located at $\theta$. Intuitively, when the sender does not know the receiver's location he is better off attracting a local receiver than one who is far away, and so in this case the sender is better off revealing his type truthfully by sending message $m^* = q$.[9]

[9]Communication parametrizations imply a 'meaning' of the message to senders. The current parametrization means that by observing message $m$ the receiver understands that with some probability the message originates from an uninformed sender located at position $q = m$. Other rules such as $c_\theta = \left\{m : d\left(\theta + \frac{\pi}{2}, m\right) \leq \overline{\Delta}\right\}$ for example are

In summary, when the receiver's location is known, the sender draws a message within $c_\theta$; otherwise the sender prefers to reveal his location truthfully. The optimal messaging policy induces density

$$f_{m^*|\theta,q,\alpha} = \alpha\phi\left(m, q, \theta, \alpha\right) + \left(1 - \alpha\right)\delta\left(m - q\right) \tag{6}$$

where $\phi\left(m, q, \theta, \alpha\right)$ describes the mixing message policy of an informed sender, and $\delta\left(\cdot\right)$ is the Dirac-delta function representing a mass point.[10]

## 3.2 Equilibrium Outcome

Given the messaging policy discussed above, we now derive an upper bound on the level of information acquisition for the sender. From the previous section, an informed sender prefers to send some message $m \in c_\theta$ to induce a match, and an uninformed sender prefers to send message $m = q$. Assuming $c_\theta$ is non-empty in equilibrium, it follows that the sender's ex-ante utility is given by

$$E\left(U^S\right) = \alpha E\left(v^S - d\left(\theta, q\right)\right) + \left(1 - \alpha\right) Pr\left(q \in c_\theta\right) E\left(v^S - d\left(\theta, q\right) \middle| q \in c_\theta\right) \tag{7}$$

The first term arises because all informed types are able to attract the receiver when $c_\theta$ is non-empty and the second term captures the fact that uninformed types are only able to attract the receiver in some circumstances. The sender's utility is increasing in the level of information acquisition as long as $c_\theta$ is non-empty because the sender is willing to match with any

also possible but are less intuitive and often yield similar payoffs.

[10] It is useful to note that while $f_{m^*|\theta,q,\alpha}$ includes $\theta$ in its notation, this does not imply that the sender always knows the receiver's location. Parameter $\theta$ is 'given' in $f_{m^*|\theta,q,\alpha}$ in the sense of the data generation process rather than in the sense of the information set.

receiver type $\left(v^S > \pi r\right)$. Consider the receiver's ex-ante utility,

$$E\left(U^R\right) = \alpha E\left(v^R - d\left(\theta, q\right)\right) + \left(1 - \alpha\right) Pr\left(q \in c_\theta\right) E\left(v^R - d\left(\theta, q\right) \middle| q \in c_\theta\right) \tag{8}$$

As before, the first term captures the fact that any informed sender is able to ensure a match in equilibrium, and the second one captures the fact that with probability $1 - \alpha$ the sender induces a match if and only if $q \in c_\theta$. Based on these two identities and the communication policy from the previous section, we establish the following result:

**LEMMA 1** (*Willful Ignorance*) *The level of information acquisition associated with the* sender*'s first-best payoff is given by*

$$\overline{\alpha}^* = \left(\frac{v^R}{\pi r - v^R}\right)^2 \tag{9}$$

The intuition for Lemma 1 is that the expected receiver utility conditional on any message is always bounded below by zero, which means that the ex-ante utility must also be bounded below by zero. Hence, if the first-best level of information acquisition $\alpha^*$ were to be strictly above $\overline{\alpha}^*$ then there must exist some message $m_\theta$ that yields negative expected utility to the receiver. However, this is in contradiction with the strategic assumption of our receiver's behavior. Interpreted differently, $\overline{\alpha}^*$ is the highest level of information acquisition that enables an informed sender to pool with other desirable types to induce a match. If in contrast $\alpha$ were strictly higher than $\overline{\alpha}^*$ then $c_\theta$ would be empty, i.e. no message could ensure a match in equilibrium. Finally, threshold $\overline{\alpha}^*$ falls between zero and one and is increasing in the receiver's valuation $v^R$ and decreasing in the market differentiation parameter $r$. Clearly, the sender is able to engage in higher levels of information acquisition as the receiver values a match more. Relatedly, the information does not depend on $v^S$ because the sender is always better off obtaining a match, and so the receiver's expected utility is the only relevant constraint. While other equilibria exist, we will later show that the sender's first-best outcome is focal.

We now show that there exists a messaging policy $m^*(\cdot)$ that can implement the information acquisition level $\overline{\alpha}^*$ while keeping region $c_\theta$ non-empty:

**THEOREM 1** *The sender can attain his first-best outcome by selecting the message policy that induces the probability density function*

$$f^*_{m|\theta,q} = (1-\alpha^*)\left(\frac{v^R - d(\theta,m)}{\pi\left(\pi r - 2v^R\right)}\mathbf{1}\left(d(\theta,m) \leq v^R\right) + \delta(m-q)\right) \tag{10}$$

*where $\alpha^* = \overline{\alpha}^*$ is the utility-maximizing level of information acquisition of the sender.*

The optimal communication policy for the sender involves sampling from different messages at different rates. The informed sender prefers to send attractive messages to the receiver, and samples from more attractive messages more frequently than from less attractive ones. Mixing across messages enables informed senders to pool with all attractive uninformed types, and allows attaining the sender's first-best level of information acquisition.

The result above follows from the following considerations. First, an informed sender has an incentive to pool with attractive uninformed types to the largest possible extent. Because attractive truth-telling types send messages in region $s_{\phi} = \left\{ m : d\left(\theta, m\right) \leq v^{R} \right\}$, informed types prefer to pool over the same region. Second, in the case of informed senders we need only consider communication policies that are invariant to the sender's location. In order to determine policy $f_{m^{*}|\theta,q,\alpha}$ in equation (6) we need only look for function $\phi\left(\cdot\right)$ from set

$$\Phi = \left\{ \phi'\left(m, \theta, \alpha\right) : E_{q}\left( U^{R} \middle| m \in s_{\phi}, \theta, \alpha \right) = 0 \right\} \tag{11}$$

which is independent of the sender's location $q$. The solution is attainable by setting the receiver's expected utility equal to zero point by point, and the result follows.

Depending on the receiver's beliefs, there may exist other equilibria. For example, if the receiver believes that the sender mixes among messages uniformly, then the sender may prefer to engage in such a policy. However, equilibria that do not attain the sender's first-best level of information acquisition are not robust to forward induction. To see this, consider an equilibrium outcome with fixed beliefs $\widehat{f_{m|\theta,q,\alpha}}$, which induces a level of $\alpha' < \overline{\alpha}^{*}$. Now suppose the sender deviates from this outcome and chooses level $\alpha'' \in (\alpha', \overline{\alpha}^{*}]$ instead. Under forward induction the receiver ascribes strategic behavior to the sender, and so is willing to 'revisit' her beliefs in order to rationalize the sender's choice of information acquisition. Consequently, any equilibrium with level $\alpha' < \overline{\alpha}^{*}$ does not survive forward induction because by choosing level $\overline{\alpha}^{*}$ instead, the sender can induce more advantageous beliefs. This is summarized in the following corollary.

**COROLLARY 1** *Only the equilibrium outcome associated with the sender's first-best level of information acquisition survives forward induction.*

In sum, forward induction allows us to rule out other equilibria that yield lower levels of information acquisition as well as different payoff levels. Theorem 1 also has implications to the payoff of the receiver:

**COROLLARY 2** *The sender's first-best information acquisition policy makes the receiver's ex-ante utility, and expected utility conditional on any given message, equal to zero.*

This result follows from the discussion above: by selecting his messaging policy appropriately the sender is able to ensure that the receiver always expects to earn zero utility. If the receiver were to expect a higher utility upon receiving a given message, the sender could alter his mixing distribution to increase the information acquisition level as well as his payoffs.

As a result, payoffs are given by

$$E\left(U^R\right)_{\alpha=\overline{\alpha}^*} = 0 \tag{12}$$

and

$$E\left(U^S\right)_{\alpha=\overline{\alpha}^*} = \frac{v^S - v^R}{\pi r - v^R} v^R \tag{13}$$

where, as expected, the sender is better off as $v^S$ and $v^R$ increase and as $r$ decreases.

We now consider the case in which the sender does not hold ex-ante transparent motives, i.e. $v^S < \pi r$. For example, in this case the sender may face high communication costs.

# 4 Costly Persuasion / Low Sender Valuation

In many contexts senders are required to incur a communication cost, say $c > 0$, in order to send a message to receivers. In this case engaging in communication yields gross utility $v^{S'} \equiv v^S - c$ for the sender. As communication costs increase, the match utility of the sender becomes lower and as a result he may no longer want to engage in communication with all receivers. For this reason we now allow the sender not to communicate in case he prefers to avoid a match with an unattractive receiver.

Figure 3: Partitions of the Parameter Space

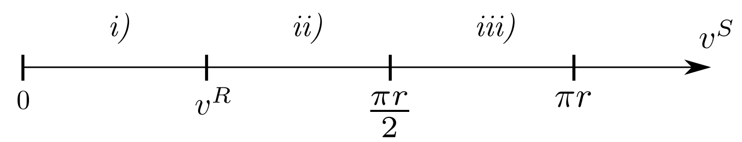



Figure 3 divides the parameter space into different partitions according to the value of $v^S$. In this section we characterize the cases in which the sender derives utility $v^S < \pi r$ from a match, i.e. the sender no longer holds ex-ante transparent motives. As before, we consider the case of decisive communication when $v^R < E\left(d\left(\theta, q\right)\right)$.

## 4.1 Region *i)* $v^S < v^R$

When communication costs are high, or equivalently when $v^S$ is low, the sender does not engage in misrepresentation because all attractive receiver types profit from a match. Consequently the sender engages in full information acquisition and reveals his type whenever his match utility is above zero. If an informed sender's match value is too low, then he prefers not to communicate.

**LEMMA 2** *When $v^S < v^R$ the sender engages in full information acquisition and the sender and receiver ex-ante payoffs are given by, respectively,*

$$
\begin{aligned}
E\left(U^S\right) &= Pr\left(v^S \geq d(\theta, q)\right) E\left(v^S - d(\theta, q) \middle| v^S \geq d(\theta, q)\right) \\
&= \frac{\left(v^S\right)^2}{2\pi r}
\end{aligned}
\tag{14}
$$

and

$$
\begin{aligned}
E\left(U^R\right) &= Pr\left(v^S \geq d(\theta, q)\right) E\left(v^R - d(\theta, q) \middle| v^S \geq d(\theta, q)\right) \\
&= \frac{v^S\left(2v^R - v^S\right)}{2\pi r}
\end{aligned}
\tag{15}
$$

In this case the comparative statics behave as expected and in particular the receiver is better off as $v^S$ increases in this region because this leads to a higher likelihood of a match.

## 4.2 Region *ii)* $v^R < v^S < \frac{\pi r}{2}$

When $v^S$ is slightly higher there exist unattractive types of senders that prefer to misrepresent themselves in order to induce matches. However, in this region uninformed senders prefer not to communicate because they expect negative utility from matches. As a result, the receiver understands that if she receives a message it must originate from an informed sender. Upon reception of a message the receiver expects match utility

$$E\left(U^R \middle| received\, message\right) = E\left(v^R - d(\theta,q) \middle| v^S \geq d(\theta,q)\right) = v^R - \frac{v^S}{2} \tag{16}$$

which yields the following result:

**LEMMA 3** *When $v^R < v^S < \frac{\pi r}{2}$ communication and matches take place if and only if $2v^R \geq v^S$, in which case the sender engages in full information acquisition and payoffs are equal to those of case i).*

When $2v^R \geq v^S$ the sender engages in full information acquisition because that decision has no bearing on the receiver's expected utility of a match, conditional on receiving a message. However, when $2v^R < v^S$ no matches occur because the sender's incentive for misrepresentation is too great. The reason is that under cheap-talk communication the sender is unable to commit not to send attractive messages when the value of the match is low.

## 4.3 Region *iii)* $\frac{\pi r}{2} < v^S < \pi r$

In this case uninformed senders are willing to communicate, but informed senders located far from the receiver's location may not be. This case is similar to the main model, with the added complexity that now informed types may prefer not to engage in communication. The receiver's utility conditional on receiving a message is

$$E\left(U^R\middle| message\right) = \gamma E\left(v^R - d(\theta,q)\middle| v^S \geq d(\theta,q)\right) + (1-\gamma)\, Pr\left(v^R \geq d(\theta,q)\right) E\left(v^R - d(\theta,q)\middle| v^R \geq d(\theta,q)\right) \tag{17}$$

where $\gamma$ is the probability that the sender is informed conditional on preferring to communicate; $\gamma = \frac{\alpha Pr\left(v^S \geq d(\theta,q)\right)}{\alpha Pr(v^S \geq d(\theta,q)) + 1 - \alpha}$. Equating $E\left(U^R\middle| message\right)$ to zero yields an upper bound on the sender's level of information acquisition

$$\overline{\alpha}'^* = \min\left\{1, \left(\frac{v^R}{v^S - v^R}\right)^2\right\} \tag{18}$$

By the same methodology used to prove Lemma 1 and Theorem 1, we find the following results:

**LEMMA 4** *There exists a message policy that enables the sender to attain level of information acquisition $\overline{\alpha}'^*$ when $\frac{\pi r}{2} < v^S < \pi r$. Forward induction equilibrium payoffs are given by*

$$\begin{aligned} E\left(U^R\right) &= \overline{\alpha}'^* Pr\left(v^S \geq d(\theta,q)\right) E\left(v^R - d(\theta,q)\middle| v^S \geq d(\theta,q)\right) \\ &+ \left(1-\overline{\alpha}'^*\right) Pr\left(v^R \geq d(\theta,q)\right) E\left(v^R - d(\theta,q)\middle| v^R \geq d(\theta,q)\right) \\ &= 0 \end{aligned} \tag{19}$$

*and*

$$\begin{aligned} E\left(U^S\right) &= \overline{\alpha}'^* Pr\left(v^S \geq d(\theta,q)\right) E\left(v^S - d(\theta,q)\middle| v^S \geq d(\theta,q)\right) \\ &+ \left(1-\overline{\alpha}'^*\right) Pr\left(v^R \geq d(\theta,q)\right) E\left(v^S - d(\theta,q)\middle| v^R \geq d(\theta,q)\right) \\ &= \frac{v^R v^S}{\pi r} \end{aligned} \tag{20}$$

As in the main model, the sender is able to appropriate all of the receiver's expected utility by setting $\alpha = \overline{\alpha}'^{*}$.

In this parameter region both the cost as well as the content of the message act as signals to the receiver. The mere presence of communication is relevant for the receiver because she understands that not all sender types are willing to communicate. The content of the message provides further information: if the message is close to the receiver's location then she weighs the relative probabilities of types of senders, but when the message is far away the receiver understands the sender is uninformed and takes appropriate action. Lemma 4 unites prior work on informative and dissipative communication, allowing these mechanisms to have complementary rather than substitute roles.

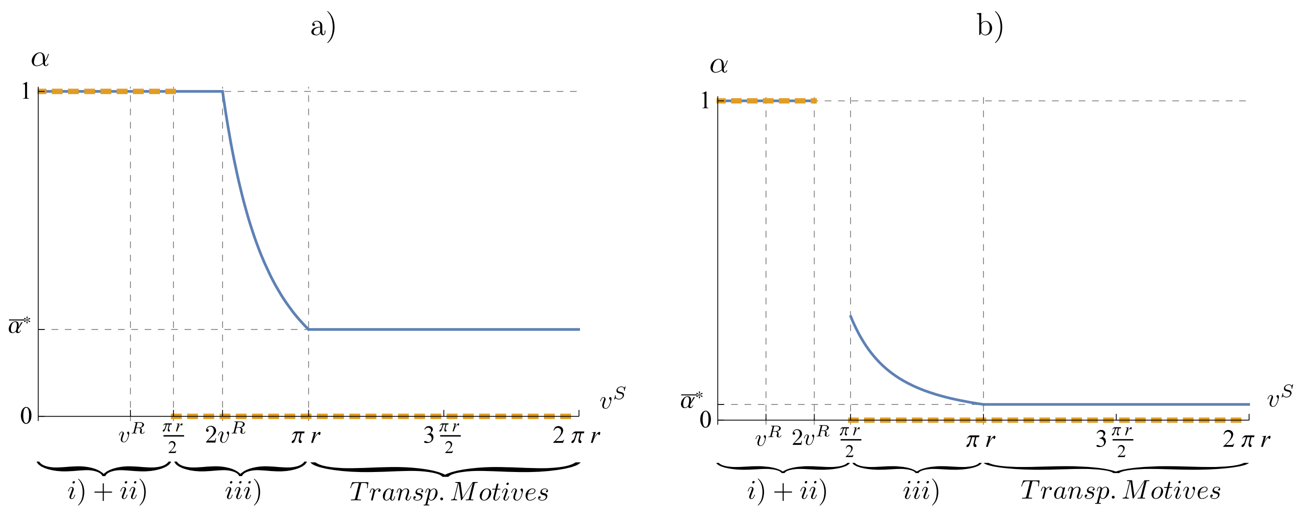
# 5 Discussion and Welfare Implications

Having characterized the equilibrium outcomes across different parameter regions, we now present general results on the first-best information levels:

**THEOREM 2** *The receiver's first-best level of information acquisition is always less or equal to the sender's. Moreover, both parties' preferred levels weakly decrease in $v^S$ and $r$ and increase in $v^R$.*

This is because the sender always prefers a higher level of information acquisition in order to engage in persuasive communication. However, unless information acquisition is necessary for a match to occur, the receiver prefers to be approached only by uninformed senders, who in turn prefer to communicate truthfully. Figure 4 summarizes the receiver and sender first-best information levels.

Figure 4: First-Best Information Levels for Sender and Receiver




Note: In both panels, $r = 1$. Panel a) $v^R = \frac{\pi r}{2} - \frac{1}{2}$; Panel b) $v^R = \frac{\pi r}{2} - 1$. Solid and dashed lines represent first-best levels of information acquisition for sender and receiver, respectively.

The left panel of Figure 4 shows that when $v^S < \frac{\pi r}{2}$, information acquisition is necessary to incentivize the sender to engage in communication, and so both parties prefer $\alpha = 1$. However, as $v^S$ increases past this range the sender has an incentive to tailor the message, and as a result the receiver prefers to avoid being identified. In general, the receiver's preferences over information collection are as follows: if information acquisition is a necessary condition to incentivize communication then the receiver prefers full disclosure. Otherwise, the receiver prefers complete privacy. In contrast, the sender always prefers high levels of information but is forced to reduce the information level as $v^S$ increases in order to satisfy the receiver's participation constraint. After $v^S > \pi r$ the sender is willing to attract any receiver, and so the actual level of $v^S$ no longer affects the level of information acquisition.

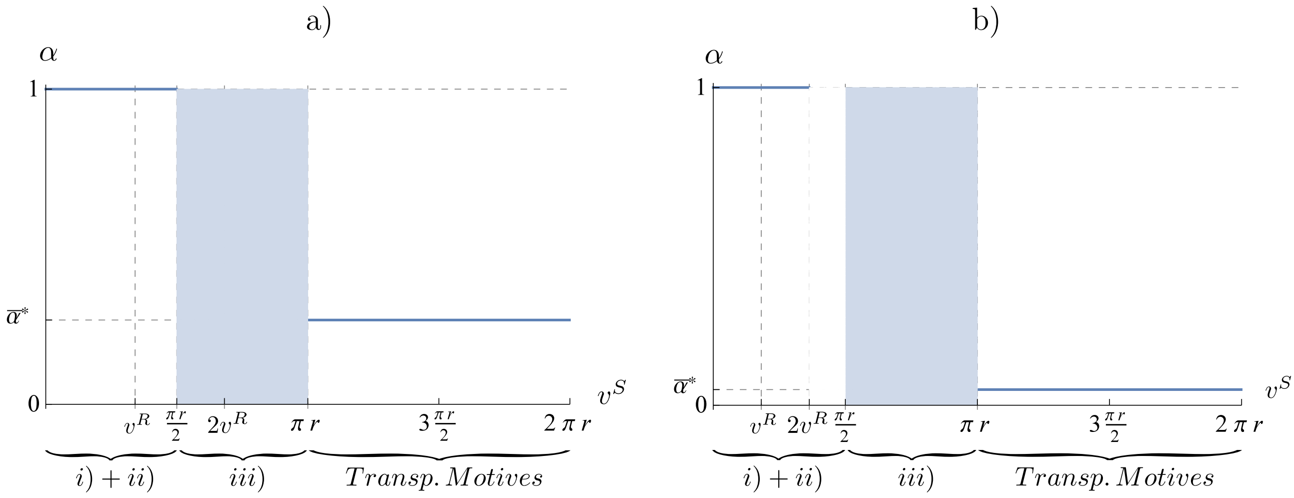
The right panel of Figure 4 depicts the case of a lower $v^R$, in which case region $v^S \in \left(2v^R, \frac{\pi r}{2}\right)$ with no matches emerge. In this region uninformed senders are not willing to communicate whereas informed senders may be, but cannot commit not to take advantage of the information they collect. Therefore, receivers expect negative utility from informed senders and no matches emerge. Theorem 2 has additional implications in case the receiver is also uncertain about the sender's gross valuation $v^S$:

**COROLLARY 3** *Suppose the sender's gross valuation* $v^S \in [\underline{v}, \overline{v}]$ *is uncertain to the receiver. In that case there exists a belief* $\widehat{v^S} \in \left[\underline{v}, v^S\right]$ *that is (weakly) more favorable to the sender.*

This result describes a belief ordering over sender types. The sender prefers to be believed to derive low gross value from matches in order to engage in information acquisition to a higher extent. Hence, if senders were able to communicate the value of $v^S$ credibly we should expect them to disclose them in ascending order, much as in the spirit of Milgrom (1981). However, if there is no way of credibly communicating $v^S$ then cheap-talk mechanisms are ineffective because all sender types prefer to claim a low value of $v^S$. We now inspect the case of joint welfare maximization:

**COROLLARY 4** *The level of information acquisition that maximizes total welfare is equal to one when* $v^S < v^R$ *and equal to* $\overline{\alpha}^*$ *when* $v^S > \pi r$. *In the intermediate range* $v^S \in \left(v^R, \pi r\right)$ *all information levels yield the same level of total welfare.*

Figure 5: Welfare-Maximizing Levels of Information




Note: In both panels, $r = 1$. Panel a) $v^R = \frac{\pi r}{2} - \frac{1}{2}$; Panel b) $v^R = \frac{\pi r}{2} - 1$.The solid line represents welfare-maximizing level of information, and in the solid region all levels yield the same joint welfare.

Figure 5 depicts the first-best level of information. In the left panel information acquisition is equal to 1 when $v^S < \frac{\pi r}{2}$, consistent with the results described in Figure 4. However, in region $v^S \in \left(\frac{\pi r}{2}, \pi r\right)$ changing the level of information acquisition transfers utility efficiently between the receiver and the sender while keeping the match probabilities constant. When $v^S > \pi r$, total utility is maximized when $\alpha = \overline{\alpha}^*$ because the sender has much to gain from a match.

The right panel of Figure 5 considers a case with a lower value of $v^R$. It illustrates that when $v^S \in \left(2v^R, \frac{\pi r}{2}\right)$ no level of information acquisition enables a match because the sender cannot commit not to tailor the message to his advantage.

# 6 Concluding Remarks

We propose a model of communication in which the sender is able to engage in information acquisition about the receiver's preferences. The main result is that the sender may prefer to remain in a state of partial willful ignorance in order to ensure credibility. When the sender features ex-ante transparent motives he prefers to remain partially ignorant about the receiver's preferences and is able to attain his first-best payoff in the forward-induction equilibrium. In contrast, the receiver would be better off shrouding her preferences altogether in this case.

When the sender's valuation is low, information acquisition may be essential for matches to take place. In this case both parties benefit from information acquisition and prefer the highest possible level. Finally, in an intermediate range different levels of information efficiently transfer payoffs between the agents. We uncover two additional results. First, the sender's first-best outcome always maximizes joint welfare. Second, dissipative and cheap-talk communication mechanisms may complement each other rather than act purely as substitutes.

Our results are relevant to matching markets and shed light on current market trends and policy debates related to consumer privacy, personalized communication and online advertising in particular. We have found that information acquisition increases consumers' welfare only when it is pivotal for communication. For example, consumers may be better off sharing their preferences with niche firms but should shroud them from those willing to attract the average consumer. Our results also point to the flip-side of obtaining better information, which is essentially the deterioration of communication credibility.

Another implication of our model is that receivers also have preferences over the amount of information available to senders. In settings such as the job and dating markets, the receiver (e.g. a firm comparing applicants' vitae or an individual being romantically pursued) may have an incentive not to share too much information about what she is looking for, because the sender may use such information to persuade her that he possesses the skills or shares the right set of interests that ensure a successful match. In short, agents should provide only the information necessary to peak interest, but no further information that may be used for misrepresentation by their suitors.

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# A Appendix

## A.1 Main Model

### A.1.1 LEMMA 1

We assume there exists a non-empty region $c_\theta$ such that the receiver is willing to match as long as $m \in c_\theta$. In this case informed senders cannot commit not to send an attractive message to the receiver. Uninformed senders, however, reveal their own types. The receiver's ex-ante utility is given by

$$E\left(U^R\right) = \alpha E\left(v^R - d\left(\theta, q\right)\right) + \left(1-\alpha\right) Pr\left(q \in c_\theta\right) E\left(v^R - d\left(\theta, q\right)\middle| q \in c_\theta\right) \tag{21}$$

where the first term arises because all informed types are able to attract the receiver when $c_\theta$ is non-empty. The second term captures the fact that uninformed types are only able to attract the receiver in some circumstances.

The first term in $E\left(U^R\right)$ is negative since $v^R < E\left(d\left(\theta, q\right)\right) = \frac{\pi r}{2}$ because $E\left(d\left(\theta, q\right)\right) = E\left(d\left(\theta, q\right)\middle| q\right) = E\left(d\left(\theta, q\right)\middle| \theta\right)$ and $d\left(\theta, q\right) \sim U\left(0, \frac{\pi r}{2}\right)$. The second term is positive because the receiver has access to an outside option with value normalized to zero. Hence, conditional on each message the receiver is able to secure a payoff of at least zero, and so ex-ante payoffs (averaged across all message paths) are bounded below by zero as well.

The sender attains ex-ante payoffs

$$E\left(u^S\right) = \alpha E\left(v^S - d\left(\theta, q\right)\right) + \left(1-\alpha\right) Pr\left(q \in c_\theta\right) E\left(v^S - d\left(\theta, q\right)\middle| q \in c_\theta\right) \tag{22}$$

where the first term is positive and higher than the second because $v^S > \pi r$

and $E\left(v^S-d(\theta,q)\right)=Pr(q\in c_\theta)E\left(v^S-d(\theta,q)\middle|q\in c_\theta\right)+Pr(q\notin c_\theta)E\left(v^S-d(\theta,q)\middle|q\notin c_\theta\right)$ where both terms are positive. Hence, sender payoffs increase in $\alpha$.

We now show that in the first-best equilibrium for the sender, $c_\theta = \left\{m : v^R - d(\theta, m) \geq 0\right\}$ or equivalently $c_\theta = \left\{m \in \left[\theta - \frac{v^R}{r}, \theta + \frac{v^R}{r}\right]\right\}$.

First, suppose $\alpha = 0$ and let $c_\theta^{'}$ be a set outside interval $\left[\theta - \frac{v^R}{r}, \theta + \frac{v^R}{r}\right]$. Clearly the receiver would prefer not to match with any uninformed type $q \in c_\theta^{'}$ under perfect information. The only case in which the receiver considers types in this region is when $\alpha > 0$. In this case attractive informed sender types may be pooling in region $c_\theta^{'}$ such that the receiver is willing to match when she receives a message $m \in c_\theta^{'}$. Let $Q$ be a set of such informed types. We now show that a more advantageous equilibrium exists for informed sender in $q \in Q$.

Suppose the informed senders in $Q$ shift the mixing in set $c_\theta^{'}$ into region $c_\theta$: in this case the receiver still matches in face of the same informed sender types, but not when facing *unattractive* uninformed types $q \in c_\theta^{'}$. This increases the ex-ante utility of the receiver because she faces informed types as before, but no longer matches with unattractive uninformed senders in $c_\theta^{'}$. Hence, the sender becomes better off because he is able to increase the level of information acquisition. Finally, appropriate beliefs ensure equilibrium existence.

We have shown that only informed types, and uninformed sender types in region $c_\theta$, attract the receiver in the sender's first-best equilibrium. We now show that *all* such types attract the receiver.

In equilibrium informed senders never pool in a region such that $E\left(U^R(m)\right)$ is strictly negative because they always prefer to induce a match. Hence, there cannot be an uninformed type $q \in c_\theta$ that does not attract the receiver because informed types would not pool to such an extent as to make $m = q \in c_\theta$ unattractive. This completes the argument that all informed types, and uninformed types in $c_\theta$ are able to attract the receiver in the sender's first-best equilibrium.

Solving for the level $\alpha$ that makes the receiver attain zero ex-ante utility yields the first-best information acquisition level for the sender:

$$\begin{aligned} E\left(U^R\right) &= \alpha E\left(v^R - d(\theta,q)\right) + (1-\alpha) Pr(q\in c_\theta) E\left(v^R - d(\theta,q) \middle| q\in c_\theta\right) \geq 0 \\ &= \alpha\left(v^R - \tfrac{\pi r}{2}\right) + (1-\alpha)\left(\int_{\theta-\frac{v^R}{r}}^{\theta} v^R - r(\theta-q)\tfrac{1}{2\pi}dq + \int_{\theta}^{\theta+\frac{v^R}{r}} v^R - r(q-\theta)\tfrac{1}{2\pi}dq\right) \geq 0 \\ &= \alpha\left(v^R - \tfrac{\pi r}{2}\right) + (1-\alpha)\frac{\left(v^R\right)^2}{2\pi r} \geq 0 \\ &\Leftrightarrow \alpha \leq \left(\frac{v^R}{\pi r - v^R}\right)^2 \end{aligned} \tag{23}$$

When $\alpha > \left(\frac{v^R}{\pi r - v^R}\right)^2$ the receiver's ex-ante utility is negative, which is a contradiction. Rather, at that level of information acquisition, informed types are no longer able to pool successfully and $c_\theta$ becomes empty. In this case senders are worse off and the market breaks down.

### A.1.2 THEOREM 1

We show the result by ensuring the receiver earns expected utility equal to zero point by point in region $c_\theta$. Let $s_\phi$ be the support of function $\phi(m, \theta, \alpha)$ such that

$$f_{m^*|\theta,q,\alpha} = \alpha\phi(m,\theta,\alpha) + (1-\alpha)\delta(m-q) \tag{24}$$

because informed senders may decide to mix in a sub-region of $c_\theta$ (see proof of Lemma 1). Ensuring that the receiver expects zero utility over region $s_\phi$

together with Bayes rule yields

$$E\left[U^R\middle| m\in s_\phi,\theta,\alpha\right] = \int_0^{2\pi}\left(v^R-d(\theta,q)\right)f_{m|\theta,q,\alpha}\,dq \tag{25}$$

$$= \int_0^{2\pi}\left(v^R-d(\theta,q)\right)\frac{\alpha\phi(m,\theta,\alpha)+(1-\alpha)\delta(m-q)}{f_{m|\theta,\alpha}}f_q dq=0\,\forall m\in s_\theta$$

and it remains only to solve the equation above w.r.t. function $\phi(m,\theta,\alpha)$:

$$\phi(m,\theta,\alpha) = \frac{1-\alpha}{\alpha}\frac{v^R-d(\theta,m)}{\pi\left(\pi r-2v^R\right)}\mathbf{1}(m\in s_\phi) \tag{26}$$

Finally, the goal of the sender is to choose the highest possible level of information acquisition that allows an informed type to induce a match. Hence, the sender's utility maximization problem is given by

$$\max_{\alpha\in[0,1]} \quad \alpha$$

$$s.t. \quad \phi(m,\theta,\alpha) \geq 0,\ \forall m\in[\theta-\Delta,\theta+\Delta] \tag{27}$$

$$\int_0^{2\pi} f_{m^*|\theta,q,\alpha}dm^* = 1 \tag{28}$$

where $[\theta-\Delta,\theta+\Delta]$ is a guessed parametrization of $s_\phi$, under the intuition that the sender is always better off pooling with more attractive uninformed types first, located near $\theta$. Expressions (27) and (28) follow from the fact that $f_{m^*|\theta,q,\alpha}$ must be a valid probability density function. Condition (27) implies $v^R\geq d(\theta,m)\ \forall m\in s_\phi$. Condition (28) implies

$$\frac{1-\alpha}{\alpha}\frac{1}{\pi\left(\pi r-2v^R\right)}\int_{\theta-\Delta}^{\theta+\Delta} v^R-d(\theta,m)\,dm = 1$$

$$\Leftrightarrow \frac{1-\alpha}{\alpha}\frac{1}{\pi\left(\pi r-2v^R\right)}\left(2\Delta v^R - r\int_{\theta-\Delta}^{\theta}(\theta-m)\,dm - r\int_{\theta}^{\theta+\Delta}(m-\theta)\,dm\right) = 1$$

$$\Leftrightarrow \alpha = \frac{\Delta}{\pi-\Delta}\frac{2v^R-r\Delta}{(\pi+\Delta)r-2v^R} \tag{29}$$

The right-hand side of (29) is concave in $\Delta$, and its unique maximizer is

$$\Delta^* = \frac{v^R}{r} \tag{30}$$

such that the support of $\phi(m, \theta, \alpha)$ is equal to $c_\theta$. Finally, substituting $\Delta^*$ into (29) yields $\alpha^* = \overline{\alpha}^*$, which completes the proof.

### A.1.3 COROLLARY 1

Corollary 1 is easily shown by contradiction. Fix some equilibrium beliefs associated with a forward-induction equilibrium level $\alpha' < \overline{\alpha}^*$. Under forward induction the receiver is willing to 'revisit' her beliefs over the messaging policy upon observation of an 'unexpected' $\alpha$. Hence, a sender who deviates to level $\alpha'' = \overline{\alpha}^*$ can increase payoffs by inducing beliefs $\widehat{f_{m^*|\theta,q,\alpha}} = f_{m^*|\theta,q,\alpha}$, as defined in equation (10), and therefore no level of level $\alpha' < \overline{\alpha}^*$ survives forward induction.

### A.1.4 COROLLARY 2

The receiver's payoffs follow directly from the derivation of Theorem 1. The sender's payoffs are given by

$$\begin{aligned}
E\left(U^S\right) &= \overline{\alpha}^* E\left(v^S - d(\theta,q)\right) + (1-\overline{\alpha}^*) Pr\left(q \in \left[\theta - \tfrac{v^R}{r}, \theta + \tfrac{v^R}{r}\right]\right) E\left(v^S - d(\theta,q) \middle| q \in \left[\theta - \tfrac{v^R}{r}, \theta + \tfrac{v^R}{r}\right]\right) \\
&= \overline{\alpha}^* \left(v^S - \tfrac{\pi r}{2}\right) + (1-\overline{\alpha}^*) \int_{\theta - \frac{v^R}{r}}^{\theta + \frac{v^R}{r}} \left(v^S - d(\theta,q)\right) \tfrac{1}{2\pi} d\theta \\
&= \overline{\alpha}^* \left(v^S - \tfrac{\pi r}{2}\right) + \tfrac{1-\overline{\alpha}^*}{2\pi} \int_{\theta - \frac{v^R}{r}}^{\theta} v^S - r(\theta - q) d\theta + (1-\overline{\alpha}^*) \int_{\theta}^{\theta + \frac{v^R}{r}} v^S - r(q-\theta) d\theta \\
&= \tfrac{v^S - v^R}{\pi r - v^R} v^R
\end{aligned}$$

## A.2 Costly Persuasion / Low Sender Valuation

### A.2.1 LEMMA 2

When $v^S < v^R < \frac{\pi r}{2}$, the sender only communicates when the receiver is nearby. Given that all senders who find a desirable receiver are also attractive, the sender is better off engaging in full information acquisition, and the receiver is willing to match whenever she receives a message. Payoffs are given by

$$\begin{aligned} E\left(U^S\right) &= Pr\left(v^S \geq d\left(\theta, q\right)\right) E\left(v^S - d\left(\theta, q\right) \middle| v^S \geq d\left(\theta, q\right)\right) \\ &= \frac{1}{2\pi}\left(\int_{q-\frac{v^S}{r}}^{q} v^S - r\left(q-\theta\right) d\theta + \int_{q}^{q+\frac{v^S}{r}} v^S - r\left(\theta - q\right) d\theta\right) \\ &= \frac{\left(v^S\right)^2}{2\pi r} \end{aligned} \tag{31}$$

and

$$\begin{aligned} E\left(U^R\right) &= Pr\left(v^S \geq d\left(\theta, q\right)\right) E\left(v^R - d\left(\theta, q\right) \middle| v^S \geq d\left(\theta, q\right)\right) \\ &= \frac{2v^R - v^S}{2\pi r} v^S \end{aligned} \tag{32}$$

### A.2.2 LEMMA 3

As in the previous case, when $v^R < v^S < \frac{\pi r}{2}$ the sender only engages in communication when he is informed of an attractive receiver. Upon receiving a message, the receiver understands that it must originate from an informed sender, and so expects utility

$$\begin{aligned} E\left(U^R \middle| message\right) &= E\left(v^R - d(\theta, q) \middle| v^S \geq d(\theta, q)\right) \\ &= \frac{\int_{q-\frac{v^S}{r}}^{q} v^R - r(q-\theta)\, d\theta + \int_{q}^{q+\frac{v^S}{r}} v^R - r(\theta - q)\, d\theta}{2\pi Pr\left(v^S \geq d(\theta, q)\right)} \\ &= v^R - \frac{v^S}{2} \end{aligned} \tag{33}$$

because $Pr\left(v^S \geq d(\theta, q)\right) = \frac{2\frac{v^S}{r}}{2\pi} = \frac{v^S}{\pi r}$. Hence, communication occurs if and only if $v^R \geq \frac{v^S}{2}$. Because the sender is always understood to be informed when communication takes place, he is better off engaging in full information acquisition in this case. In contrast, the market breaks down whenever $v^R < \frac{v^S}{2}$.

### A.2.3 LEMMA 4

The ex-ante receiver utility is given by

$$E\left(U^R\right) = Pr(Informed)E\left(v^R - d(\theta, q) \middle| q \in c_\theta^{'}\right) + Pr(Uninformed)Pr(q \in c_\theta)E\left(v^R - d(\theta, q) \middle| q \in c_\theta\right) \tag{34}$$

by the same arguments of the proof of Lemma 1. Informed sender types are willing to communicate if and only if $v^S - d(\theta, q) \geq 0$, and so $c_\theta^{'} =$

$\left[\theta - \frac{v^S}{r}, \theta + \frac{v^S}{r}\right]$. The upper bound on $\alpha$ is derived by ensuring $E\left(U^R\right) \geq 0$:

$$\begin{aligned}
E\left(U^R\right) &= Pr\left(Informed \wedge q \in c_\theta^{'}\right) E\left(v^R - d(\theta,q) \middle| q \in c_\theta^{'}\right) + (Pr(Uninformed \wedge q \in c_\theta)) Pr(q \in c_\theta) E\left(v^R - d(\theta,q) \middle| q \in c_\theta\right) \\
&= \frac{1}{2\pi}\left(\alpha \int_{\theta - \frac{v^S}{r}}^{\theta + \frac{v^S}{r}} v^R - d(\theta,q)\, dq + (1-\alpha) \int_{\theta - \frac{v^R}{r}}^{\theta + \frac{v^R}{r}} v^R - d(\theta,q)\, dq\right) \\
&= \frac{\left(v^R\right)^2 - \alpha\left(v^S - v^R\right)^2}{2\pi r} \geq 0 \\
&\Leftrightarrow \alpha \leq \left(\frac{v^R}{v^S - v^R}\right)^2
\end{aligned}$$

Unlike in Lemma 1, the information acquisition level may be equal to one, which happens whenever $2v^R > v^S$. In this case informed senders are attractive to the receiver such that the forward induction equilibrium supports full information.

It remains to show that there exists a message policy that is able to implement level $\overline{\alpha}^{'} = \min\left\{1, \left(\frac{v^R}{v^S - v^R}\right)^2\right\}$. When $2v^R < v^S$, following the same steps in the proof of Theorem 1 we verify that $\overline{\alpha}^{'}$ is attainable by communication policy

$$f_{m^*|\theta,q,\alpha} = \alpha\phi^{'}(m,\theta,\alpha) + (1-\alpha)\,\delta(m-q) \tag{35}$$

where

$$\phi^{'}(m,\theta,\alpha) = r\frac{1-\alpha}{\alpha}\frac{v^R - d(\theta,m)}{v^S\left(2v^R - v^S\right)}\mathbf{1}\left(m \in \left[\theta - \frac{v^R}{r}, \theta + \frac{v^R}{r}\right]\right), \tag{36}$$

$\alpha = \overline{\alpha}^{'}$ and such that the receiver payoffs are equal to zero, and the sender payoffs are given by

$$
\begin{aligned}
E\left(U^S\right) &= \overline{\alpha}'^* Pr\left(v^S \geq d(\theta,q)\right) E\left(v^S - d(\theta,q) \middle| v^S \geq d(\theta,q)\right) \\
&+ \left(1-\overline{\alpha}'^*\right) Pr\left(v^R \geq d(\theta,q)\right) E\left(v^S - d(\theta,q) \middle| v^R \geq d(\theta,q)\right) \\
&= \frac{v^R v^S}{\pi r}
\end{aligned} \tag{37}
$$

A detailed proof of these results is available from the authors.

## A.3 Welfare Analysis

### A.3.1 THEOREM 2

Theorem 2 follows from the results in Lemmas 2-4. In cases *i*) and *ii*) information acquisition is necessary for matches to take place, and full information is best for both parties. The exception occurs in case *ii*) when $2v^R < v^S$, under which no level of information acquisition can result in communication. In region *iii*) the first-best information acquisition level for the sender is equal to $\min\left\{1, \left(\frac{v^R}{v^S - v^R}\right)^2\right\}$, whereas the receiver prefers complete privacy, which in turn results in truth-telling. Similarly, when $v^S > \pi r$ the sender's first-best level of information acquisition is equal to $\left(\frac{v^R}{\pi r - v^S}\right)^2$, and to zero for the receiver. Comparative statics imply that the information acquisition levels weakly decrease in $v^S$ and $r$ and increase in $v^R$.

### A.3.2 COROLLARY 3

Corollary 3 follows from the collection of the information acquisition levels described in Theorem 2: under uncertainty over sender types $v^S$, the sender benefits from inducing the lowest possible belief in order to induce a match under full information. Hence, no cheap-talk separation is possible across types $v^S$.

### A.3.3 COROLLARY 4

The result follows from the information acquisition levels in Theorem 2. When $v^S < v^R$ both parties depend on information acquisition to match, and so the first-best outcome is full information. In the intermediate range $v^R < \frac{\pi r}{2} < v^S < \pi r$ joint welfare is given by

$$\begin{aligned} E\left(U^R\right) + E\left(U^S\right) &= \tfrac{1}{2\pi}\left(\alpha \int_{\theta-\frac{v^S}{r}}^{\theta+\frac{v^S}{r}} v^R - d(\theta,q)dq + (1-\alpha)\int_{\theta-\frac{v^R}{r}}^{\theta+\frac{v^R}{r}} v^R - d(\theta,q)dq\right) \\ &+ \tfrac{1}{2\pi}\left(\alpha \int_{\theta-\frac{v^S}{r}}^{\theta+\frac{v^S}{r}} v^S - d(\theta,q)dq + (1-\alpha)\int_{\theta-\frac{v^R}{r}}^{\theta+\frac{v^R}{r}} v^S - d(\theta,q)dq\right) \\ &= \frac{v^R v^S}{\pi r} \end{aligned}$$

where the result does not depend on the level of information. When $v^S > \pi r$ joint welfare is given by

$$\begin{aligned} E\left(U^R\right) + E\left(U^S\right) &= \tfrac{1}{2\pi}\left(\alpha \int_{0}^{2\pi} v^R - d(\theta,q)dq + (1-\alpha)\int_{\theta-\frac{v^R}{r}}^{\theta+\frac{v^R}{r}} v^R - d(\theta,q)dq\right) \\ &+ \tfrac{1}{2\pi}\left(\alpha \int_{0}^{2\pi} v^S - d(\theta,q)dq + (1-\alpha)\int_{\theta-\frac{v^R}{r}}^{\theta+\frac{v^R}{r}} v^S - d(\theta,q)dq\right) \\ &= \frac{1}{\pi r}\left(v^R v^S + \alpha\left(v^S - \pi r\right)\left(\pi r - v^R\right)\right) \end{aligned}$$

which is increasing in $\alpha$.