Tailored Cheap Talk*

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Abstract

We consider a cheap-talk setting in which the sender can use information about the receiver's preferences to tailor communication. Better information increases the persuasion ability of the sender, but also results in receiver skepticism. When the sender's motives are transparent to the receiver, trade always relies on the sender not being too well-informed about the receiver's preferences. In the case of horizontal matching, trade breaks down when communication costs are intermediate, whenever the agents' valuations differ sufficiently. Moreover, the cost and content of the communication may affect market outcomes through concurrent mechanisms.

Both sender and social planner prefer the maximum amount of information that allows trade, whereas the receiver prefers complete privacy, unless information revelation is necessary to induce trade. Finally, the sender's first-best strategy involves pooling with undesirable sender types. The implications are discussed in the contexts of matching markets, including online advertising, sales, dating and job search.

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1 Introduction

Tailoring, the act of using information about the receiver's preferences to improve persuasion, is a basic matching ingredient in markets. In modern advertising markets, for example, firms tailor their communication in real time, according to users' search terms, demographics, cell phone usage/locations, browsing behaviors, device characteristics, etc. In job markets, applicants learn about employers and the activities they are expected to perform, and well-prepared applicants are likely to use such pieces of information during job interviews. Negotiators, salespeople, and romantic pursuers frequently use information about their counterparts to convince them of high match values.

We investigate this phenomenon by considering a communication game in which a persuader tries to elicit a desired action from an agent by means of a compelling argument. In order to understand which arguments may indeed be compelling, the sender can use information about the receiver's preferences prior to the communication stage. This simple arrangement can lead to complex consequences because the receiver understands that the communication may have been appropriately tailored to appear persuasive.

We focus on cheap-talk communication settings (Crawford and Sobel, 1982) with transparent motives, that is, the receiver is aware of the action the sender would like to induce (e.g., online advertisers seek clicks, job applicants seek placements). At the core of our analysis is a tension between information and communication effectiveness: when the sender is better informed, he is able to cater the communication to the receiver's preferences better. However, more information also increases the likelihood that the communication was tailored, resulting in receiver skepticism.

Our first result shows that, in general, there exists an information threshold beyond which trade breaks down, independently of the communication policy employed by the sender. The intuition is that, under cheap-talk communication, the sender lacks the commitment power to not say whatever he believes the receiver would like to hear. As a result, access to better information may hurt both agents.

We then investigate a matching setting in which agents are uncertain of each others' horizontal matching preferences.[1] An equilibrium refinement, motivated by Farrell and Rabin (1996), allows us to investigate the sender's communication policy: we find that uninformed senders communicate their types, whereas informed senders prefer to pool with uninformed senders at different rates, in order to shroud tailoring activities from the receiver. We find the sender and the social planner are better off at higher levels of information, up to a threshold. Receivers, on the other hand, prefer complete privacy, in order to induce truth-telling by senders.

We also consider the case of costly communication. As expected, these costs are irrelevant when they are low, but effectively determine matches when they are high. The market outcomes are less intuitive at intermediate cost levels: when the agents' valuations are far apart, uninformed senders are discouraged from communication at a higher rate than unattractive informed senders. As a result, the receiver understands that the message is likely to have been tailored, and trade breaks down at intermediate communication cost levels. When the agents' valuations are relatively similar, however, the cost and content of communication affect market outcomes through concurrent mechanisms: the content of the message affects the types of uninformed senders who are able to induce matches, whereas the communication cost affects the types of informed senders that are willing to communicate.

By allowing a more general belief structure, we find that the sender's first-best outcome is achieved by pooling with both attractive and unattractive uninformed sender types. The intuition underlying the receiver's beliefs that support such outcomes is better explained by use of examples of a dynamic nature: an expert adviser may prefer to answer questions about topics he is well-informed of in a less than perfect fashion, in order to weaken the receiver's discernment when topics he does not master are touched upon.[2]

[1] We later explain that markets often devise ways to verify vertical attributes, and that cheap talk is often most relevant about horizontal ones.

[2] We refer to the sender and receiver as male and female, respectively.

We also inspect the case in which the sender features non-transparent motives, and find that the receiver prefers to share information whenever it is pivotal to induce trade. In such cases, the receiver prefers full information revelation.

In our model, both the sender and the receiver care about the types of their counterparts. The intuitive reason for this is that interactions in matching markets do not typically terminate immediately after a match takes place, but are often followed by post-match allocation stages. For example, a consumer who clicks on an online advertiser's link has to subsequently decide whether to buy a product. Because of this, the advertiser prefers inducing clicks from consumers who are more likely to buy, and consumers would also like to click on advertisements of firms carrying offerings they like. Similarly, salespeople would like to attract high-potential buyers, and employers and employees negotiate over streams of future payoffs. While different markets use different allocation rules (e.g. auctions in real estate, bargaining in automobile sales), the common aspect is that agents hold expectations over future payoffs in case a match takes place. Rather than modeling the ultimate payoff-splitting rules, we focus on the payoffs agents *expect* to earn if a match is produced. This assumption allows us to focus on the role of information in persuasion separately from the sender's increased ability to extract value.[3]

Despite the extensive literature available on cheap-talk communication, following Crawford and Sobel (1982), limited attention has been paid to the implications of the information environment. Seidmann (1990) shows that receiver's information regarding her own type helps her distinguish senders, and in this case cheap-talk can be informative even when senders agree on the attractiveness ranking over the receiver's actions. Watson (1996) considers the case in which sender confusion may lead him to prefer to reveal his type truthfully. Barreda (2013) ex-

[3]In some contexts it may be lawful for a sender to use his informational advantage to appropriate additional value from the receiver. Our main results are robust to these cases as long as the receiver is strategic. Our modeling assumptions also ensure that communication strategies do not depend on the sender's need to avoid the hold-up problem of fully extracting consumer surplus (see Anderson and Renault, 2006), although it is not incompatible with such settings (see Bagwell and Ramey (1993) and Gardete (2013) for examples of cheap-talk interactions followed by search).

tends the Crawford and Sobel (1982) framework to allow the receiver to also hold private information about the state of nature. She finds that the receiver's private information may hurt communication, in some cases making both agents worse off than if the receiver had no information to start with. Our paper incorporates ingredients of each of the papers mentioned above. Like Seidmann (1990), the central part of our paper concerns itself with the case of transparent motives, in which all sender-types have the same rank-preferences over the receiver's actions.[4] In our horizontal matching application, uninformed senders prefer to reveal their types, much like the confused senders in Watson (1996). Finally, our model also includes privately-informed receivers, like in Barreda (2013). Our main contribution to the literature is that we characterize market outcomes as a function of the sender's informational advantage. In particular, in our setting the sender may be informed about the receiver's privately-known type, which allows him to tailor his communication strategy.

As in our case, Che, Dessein, and Kartik (2013) consider a situation in which the sender has an incentive to tailor the communication to the receiver's preferences. However, the underlying mechanism is different: in their setting, the sender is perfectly informed about the receiver's preferences, and his communication policy depends on the characteristics of the distribution of projects available to the receiver.

Our results also speak to the interaction between cheap-talk and costly signaling. Austen-Smith and Banks (2000) and Kartik (2007) consider the case of multidimensional signaling via cheap-talk (Crawford and Sobel, 1982) and burned money (Kihlstrom and Riordan, 1984; Milgrom and Roberts, 1986). They show that money burning increases information transmission and that, except in knife-edge cases, it cannot expand the set of environments in which cheap talk is credible. Our setting is somewhat different in that, in our case, the sender decides whether to incur a communication cost in order to communicate with the receiver. Investi-

[4] We consider the case of non-transparent motives in Section 5. See Bagwell and Ramey (1993), Chakraborty and Harbaugh (2010) and Gardete (2013) for examples of other papers in which the sender also features transparent motives.

gating this setting produces the novel result that both the cost and the content of communication can be informative concurrently, and moreover, the market may break down at intermediate communication cost levels.[5]

Because cheap-talk settings commonly feature multiple equilibria, it is often challenging to characterize the sender's communication policies. As a result, the literature's main focus has been to describe the conditions under which credible information transmission can take place, usually with limited bearing on the sender's communication policy. We use the insight by Farrell and Rabin (1996) that agents' initial interpretations of messages are likely to be their usual meanings, but that nonetheless agents are still able to assess the sender's incentives for picking particular messages. The refinement allows us to focus the analysis on a set of beliefs that generates interpretable communication policies by the sender. The focus on the content of communication has also recently been addressed by Sobel (2016), who provides different definitions of lying and deception. In our case, we show that attractive informed senders attain the first-best outcome by engaging in lying with no deception, i.e., they prefer to misreport their types, while leading the receiver to make ex post beneficial decisions. These beliefs benefit the sender because, as a result, he is able to induce matches in cases where he is uninformed while moderately unattractive.

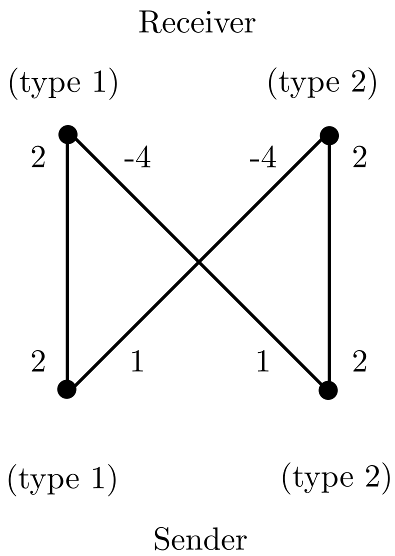
The following section presents a motivating example. The general model is presented in Section 3, and the case of horizontal matching is presented in Section 4. Section 5 extends the analysis to consider the case of non-transparent motives. Section 6 provides a discussion about the impact of the observability of the information level by the agents, and Section 7 concludes.

[5]There also exists an extensive literature on disclosure, including Anderson and Renault (2006), Ostrovsky and Schwarz (2010), Rayo and Segal (2010), Kamenica and Gentzkow (2011), de Cornière and de Nijs (2016) and Shen and Villas-Boas (2017). This paper (and the cheap-talk literature in general) is different in that, by uncoupling commitment from communication, it allows the persuader to send any message, in contrast to having him solely decide on which information to disclose.

2 Motivating Example

Consider a case in which a receiver decides whether to match with a sender. Both the receiver and the sender belong to one of two possible types, with equal probabilities, and agents' types are their own private information. We depict the matching payoffs in Figure 1. If no match occurs, both parties receive payoffs equal to zero.

Figure 1: Example of potential matching configurations and payoffs with two types of receivers and senders



Both senders and receivers prefer to be matched with counterparts of the same type. Clearly, in the absence of additional information, the receiver prefers never to match with the sender because she expects an average payoff of -1. Allowing the sender to communicate, however, enables trade: by revealing his type, the sender is able to attract his preferred receiver, and earn an average payoff of 1. The receiver is willing to match with the sender whenever she observes a message equal to her type, also earning an average payoff of 1.

What happens if, additionally, the sender is informed about the receiver's type? In this case trade completely breaks down because the sender cannot commit not to say whatever he thinks the receiver would like to hear. If, as before, the receiver were to match with the sender whenever she observed a message equal to her own type, then the sender could simply send the message the receiver preferred, and the receiver would earn an average payoff of -1 as a result.

Despite the starkness of the result, trade can still be supported as long as the sender is not perfectly informed about the receiver's preferences. Suppose that the sender is able to identify the receiver with 1/2 probability. In this case, sender 1 is always able to induce a match from receiver 1, and moreover can induce a match from receiver 2 half of the time, earning an average payoff of 1.25. The receiver, on the other hand, matches only when her type is communicated, and earns 0 utility on average. In fact, 1/2 is the highest level of information that can support trade in this market.

In the next section we analyze the case of a general persuasion setting in which the sender exhibits transparent motives. Both agents are endowed with matching preferences and a joint distribution over types. We characterize an upper bound on the information level that allows trade, and provide an accessory result that ensures existence of trade.

3 General Model

3.1 Preliminaries

Consider a persuasion setting in which a sender attempts to induce a match from a receiver through communication. The receiver and the sender can earn matching payoffs $u^R(\theta, q)$ and $u^S(\theta, q)$, where $\theta \in \Theta$ and $q \in Q$ index the receiver and sender types, respectively. If the sender fails to induce a match by the receiver, both parties earn payoffs of zero.[6]

Agents' types are distributed according to the joint probability distribution $F_{\theta,q}$, which is common knowledge. The sender is informed about the receiver's type θ with probability α. The level of information α is public, but the actual realization of whether the sender becomes informed is his own private information.[7]

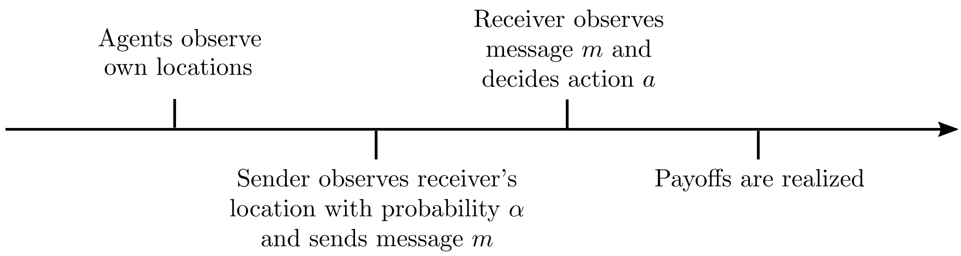
[6] Our subsequent analysis is consistent with $\Theta, Q \subseteq \mathbb{R}$. Applying the results to the countable case is straightforward.

[7] See Section 5 for a robustness discussion about the observability of the level of information

Finally, persuasion is conducted by use of a message $m \in Q$, used by the sender to persuade the receiver of the merits of the match.

The timing of the game is given in Figure 2. First, the sender and the receiver

Figure 2: Timing



learn their respective types. Second, the sender learns the receiver's location with probability α, and decides on a message m. Upon observing the message, the receiver decides whether to match, and payoffs are realized. We focus on perfect Bayesian equilibria (PBE), such that agents maximize their utilities given their beliefs, and on-equilibrium path beliefs are given by Bayes' rule at each information set. The latter requirement implies that the receiver's beliefs about the sender's location are consistent with the distribution induced by the communication policy. The receiver forms beliefs about the sender's type conditional on two pieces of information: her own type θ and the sender's message m. PBE implies

$$\widehat{F_{q|\theta,m,\alpha}} = \frac{F_{m^*|\theta,q,\alpha} . F_{q|\theta}}{F_{m^*|\theta,\alpha}}. \tag{1}$$

Distribution $\widehat{F_{q|\theta,m,\alpha}}$ denotes the receiver's beliefs, and $F_{m^*|\theta,q,\alpha}$ is the probability distribution induced by informed and uninformed senders' equilibrium communication policies, $m_I^*(\theta, q)$ and $m_U^*(q)$, respectively.[8] Distribution $F_{q|\theta}$ captures the fact that agents' types may be correlated, and moreover, some receiver/sender type combinations may appear more frequently than others.

by the receiver.

[8]We include α in $\widehat{F_{q|\theta,m,\alpha}}$ and $F_{m^*|\theta,q,\alpha}$ in order to stress that the beliefs and communication policies may vary with the information level.

We focus on cases in which 1) the sender has transparent motives and 2) communication is decisive. The first criterion focuses on cases where the sender is willing to match with all receiver types. The criterion is operationalized by

$$E\left(u^S(\theta,q)\middle|\, q\right) \geq 0,\ \forall q \in Q \tag{2}$$

Hence, transparent motives imply that the receiver understands that the sender always benefits from inducing a match.

The requirement that communication is decisive ensures the receiver needs to, and can, be persuaded to match. It is operationalized by

$$E\left(u^R(\theta,q)\middle|\, \theta\right) < 0,\ \forall \theta \in \Theta \tag{3}$$

and

$$\exists \theta \in \Theta, Q' \subset Q :\ E\left(u^R(\theta,q)\middle|\, \theta, q \in Q'\right) \geq 0 \tag{4}$$

The first inequality guarantees we focus on receivers who need to be persuaded. The second inequality ensures that communication can induce trade from strategic agents, at least in some cases. Finally, we ignore 'babbling' equilibria throughout the analysis, in which the receiver always ignores the message of the sender, ascribing it to uninformative randomization, and the sender becomes indifferent across all possible communication policies, including the one the receiver expects.

3.2 Analysis

The receiver matches with the sender if and only if

$$U_\theta^R(m) \equiv E\left(u^R(\theta,q)\middle|\, \theta, m\right) \geq 0 \tag{5}$$

The receiver uses beliefs $\widehat{F_{q|\theta,m,\alpha}}$ to assess the merits of the match, according to the expression above. As with several signaling settings, the space of beliefs $\widehat{F_{q|\theta,m,\alpha}}$ can be intractably large. Because of this, we instead focus on the space of messages that induces matches by the receiver. We start by defining set $M_\theta \subseteq Q$, which we refer to as the set of persuasive messages. Formally, it is given by $M_\theta \equiv \left\{m : U_\theta^R(m) \geq 0\right\}$, i.e., it contains the messages that induce matches by receiver θ. Clearly, M_θ is directly induced by beliefs $\widehat{F_{q|\theta,m,\alpha}}$, generally on a many-to-one fashion. A message m is said to be persuasive for receiver θ if it belongs to set M_θ.

We say an equilibrium supports trade (or alternatively, is a matching equilibrium) if and only if there exists a set of receiver beliefs and consistent communication policies that induce a non-empty set of equilibrium persuasive messages M_θ for some receiver θ.

Before observing the sender's message, the receiver's ex-ante utility is given by

$$EU^R = \alpha E\left(u^R(\theta, q)\middle|\,\theta\right) + (1-\alpha)\,Pr\left(q \in C_\theta\right) E\left(u^R(\theta, q)\middle|\,\theta, C_\theta\right) \tag{6}$$

The first and second terms capture the expected utilities of facing an informed and an uninformed sender, respectively.

Assuming M_θ is non-empty, the first term is given directly by the expected utility of matching with the sender, given that the latter is informed. The reason is that, by not deviating from sending a message inside region M_θ, an informed sender can induce a match with certainty. Moreover, the assumption that communication is decisive implies that this term is negative.

Unlike informed senders, uninformed senders may not be able to secure matches with certainty, and so the second term of expression (6) is probabilistic: it introduces set $C_\theta \subseteq Q$, which is defined as $C_\theta = \{q : m_U^*(q) \in M_\theta\}$, i.e., it is comprised of the uninformed sender types who send persuasive messages to receiver θ in equilibrium. Intuitively, this term is positive whenever trade occurs in the market.

Given the elements described above, it is possible to derive the following result:

Theorem 1 (*Willful Ignorance*) *Under transparent motives, matches occur only if the level of information is at most* $\overline{\alpha}$*, defined as*

$$\overline{\alpha} \equiv \max_{\theta \in \Theta} \left\{ -\frac{\int_{u^R(\theta,q)\geq 0} u^R(\theta, q)\, dF_{q|\theta}}{\int_{u^R(\theta,q)<0} u^R(\theta, q)\, dF_{q|\theta}} \right\} \in (0, 1) \tag{7}$$

The result implies that trade can only arise in equilibrium when the information level falls below $\overline{\alpha}$, independently of the communication policy employed by the sender. Moreover, under transparent motives, the information threshold does not depend on the utility the sender derives from a match. The only limiting factor is the relative difference between the receiver's (probability weighted) expected utilities of facing an attractive vs. an unattractive sender.

The underlying intuition for this result is that, when the information level increases, the receiver is required to be compensated either in terms of the frequency of facing an attractive sender, or the expected payoffs, in order to be willing to match. The reason is that, under cheap-talk communication, informed senders cannot commit not to tailor their communication. The higher the level of information, the higher the probability that the receiver faces such a sender. When α is very high, 'too many' informed senders take advantage of the set of persuasive messages M_θ, and no consistent beliefs can support trade as result.

Without imposing further structure, testing existence of matching equilibria requires a full inspection of the agents' incentive compatibility constraints, for general receiver beliefs. A central reason is that uninformed senders can communicate in relatively arbitrary ways.

Related to this, Theorem 1 implies that, in some cases, the occurrence of trade can nonetheless be established:

Corollary 1 *Suppose there exist receiver beliefs that induce separation by uninformed senders. In that case, condition $\alpha \leq \overline{\alpha}$ is necessary and sufficient for the existence of a matching equilibrium.*

The result follows from Theorem 1. The highest level of information produces trade when, at a minimum, uninformed senders are able to report whether they are attractive or unattractive to the receiver, or alternatively, they report their types truthfully. In this case, the receiver matches with uninformed types only when they provide positive match utility. This allows informed senders to pool with attractive uninformed senders efficiently, and maximize the level of information the receiver is willing to bear. It follows that, as long as uninformed senders report their types truthfully, there exist beliefs and an incentive-compatible communication policy by informed senders that generate trade, as long as $\alpha \leq \overline{\alpha}$.

Due to its general specification, the model above lends itself to little additional insight. In the following section, we consider an application in which agents are located along a preference circle.[9] As such, each agent has a unique optimal matching profile, and so the model captures the role of tailoring in a horizontal differentiation setting. We allow agents to differ on an observable vertical dimension, which in the next section we argue is often easier to commit to and verify than the horizontal one.[10] The matching application allows a precise characterization of 1) welfare comparative statics on the level of information; 2) equilibrium outcomes when communication is costly; 3) characterization of sender communication policies; and 4) the case of non-transparent motives.

[9] Somewhat related to our work, Spector (2000) and Filipovich (2008) extend the Crawford and Sobel (1982) setting by investigating circular action and state spaces, respectively.

[10] In some cases, seemingly vertically-differentiated settings are effectively horizontally differentiated, because they involve a tradeoff across vertical attributes (e.g., Bagwell and Ramey (1993) and Gardete (2013)). Also, note that traditional horizontal differentiation à la Hotelling (1929) would not be horizontal in our setting, because senders nearest the mean receiver location would be ex-ante more attractive than others.

4 Horizontal Matching

4.1 Preliminaries

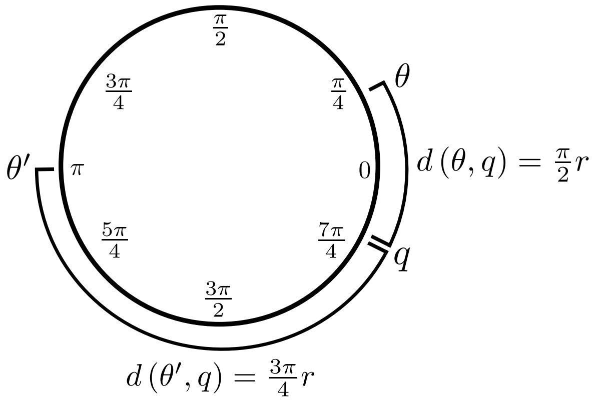
As before, there exists a sender and a receiver of a message. Upon observing the message, the receiver decides whether to match. If the receiver matches, she earns utility $u^R = v^R - d(\theta, q)$ and the sender earns utility $u^S = v^S - d(\theta, q)$, where v^R and v^S are positive and $d(\theta, q)$ is a distance function. Failure to induce a match from the receiver yields the value of the outside option to both parties, with payoffs normalized to zero. The sender and the receiver are independently located along a preference circle with uniform probability: the receiver's location is given by $\theta \sim U[0, 2\pi)$ and the sender's location is given by $q \sim U[0, 2\pi)$.

The vertical components v^R and v^S are observable to both agents, and can be interpreted according to two different perspectives. First, they capture each agents' relative preference for a match versus the outside option. For example, in a bargaining situation between a union and management, gross utilities v^R and v^S measure the relative preference for a given proposal, versus prolonging negotiations (i.e., the outside option).

A second perspective is that v^R is earned by the receiver because of the sender's type, and vice-versa. For example, a professional vitae can be seen as credible information of an applicant's average value for a potential employer. As such, v^R captures the average utility the employer should expect if she hires the applicant. This interpretation applies to a number of settings because verifiability mechanisms often arise in markets to measure vertical dimensions. These include contacting referrals of job applicants, posting price information through marketing materials, sharing pictures in the housing and dating markets, etc. In turn, cheap-talk communication often applies to horizontal components, which are less systematic and harder to verify: organizational and social skills in job markets, product fit in retail markets, preferences and values in dating markets, etc.

The uncertain horizontal dimension is captured by the distance function $d(\theta, q)$, which represents the preference mismatch between the sender and the receiver. It can be intuitively understood as the shortest angular distance between θ and q, multiplied by scalar $r > 0$.[11] Figure 3 provides a representation. Parameter r

Figure 3: Illustration of the distance function $d(\theta, q)$



captures the market differentiation level and simultaneously affects the match values of the sender and receiver by introducing a distance penalty. For example, in markets with large differentiation, which may be 'thin' and/or exhibit long tails, r is large and parties expect to earn relatively low payoffs from matches on average.

As with the general model, the sender's message is denoted by m and lies in the circular domain $[0, 2\pi)$, and the publicly-observed information level is $\alpha \in [0, 1]$. We introduce a communication cost parameter $c \geq 0$, such that the sender may need to incur a cost in order to send message m. The timing of the game is the same as before, the only difference being that the sender may prefer not to send a message, in order to avoid incurring communication cost c.

As explained before, in cheap-talk models the space of receiver beliefs is generally large and often unproductive as a starting point for the analysis. To illustrate this, consider the extreme example in which receivers believe attractive senders select messages equal to "the rational number closest to their own locations, for which the second decimal digit is a prime number." Despite the arbitrariness of this rule, informed senders would then have an incentive to pool at these locations, giving rise to informative equilibrium outcomes. While theoretically valid, these

[11] Formally, it is given by $d(\theta, q) = r. \cos^{-1}(\cos(\theta - q))$.

outcomes are unappealing in terms of interpretation of the messages they produce. In order to be able to make predictions about the content of messages in a realistic fashion, we borrow an insight from Farrell and Rabin (1996), who argue that the literal meaning of messages is a focal starting point for receivers:

"*People don't usually take the destructively agnostic attitude that 'I won't presume that the words mean what they have always meant.' Rather, people take the usual or literal meaning seriously. This doesn't mean they believe whatever they hear; rather, they use the usual meanings as a starting point and then assess credibility, which involves asking questions such as, 'Why would she want me to think that?'*" - Farrell and Rabin (1996)

We use the statement above to suggest that it is focal for receivers to interpret the distance of a message to their location as a claim about the value of the match. We adopt the following refinement:

Refinement (*Farrell and Rabin*): *An equilibrium satisfies refinement FR if, whenever a receiver θ is willing to match upon receiving a message at distance $d(\theta, m) = \Delta_1$, then she is also willing to match if she receives a message at a shorter distance, i.e., in cases where $d(\theta, m) = \Delta_2$, $\forall\, \Delta_2 \in (0, \Delta_1)$.*

The statement provides a belief ordering that induces a set of persuasive messages $M_\theta = \{m : d(\theta, m) \leq \Delta\}$, for some $\Delta \geq 0$. In the respective equilibria, messages can be interpreted as claims about match values. Finally, in cases in which the receiver prefers never to match, we define M_θ as empty whenever $\Delta = 0$.[12]

[12] The refinement provides a focal point for beliefs and messages, as it rules out equilibria obtained from relabeling messages in a one-to-one fashion. Another advantage is that, as we describe later, the restriction rules out uninformed types coordinating around a particular location on the circle, which is attractive given the absence of a randomization device in our model. While Δ may depend on parameters, we investigate cases where m and θ are excluded from it, in order to keep the restriction from unraveling. Finally, in principle, the distance function Δ can depend on the model's parameters in complex ways. We impose a Markov-perfect equilibrium refinement on distance Δ, (Maskin and Tirole, 2001), such that it cannot depend on payoff-irrelevant factors, including the numerical properties of the model parameters.

Focusing on a specification of Δ facilitates the analysis and allows us to characterize the agents' welfares as a function of the information level.

First, according to Theorem 1, no matches can occur beyond information level $\overline{\alpha}$. Second, senders are attractive to the receiver as long as they are located in set $\left\{q : d(\theta, q) \leq v^R\right\}$. This informs the candidate parameterization $\Delta = \mathbf{1}(\alpha \leq \alpha_1).v^R$, where the information component is motivated by Theorem 1, whereas v^R is motivated by the set of attractive senders. We consider the general case of Δ in Section 4.4.

Suppose that, according to the schema by Sobel (2016), we define $m = q$ as telling the truth, and $m \neq q$ as lying. Then, given M_θ, the sender is said to deceive (and lie) if he sends a message in set $\left\{m : d(\theta, m) \leq v^R\right\}$ when in reality he is located in set $\left\{q : d(\theta, q) > v^R\right\}$.[13]

In this section, we also ignore welfare-destructive beliefs that imply the sender communicates in such a way as to never produce matches, even though there exist communication policies that would make matching attractive to both parties. We refer to these cases as fatalistic equilibria. These outcomes are related to the babbling equilibrium in that the sender becomes indifferent among all communication strategies, including the one the receiver expects, because no message can produce matches. The difference to babbling equilibria is that, albeit also inconsequential, communication can be informative in fatalistic equilibria.[14]

We maintain the assumption that communication is decisive, which in the current model implies $v^R - E_q\left(d(\theta, q)\right|\theta) = v^R < \frac{\pi r}{2}$. We also keep the assumption about the sender holding transparent motives, which implies $v^S \geq \pi r$. We extend the analysis to the case of non-transparent motives in Section 5.

[13]Note that assessing deception requires considering the receiver's beliefs, which are summarized by M_θ.

[14]The restriction is not required for equilibrium analysis, but greatly increases the intuition and clarity of the results. We also include fatalistic equilibria in the analysis in Section 4.4.

4.2 Analysis

Consider first the case of costless communication. Immediately before sending the message, the sender can be in one of two states: with probability α he knows the receiver's location, and with probability $1-\alpha$ he does not. When the sender knows the location of the receiver, his match utility is given by

$$u^S_{Info} = \max_m \mathbf{1}(m \in M_\theta)\left(v^S - d(\theta, q)\right) \tag{8}$$

In this case, the sender is indifferent across messages in M_θ because all of them are successful in inducing his preferred action.

For some generic maximum matching distance Δ, an uninformed sender chooses the message that maximizes his expected utility:

$$\begin{aligned} u^S_{No\,Info} &= \max_m E\left(\mathbf{1}(m \in M_\theta)\left(v^S - d(\theta, q)\right)\Big| q\right) \\ &= \max_m \frac{1}{2\pi}\int_{d(\theta,m)\leq\Delta} v^S - d(\theta, q)\, d\theta \\ &= \max_m \frac{1}{2\pi}\int_{m-\frac{\Delta}{r}}^{m+\frac{\Delta}{r}} v^S - d(\theta, q)\, d\theta \end{aligned} \tag{9}$$

Differentiation w.r.t. m yields the first-order condition

$$d\left(m^* + \frac{\Delta}{r}, q\right) = d\left(m^* - \frac{\Delta}{r}, q\right) \tag{10}$$

The optimal message is given by $m^* = q$ and minimizes the expected distance to a receiver located at θ. Intuitively, when the sender does not know the receiver's location, he is better off attracting a local receiver than one located far away. Consequently, in this case the sender is better off revealing his type truthfully.

Together with Theorem 1 and Corollary 1, it is possible to establish the following result:

Corollary 2 (*Willful Ignorance – Circle*) *Under transparent motives, trade occurs if and only if*

$$\alpha \leq \overline{\alpha} \equiv \left(\frac{v^R}{\pi r - v^R} \right)^2 \tag{11}$$

and the set of persuasive messages is given by

$$M_\theta = \left\{ m : \, d(\theta, m) \leq \mathbf{1}(\alpha \leq \overline{\alpha}) \, .v^R \right\}.$$

This first part of the result is a direct application of Theorem 1 and Corollary 1: because uninformed senders report their types truthfully, existence is assured as long as $\alpha \leq \overline{\alpha}$. As for the second part, as we explain in the appendix, information cutoffs falling below $\overline{\alpha}$ can only be induced by fatalistic beliefs, i.e., the receiver would need to believe that, at particular levels of information, the sender communicated 'in the worst possible way' for both parties: attractive informed senders would pool with unattractive uninformed ones, and unattractive informed senders would pool with attractive uninformed ones, so as to always make the utility from matching negative. However, the sender should be able to convince the receiver otherwise, because no party can benefit from such communication policy.

Because our focus is also on the characterization of the content of messages, we derive a communication policy that can generate the set of persuasive messages M_θ. We have already established that when the receiver's location is known, the sender selects a message within M_θ; otherwise, the sender prefers to reveal his location truthfully. The optimal communication policy induces a probability density

$$f_{m^*|\theta,q,\alpha} = \alpha \phi_{m|\theta,q,\alpha} + (1 - \alpha) \, \delta(m - q) \tag{12}$$

where $\phi_{m|\theta,q,\alpha}$ describes a mixing message policy of an informed sender, and $\delta(\cdot)$ is the Dirac-delta function.

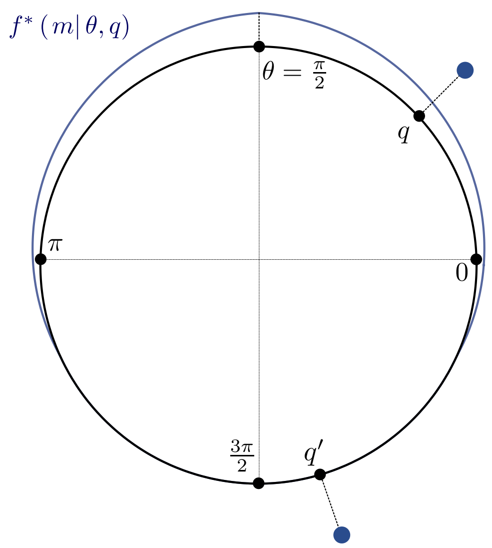
We obtain the following result:

Theorem 2 (*Communication Policy*) *The equilibrium set of persuasive messages* $M_\theta = \left\{m : d(\theta, m) \leq \mathbf{1}(\alpha \leq \overline{\alpha}) . v^R\right\}$ *can be induced by communication policy (and consistent receiver beliefs)*

$$f^*_{m|\theta,q,\alpha} = \alpha \frac{1-\overline{\alpha}}{\overline{\alpha}} \frac{v^R - d(\theta, m)}{\pi (\pi r - 2v^R)} \mathbf{1}\left(d(\theta, m) \leq v^R\right) + (1-\alpha)\,\delta(m-q), \; \alpha \leq \overline{\alpha}. \tag{13}$$

The results states that the set of persuasive messages M_θ can be induced by a communication policy that samples from different messages at different rates. Figure 4 presents an example. Informed senders sample from persuasive messages

Figure 4: Example of sender's communication policy



Note: The informed sender's communication policy assigns higher weight to messages near the receiver along support $\{m : d(\theta, m) \leq v^R\}$. Uninformed senders located at q or q', in particular, reveal their locations, represented by mass points.

near the receiver's location more frequently, while uninformed senders prefer to reveal their locations. The support of the mixing region of informed senders is given by $\{m : d(\theta, m) \leq v^R\}$. Mixing across this region enables informed senders to pool with attractive uninformed types to the largest extent possible, conditional on α. While informed senders are always successful in inducing matches, this does not hold for uninformed ones, as shown by the case of sender q' in Figure 4. Hence, the receiver should be doubtful of a seemingly persuasive message, but can always trust the credibility of an unappealing one.

Finally, we characterize comparative statics as a function of the level of information. The sender's ex-ante utility is given by

$$E\left(U^S\right) = \alpha E\left(v^S - d\left(\theta, q\right)\Big| q\right) + \left(1-\alpha\right) Pr\left(q \in C_\theta\right) E\left(v^S - d\left(\theta, q\right)\Big| q, q \in C_\theta\right) \tag{14}$$

where C_θ is induced by M_θ, and is given by $\left\{q : \, d\left(\theta, q\right) \leq \mathbf{1}\left(\alpha \leq \overline{\alpha}\right).v^R\right\}$. Similarly, the receiver's ex-ante utility is given by

$$E\left(U^R\right) = \alpha E\left(v^R - d\left(\theta, q\right)\Big| \theta\right) + \left(1-\alpha\right) Pr\left(q \in C_\theta\right) E\left(v^R - d\left(\theta, q\right)\Big| \theta, q \in C_\theta\right) \tag{15}$$

Under transparent motives (i.e. $v^S \geq \pi r$), analysis of expressions (14) and (15) provides the following result

Proposition 1 (*Comparative Statics*) The receiver is better off with a lower level of information, whereas the sender prefers a higher level, up to $\overline{\alpha}$.

As the level of information increases, so does the frequency with which informed senders pool with attractive uninformed ones. Hence, the receiver prefers full privacy, whereas the sender prefers information level $\overline{\alpha}$, i.e., the maximum amount of information the receiver is willing to bear. We later describe the comparative statics more generally, including the case in which the sender does not hold transparent motives and also investigate the social planner's optimal information level.

4.3 Costly Communication

In most settings, the sender decides whether he should incur a cost $c > 0$ in order to communicate with receiver. When c is high enough, two forces are in play. First, uninformed senders may refrain from communication. Second, informed senders located far away from the receiver may also refrain from communication. The first effect is generally negative for the receiver whereas the second one is positive. The rates at which each type of sender refrains from communication may be different, leading to different possible outcomes.

We first examine a case in which the market may collapse because uninformed senders refrain from communication at intermediate levels of c. Define c_U^* as the cost threshold that makes uninformed senders indifferent between communicating and not. Moreover, because distant informed senders refrain from communicating as the cost increases, there exists a cost level c_R^* that makes the receiver indifferent between matching exclusively with informed senders and not matching. When the agents' valuations are sufficiently different, we obtain the following result:

Proposition 2 (*Existence of Trade and Costly Communication*) *Trade cannot occur when the communication cost is intermediate*, $c \in (c_U^*, c_R^*)$, *if* $v^S \geq \frac{4\pi r - v^R}{2(\pi r - v^R)} v^R$.

When v^S is relatively higher than v^R, the forces mentioned above play out in the following way. On one hand, uninformed senders refrain from communication because the probability of trade is low, given the low value of v^R. On the other hand, too many unappealing informed senders prefer to communicate, because they can earn a high gross match utility v^S if they can induce a match. In this case, trade breaks down irrespective of the level of information available to the sender.

When the communication cost falls outside interval (c_U^*, c_R^*), trade resumes. The intuition for low communication costs is the same as when $c = 0$. When communication costs are high, $c > c_R^*$, the receiver is also willing to match because senders can credibly signal that they provide enough utility from matches. In this case, trade relies solely on the communication cost and not on content, which is in line with the literature on money burning.

When the valuations of the agents are not too different, however, both the content and cost of communication act through concurrent mechanisms:

Proposition 3 *(Concurrent Roles of Cost and Content) The communication cost affects the types of informed senders willing to communicate, whereas the content of the message affects the types of uninformed senders who are able to induce matches, if* $v^S < \frac{2(\pi r)^2 - \left(v^R\right)^2}{2(\pi r - v^R)}$.

The intuition for this result is as follows: at certain levels of the communication cost, a few unattractive informed senders may refrain from communication. However, the remaining senders still need to convey attractive messages in order to induce matches, which uninformed senders may fail to do. In this case, the receiver's beliefs as well as the match probabilities are affected by both the content and the cost of communication: the cost of communication determines the types of *informed* senders in the market, whereas the content of communication is used by the receiver to identify *uninformed* senders who do not merit a match.

4.4 Generalized Matching Distance

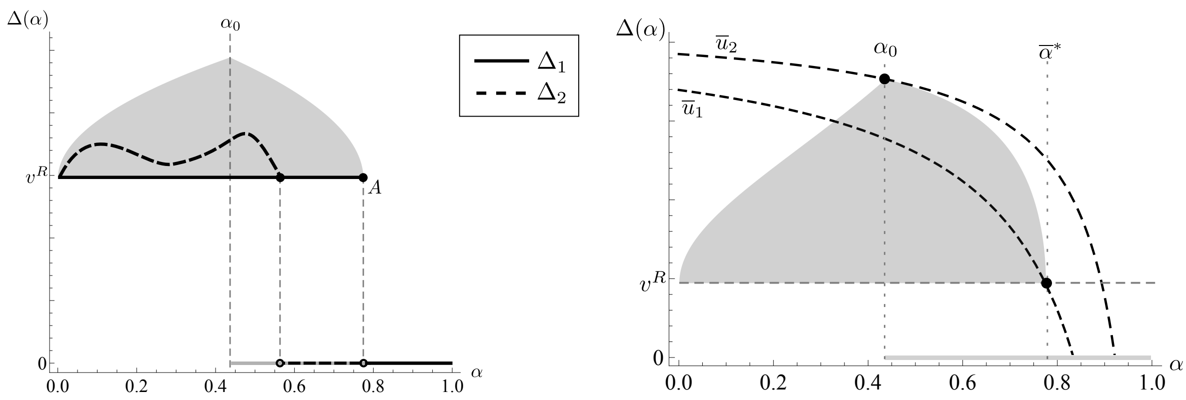
We have characterized situations in which the receiver's beliefs induce a critical set $M_\theta = \{m : d(\theta, m) \leq \Delta\}$, where $\Delta = \mathbf{1}(\alpha \leq \overline{\alpha})\, v^R$. In general, threshold Δ is a function of the receiver's beliefs about the sender's communication policies at different parameter values. For example, in some settings the receiver may have reason to believe that the sender changes the communication policy as a function of the information level, such that Δ is a more complex function of α. We provide a general characterization of beliefs and consistent communication policies when Δ is allowed to depend on the level of information. We focus on sequential equilibria, as defined by Kreps and Wilson (1982), in order to impose restrictions on the allowable off-equilibrium path beliefs.

We obtain the following result:

Theorem 3 (*Belief Characterization*) *Under the sequential equilibrium refinement, the set of maximum matching distances is characterized by* $\mathcal{S}_\Delta$ *(defined in the appendix).*

Theorem 3 establishes the set of distances $\mathcal{S}_\Delta$ that can be induced by receiver's beliefs. The left panel of Figure 5 depicts set $\mathcal{S}_\Delta$ for different levels of the information level α.

Figure 5: (left) Set $\mathcal{S}_\Delta$ as a function of α; (right) Sender's indifference curves



The shaded area describes the matching distances that can be supported by beliefs, as a function of the level of information α. It is generated by the potential communication policies of senders, and point A denotes the sender's first-best level of information analyzed in the previous section.

The main intuition for the triangle-like shape of $\mathcal{S}_\Delta$ is as follows. First, note that no strictly positive distance can occur below v^R in equilibrium. If the receiver faced only uninformed senders, she would match upon receiving messages within distance v^R. If the matching distance fell below v^R, however, this would imply that unattractive informed senders would have pooled within the same region to the extent of rendering some messages unattractive, which would yield a contradiction. Strikingly, the receiver may be willing to entertain messages farther away than v^R.

This occurs whenever enough attractive informed senders select messages outside interval $\left\{m : d(\theta, m) \leq v^R\right\}$. For example, when α increases from low levels, unattractive informed senders can pool close to the receiver's location with no effect to the matching distance. Attractive informed senders can pool just outside region $d(\theta, m) \leq v^R$ and increase the matching distance as a result. As α increases, the size of the set of informed senders that can pool outside region $d(\theta, m) \leq v^R$ increases as well, and the greatest matching distance is given by

$$\overline{\Delta} = \frac{\pi r v^R}{\pi r - v^R} \tag{16}$$

which is attained at information level α_0, as shown in Figure 5. Beyond α_0, the maximum matching distance decreases because too many unattractive informed types pool with attractive uninformed senders within region $d(\theta, m) \leq v^R$ and, as a result, attractive informed senders cannot pool as far as before.

We depict two belief paths on the left panel of Figure 5. Path Δ_1 was the one examined in the previous section: receivers are willing to match as long as they observe a message within distance v^R and $\alpha \leq \overline{\alpha}$. Path Δ_2 is non-monotonic on α, up to $\overline{\alpha}$. It is consistent with the belief that informed senders change their communication strategies as a function of the information level.

We plot the sender's indifference curves in the right panel of Figure 5. Clearly, the sender can do better than with level of information $\overline{\alpha}$:

Proposition 4 (*Sender's first-best*) *The sender's first-best outcome is achieved at level of information $\alpha_0 < \overline{\alpha}$ (defined in the appendix), under beliefs that induce the maximum matching distance $\overline{\Delta}$.*

Beliefs allowing, the result above states that the sender's first-best information level falls strictly below $\overline{\alpha}$, even if the receiver is willing to trade at higher information levels. The underlying intuition is that, by limiting the level of information to $\overline{\alpha}$, the sender is able to maximize the matching distance Δ.

Uninformed senders reveal their types as before. However, in this case, informed senders engage in a more complex communication policy: first, unattractive informed senders pool with attractive uninformed senders to the furthest extent possible; and second, attractive informed senders pool with unattractive uninformed ones, just enough to make them attractive.

Why does the sender benefit from the receiver believing that he pools with unattractive uninformed senders, whenever he is attractive and informed? The reason is that such beliefs increase the probability of a match occurring when the sender is slightly unattractive to the receiver. The result survives most equilibrium refinements as well as changes to the modeling assumptions. As such, it is worth considering the implications to market settings.

For example, consider the case of a job applicant who is asked a series of questions about himself during a job interview. The questions are subjective, and so allow for a number of possible correct answers, some more acceptable than others. In our model, the sender has an incentive to report truthfully whenever he does not know which answer the employer would prefer to hear, i.e., truth-telling is better than a shot in the dark. However, how should the sender answer questions he knows exactly how to respond? If he always says what the receiver would like to hear (i.e., send a message in set $\left\{m : d(\theta, m) \leq v^R\right\}$), it becomes easy for the receiver to distinguish pandering from truth-telling: specifically, any message outside this region is clearly truthful but unconvincing. However, if the sender answers questions less perfectly (i.e., samples from messages on the neighborhood of $\left\{m : d(\theta, m) \leq v^R\right\}$), it decreases the receiver's signal-to-noise ratio in cases where the sender does not know the correct answer. Under consistent beliefs, the job applicant gains from communicating less perfectly about questions he masters, thus granting himself leeway for the questions he is not informed of. The same logic applies to an expert or a political agent, who may prefer to be relatively guarded about aspects they are knowledgeable of, so as not to endanger their reputation, the chances of closing a deal, or of achieving a higher position in an organization, because of how they subsequently perform while discussing aspects they are less aware of.

5 Non-Transparent Motives

We now investigate cases in which v^S can be lower than πr, such that the sender may not always want to induce a match. We investigate the case where $c = 0$, and $M_\theta = \left\{m : d(\theta, m) \leq \mathbf{1}(\alpha \leq \overline{\alpha}) . v^R\right\}$. We consider the effects of the information level on the welfare of the sender, the receiver, and of the social planner.

When the sender does not hold transparent motives $\left(v^S < \pi r\right)$, he is no longer willing to communicate in some cases: informed senders prefer not to communicate with distant receivers, and become indifferent at locations

$$\begin{aligned} & E\left(u^S \middle| Informed\right) &= 0 \\ \Leftrightarrow \quad & v^S - d(\theta, q_{Indifferent}) &= 0 \\ \Leftrightarrow \quad & d(\theta, q_{Indifferent}) &= v^S \end{aligned} \tag{17}$$

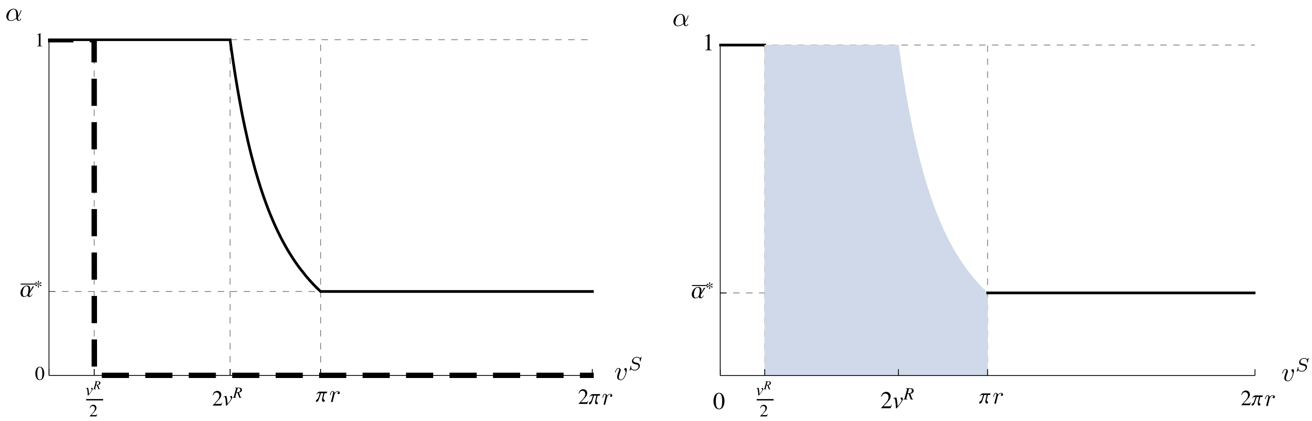
As v^S decreases, the sender is able to increase the level of information beyond $\overline{\alpha}$, taking advantage of the fact that the receiver knows that messages no longer originate from distant informed senders. In the appendix, we show that when $v^S \leq 2v^R$, the sender is able to collect perfect information about the receiver because the latter understands only few unappealing informed senders engage in communication. Formally, the first-best level of information of the sender is given by

$$\alpha_S^* = \begin{cases} 1, & v^S \leq 2v^R \\ \max_{v \in \{v^S, \pi r\}} \left(\frac{v^R}{v - v^R}\right)^2, & v^S > 2v^R \end{cases}$$

as depicted by the solid line on the left panel of Figure 6. Intuitively, the sender's first-best level of information is the highest one that ensures the receiver earns non-negative ex-ante utility.

We have already shown that, under transparent motives, the receiver prefers not to disclose any information, so as to induce perfect revelation by the sender. However, when the valuation of the sender is low $\left(v^S < \frac{v^R}{2}\right)$, the sender cares about the quality of the match enough that he is only willing to communicate if he is informed. In this case, the receiver prefers to reveal all information in order

Figure 6: First-best information levels



Note: Solid and dashed lines on the left panel represent first-best information levels for sender and receiver, respectively. The right panel represents the correspondence that maximizes joint welfare.

to maximize the likelihood of matching. We depict the result in the dashed line of the left panel of Figure 6.

Consider now the cases of regulators overseeing market interactions that involve persuasion, of private firms managing two-sided markets, and of search engines matching advertisers to buyers. These agents may prefer to affect the amount of information available in order to maximize joint welfare. The optimal information policy depends on the sender's valuation, and is depicted in the right panel of Figure 6.

When $v^S < \frac{v^R}{2}$, the sender is only willing to communicate if he is informed. In this case, both agents prefer there to be perfect information about the receiver's preferences. When the sender's valuation is high, $v^S > \pi r$, the sender's high value from matching offsets possible ex-post losses from the receiver, and so the optimal information level is equal to the sender's first-best level of information, $\overline{\alpha}$. Finally, in the intermediate range $v^S \in \left[\frac{v^R}{2}, 2v^R\right]$, changes to the level of information exactly transfer utility between the agents, as long as the receiver's participation constraint is satisfied, and no aggregate welfare effects result. The intuition for this case is that when the information level increases from low levels, a few informed sender types that are unattractive to the receiver become informed and are able to induce matches, and the sender's incremental gains equal the receiver's losses.

Overall, the fact that the sender's optimal information level is also optimal for the social planner derives from two reasons. First, trade generates a net surplus in the market and second, the receiver's ex-ante payoff is bounded below by zero, the value of the outside option. We summarize these results by the following proposition:

Proposition 5 *(Welfare Comparative Statics) The sender always prefers the maximum amount of information level the market can bear. The receiver prefers full privacy, unless communication is pivotal for communication, in which case she prefers full disclosure. The optimal level of information for the sender is also optimal for the social planner.*

Finally, it is worth interpreting the results under the perspective that the parties' valuations depend on the types of their counterparts. In this case, valuation v^S $\left(v^R\right)$ is interpreted as the commonly-known ability of the receiver (sender) to provide a high match value to her counterpart. Our results imply that high-type receivers (who provide a high v^S to the sender in case of a match) should refrain from disclosing preference information because they will always be communicated to, whereas low-type receivers (low v^S) prefer to reveal their preferences completely in order to induce communication by interested senders. This is consistent with the dating market for example, where v^S is an observable measure of attractiveness of the receiver. In this case, the receiver should refrain from communicating, i.e. sharing likes and dislikes for example, in order to elicit truthful communication about the sender's preferences. If in contrast the receiver announced her type, she would receive exactly what she would like to hear, but be able to trust little of it. In comparison, a receiver that produces a lower match value to the sender is better off sharing personal information in order to elicit communication. Similarly, a firm posting an attractive job opening is better off not describing the position in too much detail, a 'high-type customer' prefers to hide her preferences from a salesperson or advertiser, etc.

6 Observability of the Information Level

It is worth considering the case in which the level of information is unobserved by the receiver. For example, if the sender could select the level of information in this case, he would always prefer perfect information, because such action would not affect the receiver's beliefs. The observability assumption does not affect our results when sender's valuation is low $\left(v^S \leq 2v^R\right)$. However, it implies that the market would break down when the sender's valuation is high $\left(v^S > 2v^R\right)$. We now explain that it is possible to avoid the market from collapsing as long as the receiver is imperfectly informed about the information level selected by the sender.

Consider the case in which the receiver observes the level of information selected by the sender, α^*, with noise, i.e. she observes $\alpha' = \alpha^* + \varepsilon$ where ε is a nuisance parameter. The results from Bagwell (1995) imply that, in our case, the pure strategy equilibrium on α can never induce a match. The intuition is as follows: if the sender selects level of information α^*, Nash equilibrium requires the receiver to best-respond to that same level. But, given the receiver's policy, the choice of α^* would never arise in equilibrium because the sender would be better off deviating to full information ($\alpha = 1$). Van Damme and Hurkens (1997) find that a mixed strategy equilibrium on α exists nonetheless, and argue that such outcome has attractive properties. Importantly, they find that the case of perfect observability of α yields the limit payoffs of a mixed strategy equilibrium as the noise approaches zero. The implication is that our model characterizes the limit of cases with imperfectly-observed α, as the variance of ε approaches zero.

Our results also characterize the limit outcome when the sender's choice of information level is affected by his own private information, as shown by Maggi (1999). For example, let κ be a sender's stochastic privately-known cost of collecting information. In this case, the receiver reacts to policy $\alpha^*(\kappa)$ in equilibrium rather than to a constant level of information, and so she incorporates the noisy signal α' in her equilibrium response. Maggi (1999) shows that, as long as the amount of noise is small, there exist equilibria close to the ones we characterize in this paper.

In some other cases our results apply directly. Suppose α is communicated with a high enough probability, in which case it is perfectly observed by the receiver. In this case, the sender prefers not to increase the level of information beyond α^*.[15]

7 Conclusion

We have investigated a cheap-talk setting in which the sender can use the information available to tailor the communication to the receiver's preferences. At the center of our results is the fact that, while in principle the sender could benefit from higher levels of information, these in turn make the receiver skeptical of the merits of the message. As a result, trade always breaks down if the sender is sufficiently likely to be informed.

We consider a horizontal matching setting, and show that, both the sender and the social planner always prefer the maximum amount of information that can support trade. The receiver, on the other hand, prefers complete privacy unless if information is pivotal for communication, in which case she prefers full disclosure.

Our results are relevant to multiple matching markets and related policy debates. For instance, we have found that information revelation by consumers increases their welfare only when such information is pivotal in inducing communication. Hence, in advertising markets, consumers may be better off disclosing their preferences to niche firms, but should shroud them from others willing to engage in mass market communication. Our model also applies to settings in which the receiver selects the amount of information that is observed by the sender. In settings such as the job and dating markets, the receiver (e.g. a firm comparing applicants' vitae or an individual being romantically pursued) may have an incentive not to share too much information about what they are looking for, because the sender may use such information to claim he possesses the skills or shares the right set of interests that ensure successful matches.

[15] Van Damme and Hurkens (1997) also discuss this case, and mention a formal treatment by Chakravorti and Spiegel (1993).

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A Appendix

Theorem 1

The receiver has access to an outside option, and so her expected utility is bounded below by zero upon receiving any message, i.e., receiver θ's expected utility conditional on receiving message m is given by

$$U^R(m) \equiv \max\left\{E\left(u^R(\theta,q)\middle|\theta,m\right),0\right\} \tag{18}$$

Averaging over all messages, the receiver's ex-ante utility is given by

$$EU^R = \alpha E\left(u^R(\theta,q)\middle|\theta\right) + (1-\alpha)\,Pr\,(q\in C_\theta)\,E\left(u^R(\theta,q)\middle|\theta,C_\theta\right) \tag{19}$$

where $C_\theta = \{q : m_U^*(q) \in M_\theta\}$ and $M_\theta \equiv \left\{m : u^R(m) \geq 0\right\}$, and $u^R(m) \equiv E\left(u^R(\theta,q)\middle|\theta,m\right)$. It follows that

$$EU^R = E\left(U^R(m)\right) \tag{20}$$

and because $U^R(m) \geq 0\,\forall m \in Q$, the receiver's ex-ante payoff EU^R is also bounded below by zero.

Rearranging inequality (19) yields:

$$\begin{aligned}
\alpha E\left(u^R(\theta,q)\middle|\theta\right) + (1-\alpha)\,Pr\,(q\in C_\theta)\,E\left(u^R(\theta,q)\middle|\theta,C_\theta\right) &\geq 0\\
\alpha\int_Q u^R(\theta,q)\,dF_{q|\theta} + (1-\alpha)\int_{C_\theta} u^R(\theta,q)\,dF_{q|\theta} &\geq 0\\
\alpha \leq \frac{1}{1-\frac{\int_Q u^R(\theta,q)dF_{q|\theta}}{\int_{C_\theta} u^R(\theta,q)dF_{q|\theta}}} & \qquad (21)
\end{aligned}$$

where the right-hand side of (21) depends on α through C_θ. The assumption that communication is decisive implies that expression $\int_Q u^R(\theta,q)\,dF_{q|\theta}$ is negative, making the right-hand side of (21) increasing in $\int_{C_\theta} u^R(\theta,q)\,dF_{q|\theta}$. Clearly, the support that maximizes $\int_{C_\theta} u^R(\theta,q)\,dF_{q|\theta}$ is given by $C_\theta^* = \left\{q : u^R(\theta,q) \geq 0\right\}$, such that the integral sum is made across positive integrands. Decomposing $\int_Q u^R(\theta,q)\,dF_{q|\theta}$ as $\int_{C_\theta^*} u^R(\theta,q)\,dF_{q|\theta} + \int_{Q\setminus C_\theta^*} u^R(\theta,q)\,dF_{q|\theta}$ completes the proof.

Corollary 1

Suppose uninformed senders reveal their types (or alternatively, can communicate whether they are attractive to the receiver or not). Then, Theorem 1 implies that it is possible for the informed senders to pool with the uninformed senders while respecting the receiver's individual rationality constraint at each message. To see this, note that Theorem 1 implies that the total expected utility provided by attractive uninformed senders is high enough to balance the net disutility of informed ones, as long as $\alpha \leq \overline{\alpha}$.

Corollary 2

The result follows directly from Theorem 1 and Corollary 1.

Theorem 2

We show this result by first considering a communication strategy that provides the receiver zero expected utility, point-by-point, at $\alpha = \overline{\alpha}$. Then, we show that the resulting communication policy of informed senders can be used at all levels of information below $\overline{\alpha}$, and induces $M_\theta = \left\{m : \, d\left(\theta, m\right) \leq \mathbf{1}\left(\alpha \leq \overline{\alpha}\right).v^R\right\}$.

We focus on communication policies of the form

$$f_{m^*|\theta,q,\alpha} = \alpha\phi_{m|\theta,\alpha} + \left(1-\alpha\right)\delta\left(m-q\right). \tag{22}$$

As discussed in the text, we restrict ourselves to the class of informed sender p.d.f.'s $\phi_{m|\theta,\alpha}$, invariant to the sender's location q.

When $\alpha = \overline{\alpha}$, the receiver cannot do better than earn zero matching utility in region M_θ. Together with Bayes rule, it follows that

$$E\left(u^R\left(\theta, q\right)\Big|\,\theta, m \in S_\phi, \alpha = \overline{\alpha}\right) = \int_0^{2\pi}\left(v^R - d(\theta,q)\right)d\widehat{F_{q|\theta,m,\alpha}}\Big|_{\alpha=\overline{\alpha}} = 0 \tag{23}$$

$$\Leftrightarrow \int_0^{2\pi}\left(v^R - d(\theta,q)\right)\frac{\left(\overline{\alpha}\phi_{m|\theta,\alpha} + (1-\overline{\alpha})\delta(m-q)\right)}{f_{m|\theta,\alpha}}\frac{1}{2\pi}dq = 0 \; \forall m \in S_\phi \tag{24}$$

Solving for $\phi_{m|\theta,\alpha}$ yields:

$$\phi_{m|\theta,\alpha} = \frac{1-\overline{\alpha}}{\overline{\alpha}} \frac{v^R - d(\theta, m)}{\pi(\pi r - 2v^R)} \mathbf{1}(m \in S_\phi) \tag{25}$$

where S_ϕ is the support of $\phi_{m|\theta,\alpha}$. The informed sender's p.d.f. candidate is hereafter referred to by $\phi_{m|\theta}$, because it is invariant to the information level α. In order for $\phi_{m|\theta}$ to be a density, it has to be positive over its support, and integrate to one. Assuming support $S_\phi = \left\{m : d(\theta, m) \leq v^R\right\}$ ensures both conditions.[16]

It follows that the ex-ante communication p.d.f. $\alpha\phi_{m|\theta} + (1-\alpha)\delta(m-q)$ also integrates to one. It remains to show that the receiver expects (weakly) positive utility whenever $m \in M_\theta$, and strictly negative utility whenever $m \notin M_\theta$. Bayes rule implies the receiver's consistent beliefs are given by:

$$\widehat{f_{q|\theta,m,\alpha}} = \frac{\left(\alpha\frac{1-\overline{\alpha}}{\overline{\alpha}}\frac{v^R-d(\theta,m)}{\pi(\pi r-2v^R)}\mathbf{1}(m \in S_\phi) + (1-\alpha)\delta(m-q)\right)}{f_{m|\theta,\alpha}}\frac{1}{2\pi} \tag{26}$$

The receiver's expected utility, conditional on observing message m, becomes

$$\int_0^{2\pi}\left(v^R-d(\theta,q)\right)\widehat{f_{q|\theta,m,\alpha}}dq= \tag{27}$$

$$\propto(1-\alpha)\left(v^R-d(\theta,m)\right)-\alpha\frac{1-\overline{\alpha}}{\overline{\alpha}}\left(v^R-d(\theta,m)\right)\mathbf{1}\left(m\in S_\phi\right) \tag{28}$$

where the proportionality sign follows from eliminating the positive constants on the right-hand side. Whenever $m \in M_\theta$, expression (28) becomes

$$(1-\alpha)\left(v^R - d(\theta, m)\right) \geq \alpha\frac{\pi r\left(\pi r - 2v^R\right)}{\left(v^R\right)^2}\left(v^R - d(\theta, m)\right) \tag{29}$$

$$\Leftrightarrow 1 \geq \left(1 + \frac{\pi r\left(\pi r - 2v^R\right)}{\left(v^R\right)^2}\right)\alpha \tag{30}$$

such that the receiver expects positive match utility, because (30) attains equality exactly at the highest level of the information level, $\alpha = \overline{\alpha}$. Whenever $m \notin S_\phi$, we require the receiver's expected utility to be strictly negative. Expression (28)

[16]Note that the assumption that communication is decisive implies that $\pi r - 2v^R > 0$.

becomes

$$(1-\alpha)\left(v^R - d(\theta, m)\right) < 0 \tag{31}$$

which holds for all $m \notin S_\phi$ for all information levels, concluding the proof.

Proposition 1

In the text we have established $C_\theta = \left\{q : d(\theta, q) \leq \mathbf{1}(\alpha \leq \overline{\alpha}) .v^R\right\}$, such that no trade takes place when $\alpha > \overline{\alpha}$, in which case both agents earn zero payoffs. Assuming $\alpha \leq \overline{\alpha}$, the receiver's ex-ante surplus is given by

$$EU^R = \alpha E\left(v^R - d(\theta, q)\right) + (1-\alpha) Pr\left(d(\theta, q) \leq v^R\right) E\left(v^R - d(\theta, q)\middle| d(\theta, q) \leq v^R\right) \tag{32}$$

where the first term is negative (communication is decisive), and the second term is positive. Hence, the receiver is better off with a lower level of information. When $\alpha = 0$, she earns

$$\int_{\theta-\frac{v^R}{r}}^{\theta+\frac{v^R}{r}} \left(v^R - d(\theta, q)\right) \frac{1}{2\pi} dq \tag{33}$$

$$= \frac{\left(v^R\right)^2}{r} \tag{34}$$

which is strictly great than zero.

As for the sender, his ex-ante surplus when $\alpha \leq \overline{\alpha}$ is given by

$$EU^S = \alpha E\left(v^S - d(\theta, q)\right) + (1-\alpha) Pr\left(d(\theta, q) \leq v^R\right) E\left(v^S - d(\theta, q)\middle| d(\theta, q) \leq v^R\right) \tag{35}$$

where the first term is positive and dominates the second one because of transparent motives. Because trade breaks down at $\alpha > \overline{\alpha}$, the sender is better off at information level $\overline{\alpha}$.

Proposition 2

We first set up some notation. The level c_U^* that makes uninformed senders indifferent between communicating and not, is equal to

$$\begin{aligned} & EU^S_{Uninformed} - c_U^* && = 0 \\ \Leftrightarrow\quad & \tfrac{1}{2\pi}\int_{q-\frac{v^R}{r}}^{q+\frac{v^R}{r}} v^S - d\left(\theta,q\right) d\theta - c_U^* && = 0 \\ \Leftrightarrow\quad & c_U^* && = \frac{2v^S - v^R}{2\pi r} v^R \end{aligned}$$

Also, consider the case in which the receiver is willing to match when only informed senders remain. This threshold is given by

$$\begin{aligned} & EU^R_{Communication} && = 0 \\ \Leftrightarrow\quad & \tfrac{1}{2\pi}\int_{\theta-\frac{v^S-c_R^*}{r}}^{\theta+\frac{v^S-c_R^*}{r}} v^R - d\left(\theta,q\right) dq && = 0 \\ \Leftrightarrow\quad & c_R^* && = v^S - 2v^R \end{aligned}$$

When $c_U^* < c_R^*$, there exists an intermediate cost level such that receivers are not willing to match. Solving the inequality w.r.t. v^S completes the proof.

Proposition 3

Here we prove that the communication cost can make some informed senders refrain from communication while uninformed ones are still willing to communicate. The statement about the content of communication trivially follows from $M_\theta = \left\{m : d\left(\theta, m\right) \leq \mathbf{1}\left(\alpha \leq \overline{\alpha}\right).v^R\right\}$, and uninformed senders revealing their types.

Suppose $c < c_U^*$ such that uninformed senders participate. The farthest informed sender from the receiver is indifferent about communicating when $c = c_I^* \equiv v^S - \pi r$.

When $c_I^* < c_U^*$, there exists a cost region $c \in (c_I^*, c_U^*)$ such that distant informed senders refrain from communication, whereas uninformed ones still communicate, but may not find matches because of the content of their messages.

Theorem 3

Set $\mathcal{S}_\Delta$ is given by

$$\mathcal{S}_\Delta \equiv \{\Delta = 0 \wedge \alpha \geq \alpha_0\} \cup \left\{ v^R \leq \Delta \leq \begin{cases} v^R \left(1 + \sqrt{\frac{\alpha}{1-\alpha}}\right), & \alpha \leq \alpha_0 \\ v^R + \sqrt{\frac{(v^R)^2 - \alpha(\pi r - v^R)^2}{1-\alpha}}, & \alpha_0 < \alpha \leq \overline{\alpha} \end{cases} \right\}$$

where $\alpha_0 \equiv \frac{\left(v^R\right)^2}{(\pi r - v^R)^2 + (v^R)^2}$.

A set of beliefs $\widehat{f_{q|\theta,m,\alpha}}$ induces a maximum distance Δ such that the receiver is willing to match as long as $d(\theta, m) \leq \Delta$. We characterize the set of distances $\mathcal{S}_\Delta$ that can be induced by on-equilibrium path beliefs. We then apply the sequential equilibrium restriction to characterize the admissible set of off-equilibrium path beliefs.

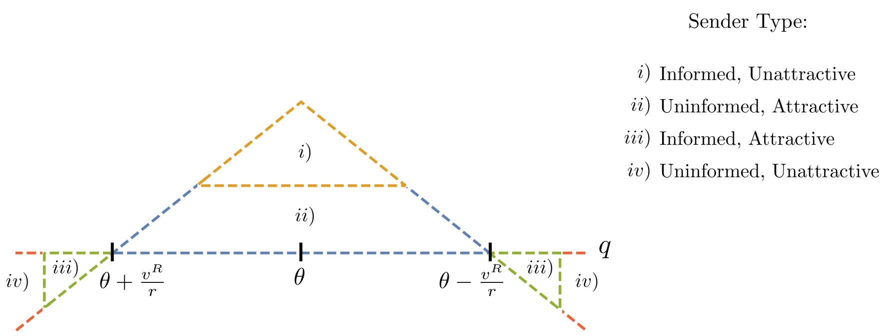
- Region $\mathcal{S}_\Delta$ contains two sub-regions. We consider the triangle-like sub-region first, namely:

$$v^R \leq \Delta \leq \begin{cases} v^R \left(1 + \sqrt{\frac{\alpha}{1-\alpha}}\right), & \alpha \leq \alpha_0 \\ v^R + \sqrt{\frac{(v^R)^2 - \alpha(\pi r - v^R)^2}{1-\alpha}}, & \alpha_0 < \alpha \leq \overline{\alpha}^* \end{cases}$$

The lower bound v^R follows from the fact that messages in set $\left\{m : d(\theta, m) \leq v^R\right\}$ are always persuasive in equilibrium. We have already shown that uninformed senders report their types, whereas informed senders are indifferent among messages that the receiver deems attractive according to his beliefs. In region $d(\theta, m) \leq v^R$, the receiver earns positive utility from being matched with an uninformed type, and (unattractive) informed types would never pool in this region to the extent of making a message unappealing in equilibrium. Hence, if strictly positive, the distance between the message and the receiver's location that generates a match in equilibrium must be at least v^R.

The upper bounds on Δ follow from the following considerations. First, informed senders may pool with attractive and unattractive uninformed senders to different extents. When α is low, all informed senders are able to pool with attractive uninformed senders and produce matches. An alternative policy is for unattractive informed senders to pool with attractive uninformed senders, while attractive informed senders pool with uninformed senders located just outside region $d(\theta, q) \leq v^R$. In this case, under consistent beliefs, informed senders are able to secure matches by sending messages outside region $\left\{m : d(\theta, m) \leq v^R\right\}$. Figure 7 depicts the receiver's match utility, which decreases with the distance to

Figure 7: Expected Utility for a Given Communication Policy



Note: q increases from right to left in order to be consistent with the counter-clockwise angular definition.

the sender's location, and the areas of the triangular regions constitute expected match utilities for the receiver. For example, a receiver earns utility equal to the area of triangle *ii*) - partially hidden by region i) - if she matches with an attractive uninformed sender with uniform probability. Her matching utility may also be negative if she matches with uninformed senders located far away, as depicted by triangular regions *iv*), partially covered by regions *iii*). Moreover, the area of region *i*) depicts the receiver's *disutility* from matching with unattractive *uninformed* senders and the area of region *iii*) depicts the utility from matching with attractive informed senders.

When attractive informed senders pool with unattractive uninformed senders as depicted by regions *iii*) and *iv*), the receiver is willing to match for some messages beyond $\theta \pm \frac{v^R}{r}$. The maximum matching distance an informed sender can induce is attained by pooling with unattractive uninformed senders, so as to provide exactly zero match utility to the receiver over such messages. Multiple mixing schemes can accomplish this, but the maximum distance is always attained by providing zero utility to the receiver over the mixing region.

At low levels of α, the maximum matching distance is obtained by calculating the width of each of the $iii)$ triangles that guarantees that the sender receives zero matching utility to the furthest extent possible. The utility from being matched with an attractive informed sender - area of $iii)$ - is equal to

$$\frac{\alpha}{2\pi}\int_{\theta-\frac{v^R}{r}}^{\theta+\frac{v^R}{r}} v^R - d(\theta,q)\,dq = \frac{\alpha\left(v^R\right)^2}{2\pi r}$$

The utility of matching with unattractive uninformed senders over some distance ω - region $iv)$ - is equal to

$$\frac{1-\alpha}{2\pi}\left(\int_{\theta-\frac{\omega}{r}}^{\theta-\frac{v^R}{r}} v^R - d(\theta,q)\,dq + \int_{\theta+\frac{v^R}{r}}^{\theta+\frac{\omega}{r}} v^R - d(\theta,q)\,dq\right) = -\frac{(1-\alpha)\left(\omega-v^R\right)^2}{2\pi r}$$

Hence, the maximum distance to which attractive informed types can pool, while providing zero expected utility to the receiver, is given by

$$\begin{aligned} \tfrac{\alpha\left(v^R\right)^2}{2\pi r} - \tfrac{(1-\alpha)\left(\omega-v^R\right)^2}{2\pi r} &= 0 \\ \Leftrightarrow \qquad \omega &= v^R\left(1+\sqrt{\frac{\alpha}{1-\alpha}}\right) \end{aligned}$$

Hence, for low levels of α, there exist communication policies (and consistent beliefs) that induce matches up to messages $\left\{m : d(\theta,m) \leq v^R\left(1+\sqrt{\frac{\alpha}{1-\alpha}}\right)\right\}$.

As α increases, so does the area of region $i)$, which eventually matches the area of region $ii)$. At this point, the expected utility from receiving a message in region $\left\{m : d(\theta,m) \leq v^R\right\}$ yields zero utility to the receiver. The utility earned from matching with unattractive informed senders is given by[17]

$$\frac{\alpha}{2\pi}\left(\int_{0}^{\pi-\frac{v^R}{r}} v^R - r(\pi-q)\,dq + \int_{\pi+\frac{v^R}{r}}^{2\pi} v^R - r(q-\pi)\,dq\right) = -\frac{\alpha\left(\pi r-v^R\right)^2}{2\pi r}$$

Finally, the utility earned from attractive uninformed senders is equal to $\frac{(1-\alpha)\left(v^R\right)^2}{2\pi r}$.

[17] In the calculation above we used $\theta=\pi$ for parsimony, but any θ can be used, provided the correct distance function is also used.

The receiver expects zero utility from a message in set $\left\{m : d(\theta, m) \leq v^R\right\}$ when

$$\begin{aligned} -\tfrac{\alpha\left(\pi r - v^R\right)^2}{2\pi r} + \tfrac{(1-\alpha)\left(v^R\right)^2}{2\pi r} &= 0 \\ \Leftrightarrow \qquad \alpha &= \frac{\left(v^R\right)^2}{\left(\pi r - v^R\right)^2 + \left(v^R\right)^2} \equiv \alpha_0. \end{aligned}$$

When α increases beyond α_0, the matching utility in region $\left\{m : d(\theta, m) \leq v^R\right\}$ yields negative matching utility for the receiver, which cannot happen in equilibrium as we explained before. In this case the maximum matching distance is attained by partial pooling of attractive senders into the inner region, to ensure the receiver earns zero matching utility throughout. The net surplus can still be spread on region *iv*) as before.

The net surplus available for mixing outside region $\left\{m : d(\theta, m) \leq v^R\right\}$ by attractive informed senders is equal to

$$-\underbrace{\frac{\alpha\left(\pi r - v^R\right)^2}{2\pi r}}_{(Unattractive\,Informed)} + \underbrace{\frac{(1-\alpha)\left(v^R\right)^2}{2\pi r}}_{(Attractive\,Uninformed)} + \underbrace{\frac{\alpha\left(v^R\right)^2}{2\pi r}}_{(Attractive\,Informed)} = \frac{\left(v^R\right)^2 - \alpha\left(\pi r - v^R\right)^2}{2\pi r} \tag{36}$$

Relatedly, note that this surplus is equal to zero when $\alpha = \overline{\alpha}$, which is expected given the results of Theorem 1. Up to $\overline{\alpha}$, the maximum matching distance is given by ω' according to

$$\begin{aligned} \tfrac{\left(v^R\right)^2 - \alpha\left(\pi r - v^R\right)^2}{2\pi r} - \tfrac{(1-\alpha)\left(\omega' - v^R\right)^2}{2\pi r} &= 0 \\ \Leftrightarrow \qquad \omega' &= v^R + \sqrt{\frac{\left(v^R\right)^2 - \alpha\left(\pi r - v^R\right)^2}{1-\alpha}}. \end{aligned}$$

Finally, the upper bound on Δ is continuous at point $\alpha = \alpha_0$, and moreover $\overline{\alpha}^* > \alpha_0$.

- We now consider region

$$\{\Delta = 0 \wedge \alpha \geq \alpha_0\}$$

There exist beliefs under which the receiver is unwilling to match with the sender, independently of the message she observes. Consider first the case $\alpha = 0$. In this case no informed senders exist, and aside from babbling equilibria, the matching region is equal to $\left\{m : d(\theta, m) \leq v^R\right\}$.

We now describe that there exist beliefs such that no persuasive messages exist when $\alpha \geq \alpha_0$. When α is high enough, it suffices that the receiver believes that attractive informed senders pool with unattractive uninformed senders, and that unattractive informed senders pool with attractive uninformed senders. Unlike babbling equilibria, in this case communication is informative because messages do affect the receiver's beliefs. However, no matches ever take place. As a result, informed senders are willing to communicate according to the receiver's beliefs, and a fatalistic equilibrium results. When $\alpha < \alpha_0$, these beliefs do not exist. The reason is that there exist too few unattractive informed types to offset attractive uninformed types, so as to make all messages unattractive.[18]

Finally, under the sequential equilibrium refinement, off-equilibrium path beliefs are also required to fall within $\mathcal{S}_\Delta$. Consider an equilibrium that implements an information level α^*, and let Λ be a totally mixed distribution over α. Sequential equilibrium implies the sender should best-respond to all possible outcomes of Λ and the receiver should hold consistent beliefs, which is the exact characterization of $\mathcal{S}_\Delta$.

Proposition 4

The sender's first-best outcome is characterized by an information level α, a communication policy m^* and a maximum distance (induced by receiver's beliefs). Assuming, as before, that the sender's expected payoff is given by

$$EU^S = \alpha E\left(v^S - d(\theta, q)\right) + (1-\alpha)\, Pr(q \in C_\theta)\, E\left(v^S - d(\theta, q)\middle| q \in C_\theta\right) \tag{37}$$

where $C_\theta = \{m : d(\theta, m) \leq \Delta\}$, the sender's first-best outcome is attained by solving problem

$$\begin{aligned} \max_{\Delta, \alpha \in (0,1)} \quad & EU^S \\ s.t. \quad & EU^R \geq 0 \end{aligned}$$

[18] The proof is straightforward and is available from the authors.

where the constraint is equal to

$$EU^R = \alpha E\left(v^R - d(\theta, q)\right) + (1-\alpha) Pr(q \in C_\theta) E\left(v^R - d(\theta, q) \middle| q \in C_\theta\right) \geq 0 \quad (38)$$

Finally, note that $Pr(q \in C_\theta) E(v^j - d(\theta, q) | q \in C_\theta), \ j \in \{R, S\}$ yields

$$\frac{1}{2\pi} \int_{q-\frac{\Delta}{r}}^{q+\frac{\Delta}{r}} v^j - d(\theta, q)\, d\theta = \frac{(2v^j - \Delta)\Delta}{2\pi r} \quad (39)$$

Restriction $EU^R \geq 0$ must bind in equilibrium. If this were not the case, then the sender would be able to alter his mixing strategy in order to create slack for messages within a fixed distance Δ, and increase α as well as his utility as a result. Rewriting this condition yields

$$\begin{aligned} & E\left(U^R\right) && = 0 \\ \Leftrightarrow \quad & \alpha\left(v^R - \tfrac{\pi r}{2}\right) + (1-\alpha) \tfrac{(2v^R - \Delta)\Delta}{2\pi r} && = 0 \\ \Leftrightarrow \quad & \alpha && = \frac{\left(2v^R - \Delta\right)\Delta}{(\pi r - \Delta)(\pi r - 2v^R + \Delta)} \end{aligned} \quad (40)$$

We analyze the two relevant regions of $\mathcal{S}_\Delta$ in turn.

- $\alpha \leq \alpha_0$

In this region $\Delta \in \left[v^R, v^R\left(1 + \sqrt{\frac{\alpha}{1-\alpha}}\right)\right]$. Plugging (40) in the sender's utility function and taking a derivative w.r.t. Δ reveals that the sender's utility is increasing in Δ. Hence, constraint $\Delta \leq v^R\left(1 + \sqrt{\frac{\alpha}{1-\alpha}}\right)$ is binding. Equalities $\Delta = v^R\left(1 + \sqrt{\frac{\alpha}{1-\alpha}}\right)$ and (40) imply:

$$\begin{aligned} \alpha^* &= \alpha_0 \\ \Delta^* &= \frac{\pi r v^R}{\pi r - v^R} \end{aligned}$$

which falls in the correct region.

- $\alpha \geq \alpha_0$

In this region $\Delta \in \left[v^R, v^R + \sqrt{\frac{(v^R)^2 - \alpha(\pi r - v^R)^2}{1-\alpha}}\right]$. Along path $\Delta = v^R + \sqrt{\frac{(v^R)^2 - \alpha(\pi r - v^R)^2}{1-\alpha}}$, the receiver earns zero utility, and so the upper bound on Δ is equal to restriction $EU^R = 0$.

Moreover, there is a tradeoff between Δ and α:

$$\frac{d\Delta}{d\alpha}=-\frac{\pi r\left(\pi r-2v^R\right)}{(1-\alpha)^2}\frac{1}{2\sqrt{\frac{(v^R)^2-\alpha(\pi r-v^R)^2}{1-\alpha}}}<0 \tag{41}$$

Plugging the restriction $EU^R=0$, w.r.t. α, into the sender's utility function, yields

$$EU^S\Big|_{EU^R=0}=\frac{\Delta\left(v^S-v^R\right)}{\pi r-2v^R+\Delta}$$

which increases in Δ. Hence, the sender's utility is decreasing in the level of information along $EU^R=0$, and $\alpha=\alpha_0$. The sender's communication policy can be found relatively easily, through the same methods used in Theorem 2.

Proposition 5

We focus on the characterization of joint welfare. The remaining proofs are straightforward, and available from the authors.

When $v^S>\pi r$, joint welfare is given by

$$EU^R+EU^S=\tfrac{1}{\pi r}\left(v^Rv^S+\alpha\left(v^S-\pi r\right)\left(\pi r-v^R\right)\right)$$

which is increasing in α.

When $v^S\in\left(\frac{v^R}{2},\pi r\right)$, joint welfare is given by

$$EU^R+EU^S=\tfrac{v^Rv^S}{\pi r}$$

which is independent of the level of information α, although the receiver's rationality constraint requires $\alpha\leq\overline{\alpha}$.

Finally, when $v^S<\frac{v^R}{2}$, uninformed senders do not communicate. Hence, matches only take place if the sender is nearby the receiver, such that joint welfare increases in α:

$$EU^R+EU^S=\alpha\frac{v^Rv^S}{\pi r}$$