A Model of Safe Asset Determination

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~75min
US Treasury bonds have been the world safe asset for a long time

- Safe asset portfolios tilted towards US Treasury bonds
- “Convenience yield” on US Treasury bonds
- Higher premium in bad states ("negative $\beta$") & flight to quality
  - Persists despite a high US debt/GDP ratio
Motivation

- US Treasury bonds have been the world safe asset for a long time
  - Safe asset portfolios tilted towards US Treasury bonds
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- Main idea: Safety is *endogenous*—when investors believe an asset is safe, their actions can make that asset safe
Preview

Model:

- Lack of common knowledge to model coordination game
- Key trade-off between
  - *default/rollover risk* (strategic complementarity): more investors buying bond makes country safer, so repayment more likely
  - *market depth/liquidity* (strategic substitutability): for fixed issuance size, more investors buying lowers returns conditional on repayment
- Main results: for safe asset,
  - size (float) matters and can lead to an advantage, and
  - relative rather than absolute fundamentals matter

Some Applications:

1. Negative \( \beta \) of safe asset
2. Euro-bonds:
   - German bund currently safe asset within the Euro area
   - Many proposals to create a Euro-bond to serve as a Euro safe asset
   - Euro-bonds can help overcome coordination problems
3. Strategic behavior in debt issuance to influence safe-asset status
   - If size advantageous, possible rat-race to issue more and more bonds
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Literature

- International finance, economic history literature on reserve currency
  - Eichengreen (many), Krugman (1984), Frankel (1992)
  - Store of value, medium of exchange, unity of account, multiple equilibria

- Shortages of store of value
  - Multiplicity: Samuelson (1958) on money, Diamond (1965) on govt debt
  - No formal models of endogenous determination of which asset is chosen as store of value

- Sovereign debt rollover risk and global games
  - Highlight strategic substitution in asset market
    - Complement to Goldstein and Pauzner (2005) in bank runs
Model Setup

**Investors** $(j)$:
- Measure $1 + f$ of investors with one unit of funds each
- Risk neutral, each investor **must** invest his funds in sovereign debt

**Countries/debt** $(i)$:
- Two countries, debt size $s_1 = 1$ and size $s_2 = s < 1$
- Debt of face value of $s_i$ (exogenous) issued at endogenous price $p_i$
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**Default:**
- Default if surplus plus bond proceeds insufficient for obligations
  \[
  \underbrace{s_i \theta_i + s_i p_i} < \underbrace{s_i}_{\text{total funds available}} \overbrace{s_i}^{\text{debt obligations}}
  \]
  \(\Rightarrow\) Given price \(p_i\), default decision depends on \(\theta_i\)
- Fundamental ("surplus") \(s_i \theta_i\)
  - Foreign denominated debt: true surplus plus foreign reserves
  - Domestic denominated debt: true surplus plus any resources CB is willing to provide to forestall a rollover crisis
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- Recovery in default=0; Static model; Real model
Multiple Equilibria in a Special Case

- No default (i.e. repayment of 1 for each unit of bond) if,
  \[ s_i p_i \geq s_i (1 - \theta_i) \]

- Suppose sufficient funding for both countries:
  \[
  \frac{1 + f}{\text{total funds available}} \geq \frac{(1 - \theta_1) + s(1 - \theta_2)}{\text{funding needs}}
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Possible equilibria
1. Country 1 is safe (=safe asset), country 2 defaults:
\[ p_1 = 1 + f, \quad p_2 = 0 \]
Investor return = \[ \frac{1}{1 + f} \]
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3. Both countries safe, two safe assets with equalized returns
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- Suppose sufficient funding for both countries:
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  total funds available \hspace{1cm} funding needs

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4. If \( s = 0 \), country 1 is safe (Japan?)
Lack of Common Knowledge & Coordination Failure

Lack of common knowledge:

- Unobserved relative fundamentals (higher $\tilde{\delta} \Rightarrow$ country 1 stronger):
  
  \[
  1 - \theta_1 = (1 - \theta) \exp(-\tilde{\delta}) ; \\
  1 - \theta_2 = (1 - \theta) \exp(\tilde{\delta}) .
  \]

- Each investor receives a noisy private signal before investing
  
  \[
  \delta_j = \tilde{\delta} + \epsilon_j
  \]

- Take $\tilde{\delta} \in [-\bar{\delta}, \bar{\delta}]$ (any cdf, but wide support) & $\epsilon_j \sim \mathbb{U}[-\sigma, \sigma]

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Timing assumptions:

- Investors place market orders to buy debt
- Country default decision after orders are submitted
Returns for $\tilde{\delta} = 0$ (Default or Rollover Risk)

Given proportion $x$ investing in country 1, no default if:

$$x \geq \frac{1 - \theta_1 (\tilde{\delta})}{1 + f} \quad \text{(country 1)}$$

$$1 - x \geq \frac{s \left(1 - \theta_2 (\tilde{\delta})\right)}{1 + f} \quad \text{(country 2)}$$
Returns for $\tilde{\delta} = 0$ (Liquidity)

- Liquidity/market depth: country 2 price-rises/return-falls faster
- Given proportion $x$ investing in country 1, conditional returns are

$$\frac{1}{p_1} = \frac{1}{(1 + f) x} \quad \text{and} \quad \frac{1}{p_2} = \frac{s}{(1 + f)(1 - x)}$$
Threshold Equilibrium:

- Let $\phi(\delta_j)$ be investment in country 1 of agent with signal $\delta_j$.
- A threshold strategy is given by:
  
  If $\delta_j > \delta^*$ invest in country 1 i.e. $\phi=1$; otherwise country 2 i.e. $\phi=0$.

- The equilibrium cutoff $\delta^*$ is determined by indifference of marginal investor with signal $\delta_j = \delta^*$. 

How restrictive are threshold strategies?

- We can prove that the threshold equilibrium is the unique equilibrium among monotone strategies (i.e., $\phi'(\cdot) \geq 0$).
- For non-monotone strategies, other equilibria might exist (we will come back to this).
Strategy Space

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Indifference & Expected Returns

Marginal Investor Indifference:

- Marginal investor $\delta_j = \delta^*$ does not know other investors’ signals
  - He asks, suppose fraction $x \in [0, 1]$ of investors have signals $> \delta_j$
  - Marginal agent backs out true $\tilde{\delta}$ for given $x$ as follows
    \[
    \tilde{\delta} = \delta^* + (2x - 1)\sigma
    \]

- When $\sigma \to 0$ only strategic uncertainty (uncertainty about relative position $x$) remains
- Global games result: $x \sim \mathbb{U}[0, 1]$ from the view of marginal investor, for any prior of $\tilde{\delta}$
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Expected returns
- Integrating over possible values of $x \sim \mathcal{U} [0, 1]$ gives expected profits

$$\Pi_1 = \int_0^1 \frac{1}{(1-\theta)e^{-\delta^*}} \frac{1}{1+f} dx = \frac{1}{1+f} \left( \ln \frac{1+f}{1-\theta} + \delta^* \right)$$

$$\Pi_2 = \int_0^1 \frac{s}{1+f} \frac{1}{1+f-\theta e^{\delta^*}} dx = \frac{s}{1+f} \left( -\ln s + \ln \frac{1+f}{1-\theta} - \delta^* \right)$$
Expected Returns

For any agent, expected returns linked to graph:

- $\Pi_1 = \text{Integral under green curve}$
- $\Pi_2 = \text{Integral under red curve}$
Equilibrium Threshold

- Threshold $\delta^*$ characterized by equalized expected returns

$$\Pi_1 (\delta^*) = \Pi_2 (\delta^*)$$

- Solving for $\delta^*$ (recall that $s \in (0, 1]$)

$$\delta^* = -\frac{1-s}{1+s} \cdot z + \frac{-s \ln s}{1+s}$$

negative, liquidity \hspace{1cm} positive, rollover

where we define aggregate funding conditions

$$z \equiv \ln \frac{1+f}{1-\theta} > 0$$
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- High $z$ means high savings (“savings glut”), good average fundamentals
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  \[ z \equiv \ln \frac{1 + f}{1 - \theta} > 0 \]

- High $z$ means high savings ("savings glut"), good average fundamentals

- The lower $\delta^*$, the ex-ante safer is country 1
High \( z \) World: Liquidity Effect dominates

- Country 1 is safe asset if fundamental \( \tilde{\delta} > \delta^* \)
- \( \delta^* < 0 \) implies that the large country enjoys premium
Low $z$ World: Rollover Risk Effect dominates

Country 1 is safe asset if fundamental $\tilde{\delta} > \delta^*$

$\delta^* < 0$ implies that the large country enjoys premium
When will world switch?

Currently: In high $z$ world (savings glut)

- US Treasury size: Debt = $12.7tn, (CB money $\approx$ $4.6tn) : maximum liquidity for the world
- Even if US fiscal position is worse than others (i.e. $\delta^* < 0$)
- ... Switch not on the horizon

Unless macro moves to low $z$ world

- US Treasury size becomes a concern – can the country rollover such a large debt?
- Investors may start coordinating on countries with (a bit) smaller debt size
- Germany? Debt = $1.5tn
A Historical Perspective: Size helps

- UK government debt was safe asset until sometime after WWI
  - US GDP exceeds UK GDP by 1870
  - In 1890, UK Govt Debt $\approx 3 \times$ US Govt Debt
  - UK Debt/UK GDP $= 0.43$ vs. US Debt/US GDP $= 0.10$ (so safer)
Relative Fundamentals vs Absolute Fundamentals

Relative fundamentals/GE in safe assets is central to our model

- Take model with no coordination, where repayment is equal to surplus ($\theta$) and world interest rate is normalized to zero.

$$p_1 = \mathbb{E} \left[ \min (\theta_1, 1) \right], \quad p_2 = \mathbb{E} \left[ \min (\theta_2, 1) \right]$$

- Our model without safe alternative (e.g. for threshold $\delta^* = 0$)

$$\theta_1 > \theta_2 \implies p_1 = 1 + f, \quad p_2 = 0$$
$$\theta_1 < \theta_2 \implies s \cdot p_2 = 1 + f, \quad p_1 = 0$$

- US fiscal position is weaker now than before, but still better than everyone else

- Same for Germany within Eurozone
Introducing recovery

- Take an extreme case where country 1 is a.s. safe asset, $\tilde{\delta} \gg \delta^*$
- Say $s = 1$ & suppose recovery in default is $l_i$
- Country 1 bond price and return ($R$)

$$p_1 = 1 + f - p_2 \quad R = \frac{1}{p_1} = \frac{1}{1 + f - p_2}$$

- Country 2 bond price $p_2 = \frac{l_2}{R}$ (has to offer same return)
- Solving:

$$p_1 = \frac{1 + f}{1 + l_2} \quad \text{and} \quad p_2 = \frac{1 + f}{1 + l_2} l_2$$
Negative $\beta$

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Shocks to recovery values:
- Shock to $l_1 \downarrow$ has no effect on anything, but $l_2 \downarrow \Rightarrow p_1 \uparrow$ and $p_2 \downarrow$
- Say shocks to average fundamentals hurt $l_1, l_2$ equally:
  - Reduces $p_2$, increases $p_1 \Rightarrow$ Safe asset has negative $\beta$
Negative $\beta$

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- Say shocks to average fundamentals hurt $l_1, l_2$ equally:
  - Reduces $p_2$, increases $p_1$ ⇒ Safe asset has negative $\beta$
- Lehman shock: Negative shock to US and world fundamentals
  - Treasury yields fall (alternatives rise)
Negative $\beta$: Uncertainty about $\theta$

$\theta \sim U[0.1, 0.6], s=0.9, f=0.1, l=0.7$

Country 1 $\beta_1 = \frac{\text{Cov}(p_1, \theta)}{\text{Var}(\theta)}$, as function of relative fundamental $\delta$. 

Really safe assets OR What about Switzerland?

“Full-commitment” assets:

- What if there were “full-commitment” safe assets available?
  - Switzerland: Debt = $127bn, (CB money ≈ $500bn)
  - Denmark: Debt = $155bn
- US: Debt = $12.7tn, (CB money ≈ $4.6tn)
- **Implicit assumption** in our analysis is that substantially all of safe asset demand is satisfied by debt subject to rollover risk
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Tweaking the model:

- Say $s$ is quantity of “full-commitment” assets and define
  $$\hat{f} \equiv f - p_s s$$

- Suppose $s = 1$, so return of “winner” is
  $$\frac{1}{1 + \hat{f}}$$

- Price $p_s$ set using the expected return from investing in country 1,2

- Thus, same analysis as before with adjusted total demand of $\hat{f}$
Mixed strategies & Joint Safety

- Monotone strategies: Only one safe asset with threshold equilibrium
  - \( \phi(\delta_j) = 1 \) if \( \delta_j > \delta^* \), otherwise 0 (recall \( \phi \) investment in country 1)
Mixed strategies & Joint Safety

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  \( \Rightarrow \) Approximate mixed strategies via “oscillating” strategy:
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  - \( \Rightarrow \) Approximate mixed strategies via “oscillating” strategy:
    - \( \varphi(\delta_j): 0,1,0,1,0,1,... \) in a non-monotone fashion (approximates “mixing” strategies when \( \sigma \to 0 \))
    - Then, for high \( z > z_{HL} \), joint safety for values of \( \tilde{\delta} \) in GRAY
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![Graph](Image)
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Mixed strategies & Joint Safety: Discussion

Interpretation:

- In limit, “oscillating” strategy equivalent to everyone on $[\delta_L, \delta_H]$ investing $\phi = \frac{1}{1+s}$ in country 1
- Mixed strategy a quite natural outcome
  - Everything “normal” (both countries survive) for $\tilde{\delta} \in [\delta_L, \delta_H]$
  - Only in extreme cases $\tilde{\delta} \notin [\delta_L, \delta_H]$ are we in a “winner takes all” world
- Mixed strategy needs joint survival possibility (high enough $z$!) to materialize

Extensions

- Mixed strategy used for positive recovery result above
- Can also incorporate positive prices from inelastic local demand
Sovereign Choices

- Security design as coordination

- Debt size ($s$), fundamentals ($\theta$), are choice variables
  - Externalities in model
  - Role for coordination
Common Bond as Coordination Device

Policy proposals to create a Euro-area safe asset

- Proceeds to all countries, so all countries get some seignorage
- Flight to quality is a flight to all, rather than just German Bund

\[
\text{Countries issue two bonds:}
\]

- A common bond of total size \( \alpha (1 + s) \) made up of shares \( \frac{1}{1+s} \) large and \( \frac{s}{1+s} \) small bonds

- Each country issues individual bond of size \( (1-\alpha)s_i \)

\[
\Rightarrow \text{Total amount of face-value offered still } (1+s)
\]

- Common bond is pooled bond (essentially a size-based “bundle”), for which each country is responsible for paying only its respective share

- No cross-default provisions (structure is closest to Euro-Safe-Bonds, or ESBies)

Moral hazard:

- We set aside moral hazard considerations which are likely first-order
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Why might this work?

▶ In basic model ($\alpha = 0$) no default if,

$$s_ip_i \geq s_i(1 - \theta_i)$$

▶ Suppose global funds exceeds funding need:

$$\underbrace{1 + f}_{\text{total funds available}} \geq \underbrace{(1 - \theta_1) + s(1 - \theta_2)}_{\text{sum of individual funding needs}}$$

▶ When $\alpha = 1$, joint survival / no country defaults if

$$\underbrace{1 + f}_{\text{total funds available}} \geq \underbrace{(1 - \theta_1) + s(1 - \theta_2)}_{\text{funding need of common bond}}$$
Why might this work?

- In basic model ($\alpha = 0$) no default if,

$$s_i p_i \geq s_i (1 - \theta_i)$$

- Suppose global funds exceeds funding need:

$$1 + f \geq (1 - \theta_1) + s (1 - \theta_2)$$

- When $\alpha = 1$, joint survival / no country defaults if

$$1 + f \geq (1 - \theta_1) + s (1 - \theta_2)$$

- Security design coordinates investor actions
  - Flight to the safe asset generates stable funding for both countries
  - Note: investors cannot coordinate privately (e.g., diversified mutual fund)
Common Bond Equilibrium

2-Stage game:
1. Investors buy common bonds on offer with funds $f - \hat{f} > 0$
2. Investors receive private signal and use remaining funds $1 + \hat{f}$ to buy individual bonds
Common Bond Equilibrium

2-Stage game:
1. Investors buy common bonds on offer with funds $f - \hat{f} > 0$
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Equilibrium by working backwards:
- Stage 2: investors with $\hat{f}$
  - Game almost same as before, except country fiscal surplus now includes common bonds proceeds from Stage 1
- Stage 1: Investors equalize expected returns between common bond and their informed investment in Stage 2

$$\mathbb{E}[R_c] = \mathbb{E}[R_{stage2}]$$
Common Bond Equilibria as Function of $\alpha$

$s=0.5, z=1.$

- High $\alpha > $ $\alpha^*$ $\Rightarrow$ joint safety equilibrium always
- Low $\alpha < $ $\alpha_{HL}$ $\Rightarrow$ single safe asset, threshold equilibrium
- For $\alpha \in [\alpha_{HL}, \alpha^*]$ both equilibria are possible
Sovereign Choices

- Security design as coordination

- **Debt size** ($s$), **fundamentals** ($\theta$), are choice variables
  - Externalities in model
  - Role for coordination
Debt Size & Crowding Out

- Suppose country $i$ can choose (float) size, $S_i$
  - Surplus is adjusted to $\theta_i S_i$ - i.e. keep tax revenues to debt constant
Debt Size & Crowding Out

- Suppose country $i$ can choose (float) size, $S_i$
  - Surplus is adjusted to $\theta_i S_i$ - i.e. keep tax revenues to debt constant

- Given $S_i$, easy to solve for threshold equilibrium:
  \[
  \delta^* = \frac{S_2 - S_1}{S_1 + S_2} z + \frac{S_1 \ln S_1 - S_2 \ln S_2}{S_1 + S_2}
  \]

- Effect of increasing size $S_1$ on threshold $\delta^*$:
  \[
  h(S_1, S_2; z) \equiv \frac{\partial \delta^* (S_1, S_2)}{\partial S_1} = \frac{\left[ S_1 + S_2 (\ln S_1 + \ln S_2 + 1 - 2z) \right]}{(S_1 + S_2)^2}
  \]

  - Decreasing in $z$, negative for large $z$

- Expanding US debt can increase US safe asset status
  - Decreases other country’s position
Endogenous Size Choices:

- Suppose country 1, 2 have “natural” debt size \((s_1^*, s_2^*)\)

- Country \(i\) chooses new size \(S_i\) subject to adjustment costs

- **Objective:** reduce default probability subject to adj costs:
  - Country 1:
    \[
    \max_{S_1} -\delta^* (S_1, S_2) - c(S_1 - s_1^*)
    \]
  - Country 2:
    \[
    \max_{S_2} \delta^* (S_1, S_2) - c(S_1 - s_1^*).
    \]

- **Equilibrium:**
  \[
  h(S_1, S_2; z) = c'(S_1 - s_1^*) \quad \text{and,} \quad h(S_2, S_1; z) = c'(S_2 - s_2^*). \]
Equilibrium via Phase Diagram

- High $z$ case; $\delta^* = 0$ along diagonal

\[ -\frac{\partial \delta^*}{\partial S_2} = h(S_2, S_1) = 0 \]

\[ \frac{\partial \delta^*}{\partial S_1} = h(S_1, S_2) = 0 \]

$A$: Rat Race

$B'$

$B$: Top Dog

$45^\circ$
Equilibrium via Phase Diagram

- $\delta^* = 0$ along diagonal

$h(S_1, S_2) = 0\quad h(S_2, S_1) = 0$
Conclusion

- With a shortage of truly safe assets (Switzerland), safety is endogenous

- US govt debt is safe asset because
  - Good relative fundamentals
  - Debt size is large, world in high demand for safe asset (savings glut)
    - Nowhere else to go

- Economics of safe asset suggest that there can be gains from coordination
  - Eurobonds as coordinated security-design

- Theoretical contribution: non-monotone (oscillating) equilibrium in global games setting

- Future work: one large country, n small countries
More details on the oscillating equilibrium

The non-monotone equilibrium:

- Let $\phi(y)$ be investment in country 1 of agent with signal $y$
- Consider $\phi(y)$ oscillating on $(\delta_L, \delta_H)$

$$
\phi(y) = \begin{cases} 
0, & y < \delta_L \\
1, & y \in [\delta_L, \delta_L + k\sigma] \\
0 & y \in [\delta_L + k\sigma, \delta_L + 2\sigma] \\
1, & y \in [\delta_L + 2\sigma, \delta_L + (2+k)\sigma] \\
\vdots & \vdots \\
0, & y \in [\delta_H - (2-k)\sigma, \delta_H] \\
1, & y > \delta_H
\end{cases}
$$

- Suppose joint safety for $\delta \in (\delta_L, \delta_H)$. Then a no-arbitrage condition between bond prices has to hold $\Rightarrow k = \frac{2}{1+s}$
An oscillating strategy

Example: $z = 1, s = 1/4$ ⇒ Thresholds: $\delta_L = -0.37, \delta_H = -0.12$

- Full oscillations on $(\delta_L, \delta_H)$: $n = 2$
- **Joint safety**: no-arbitrage condition requires investment splits of $\frac{1}{1+s} = 80\%$ to country 1 and $\frac{s}{1+s} = 20\%$ to country 2 on $(\delta_L, \delta_H)$
An oscillating strategy

Example: $z = 1, s = 1/4 \Rightarrow$ Thresholds: $\delta_L = -0.37$, $\delta_H = -0.12$

Full oscillations on $(\delta_L, \delta_H)$: $n = 2$

**Joint safety**: no-arbitrage condition requires investment splits of $\frac{1}{1+s} = 80\%$ to country $1$ and $\frac{s}{1+s} = 20\%$ to country $2$ on $(\delta_L, \delta_H)$

Let us look at the incentives of different investors...
An oscillating strategy

Consider marginal agent $\delta_j = \delta_L$ who thinks he has the most positive signal, i.e. $x = 0$

Investment in country 1 conditional on $x$, $\int_{\delta-x}^{\delta+x} 2\sigma \frac{\phi(y)}{2\sigma} dy$, increasing in $x$
An oscillating strategy

Consider marginal agent $\delta_j = \delta_L$ who thinks he has the median signal, i.e. $x = \frac{1}{2}$

Investment in country 1 conditional on $x$, $\int_{\delta - (1-x)\sigma}^{\delta + x\sigma} \frac{\phi(y)}{2\sigma} dy$, increasing in $x$
An oscillating strategy

Consider marginal agent $\delta_j = \delta_L$ who thinks he has the most negative signal, i.e. $x = 1$

Investment in country 1 conditional on $x$, $\int_{\delta-(1-x)2\sigma}^{\delta+x2\sigma} \frac{\phi(y)}{2\sigma} dy$, increasing in $x$
An oscillating strategy

Consider interior agent $\delta_j = \delta_L + k\sigma$ who thinks he has the most positive signal, i.e. $x = 0$

Investment in country 1 conditional on $x$: $\int_{\delta-(1-x)2\sigma}^{\delta+x2\sigma} \frac{\phi(y)}{2\sigma} dy = \frac{1}{1+s}$
An oscillating strategy

Consider interior agent $\delta_j = \delta_L + k\sigma$ who thinks he has the median signal, i.e. $x = 1/2$

Investment in country 1 conditional on $x$: 

$$\int_{\delta-(1-x)2\sigma}^{\delta+x2\sigma} \frac{\phi(y)}{2\sigma} dy = \frac{1}{1+s}$$
An oscillating strategy

Consider interior agent $\delta_j = \delta_L + k\sigma$ who thinks he has the most negative signal, i.e. $x = 1$

Investment in country 1 conditional on $x$: $\int_{\delta-(1-x)2\sigma}^{\delta+x2\sigma} \frac{\phi(y)}{2\sigma} dy = \frac{1}{1+s}$
An oscillating strategy

Consider interior agent $\delta_j = \delta_L + k\sigma$ who thinks he has the most negative signal, i.e. $x = 1$.

- Investment in country 1 conditional on $x$:  
  $$\int_{\delta - (1-x)2\sigma}^{\delta + x2\sigma} \frac{\phi(y)}{2\sigma} dy = \frac{1}{1+s}$$

- Interior agent’s expectation of investment in country 1 independent of relative position $x$. 
An oscillating strategy: Incentives

$\Pi_1(\delta) - \Pi_2(\delta)$

$\phi(\delta)$

Indifference only on $(\delta_L + k\sigma, \delta_H - (2 - k)\sigma)$
An oscillating strategy: Equilibrium

- Indifference of marginal agent $\delta_L$ requires $\Pi_2(\delta_L) = \Pi_1(\delta_L)$:
  - Country 2 safe for sure, so always certain return
  - Country 1 not always safe in the eyes of marginal agent (strategic uncertainty), so require higher expected return conditional on survival

- Solving, we have

$$
\delta_L = -z - \ln \left\{ \frac{s^s}{(1 + s)^{1+s}} \right\}
$$
$$
\delta_H = z + \ln \left\{ \frac{1}{(1 + s)^{\frac{1+s}{s}}} \right\}
$$

- Note that both $\delta_H$ and $\delta_L$ are independent of $\sigma$ (need specific $\sigma$’s to make sure that there exists $n$ s.t. $\delta_H = n \cdot 2\sigma + k$)
  - As we are taking $\sigma \rightarrow 0$, this is wlog

- Define unique $z_{HL}$ so that $\delta_L(z_{HL}) = \delta_H(z_{HL})$. Then oscillating equilibrium possible for $z > z_{HL}$. 