On the Relation between Earnings Smoothing and Investment Efficiency*

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*Job Market Paper.
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Abstract

Empirical evidence documents that managers exploit reporting discretion to convey incremental information. When the manager makes both disclosure and investment choices, and is privately informed about both current earnings (hard information) and future investment profitability (soft information), I show that the equilibrium reporting bias is a function of the latter. By smoothing earnings and distorting investment, the manager credibly signals soft information. The extent of smoothing determines the amount of soft information revealed by earnings, thereby affecting the residual signaling distortion borne by investment. I find that changes in misreporting costs and in the precision of accounting information may have, through their impact on earnings management, non-monotonic effects on investment efficiency. Further, greater managerial myopia may actually be associated with better long-term performance. I also provide testable predictions regarding the effect of observable firm’s characteristics on the earnings response coefficient and the amount of reporting bias.

Keywords: reporting discretion, reporting bias, earnings management, capital expenditure, earnings smoothing, investment efficiency, misreporting cost, accounting quality, earnings response coefficient.

JEL classification: D61, D82, G14, M41, M48.
1 Introduction

The flexibility of Generally Accepted Accounting Principles allows managers to exercise a certain level of discretion when reporting financial performance to the public. Seemingly, the predominant view on earnings management is that opportunistic managers manipulate financial statements to achieve personal goals, such as disguising the firm’s actual performance (e.g., Teoh et al. (1998), Nelson et al. (2002), Hribar and Jenkins (2004), Badertscher (2011)), meeting-or-beating benchmarks (e.g., Burgstahler and Dichev (1997), Degeorge et al. (1999), Matsumoto (2002)), or increasing their bonus payments (e.g., Healy (1985), Holthausen et al. (1995), Guidry et al. (1999)). Opportunistic earnings manipulations are likely to diminish the quality of financial information. For this reason, standard setters usually regard the absence of bias as a characteristic that contributes to the usefulness of financial reports (see the Financial Accounting Standards Board’s Statement of Financial Concepts No. 8, QC14).

Nonetheless, the empirical accounting literature has also documented evidence suggesting that, in fact, reporting discretion may play a positive role in financial markets. For instance, Subramanyam (1996) finds that discretionary accruals are priced by the market and positively associated with future profitability; Louis and Robinson (2005) show that higher discretionary accruals increase the positive market reaction to a subsequent stock split announcement; Badertscher et al. (2012) conclude that, in the absence of meet-or-beat incentives, restated earnings are less predictive of future cash flows than the originally reported numbers. Taken as a whole, this research supports the idea that, at least under some circumstances, managers actually exploit reporting discretion to incorporate private forward-looking information that unbiased disclosures would not be able to capture.¹

Consistent with the latter strand of literature, this paper shows that the reporting bias can convey value-relevant information. More specifically, my main research question is twofold. First, under what circumstances does a manager engage in costly misreporting to credibly

¹Hunt et al. (2000), Tucker and Zarowin (2006), Louis and White (2007), Bowen et al. (2008), Gunn (2016), and Perotti and Windisch (2016) also corroborate empirically the hypothesis that reporting discretion allows managers to signal private information.
communicate his private information about an investment opportunity? Second, how do observable firm’s characteristics affect earnings smoothness and investment efficiency?

**Model overview.** To address these questions, I study a multi-dimensional signaling model in which a manager jointly takes financial reporting and investment decisions. The firm has assets in place that generate cash flows over time according to an exogenous stochastic process. In addition, the firm faces an investment opportunity, which generates additional cash flows at a late point in time. Two factors determine the investment return: the level of capital expenditure, an endogenous variable that the manager chooses; and investment productivity, an exogenously determined stochastic parameter that affects the marginal profitability of capital expenditure. The manager is endowed with two-dimensional private information, consisting in a hard signal about the current performance of the assets in place, and a soft signal about investment productivity. Conditional on these pieces of information, the manager reports his hard signal to the market and chooses the capital expenditure level. His objective function reflects short-term stock price incentives as well as long-term value. Communication need not be truthful, in that the manager may introduce a bias in his earnings report, albeit at a personal cost. Soft information is instead completely unverifiable, implying that it cannot be disclosed credibly to the public. As a consequence, the manager must engage in costly signaling in order to credibly convey his private soft information. Signaling takes place through both costly earnings manipulations and investment distortions.

**Summary of main findings.** In equilibrium, the extent of earnings manipulation is a function of the manager’s private soft information, signifying that the reporting bias itself has incremental information content relative to unmanaged earnings (Proposition 1). This result originates from the interaction between the two signaling devices – reported earnings and capital expenditure. The intuition is as follows. As the investment scale increases, the manager’s incentives to signal soft information about investment productivity become larger. Since soft information affects capital expenditure, the reporting bias is ultimately a function
of soft information.

The manager’s equilibrium misreporting behavior may take the particular form of earnings smoothing (Proposition 2). Prior literature has empirically documented earnings smoothing (e.g., Beidleman (1973), Ronen and Sadan (1981), Healy (1985), DeFond and Park (1997)), and earnings smoothness is thought to be related to earnings quality (Dechow et al. (2010)). Intuitively, there are two opposing forces at work that influence the manager’s reporting strategy. First, the misreporting cost incentivizes truthtelling. Second, managerial short-termism incentivizes misreporting. In principle, if unmanaged earnings are higher, then the manager would like to increase reported earnings to avoid the misreporting cost. However, while higher reported earnings indicate better current performance, if not paired with greater capital expenditure they also imply unfavorable soft information. Thus, for any given level of capital expenditure, the manager has incentives to underreport any above-average earnings realization, so as to let the market infer higher investment productivity. This signaling explanation of earnings smoothing finds support in the survey evidence from Graham et al. (2005). According to the executives they interviewed, conveying higher growth prospects is one of the most common motivations to smooth earnings.

My analysis unveils an endogenous relation between earnings smoothing and investment efficiency. For the manager to signal his private information in a credible manner, he has to incur dissipative costs. Therefore, in models where investment is a signaling device, it is usually the case that unobservability of the manager’s information leads to a lower ex ante expected investment return, relative to a situation in which his information is public (e.g., see Kanodia and Lee (1998)). Surprisingly, in my model investment is not necessarily inefficient. Indeed, I show that there exist parameter values such that the manager’s equilibrium investment strategy does not suffer from any signaling distortion (Lemma 1). This occurs when the manager’s incentives to report truthfully (to avoid the misreporting cost) and his incentives to misreport (to signal favorable soft information) exactly offset each other. When these countervailing incentives are of equal magnitude, reported earnings cease to be
a function of unmanaged earnings. At the same time, they become a perfect public signal of the manager’s soft information. Given that one signaling device – reported earnings – bears the entire signaling cost, the other signaling device – capital expenditure – does not need to be distorted. In this way, full investment efficiency is restored even when the manager’s information is private.

Comparative statics on investment efficiency yields interesting non-monotonic predictions (Proposition 3). For instance, a higher misreporting cost is beneficial at first, but once it reaches a certain threshold, a further increase lowers investment efficiency (Propositions 3(2a) and 3(1a), respectively). In a similar fashion, higher precision of accounting information may be beneficial or detrimental for investment efficiency, depending on the case (Propositions 3(1b) and 3(2a), respectively). Furthermore, investment efficiency may be non-monotonic even in managerial myopia, the latter being defined as the relative importance of short-term versus long-term value in the manager’s objective function. This finding is at odds with the common wisdom, which might suggest that executives’ short-term focus sacrifices the firm’s value. In contrast, I show that greater myopia may be associated with better investment efficiency (Proposition 3(1c)).

A growing body of empirical research has considered the impact of financial reporting quality on firm value (e.g., Bens and Monahan (2004), Biddle and Hilary (2006), McNichols and Stubben (2008), Biddle et al. (2009), Beatty et al. (2010), Francis and Martin (2010), Bushman et al. (2011), García Lara et al. (2016)). The general approach in this literature is to posit a causal relation: by reducing information asymmetries between managers and shareholders or external capital suppliers, greater financial reporting quality mitigates the investment inefficiencies due to moral hazard and adverse selection. A contribution of this paper is to show that, on the contrary, both informational and investment efficiency are determined endogenously. My model generates testable predictions regarding the effects of observable firm-specific characteristics – such as the volatility and persistence of cash flows, misreporting costs, the precision of accounting information, and the uncertainty about future
profitability – on the extent of earnings smoothing (Corollary 1) and investment efficiency (Proposition 3).

Last, my comparative statics results suggest possible empirical tests involving the earnings response coefficient (Corollary 2) and the amount of reporting bias (Corollary 3). Among other results, I predict that the earnings response coefficient may be non-monotonic in the volatility of cash flows (Corollaries 2(1b) and 2(1a)). Typically, analytical models of disclosure conclude that greater cash flows volatility is associated with a stronger market reaction to earnings (e.g., Holthausen and Verrecchia (1988), Fischer and Verrecchia (2000)). My prediction that greater cash flows volatility may, in fact, reduce the market reaction to earnings news is consistent with the empirical findings in Hunt et al. (2000).

Structure of the paper. The paper is structured as follows. Section 2 summarizes the related literature. Section 3 introduces the model and notation. Section 4 solves for the unique linear equilibrium and establishes the equilibrium property that the reporting bias contains value-relevant forward-looking information. Section 5 provides conditions under which earnings smoothing occurs in equilibrium and presents several empirical predictions. Finally, Section 6 concludes.

2 Related Literature

My paper intersects the following four broad areas of theoretical accounting research.

Costly misreporting. Stein (1989), Fischer and Verrecchia (2000), Dye and Sridhar (2004) study models of costly earnings management. These papers do not capture the feature that the reporting bias conveys value-relevant information, as in their equilibria bias is independent of the underlying value.
**Earnings smoothing.** Various forms of earnings smoothing can arise in “internal” agency settings between management and current shareholders (Lambert (1984), Dye (1988), Fudenberg and Tirole (1995), Evans and Sridhar (1996), Demski (1998)), or in “external” agency settings between current shareholders and external parties, such as prospective shareholders or lenders. For the case of external agency, earnings smoothing obtains under uncertainty about the variance of the firm’s terminal value (Trueman and Titman (1988), Kirschenheiter and Melumad (2002), Beyer (2009)), with manager’s risk aversion (Sankar and Subramanyam (2001)), or with exogenously specified managerial incentives to follow a smooth earnings path (Ewert and Wagenhofer (2015, 2016)). By contrast, I abstract away from internal agency issues and do not assume uncertainty about the second moment. Besides, I make the assumption of manager’s risk neutrality without imposing an exogenous preference for smoothing. I find that smoothing incentives can arise endogenously from the interaction between financial reporting and investment.

**Welfare implications of reporting discretion.** It has been shown that reporting discretion can improve firm value in internal agency settings (Verrecchia (1986), Dye and Verrecchia (1995), Arya et al. (1998), Demski and Frimor (1999), Dutta and Gigler (2002), Liang (2004)), under noisy measurement of investment (Liang and Wen (2007)), or in a multi-firm economy (Dye and Sridhar (2008)). Ewert and Wagenhofer (2005) argue that less accounting discretion encourages real earnings management. The distinctions between real earnings management and the capital expenditure in my model are that real earnings management is unobservable by the market and that there is no private information about the (negative) consequences of engaging in it. Stocken and Verrecchia (2004) find that removing reporting discretion alters the manager’s choice of the precision of accounting information, in turn affecting the efficiency with which external investors allocate resources to the firm. Here, instead, the precision of accounting information is exogenous. Moreover, it is the manager who selects the capital expenditure: as opposed to external investment responding to the
financial report, here report and investment are complementary sources of information.

**Signaling and disclosure.** The signaling of private managerial information has been analyzed in the context of mandatory disclosure (Kanodia and Lee (1998), Kanodia et al. (2005), Guttman et al. (2006), Budanova et al. (2016), Jiang and Yang (2016)) and voluntary disclosure (Beyer and Guttman (2012), Einhorn and Ziv (2012), Cianciaruso and Sridhar (2015)). With the exception of Kanodia and Lee (1998), these are models where the manager transmits his private information only through one signaling device. Kanodia and Lee (1998), however, do not allow for earnings manipulation.

### 3 Model

A firm lives for three dates, $t \in \{1, 2, 3\}$. The firm is run by an initial owner/manager who takes both financial reporting and investment decisions. The firm’s shares are traded on the capital market at dates 1 and 2, whereas at date 3 the firm is liquidated. All players – the manager and outside investors – are risk neutral. I next describe the model and comment its salient features. Appendix A contains a summary of the notation used.

#### 3.1 Model Description

**Cash Flows.** Throughout, I assume that all random variables are jointly normally distributed. The firm’s cash flows derive from its assets in place and from an investment opportunity. While the manager’s investment decision affects the distribution of cash flows from investment, the cash flows from assets in place derive from existing assets and, therefore, their distribution is taken as exogenous.

At each date $t \in \{2, 3\}$, the assets in place generate cash flows $\tilde{x}_t$. Cash flows are given by the following process,

$$\tilde{x}_t = \mu_0 + \sqrt{\rho_0} \tilde{\theta} + \sqrt{1 - \rho_0} \tilde{\eta}_t,$$
where $\tilde{\theta}$ is a permanent (i.e., time-independent) component and $\tilde{\eta}_t$ is a transitory (i.e., time-specific) component.\(^2\) I assume that the random variables $\left(\tilde{\theta}, \tilde{\eta}_t\right)$ are independent, with $E\left[\tilde{\theta}\right] = E\left[\tilde{\eta}_t\right] = 0$ and $\text{Var}\left[\tilde{\theta}\right] = \text{Var}\left[\tilde{\eta}_t\right] = \sigma_0^2$. In this way, $\mu_0$ parametrizes the mean of cash flows from assets in place, $\sigma_0$ their volatility, and $\rho_0$ their correlation across dates.

The precision (i.e., the inverse of the variance) of cash flows from the assets in place, is denoted $\tau_0 \equiv \sigma_0^{-2}$.

The cash flows from investment are a function of investment productivity, denoted $\tilde{\xi}$, and of the capital expenditure, denoted $k$. The manager has control over capital expenditure: $k$ is chosen at date 1, implemented at date 2, and the investment yields cash flows at date 3. This lag between the investment decision and its implementation reflects the notion that capital is fixed in the short run, in that it may take a significant amount of time to adjust a production process or to set up a new line of business. The capital expenditure decision taken at date 1 is irreversible (that is, it cannot be modified before the implementation at date 2).\(^3\) If the manager chooses capital expenditure $k$, then the gross investment return is distributed as

$$\tilde{\xi} k - \frac{1}{2} k^2.$$ 

Investment productivity is given by

$$\tilde{\xi} = \mu_k + 1 + \sigma_k \left[\left(1 - \delta_k\right) \frac{\tilde{x}_3 - \mu_0}{\sigma_0} + \delta_k \tilde{u}\right],$$

where $\tilde{u}$ is independent of $\left(\tilde{\theta}, \left\{\tilde{\eta}_t\right\}_{t=1}^3\right)$, and such that $E\left[\tilde{u}\right] = 0$, $\text{Var}\left[\tilde{u}\right] = 1$. With this notation, $\mu_k + 1$ is the mean of investment productivity and $\sigma_k$ is its volatility. Throughout, I assume $\mu_k > 0$. Also, note that investment productivity $\tilde{\xi}$ is potentially correlated with the date-3 cash flows from the assets in place. That is, the return on investment, which is realized at date 3, is affected by the prevailing productivity of the firm’s assets in place.

\(^2\)Throughout, I use tildes to denote random variables and drop the tilde when referring to their realizations.

\(^3\)Irreversibility of the capital expenditure decision ensures that, in equilibrium, $k$ is a function of only the information available to the owner at date 1.
The correlation between the date-3 cash flows from the assets in place $\tilde{x}_3$ and investment productivity is given by $1 - \delta_k$, where the parameter $\delta_k$ represents the relevance of soft information, relative to hard information. As $\delta_k$ increases, investment productivity becomes more sensitive to soft information and less sensitive to hard information.

The net investment return, denoted $\tilde{y}$, is simply the gross return less the capital expenditure $k$,

$$\tilde{y} = \left(\tilde{\xi} - 1\right) k - \frac{1}{2} k^2.$$ 

Overall, the firm’s liquidating dividends, denoted by $\tilde{L}$, are given by the sum of all cash flows from the assets in place and the net investment return,

$$\tilde{L} = \tilde{x}_2 + \tilde{x}_3 + \tilde{y}.$$ 

**Stock prices.** Since all players are risk-neutral, prices at each date $t$ are determined as the expectation of the liquidating dividends $\tilde{L}$ conditional on all publicly available information until that point in time, that is,

$$P_t = \mathbb{E} \left[ \tilde{L} \mid \mathcal{I}_{p,t} \right],$$

where $\mathcal{I}_{p,t}$ denotes the outside investors’ information set at date $t$ (the subscript $p$ stands for “public”). Below, I describe the content of the public information set $\mathcal{I}_{p,t}$, as well as what information the manager privately knows.

**Timeline and information structure.** The sequence of events in this game is depicted in Figure 1. At date 2, the firm’s assets in place generate cash flows $\tilde{x}_2$. At date 1, the accounting system produces a noisy signal $\tilde{s}_1$ about $\tilde{x}_2$, given by

$$\tilde{s}_1 = \tilde{x}_2 + \tilde{\epsilon}_1,$$
where $\tilde{\varepsilon}_1$ is a noise component with $E[\tilde{\varepsilon}_1] = 0$ and $\text{Var}[\tilde{\varepsilon}_1] = \tau_{\varepsilon}^{-1}$. The inverse of the variance of the noise component, $\tau_{\varepsilon}$, will be referred to as the precision of accounting information. At date 1, the manager observes the realization of $\tilde{s}_1$ and reports it to the market. The manager may privately take actions aimed at misreporting the realized $\tilde{s}_1$, albeit at a cost (I provide the details of the manager’s payoff below). The extent of misreporting is denoted by $b$, and will be referred to as bias. The manager’s report, $r_1$, will be referred to as reported earnings. By definition, we have

$$r_1 = s_1 + b.$$

In addition to the observing the realization of $\tilde{s}_1$, at date 1 the manager privately learns the realization of the component $\tilde{u}$ of investment productivity. I assume that $\tilde{u}$ is soft information and, as such, cannot be disclosed credibly to the market.

Given his private two-dimensional information $(s_1, u)$, the manager takes both the reporting decision $r_1$ and the private investment decision $k$. Then, the firm issues financial statements, which publicly reveal the earnings report $r_1$. The stock price $P_1$ is formed conditional on $r_1$, which at that point in time is the only information available to investors. Formally, outside investors’ date-1 information set is $\mathcal{I}_{p,1} = \{r_1\}$. Note that at date 1 the market does not observe the investment decision $k$ – it will become public at date 2, when the investment is implemented and recognized by the accounting system as capital expenditure.

At date 3, cash flows from the assets in place $\tilde{x}_3$ are realized. At date 2 the accounting system produces a signal $\tilde{s}_2 = \tilde{x}_3 + \tilde{\varepsilon}_2$, where $(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2)$ are identically distributed. Again, the manager privately observes the realization of $\tilde{s}_2$ and reports $r_2$ to the market. To capture the feature that GAAP rules impose dynamic constraints on misreporting, and in line with prior literature (e.g., Sankar and Subramanyam (2001), Kirschenheiter and Melumad (2002), Ewert and Wagenhofer (2015, 2016)), I assume that date-1 accounting noise and bias reverse in the subsequent reporting period. Hence, the manager does not have reporting discretion at date 2: if the realized $\tilde{s}_2 = s_2$, and if the accounting noise and bias are, respectively, $\varepsilon_1$
and $b$, then at date 2 the manager must report

$$r_2 = s_2 - \varepsilon_1 - b.$$  \hspace{1cm} (1)

The firm’s date-2 financial statements reveal three pieces of information: the realized cash flows, $x_2$; the capital expenditure, $k$; and reported earnings, $r_2$. Therefore, the date-2 stock price equals the expected liquidating dividends conditional on $\mathcal{I}_{p,2} = \{r_1, x_2, k, r_2\}$.

At date 3 cash flows $\tilde{x}_3$ from the assets in place and the net investment return $\tilde{y}$ are realized. The firm is liquidated and the sum of all cash flows is paid out to shareholders as dividends.

All the random variables $\theta, \{\tilde{\eta}_t\}_{t=1}^3, \tilde{u}, \{\tilde{\varepsilon}_t\}_{t=1}^2$ are mutually independent.

**Preferences.** Following the extant theoretical research on earnings management (e.g., Stein (1989), Fischer and Verrecchia (2000) and Dye and Sridhar (2004)), I assume a quadratic cost of misreporting. If the realized signal $\tilde{s}_1 = s_1$ and the manager chooses to bias earnings by the amount $b$, then the manager bears a personal misreporting cost of

$$\frac{1}{2} \gamma b^2,$$

where, by construction, $b = r_1 - s_1$. Among other things, this misreporting cost may capture the effort cost, the reputation loss, as well as regulatory and litigation risks resulting from the misrepresentation of financial information. The parameter $\gamma$ measures the magnitude of all these effects. A higher $\gamma$ may be the consequence of stricter regulation, such as the Sarbanes-Oxley Act of 2002, or tighter accounting standards (Ewert and Wagenhofer (2005)). I refer to $\gamma$ as the marginal cost of misreporting.

I capture short-termism by assuming that the manager values a convex combination of the short-term price $P_2$ and the liquidating dividend $L$. Letting $\mathcal{I}_{m,t}$ denote the manager’s information set at date $t$ (the subscript $m$ stands for “manager”), the manager’s preferences
Accounting system produces signal $s_1$ about date-2 cash flows from assets in place $\tilde{x}_2$.

Manager privately observes $s_1$ and soft information $u$.

Manager issues public report $r_1$ about $s_1$ and privately takes the irreversible investment decision $k$.

Stock price $P_1$ is formed conditional on $\{r_1\}$.

Cash flows from assets in place $x_2$ are realized.

Investment decision $k$ is implemented.

Accounting system produces signal $s_2$ about date-3 cash flows from assets in place $\tilde{x}_3$.

Manager privately observes $s_2$ and issues report $r_2$ about $e_2$.

Financial statements publicly reveal cash flows $x_2$, capital expenditure $k$, and reported earnings $r_2$.

Stock price $P_2$ is formed conditional on $\{r_1, x_2, k, r_2\}$.

Cash flows from assets in place $x_3$ and net investment return $y$ are realized.

Firm is liquidated and total cash flows $x_2 + x_3 + y$ are paid out as dividends.

Figure 1: Timeline of events.
at date 1 can then be represented as

\[
E \left[ \alpha \tilde{P}_2 + (1 - \alpha) \tilde{L} \mid \mathcal{I}_{m,1} \right] - \frac{2}{2} b^2, \quad (2)
\]

where \( \mathcal{I}_{m,1} = \{ s_1, u, r_1, k \} \). Observe that, apart from conditioning on his private information \((s_1, u)\), in (2) the manager also conditions on reported earnings \(r_1\) and capital expenditure \(k\), as he chooses these variables.

The parameter \( \alpha \in (0, 1) \) denotes the degree of managerial myopia. As \( \alpha \) increases, the manager values relatively more the short-term stock performance rather than the firm’s actual value. This formulation is common in the literature and can motivated in multiple ways that are analytically equivalent.\(^4,5\)

### 3.2 Discussion of Main Modeling Features

In the current subsection, I highlight (four of) the main features of my model in order to discuss how they relate to and differ from the existing literature.

**Soft information.** Previous research has investigated the implications of communication restrictions when the manager possesses private information which is not recognized by the accounting system. In the contracting framework of Demski (1998), constraints on the message space lead to the inapplicability of the revelation principle. In my model, as is the case for Stocken and Verrecchia (2004) and Ewert and Wagenhofer (2015, 2016), the assumption that the manager’s soft information is soft precludes its credible disclosure to

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\(^4\)Similar to Gigler et al. (2014), I could assume that at date 1 it is common knowledge among all players that the initial owner may be either of the following two types: with probability \( \alpha \), he is a short-term investor, who must sell his holdings at date 2; with the remaining probability \( 1 - \alpha \), he is a long-term investor who, instead of trading shares, values consumption of the liquidating dividends. Alternatively, the initial owner could sell a known fraction \( \alpha \) of his shares at date 2 (e.g., Dye and Sridhar (2007)), or he could face a takeover at date 2 with probability \( \alpha \) (e.g., Stein (1989)). In either case, the resulting owner’s preferences would be as described in the main text.

\(^5\)Consistent with the practice of lockup agreements, which prohibit insiders from selling their shares in the months immediately following an initial public offering, I assume that the owner cannot trade in the early phase of the firm’s life (date 1).
the market. This inability to credibly disclose private soft information creates incentives for the manager to engage in costly signaling: for the manager to credibly convey his soft information, he must take dissipative actions. In Stocken and Verrecchia (2004) and Ewert and Wagenhofer (2015, 2016) the only dissipative action is misreporting, whereas in this paper I allow the manager to both misreport and choose an inefficient investment level.

**Misreporting cost.** The assumption that the misreporting cost is proportional to the square of the bias also appears in Stein (1989) and Fischer and Verrecchia (2000). Since in Stein (1989) the manager’s preferences are common knowledge, in equilibrium the manager adds a constant bias to his private signal. Consequently, the market is able to perfectly infer the manager’s private signal by subtracting the equilibrium bias from reported earnings. To prevent full revelation, Fischer and Verrecchia (2000) assume that the market is uncertain about the manager’s marginal benefit from manipulating the price. In Dye and Sridhar (2004), the manager’s benefit from price manipulation is known, but reports are not fully revealing because the market is uncertain about the manager’s cost of misreporting. By contrast, in my model both the weights on prices in the manager’s objective function and the misreporting cost are commonly known. Nevertheless, reported earnings $r_1$ are not fully revealing: in order to affect the market’s perception of investment productivity, the manager biases the report by an amount which is a function of the private soft information $\tilde{u}$. Put differently, it is the soft information conveyed through earnings manipulation that makes reported earnings a noisy signal of hard information. This feature distinguishes my results from the three abovementioned papers, where the equilibrium bias is uninformative about firm value.

**Correlation.** From Benchmark 2 in Section 4.1 it will be clear that, in my model, a necessary condition for the reporting bias to convey private soft information is that the cash flows from the assets in place be both intertemporally correlated and correlated with investment productivity. The latter assumption differs from Liang and Wen (2007), who
assume that the performance of the assets in place and the productivity of the investment opportunity are independent. As to the former assumption, Kirschenheiter and Melumad (2002) also model cash flows as a process with a permanent and transitory components. Their earnings smoothing result relies on the variance of the transitory components being uncertain. In contrast, here such a variance is fixed and earnings smoothing stems from the interaction between financial reporting and the investment decision.

Multiple sources of information. Prior literature on earnings management has typically focused on the representative case in which earnings are the sole source of information for investors. In this paper, I relax this assumption by letting some cash flows be realized and publicly observable at an interim date. Moreover, the market price also conditions on capital expenditure. These features capture the idea that, in reality, financial statement users form beliefs by combining multiple sources of information, such as the income statement, balance sheet, and statement of cash flows.\footnote{In principle, I could allow for part of the date-1 cash flows to be realized at the same date 1, so that the stock price would be formed conditional on earnings and cash flows even at that point in time. This, however, would add an extra layer of complexity to the analysis without altering the substance of the results.}

4 Analysis

In line with prior research involving rational expectations equilibria, I focus my attention on equilibria in linear strategies. Here, a linear equilibrium takes the following form,

\[
\begin{align*}
    r_1 &= A_r + B_r (s_1 - \mu_0) + C_r u \\
    k &= A_k + B_k (s_1 - \mu_0) + C_k u
\end{align*}
\]

where \((A_r, B_r, C_r, A_k, B_k, C_k)\) are endogenous parameters to be determined. In words, equation (3) states that the manager’s reporting strategy \((r_1)\) and investment strategy \((k)\) are linear function of the manager’s date-1 private information, \((s_1, u)\). Three remarks are in order. First, given a linear date-1 reporting strategy, the date-2 reporting strategy \((r_2)\) is
also linear, due to the intertemporal constraint (1). Second, linearity of the manager’s reporting and investment strategies does not imply that prices are a linear function of public information. As a matter of fact, the pricing equation will have quadratic terms because the investment return \( \tilde{y} \) involves products of random variables. Third, since both \( \tilde{s}_1 \) and \( \tilde{u} \) are normally distributed, the equilibrium investment \( \tilde{k} \) will be negative with positive probability. Negative investment is allowed to avoid the technical issues associated with the truncation of normals (Kanodia et al. (2005)). Still, the probability of investment taking negative values can be made arbitrarily small by assuming that the mean of \( \tilde{k} \) is sufficiently high relative to its variance (e.g., see Stocken and Verrecchia (2004), Dye and Sridhar (2008), and Liang and Wen (2007)).

The goal of this section is to establish existence (and uniqueness) of an equilibrium in which the coefficient \( C_r \neq 0 \), signifying that, in equilibrium, the reporting bias

\[
\tilde{b} = \tilde{r}_1 - \tilde{s}_1 = A_r - B_r \mu_0 + (B_r - 1) \tilde{s}_1 + C_r \tilde{u} \tag{4}
\]

is correlated with firm value and, in particular, it incorporates the manager’s private soft information \( \tilde{u} \).

To fix ideas, before I begin solving the main model, I briefly analyze several benchmark settings in which the latter feature of the equilibrium actually does not arise (i.e., \( C_r = 0 \) in these benchmarks).

### 4.1 Benchmarks

The result that equilibrium reporting bias is a function of the manager’s private soft information originates from the combined effect of four assumptions. I argue this formally in Section 4.2 below. The purpose of the benchmarks in this section is to show that whenever one of these assumptions is not satisfied, we have instead \( C_r = 0 \). To emphasize the primary
role of these four assumptions, I now state them formally.

**Assumption 1** *Capital expenditure $k$ is endogenous.*

**Assumption 2** *The date-1 signal $\tilde{s}_1$ is informative about investment productivity $\tilde{\xi}$ (i.e., $\rho_0 > 0$ and $\delta_k < 1$).*

**Assumption 3** *The manager partially values the date-2 stock price $\tilde{P}_2$ (i.e., $\alpha > 0$).*

**Assumption 4** (1) *The manager partially values consumption of the liquidating dividends $\tilde{L}$ (i.e., $\alpha < 1$); and (2) soft information $\tilde{u}$ is relevant (i.e., $\delta_k > 0$).*

Each of the following benchmarks deals with a situation in which a single one of the above Assumptions 1-4 is violated, while all others are satisfied.

**Benchmark 1 (Assumption 1 violated)** In equilibrium, the reporting bias is proportional to capital expenditure. When investment is endogenous, in the sense that the manager can choose the capital expenditure, the choice of $k$ signals his private information about investment productivity, which includes the soft information $u$. We then have that the reporting bias depends on $u$ because it depends on a variable, $k$, which itself is a function of $u$. Oppositely, when investment is exogenously given, $k$ cannot signal the manager’s private information, and consequently the bias is not a function of $u$.

**Benchmark 2 (Assumption 2 is violated)** In this model, the manager issues a date-1 report that deviates from his hard information in order to manipulate the market’s beliefs about soft information $u$. If both reported earnings $r_1$ and capital expenditure $k$ are functions of both financial and soft information $(s_1, u)$, then in equilibrium the market combines these two sources of information to infer $u$. Yet, if the date-1 signal is uninformative about investment productivity – be it because all cash flows from the assets in place are transitory ($\rho_0 = 0$), or because they are uncorrelated with investment productivity ($\delta_k = 1$) – then capital expenditure will only depend on $u$. This implies that the market disregards $r_1$, $s_1$, and $u$.
as capital expenditure alone suffices to infer the soft information. In turn, this effectively removes all incentives for the manager to misreport earnings, so that in equilibrium \( b = 0 \).

**Benchmark 3 (Assumption 3 is violated)** If the manager is fully forward looking \((\alpha = 0)\), then he is not interested in manipulating the market’s perception of soft information. As in Benchmark 2, we have no misreporting.

**Benchmark 4 (Assumption 4(1) is violated)** When the manager is fully myopic \((\alpha = 1)\), he does not bear the monetary consequences of his investment decision. It follows that the manager cannot credibly signal his private soft information \( u \) and we obtain a situation similar to Benchmark 1, in which \( k \), and therefore \( r_1 \), do not depend on \( u \).

**Benchmark 5 (Assumption 4(2) is violated)** If the cash flows from the assets in place are perfectly correlated with investment productivity \((\delta_k = 0)\), then the soft information \( u \) is irrelevant in terms of the firm’s value. In equilibrium, capital expenditure does not signal such value-irrelevant information, and hence \( u \) is not incorporated in the reporting bias.

### 4.2 Equilibrium of the Main Model

I solve for the (unique) equilibrium of the game using the standard “guess and verify” method. The initial step consists in supposing the existence of an equilibrium where the market conjectures that the manager’s reporting and investment strategies are of the linear form in \((3)\). The second step involves determining the market’s date-2 expectation of the firm’s value, \( P_2 \), taking as given the market’s conjecture of the manager’s strategies. In the third step, I compute the manager’s date-1 expected payoff in \((2)\), taking as given the date-2 price \( P_2 \). Lastly, I find the manager’s optimal choices of \((r_1,k)\) and solve for the endogenous parameters \((A_r,B_r,C_r,A_k,B_k,C_k)\) by equating these coefficients with the corresponding coefficients in the manager’s best replies. All proofs can be found in Appendix B.
At date 2, the market’s information set is \( \mathcal{I}_{p,2} = \{ r_1, x_2, k, r_2 \} \). As it turns out, in equilibrium the manager’s reporting and investment choices \((r_1, k)\) reveal the manager’s date-1 private information. This is due to the fact that the manager possesses two-dimensional private information, \((s_1, u)\), and he has access to two signaling devices, \((r_1, k)\): taking as given the coefficients \((A_r, B_r, C_r, A_k, B_k, C_k)\), the market can solve the system of two equations (3) for the two unknowns \((s_1, u)\) as a function of \((r_1, k)\), both of which are observable at date 2. In particular, given a pair \((r_1, k)\), the market infers that the manager’s soft information is

\[
u = \frac{B_r (k - A_k) - B_k (r_1 - A_r)}{B_r C_k - B_k C_r} \equiv \hat{u}(r_1, k).
\]

(5)

Equation (5) is of central importance for my analysis. It implies that the manager can affect the perception of soft information (which is an imperfect signal of investment productivity), both through the reporting choice and the capital expenditure decision. This is formally captured by the market’s belief \(\hat{u}(r_1, k)\) being a function of both \(r_1\) and \(k\). On the other hand, note that the market’s expectation of future cash flows from of the assets in place is beyond the manager’s control, because at date 2 the cash flows information \(x_2\) replaces the noisy reported earnings \(r_1\).

The market’s knowledge at date 2 of the realized cash flows \(x_2\) and the intertemporal reporting constraint (1) also allow the market to pin down the date-2 signal,

\[
s_2 = r_2 + r_1 - x_2.
\]

(6)

Consequently, in equilibrium the market’s date-2 information set \(\mathcal{I}_{p,2}\) is informationally equivalent to \(\{ r_1, x_2, k, r_2, s_1, u, \varepsilon_1, s_2 \}\), and the date-2 stock price is determined as

\[
P_2 = \mathbb{E} \left[ \hat{L} \mid r_1, k, x_2, s_2 = r_2 + r_1 - x_2, u = \hat{u}(r_1, k) \right].
\]

(7)

In equilibrium, it is the case that the equation system (3) is invertible, a condition which boils down to the difference \(B_r C_k - B_k C_r\) in the denominator of (5) being non-zero.
Taking the pricing function in (7) as given, by the law of iterated expectations the manager’s date-1 maximization program can be written (omitting a constant that is independent of \( r_1 \) and \( k \)) as

\[
\max_{r_1, k} \left\{ \alpha E \left[ \tilde{\xi} \mid r_1, k, s_1, u = \hat{u}(r_1, k) \right] + (1 - \alpha) E \left[ \tilde{\xi} \mid s_1, u \right] - 1 \right\} \left\{ k - \frac{1}{2} k^2 - \frac{\gamma}{2} (r_1 - s_1)^2 \right\}. \tag{8}
\]

Equation (8) states that at date 1 the manager chooses capital expenditure \( k \) to maximize the net investment return as if investment productivity were given by a convex combination of: \( E \left[ \tilde{\xi} \mid s_1, u \right] \), which is the actual productivity estimated by the manager based on his date-1 information; and \( E \left[ \tilde{\xi} \mid r_1, k, s_1, u = \hat{u}(r_1, k) \right] \), which is the manager’s date-1 expectation of the market’s date-2 perception of productivity. The incentives to manipulate earnings arise from the fact that reported earnings \( r_1 \) affect the market’s perception of soft information. A crucial aspect is that the magnitude of these incentives is proportional to the capital expenditure, as the latter scales the investment return. Therefore, greater capital expenditure increases the manager’s marginal benefit of misreporting.

My first major result is that, in equilibrium, discretionary accruals convey (at least partially) the manager’s private soft information.

**Proposition 1** There exists a unique equilibrium in which the manager’s reporting and investment strategies take the linear form in (3). In this equilibrium, the reporting bias \( b \) is a function of the manager’s private soft information \( u \), that is, \( C_r \neq 0 \).

The intuition for Proposition 1 is as follows. Assumption 2 implies that the market uses reported earnings \( r_1 \) to assess soft information, as capital expenditure alone is insufficient to infer \( u \). This fact and the short-termism Assumption 3 jointly imply that the manager has incentives to manipulate earnings. By the previous discussion, the investment scale \( k \) determines the extent of misreporting; and, by Assumption 4, the equilibrium \( k \) is a function of soft information \( u \), since \( u \) affects the investment return and the manager partially cares about the liquidating dividends. Hence, the equilibrium reporting bias must also be a
function of $u$.

5 Empirical Predictions

This section examines the properties of the linear equilibrium. I provide testable empirical predictions regarding earnings smoothness (Section 5.1), the earnings response coefficient (Section 5.3), and the amount of reporting bias (Section 5.4). Moreover, Section 5.2 studies how investment efficiency varies with several observable firm’s characteristics, with particular emphasis on the real effects of increases in the marginal cost of misreporting, the precision of accounting information, and managerial myopia. My analysis documents an endogenous link between changes in the manager’s incentives to smooth earnings and in the investment return.

5.1 Earnings Smoothness

Earnings smoothing is a particular form of earnings management. It is typically defined as the practice of shifting the recognition of income from the current to the next reporting period, if the manager’s hard information is higher than average, or from the next to the current period, if hard information is lower than average (e.g., see Trueman and Titman (1988), Fudenberg and Tirole (1995), and Sankar and Subramanyam (2001)). In the linear equilibrium of my model, earnings smoothing is formally equivalent to the coefficient $B_r$ in (3) satisfying

$$B_r \in (0, 1).$$

To see the equivalence between (9) and the notion of earnings smoothing stated previously, suppose that (9) is satisfied and consider the following: if $s_1 - \mu_0 > 0$, then at date 1 the manager only reports the fraction $B_r$ of the positive earnings surprise and, by the intertemporal reporting constraint (1), the recognition of the remaining fraction $1 - B_r$ is delayed until date 2; vice versa, if $s_1 - \mu_0 < 0$, then the manager reports only the fraction $B_r$ of
the negative surprise by “borrowing” the remaining fraction $1 - B_r$ from future reported earnings.

Proposition 2 below, my second major result, establishes that if the marginal cost of misreporting is sufficiently high, then in equilibrium the manager smooths earnings.

**Proposition 2** The coefficient $B_r$ of the equilibrium reporting strategy in (3) is

$$B_r = \frac{\gamma - \alpha \varphi^2}{\gamma + \alpha^2 \varphi^2} \in \left( -\frac{1}{\alpha}, 1 \right),$$

(10)

where

$$\varphi \equiv \sigma_k (1 - \delta_k) \rho_0 \frac{\sigma_0}{\sigma_0^2 + \sigma_0^2}. \quad (11)$$

Hence, if the marginal cost of misreporting $\gamma > \alpha \varphi^2$, then earnings smoothing takes place in equilibrium (i.e., $B_r \in (0, 1)$).

The result above is best understood by comparing the equilibrium of the main model to the extreme case in which the manager has no reporting discretion, which corresponds to the limit as the marginal cost of misreporting $\gamma \to \infty$. Without misreporting, the coefficient $B_r$ would be exactly equal to one (as $r_1 = s_1$ in that case). In contrast, for any finite $\gamma$, the following trade-off arises which prompts the manager to underreport positive surprises in the manager’s hard information, $s_1 - \mu_0$. Indeed, if $s_1$ increases, reporting higher earnings has both a positive and a negative effect. The positive effect comes from the fact that a higher $r_1$ reduces the misreporting cost, since reported earnings are closer to the manager’s hard information. The negative effect is that a higher $r_1$ induces a more pessimistic market’s belief about the soft information $u$. This follows from the consideration that the manager rationally increases capital expenditure when either $s_1$ or $u$ are higher. Therefore, if a higher report is not associated with a corresponding increase in capital expenditure, then the market correctly deduces that the more favorable hard information $s_1$ must have been compensated
by more unfavorable soft information $u$.\textsuperscript{8} As part of the manager’s objective is to increase the market’s perception of soft information, for any fixed $k$ he responds to this trade-off by attenuating the reported surprise.

The marginal cost of misreporting $\gamma$ in the numerator of (10) measures the manager’s incentives to report truthfully, whereas managerial myopia $\alpha$ and $\varphi$ in (11) reflect his incentives to improve the market’s assessment of soft information. Intuitively, if $\gamma$ is higher, then the manager has more incentives to issue a report closer to his signal. If $\alpha$ is lower, then the manager’s incentives to misreport are lower because he is less concerned about the short-term price. To comprehend the meaning of $\varphi$ in the present context, note that $\varphi$ is the regression coefficient of investment productivity $\xi$ on the surprise component of the manager’s hard information $s_1 - \mu_0$, that is,

$$
E \left[ \xi \mid s_1 \right] = \mu_k + 1 + \varphi (s_1 - \mu_0).
$$

If $\varphi$ is smaller, then earnings information is less relevant in terms of assessing investment productivity, so that the manager’s investment strategy becomes less sensitive to $s_1$. Recall, at this point, that higher reported earnings without a corresponding increase in capital expenditure are indicative of better current performance and worse soft information. The market’s negative inference about soft information is what creates the manager’s incentives to underreport earnings surprises. The fact that a lower $\varphi$ depresses the manager’s incentives to misreport can be seen by means of the following example. Suppose that the manager’s investment strategy were insensitive to hard information (i.e., $\varphi = 0$). Then, the market would not expect that higher earnings lead to greater capital expenditure. In turn, this would imply no negative inference about soft information by the market and, as a consequence, all misreporting motives would vanish.

When $\gamma$ is large enough relative to $\alpha$ and $\varphi$, the incentive to report truthfully dominate the incentives to misreport. For such parameter values, $B_r > 0$ and hence we obtain earn-

\textsuperscript{8}In equilibrium, the market’s date-2 belief $\hat{u} (r_1, k)$ about $u$ is, for a fixed $k$, a decreasing function of $r_1$.\textsuperscript{23}
ings smoothing. Contrarily, when the incentives to misreport dominate, \( B_r < 0 \) and the equilibrium reported earnings are decreasing in the surprise \( s_1 - \mu_0 \).

Next, I perform comparative statics on the extent of earnings smoothing. Whenever the parameter values are such that earnings smoothing indeed takes place (i.e., \( B_r \in (0, 1) \)), a lower \( B_r \) is suggestive of a more pronounced smoothing. Hence, I measure the extent of smoothing as \( 1 - B_r \). In this way, we have \( 1 - B_r = 0 \) when no smoothing occurs (\( B_r = 1 \)) and greater smoothing as \( B_r \) decreases.

**Corollary 1** Suppose that in equilibrium earnings smoothing occurs (i.e., \( B_r \in (0, 1) \)). Then, the extent of earnings smoothing \((1 - B_r)\) is:

1. Decreasing in the marginal cost of misreporting \( \gamma \) and the relevance of soft information \( \delta_k \);
2. Increasing in managerial myopia \( \alpha \), the precision of accounting information \( \tau_\varepsilon \), the intertemporal correlation of cash flows from the assets in place \( \rho_0 \), the uncertainty about investment productivity \( \sigma_k \);
3. Decreasing (increasing) in the volatility of cash flows from the assets in place \( \sigma_0 \), if cash flows are more (less) volatile than the accounting noise, that is, \( \sigma_0 > \sigma_\varepsilon \) (\( \sigma_0 < \sigma_\varepsilon \)).

The intuitions for the comparative statics results with respect to the marginal cost of misreporting and managerial myopia are immediate, since these variables influence directly the incentives to report truthfully and to misreport, respectively. All other variables affect the incentives to misreport only indirectly through \( \varphi \). The manager’s current hard information lose relevance for investment productivity (i.e., \( \varphi \) is lower) when: the uncertainty about investment productivity is lower, because a decrease in \( \sigma_k \) scales down the impact on investment productivity of changes in the manager’s hard information; soft information becomes more relevant, at the expense of the relevance of earnings (\( \delta_k \) increases); earnings are less persistent, because a lower \( \rho_0 \) reduces the ability of current earnings to predict the
date-3 cash flows from the assets in place; or hard information is noisier ($\sigma_2^2$ is higher). Interestingly, the volatility of cash flows from the assets in place has a non-monotonic effect on $\varphi$. Changes in $\sigma_0$ have a twofold effect on the association between hard information $\tilde{s}_1$ and the component of investment productivity with which they comove, $(\tilde{x}_3 - \mu_0) / \sigma_0$. All else equal, as $\sigma_0$ increases, the posterior expectation of the future cash flows from the assets in place $\tilde{x}_3$ becomes more sensitive to current hard information $\tilde{s}_1$. The intuition is that when the prior belief is less precise, earnings have more incremental information relative to the prior and, accordingly, the posterior assigns to them more weight. At the same time, a higher $\sigma_0$ at the denominator of $(\tilde{x}_3 - \mu_0) / \sigma_0$ attenuates any deviation of $\tilde{x}_3$ from its expected value. Which of these two forces prevails determines whether $\varphi$ increases or decreases following a change in $\sigma_0$.

5.2 Investment Efficiency

At this point, we are in a position to examine the connection between earnings smoothing and investment efficiency. As usual in signaling models, unobservability of the manager’s information leads to distortions in his actions. To begin with, I solve for the reporting and investment strategies in the “first-best” scenario, where there is no private information. Then, I compare the first-best investment with that of the main model. The equilibrium investment strategy in the main model is referred to as “second-best”, since it corresponds to a scenario where the manager’s information is private.

First-best investment. We are in the first-best setting whenever soft information $u$ is public. Whether hard information $s_1$ is public is irrelevant, as at date 2 cash flows information $x_2$ becomes available, and earnings $s_1$ are only a noisy version of $x_2$. Since at date 2 investment productivity is common knowledge, the manager has no incentive to bias earnings. It follows that the first-best equilibrium reporting strategy is $r_1^{FB} = s_1$ (the superscript $FB$ is a mnemonic for “first-best”). Besides there being no distortion in reported earnings,
in the first best there is no distortion in capital expenditure, which equals the (commonly known) net investment productivity,

\[ k^{FB} = E \left[ \tilde{\xi} - 1 \middle| s_1, u \right]. \]

**Second-best investment.** In the main model, one shows that the equilibrium investment from (3) is given by

\[ k = \psi k^{FB}, \]

where

\[ \psi \equiv \frac{\gamma (1 + \alpha)}{\gamma + \alpha^2 \varphi^2} \]

(12)

and \( \varphi \) is defined in (11). The endogenous parameter \( \psi \) reflects the investment distortion due to signaling of soft information. By Proposition 2, we know that the second-best reporting strategy is always distorted, as \( B_r < 1 \) implies a deviation from truthful reporting. Remarkably, however, the second-best investment strategy need not suffer from any signaling distortion, as \( \psi \) may be equal to one, in which case \( k = k^{FB} \). Observe that \( \psi = 1 \) if and only if \( B_r = 0 \). That is, the second best achieves first-best investment efficiency when the manager’s incentives to report truthfully (so as to avoid the misreporting cost) and his incentives to misreport (so as to signal more favorable soft information) precisely offset each other. When these two opposing effects cancel out, the date-1 reporting strategy becomes independent of the actual hard information \( s_1 \) and, as a result, reported earnings \( r_1 \) become a perfect signal of soft information. Given these considerations, the intuition for why \( \psi = 1 \) implies first-best investment efficiency is as follows. While generally (i.e., when \( \psi \neq 1 \)) the manager uses two costly signaling devices (\( r_1 \) and \( k \)) to credibly convey his private soft information, when \( \psi = 1 \) he uses only \( r_1 \) to signal \( u \). Since \( r_1 \) alone fully reveals \( u \), the manager does not have to also distort capital expenditure for signaling purposes.

In general, \( \psi \) can take any value in the interval \((0, 1 + \alpha)\). Therefore, the expected
equilibrium investment $E[\hat{k}|s_1,u]$ can be, depending on the parameters, greater than the expected first-best investment $E[\hat{k}^{FB}|s_1,u]$ (overinvestment) or lower (underinvestment). In either case, investment is inefficient. To capture deviations from the first best, regardless of whether such deviations are due to overinvestment or underinvestment, I follow Stocken and Verrecchia (2004) and define investment efficiency ($IE$) as the ratio between the expected net investment return in the second best ($\tilde{y}$) and the same return in the first best ($\tilde{y}^{FB}$),

$$IE \equiv \frac{E[\tilde{y}]}{E[\tilde{y}^{FB}]}.$$ 

A higher $IE$ corresponds to greater investment efficiency. $IE$ takes values in $(0, 1]$ and is exactly equal to one for $\hat{k} = \hat{k}^{FB}$. The following Lemma 1 provides a convenient characterization of $IE$ in terms of the investment distortion $\psi$ in (12).

**Lemma 1** *In the unique linear equilibrium, investment efficiency $IE$ is given by*

$$IE = 2\psi - \psi^2,$$

*where $\psi$ is defined in (12). Moreover, at $\psi = 1$ the second-best investment strategy is first-best efficient (that is, $IE$ is maximized and equal to 1).*

Lemma 1 shows that investment efficiency is a hump-shaped function of the investment distortion $\psi$. A $\psi < 1$ occurs when the incentives to misreport outweigh the incentives to report truthfully (equivalently, when the coefficient $B_r < 0$). In this region of parameters, an increase in $\psi$ enhances investment efficiency. Vice versa, when $\psi > 1$ (equivalently, when $B_r > 0$) an increase in $\psi$ reduces efficiency. As noted before, first-best efficiency is replicated in the second-best setting when the incentives to misreport and to tell the truth are perfectly balanced (i.e., when $\psi = 1$ or, equivalently, $B_r = 0$). This leads us to my next set of major results, Proposition 3, in which I analyze how changes in the parameters of the model affect investment efficiency through their effect on the investment distortion $\psi$. 
Proposition 3 (1) If the marginal cost of misreporting $\gamma > \alpha \varphi^2$\footnote{$\varphi$ is defined in (11).}, then investment efficiency $IE$:

(a) Is decreasing in the marginal cost of misreporting $\gamma$ and the relevance of soft information $\delta_k$;

(b) Is increasing in precision of accounting information $\tau_\varepsilon$, the intertemporal correlation of cash flows from the assets in place $\rho_0$, and the uncertainty about investment productivity $\sigma_k$;

(c) May be increasing in managerial myopia $\alpha$, for intermediate or high values of $\alpha$, whereas it is decreasing in $\alpha$ for low values of $\alpha$;

(d) Is decreasing (increasing) in the volatility of cash flows from the assets in place $\sigma_0$, if cash flows are more (less) volatile than the accounting noise, that is, $\sigma_0 > \sigma_\varepsilon$ ($\sigma_0 < \sigma_\varepsilon$).

(2) If instead the marginal cost of misreporting $\gamma < \alpha \varphi^2$, then:

(a) Parts (1a, 1b, 1d) of this proposition hold with reverse sign;

(b) $IE$ is decreasing in $\alpha$.

Part (1) of Proposition 3 represents a situation in which the incentives to tell the truth are greater than those to manipulate earnings ($B_r > 0$). Instead, part (2) of the proposition deals with a situation with reverse incentives ($B_r < 0$). Hence, the first conclusion to be drawn from Proposition 3 is that all predictions on investment efficiency crucially depend on which of these two contrasting incentives prevails in equilibrium. Recall that first-best investment efficiency obtains when the two incentives have equal magnitude, so that in equilibrium reported earnings bear all the signaling distortions (the coefficient $B_r = 0$), whereas investment is first-best efficient. If instead $B_r \neq 0$, then the same variation in one of
the model parameters may be detrimental or beneficial for efficiency depending on whether it exacerbates or reduces the gap between the strengths of the two incentives, respectively.

The second conclusion from Proposition 3 concerns the marginal cost of misreporting and the precision of accounting information. While perhaps the commonly held belief is that policies aimed at increasing misreporting penalties or information quality can improve efficiency by reducing information asymmetries, surprisingly I find that, in certain cases, such policies can inadvertently have the exactly opposite effect. Specifically, a greater marginal cost of misreporting decreases efficiency, if the misreporting cost is already relatively high (part (1a) of the proposition), whereas it enhances efficiency, if the misreporting cost is relatively low (part (2a) of the proposition). The intuition behind these two opposite results is as follows. A higher $\gamma$ incentivizes the manager to report truthfully. As pointed out previously, whether lower reporting discretion is desirable relies on whether it achieves a better balance between the truthtelling incentives and the incentives to misreport. An analogous intuition explains the comparative statics with respect to the precision of accounting information (parts (1b) and (2a) of the proposition). If $\tau$ increases, then the manager’s investment strategy becomes more sensitive to earnings information. Then, for a given $k$, higher reported earnings are indicative of even worse soft information, so that the incentives to underreport earnings surprises are greater. Investment efficiency requires more misreporting, if reporting discretion is relatively low (part (1b)), and less misreporting, if reporting discretion is relatively high (part (2a)). Figure 2 illustrates the non-monotonic behavior of investment efficiency with respect to the marginal cost of misreporting and the precision of accounting information.

Further, Proposition 3 sheds light on the impact of managerial myopia on investment efficiency. At first blush, one might think that a higher myopia necessarily leads to a less efficient investment, as stronger incentives to manipulate the date-2 stock price induce larger signaling distortions. While this negative effect of a higher $\alpha$ is present, it is only a part of the intuition. Interestingly, as part (1c) of the proposition states and Figure 3 shows, greater myopia can actually be associated with better investment efficiency. When the incentives to
Figure 2: Investment efficiency ($\mathcal{I}_E$) as a function of the marginal cost of misreporting ($\gamma$) for different levels of the precision of accounting information ($\tau \varepsilon$). The model is parametrized as follows: $\alpha = 0.75$, $\sigma_0 = \sigma_k = 10$, $\rho_0 = 0.6$, and $\rho_k = 0.5$.

report truthfully dominate those to misreport (i.e., when the coefficient $B_r > 0$), a higher $\alpha$ improves the balance by incentivizing misreporting. This shifts part of the signaling distorting from capital expenditure to reported earnings, thereby alleviating the investment distortion. The latter indirect effect may be so large as to offset the negative direct effect of a higher $\alpha$ on investment.

5.3 Earnings Response Coefficient

Among all dates, date 1 is the most appropriate for studying the earnings response coefficient (ERC), as this is the only date at which: (1) earnings information is available to the public; and (2) there is asymmetric information between the manager and outside investors.\(^{10}\) In

\(^{10}\)At date 0, everyone is equally uninformed; at date 2, by observing the financial statements investors can perfectly infer the owner’s private information; at date 3, all cash flows are distributed to shareholders.
Figure 3: Investment efficiency \((ΙΕ)\) as a function of managerial myopia \((α)\) for different levels of the marginal cost of misreporting \((γ)\). The model is parametrized as follows: \(σ_0 = σ_k = 10, \rho_0 = 0.6, \rho_k = 0.5, \text{and } τ_ε = 0.1.\)

equilibrium, the date-1 pricing equation takes the following form,

\[
P_1 - E \left[ \hat{P}_1 \right] = a_P + b_P (\tilde{r}_1 - E[\tilde{r}_1]) + c_P (\tilde{r}_1 - E[\tilde{r}_1])^2,
\]

where \((a_P, b_P, c_P)\) are endogenous coefficients. Equation (13) expresses the price reaction, \(P_1 - E \left[ \hat{P}_1 \right]\), as a function of the earnings surprise, \(r_1 - E[\tilde{r}_1]\). The quadratic term in (13) implies that the price reaction to the earnings surprise depends on the realization of \(\tilde{r}_1\). Therefore, we focus on the average ERC, defined as the expected price reaction,

\[
E \left[ \frac{\partial \left( P_1 - E \left[ \hat{P}_1 \right] \right)}{\partial (r_1 - E[\tilde{r}_1])] \right] = b_P,
\]
where the expectation is taken with respect to reported earnings $\tilde{r}_1$.

I now perform comparative statics on the date-1 average ERC.

**Corollary 2**

(1) If the marginal cost of misreporting $\gamma$ is sufficiently larger than $\frac{\sigma_k^2}{\sigma_0^2}$, then the average ERC is:

(a) Decreasing in the volatility of cash flows from the assets in place $\sigma_0$, if cash flows are more volatile than the accounting noise (i.e., $\sigma_0 > \sigma_\varepsilon$) and the expected investment productivity $\mu_k$ is sufficiently high;

(b) Increasing in $\sigma_0$, if cash flows are less volatile than the accounting noise (i.e., $\sigma_0 < \sigma_\varepsilon$) or if the expected investment productivity $\mu_k$ is sufficiently low;

(c) Decreasing in managerial myopia $\alpha$ and the relevance of soft information $\delta_k$;

(d) Increasing in the precision of accounting information $\tau_\varepsilon$, the intertemporal correlation of cash flows from the assets in place $\rho_0$, the expected investment productivity $\mu_k$, the uncertainty about investment productivity $\sigma_k$.

(2) If the marginal cost of misreporting $\gamma < \alpha \varphi^2$, then the average ERC is negative.

Part (1) of Corollary 2 considers the case in which the incentives to tell the truth unequivocally dominate the incentives to manipulate earnings. More specifically, it requires that misreporting be so costly as to imply that variations in reported earnings are likely to reflect variations in hard information, as opposed to variations in soft information. Corollaries

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Note that, since the earnings surprise $\tilde{r}_1 - E[\tilde{r}_1]$ is normally distributed and zero-mean, we have

$$b_P = \frac{\text{Cov}\{\tilde{P}_1 - E[\tilde{P}_1], \tilde{r}_1 - E[\tilde{r}_1]\}}{\text{Var}\{\tilde{r}_1 - E[\tilde{r}_1]\}}.$$  

Indeed, by the properties of the normal distribution, the third central moment is zero, whence $\text{Cov}\{\tilde{r}_1 - E[\tilde{r}_1], (\tilde{r}_1 - E[\tilde{r}_1])^2\} = 0$. In other words, the average ERC can be consistently estimated from a linear regression of the price reaction on the earnings surprise: omitting the quadratic term is without loss of generality, because it is uncorrelated with the regressor.

Since the term $\alpha \varphi^2$ is bounded above by the ratio of variances $\sigma_k^2/\sigma_0^2$, the condition $\gamma > \sigma_k^2/\sigma_0^2$ implies $\mathcal{B}_r > 0$.

In the limit as misreporting becomes infinitely costly, we have $r_1 = e_1$. 

---
2(1a,1b) collectively state that the average ERC may be non-monotonic in the volatility of cash flows. The firm’s liquidating dividends have two components: the stream of cash flows from the assets in place and the investment return. While higher prior uncertainty implies, ceteris paribus, that the expectation of cash flows from the assets in place becomes more sensitive to earnings information ($r_1$), the opposite may happen for the expected investment return. The reason is that the coefficient $\varphi$, which captures the association between hard information and investment productivity, is non-monotone in $\sigma_0$, as discussed in Section 5.1. The average ERC increases in $\sigma_0$ (Corollary 2(1b)) when most of the firm’s cash flows derive from the firm’s assets in place ($\mu_k$ is relatively small) or when $\varphi$ increases in $\sigma_0$ ($\sigma_0 < \sigma_\varepsilon$). In the former case, the stock price is mostly driven by the expectation of the assets in place, which becomes more sensitive to earnings as $\sigma_0$ increases. In the latter case, even if firm value relies substantially on the investment return, the expected investment productivity, and hence the expected investment return, becomes more sensitive to earnings as $\sigma_0$ increases. For the average ERC to be decreasing in $\sigma_0$ (Corollary 2(1a)), the sensitivity of expected investment return has to be decreasing in $\sigma_0$ ($\sigma_0 > \sigma_\varepsilon$), and investment return has to constitute a significant part of firm value ($\mu_k$ is relatively large). In this way, despite the expected cash flows from the assets in place becoming more sensitive to earnings as $\sigma_0$ increases, the expected investment return becomes less sensitive, and the magnitude of this negative effect is large enough to dominate the positive effect.

Corollary 2(1c) states that as the relevance of soft information or managerial myopia increase, the association between stock returns and reported earnings surprises becomes more muted. In either case, the result holds because the expected net investment return becomes less sensitive to current earnings information. A high $\delta_k$ implies that the net investment return $\tilde{y}$ is mainly driven by soft information, and hence hard information $s_1$ is relatively less predictive of $\tilde{y}$. The impact of a higher $\alpha$ is subtler. As $\alpha$ increases, the signaling distortion of investment becomes more severe and, in particular, the expected
net investment return approaches zero. Consequently, the covariance between investment return and current earnings decreases. Corollary 2(1d) follows from standard arguments.

In Part (2) of the corollary, the incentives to report truthfully are weaker than those to misreport. Reported earnings are a decreasing function of hard information ($B_r < 0$) and, in equilibrium, the market correctly interprets low reported earnings as good news. This result thus provides a novel explanation for the empirical evidence on negative ERCs (e.g., Schroeder (1995)).

### 5.4 Amount of Bias

The expected bias $E[b]$ is negative in equilibrium, as the manager tends to underreport earnings in order to credibly signal his soft information. Thus, a lower (more negative) expected bias is associated with greater manipulation. We can then take $-E[b]$ as a measure of the equilibrium amount of bias. Corollary 3(1) below states that, under those circumstances in which the manager engages in earnings smoothing, the amount of bias behaves in the same manner as the extent of earnings smoothing.

**Corollary 3** If the marginal cost of misreporting $\gamma > \alpha \varphi^2$, then the equilibrium amount of bias ($-E[b]$) is:

1. Such that for all parameters $\pi \in \{\gamma, \delta_k, \alpha, \tau_\epsilon, \rho_0, \sigma_k, \sigma_0\}$ we have $\left[ \frac{\partial (-E[b])}{\partial \pi} \right] = \text{sign} \left[ \frac{\partial (1-b)}{\partial \pi} \right]$;

and

2. Increasing in the expected investment productivity $\mu_k$.

A higher investment scale magnifies the incentives to underreport earnings for the purpose of signaling soft information. As the expected investment productivity increases, the expected capital expenditure also increases. Hence, a higher $\mu_k$ implies a greater amount of bias in equilibrium, as per part (2).

---

14 For $\gamma$ high enough, Proposition 3(1c) states that investment efficiency is decreasing in $\alpha$. 

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6 Conclusion

The empirical literature has found evidence that managers exploit their reporting discretion to convey value-relevant information. I show that this behavior can be rationalized in a theoretical model that combines earnings management with an investment decision. In equilibrium, the reporting bias is a function of private information about future investment productivity that is not yet recognized by the accounting system. The manager uses both financial reporting and capital expenditure to signal soft information. Earnings management may take the particular form of earnings smoothing, whereby the manager shifts the recognition of income across periods. First-best investment efficiency is achieved in the extreme case where the manager smooths earnings to the extent that reported earnings do not depend on current (unmanaged) earnings. My analysis provides testable predictions regarding non-monotonic effects of observable firm’s characteristics on earnings smoothing, investment efficiency, the earnings response coefficient, and the amount of reporting bias.
References


Appendix

A  Notation

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Variable Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t \in {1, 2, 3}$</td>
<td>Time index.</td>
</tr>
<tr>
<td>$\tilde{x}_t$</td>
<td>Cash flows from assets in place at date $t \in {2, 3}$.</td>
</tr>
<tr>
<td>$\tilde{\theta}$</td>
<td>Permanent component of $\tilde{x}_t$.</td>
</tr>
<tr>
<td>$\tilde{\eta}_t$</td>
<td>Transitory component of $\tilde{x}_t$.</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>Mean of $\tilde{x}_t$.</td>
</tr>
<tr>
<td>$\rho_0 \in (0, 1)$</td>
<td>Intertemporal correlation of $\tilde{x}_t$.</td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>Volatility of $\tilde{x}_t$.</td>
</tr>
<tr>
<td>$\tau_0$</td>
<td>Precision of $\tilde{x}_t$ ($\tau_0 = \sigma_0^{-2}$).</td>
</tr>
<tr>
<td>$k$</td>
<td>Capital expenditure.</td>
</tr>
<tr>
<td>$\tilde{\xi}$</td>
<td>Investment productivity.</td>
</tr>
<tr>
<td>$\tilde{u}$</td>
<td>Manager’s private soft information, a signal of $\tilde{\xi}$.</td>
</tr>
<tr>
<td>$\tilde{\xi}k - 0.5k^2$</td>
<td>Gross investment return.</td>
</tr>
<tr>
<td>$\tilde{y}$</td>
<td>Net investment return, given by $\tilde{y} = (\tilde{\xi} - 1)k - 0.5k^2$.</td>
</tr>
<tr>
<td>$\mu_k + 1$</td>
<td>Mean of $\tilde{\xi}$, where $\mu_k &gt; 0$.</td>
</tr>
<tr>
<td>$\sigma_k$</td>
<td>Uncertainty about $\tilde{\xi}$ ($\sigma_k = \text{Var}[\tilde{\xi}]$).</td>
</tr>
<tr>
<td>$\delta_k \in (0, 1)$</td>
<td>Relevance of soft information ($\delta_k = 1 - \text{Corr}[\tilde{x}_3, \tilde{\xi}]$).</td>
</tr>
<tr>
<td>$\tilde{\varepsilon}_t$</td>
<td>Accounting noise, $t \in {1, 2}$.</td>
</tr>
<tr>
<td>$\tau_\varepsilon$</td>
<td>Precision of accounting information ($\tau_\varepsilon = 1/\text{Var}[\varepsilon_t]$).</td>
</tr>
<tr>
<td>$\tilde{s}_t$</td>
<td>Manager’s private hard information, given by $\tilde{s}<em>t = \tilde{x}</em>{t+1} + \tilde{\varepsilon}_t$, $t \in {1, 2}$.</td>
</tr>
<tr>
<td>$b$</td>
<td>Reporting bias.</td>
</tr>
<tr>
<td>$r_t$</td>
<td>Reported earnings, given by $r_1 = s_1 + b$, $r_2 = s_2 - \varepsilon_1 - b$, $t \in {1, 2}$.</td>
</tr>
<tr>
<td>$\tilde{L}$</td>
<td>Liquidating dividends, distributed to shareholders at $t = 3$, given by $\tilde{L} = \tilde{x}_2 + \tilde{x}_3 + \tilde{y}$.</td>
</tr>
<tr>
<td>$P_t$</td>
<td>Firm’s stock price at date $t \in {1, 2}$.</td>
</tr>
<tr>
<td>$\alpha \in (0, 1)$</td>
<td>Managerial myopia, defined as the manager’s preference for short-term performance ($P_2$) relative to long-term ($L$).</td>
</tr>
<tr>
<td>$\gamma &gt; 0$</td>
<td>Marginal cost of misreporting.</td>
</tr>
</tbody>
</table>
B Proofs

Proof of Proposition 1. Using equations (5), (6), and the fact that
\[
\begin{pmatrix}
\tilde{x}_3 - \mu_0 \\
\tilde{x}_2 - \mu_0 \\
\tilde{s}_2 - \mu_0
\end{pmatrix}
\sim \mathcal{N}
\begin{bmatrix}
0 \\
0
\end{bmatrix},
\begin{bmatrix}
\frac{1}{\tau_0} & \frac{\rho_0}{\tau_0} & \frac{1}{\tau_0} \\
\frac{\rho_0}{\tau_0} & \frac{1}{\tau_0} & \frac{\rho_0}{\tau_0} \\
\frac{1}{\tau_0} & \frac{\rho_0}{\tau_0} & \frac{1}{\tau_0} + \frac{1}{\tau_}\epsilon
\end{bmatrix},
\]
we have the date-2 markets expectations
\[
\begin{align*}
\mathbb{E}[\tilde{x}_2 | \mathcal{I}_{p,2}] &= x_2, \\
\mathbb{E}[\tilde{x}_3 | \mathcal{I}_{p,2}] &= \mu_0 + \frac{\tau_0 \rho_0 (x_2 - \mu_0) + (1 - \rho_0^2) \tau_\epsilon (r_2 + r_1 - x_2 - \mu_0)}{\tau_0 + (1 - \rho_0^2) \tau_\epsilon}, \\
\mathbb{E}[\tilde{u} | \mathcal{I}_{p,2}] &= \hat{u} (r_1, k).
\end{align*}
\]

It follows that the manager’s date-1 expectations of the market’s date-2 expectations are
\[
\begin{align*}
\mathbb{E}[\mathbb{E}[\tilde{x}_2 | \mathcal{I}_{p,2}] | \mathcal{I}_{m,1}] &= \mu_0 + \frac{\tau_\epsilon}{\tau_0 + \tau_\epsilon} (s_1 - \mu_0), \\
\mathbb{E}[\mathbb{E}[\tilde{x}_3 | \mathcal{I}_{p,2}] | \mathcal{I}_{m,1}] &= \mu_0 + \frac{\rho_0 \tau_\epsilon}{\tau_0 + \tau_\epsilon} (s_1 - \mu_0), \\
\mathbb{E}[\mathbb{E}[\tilde{u} | \mathcal{I}_{p,2}] | \mathcal{I}_{m,1}] &= \hat{u} (r_1, k).
\end{align*}
\]

Note that the date-2 price is given by
\[
P_2 = x_2 + \mathbb{E}[\tilde{x}_3 | \mathcal{I}_{p,2}] + \mathbb{E}\left[ \sqrt{\rho_0 \theta} \right] | \mathcal{I}_{p,2}
+ \left\{ \mu_k + \frac{\sigma_k}{\sigma_0} (1 - \delta_k) \mathbb{E}[\tilde{x}_3 - \mu_0 | \mathcal{I}_{p,2}] + \omega \hat{u} (r_1, k) \right\} k - \frac{1}{2} k^2,
\]
where \( \omega \equiv \sigma_k \delta_k. \)

Hence, from the manager’s perspective at date 1,
\[
\mathbb{E}[\tilde{P}_2 | \mathcal{I}_{m,1}] = constant + \left[ \mu_k + \varphi (s_1 - \mu_0) + \omega \frac{-A_k B_r - B_k (r_1 - A_r)}{\Delta} \right] k - \frac{1}{2} \left( 1 - \frac{2 \omega B_r}{\Delta} \right) k^2,
\]
where the constant is independent of the manager’s date-1 choice variables \((r_1, k)\), \(\varphi\) is defined in (11), and

\[
\Delta \equiv B_r C_k - B_k C_r.
\]

Also,

\[
E \left[ \tilde{L} \mid I_{m,1} \right] = \text{constant} + [\mu_k + \varphi (s_1 - \mu_0) + \omega u] k - \frac{1}{2} k^2.
\] (A1)

The first-order condition (FOC) of the objective function (2) with respect to capital expenditure \(k\) gives

\[
k = \frac{\mu_k + \varphi (s_1 - \mu_0) + \omega \left[ \alpha \frac{A_r B_k - A_k B_r}{\Delta} + (1 - \alpha) u \right] - \frac{\alpha \omega B_k}{\Delta} r_1}{1 - 2 \frac{\omega B_r}{\Delta}}.
\] (A2)

whereas its FOC respect to the date-1 report \(r_1\) gives

\[
r_1 = \mu_0 + (s_1 - \mu_0) - \frac{\alpha \omega B_k}{\Delta} k.
\] (A3)

Plugging (A3) into (A2) yields

\[
k = \frac{\mu_k - \frac{\alpha \omega B_k}{\Delta} \mu_0 + (\varphi - \frac{\alpha \omega B_k}{\Delta}) (s_1 - \mu_0) + \omega \left[ \alpha \frac{A_r B_k - A_k B_r}{\Delta} + (1 - \alpha) u \right] - \frac{\alpha \omega B_k}{\Delta} r_1}{1 - 2 \frac{\omega B_r}{\Delta} - \frac{\alpha^2 \omega^2 B_r^2}{\Delta^2 \gamma}}.
\]

By equating the coefficients of (3) and (A2), we obtain

\[
C_k = \frac{(1 - \alpha) \omega}{1 - 2 \frac{\alpha \omega B_r}{\Delta} - \frac{\alpha^2 \omega^2 B_r^2}{\Delta^2 \gamma}},
\] (A4)

\[
B_k = \frac{\varphi - \frac{\alpha \omega B_k}{\Delta}}{1 - 2 \frac{\alpha \omega B_r}{\Delta} - \frac{\alpha^2 \omega^2 B_r^2}{\Delta^2 \gamma}} = \chi C_k,
\] (A5)

where

\[
\chi \equiv \frac{\varphi - \frac{\alpha \omega B_k}{\Delta}}{(1 - \alpha) \omega}.
\]
From (A3), we know that

\[ B_r = 1 - \frac{\alpha \omega B_k}{\Delta \gamma} B_k, \]  
(A6)

\[ C_r = -\frac{\alpha \omega B_k}{\Delta \gamma} C_k, \]  
(A7)

so that

\[ \Delta = c_k. \]  
(A8)

Using (A5) and (A8), equation (A6) simplifies to

\[ B_r = 1 - \frac{\alpha \omega \chi^2 C_k}{\gamma}. \]  
(A9)

Exploiting (A5) and (A9), equation (A4) boils down to

\[ C_k = \frac{\gamma (1 - \alpha) \omega C_k}{(\gamma + \alpha^2 \omega^2 \chi^2) C_k - 2 \alpha \omega \gamma}, \]

which, solving for \( C_k \) as a function of \( \chi \), yields

\[ C_k = \frac{\gamma (1 + \alpha) \omega}{\gamma + \alpha^2 \omega^2 \chi^2}. \]

Equations (A5) and (A8) imply

\[ \chi = \frac{\varphi}{\omega}, \]

and hence

\[ C_k = \omega \psi, \]  
(A10)

\[ B_k = \varphi \psi, \]  
(A11)
where $\psi$ is defined in (12). Knowing the equilibrium coefficients $B_k$ and $C_k$, we can determine $B_r$ and $C_r$ from, respectively, (A6) and (A7). Their equilibrium values are

\[
B_r = \frac{\gamma - \alpha \varphi^2}{\gamma + \alpha^2 \varphi^2}, \quad (A12)
\]

\[
C_r = -\frac{\alpha (1 + \alpha) \omega \varphi}{\gamma + \alpha^2 \varphi^2}. \quad (A13)
\]

By equating the intercepts of (3) with those of (A2) and (A3), we find that

\[
A_k = \frac{\mu_k - \frac{\alpha \omega B_k}{\Delta} \mu_0 + \omega \alpha \frac{A_r B_k - A_k B_r}{\Delta}}{1 - 2 \frac{\alpha \omega B_r}{\Delta} - \frac{\alpha^2 \omega B_r^2}{\Delta^2} \gamma} = \frac{C_k}{(1 - \alpha) \omega} \left[ \mu_k - \frac{\alpha \omega B_k}{\Delta} \mu_0 + \omega \alpha \frac{A_r B_k - A_k B_r}{\Delta} \right] = \frac{C_k}{(1 - \alpha) \omega} \left[ \mu_k - \frac{\alpha \omega B_k}{\Delta} \mu_0 + \omega \alpha A_r C_k - A_k \left( 1 - \frac{\alpha \omega^2 C_k}{\gamma} \right) \right],
\]

\[
A_r = \mu_0 - \frac{\alpha \omega B_k}{\Delta} A_k = \mu_0 - \frac{\alpha \varphi}{\gamma} A_k,
\]

which further simplify to

\[
A_k = \psi \mu_k, \quad (A14)
\]

\[
A_r = \mu_0 - \frac{\alpha (1 + \alpha) \varphi}{\gamma + \alpha^2 \varphi^2} \mu_k. \quad (A15)
\]

Finally, we need to check that the FOCs indeed correspond to a maximum. To this purpose, it suffices to show that the second-order condition is satisfied. The Hessian matrix of the objective function in (2) is

\[
\mathcal{H} = 
\begin{pmatrix}
- \left( 1 - 2 \frac{\alpha \omega B_r}{\Delta} \right) & -\frac{\alpha \omega B_r}{\Delta} \\
-\frac{\alpha \omega B_k}{\Delta} & -\gamma
\end{pmatrix}
= 
\begin{pmatrix}
-\frac{\gamma (1-\alpha)+2\alpha^2 \varphi^2}{\gamma (1+\alpha)} & -\alpha \varphi \\
-\alpha \varphi & -\gamma
\end{pmatrix}.
\]

The diagonal elements of $\mathcal{H}$ are negative and its determinant,

\[
\det \mathcal{H} = \frac{(1 - \alpha) (\gamma + \alpha \varphi^2)}{1 + \alpha},
\]

is positive. Thus, $\mathcal{H}$ is negative definite and the objective function is strictly concave. \qed
Proof of Proposition 2. The expression for the equilibrium $B_r$ is provided in (A12). The remaining claims follow by inspection. ■

Proof of Corollary 1. Proof of Parts (1,2). Immediate.

Proof of Part (3). The extent of earnings smoothing $(1 - B_r)$ is increasing in $\varphi$. Observe that
\[ \frac{\partial \varphi}{\partial \sigma_0} = \sigma_k (1 - \delta_k) \rho_0 \frac{\sigma_0^2 - \sigma_0^2}{(\sigma_0^2 + \delta_0^2)^2}, \]
which is negative if and only if $\sigma_0 > \sigma_{\varepsilon}$. ■

Proof of Lemma 1. The first-best investment strategy, $k^{FB}$, maximizes (A1) with respect to $k$. Therefore,
\[ k^{FB} = \mu_k + \varphi (s_1 - \mu_0) + \omega u. \]
By (A10), (A12), and (A14), the second-best investment strategy is
\[ k = \psi [\mu_k + \varphi (s_1 - \mu_0) + \omega u] = \psi k^{FB}. \]
Then, the second-best net investment return is
\[ E [\tilde{y}] = E [E [\tilde{y} | s_1, u]] = E \left[ E \left[ (\tilde{k} - 1) \tilde{k} - \frac{1}{2} \tilde{k}^2 \right] | s_1, u \right] \]
\[ = \left( \psi - \frac{1}{2} \psi^2 \right) E \left[ [\mu_k + \varphi (\tilde{s}_1 - \mu_0) + \omega \tilde{u}]^2 \right] \]
\[ = 2 \left( \psi - \frac{1}{2} \psi^2 \right) E \left[ \tilde{y}^{FB} \right]. \]
Investment efficiency $IE$ is a concave quadratic function which is maximized at $\psi = 1$. One sees that $\psi = 1$ if and only $B_r = 0$. ■

Proof of Proposition 3. By Lemma 1, $IE$ is increasing in $\psi$, if $\psi < 1$ (equivalently, $B_r < 0$), and decreasing in $\psi$, if $\psi > 1$ (equivalently, $B_r > 0$). As a consequence, all parts of the proposition except for parts (1c,2b) follow from the facts that
\[ \frac{\partial \psi}{\partial \gamma} > 0 \text{ and } \frac{\partial \psi}{\partial \varphi} < 0. \]
As to part (1c), note that

\[
\frac{\partial \psi}{\partial \alpha} = \gamma - 2\alpha \varphi^2 - \alpha^2 \varphi^2. 
\]

Hence, if \( \gamma > \alpha \varphi^2 \) (which implies \( \partial I\!E/\partial \psi < 0 \)) and \( \gamma < 2\alpha \varphi^2 + \alpha^2 \varphi^2 \) (which implies \( \partial \psi/\partial \alpha < 0 \)), then

\[
\frac{\partial I\!E}{\partial \alpha} = \frac{\partial I\!E}{\partial \psi} \frac{\partial \psi}{\partial \alpha} > 0. 
\]

The condition \( \gamma < 2\alpha \varphi^2 + \alpha^2 \varphi^2 \) requires \( \alpha > 0 \), hence the requirement that the value of \( \alpha \) be intermediate or high. If instead \( \gamma > 2\alpha \varphi^2 + \alpha^2 \varphi^2 \) (which implies \( \partial \psi/\partial \alpha > 0 \)), then \( \partial I\!E/\partial \alpha < 0 \).

Last, I prove part (2b), if \( \gamma < \alpha \varphi^2 \) (which implies \( \partial I\!E/\partial \psi > 0 \)), then \( \gamma < 2\alpha \varphi^2 + \alpha^2 \varphi^2 \) (which implies \( \partial \psi/\partial \alpha < 0 \)). Overall, we have \( \partial I\!E/\partial \psi < 0 \). \( \blacksquare \)

**Proof of Corollary 2.** The average date-1 ERC equals \( \text{Cov} \left[ \tilde{P}_1, \tilde{r}_1 \right] / \text{Var} [\tilde{r}_1] \). We have

\[
\text{Cov} \left[ \tilde{P}_1, \tilde{r}_1 \right] = \text{Cov} \left[ \text{E} \left[ \tilde{L} \bigg| \tilde{r}_1 \right], \tilde{r}_1 \right] = \text{Cov} \left[ \tilde{L}, \tilde{r}_1 \right],
\]

where the first equality follows from the definition of the date-1 price (\( P_1 = \text{E} \left[ \tilde{L} \bigg| r_1 \right] \)) and the second equality from the law of total covariance.\(^{15}\) Applying (3) and the law of total covariance once more,

\[
\text{Cov} \left[ \tilde{L}, \tilde{r}_1 \right] = \text{E} \left[ \text{Cov} \left[ \tilde{L}, \tilde{r}_1 \bigg| \tilde{s}_1, \tilde{u} \right] \right] + \text{Cov} \left[ \text{E} \left[ \tilde{L} \bigg| \tilde{s}_1, \tilde{u} \right], \text{E} \left[ \tilde{r}_1 \bigg| \tilde{s}_1, \tilde{u} \right] \right] 
= \text{Cov} \left[ \text{E} \left[ \tilde{L} \bigg| \tilde{s}_1, \tilde{u} \right], B_r \left( \tilde{s}_1 - \mu_0 \right) + C_r \tilde{u} \right].
\]

\(^{15}\)By the law of total covariance,

\[
\text{Cov} \left[ \tilde{L}, \tilde{r}_1 \right] = \text{E} \left[ \text{Cov} \left[ \tilde{L}, \tilde{r}_1 \bigg| \tilde{r}_1 \right] \right] + \text{Cov} \left[ \text{E} \left[ \tilde{L} \bigg| \tilde{r}_1 \right], \text{E} \left[ \tilde{r}_1 \bigg| \tilde{r}_1 \right] \right] 
= 0 + \text{Cov} \left[ \text{E} \left[ \tilde{L} \bigg| \tilde{r}_1 \right], \tilde{r}_1 \right].
\]
At this point, observe that
\[
\begin{aligned}
E \left[ \hat{L} \big| s_1, u \right] &= 2\mu_0 + \frac{(1 + \rho_0) \tau_\varepsilon}{\tau_0 + \tau_\varepsilon} (s_1 - \mu_0) + \left( \psi - \frac{1}{2} \psi^2 \right) \left[ \mu_k + \varphi (s_1 - \mu_0) + \omega u \right]^2 \\
&= 2\mu_0 + \frac{(1 + \rho_0) \tau_\varepsilon}{\tau_0 + \tau_\varepsilon} (s_1 - \mu_0) \\
&\quad + \left( \psi - \frac{1}{2} \psi^2 \right) \left\{ \mu_k^2 + [\varphi (s_1 - \mu_0) + \omega u]^2 + 2\mu_k [\varphi (s_1 - \mu_0) + \omega u] \right\},
\end{aligned}
\]
and that \( \text{Cov} \left[ \tilde{s}_1 - \mu_0, [\varphi (s_1 - \mu_0) + \omega u]^2 \right] = 0 \) (because \( \tilde{s}_1 - \mu_0 \) and \( \tilde{u} \) are independent, normally distributed, and with zero mean). Also, note that
\[
\text{Var} \left[ \tilde{r}_1 \right] = B_r^2 \text{Var} \left[ \tilde{s}_1 \right] + C_r^2,
\]
as \( \text{Var} \left[ \tilde{u} \right] = 1 \) by definition. It follows that the average ERC is given by
\[
E \left[ \hat{\text{ERC}} \right] = \frac{(1 + \rho_0) \tau_\varepsilon}{\tau_0 + \tau_\varepsilon} B_r \text{Var} \left[ \tilde{s}_1 \right] + 2 \left( \psi - \frac{1}{2} \psi^2 \right) \mu_k (\varphi B_r \text{Var} \left[ \tilde{s}_1 \right] + \omega C_r). \]

Plugging in the equilibrium values of \( B_r \) and \( C_r \) from (A12) and (A13), respectively, we obtain
\[
E \left[ \hat{\text{ERC}} \right] = \left[ \frac{(1 + \rho_0) \tau_\varepsilon}{\tau_0 + \tau_\varepsilon} \right] \cdot A_{\text{ERC}} + \left[ 2\mu_k \left( \psi - \frac{1}{2} \psi^2 \right) \varphi \right] \cdot B_{\text{ERC}},
\]
where
\[
A_{\text{ERC}} \equiv \frac{(\gamma - \alpha \varphi^2) (\gamma + \alpha^2 \varphi^2)}{(\gamma - \alpha \varphi^2)^2 + \alpha^2 (1 + \alpha)^2 \omega^2 \varphi^2 \frac{\tau_0 \tau_\varepsilon}{\tau_0 + \tau_\varepsilon}},
\]
\[
B_{\text{ERC}} \equiv \frac{(\gamma + \alpha \varphi^2) \left[ (\gamma - \alpha \varphi^2) - \alpha (1 + \alpha) \omega^2 \varphi^2 \frac{\tau_0 \tau_\varepsilon}{\tau_0 + \tau_\varepsilon} \right]}{(\gamma - \alpha \varphi^2)^2 + \alpha^2 (1 + \alpha)^2 \omega^2 \varphi^2 \frac{\tau_0 \tau_\varepsilon}{\tau_0 + \tau_\varepsilon}}.
\]

\textbf{Proof of Part (1).} Let \( \pi \) be one of the parameters in \( \{ \mu_k, \rho_0, \tau_\varepsilon, \delta_k, \sigma_k, \tau_0, \alpha \} \). Notice that
\[
\begin{aligned}
\lim_{\gamma \to \infty} A_{\text{ERC}} &= \lim_{\gamma \to \infty} B_{\text{ERC}} = 1, \\
\lim_{\gamma \to \infty} \frac{\partial A_{\text{ERC}}}{\partial \pi} &= \lim_{\gamma \to \infty} \frac{\partial B_{\text{ERC}}}{\partial \pi} = 0.
\end{aligned}
\]
Therefore, in the limit as $\gamma \to \infty$,

$$\frac{\partial \mathbb{E}[\tilde{ER}_C]}{\partial \pi} \approx \frac{\partial \left[ \frac{(1+\rho_0)\tau_e}{\tau_0+\tau_e} \right]}{\partial \pi} \cdot 1 + \left\{ \lim_{\gamma \to \infty} \frac{\partial \left[ 2\mu_k \left( \psi - \frac{1}{2} \psi^2 \right) \varphi \right]}{\partial \pi} \right\} \cdot 1$$

$$= \frac{\partial \left[ \frac{(1+\rho_0)\tau_e}{\tau_0+\tau_e} \right]}{\partial \pi} + 2 \left( \lim_{\gamma \to \infty} \left[ \frac{\partial \mu_k}{\partial \pi} \left( \psi - \frac{1}{2} \psi^2 \right) + \mu_k (1-\psi) \frac{\partial \psi}{\partial \pi} \right] \right) \varphi$$

$$+ 2\mu_k \frac{\partial \left[ \frac{(1+\rho_0)\tau_e}{\tau_0+\tau_e} \right]}{\partial \pi} \cdot 1 + 2 \left[ \frac{\partial \mu_k}{\partial \pi} \frac{1-\alpha^2}{2} - \mu_k \alpha \cdot \lim_{\gamma \to \infty} \frac{\partial \psi}{\partial \pi} \right] \varphi + 2\mu_k \frac{1-\alpha^2}{2} \frac{\partial \varphi}{\partial \pi},$$

where

$$\lim_{\gamma \to \infty} \frac{\partial \psi}{\partial \pi} = \begin{cases} 0 & \text{if } \pi \in \{\mu_k, \rho_0, \tau_e, \delta_k, \sigma_k, \tau_0\} \\ 1 & \text{if } \pi = \alpha \end{cases}.$$

The claims then follow by inspection (recall that $\mu_k > 0$).

**Proof of Part (2).** If $\gamma < \alpha \varphi^2$, then both $\mathcal{A}_{ER_C}, \mathcal{B}_{ER_C} < 0$, and so $\mathbb{E}[\tilde{ER}_C] < 0$. ■

**Proof of Corollary 3.** By (A15),

$$-\mathbb{E}[\tilde{b}] = -(\mu_0 - A_r) = \frac{\alpha (1+\alpha) \varphi}{\gamma + \alpha^2 \varphi^2} \mu_k.$$  

Comparative statics of the amount of bias with respect to $\mu_k$ and $\gamma$ is immediate (recall that $\mu_k > 0$). As for the other parameters, note that if $\gamma > \alpha \varphi^2$, then

$$\frac{\partial \left( -\mathbb{E}[\tilde{b}] \right)}{\partial \varphi} = \frac{\gamma - \alpha^2 \varphi^2}{(\gamma + \alpha^2 \varphi^2)^2} > 0,$$

$$\frac{\partial \left( -\mathbb{E}[\tilde{b}] \right)}{\partial \alpha} = \frac{\gamma + 2\alpha \gamma - \alpha^2 \varphi^2}{(\gamma + \alpha^2 \varphi^2)^2} > 0.$$  

The signs of these derivatives are the same as the signs of $\partial (1 - B_r) / \partial \varphi$ and $\partial (1 - B_r) / \partial \alpha$. ■