Timely Loan Loss Provisioning: A Double-Edged Sword?*

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Abstract

We investigate the impact of loan loss provisioning on the prudential regulation of banks. We study two provisioning models: an incurred loss model and an expected loss model. Relative to the incurred loss model, an expected loss model improves efficiency as it allows the bank regulator to intervene in the bank’s operations in a timely manner to curb inefficient ex-post asset-substitution. However, taking real effects into account, our analysis uncovers a potential cost of the expected loss model. We show that when the bank is highly leveraged, it responds to timely regulatory intervention under the expected loss model by originating riskier loans so that timely intervention induces timelier risk-taking. By appropriately tailoring the bank’s capital requirements to the expected loss model, the regulator may improve the efficiency of the expected loss model.

Keywords: Capital Requirements; Loan Loss Provisioning; Real Effects, Current Expected Credit Loss Model; Incurred Loss Model.

JEL codes: G21, G28, M41, M48.

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1 Introduction

The recent adoption of the Current Expected Credit Loss model (CECL) for the recognition and measurement of credit losses for loans and debt securities is arguably one of the most consequential accounting changes to impact banks and financial institutions (FASB, 2019). Under the new standard, banks are required to replace the “incurred loss” model with an “expected loss” model. A key difference between the two accounting models is that under the incurred loss model, banks “delay recognition of credit losses until they have been incurred,” whereas under the expected loss model, banks must recognize “the full amount of credit losses that are expected.” (FASB, 2016) While the Financial Accounting Standards Board (FASB) argue that the new standard results in “more timely and relevant information,” others have argued that it “could actually produce negative economic consequences.” (Quaadman, 2019) Most notably, banks are concerned that the forecasts of future credit losses are often unreliable and false loss recognition may lower bank capital ratios and reduce lending to the economy, thus making credit supplies procyclical and worsening economic downturns. To shed some light on this important debate, we develop an economic model to study the trade-offs in moving from an “incurred loss” model to an “expected loss” model.

In our model, a representative bank is subject to shareholder-debtholder conflicts. The bank’s shareholders have incentives to take excessive risk by either: 1) increasing the ex ante risk of the bank’s loan portfolio via exerting less effort to screen borrowers, and/or 2) engaging in ex post asset substitution to replace low-risk loans with high-risk ones. To discipline excessive risk-taking, a regulator imposes minimum capital requirements. Importantly, the level of capital depends on the information on loan losses reported by the bank’s accounting system. Under an incurred loss model, the bank does not report a loan loss until it is realized, whereas under the expected loss model, the bank must additionally provide an early but imprecise report of the expected loan loss. Recognizing a large loan loss erodes the bank’s capital taking the bank close to, or even below,
its regulatory capital requirement, thus triggering regulatory intervention. The regulator can then choose whether to liquidate or restructure the bank’s loan portfolio before it pays off.

We first study a setting in which the bank’s \textit{ex ante} choice of loan risk is kept fixed. We show that timely recognition of loan losses under the expected loss model is always beneficial. Such a benefit arises because the regulator may utilize the timely information to curb the bank’s \textit{ex post} asset-substitution. Whenever the bank recognizes a large loan loss in the interim, the regulator rationally anticipates that the bank has a strong incentive to asset-substitute and thus liquidates the bank early. More importantly, we show that the benefit of timely loss recognition is strictly positive, despite the possibility of false alarm costs caused by the imprecise information inherent in an expected loss model. The reason is that a rational and benevolent regulator fully internalizes the false-alarm costs and chooses an action that results in the highest surplus in expectation. In other words, our analysis shows that the usual false alarm arguments are not sufficient to overturn the benefits of timeliness offered by expected loss models.

However, once we consider the real effects of timely loss recognition on banks’ risk choices, our model shows that early intervention is a double-edged sword. While timely intervention always curbs \textit{ex post} asset substitution, it may induce the bank to originate safer or riskier loans. In particular, when the bank’s leverage is high, it invests in riskier loans under the expected loss model. This results in a surplus loss that outweighs the benefit of timely loss recognition, thereby making the expected loss model inferior to the incurred loss model.

To understand the risk-disciplining effect of early intervention, note that under the incurred loss model, given the bank’s anticipation of future asset substitution, its incentives to screen borrowers \textit{ex ante} decrease. In contrast, under the expected loss model, since the bank is restrained from switching to riskier loans later, its incentives to originate safer loans increase.

The risk-aggravating effect of early intervention is more subtle. Note that under the incurred
loss model, the bank prefers to defer taking excessive amounts of risk until it receives more precise information on the performance of its loan portfolio. However, under the expected loss model, the option value of waiting is constrained by regulatory intervention. Anticipating this, the bank shifts the timing of its risk-taking earlier by originating riskier loans under the expected loss model.

In equilibrium, the bank trades off these two real effects in determining how much risk to take \textit{ex ante}. We show that, when the bank is highly leveraged, the risk-aggravating effect of early intervention dominates its risk-disciplining effect, inducing the bank to originate riskier loans. Intuitively, a highly-leveraged bank has a strong incentive to asset-substitute. When that possibility is constrained by early intervention, the bank responds by building up more risk in its loan portfolio \textit{ex ante} so that timely intervention triggers even timelier risk-taking by the bank. We find that, when we take into account the real effects of accounting measurements, the expected loss becomes inferior when banks are heavily leveraged.

An implication of our model is that the regulator should take into account such real effects when designing the capital requirement policy that, in turn, governs the bank’s leverage. Stated differently, changing the accounting method for loan loss provisioning requires the bank regulator to simultaneously adjust the capital adequacy ratios. We show that by appropriately tailoring the bank’s capital requirements to information about loan losses, the regulator can improve the efficiency of the expected loss model.

Our paper contributes to the banking and accounting literature on loan loss provisioning. Given the extensive size of this literature, we refer interested readers to two recent surveys by Beatty and Liao (2014) and Acharya and Ryan (2016). Most of the papers in this literature focus on empirically examining the effect of loan loss provisioning on banks’ behavior. For instance, Beatty and Liao (2011) find that delays in loan loss recognition make banks’ lending more procyclical. Bushman and Williams (2012) document that timely recognition of expected future loan losses is associated
with enhanced risk-taking disciplining. Similarly, Bushman and Williams (2015) find that delayed expected loan loss recognition (DELR) is positively associated with banks’ vulnerability to stock market liquidity risk, downside tail risk of individual banks and codependence of downside tail risk among banks. Akins, Dou and Ng (2019) report that timely loan loss recognition constrains corruption in bank lending and thus improves the quality of loans. To the best of our knowledge, our paper is the first paper that develops an analytical framework of the impact of loan loss provisioning on banks’ risk-taking behavior, financial stability and prudential regulation. In line with the prior empirical evidence, we find that timely recognition of loan losses helps to curb inefficient ex-post asset-substitution. However, our analysis also uncovers a potential cost of the timely loss recognition and shows that it may induce banks to originate riskier loans ex ante. An implication of our result is that empirical research examining the impact of loan loss provisioning should classify banks’ risk decisions by the time they are undertaken (e.g., screening effort at loan origination versus asset substitution after learning bad news).

The regulator sets capital requirements. The bank raises deposits \( D \), invests \( A \) in a loan portfolio and makes an initial risk decision \( q \).

The bank learns information \( s_1 \) and reports it according to the loan loss provisioning rule. The regulator chooses interventions upon a capital requirement violation. If not liquidated, the bank makes an asset-substituting decision \( r \).

The state of the world \( s \) is realized. If the bank has not been liquidated, the regulator chooses interventions upon a capital requirement violation.

The payoffs of the loans \( \pi \) are realized.

Figure 1: Timeline of the baseline model

Bertomeu, Mahieux, and Sapra (2018) study the joint determination of optimal capital requirement policy and accounting measurement rules in disciplining banks’ risk-taking and stimulating bank lending. Lu, Sapra and Subramanian (2019) study the optimal use of mark-to-market accounting in implementing capital requirements, when there are both asymmetric information and shareholder-debtholder agency conflicts. Our paper focuses on examining the impact of a specific yet critical accounting item, loan loss provisioning. Our analysis obtains new insights into the “timing” aspects of accounting measurements, bank regulation and banks’ behavior. We find that timely loan loss recognition not only facilitates timely regulatory intervention, but also induces timelier risk-taking by banks. Neither loan loss provisioning nor its timing aspect are examined in any of the aforementioned studies.

In Section 2, we describe our model. In Section 3, we analyze the model. Section 4 concludes. The Appendix contains the proofs of all our results.

2 The Model

We examine an environment with four dates that includes a representative bank and a bank regulator. Figure 1 summarizes the sequence of events.

At \( t = 0 \), the bank is endowed with an amount of exogenous equity \( E \) and the bank’s shareholders (hereafter the bank) chooses the size \( A \in [0, \bar{A}] \) and risk \( q \in [0, 1] \) of investments in a loan portfolio. We will sometimes refer to \( A \) as the bank size. \( \bar{A} \) denotes the maximum bank size and is chosen to
be sufficiently large. For a bank size $A > E$, the bank borrows $D = A - E$ from depositors. We assume that deposits are fully insured and we normalize the risk-free deposit rate to zero.

We model the return on the loan portfolio as follows. The outcome of each loan is binary: either the loan succeeds or it defaults. The bank chooses a costly effort $q$ to screen risky borrowers. Absent any screening effort ($q = 0$), the bank always generates a “high-risk loan” that returns $\beta$ with probability $\tau \in (0, 1)$ and 0 with probability $1 - \tau$. To improve the performance of the loan portfolio, the bank exerts effort $q$ at a cost of $C(q)$ where the cost function $C(.)$ satisfies the standard properties: $C(0) = 0$, $C(1) = \infty$, $C'(0) = 0$, $C''(1) = \infty$, and $C'' > 0$. Conditional on a choice of $q > 0$, the bank receives the high-risk loan with probability $1 - q$ and the low-risk loan with probability $q$. The low-risk loan returns $\alpha$ with probability $s$ and 0 with probability $1 - s$. The random variable $s$ represents the (inverse) default risk of the low-risk loan and has a distribution $H(.)$ and a density $h(.)$ with full support on $[\tau, 1]$ so that the low-risk loan always has a lower default risk than the high-risk loan. This assumption guarantees that the bank’s effort $q$ reduces the default risk of the loan portfolio by decreasing the likelihood of high-risk loans in its loan portfolio. To reflect the risk-return trade-off, we assume that $\beta > \alpha > 1$ such that the bank demands a higher interest rate on the high-risk loan.

In originating its loan portfolio, the bank is subject to a minimum capital requirement, $\gamma \in (0, 1)$, imposed by the regulator:

$$\frac{E}{A} \geq \gamma. \quad (1)$$

In the expression of the capital ratio in (1), the numerator is the accounting value of the bank’s equity and the denominator is the accounting value of the bank’s assets. The asset side of the bank’s balance sheet contains the loan portfolio and the liability side of the balance sheet contains the deposits. Under the “effective interest rate” method of the FASB rules, the initial values of both the loans and the deposits are recognized at the initial contractual principal, $A$ and $D$, respectively.
Therefore, the value of total assets is $A$ and the value of total liabilities is $D$. By the accounting identity, the equity value is $A - D = E$.\footnote{We consider the case that the regulator uses a simple leverage ratio to regulate capital. This is consistent with the newly proposed Basel III framework. In particular, Section V of Basel Committee (2010) provides a discussion of the use of leverage ratios in Basel III, that “the Committee agreed to introduce a simple, transparent, non-risk based leverage ratio that is calibrated to act as a credible supplementary measure to the risk based capital requirements.” Under the previous Basel II regulatory framework, bank regulators may use some risk-weighted measure of assets to regulate capital. By Basel II, assets are partitioned into different groups based on their risk and these different groups are assigned different weights. However, in the actual implementations of Basel II, the exact partitions of assets are very coarse. For example, the risk weight for most commercial loans and investment exposures held by banks is 1. If we assign a weight of 1 to the loan portfolio, the resulting risk-weighted capital ratio is identical to the simple leverage ratio we have in the model.}

At $t = 1$, the bank observes the type of the loan it generates. Conditional on obtaining a low-risk loan, the bank receives some early information $s_1$ regarding the default risk $s$. The random variable $\tilde{s}_1$ has a distribution $G(.)$ and a density $g(.)$ with full support on $[\tau, 1]$. We interpret $s_1$ as new information about a non-incurred loan value change that arrives at the intermediate date, and reflects the change in expectation of future loan losses. The arrival of such information affects the bank’s assessment of the default likelihood and future loan losses. Formally, we model the bank’s updated assessment of loan performance using the posterior distribution of $s$ given $s_1$, denoted by $F(s|s_1)$ with a density $f(s|s_1)$. While in our analysis we consider a general distribution for $s$ and $s_1$, we use the following linear-uniform structure for making numerical examples and plots:

$$\tilde{s} = w\tilde{s}_1 + (1 - w)\tilde{s}_2,$$

where $\tilde{s}_1$ and $\tilde{s}_2$ are i.i.d. random variables that are both uniformly distributed in the interval $[\tau, 1]$. The state $\tilde{s}$ is a weighted average of $\tilde{s}_1$ and $\tilde{s}_2$, with the weight on $\tilde{s}_1$, $w \in [0, 1]$. The first component of $\tilde{s}$, $\tilde{s}_1$, is realized and observed by the bank at $t = 1$ but its second component $\tilde{s}_2$ is not realized until $t = 2$. In other words, observing $s_1$ at $t = 1$ reveals some information about $s$. The weight on $\tilde{s}_1$, $w$, thus captures the precision of $s_1$ in forecasting $s$. At $w = 0$, observing $s_1$ reveals no information regarding $s$, whereas at $w = 1$, observing $s_1$ reveals $s$ perfectly.
An important goal of our paper is to determine whether the bank’s accounting system should report the early information in its financial statements, i.e., by taking a provision for the expected loan losses. We study two accounting systems. We refer to the accounting system in which the bank does not report $s_1$ and hence does not recognize loan losses early as an “incurred loss model (IL)” while the accounting system in which the bank reports $s_1$ and therefore recognizes loan losses early as “the expected loss model (EL).” Our model therefore provides a parsimonious and tractable way to capture a key difference between an incurred loss model and an expected loss model. The expected loss model requires a more timely recognition of non-incurred loan losses whereas the incurred loss model does not recognize loan losses until they are incurred.

The loan loss recognition model matters in our environment because the level of capital depends on the loan losses reported by the bank’s accounting system. Under the incurred loss model, since the bank delays the recognition of loan losses, it always satisfies the capital requirement at $t = 1$. In contrast, under the expected loss model, timely recognition of loan losses leads to a write-down of the bank’s loan, which, in turn, lowers its capital ratio. As it turns out, the bank in equilibrium always invests up to the capital requirement at $t = 0$ so that the recognition of any loan losses at $t = 1$ causes the bank to violate the capital requirement. At the end of the model setup, we provide a detailed account on how the recognition of loan losses at $t = 1$ and $t = 2$ affects the bank’s balance sheet and capital ratios. We assume that a capital requirement violation triggers regulatory intervention. In practice, a regulator has broad discretion in terms of what actions to take ranging from being passive, thereby allowing the bank to continue its operations to a reorganization, partial asset sale, a reduction in the scope of the bank or even liquidation. For simplicity, we focus on two possible actions: continuation or liquidation. Given the information $s_1$, the regulator optimally liquidates the bank’s loans if the total expected payoffs from liquidation exceed the total payoffs from continuation.
Note that we assume that the regulator can only commit to a minimum capital requirement rule such that the regulator may intervene in a bank only when the bank’s capital ratio falls below the minimum capital adequacy ratio; yet, the regulator cannot commit to the action taken after a capital requirement violation. This set of assumptions is consistent with the practice of prudential regulation (Dewatripont and Tirole, 1994). For instance, in the United States, the Federal Reserve only sets minimum capital requirements (e.g., to be well-capitalized, a bank holding company must have a Tier-1 capital ratio of at least 6%) but never firmly specifies any intervening action after a capital requirement violation. Such lack of commitment by bank regulators has been examined extensively in the literature (e.g., Bagehot, 1873; Mailath and Mester 1994; Freixas, 1999).

We assume that the regulator learns the type of the loan at the liquidation stage. Furthermore, we assume that loan liquidation is costly: if a low-risk loan is liquidated, the liquidation payoff is \( L_\alpha < \alpha \) whereas if a high-risk loan is liquidated, the liquidation payoff is \( L_\beta < \beta \). Furthermore, liquidating risky subprime loans entails losses in expectation and regulators often recover very little residual value. We therefore assume that:

**Assumption 1:** \( \tau \beta > L_\beta \).

In case of continuation at \( t = 1 \), based on its updated assessment of the loan performance \( F(s|s_1) \), the bank may engage in asset substitution \( r \in \{0, 1\} \). The variable \( r = 0 \) implies no asset substitution so that the bank does not change its original loan portfolio whereas the variable \( r = 1 \) implies that the bank changes its original loan portfolio by substituting the low-risk loans in the loan portfolio with high-risk loans. That is, conditional on \( r = 1 \), the bank always receives the high-risk loan.

At \( t = 2 \), conditional on continuation of the bank at \( t = 1 \), the default risk \( s \) is realized and observed by the bank. Since the default risk is realized, the bank reports \( s \), regardless of whether the bank follows the incurred loss model or the expected loss model. The recognition of loan
losses reduces the bank’s capital ratio below the minimum capital ratio, in which case the regulator decides, again, whether to liquidate the bank.

At $t = 3$, the terminal payoffs of the loans are realized. The payoff $\pi$ of the bank’s loan portfolio is as follows. If the loan is low-risk and is liquidated, $\pi = L_\alpha$, but if the loan is low-risk and is continued, $\pi = \alpha$ with probability $s$ and $\pi = 0$ with probability $1 - s$. If the loan is high-risk and is liquidated, $\pi = L_\beta$ whereas in case of continuation, $\pi = \beta$ with probability $\tau$ and $\pi = 0$ with probability $1 - \tau$. The regulator compensates depositors if the bank fails, i.e., $\pi < D$, with a lump sum payment which we assume is financed via a frictionless *ex ante* tax.

Finally, to create a demand for prudential regulation, we impose the following two assumptions throughout our entire analysis:

**Assumption 2:** A high-risk loan is value-destroying whereas a low-risk loan is value-creating. This assumption ensures that the regulator has an incentive to discipline the bank from investing in high-risk loans. It turns out that a sufficient condition for Assumption 2 is

$$L_\alpha \geq 1 > \tau \beta. \quad (3)$$

The first part of the inequality ensures that the low-risk loan generates at least zero net present value (NPV) even in case of liquidation whereas the second part ensures that the high-risk loan always generates negative NPV.

**Assumption 3:** The bank always prefers to invest in the high-risk loan if it lends the maximal extent, i.e., $A = \tilde{A}$.

This assumption rules out the degenerate case in which the bank can achieve the first-best by choosing the highest leverage and lending the maximal extent, thus making the capital requirement
constraint undesirable. A sufficient condition for this assumption is

\[ \tau (\beta - 1) > \left( \int_\tau^{L_\alpha} f (s|s_1) \, ds \right) (L_\alpha - 1) + \left( \int_{L_\alpha}^1 s f (s|s_1) \, ds \right) (\alpha - 1), \tag{4} \]

for any \( s_1 \in [\tau, 1] \).

A bank balance sheet interpretation  We now provide a detailed account on how the recognition of loan losses at \( t = 1 \) and \( t = 2 \) affects the bank’s balance sheet and capital ratio. Recall that at \( t = 0 \), under the effective interest rate method, the value of the bank’s assets \( A_0 = A \) and the value of equities \( E_0 = E \). As we will show in our later analysis, the capital requirement also binds at \( t = 0 \) in equilibrium:

\[ \frac{E_0}{A_0} = \frac{E}{A} = \gamma. \tag{5} \]

At \( t = 1 \), after observing its loan type, the bank considers taking a provision for loan losses. Consider first the case that the bank generates a high-risk loan. Since the default risk \( \tau \) is realized, the bank measures the value of the loan by discounting the expected future cash flows from the loan \( (A \beta \tau) \) to their present value using the loan’s effective interest rate \( (\beta - 1) \):

\[ A_1 = \frac{A \beta \tau}{1 + \beta - 1} = A \tau. \tag{6} \]

Comparing \( A_1 \) to the carrying value of the loan at \( t = 0 \) gives the amount of impairment on the loan, \( A_0 - A_1 = A (1 - \tau) \). After recognizing the impairment loss, the bank’s equity becomes \( E_1 = E - A (1 - \tau) \). The bank’s capital ratio changes to

\[ \frac{E_1}{A_1} = \frac{E - A (1 - \tau)}{A - A (1 - \tau)} < \frac{E}{A} = \gamma. \tag{7} \]
Upon the capital requirement violation, the regulator intervenes and decides which action to take. As we will show later, since the liquidation payoff of a high-risk loan is smaller than the continuation payoff, the regulator never liquidates the high-risk loan. Recognizing the loan losses for the high-risk loan thus does not affect any of our analysis.

Consider now the case that the bank generates a low-risk loan. Under the incurred loss model, since the bank delays the recognition of loan losses, its asset and equity value stay unchanged, i.e., $A_1 = A$ and $E_1 = E$. The bank thus always satisfies the capital requirement,

$$\frac{E_1}{A_1} = \frac{E}{A} = \gamma.$$  

(8)

As a result, the regulator cannot intervene at $t = 1$ under the incurred loss model.

Under the expected loss model, based on the information $s_1$, the bank measures the value of the loan by discounting the expected future cash flows from the loan $(A\alpha E[s|s_1])$ using the loan’s effective interest rate $(\alpha - 1)$:

$$A_1 = \frac{A\alpha E[s|s_1]}{1 + \alpha - 1} = AE[s|s_1].$$  

(9)

Comparing $A_1$ to the carrying value of the loan at $t = 0$ gives the amount of impairment on the loan, $A_0 - A_1 = A(1 - E[s|s_1])$. The impairment loss in turn reduces the bank’s equity to $E_1 = E - A(1 - E[s|s_1])$. The bank’s capital ratio changes to

$$\frac{E_1}{A_1} = \frac{E - A(1 - E[s|s_1])}{A - A(1 - E[s|s_1])} < \frac{E}{A} = \gamma.$$  

(10)

Note that under the expected loss model, reporting $s_1$ at $t = 1$ triggers a capital requirement violation and regulatory intervention, as described in the model setup.

Finally, at $t = 2$, if the bank generates a high-risk loan, the recognition of loan losses is the
same as that in \( t = 1 \). The bank violates the capital requirement but the regulator chooses not to liquidate the bank. If the bank generates a low-risk loan, the bank recognizes an impairment loss of \( A(1 - s) \) since the default risk \( s \) is realized. This in turn reduces both the loan and the bank’s equity by \( A(1 - s) \), i.e., \( A_2 = A - A(1 - s) \) and \( E_2 = E - A(1 - s) \). The bank’s capital ratio changes to

\[
\frac{E_2}{A_2} = \frac{E - A(1 - s)}{A - A(1 - s)} < \frac{E}{A} = \gamma. \tag{11}
\]

That is, under both the incurred and the expected models, reporting \( s \) at \( t = 2 \) triggers a capital requirement violation and regulatory intervention, as described in the model setup.

3 Analysis

3.1 Exogenous loan risk

As a benchmark, we first solve a variant of our model treating the bank’s \textit{ex ante} risk \( q \) as exogenous.

3.1.1 Incurred loss model with exogenous risk

We analyze the incurred loss model in which the bank does not report \( s_1 \) so that the regulator can only intervene at \( t = 2 \) after the default risk \( s \) is realized. We solve the model using backward induction. At \( t = 2 \), given default risk \( s \), the regulator decides whether to liquidate the bank by comparing the total expected payoff from liquidation with the total payoffs from continuation. If the bank’s loan turns out to be high-risk, it generates \( L_\beta \) upon liquidation and \( \tau \beta > L_\beta \) upon continuation. Therefore, the regulator never liquidates the high-risk loan. On the other hand, if the loan turns out to be low-risk, it generates \( L_\alpha \) upon liquidation and \( s \alpha \) upon continuation.
Therefore, the regulator liquidates the low-risk loan if and only if

\[ s < \frac{L_\alpha}{\alpha}. \tag{12} \]

Next, we solve for the bank’s asset-substituting decision \( r \) at \( t = 1 \) conditional on the early information \( s_1 \). Denote the bank’s shareholders’ (hereafter the bank’s) expected payoff by \( U(r|s_1) \). If the bank has already received a high-risk loan, the asset-substitution decision is moot. If the bank receives a low-risk loan and chooses not to engage in asset substitution \( (r = 0) \), its payoff is:

\[
U(0|s_1) = \left[ \int_{0}^{1} s f(s|s_1) \, ds \right] (A\alpha - (A - E)) + \Pr \left[ s < \frac{L_\alpha}{\alpha} | s_1 \right] (AL_\alpha - (A - E)). \tag{13}
\]

Note that the bank’s payoff depends on the regulator’s liquidation decision at \( t = 2 \). If \( s \geq L_\alpha / \alpha \) so that the regulator does not liquidate the bank, the bank receives a net payoff of \( A\alpha - (A - E) \) after repaying depositors with probability \( s \), and receives 0 with probability \( 1 - s \). But if \( s < L_\alpha / \alpha \), the regulator liquidates the bank so that the bank receives the certain liquidation payoff \( AL_\alpha - (A - E) \) after repaying depositors. Note that \( AL_\alpha - (A - E) > 0 \) because \( L_\alpha \geq 1 \).

If the bank engages in asset substitution so that \( r = 1 \), its payoff is

\[
U(1|s_1) = \tau(A\beta - (A - E)). \tag{14}
\]

Conditional on obtaining a high-risk loan, the bank receives \( A\beta - (A - E) \) after repaying depositors with probability \( \tau \), and receives 0 with probability \( 1 - \tau \). The bank chooses asset substitution if and only if

\[
U(1|s_1) > U(0|s_1), \tag{15}
\]
which reduces into

$$\tau(A\beta - (A - E)) > \left( \left[ \int_{\frac{L_\alpha}{\alpha}}^{1} s f(s|s_1) ds \right] (A\alpha - (A - E)) + \Pr \left[ s < \frac{L_\alpha}{\alpha} | s_1 \right] (A\alpha L - (A - E)) \right).$$

The left hand side of (16) represents the bank’s payoff from investing in a high-risk loan and does not depend on $s_1$. As $s_1$ increases, the posterior distribution of $s$ shifts to the right so that the right hand side of (16) which represents the bank’s payoff from investing in a low-risk loan increases in $s_1$. As a result, there exists a unique cutoff $\bar{s}_1(A)$ such that the bank engages in asset substitution if and only if its early information regarding the low-risk loan’s performance is below the cutoff, i.e., $s_1 < \bar{s}_1(A)$. We formally state the bank’s equilibrium asset-substituting decision in the following lemma.

**Proposition 1** Conditional on the early information $s_1$ and given a bank of size $A$, there exists a unique threshold $\bar{s}_1(A) \in [\tau, 1]$ such that the bank makes the asset-substituting decision ($r = 1$) if and only if $s_1 < \bar{s}_1(A)$, where $\bar{s}_1(A)$ is given by

$$\tau = \left[ \int_{\frac{L_\alpha}{\alpha}}^{1} s f(s|\bar{s}_1(A)) ds \right] \frac{A\alpha - (A - E)}{A\beta - (A - E)} + \Pr \left[ s < \frac{L_\alpha}{\alpha} | \bar{s}_1(A) \right] \frac{A\alpha L - (A - E)}{A\beta - (A - E)}. \quad (17)$$

The threshold $\bar{s}_1(A)$ increases in the size of the bank $A$, decreases in the face value of the low-risk loan $\alpha$ and increases in the face value of the high-risk loan $\beta$.

Proposition 1 is intuitive. It states that when the bank expects the performance of its loan portfolio to deteriorate, its incentives to engage in asset substitution increase. Furthermore, such incentives become sharper when either the relative payoff of the high-risk loan to the low-risk loan $\beta/\alpha$ increases and/or the bank’s leverage $A$ increases. The latter result suggests a beneficial role for capital requirements in curbing asset substitution: a higher capital ratio $\gamma$ (equivalently, a lower
A) induces the bank to set a lower leverage, thus weakening its asset-substituting incentive in the interim. To highlight this role, the following corollary characterizes the effect of asset size $A$ on the asset-substitution threshold.

**Corollary 1** There exist two cutoffs on the bank’s size $A_{\text{max}}$ and $A_{\text{min}}$ such that:

\[
\begin{align*}
\bar{s}_1(A) &= \tau & \text{if } A \in [E, A_{\text{min}}]; \\
\bar{s}_1(A) &\in (\tau, 1) \text{ and } \frac{\partial \bar{s}_1(A)}{\partial A} > 0 & \text{if } A \in (A_{\text{min}}, A_{\text{max}}); \\
\bar{s}_1(A) &= 1 & \text{if } A \in [A_{\text{max}}, \bar{A}].
\end{align*}
\]

The cutoffs $A_{\text{max}}$ and $A_{\text{min}}$ increase in the face value of the low-risk loan $\alpha$ and decrease in the face value of the high-risk loan $\beta$.

Corollary 1 is a direct consequence of Proposition 1. Figure 2 provides a graphical illustration of Corollary 1. It shows that when the leverage of the bank is extremely low (i.e., the bank faces a tight capital requirement), the bank never engages in asset substitution, whereas when the leverage is extremely high, the bank always chooses to asset-substitute. For intermediate values of leverage, the bank’s assessment of its loan performance matters and the bank engages in asset substitution if and only if such assessment deteriorates.

Finally, we solve for the bank’s choice of asset size $A$ at $t = 0$ given its equilibrium asset substitution strategy at date 1. The bank chooses $A$ to maximize

\[
U(q) = \int_{\tau}^{\bar{s}_1(A)} \left[ U(1|s_1) g(s_1) ds_1 \right] \underbrace{\int_{\bar{s}_1(A)}^1 \left[ qU(0|s_1) + (1-q) U(1|s_1) \right] g(s_1) ds_1}_{\text{expected payoff given no asset substitution}} - C(q).
\]

Note that if $s_1 < \bar{s}_1(A)$, the bank will engage in asset substitution and thus always generate a high-risk loan, regardless of its initial risk choice. If $s_1 \geq \bar{s}_1(A)$, the bank will not asset substitute.
Figure 2: The effect of leverage on the asset-substituting threshold for $A \in (A_{\min}, A_{\max})$.

and its payoff depends on its initial risk choice $q$. With probability $q$, the bank generates a low-risk loan and receives a payoff of $U(0|s_1)$, whereas with probability $1-q$, the bank generates a high-risk loan and receives a payoff of $U(1|s_1)$. The bank is also subject to the capital requirement, $\frac{E}{A} \leq \gamma$, which essentially sets an upper-bound on the bank’s asset size, i.e., $A \leq \frac{E}{\gamma}$. Differentiating $U(q)$ with respect to $A$ yields:

\[
\begin{align*}
(1-q)\tau(\beta-1) + q \int_{s_1(A)}^{1} \left[ \int_{\frac{L_\alpha}{\alpha}}^{1} s f(s|s_1) \, ds \right] (\alpha - 1) &+ \Pr \left[ s < \frac{L_\alpha}{\alpha} | s_1 \right] (L_\alpha - 1) \, g(s_1) \, ds_1 \\
+ q \int_{\tau}^{s_1(A)} \tau(\beta-1)g(s_1) \, ds_1 > 0.
\end{align*}
\]

Therefore, the bank always chooses to invest up to the maximum allowed under the capital requirement, i.e., $A^* = \frac{E}{\gamma}$. It turns out that throughout our analysis, the bank always sets $A^* = \frac{E}{\gamma}$, making the bank’s choice of its asset size $A$ isomorphic to the minimum capital ratio $\gamma$ set by the regulator. For brevity, in future analyses, we therefore omit discussions of the bank’s choice of its size.
3.1.2 Expected loss model with exogenous risk

We now analyze the expected loss model in which the bank reports $s_1$ so the regulator can either intervene early at $t = 1$ based on $s_1$ or late at $t = 2$ based on $s$. We again solve the model using backward induction. Note that the regulator’s liquidation decision at $t = 2$ is the same as that in the incurred loss model because under both models, the regulator receives the full information $s$ about the default risk. Facing the same liquidation policy at $t = 2$, the bank’s asset-substituting decision also stays the same across both models. Nonetheless, there exists a key difference between the two loan loss models: under the expected loss model, since the recognition of the expected loan loss $s_1$ causes the bank to violate the capital requirement, the regulator may choose to intervene early at $t = 1$.

More specifically, conditional on a high-risk loan, the regulator never liquidates, as explained previously. Conditional on a low-risk loan, two key factors come into play when the regulator decides whether to liquidate: 1) the information $s_1$ and 2) the bank’s future asset-substituting decision $r$. While the regulator receives a constant payoff $L_\alpha$ from liquidation, her expected payoff from continuation depends on both $s_1$ and $r$. If $s_1 \geq \bar{s}_1(A)$, the regulator rationally anticipates that the bank will keep the low-risk loan whose expected surplus is

$$
\left[ \int_{L_\alpha}^{s} sf(s|s_1) \, ds \right] \alpha + \Pr \left[ s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha > L_\alpha.
$$

That is, absent the asset-substitution problem, the regulator should never liquidate the bank early based on the timely but imperfect information $s_1$. This is because, the regulator can always postpone the decision to a later date (i.e., $t = 2$) when better information arrives (i.e., the default risk $s$ is fully realized).

However, given the bank’s asset substitution incentives, early intervention becomes desirable to
curb such behavior. To see this, note that when the loan performance deteriorates, i.e., \( s_1 < \bar{s}_1(A) \), the regulator anticipates that the bank will switch to the high-risk loan, which generates an expected surplus of \( \tau \beta \). Since \( L_\alpha \geq 1 > \tau \beta \) (Assumption 2), by curbing asset substitution, early intervention generates expected surplus gains. This represents the social benefit of timely intervention. Note, however, that the regulator faces a non trivial trade-off in intervening early in the bank’s operations: early intervention is imperfect as it relies on imprecise information \( s_1 \) about future default risk \( s \). In fact, had the bank not engaged in asset substitution, there would be a false alarm cost from early intervention, which is given by:

\[
\int_{\frac{L_\alpha}{\alpha}}^{1} \int_{\tau}^{\bar{s}_1} (L_\alpha - \alpha s) f(s|s_1) g(s_1) ds ds_1 < 0. \tag{21}
\]

That is, the regulator may liquidate a bank that is financially sound (\( s > L_\alpha/\alpha \)) but nevertheless receives some bad interim information (\( s_1 < \bar{s}_1(A) \)), resulting in a surplus loss. However, a rational and benevolent regulator will trade off the false-alarm cost against the benefit of curbing asset substitution and choose an action that results in the highest social surplus in expectation. We formally state the regulator’s liquidation decision at \( t = 1 \) in the following proposition.

**Proposition 2** Under the expected loss model, conditional on the early information \( s_1 \), the regulator liquidates the bank at \( t = 1 \) if and only if the bank has a low-risk loan and \( s_1 < \bar{s}_1(A) \).

### 3.1.3 Surplus comparison with exogenous risk

With the equilibrium characterized, for a given leverage \( A \), we compare the surplus between under the incurred loss model and the expected loss model, holding the bank’s ex ante risk choice \( q \) fixed.
The surplus under incurred loss is

\[ W_{IL}(A) = (1 - q)A\tau \beta + q \left( A \int_{\tau}^{s_1(A)} \tau \beta f(s_1) ds_1 \right. \\
+ \left. A \int_{\tau}^{1} \left( \int_{L_0 / \alpha}^{L_0} sf(s_1) ds \right) \alpha + \Pr \left( s < \frac{L_0}{\alpha} | s_1 \right) f(s_1) ds_1 \right) - A. \]  

(22)

With probability \(1 - q\), the bank gets a high-risk loan and the expected surplus is \(\tau \beta\), whereas with probability \(q\), the bank gets a low-risk loan. The bank keeps the low-risk loan if \(s_1 \geq \bar{s}_1(A)\) and switches to a high-risk one if \(s_1 < \bar{s}_1(A)\).

Under the expected loss model, the surplus is

\[ W_{EL}(A) = (1 - q)A\tau \beta + q \left( A \int_{\tau}^{s_1(A)} L_\alpha f(s_1) ds_1 \right. \\
+ \left. A \int_{\tau}^{1} \left( \int_{L_0 / \alpha}^{L_0} sf(s_1) ds \right) \alpha + \Pr \left( s < \frac{L_0}{\alpha} | s_1 \right) f(s_1) ds_1 \right) - A. \]  

(23)

The only difference from the incurred loss model is that, if the bank gets a low-risk loan which occurs with probability \(q\) and \(s_1 < \bar{s}_1(A)\), the bank is liquidated under the expected loss model whereas the bank is allowed to continue and asset-substitute under the incurred loss model. Recognizing this difference, we obtain:\(^2\)

\[ W_{EL}(A) - W_{IL}(A) = qA \left[ \int_{\tau}^{s_1(A)} (L_\alpha - \tau \beta) f(s_1) ds_1 \right] \geq 0. \]  

(24)

The expected loss model always dominates the incurred loss model. Apparently, since the expected loss model dominates for all ranges of asset sizes \(A\), it also dominates when the regulator sets the capital requirement policy optimally. We formally state this result in the following proposition and

\(^2\)The inequality is strict if \(\bar{s}_1(A) > \tau\) (i.e., \(A \in [A_{\min}, \bar{A}]\) from Corollary 1).
Proposition 3. Fixing the bank’s ex ante risk $q$ of the loan portfolio, the expected loss model always dominates the incurred loss model.

Proposition 3 implies that the additional information revealed by the expected loan loss model generates an expected benefit because it allows the regulator to intervene in a more timely manner in banks’ operations to curb inefficient asset substitution. Interestingly, such benefit cannot be overturned by the false-alarm cost stemming from using the imperfect information of the expected loss model. The reason is that, the regulator rationally takes into account the (im)precision of the early information and therefore internalizes such false alarm costs in determining whether to intervene. The preceding result therefore supports claims made by proponents of expected loss models who have argued that by providing more timely information about the performance of banks’ loans, expected loss models would curb banks’ excessive risk taking behavior. Proposition 3 confirms those views as long as the ex ante riskiness of the banks’ loan portfolios are kept fixed. However, in the presence of real effects, banks may respond to changes in the regulator’s intervention strategy that, in turn, depend on the loan loss provisioning model. We next investigate the impact
of the loan loss provisioning models on banks’ ex ante risk portfolios.

3.2 Endogenous loan risk

We now analyze the full model in which the bank can choose the riskiness $q$ of the loans at the origination stage.

3.2.1 Incurred loss model with endogenous risk

We start with the incurred loss model. For a given bank size $A$, the bank chooses risk $q$ that solves

$$q_{IL}^* \in \arg \max_{q \in [0,1]} U(q) = \int_{\tilde{s}_1(A)} U(1|s_1) g(s_1) ds_1 + \int_{\tilde{s}_1(A)} [qU(0|s_1) + (1-q)U(1|s_1)] g(s_1) ds_1 - C(q).$$

Recall that if $s_1 < \tilde{s}_1(A)$, the bank’s payoff is independent of its initial risk choice because the bank will engage in asset substitution and change the loan portfolio into a high-risk one. Thus, the ex ante risk choice only matters when the bank does not asset substitute. The higher the likelihood of interim asset substitution, the lower the bank’s incentives to engage in costly ex ante screening, i.e., $q_{IL}^*$ decreases in $\tilde{s}_1(A)$. As the bank’s leverage becomes very large, i.e., $A \in [A_{\text{max}}, \bar{A}]$ so that $\tilde{s}_1(A) = 1$, the bank always engages in interim asset substitution making the ex ante risk decision moot. In this case, since screening is costly, the bank chooses not to screen the borrowers ex ante so that $q_{IL}^* = 0$.

The first-order condition of the preceding equation with respect to $q$ yields:

$$\int_{\tilde{s}_1(A)} [U(0|s_1) - U(1|s_1)] g(s_1) ds_1 = C'(q_{IL}^*).$$

The right hand side of equation (26) captures the marginal cost of screening borrowers whereas the left hand side captures the marginal benefit of screening stemming from reducing the future
default risk. To see the latter effect, note that from equation (16),
\[
U(0|s_1) - U(1|s_1) = \left( \int_{\frac{L_\alpha}{\alpha}}^1 s f(s|s_1) ds \right) (A\alpha - (A - E)) + \Pr \left[ s < \frac{L_\alpha}{\alpha} | s_1 \right] (AL_\alpha - (A - E)) - \tau(A\beta - (A - E)),
\]
which is the incremental payoff from investing in the low-risk loan vs. the high-risk loan. It is positive because, by the incentive-compatibility constraint (16), the bank strictly prefers the low-risk loan for \( s_1 \geq \bar{s}_1 \).

We formally state the bank’s *ex ante* risk choice in the following proposition.

**Proposition 4** Under the incurred loss model, the bank makes the risk choice \( q_{IL}^* \) such that, for \( A \in [E, A_{\text{max}}) \), \( q_{IL}^* \in (0, 1) \) where \( q_{IL}^* \) solves equation (26), whereas for \( A \in [A_{\text{max}}, \bar{A}] \), \( q_{IL}^* = 0 \). The risk choice \( q_{IL}^* \) decreases in the size of the bank \( A \), increases in the face value of the low-risk loan \( \alpha \) and decreases in the face value of the high-risk loan \( \beta \).

### 3.2.2 Expected loss model with endogenous risk

The bank chooses risk \( q \) that solves
\[
q_{EL}^* \in \arg \max_{q \in [0,1]} U(q).
\]

Under the expected loss model, the bank’s payoff is given by
\[
U(q) = \int_{\tau}^{\bar{s}(A)} [q(AL_\alpha - (A - E)) + (1 - q)U(1|s_1)] g(s_1) ds_1 + \int_{\bar{s}_1(A)}^1 [qU(0|s_1) + (1 - q)U(1|s_1)] g(s_1) ds_1 - C(q).
\]

Note that when \( s_1 < \bar{s}_1(A) \), the bank’s expected payoff under the expected loss model differs from that under the incurred loss model in (25). Under the expected loss model, if the regulator expects
the performance of the bank’s portfolio to deteriorate, the regulator intervenes and liquidates the bank with low-risk loan in order to prevent the bank from engaging in asset substitution. Using this equilibrium property, we obtain that for \( s_1 < \bar{s}_1(A) \), the bank’s payoff is

\[
q(AL_\alpha - (A - E)) + (1 - q)(A\beta - (A - E)).
\]  

(29)

Taking the first-order condition yields:

\[
\int_{\tau}^{s_1(A)} [(AL_\alpha - (A - E)) - \tau(A\beta - (A - E))] g(s_1) ds_1 + \int_{s_1(A)}^1 [U(0|s_1) - U(1|s_1)] g(s_1) ds_1 = C'(q^*_EL).
\]

(30)

To compare the risk choices across the loan loss models, we plug the first-order condition (26) on \( q^*_IL \) into the first order condition (30) to obtain:

\[
\int_{\tau}^{s_1(A)} [(AL_\alpha - (A - E)) - \tau(A\beta - (A - E))] g(s_1) ds_1 = C'(q^*_EL) - C'(q^*_IL).
\]

(31)

Since \( C'' > 0 \), \( q^*_EL \geq q^*_IL \) if and only if

\[
\frac{AL_\alpha - (A - E)}{\text{risk disciplining effect}} - \frac{\tau(A\beta - (A - E))}{\text{risk aggravating effect}} \geq 0.
\]

(32)

The left-hand side of (32) captures two supplementary forces at play in the region of \( s_1 \in [\tau, \bar{s}_1(A)] \) under the expected loss model that are not present in the incurred loss model. The first term is positive and captures the risk-disciplining effect of early intervention. By reducing asset substitution, early intervention increases the bank’s incentives to engage in costly screening as explained above. This is precisely the argument made by proponents of expected loss models: by reducing \textit{ex post} asset substitution, regulatory intervention disciplines \textit{ex ante} risk taking. However, when real
effects are taken into account, our model shows that early intervention is a double-edged sword: while timely intervention curbs asset substitution, it could lead to timelier risk-taking. The latter effect is captured by the second term of (32). To understand this risk-aggravating effect, note that under the incurred loss model, the bank prefers to defer taking excessive amounts of risk until it receives more precise information on the performance of its loan portfolio. However, under the expected loss model, the option value of waiting for more precise information diminishes due to regulatory intervention that preempts asset substitution. Anticipating this, the bank shifts the timing of its risk-taking earlier by originating riskier loans \textit{ex ante} under the expected loss model. In equilibrium, the bank trades off these two effects in determining how much risk to take \textit{ex ante}.

We next show that whether the risk-aggravating effect exceeds the risk-disciplining effect depends on the bank’s leverage. It is straightforward to verify that (32) holds if and only if

$$A \leq A_r \equiv \frac{1 - \tau}{\tau \beta + (1 - \tau) - L} E$$

(33)

If the bank has a low leverage \((A < A_r)\) so that the risk-aggravating effect is not too severe, the bank chooses a lower risk under the expected loss model, i.e., \(q_{EL}^* > q_{IL}^*\). Otherwise, if the bank has a high leverage \((A > A_r)\) so that the risk-aggravating effect dominates, the bank chooses a higher risk under the expected loss model, i.e., \(q_{EL}^* < q_{IL}^*\). Intuitively, a highly-leveraged bank has a strong incentive to asset-substitute. When that possibility is eliminated by regulatory intervention, the bank responds by building up more risk in its loan portfolio \textit{ex ante}.

Finally, when \(A\) is sufficiently large and close to \(\bar{A}\), the first-order condition (30) becomes:

$$\int^1_\tau \left[ \bar{A} (L - \tau \beta) - (1 - \tau) (\bar{A} - E) \right] g(s_1) ds_1 < 0.$$  

(34)

\footnote{Note that using Assumption 3, we can show that \(\tau \beta + (1 - \tau) - L > 0\) so that \(A_r\) is strictly positive.}
Note that we used the property that as $A \to \tilde{A}$, the threshold $\tilde{s}_1(A) \to 1$. An application of the intermediate value theorem thus suggests that, there exists some cutoff $A_e \in (E, A_{\text{max}})$ such that for $A \geq A_e$, we have $q^*_{EL} = 0$. The cutoff $A_e$ is defined by

$$
\int_{\tau}^{\tilde{s}_1(A_e)} [A_e (L_{\alpha} - \tau \beta) - (1 - \tau) (A_e - E)] g(s_1) ds_1 + \int_{\tilde{s}_1(A_e)}^{1} \frac{\partial U(0|s_1)}{\partial q} g(s_1) ds_1 = 0. \tag{35}
$$

In other words, whenever when $A \geq A_{\text{max}}$, the region in which $q^*_{EL} = 0$ is larger than that of $q^*_{IL} = 0$. We formally state these results in the following proposition.

**Proposition 5** Under the expected loss model, the bank makes the risk choice $q^*_{EL}$ such that, if $A \in [E, A_e)$, $q^*_{EL} \in (0, 1)$ and solves equation (30). Otherwise, for $A \in [A_e, \tilde{A}]$, $q^*_{EL} = 0$. In addition, $q^*_{EL} < q^*_{IL}$ if and only if $A > A_r$. For $A > A_r$, the risk choice $q^*_{EL}$ decreases in the size of the bank $A$, increases in the face value of the low-risk loan $\alpha$ and decreases in the face value of the high-risk loan $\beta$.

Proposition 5 implies that when real effects are taken into account, there are potential costs of adopting the expected loss model. While timely regulatory intervention provides discipline in terms of curbing ex post inefficient asset substitution, banks may respond to such intervention by originating riskier loan portfolios that potentially reduce surplus. We next investigate whether the costs triggered by such real effects may outweigh the benefits of timely intervention under an expected loss model.

### 3.2.3 Surplus comparison with endogenous risk

To compare the results with endogenous risk choice with those with exogenous risk choice (Proposition 3), we first compare the surplus between the two models for all ranges of asset sizes $A$. 

27
Proposition 6  With the risk choice endogenized, there exist two thresholds $A_r, A_e$ where $A_r \leq A_e$, such that:

1. if $A \leq A_r$, the expected loss model dominates the incurred loss model;

2. if $A \geq A_e$, the incurred loss model dominates the expected loss model.

Proposition 6 shows that, when we take into account the real effects of accounting measurements, the more timely information under the expected loss model is no longer always socially desirable. While the expected loss model remains superior when banks’ leverages are low, it becomes inferior when banks are heavily leveraged. To see the latter, consider the case that $A \geq A_e$ so that $q^{*}_{EL} = 0 < q^{*}_{IL}$. When the bank is highly leveraged, under the expected loss model, the bank always chooses the high-risk loan whereas under the incurred loss model, the bank chooses the low-risk loan with some probability, thus resulting in a higher surplus.

The fact that the expected loss model improves the surplus if the leverage is sufficiently low and reduces the surplus if the leverage is sufficiently high suggests there exists a unique cutoff such that the expected loss model reduces the surplus when the leverage is above the cutoff. Although the complexity of our model prevents us from obtaining an analytical proof, numerical analyses suggest that such a conjecture is indeed true, as illustrated in Figure 4 which shows that $W_{EL} < W_{IL}$ if and only if the leverage $A$ is large.

3.2.4 Optimal capital requirement under different loan loss models

An implication of Proposition 6 is that the regulator should take into account the real effects of loan loss provisioning when designing the capital requirement policy that, in turn, governs the bank’s leverage. Stated differently, changing the accounting method for loan loss provisioning requires the bank regulator to simultaneously adjust the capital adequacy ratios. To shed some light on how
the regulator should make such adjustments, we next solve for the optimal capital ratios under the two loan loss models.

Recall that the bank always chooses to invest up to the maximum allowed under the capital requirement, i.e., $A^* = \frac{E}{\gamma}$. Thus choosing the capital ratio $\gamma$ is isomorphic to choosing the bank size $A$. For expositional purposes, we describe the regulator’s choice of $\gamma$ as if she were choosing $A$. We first reproduce the surplus under the incurred loss model, equation (22), here:

$$\begin{align*}
W_{IL}(A) &= (1 - q_{IL}^*) A \tau \beta + q_{IL}^* \left( A \int_{s(A)}^{s_1(A)} \tau \beta g(s_1) ds_1 ight.
\phantom{A \int_{s(A)}^{s_1(A)} \tau \beta g(s_1) ds_1} + A \int_{s_1(A)}^{1} \left( \int_{\frac{s_1(A)}{\alpha}}^{1} sf(s | s_1) ds \right) \alpha + \Pr \left[ s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha g(s_1) ds_1 \left. \right) - A. \quad (36)
\end{align*}$$

Taking the first-order condition yields

$$\begin{align*}
\frac{\partial W_{IL}}{\partial A} &= NPV_{IL}(s_1(A)) - q_{IL}^* \frac{\partial s_1}{\partial A} \left( \int_{\frac{s_1(A)}{\alpha}}^{1} sf(s | s_1(A)) ds \right) \alpha + \Pr \left[ s < \frac{L_\alpha}{\alpha} | s_1(A) \right] L_\alpha - \tau \beta \right) g(s_1(A)) \right) \\
&\phantom{= NPV_{IL}(s_1(A)) - q_{IL}^* \frac{\partial s_1}{\partial A} \left(} + A \frac{\partial q_{IL}^*}{\partial A} \left( \int_{s_1}^{1} \left( \int_{\frac{s_1}{\alpha}}^{1} sf(s | s_1) ds \right) \alpha + \Pr \left[ s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha - \tau \beta \right) g(s_1) ds_1 \right). \quad (37)
\end{align*}$$
The first term in $\frac{\partial W_{IL}}{\partial A}$,

$$NPV_{IL}(s_1(A)) = (1 - q_{IL}^*) \tau \beta + q_{IL}^* \left( \int_{\tau}^{s_1} \tau \beta g(s_1) ds_1 \right) + \int_{s_1}^{1} \left[ \int_{L_\alpha}^{s_1} s f(s|s_1) ds \right] \alpha + \Pr \left[ s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha g(s_1) ds_1 - 1, \quad (38)$$

measures the per-unit NPV from the bank’s loan portfolio and represents the potential social benefit of increasing the bank’s size. It is straightforward to verify the NPV is positive if and only if the asset size is sufficiently small. The reason is that, if the bank is highly leveraged, it will convert its entire loan portfolio into high-risk in the interim, which generates a negative NPV in expectation by Assumption 2.

The other two terms in $\frac{\partial W_{IL}}{\partial A}$ are both negative and represent the social costs of increasing the bank’s size. In particular, the second term captures the effect of increasing the asset size in motivating the bank to asset substitute _ex post_ (i.e., $\frac{\partial s_1}{\partial A} > 0$) whereas the third term captures the effect of increasing the asset size in discouraging the bank from exerting screening effort _ex ante_ (i.e., $\frac{\partial q_{IL}^*}{\partial A} < 0$).\(^4\)

The regulator sets the optimal capital requirement ratio by trading off the above benefit and the costs. We denote the optimal bank size under the incurred loss model by $A^*_{IL}$ which solves the first-order condition (37). The optimal bank size under the expected loss model can be similarly derived. We denote it by $A^*_{EL}$.\(^5\) Because $A^*_{IL}$ and $A^*_{EL}$ are defined by implicit solutions to differential equations, in general, we are unable to compare them. However, we next derive a sufficient condition under which $A^*_{EL} > A^*_{IL}$, i.e., the regulator should lower the capital ratio in response to the adoption

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\(^4\)Mathematically, the second term in $\frac{\partial W_{IL}}{\partial A}$ is negative because 1) from Proposition 1, $\frac{\partial s_1}{\partial A} > 0$ and 2) $\left[ \int_{L_\alpha}^{s_1} s f(s|s_1) ds \right] \alpha + \Pr \left[ s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha > \tau \beta$ (Assumption 2). The third term is negative because, 1) from Proposition 4, $\frac{\partial q_{IL}^*}{\partial A} < 0$ and 2) $\left[ \int_{L_\alpha}^{s_1} s f(s|s_1) ds \right] \alpha + \Pr \left[ s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha > \tau \beta$.

\(^5\)The first-order condition that defines $A^*_{EL}$ is complicated and given in the proof of Proposition 7.
of the expected loss model.

**Proposition 7** If the surplus functions are sufficiently concave, then the regulator sets tighter capital requirements under the incurred loss model than under the expected loss model, i.e., \( A_{IL}^* < A_{EL}^* \) so that the expected loss model dominates the incurred loss model, i.e., \( W_{EL}(A_{EL}^*) > W_{IL}(A_{IL}^*) \).

Proposition 7 states that when the surplus functions are sufficiently concave, the regulator may be able to relax the capital requirements when banks use the expected loss model for loan loss provisioning. More importantly, provided that the capital requirements are optimally set, the expected loss model generates higher surplus than the incurred loss model. While Proposition 6 states that, fixing the capital requirement policy, the additional information released under expected loss may impair surplus, Proposition 7 shows that such adverse effect can be overturned if the regulator can appropriately fine-tune the capital ratio in response to the loan loss model. Under the optimal capital requirements, the expected loss model supplies the regulator with valuable early warning signals on banks’ loan performance, thus facilitating regulatory intervention and improving the social surplus. Indeed, given the better information under expected loss, the regulator can actually relax the capital constraint since the regulation has become more effective. These results echo the recent call for better coordination between accounting and bank regulation.

While Proposition 7 provides a general condition under which the regulator may loosen the capital requirements under the expected loss model, the complexity of our general model structure prevents us from expressing this general condition in terms of exogenous parameters. To provide further policy implications, we turn to some numerical analysis of the optimal capital requirements. Toward that end, we use the linear-uniform information structure of \( s_1 \) and \( s \) given in the model setup section, equation (2):

\[
\bar{s} = w\bar{s}_1 + (1 - w)\bar{s}_2.
\]  (39)
Recall from the model setup discussion that, $w$ measures the precision of $s_1$ in forecasting $s$, thus representing the precision of the additional interim information reported under the expected loss model. Figure 5 illustrates the comparison between the optimal capital requirements, $\{A_{EL}^*, A_{IL}^*\}$ in terms of two key parameters, the precision of expected loss model $w$ and the face value of the high-risk loan $\beta$. The figure shows that the regulator should loosen the capital requirement under the expected loss model in the lower-right region when the expected loss model reports more precise interim information (high $w$) and/or the high-risk loan is less attractive to the bank (low $\beta$). Intuitively, when the expected loss model provides more precise interim information, it is more effective in facilitating regulatory intervention and curbing ex-post asset-substitution. On other hand, when the bank finds the high-risk loan more appealing to invest in, the excessive risk-taking problem becomes sufficiently severe and requires the regulator to set an extremely stringent capital requirement under any loan loss provisioning models, i.e., both $A_{EL}^*$ and $A_{IL}^*$ approach $A_{min}$, the level of capital requirement that eliminates the bank’s incentive to invest in the high-risk loan. In this case, the optimal capital requirements are the same across the two loan loss provisioning models.

4 Conclusion

We develop an analytical framework to examine the impact of loan loss provisioning on the prudential regulation of banks. The loan loss provision is arguably the largest accrual item on banks’ financial statements and thus “plays a prominent role in much of the bank accounting literature” (Beatty and Liao, 2014). Indeed, there has been a large amount of empirical evidence on the association between loan loss provisioning and banks’ behavior. Yet, somewhat surprisingly, relatively little is known about the exact mechanism through which loan loss provisioning interacts with prudential regulation and affects financial stability. We believe that, to better inform policy debates, it
is crucial to have a solid conceptual understanding of the role of loan loss provisioning. In this light, our paper shows that the timely recognition of loan losses through provisioning is a double-edged sword. It yields a benefit of facilitating timely regulatory intervention that curbs inefficient *ex-post* asset-substitution, but, at the same time, also triggers timelier risk-taking by banks, who respond to regulation by originating riskier loans *ex-ante*. Taking into account these countervailing effects, we argue that it is important to appropriately tailor capital requirements to loan loss provisioning models, in order to seize the full benefit of timely loan loss recognition.

In order to build a model as parsimonious as possible, we have left out some potentially important aspects of loan loss provisioning. One such aspect is banks’ discretion in reporting expected loan losses. Banks are often criticized for exercising a large amount of discretion in estimating and disclosing their loan losses, even under the incurred loss model. Arguably, such discretion can only get larger when it comes to estimating expected loan losses. To the extent that banks’ discretionary reporting of loan loss provisions hides loan loss information from regulators, it may result in an
additional cost of the expected loss model. How such a cost of reporting discretion may be traded against the cost and the benefit identified in our model is an interesting avenue we leave to future research.

Another important aspect of loan loss provisioning that we do not consider is how it may affect the systemic risk of the banking industry (e.g., procyclicality of bank lending). In the current debate, there are concerns that the incurred loss model may have contributed to procyclicality and adopting the new expected loss model may help to mitigate procyclicality. Extending our single-bank model to include multiple banks may thus shed light on the systemic impact of loan loss provisioning, which is an interesting avenue left for future research.
References


Appendix: proofs

Proof. of Proposition 1: The existence of the cutoff $\tilde{s}_1$ has been proved in the main text. We now derive some comparative statics on $\tilde{s}_1 \in (\tau, 1)$, which is defined such that

$$
\tau = \left[ \int_{\frac{L_\alpha}{\alpha}}^{1} sf(s|\tilde{s}_1) \, ds \right] \frac{A\alpha - (A - E)}{A\beta - (A - E)} + \text{Pr} \left[ s < \frac{L_\alpha}{\alpha}|\tilde{s}_1 \right] \frac{AL_\alpha - (A - E)}{A\beta - (A - E)}. \tag{40}
$$

which is equivalent to

$$
\tau (A\beta - (A - E)) = \left[ \int_{\frac{L_\alpha}{\alpha}}^{1} sf(s|\tilde{s}_1) \, ds \right] (A\alpha - (A - E)) + \text{Pr} \left[ s < \frac{L_\alpha}{\alpha}|\tilde{s}_1 \right] (AL_\alpha - (A - E)). \tag{41}
$$

Taking the derivative of (41) with respect to $A$, we get

$$
\tau (\beta - 1) = \left[ \frac{\partial \tilde{s}_1}{\partial A} \int_{\frac{L_\alpha}{\alpha}}^{1} \frac{\partial f(s|\tilde{s}_1)}{\partial s_1} \, ds \right] (A\alpha - (A - E)) + \left[ \frac{\partial \tilde{s}_1}{\partial A} \int_{\tau}^{\frac{L_\alpha}{\alpha}} \frac{\partial f(s|\tilde{s}_1)}{\partial s_1} \, ds \right] (AL_\alpha - (A - E))
$$

$$
+ \left[ \int_{\frac{L_\alpha}{\alpha}}^{1} sf(s|\tilde{s}_1) \, ds \right] (\alpha - 1) + \text{Pr} \left[ s < \frac{L_\alpha}{\alpha}|\tilde{s}_1 \right] (L_\alpha - 1), \tag{42}
$$

which is equivalent to

$$
\frac{\partial \tilde{s}_1}{\partial A} = \frac{\tau (\beta - 1) - \left[ \int_{\frac{L_\alpha}{\alpha}}^{1} sf(s|\tilde{s}_1) \, ds \right] (\alpha - 1) - \text{Pr} \left[ s < \frac{L_\alpha}{\alpha}|\tilde{s}_1 \right] (L_\alpha - 1) \left[ \int_{\frac{L_\alpha}{\alpha}}^{1} \frac{\partial f(s|\tilde{s}_1)}{\partial s_1} \, ds \right] (A\alpha - (A - E)) + \left[ \int_{\tau}^{\frac{L_\alpha}{\alpha}} \frac{\partial f(s|\tilde{s}_1)}{\partial s_1} \, ds \right] (AL_\alpha - (A - E))}{\left[ \int_{\frac{L_\alpha}{\alpha}}^{1} \frac{\partial f(s|\tilde{s}_1)}{\partial s_1} \, ds \right] (A\alpha - (A - E))}. \tag{43}
$$

Hence, we have $\frac{\partial \tilde{s}_1}{\partial A} \geq 0$.

Taking the derivative of (41) with respect to $\beta$, we get

$$
\tau A = \left[ \frac{\partial \tilde{s}_1}{\partial \beta} \int_{\frac{L_\alpha}{\alpha}}^{1} \frac{\partial f(s|\tilde{s}_1)}{\partial s_1} \, ds \right] (A\alpha - (A - E)) + \left[ \frac{\partial \tilde{s}_1}{\partial \beta} \int_{\tau}^{\frac{L_\alpha}{\alpha}} \frac{\partial f(s|\tilde{s}_1)}{\partial s_1} \, ds \right] (AL_\alpha - (A - E)), \tag{44}
$$
which is equivalent to
\[
\frac{\partial s_1}{\partial \beta} = \frac{\tau A}{\int_{\frac{L_0}{\alpha}}^{1} \frac{\partial s f(s \mid s_1)}{\partial s_1} ds} (A \alpha - (A - E)) + \left[ \int_{\tau}^{\frac{L_0}{\alpha}} \frac{\partial f(s \mid s_1)}{\partial s_1} ds \right] (AL_\alpha - (A - E)).
\] (45)

Hence, we have \( \frac{\partial s_1}{\partial \beta} \geq 0 \).

Finally, taking the derivative of \( (41) \) with respect to \( \alpha \), we get
\[
0 = \left[ \frac{\partial s_1}{\partial \alpha} \int_{\frac{L_0}{\alpha}}^{1} \frac{L_2}{\alpha} f \left( \frac{L_\alpha}{\alpha} \mid s_1 \right) ds + \frac{L_2}{\alpha^3} f \left( \frac{L_\alpha}{\alpha} \mid s_1 \right) \right] (A \alpha - (A - E)) + \left[ \int_{\frac{L_0}{\alpha}}^{1} sf(s \mid s_1) ds \right] A
\]
\[
+ \left[ \frac{\partial s_1}{\partial \alpha} \int_{\tau}^{\frac{L_0}{\alpha}} \frac{L_2}{\alpha} f \left( \frac{L_\alpha}{\alpha} \mid s_1 \right) ds + \frac{L_2}{\alpha^2} f \left( \frac{L_\alpha}{\alpha} \mid s_1 \right) \right] (AL_\alpha - (A - E)),
\] (46)

which is equivalent to
\[
\frac{\partial s_1}{\partial \alpha} = \frac{L_2}{\alpha^3} f \left( \frac{L_\alpha}{\alpha} \mid s_1 \right) (A \alpha - (A - E)) - \left[ \int_{\frac{L_0}{\alpha}}^{1} sf(s \mid s_1) ds \right] A + \frac{L_2}{\alpha^2} f \left( \frac{L_\alpha}{\alpha} \mid s_1 \right) (AL_\alpha - (A - E))
\]
\[
\left[ \int_{\frac{L_0}{\alpha}}^{1} \frac{\partial s f(s \mid s_1)}{\partial s_1} ds \right] (A \alpha - (A - E)) + \left[ \int_{\tau}^{\frac{L_0}{\alpha}} \frac{\partial f(s \mid s_1)}{\partial s_1} ds \right] (AL_\alpha - (A - E)).
\] (47)

Hence, we have \( \frac{\partial s_1}{\partial \alpha} \leq 0 \). ■

**Proof.** of Corollary 1: At one extreme, \( A = E \) (the lower bound for \( A \)), condition (16) becomes
\[
\tau \beta > \left[ \int_{\frac{L_0}{\alpha}}^{1} sf(s \mid s_1) ds \right] \alpha + \Pr \left[ s < \frac{L_\alpha}{\alpha} \mid s_1 \right] L_\alpha.
\] (48)

Inequality (48) is never satisfied by Assumption 1:
\[
\tau \beta < 1 \leq \left[ \int_{\frac{L_0}{\alpha}}^{1} sf(s \mid s_1 = \tau) ds \right] \alpha + \Pr \left[ s < \frac{L_\alpha}{\alpha} \mid s_1 = \tau \right] L_\alpha.
\] (49)

That is, at \( A = E \), the bank always makes the no asset-substituting decision such that \( s_1 = \tau \).

Intuitively, the bank fully internalizes the surplus consequences of the risk decision and makes the
efficient no asset-substituting decision.

At the other extreme of \( A = \bar{A} \), by Assumption 3, the bank always makes the asset-substituting decision such that \( \bar{s}_1 = 1 \). Notice that from the bank’s incentive-compatibility constraint (16), this requires that

\[
\tau(\bar{A}\beta - (\bar{A} - E)) > \left[ \int_{L_0}^1 sf(s|s_1 = 1) \, ds \right] (\bar{A}\alpha - (\bar{A} - E)) + \Pr \left[ s < \frac{L_0}{\alpha} | s_1 = 1 \right] (\bar{A}\alpha L_0 - (\bar{A} - E)).
\]

(50)

Since \( \int_{L_0}^1 sf(s|s_1 = 1) \, ds \) is increasing in \( \bar{s}_1 \), the right-hand side of (17) is increasing in \( \bar{s}_1 \), and by the intermediate value theorem, there exists some \( A_{\min} > E \) such that \( \bar{s}_1(A_{\min}) = \tau \) and some \( A_{\max} < \bar{A} \) such that \( \bar{s}_1(A_{\max}) = 1 \). For \( A \in [A_{\min}, A_{\max}] \), \( \bar{s}_1(A) \in [\tau, 1] \); for \( A < A_{\min} \), \( \bar{s}_1(A) = \tau \); for \( A > A_{\max} \), \( \bar{s}_1(A) = 1 \). The variable \( A_{\max} \) is defined such that

\[
\tau = \left[ \int_{L_0}^1 sf(s|s_1 = 1) \, ds \right] \frac{A_{\max}\alpha - (A_{\max} - E)}{A_{\max}\beta - (A_{\max} - E)} + \Pr \left[ s < \frac{L_0}{\alpha} | s_1 = 1 \right] \frac{A_{\max}L_0 - (A_{\max} - E)}{A_{\max}\alpha L_0 - (A_{\max} - E)},
\]

(51)

which is equivalent to

\[
\tau(A_{\max}\beta - (A_{\max} - E)) = \left[ \int_{L_0}^1 sf(s|s_1 = 1) \, ds \right] (A_{\max}\alpha - (A_{\max} - E)) + \Pr \left[ s < \frac{L_0}{\alpha} | s_1 = 1 \right] (A_{\max}L_\alpha - (A_{\max} - E)).
\]

(52)

Taking the derivative of (52) with respect to \( \beta \), we get

\[
\tau(\beta - 1) \frac{\partial A_{\max}}{\partial \beta} + \tau A_{\max} = \frac{\partial A_{\max}}{\partial \beta} \left( \int_{L_0}^1 sf(s|s_1 = 1) \, ds(\alpha - 1) + \Pr \left[ s < \frac{L_0}{\alpha} | s_1 = 1 \right] (L_\alpha - 1) \right),
\]

(53)
which is equivalent to

\[ \frac{\partial A_{\text{max}}}{\partial \beta} = \frac{\tau A_{\text{max}}}{\int_{\frac{L_{\alpha}}{\alpha}}^{1} s f(s|s_1 = 1) ds(\alpha - 1) + \Pr \left[ s < \frac{L_{\alpha}}{\alpha}|s_1 = 1 \right] (L_{\alpha} - 1) - \tau(\beta - 1)} < 0. \tag{54} \]

Taking the derivative of (52) with respect to \( \alpha \), we get

\[ \frac{\partial A_{\text{max}}}{\partial \alpha} = -\int_{\frac{L_{\alpha}}{\alpha}}^{1} s f(s|s_1 = 1) ds A_{\text{max}} - \frac{L_{\alpha}^2}{\alpha} f\left(\frac{L_{\alpha}}{\alpha}|s_1 = 1\right) (A_{\text{max}} \alpha - (A_{\text{max}} - E)) + \int_{\frac{L_{\alpha}}{\alpha}}^{1} s f(s|s_1 = 1) ds(\alpha - 1) + \Pr \left[ s < \frac{L_{\alpha}}{\alpha}|s_1 = 1 \right] (L_{\alpha} - 1) - \tau(\beta - 1) \]

\[ + \frac{L_{\alpha} f\left(\frac{L_{\alpha}}{\alpha}|s_1 = 1\right) (A_{\text{max}} L_{\alpha} - (A_{\text{max}} - E))}{\int_{\frac{L_{\alpha}}{\alpha}}^{1} s f(s|s_1 = 1) ds(\alpha - 1) + \Pr \left[ s < \frac{L_{\alpha}}{\alpha}|s_1 = 1 \right] (L_{\alpha} - 1) - \tau(\beta - 1)} > 0. \tag{55} \]

The variable \( A_{\text{min}} \) is defined such that

\[ \tau = \left[ \int_{\frac{L_{\alpha}}{\alpha}}^{1} s f(s|s_1 = 0) ds \right] \frac{A_{\text{min}} \alpha - (A_{\text{min}} - E)}{A_{\text{min}} \beta - (A_{\text{min}} - E)} + \Pr \left[ s < \frac{L_{\alpha}}{\alpha}|s_1 = 0 \right] \frac{A_{\text{min}} L_{\alpha} - (A_{\text{min}} - E)}{A_{\text{min}} \beta - (A_{\text{min}} - E)} \tag{56} \]

Hence, similar computations yield to

\[ \frac{\partial A_{\text{min}}}{\partial \beta} < 0 \quad \text{and} \quad \frac{\partial A_{\text{min}}}{\partial \alpha} > 0. \tag{57} \]

\[ \blacksquare \]

**Proof.** of Proposition 2: See the main text. \[ \blacksquare \]

**Proof.** of Proposition 3: See the main text. \[ \blacksquare \]
Proof. of Proposition 4: We know that, for \( A \in [E, A_{\text{max}}) \), \( q_{IL}^* \) is such that

\[
\int_{s_1(A)}^{1} \left( \left[ \int_{\frac{L_o}{\alpha}}^{1} s f(s|s_1) \, ds \right] (A\alpha - (A - E)) + \Pr \left[ s < \frac{L_o}{\alpha} | s_1 \right] (AL\alpha - (A - E)) \right) g(s_1)ds_1 \\
- \int_{s_1(A)}^{1} \tau (A\beta - (A - E)) f(s_1)ds_1 = C'(q_{IL}^*). \tag{58}
\]

Using the implicit function theorem,

\[
C''(q_{IL}^*) \frac{\partial q_{IL}^*}{\partial A} \\
= \int_{s_1(A)}^{1} \left( \left[ \int_{\frac{L_o}{\alpha}}^{1} s f(s|s_1) \, ds \right] (A\alpha - (A - E)) + \Pr \left[ s < \frac{L_o}{\alpha} | s_1 \right] (AL\alpha - (A - E)) \right) \frac{\partial s_1}{\partial A} \left( \int_{\frac{L_o}{\alpha}}^{1} s f(s|\bar{s}_1(A)) \, ds \right) (A\alpha - (A - E)) + \Pr \left[ s < \frac{L_o}{\alpha} | \bar{s}_1(A) \right] (AL\alpha - (A - E)) \\
- \tau (A\beta - (A - E)) \right) g(\bar{s}_1(A)) \\
= \int_{s_1(A)}^{1} \left( \left[ \int_{\frac{L_o}{\alpha}}^{1} s f(s|s_1) \, ds \right] (A\alpha - (A - E)) + \Pr \left[ s < \frac{L_o}{\alpha} | s_1 \right] (L\alpha - 1 - \tau (\beta - 1)) \right) g(s_1)ds_1.
\]

The second step uses

\[
\left[ \int_{\frac{L_o}{\alpha}}^{1} s f(s|\bar{s}_1(A)) \, ds \right] (A\alpha - (A - E)) + \Pr \left[ s < \frac{L_o}{\alpha} | \bar{s}_1(A) \right] (AL\alpha - (A - E)) = \tau (A\beta - (A - E)).
\]

Note that assumption (4) implies

\[
\left[ \int_{\frac{L_o}{\alpha}}^{1} s f(s|s_1) \, ds \right] (A\alpha - (A - E)) + \Pr \left[ s < \frac{L_o}{\alpha} | s_1 \right] (L\alpha - 1) < \tau (\beta - 1),
\]

which implies that

\[
\frac{\partial q_{IL}^*}{\partial A} < 0.
\]
Taking the derivative of (58) with respect to $\alpha$, we get

\[
\int_{s_1(A)}^{1} \frac{\partial}{\partial \alpha} \left( \left[ \int_{\frac{L_\alpha}{\alpha}}^{1} s f (s | s_1) \, ds \right] (A\alpha - (A - E)) + \Pr \left[ s < \frac{L_\alpha}{\alpha} | s_1 \right] (AL_\alpha - (A - E)) \right) g(s_1)ds_1 \n + \frac{\partial \tilde{s}_1}{\partial \alpha} \left( \left[ \int_{\frac{L_\alpha}{\alpha}}^{1} s f (s | s_1) \, ds \right] (A\alpha - (A - E)) + \Pr \left[ s < \frac{L_\alpha}{\alpha} | s_1 \right] (AL_\alpha - (A - E)) \n + \tau (A\beta - (A - E)) \right) g(\bar{s}_1) = C''(q^*_IL) \frac{\partial q^*_IL}{\partial \alpha}. \tag{59}
\]

Hence, we get $\frac{\partial q^*_IL}{\partial \alpha} > 0$.

Lastly, taking the derivative of (58) with respect to $\beta$, we get

\[
- \int_{s_1(A)}^{1} \tau Ag(s_1)ds_1 \n + \frac{\partial \tilde{s}_1}{\partial \beta} \left( \left[ \int_{\frac{L_\alpha}{\alpha}}^{1} s f (s | s_1) \, ds \right] (A\alpha - (A - E)) + \Pr \left[ s < \frac{L_\alpha}{\alpha} | s_1 \right] (AL_\alpha - (A - E)) \n + \tau (A\beta - (A - E)) \right) g(\bar{s}_1) = C''(q^*_IL) \frac{\partial q^*_IL}{\partial \beta}. \tag{60}
\]

Hence, we get $\frac{\partial q^*_IL}{\partial \beta} < 0$. □

**Proof.** of Proposition 5: We know that

\[
C'(q^*_IL) + \int_{\tau}^{s_1(A)} [A(L_\alpha - \tau\beta) - (1 - \tau)(A - E)] g(s_1)ds_1 = C'(q^*_EL). \tag{61}
\]

Taking the derivative of (61) with respect to $A$, we get

\[
C''(q^*_EL) \frac{\partial q^*_EL}{\partial A} = C''(q^*_IL) \frac{\partial q^*_IL}{\partial A} + \int_{\tau}^{s_1(A)} [L_\alpha - 1 - \tau(\beta - 1)] g(s_1)ds_1 \n + \frac{\partial \tilde{s}_1}{\partial A} (A(L_\alpha - \tau\beta) - (1 - \tau)(A - E)) g(\bar{s}_1(A)). \tag{62}
\]
The second term on the right-hand side is negative whereas the third term on the right-hand side
is negative if and only if \( A > A_r \). Therefore, we have \( \frac{\partial q^*_E}{\partial A} < 0 \) for \( A > A_r \).

Taking the derivative of (61) with respect to \( \alpha \), we get

\[
C''(q^*_{EL}) \frac{\partial q^*_E}{\partial \alpha} = C''(q^*_{IL}) \frac{\partial q^*_L}{\partial \alpha} + \frac{\partial \bar{s}_1}{\partial \alpha} (A(L_\alpha - \tau \beta) - (1 - \tau)(A - E)) g(\bar{s}_1(A)). \tag{63}
\]

The second term on the right-hand side is positive if and only if \( A > A_r \). Therefore, we have

\( \frac{\partial q^*_E}{\partial \alpha} > 0 \) for \( A > A_r \).

Lastly, taking the derivative of (61) with respect to \( \beta \), we get

\[
C''(q^*_{EL}) \frac{\partial q^*_E}{\partial \beta} = C''(q^*_{IL}) \frac{\partial q^*_L}{\partial \beta} + \frac{\partial \bar{s}_1}{\partial \beta} (A(L_\alpha - \tau \beta) - (1 - \tau)(A - E)) g(\bar{s}_1(A))
- \int_{\tau}^{s_1} \tau A g(s_1) ds_1. \tag{64}
\]

Therefore, we get \( \frac{\partial q^*_E}{\partial \beta} < 0 \) for \( A > A_r \). 

**Proof.** of Proposition 6: The surplus is given by

\[
W_{IL}(A) = (1 - q^*_{IL}) A \tau \beta + q^*_{IL} \left( A \int_{\tau}^{s_1(A)} \tau \beta g(s_1) ds_1 + A \int_{\bar{s}_1(A)}^{1} \left( \int_{\frac{L_\alpha}{\alpha}}^{s_1} s f(s|s_1) ds \right) \alpha + \Pr \left[ s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha g(s_1) ds_1 \right) - A,
\]

and

\[
W_{EL}(A) = (1 - q^*_{EL}) A \tau \beta + q^*_{EL} \left( A \int_{\tau}^{s_1(A)} L_\alpha g(s_1) ds_1 + A \int_{\bar{s}_1(A)}^{1} \left( \int_{\frac{L_\alpha}{\alpha}}^{s_1} s f(s|s_1) ds \right) \alpha + \Pr \left[ s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha g(s_1) ds_1 \right) - A.
\]
First, consider the case that \( A \in [A_{\text{max}}, \bar{A}] \). We have \( q_{IL}^* = q_{EL}^* = 0 \), which implies that
\[
W_{IL}(A) = W_{EL}(A) = A \tau \beta - A.
\]

Second, if \( A \in [A_e, A_{\text{max}}) \), we have \( q_{EL}^* = 0 \) and \( q_{IL}^* \in (0, 1) \), which implies that
\[
W_{EL}(A) = A \tau \beta - A < 0,
\]
and
\[
W_{IL}(A) = A \tau \beta - A
\]
\[
+ q_{IL}^* \left( A \int_{\bar{s}_1}^{s_1} \left( \int_{\frac{s}{\alpha}}^{1} s f(s|s_1) \, ds \right) \alpha + \Pr \left[ s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha - \tau \beta \right) g(s_1)ds_1
\].

That is, \( W_{IL}(A) > W_{EL}(A) \). Then, if \( A \in (A_{\text{min}}, A_e] \), we have \( q_{EL}^* \geq q_{IL}^* > 0 \). Thus

\[
W_{EL}(A) - W_{IL}(A)
\]
\[
= q_{EL}^* \left( A \int_{\tau}^{\bar{s}_1} L_\alpha g(s_1)ds_1 + A \int_{\bar{s}_1}^{1} \left( \int_{\frac{s}{\alpha}}^{1} s f(s|s_1) \, ds \right) \alpha + \Pr \left[ s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha \right) g(s_1)ds_1 - A \tau \beta
\]
\[
- q_{IL}^* \left( A \int_{\tau}^{\bar{s}_1} \tau \beta g(s_1)ds_1 + A \int_{\bar{s}_1}^{1} \left( \int_{\frac{s}{\alpha}}^{1} s f(s|s_1) \, ds \right) \alpha + \Pr \left[ s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha \right) g(s_1)ds_1 - A \tau \beta
\]
\[
\geq q_{IL}^* \left( A \int_{\tau}^{\bar{s}_1} L_\alpha g(s_1)ds_1 + A \int_{\bar{s}_1}^{1} \left( \int_{\frac{s}{\alpha}}^{1} s f(s|s_1) \, ds \right) \alpha + \Pr \left[ s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha \right) g(s_1)ds_1 - A \tau \beta
\]
\[
- q_{IL}^* \left( A \int_{\tau}^{\bar{s}_1} \tau \beta g(s_1)ds_1 + A \int_{\bar{s}_1}^{1} \left( \int_{\frac{s}{\alpha}}^{1} s f(s|s_1) \, ds \right) \alpha + \Pr \left[ s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha \right) g(s_1)ds_1 - A \tau \beta
\]
\[
= q_{IL}^* A \int_{\tau}^{\bar{s}_1} (L_\alpha - \tau \beta) g(s_1)ds_1 > 0.
\]
Lastly, if \( A \in [E, A_{\text{min}}] \), we have \( q^*_L = q^*_L \) and \( \bar{s}_1 = \tau \), which implies that

\[
W_{IL}(A) = W_{EL}(A) = (1 - q^*_L) A\tau \beta
\]

\[+ q^*_L A \int_{\tau}^{1} \left( \left[ \int_{\frac{L_s}{\alpha}}^{1} s f(s | s_1) \, ds \right] \alpha + \Pr \left[ s < \frac{L_s}{\alpha} | s_1 \right] L_s \right) g(s_1) ds_1 - A. \quad (67)
\]

**Proof.** of Proposition 7: The first-order condition on \( A^*_E \) is given by:

\[
\frac{\partial W_{EL}}{\partial A} = NPV_{EL}(\bar{s}_1(A)) + q^*_L A \frac{\partial \bar{s}_1}{\partial A} \left( L_s - \left[ \int_{\frac{L_s}{\alpha}}^{1} s f(s | \bar{s}_1(A)) \, ds \right] \alpha - \Pr \left[ s < \frac{L_s}{\alpha} | \bar{s}_1(A) \right] L_s \right) g(\bar{s}_1(A))
\]

\[+ A \frac{\partial q^*_L}{\partial A} \left( \int_{\tau}^{\bar{s}_1} L_s g(s_1) ds_1 + \int_{\bar{s}_1}^{1} \left[ \int_{\frac{L_s}{\alpha}}^{1} s f(s | s_1) \, ds \right] \alpha + \Pr \left[ s < \frac{L_s}{\alpha} | s_1 \right] L_s \right) g(s_1) ds_1 - \tau \beta, \]

where

\[
NPV_{EL}(\bar{s}_1(A)) = (1 - q^*_L) \tau \beta + q^*_L \left( \int_{\tau}^{\bar{s}_1(A)} L_s g(s_1) ds_1 \right.
\]

\[+ \int_{\bar{s}_1(A)}^{1} \left( \left[ \int_{\frac{L_s}{\alpha}}^{1} s f(s | s_1) \, ds \right] \alpha + \Pr \left[ s < \frac{L_s}{\alpha} | s_1 \right] L_s \right) g(s_1) ds_1) - 1.
\]
Taking the difference between $\frac{\partial W_{EL}}{\partial A}$ and $\frac{\partial W_{IL}}{\partial A}$ gives that:

$$\frac{\partial (W_{EL} - W_{IL})}{\partial A} = \frac{\partial q_{EL}^*}{\partial A} \left( \int_{\bar{s}_1}^{s_1} L_{\alpha} g(s_1) ds_1 + A \int_{\bar{s}_1}^{\bar{s}_1} \left[ \int_{\frac{L_\alpha}{\bar{s}_1}}^{\bar{s}_1} s f(s|s_1) ds \right] \alpha + \Pr \left[ s < \frac{L_\alpha}{\bar{s}_1} \right] L_\alpha g(s_1) ds_1 - A\tau \beta \right)$$

$$- \frac{\partial q_{IL}^*}{\partial A} \left( A \int_{\bar{s}_1}^{\bar{s}_1} \tau \beta g(s_1) ds_1 + A \int_{\bar{s}_1}^{\bar{s}_1} \left[ \int_{\frac{L_\alpha}{\bar{s}_1}}^{\bar{s}_1} s f(s|s_1) ds \right] \alpha + \Pr \left[ s < \frac{L_\alpha}{\bar{s}_1} \right] L_\alpha g(s_1) ds_1 - A\tau \beta \right)$$

$$+ q_{EL}^* \left( \int_{\tau}^{\bar{s}_1} L_{\alpha} g(s_1) ds_1 + A \int_{\bar{s}_1}^{\bar{s}_1} \left[ \int_{\frac{L_\alpha}{\bar{s}_1}}^{\bar{s}_1} s f(s|s_1) ds \right] \alpha + \Pr \left[ s < \frac{L_\alpha}{\bar{s}_1} \right] L_\alpha g(s_1) ds_1 - \tau \beta \right)$$

$$- q_{IL}^* \left( A \int_{\bar{s}_1}^{\bar{s}_1} \tau \beta g(s_1) ds_1 + A \int_{\bar{s}_1}^{\bar{s}_1} \left[ \int_{\frac{L_\alpha}{\bar{s}_1}}^{\bar{s}_1} s f(s|s_1) ds \right] \alpha + \Pr \left[ s < \frac{L_\alpha}{\bar{s}_1} \right] L_\alpha g(s_1) ds_1 - \tau \beta \right)$$

$$+ q_{EL}^* \frac{\partial \bar{s}_1}{\partial A} Ag(\bar{s}_1) \left( L_{\alpha} - \left[ \int_{\frac{L_\alpha}{\bar{s}_1}}^{\bar{s}_1} s f(s|\bar{s}_1) ds \right] \alpha + \Pr \left[ s < \frac{L_\alpha}{\bar{s}_1} \right] L_\alpha g(s_1) ds_1 - \tau \beta \right)$$

$$- q_{IL}^* \frac{\partial \bar{s}_1}{\partial A} Ag(\bar{s}_1) \left( \tau \beta - \left[ \int_{\frac{L_\alpha}{\bar{s}_1}}^{\bar{s}_1} s f(s|\bar{s}_1) ds \right] \alpha + \Pr \left[ s < \frac{L_\alpha}{\bar{s}_1} \right] L_\alpha g(s_1) ds_1 - \tau \beta \right).$$

We know that for $A < A_r$, we have $q_{EL}^* > q_{IL}^*$. Hence, for $A < A_r$, we have

$$q_{EL}^* \left( \int_{\tau}^{\bar{s}_1} L_{\alpha} g(s_1) ds_1 + A \int_{\bar{s}_1}^{\bar{s}_1} \left[ \int_{\frac{L_\alpha}{\bar{s}_1}}^{\bar{s}_1} s f(s|s_1) ds \right] \alpha + \Pr \left[ s < \frac{L_\alpha}{\bar{s}_1} \right] L_\alpha g(s_1) ds_1 - \tau \beta \right)$$

$$- q_{IL}^* \left( A \int_{\bar{s}_1}^{\bar{s}_1} \tau \beta g(s_1) ds_1 + A \int_{\bar{s}_1}^{\bar{s}_1} \left[ \int_{\frac{L_\alpha}{\bar{s}_1}}^{\bar{s}_1} s f(s|s_1) ds \right] \alpha + \Pr \left[ s < \frac{L_\alpha}{\bar{s}_1} \right] L_\alpha g(s_1) ds_1 - \tau \beta \right) > 0.$$
A close to \( A_{\text{min}} \), we have

\[
q_{EL}^* \frac{\partial \bar{s}_1}{\partial A} A \bar{g}(\bar{s}_1) \left( L_\alpha - \left( \int_{\frac{L_\alpha}{\alpha}}^{1} s f(s|\bar{s}_1) \, ds \right) \alpha + \text{Pr} \left[ s < \frac{L_\alpha}{\alpha} | \bar{s}_1 \right] L_\alpha \right)
\]

\[
- q_{IL}^* \frac{\partial \bar{s}_1}{\partial A} A \bar{g}(\bar{s}_1) \left( \tau \beta - \left( \int_{\frac{L_\alpha}{\alpha}}^{1} s f(s|\bar{s}_1) \, ds \right) \alpha + \text{Pr} \left[ s < \frac{L_\alpha}{\alpha} | \bar{s}_1 \right] L_\alpha \right)
\]

\[
= (q_{EL}^* - q_{IL}^*) \frac{\partial \bar{s}_1}{\partial A} A \bar{g}(\bar{s}_1) \left( L_\alpha - \left( \int_{\frac{L_\alpha}{\alpha}}^{1} s f(s|\bar{s}_1) \, ds \right) \alpha + \text{Pr} \left[ s < \frac{L_\alpha}{\alpha} | \bar{s}_1 \right] L_\alpha \right)
\]

\[
+ q_{IL}^* \frac{\partial \bar{s}_1}{\partial A} A \bar{g}(\bar{s}_1) (L_\alpha - \tau \beta).
\]

The second term is positive while the first term is negative. The second term dominates the first term if \( q_{EL}^* - q_{IL}^* \) is sufficiently small, which holds when \( A \) close to \( A_{\text{min}} \).

Lastly, we know that

\[
C''(q_{EL}^*) \frac{\partial q_{EL}^*}{\partial A} = C''(q_{IL}^*) \frac{\partial q_{IL}^*}{\partial A} + \int_{\tau}^{s_1(A)} [L_\alpha - 1 - \tau(\beta - 1)] g(s_1) \, ds_1
\]

\[
+ \frac{\partial \bar{s}_1}{\partial A} (A(L_\alpha - \tau \beta) - (1 - \tau)(A - E)) g(s_1(A)), \quad (70)
\]

which implies that for \( A \) close to \( A_{\text{min}} \), we have \( \frac{\partial q_{EL}^*}{\partial A} > \frac{\partial q_{IL}^*}{\partial A} \). Hence, for \( A \) close to \( A_{\text{min}} \), we have

\[
\frac{\partial q_{EL}^*}{\partial A} \left( A \int_{\tau}^{s_1} L_\alpha g(s_1) \, ds_1 + A \int_{s_1}^{1} \left( \int_{\frac{L_\alpha}{\alpha}}^{1} s f(s|s_1) \, ds \right) \alpha + \text{Pr} \left[ s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha \right) g(s_1) \, ds_1 - A\tau \beta
\]

\[
- \frac{\partial q_{IL}^*}{\partial A} \left( A \int_{\tau}^{s_1} \tau \beta g(s_1) \, ds_1 + A \int_{s_1}^{1} \left( \int_{\frac{L_\alpha}{\alpha}}^{1} s f(s|s_1) \, ds \right) \alpha + \text{Pr} \left[ s < \frac{L_\alpha}{\alpha} | s_1 \right] L_\alpha \right) g(s_1) \, ds_1 - A\tau \beta
\]

\[
> 0.
\]

Therefore, for all \( A \) close to \( A_{\text{min}} \), we get

\[
\frac{\partial (W_{EL} - W_{IL})}{\partial A} > 0, \quad (71)
\]
i.e., there exists a cutoff $A_+ \in (A_{\min}, A_e)$ such that, for all $A \in [E, A_+)$, we have $\frac{\partial (W_{EL} - W_{IL})}{\partial A} > 0$.

Finally, suppose that $A_{IL}^* \in [E, A_+]$ and $\frac{\partial^2 W_{EL}}{\partial A^2} < 0$. Plugging $A = A_{IL}^*$ into $\frac{\partial W_{EL}}{\partial A}$ gives that

$$\frac{\partial W_{EL}}{\partial A}|_{A=A_{IL}^*} > \frac{\partial W_{IL}}{\partial A}|_{A=A_{IL}^*} = 0.$$  \hfill (72)

The inequality uses $\frac{\partial (W_{EL} - W_{IL})}{\partial A} > 0$ for $A < A_+$. By the concavity of $W_{EL}$, $\frac{\partial W_{EL}}{\partial A}|_{A=A_{IL}^*} > 0 = \frac{\partial W_{EL}}{\partial A}|_{A=A_{EL}^*}$ implies that $A_{IL}^* < A_{EL}^*$. Since $W_{EL}(A_{EL}^*) > W_{EL}(A_{IL}^*)$, Proposition 6 implies that $W_{EL}(A_{EL}^*) > W_{EL}(A_{IL}^*) > W_{IL}(A_{IL}^*)$. ■