“When Diversity Becomes Relevant”—A Multi-Category Utility Model of Consumer Response to Content Recommendations

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Abstract

The diversity of a set of recommendations can improve consumers’ satisfaction with the personalized recommender system. However, diversifying a list of products for a one-shot recommendation sacrifices relevance, which can reduce its value. We identify a popular scenario, sessions of online news consumption, where one can increase the diversity of recommendations over the entire session while improving the relevance of each recommendation within the session. Our approach is based on a multi-category utility model that captures consumers’ preference towards different types of content, how quickly they satiate with one type and substitute it with another, and how they trade off their own costly search efforts with selecting from the recommended articles to find new content. Taken together, these three elements enable us to characterize how utility maximizing consumers construct diverse “baskets” of content over the course of each session, and how likely they are to click on content recommended to them. We estimate this model using a clickstream dataset from a large international media outlet and apply it to determine the most relevant content at different stages of an online session. We demonstrate that by taking into account how consumers sequentially select content from different categories over time within a session, we not only recommend more diverse content over a session, but also recommend more relevant content than methods that do not incorporate such information. The diversity of the content recommended by the proposed approach closely matches the diversity sought by individual readers in their actual natural consumption—exhibiting the lowest concentration-diversification bias when compared to other personalized recommender systems. Meanwhile, the proposed approach makes 6%–14% more accurate recommendations than optimized alternatives. Using a policy simulation, we estimate that recommending content using the proposed approach would result in visitors reading 57% additional articles at the studied website, which has direct revenue implication for the publisher of this site.

Keywords: Recommender Systems, Personalization, Recommendation Diversity, Variety Seeking, Substitution, Collaborative Filters, Utility Models, Content Recommendation
1 Introduction

1.1 Online News and Content Marketing

Half of adults in United States under the age of fifty get their news predominantly online; and the fraction has been growing for several years. Therefore, online news is a significant growth area for publishers who have been struggling due to a decline in print circulation (Mitchell and Holcomb 2016). These publishers compete to attract online readers and to keep them engaged with their site because their profitability hinges on doing so effectively. The more articles visitors read on their site, the more opportunities there are for serving online ads—a revenue source that has grown amidst the general decline in ad revenue for newspapers and magazines.1 Engaged readers are also more likely to pay a subscription fee to access articles behind a paywall—a business model increasingly adopted by publishers (Edmonds et al. 2013).

Online consumers, on the other hand, are experiencing a golden-era of news (Yglesias 2013). With most publishers maintaining a comprehensive online presence, content in a wide range of topics—from the mainstream to the obscure, from politics to sports to leisure—are accessible to anyone with a browser. However, this has led to an information overload problem where a reader must find interesting content from a vast number of choices. Top news sites publish over a thousand pieces of content everyday (Marshall and Alpert 2015). Due to the difficulty of identifying potentially interesting content from these large volumes of material, a significant portion of relevant content goes undiscovered, which can lower engagement with the site.

The readers’ challenge in discovering content of interest to them and the publishers’ need to engage their readers have led to the rise of a number of intermediaries, called “content discovery platforms”. These third party companies offer a solution to the content discovery problem by providing a recommendation tool that publishers can use to help their readers discover interesting content, often in less visited parts of the site. And if the reader is intrigued by a recommendation that takes her to a different website, the originating publisher typically receives a referral fee.2 The top two content discovery platforms, Outbrain and Taboola, are used by more than 66,000 web domains including most of the top news publishers such as BBC, CNN, NBC News, ESPN, The

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1 While revenues from all types of advertisements decreased by 52% for the newspapers, revenues from digital advertising increased by 177% between 2003 and 2012 (Edmonds et al. 2013).

2 This form of digital marketing, where a firm or other website is willing to pay a referral fee to an online content provider when a link posted on their site is clicked, is the fastest growing form of “content marketing” (Content marketing generated $26.5 Billion globally in 2014 and expected to double over the next 4 years (P. Q. Media 2015)). Such digital content marketing has become the primary revenue source for many publishers (Carmody 2016).
Huffington Post, Fox News, etc., (SimilarTech 2016). Despite early successes, these practices have run into some challenges in keeping the recommendations relevant for readers (Griffith 2014). Most recommendations are determined by a combination of factors including popularity, recency, similarity to the content currently being consumed, and heuristics established by the site owner to promote certain parts of the website. To the best of our knowledge, such recommendations are not personalized for each reader based on her consumption history.

1.2 Diversity in Content Recommendation

Although not prevalent among the content discovery platforms, personalized recommender systems have been under active development for some time (Goldberg et al. 1992, Resnick et al. 1994, Murthi and Sarkar 2003, Huang et al. 2007). A number of online retailers have come to rely on them to help their consumers find items relevant to their unique preferences and to find potential customers for particular products they offer (Linden et al. 2003, Gomez-Uribe and Hunt 2015). Most of the notable improvements in recommender systems have been in the accuracy of predicting the products a consumer would like (McNee et al. 2006, Bennett and Lanning 2007). However, diversity in recommendations has also been widely recognized as an important characteristic by practitioners (Alvino and Basilico 2015, Gomez-Uribe and Hunt 2015).

In recent years, a number of papers in the Management Sciences literature have studied the effect of personalized recommender systems on the diversity of the overall sales achieved by a firm and consumption by each consumer (Fleder and Hosanagar 2009, Adomavicius and Kwon 2014, Lee and Hosanagar 2014). Fleder and Hosanagar (2009) have shown that commonly used personalized recommender systems could concentrate the overall sales of a firm—a concentration bias—by frequently recommending from a small pool of popular products. But they could broaden consumption at the individual consumer level, because products recommended to any one consumer are likely to introduce her to some products she would not have otherwise found. However, this need not match the type of diversity each individual seeks since it is a result of recommending to most consumers from a common set of popular items.

At the individual level diverse, yet relevant, recommendations lead to serendipitous discovery (Zhang and Hurley 2008, Adamopoulos and Tuzhilin 2014). Therefore, a number of approaches have been proposed to introduce diversity into personalized recommendations (Bradley and Smyth 2001, Ziegler et al. 2005, Adomavicius and Kwon 2009, Zhou et al. 2010). Typically, a list

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3 Adomavicius and Kwon (2014) have proposed a max-flow based approach to maximize the aggregate diversity of the recommendations made by a firm. Although, aggregate diversity at the firm level is an interesting topic with implication
of products, as opposed to a single product, is recommended to a consumer. A common strategy for constructing a diversified list is to select from the most relevant products found by a personalization system, such as a Collaborative Filter, a subset that maximizes a combined metric of relevance and intra-list diversity (Bradley and Smyth 2001, Ziegler et al. 2005). By choosing the weight of the combination, a practitioner can trade-off one for the other. A diversified list has several advantages. For the firm, it reduces the risk associated with trying to predict a consumer’s uncertain preferences (Bradley and Smyth 2001, Vargas and Castells 2013). For consumers, recommendations are often more helpful if they allow choosing from many different types of products as opposed to when all the recommendations belong to one category (Zhang and Hurley 2008, Hurley 2011). This is particularly true for product recommendation where only one product is likely to be selected from a set of recommendations at a consumption occasion.

Despite the outlined advantages, list diversification strategies have certain limitations. In a typical diversified list, recommendations are generated for one consumption without considering interdependencies across multiple consumptions. Without such interdependencies, the most accurate recommendation strategy would invariably be to recommend the top few products predicted to be the most relevant to a consumer irrespective of time. Therefore, any diversification attempt necessarily sacrifices accuracy. Published approaches in this space strive to maximize the diversity of the recommended list with minimum loss of accuracy. Additionally, while personalized recommender systems have always inferred consumers’ preferences for different types of products from past choices, the extant diversification strategies do not yet learn consumers’ preference for diversity. This creates a disconnect between the level of diversity that is optimal for the consumer and what is generated by the system.

These observations have motivated us to seek answers to the following research questions: both relevance and diversity are highly desired traits of recommender systems—must we always sacrifice one for the other? Are there scenarios where we can make more diverse recommendations while improving relevance for the consumer? Can we learn to recommend with the level of diversity sought by each consumer, in keeping with the spirit of personalized recommendations, instead of diversifying equally for everyone as done in list diversification?

There are certain differences between the consumption of products, such as travel packages or computers, and the consumption of content, such as news articles and videos, which suggest a different way to think about diversity when recommending content. A consumer might choose

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for the product portfolio of a firm, in the current paper we focus on the diversity in recommendations made to individual consumers, which increases user satisfaction and engagement (Ziegler et al. 2005).
only one of several offered products on a given consumption occasion; therefore, recommending one diversified list of products may be all that is necessary. But, when reading news, a reader is rarely “done” after consuming one news item, nor does the reader feel he or she needs to choose only one option among the numerous articles available in each session. A general media/news site typically offers content on a host of categories providing readers an opportunity for diverse consumption. Consumption on such media outlets is more fluid, with an individual likely reading multiple articles from several categories in a given session. Therefore, diversity over the entire set of items consumed in a given session is important to investigate and an area that has been largely ignored in the literature.

While selecting items sequentially within a session, recent consumptions could affect a reader’s preference for the next item. For example, after consuming several articles on one topic a reader might satiate and seek content in another topic, leading to a variety seeking behavior (McAlister 1982, Singh et al. 2014). Or, after consuming an article in a given topic the reader might get interested and wish to read a few more articles in the same topic. Such local behaviors could lead to diverse consumption over the session, while each consumed item remains optimal for the reader at the time of selection. Therefore, by learning how readers make sequential selections and understanding how their preferences evolve within each online session one might be able to recommend diverse content over an entire session while enhancing the relevance of each recommendation instead of sacrificing it.

We explore this idea by developing a multi-category utility based model to learn how readers construct content baskets within a session. Like many personalization strategies, the proposed approach learns each individual’s general preference towards different types of content; yet unlike most, it also learns how quickly each reader satiates with a particular type of content and how likely she is to substitute one type of content with another—capturing variety seeking. Thus, instead of preparing one diversified list of recommended items for a reader, we make a sequence of path dependent recommendations over a session. This allows us to adapt the recommendations based on the content the reader has consumed so far in the session. The combination of the proposed multi-category utility model and a sequential recommendation strategy within a session enables us to introduce diversity into personalized recommendation with the goal of maximizing the reader’s utility at all stages of consumption within a session.
2 Problem Setup

Figure 1 Example layout of the content recommendations.

We partnered with a leading content discovery platform, Outbrain, for this project. This industry partner provides online publishers a set of widgets to be placed on their webpages. When a visitor to a publisher's site loads a webpage containing the widgets, a request is sent to the recommendation server of the content discovery platform. The server responds with links to various contents that populate the widgets. Figure 1 shows an example of a news article (on top-left) along with content recommendations (on right and bottom) from a typical publisher using a content discovery platform. Three types of recommendations are shown: links with thumbnails...
highlighting a story, text links to stories, and text links to promoted content from external sites, which could be about news or about promoted products. Although we describe one publisher, a similar presentation of content and recommendations is adopted by most news websites that use a content discovery platform.

We collected the clickstreams of a random sample of visitors to one of the clients of the platform, a large international news website. We also collected the full text and metadata of the articles published, recommended, or visited on this website during the data collection period. Initial evidence suggests that indeed readers are interested in several different categories (Figure 2c). In addition, a sizeable fraction of their online sessions includes content consumption from more than one categories (Figure 2d), indicating that readers often seek a variety of content during an online session. These observations reinforce our motivation to develop a personalized recommender system that meets the needs of online news providers by incorporating variety seeking as a core part of the recommendation.

![Graphs showing content consumption patterns](image)

**Figure 2** Summary of content produced and consumed over the data collection period.

The definition of diversity requires a set of items. And the value of diversity to the consumer depends on how the set is selected. A consumer is expected to seek diversity more when she selects...
multiple items simultaneously or within a short time span than when she selects each item separately with long time gaps between selections potentially at different consumption occasions (Simonson 1990, Read and Loewenstein 1995, Read et al. 2001, Fox et al. 2005). Online news consumption is characterized by a reader visiting her preferred news source at periodic intervals and typically reading multiple articles during each visit. Therefore, each visit, or online session, denotes a single consumption occasion within which variety seeking, driven by satiation with particular topics, is likely to be the clearest. However, as preferences return to the baseline level soon after each consumption occasion (Read and Loewenstein 1995), the variety seeking behavior may get masked in the aggregated consumption behavior of a reader over a long period of time that includes multiple sessions. Therefore, in this study we focus on diversity within sets of content selected and consumed within each online session. In this regard, the boundary of our set differs from that in recent studies where diversity across the entire collection of products selected by a consumer, over several consumption occasions, has been examined (Fleder and Hosanagar 2009, Lee and Hosanagar 2014). It is also worth clarifying that the literature on variety seeking distinguishes between selection and consumption occasions. For example, consumers are observed to select more diverse products when they purchase them simultaneously than when they purchase them separately, even if in both cases purchases are for distinct consumptions over time. In our setting of online news consumption, there is rarely a separation between selection and consumption of articles.

A site owner can observe every article a web-visitor reads and can recommend the next article during each page visit. We propose to take advantage of this high resolution observation and recommendation opportunities to learn each individual’s preference for diversity within a session and to make the best recommendations at each stage of the consumption.

3 Model Development

Table 1 shows some example sessions noting the sequences of categories consumed. Besides the consumption in multiple categories in each session, there are some patterns worth noting. In most sessions, after starting by reading several articles in a given category the readers move to a different category. This suggests that they could be satiating on the first category after repeated reading, and switching to a different category afterwards. In addition, we do not observe many

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4 Please see Adomavicius et al. (2015) for a survey and a nuanced discussion of the idea.

5 Although, a few readers could be saving articles for later consumption, using apps such as Pocket or features such as “Reading Lists” in Safari browser, we do not consider those behaviors in this study.
cases where readers switch between different categories on successive consumptions. Often they continue in a single category for several consecutive consumptions before shifting to a different category—suggesting they could be avoiding the cognitive costs associated with switching between categories. We incorporate some of these intuitions in our proposed model.

<table>
<thead>
<tr>
<th>Session</th>
<th>Sequence of categories consumed (Each index represents a different category)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7 7 24 7 7 22 18 18 18</td>
</tr>
<tr>
<td>2</td>
<td>3 3 3 22 14 16</td>
</tr>
<tr>
<td>3</td>
<td>22 22 22 22 22 24</td>
</tr>
<tr>
<td>4</td>
<td>14 14 3 14 14</td>
</tr>
<tr>
<td>5</td>
<td>24 3 22 3 3</td>
</tr>
<tr>
<td>6</td>
<td>22 22 15 15 15 15 15 15</td>
</tr>
<tr>
<td>7</td>
<td>24 24 20</td>
</tr>
<tr>
<td>8</td>
<td>22 22 3 4 4 4 4 4 7 7 22 22</td>
</tr>
</tbody>
</table>

Table 1 Example sessions with consumption sequences. The sessions could belong to different readers. Category labels are given in Table 1 in the Appendix.

We model each reader as maximizing her expected utility at each step of a session by choosing a category to read in. When the reader is on a page she can choose one of the recommended articles on the page as the next item to consume, or find an article on her own elsewhere on the site potentially incurring a higher search cost. Since the reader decides to visit an article from one of these two sources before seeing the content of the article, she would have to rely on her belief about the quality of articles from two sources: recommendation vs self-guided search. Such belief is shaped by her prior experience with each source. Extant works in the Information Technology adoption literature show that users lose trust in recommendation agents when they are not satisfied with the quality of the recommendations (Komiak and Benbasat 2006, Harman et al. 2014). Therefore, we model the reader’s belief as evolving with her experiences with the recommended content. Lastly, at any stage the individual may no longer derive positive utility from continued news consumption on the site and elect to leave.

At every step the reader chooses a path based on the net expected utility it offers using a Multinomial Logit model—following the practice in discrete choice model literature (McFadden 1980). The decision tree within each session is illustrated in Figure 3. The edges are the reader’s actions, which lead the reader to one of three possible types of locations: an article, a list of recommendations to consider, or an option outside the site.

The data representation we use is a sequence of chosen categories in a session as shown in Table 1. We do not explicitly model the reader’s decision to use the homepage of the website or search.
box to find the next piece of content to consume; these actions are summarized in the search cost incurred in the self-guided mode. The utility derived from consuming content in one session is modeled as independent of any content the reader may have consumed in other sessions conditional on her preference parameters.\textsuperscript{6} However, the utility derived from consuming content in one category could depend on content she has consumed so far in other categories within the current session and is modeled using a Multi-Category Utility function.

\textbf{Figure 3 Intra-session decision tree of a reader.}

In the rest of this section we present the components of our model: the utility function, readers’ updating belief about the quality of recommendations, the effect of the position on the recommended list, and how these elements affect the value of recommender system vs. self-guided search as a means to find the next article to consume.

\subsection{3.1 Utility from Consuming Content}

Utility functions play a fundamental role in capturing a consumer’s preference in Microeconomics (Varian 2010) and in explaining their choices in Marketing (McFadden 1986, Ben-Akiva et al. 1999). Utility functions that describe multiple aspects of consumption are of particular relevance

\textsuperscript{6} Any correlation in consumer’s selection behavior from session to session is captured in the baseline preference parameters which are assumed to remain constant for the consumer over the observation period. However, the independence assumption conditional on these parameters is justified as the preference of a consumer generally returns to the baseline level soon after the consumption occasion (Read and Loewenstein 1995).
to our application (Huber 1974, Erdem and Keane 1996). When the utility generated from each attribute of a product exhibits decreasing marginal utility, a utility maximizing consumer shows variety seeking by consuming more along the attributes that have been consumed less so far. A type of multi-category utility function, known as Constant Elasticity of Substitution (CES), captures how a utility maximizing agent substitutes one type of product with another in addition to the decreasing marginal utility property (Arrow et al. 1961, Dixit and Stiglitz 1977, Baltas 2001).

Because of its flexibility in representing a variety of consumer behaviors we use the CES function to model the utility from online content consumption.

Specifically, a reader is assumed to gather utility according to the CES utility function, as shown in Equation 1, from consuming in some of $K$ content categories.

$$U(x^t_u) = \left( \sum_{k=1}^{K} a_{ku} x_{ku}^{\frac{s_u-1}{s_u}} \right)$$

Where, $a_u = \{a_{ku}\}$ is a $K \times 1$ vector of utility coefficients from different categories of content for reader $u$, $s_u$ is the Elasticity of Substitution among categories and takes a value in $(0, \infty)$, and $x_{ku}^t$ is the number of pages in category $k$ consumed by reader $u$ by the $t$'th article in the current session.

We model $a_u$ as a random parameter that follows a multivariate log-Normal distribution, i.e., $\log(a_u) \sim MVN(\mu_a, \Sigma_a)$. It captures reader $u$’s unique baseline preferences for content in different categories and is assumed to be constant over the observation period. The covariance matrix $\Sigma_a$, estimated from the readers’ choices, measures the correlation in preferences towards different categories of content. This covariance captures the idea that “consumers who like this category also like these other categories”, which is often used to convey the intuition behind collaborative filtering for generating personalized recommendations (Thompson 2008).

We model $s_u$ as a random parameter that follows a log-Normal distribution, i.e., $\log(s_u) \sim Normal(\theta_s, \sigma_s)$. There is a decreasing marginal return to additional consumption in a particular category because $\frac{s_u-1}{s_u} < 1$. When $s_u \to 0$, the right hand side of Equation (1) reduces to $\min\{x_{ku}^t\}$. In other words, the total utility is limited by the least consumed category up to that point. So a reader needs to consume in all categories to be able to increase utility; in other words, categories are perfect complements. When $s_u \to \infty$, the total utility is $\sum_k a_{ku} x_{ku}^t$, i.e., low utility contribution due to less consumption in one category can be compensated by increasing consumption in another category; in other words, categories are perfect substitutes. The indifference curves of the utility functions are shown in Figure 4.
Figure 4 Indifference curves for different values of $s_u$. Utility is derived from the consumption of products $x$ and $y$. Note that $U_0 < U_1 < U_2$.

Equation (1) provides the total utility in a session from reading $x^t_u = \{x^t_{ku}\}$ articles across the $K$ categories. To model each decision regarding what to read next we need the marginal utility from a particular category given the categories of the articles read in the session so far. Taking the partial derivative of Equation (1) with respect to $x^t_{ku}$, the additional utility from one more article in category $k$ for reader $u$, given the reader has already consumed $x^t_u$ articles, can be approximated as:

$$
\Delta U(k; x^t_u) = \left( \sum_{k'=1}^{K} a_{ku} x^t_{k'u} \frac{s_{k'u} - 1}{s^t_u} \right) \frac{1}{s^t_u - 1} a_{ku} x^t_{ku} \frac{1}{s^t_u}
$$

Here, we are approximating the marginal utility at $x^t_u$ over one additional unit of content in category $k$ by a linear function with a slope given by the derivative of (1).

Equation (2) relates the marginal utility from each category to all the consumptions made so far in the session. However, there could also be shorter term dependencies, such as inertia, among consecutive consumptions (Roy et al. 1996). We allow for this possibility by including a switching cost into the marginal utility equation whenever the reader changes the consumption category.

$$
\Delta U(k; x^t_u) = \left( \sum_{k'=1}^{K} a_{ku} x^t_{k'u} \frac{s_{k'u} - 1}{s^t_u} \right) \frac{1}{s^t_u - 1} a_{ku} x^t_{ku} \frac{1}{s^t_u} - c_{switch} \Delta k^t_u
$$

Where $\Delta k^t_u$ is a binary variable indicating whether consuming an article in category $k$ constitutes a change of category for the reader. $c_{switch} \sim Normal(\theta_{switch}, \sigma_{switch})$ is the unique category switching cost for reader $u$. The two distribution-parameters to be estimated, $\theta_{switch}$ and $\sigma_{switch}$, are shared by all readers for parsimony and to pool data across readers for more robust estimation.
3.2 Reader Belief about Recommendation Quality

If the quality of all articles were the same, an individual could fully evaluate the prospect of reading the next article, either recommended by the platform or found by a self-guided search, in each of the $K$ categories by using the formula in Equation (3). However, articles from different sources might differ in their quality and some links that appear interesting at first may lead to poor quality content (Fandrey et al. 2015, Thompson 2016, Waldman 2016). Since the reader has not yet seen the content of the article, she would have to rely on her current belief about its quality.

We assume that content pages provide different values to readers based on their quality level. For simplicity, we model two levels of quality: ‘good’ articles provide one unit of content value and ‘bad’ articles provide zero units of content value. Therefore, a good article provides utility as given in Equation (3) to the reader, whereas a bad article contributes zero utility. Only upon clicking on an article does the reader observe its true quality and proceeds to read it if it is good.\(^7\) The quality of an article recommended to reader $u$, since binary, is assumed to follow a Bernoulli distribution with parameter $p_{ru}$. The reader’s uncertainty about the quality of the recommended articles is captured by a distribution over $p_{ru}$. We assume that $p_{ru}$ follows a Beta distribution—the conjugate prior of the Bernoulli.

$$p_{ru}^\tau \sim \text{Beta}(\alpha_{ru}^\tau, \beta_{ru}^\tau)$$

(4)

The superscript $\tau$ is the index of an article consumed by reader $u$ since the start of data collection period. We assume that the reader updates her belief over time in a Bayesian manner based on the quality of the documents accessed from the recommender system. If the starting belief of the reader is described by the parameters $(\alpha_{ru}^0, \beta_{ru}^0)$, then the updated belief after reading $n_{ru}^\tau$ recommended documents of which $n_{rug}^\tau$ were good is captured by the posterior distribution with parameters $(\alpha_{ru}^\tau, \beta_{ru}^\tau)$ and calculated as:

$$\alpha_{ru}^\tau = \alpha_{ru}^0 + n_{rug}^\tau$$

$$\beta_{ru}^\tau = \beta_{ru}^0 + n_{ru}^\tau - n_{rug}^\tau$$

(5)

Based on this belief, the reader’s expected probability that a recommended article will be good, is the mean of this distribution:

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\(^7\) We classify certain page visits as “readings” or “skips” based on the delay before the next click by the reader, and use an empirically determined threshold from the distribution of the intervals between consecutive clicks.
The two parameters of the distribution determine both the mean belief about the quality, as specified in Equation (6), and the uncertainty about the quality, as the variance of the distribution is \( \text{Var}(p_{ru}^\tau) = \frac{a_{ru}^\tau \beta_{ru}^\tau}{(a_{ru}^\tau + \beta_{ru}^\tau)^2(a_{ru}^\tau + \beta_{ru}^\tau + 1)} \).

The \((a_{ru}^0, \beta_{ru}^0)\) parameters are assumed to be unique per reader allowing each reader to have an inherently different belief level about the quality of recommended links. These are treated as random parameters drawn from two known Log Normal distributions common for all the readers.

We represent a reader's belief about the quality of articles found in a self-guided manner, i.e., not through the recommendation engine, using a separate distribution:

\[
p_{zu}^\tau \sim \text{Beta}(\alpha_{zu}^\tau, \beta_{zu}^\tau)
\]

Unlike the belief about the recommendation engine, a tool whose properties may be unfamiliar to readers at the outset, the belief in their own abilities to find good articles from the website is assumed to be stable and known to the reader over the data collection period.

The selection of an article from a category depends on the expected utility from the category. For a recommended article this is computed by taking the expectation of Equation (3) over the quality of the article:

\[
\Delta U(k; x_u^t)_{p_{ru}^\tau} = \left\{ \sum_{k=1}^{K} a_{ku} x_k^t s_u^{-1} \frac{1}{s_u^{-1}} \right\} a_{ku} x_k^t \frac{1}{s_u^{-1}} p_{ru}^\tau - C_{u}^{\text{switch}} \Delta k_u^\tau
\]

The net value the reader expects from one of the recommended articles is given by

\[
\Delta V^\tau(k; x_u^t) = \langle \Delta U(k; x_u^t) \rangle_{p_{ru}^\tau} - C_{u}^\tau
\]

Where \(C_{u}^\tau\) is the cost of following a recommendation link. Conceptually, this includes the cost of clicking on a recommended link and consuming the referenced content. But, it does not include the cost of evaluating a list of recommended links and selecting one; this is modeled separately (Section 3.4). This reader specific random variable is assumed to follow a LogNormal distribution.

\[
C_{u}^\tau \sim \text{LogNormal}(\mu^\tau, \sigma^\tau)
\]

The parameters of the distribution, \(\mu^\tau\) and \(\sigma^\tau\), are shared among the readers for parsimony and are estimated from the data generated by all the readers. The expected value the reader gets from
an article she found herself, \( \langle \Delta U(k; x'_u) \rangle_{p_{fu}} \), is computed in an analogous manner by replacing \( p_{fu} \) in Equation (8) with the mean of the distribution in Equation (7) and using an analogous cost, \( C^S_{fu} \), incurred to find an article in the self-guided mode.

### 3.3 Position Bias

The position of an item in a list has been known to affect the probability of the item being selected (Day 1969, Blunch 1984). In online settings, consumers click on items higher in a list more often (Pan et al. 2007, Ursu 2015). We find evidence of this behavior in our dataset (Figure 5). This is known as the position bias and needs to be corrected to learn the true preferences of consumers from their selections (Chapelle and Zhang 2009).

![Figure 5 Frequency of clicks at different positions of a recommendation list in our dataset.](image)

We use a model, known as the Cascade Model, to capture this bias because it is relatively simple and has been shown to quite accurately capture this phenomenon (Craswell et al. 2008, Chapelle and Zhang 2009). According to this model, a reader examines links recommended in a list from top to bottom and clicks on the first link with positive expected utility. The reader does not examine any link positioned lower in the list than the one clicked, but examines every link above it and is interpreted to have chosen to not click on them, i.e., they were deemed to have non-positive expected utility. Therefore, the observation that the reader clicked on an article ranked at the \( n \)'th position is a conjunction of \( n - 1 \) binary decisions to not click on links before it and one binary decision to click on the \( n \)'th link. Modeling each decision using a Binary Logit model, the probability of this joint event can be written as:

\[
\frac{1}{1 + e^{-\Delta U_n^L(k; x'_u)}} \prod_{i=1}^{n-1} \frac{1}{1 + e^{\Delta U_i(k; x'_u)}}
\]  

\[\text{(10)}\]
Where $\Delta V^T_i (k; x^T_u)$, given by Equation (9), is the net utility the reader expects to get from a recommended article at position $i$ based on a) her current belief about the quality of recommended articles, b) the category of the article at this position, c) the total number of articles the reader has read in different categories so far, and d) the cost of following a recommendation.

### 3.4 Channel Choices

At the start of the session a reader chooses in the self-guided mode a category to read in according to a Multinomial Logit model. Since $s^0_{ku} = 0 \forall k$, the utility for the first article in each category is:

$$\Delta U(k; 0) = a(ku)^{s_{ku} - 1}$$  \hspace{1cm} (11)

After visiting the first page, the reader could take one of three possible paths: consider the recommended links on the page and click on one if found satisfactory, visit an article found by herself outside the set of recommended links, or leave the site.

The decision to evaluate the list of recommended links, as opposed to following the self-guided search after the current article, depends on the value the reader expects from the recommended articles. This could be written as:

$$\Delta V_f (\text{consider a list}) = P_k \Delta U\left( x_u \right)_{wr,s} - C_f$$  \hspace{1cm} (12)

where $P_k$ is the probability that an article in the recommended set will be in category $k$.

According to the cascade model there is a probability that the reader does not click on any of the recommended articles. Therefore, the expected value of choosing to consider the recommendation list is:

$$P(\text{click on any recommended link})\Delta V^T (x^T_u)_{k} = \left( 1 - P(\text{no click on recommended links}) \right) \Delta V^T (x^T_u)_{k} - C_f$$  \hspace{1cm} (13)

where $C_f$ is the cost of considering articles in the list. Conceptually, it includes the cost of scrolling down to the box containing the recommendations (often at the bottom of the page), evaluating the potential value of the links based on their image and text, and deciding whether to click on any one of them. This cost is modeled as a reader specific random variable following a log-Normal distribution with parameters $\mu_{\text{cons}}^r, \sigma_{\text{cons}}^r$.

Alternatively, the expected value of finding an article on her own in a self-guided manner is:
\[
\langle \Delta V^s(x_{wu}^s) \rangle_k = \langle \Delta U(x_{wu}^s) \rangle_{\text{lu}} - C_u^s = \sum_k P(k) \langle \Delta U(x_{wu}^s) \rangle_{\text{lu}} - C_u^s
\]  

(14)

The cost of reading through self-guided search, \( C_u^s \), is set to one for identification. This cost includes the effort of finding an interesting article from the website, perhaps from the homepage or by using the search functionality on the site, and any cost associated with consuming the content. The expected value of the outside option, i.e., of ending the session, is assumed to be constant and set to zero without loss of generality.

The reader chooses one of the three possible paths using a Multinomial Logit model. We summarize the data generation process within a session in Table 2. The interval between sessions is treated as exogenous.

1. For the first article of the session choose a category in \( 1 \ldots K \) using a Multinomial Logit model where the expected value of the first document in each category is given in Equation (11).
2. Choose one of the three possible paths: consider recommendation, find an article on her own in a self-guided manner, or leave the site, using a Multinomial Logit model with values of each choice given by Equations (13), (14), and zero respectively.
3. If the reader considers recommendations, the probability of clicking on an article at rank \( n \) is given by Equation (10)
   a. If the reader does not choose any recommended link, she chooses self-guided or leave-the-site paths using a Binary Logit model. The value of each path is given by Equation (14) and zero.
4. If the reader chooses the self-guided path, the probability of her choosing category \( k \) is given by a Multinomial Logit model, where the value of choosing each category is provided by Equation (3)
5. If the reader chooses to leave the site, end the session.

Table 2 Data generating process in a session.

3.5 Discussion

The underlying explanation of the diversity observed in consumption during a session is that as one consumes different types of content the marginal utilities in all categories change (as per the elasticities of substitution in Equation (2)). This changes the reader’s preference for different types of content as she progresses through the session, which manifests itself in a diverse selection of content as she maximizes her utility in each selection. Other factors, such as the belief about
the quality of articles from the recommender system and the search costs, could affect a reader’s willingness to follow recommendations. By accounting for these factors in our model we obtain a more accurate estimate of the reader’s preference towards different content and their elasticities of substitution. These two elements of our model are key in generating relevant recommendations for a reader as her consumption status changes through the session.

By contrast, most approaches in the literature to the issue of diversification alter an initial relevance ranking of products via inclusion of items further down the relevance scale without considering the dependence on recent consumptions (Bradley and Smyth 2001, Ziegler et al. 2005, Zhang and Hurley 2008, Adomavicius and Kwon 2009). Lathia et al. (2010) show that switching between algorithms over time can increase diversity because the top recommendations of various algorithms often differ. As our methodology relies on prior consumptions in a session to determine the next relevant recommendation, it has some similarity with the context-sensitive recommendation literature (Adomavicius et al. 2005, Chen 2005, Adomavicius and Tuzhilin 2011, Aggarwal 2016). However, the key difference is that in context-sensitive approaches the extrinsic attributes of the context, such as day of the week, location, whether consumption occurs with other individuals, etc. are considered in generating recommendations. In our approach the intrinsic state of the reader, based on the observed consumptions in the session, is used for generating the most relevant next recommendation. In one of the few published approaches that take into account consumptions in current sessions, Zheleva et al. (2013) learn a consumer’s mood for certain types of music. Based on this inferred mood they make the remaining recommendations in the current session to serve one aspect of a consumer’s preference. Our approach instead focuses on how a consumer’s preference shifts from one type of content to another over the course of a session.

4 Empirical Evaluation

4.1 Data Description

We collect a clickstream dataset generated by a random subset of anonymous visitors to a large international news website over a period of three months (3/1/15–5/31/15). Each visitor is tracked across her visits via web cookies. Upon a visit to any page a set of recommended links are generated by the content discovery platform. These links are presented using the widgets as described in Section 2. In this study we are only interested in the two widgets containing links to content within the site. Each widget on the article page contains six links. The rank of links within each widget, as well as any page visits via clicks on any of the recommendations is recorded. The
clicks on links to articles within the site found through self-guided search are recorded as well. Separately, we collect the full text, category, and publication date of every content page on this website either published, recommended, or visited during the observation period.

If the duration of a page visit is less than a certain threshold—empirically determined for each reader separately from the distribution of the time gaps between consecutive clicks—we assume that the page was skipped.\(^8\) The threshold averages at 31 seconds and is less than 55 seconds for 95% of the readers. If we do not observe a click for 20 mins, we assume that the reader has ended the session—following similar heuristic as in Montgomery et al. (2004).

Recommendations in the widgets containing image thumbnails have a higher click through rate (0.38% vs 0.30%, or 26% higher). We model this difference by allowing readers to have different beliefs about the quality of links with and without an image. As described in Section 3.2, by using a separate distribution we capture both a potentially different mean as well as a different level of the uncertainty in the belief about the quality of the recommendations in the image-widget. To be able to estimate the belief parameters and the cost of following recommendations we include only those readers who have clicked on at least one recommendation in the entire data collection period. Since our model relies on clicks over time by each reader we further limited our analysis to readers who visited at least one hundred articles during the data collection period. Table 3 presents some descriptive statistics of the dataset that is finally used.

<table>
<thead>
<tr>
<th>Entities</th>
<th>Counts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Readers</td>
<td>264,518</td>
</tr>
<tr>
<td>Webpage Visits</td>
<td>85,770,475</td>
</tr>
<tr>
<td>Sessions</td>
<td>23,823,339</td>
</tr>
<tr>
<td>Unique Webpages</td>
<td>211,968</td>
</tr>
<tr>
<td>Categories</td>
<td>24</td>
</tr>
</tbody>
</table>

Table 3 Descriptive statistics

4.2 Estimation

We estimate the proposed model from the aforementioned clickstream dataset. As closed form expressions do not exist for the estimates of the parameters in the model, we use an algorithm in the Markov Chain Monte Carlo (MCMC) framework (Appendix II). The variations in choice of

---

\(^8\) We use k-means clustering on inter-click durations of each reader separately to identify time spent on different types of page-views. The mid-point between the centers of two shortest clusters is used as the threshold.
category at different stages of the session help identify category coefficients \( (a_u) \) and the elasticity of substitution \( (s_u) \) for each reader. Most readers consume only in a subset of categories even over the entire data collection period. Since the category coefficients are modeled in a hierarchical Bayesian framework, the coefficient estimates for categories that a reader never consumes in are based primarily on the prior distributions that are estimated from the selections of all the readers. The frequency of category switching of each reader helps identify the category switching cost \( (C^\text{switch}_u) \). The cost of examining the recommendation list \( (C^\text{cons}_u) \) is set to be homogenous and all other reader specific parameters are treated as heterogeneous based on the BIC score.

We compare the proposed model (named Multi-Category Utility with CES and labeled MCUwCES) to two alternative models. The first explores whether readers could simply be probabilistically selecting a category to read at each step of a session according to their preference towards different categories, i.e., not taking into account the effect of one category on another in forming sets. This would be equivalent to the following utility function:

\[
U(x^t_u) = \sum_k a_{ku} x^t_{ku}
\]

Since the marginal utility for each category is \( a_{ku} \), a reader simply selects the next category to read following a Multinomial Logit model based on her relative preference towards the categories.\(^9\) As the marginal utility does not decrease with consumption we call this the Non-decreasing Marginal Utility model (labeled as NdMU).

The second model (labeled MCUwEK) uses an alternative utility function proposed by Erdem and Keane (1996) that exhibits decreasing marginal utility using a quadratic form.

\[
U(x^t_u) = \sum_k a_{ku} x^t_{ku} - a_{ku} r_u x^t_{ku}^2
\]

Where \( a_{ku} \) is still the utility parameter of category \( k \) for reader \( u \), and \( r_u \) is the risk aversion parameter of reader \( u \). When \( r_u > 0 \) the function exhibits decreasing marginal utility of consumption in a category. However, it can be verified, by taking derivatives with respect to \( x^t_{ku} \), that unlike in the CES utility function the marginal utility of one category is independent of consumptions in other categories.

\(^9\) This happens to be a model where categories are perfect substitutes for each other, although an article in each category could yield a different amount of utility for the reader.
From the log likelihoods and the BIC scores of the three models, we find that our proposed MCUwCES model best describes readers’ behavior (Table 4).

<table>
<thead>
<tr>
<th>Model with ...</th>
<th>Log likelihood</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>CES utility function (MCUwCES)</td>
<td>-127,203,796</td>
<td>254,810,537</td>
</tr>
<tr>
<td>Utility function from Erdem and Keane (1996) (MCUwEK)</td>
<td>-128,144,984</td>
<td>257,363,271</td>
</tr>
<tr>
<td>Utility function with perfect substitution (NdMU)</td>
<td>-143,261,695</td>
<td>304,478,675</td>
</tr>
</tbody>
</table>

Table 4 Log likelihoods and BICs of alternate models

Examining the reader level parameters, we find considerable heterogeneity in their elasticities of substitution and their cost of following recommendations (Figure 6). The median elasticity of substitution of 15.8 suggests that most readers could at some point substitute their preferred category for another in any one session; although there is considerable heterogeneity in the degree to which they are willing to do so. The cost of following recommendations for all the readers was lower than their cost of finding an article through the self-guided mode, which was set to one for identification. This indicates that, indeed, readers find it easier to select one of the recommended articles than to identify another article via self-guided search.

![Figure 6 Distribution of (a) Elasticity of Substitution and (b) Cost of following recommendations.](image)

Examining readers’ beliefs about the quality of recommended articles we find that readers start with different levels of belief, which changes over time as a result of their experience with recommended articles. In Figure 8, we present three readers’ history of encounters with recommendations and the evolution of their beliefs as a result. Initially in the data collection period, the first reader clicks on a small number of recommended articles. But as she finds that the articles are of good enough quality, as evidenced by her reading as opposed to skipping, she reads more recommended articles over time. The corresponding estimate of her belief about the
quality of the recommended articles increases over time. Meanwhile, the second reader starts by visiting many recommended articles—even though she is reading a small fraction of those and skipping the rest. This suggests that although she had a high belief about the quality of recommended articles at the start, she is perhaps not finding many good articles. As this pattern continues, the reader chooses fewer and fewer recommended articles. This pattern is captured in the decreasing belief of the reader. We can interpret the third reader’s behavior in a similar manner: the belief first increases as she finds good articles to read, but later decreases when she does not find many good articles. Eventually, this reader stops choosing recommended articles.

Figure 7 Sample belief evolution as a result of experience with recommendations

4.3 Evaluation

First, we evaluate the algorithm for the task of recommending a category of content. Later, we apply the proposed model to recommend specific articles, keeping in line with prior work in personalized recommender systems.

4.3.1 Recommending a category of content

For each reader, we use the complete data over the first 90% of sessions to estimate the model and the data over the final 10% of sessions to predict the category selections. The marginal utilities of the categories are computed before each page visit in the holdout sessions using Equation (3). The category with the highest marginal utility is predicted. We compare the proposed algorithm to the following alternative approaches:

1. **Random Recommendation.** We randomly select a category and use this category as prediction. This establishes a baseline level of performance in this dataset.

2. **Most Frequently Read Category.** We select the most frequently read category for each reader from the training period and use this category as prediction.
3. **Similarity Based Collaborative Filter.** We use a user-to-user similarity based Collaborative Filter, computed using Pearson’s correlation, to find the most relevant category for a reader.

4. Non-decreasing Marginal Utility model (NdMU). Keeping the rest of the model identical we replace the CES utility function with the Non-decreasing Marginal Utility function. If we remove the category switching cost from this model, it would be identical to the “Most Frequently Read Category” recommendation.

5. Quadratic marginal decreasing utility model (MCUwEK). Keeping the rest of the structure of our proposed model—decision tree, belief update, costs, etc.—identical, we replace the CES utility function with the utility function from Erdem and Keane (1996).

Each category prediction is deemed correct if the reader was observed to click on an article from the predicted category. For this evaluation we only used readers’ clicks in the self-guided mode—95.7% of all clicks. The average accuracy of each approach is computed over the test period (Table 5).

<table>
<thead>
<tr>
<th>Recommendation Approach</th>
<th>Average Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCUwCES</td>
<td><strong>37.78%</strong></td>
</tr>
<tr>
<td>MCUwEK</td>
<td><strong>34.33%</strong></td>
</tr>
<tr>
<td>Similarity Based Collaborative Filter</td>
<td>28.70%</td>
</tr>
<tr>
<td>NdMU</td>
<td>24.71%</td>
</tr>
<tr>
<td>Most frequently read category</td>
<td>20.66%</td>
</tr>
<tr>
<td>Random recommendation</td>
<td>4.16%</td>
</tr>
</tbody>
</table>

**Table 5** Average accuracies of category-level predictions

First note that our proposed models using multi-category utility functions with diminishing marginal utility over the course of the session, MCUwCES and MCUwEK, outperform all other approaches. Among these two, MCUwCES, which takes into account substitution among categories, performs better. Second, the accuracy of the Similarity Based Collaborative Filter declines as the session progresses, presumably as the reader moves from her most preferred category to her second, or third most preferred categories due to satiation (Figure 8). The

---

10 The clicks on currently recommended links are constrained by the categories the content discovery platform has already chosen to present. They are further influenced by the different types of widgets present on the page. Therefore, the accuracy of category prediction for the clicks in the recommendation channel is not directly comparable to that in the self-guided mode. When the former data points are also included in the evaluation, we find that the measured accuracies are slightly lower, but the relative performances of the different methods are similar to what is presented in Table 5 with our proposed approach outperforming all other candidates. Clicks on recommended links are separately analyzed in the policy simulation in Section 5.
Collaborative Filter constructs a fixed ranking of the categories according to their relevance to a reader; it does not take into account her prior consumption in the session. Therefore, it cannot adapt as the reader satiates on her preferred categories and moves to other categories. By contrast, MCUwCES takes into account how much the reader has consumed in each category at each stage of the session to re-compute the marginal utilities and determines the most relevant category for the next reading. As a result the performance of MCUwCES is stable across all stages of the session, as can be seen from Figure 8.

![Accuracies at Different Stages of Session](image)

**Figure 8 Performance of MCUwCES vs Collaborative Filter as the session progresses**

It is further difficult to ignore the higher accuracy of the MCUwCES approach even at the beginning of the sessions. This could be attributed to the more precise interpretation of readers' selection of content throughout the session by accounting for the effect of satiation and elasticity of substitution. For example, if a session consists of three readings in the Politics category first followed by three readings in the Sports category, a Collaborative Filter would conclude that the reader is equally interested in both, whereas MCUwCES would allow for the possibility that the reader prefers Politics over Sports because it was chosen first. And that only after several consumptions in the Politics category, when the marginal utility has sufficiently diminished, the reader has moved to the Sports category. This expansive interpretation of selections contributes to a more accurate prediction of what the reader selects to read even at the start of the session.

### 4.3.2 Recommending an article

The analysis in the previous subsection focused on recommending the most relevant category of content. However, personalized recommender systems are often used to recommend specific items to a consumer. In order to accommodate this, while taking into account multi-category consumption behavior, one may further order the items within the recommended category according to a desired relevance metric such as recency, popularity, or even the personalized
ranking produced by another item-level recommender system. In this section we take such an approach to make personalized article recommendations.

Keeping the same 90% training and 10% test data split as described in Section 4.3.1, we evaluated a popularity based approach and three competitive Collaborative Filters—chosen based on their prevalence and performance in recent academic publications:

1. **Popularity** based recommendation: The most popular articles in the last 12 hours—a window chosen to maximize the performance of this simple strategy—are recommended.

2. Similarity based Collaborative Filter: We evaluate both user-user and item-item similarity based Collaborative Filters—each using Pearson’s correlation and cosine similarity metrics (Breese et al. 1998). We present the results of the **user-user** similarity based approach with Pearson’s correlation as it performs the best on this dataset.

3. **Spreading algorithm**: Zhou et al. (2010) has proposed an algorithm, inspired by the heat diffusion mechanism from physics, to resolve the diversity-accuracy dilemma. We choose to evaluate this method on our dataset because it has been shown to successfully combine two specific algorithms—one favoring accuracy and the other favoring diversity—to achieve higher accuracy and diversity than either of the two combined algorithms.

4. Link prediction: We have implicit ratings in this study, i.e., whether someone selected an article, not how much they rated an article on a scale of say 1–5. For implicit rating datasets, link prediction approaches have been shown to be very successful (Dunlavy et al. 2011, Sahoo et al. 2012). We selected the best performing link prediction method, called **Katz-CWT**, to predict which article a user will select in a given session in the test period.11

As news content loses its value quickly with time, training the three abovementioned Collaborative Filters using only the articles published in the one-week window before each prediction leads to the best performances. The parameters of each algorithm are tuned to maximize their performance on our dataset. Then we overlay the best performing category-based diversification strategies (MCUwCES and MCUwEK) on the rankings produced by these algorithms, thereby selecting the most relevant articles from each ranking that belong to the most relevant category. Separately, we implement a well performing method that diversifies a list produced by each of the four item level recommendation methods (Ziegler et al. 2005).

---

11 Matrix Factorization based Collaborative Filters have performed well in Netflix prize competition—a three year long event where teams worldwide competed to be the first to predict Netflix movie ratings 10% more accurately than then used Cinematch algorithm at Netflix (Koren 2008). However, they did not perform well in our application—perhaps because they were designed for settings with explicit ratings. Our application context, as many other settings where only transactions are available, has only implicit ratings.
method alters the items ranked second and lower in the list to maximize a combined score of accuracy and diversity. The top one and top six (current recommendation widgets contain six links) most relevant items for each reader on a given page in a session in the test period are predicted; if the reader has clicked on one of the predicted articles, it is considered to be an accurate prediction—following the practice in the literature. These accuracies, shown in Table 6, can be interpreted as the fraction of recommended sets that contains the link the reader clicked on.

<table>
<thead>
<tr>
<th>Item-level recommendation</th>
<th>No additional Diversification</th>
<th>List Diversification (Ziegler et al. 2005)</th>
<th>Multi-Category Diversification</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Top 1</td>
<td>Top 6</td>
<td>Top 1</td>
</tr>
<tr>
<td>Popularity</td>
<td>2.42%</td>
<td>5.81%</td>
<td>5.11%</td>
</tr>
<tr>
<td>User-User</td>
<td>2.51%</td>
<td>6.00%</td>
<td>5.27%</td>
</tr>
<tr>
<td>Spreading algorithms</td>
<td>2.76%</td>
<td>6.14%</td>
<td>5.40%</td>
</tr>
<tr>
<td>(Zhou et al. 2010)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Katz-CWT (Dunlavy et al. 2011)</td>
<td>2.85%</td>
<td>6.31%</td>
<td>5.47%</td>
</tr>
</tbody>
</table>

*Table 6 Average accuracies of article-level predictions*\(^{12}\)

The Multi-Category Utility models and the Collaborative Filters learn preferences at two different levels. The former learn readers’ preference towards, satiation with, and elasticity of substitution between different types of content with the help of category labels. A significant focus is on intra-session preference evolution. Collaborative Filters, on the other hand, do not rely on attributes such as category labels. They capture from readers’ feedback the aspects of the content that are hard to attributize, such as certain styles in writing or topics that are not described by the category of the article or the words in it. The primary focus is on learning only the stable preference of each user—not on how preferences could be changing within a consumption occasion. Because of this difference in their emphases, by combining them we see 6%–11% improvement in prediction over the best performing Collaborative Filter; the gain is larger for the other compared approaches.

The strength of this simple strategy for combining the multi-category utility-based and the item-based approaches is that it can be used to take advantage of the latest advancements in

\(^{12}\) It is harder to predict the article a visitor will read than the category she will read from because the former requires more precision (one of thousands of articles vs one of twenty-four categories). Besides, the categories are stable, whereas, new articles are introduced everyday whose consumption generally declines with time. Therefore, we expect a lower accuracy while predicting a visitor’s article selection than while predicting her category selection.
collaborative filtering for recommending most relevant items to a reader, yet doing so within the category that brings highest utility for the reader at a given stage of the session.

We can gain insight into the diversity seeking properties of the algorithms by plotting the average number of categories recommended in by each algorithm against the number of categories consumed in by a reader in the test period. Figure 9(a) shows that the diversity of the content recommended by MCUwCES closely mirrors the diversity of the content read by each reader in the self-guided mode: it recommends in a small number of categories to those who consume in a narrow set of categories and in an appropriately larger number of categories to those who consume a more diverse set of content. On the other hand, the Katz-CWT Collaborative Filter recommendations are too diverse at the low end and too narrow at the high end. The list diversification strategy further diversifies the top 6 recommendations by the Collaborative Filter—recommending an even greater number of categories to readers with narrow preferences.

![Figure 9 Diversity of recommendation for different readers and for different sessions.](image)

To examine any diversification or concentration bias in the recommended content, we first measure the diversity sought by readers in their natural readings from their self-guided selections (as these are not a result of what was recommended to them by the platform). We use the Gini-coefficient to measure the degree of diversity in their natural consumptions across categories following Fleder and Hosanagar (2009). When the Gini coefficient of recommendations is higher than that of readings in the self-guided mode, it indicates a narrower set of recommendations than what web-visitors naturally read. We find that the degree of diversity in recommendations made by MCUwCES is closest to the degree of diversity sought by the readers (Table 7). What’s

---

13 For clarity, only the best performing Collaborative Filter and its diversifications, i.e., those in the last line of Table 6, are plotted. But, the results are qualitatively similar for other Collaborative Filters.
more, the Gini coefficient of recommendations tells only part of the story: it lets us compare
whether the diversity of the recommended categories is similar to the diversity of the categories
consumed, but it does not tell us whether the diversity is based on the same categories. Two
methods could recommend two different sets of categories while achieving similar Gini
coefficients, but the distribution of one of them could closely match the categories read by the
web-visitor while the other may not. To measure whether the category distribution in
recommendations is similar to that in self-guided readings, we compute the Jensen-Shannon
Divergence between the two distributions for each reader (Lin 1991)—presented in the bottom
row of Table 7. A low Jensen-Shannon Divergence score indicates that the two distributions are
similar. We find that the distribution of the categories recommended by the MCUwCES approach
is closest to the distribution in the natural reading of the individuals. Taken together, these two
results suggest that the proposed MCUwCES approach shows the least diversification-
concentration bias, an issue that many commonly used Collaborative Filters suffer from (Fleder
and Hosanagar 2009).

<table>
<thead>
<tr>
<th></th>
<th>Self-guided reading</th>
<th>Recommended content</th>
<th>Katz-CWT with top 6</th>
<th>Katz-CWT with list diversification</th>
<th>MCUwCES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average reader level Gini</td>
<td>0.85 (0.0002)</td>
<td>0.92 (0.0002)</td>
<td>0.76 (0.0003)</td>
<td>0.86 (0.0002)</td>
<td></td>
</tr>
<tr>
<td>Average Jensen-Shannon divergence from current consumption</td>
<td>0 (0.0008)</td>
<td>0.21 (0.0008)</td>
<td>0.34 (0.0011)</td>
<td>0.15 (0.0011)</td>
<td></td>
</tr>
</tbody>
</table>

Table 7 Diversity of competing recommender systems with respect to the self-guided readings

Similar superior diversity seeking properties of the MCUwCES approach at the session level is
evident from the plot of the average number of categories recommended against the number of
categories consumed during a session in the testing period (Figure 9 (b)). Although all the
algorithms recommend a progressively larger number of categories in sessions where individuals
consume a more diverse set of contents, we again see a tendency among the compared alternatives
to over-diversify in sessions that has small number of categories and, in some cases, under-
diversify those that has a broader set of categories. By comparison, the number of categories
recommended by the MCUwCES approach more closely matches the number of categories
consumed in the session.

Although Collaborative Filters are not specifically designed to make diverse recommendations,
and although they have a fixed relevance ranking over items, they make increasingly diverse
recommendations in sessions where readers are observed to consume more categories. This
primarily occurs in longer sessions. This is because when a reader consumes an article, it is
removed from consideration for future recommendations—readers rarely re-read articles in the same session, and removing already read articles improves the performance of the Collaborative Filters. Therefore, we see the list of recommended items change and the cumulative number of recommended categories grow with session length, even with a standard Collaborative Filter. But, by explicitly taking into account the reduction in the marginal utility of a category, the MCUwCES is able to more closely track the actual number of categories read as well as their identities.

5 Potential Impact on Utility and Reader Retention

The utility of an article selected through self-guided search could be higher than the utility of an article selected from recommendations due to the much larger number of choices available in the former route. On the other hand, personalized recommender systems reduce the search cost: it is easier to select one of the links presented in the current webpage than to try and find another interesting article elsewhere on the website. The lower search cost could narrow the gap between the net utilities of the articles from the two channels.

<table>
<thead>
<tr>
<th></th>
<th>Current recommendation policy</th>
<th>Proposed MCUwCES policy</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gross Utility</strong></td>
<td>6.262 (0.004)</td>
<td>6.462 (0.004)</td>
</tr>
<tr>
<td><strong>Cost</strong></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td><strong>Net Utility</strong></td>
<td>5.262 (0.004)</td>
<td>5.462 (0.004)</td>
</tr>
</tbody>
</table>

Table 8 Average estimated utilities from the selected articles under the current recommendation policy vs the proposed MCUwCES policy. Standard errors are given in parenthesis.

Average estimated gross and net utilities derived from articles selected from the two channels under the current recommendation policy at the news site confirm this hypothesis (Table 8 (a)). For all the readers, the cost of finding an article from the recommendations is lower than the cost of finding an article through self-guided search. Although, this lower cost narrows the gap, the net utility from an article found through self-guided search is still higher under the current recommendation policy employed by the website. This may not be surprising since the recommendations are not personalized for each reader; hence the categories of content recommended may not be optimal. As such, readers click on the recommendation channel only in 4.3% of the page visits in our dataset. Given the better performance of the proposed multi-category approach, one might be able to use it to increase the net utility from the recommended links through personalization and close the gap. To explore this, we carry out the following policy simulation.
After estimating the model, for each reader we simulate one additional session with up to ten articles consumed in it. The readers start by visiting an article in the self-guided mode from a category selected according to the expected utility from each category as given in Equation (11). In place of the links recommended by the content discovery platform, each widget on each page visit was populated with links to articles from the category with the highest marginal utility as per the MCUwCES model. Each reader’s belief about the quality of articles from the self-guided mode and from different types of recommendation widgets were set to be at the estimated levels at the end of the training period. Readers’ behavior in the simulated session follows the proposed model, which was found to best fit the data (Table 4): stochastic channel choice, selection of an article, and decision to exit were simulated by following the decision tree in Figure 3, and each stochastic selection was simulated using a Multinomial Logit model based on the expected utility from the choices available.

The estimated average net utility of the articles selected from the recommendations under the proposed approach (MCUwCES) is about twice as high as that achieved under the current policy employed by the website. In fact, it is higher than the utility from articles found by self-guided search (Table 8 (b)). This is a result of recommending articles from the category that yields highest utility for the reader at each stage of the session. As an aside, it might be puzzling to see that the utility from articles found by self-guided search is higher under the simulation than it was in the data collection period as only the recommendation policy is changed, after all. This is because the new recommendation policy increases the average expected utility from recommended articles, prompting users to (stochastically) select the self-guided channel only when the expected utility from that channel is even higher.

Although the ex-post utility of an article selected from the recommendations by the MCUwCES approach is higher than that from the self-guided channel (Table 8 (b)), before selection the reader must choose a channel based on the net ex-ante utility of the next article from each channel. The marginal utility from articles declines with consumption, and readers leave the site when the net ex-ante utility is not attractive compared to the outside option. By improving this expected utility from the recommendations, one should be able to retain readers longer. Figure 10 shows the average net ex-ante utility at various stages of an online session—under the current policy on the left and under the proposed MCUwCES policy on the right. We also plot the

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14 This ex-ante utility could be lower than the ex-post utility due to the belief that the probability of obtaining a good article is less than one. Additionally, as the reader (stochastically) selects the channel with higher expected utility, the mean utility of a channel conditional on its selection is higher than the unconditional mean.
probability of a session containing certain number of article views, i.e., the distribution of session lengths.

**Figure 10** The reader’s probability of exiting the site and her average net ex-ante utility from the next article from each channel at different stages of the session under (a) the currently implemented recommendation policy and (b) the proposed MCUwCES approach.

Under the proposed strategy, the average net ex-ante utility from a recommended article is significantly higher than what it is under the currently implemented strategy—making the former competitive with articles found through self-guided search. Consequently, we estimate that more page-views would lead to clicks on recommended content—6.9% instead of 4.3% currently observed, or 59% higher—if we replace the current recommendations with content from categories recommended by MCUwCES. The distribution of session lengths shifts to the right as well, meaning visitors will read more articles in a session. We estimate that, on average, 57% more articles will be read in a session (3.3 vs. 2.1).

It is curious to see that the net ex-ante utility under the proposed strategy approaches, but does not exceed, that from self-guided search. As a result, readers choose recommendations less often than the self-guided channel. This is primarily due to the readers’ belief about the quality of the recommended articles, which is shaped by their experience with recommendations in the collected data. On average they read 67% of the articles found via self-guided search, vs. only 53% of those selected from the recommendations and skip the rest. This indicates a lower perceived quality of articles from recommendations—based on the reader’s experience with the current policy—which leads to a lower belief. This belief is not altered for the simulation. We find that even if a system recommends the best possible content at each stage of the session, the effectiveness of the system would be limited if the reader has a lower quality-belief. Some model free evidence of this can be found in our dataset. 1.4% of the clicks are on links found via self-guided search but that were recommended in the previous page. In other words, in these cases,
the content discovery platform correctly predicts what the reader would select next, but the reader skips its recommendation. Including these correct predictions would increase the click through on the current recommendation channel from 4.3% to 5.7%.

Despite this limitation arising out of readers’ already formed beliefs, the results from our policy simulation suggests that using the proposed strategy one can make better personalized recommendations that would engage readers on the website for longer durations. In turn, this would lead to more monetization opportunities for the publishers.

6 Conclusion

Personalized Recommender systems strive to fulfill a dual mandate: make diverse recommendations while keeping them relevant to each consumer’s unique preference. These two goals conflict when a list of products is recommended for a single consumption episode. We identify a popular scenario, consumption of multiple pieces of content within online sessions at news websites, where these two goals could be complementary when the entire session is considered. The premise is that by learning how readers form “baskets” of different types of contents to consume in a session and by making sequential recommendations in a path dependent manner, during each web-page visit within a session, we can generate more diverse recommendations while improving accuracy.

To implement this idea, we develop a utility theoretic model of online reader behavior. By choosing an appropriate Multi-Category Utility function, namely the Constant Elasticity of Substitution, we capture the decreasing marginal utility of a category as consumption in that category continues. Decreasing marginal utility prompts the reader to seek different types of content, which results in the observed diversity in consumption. The understanding that by seeking diverse content readers are not sacrificing relevance but merely selecting the content that best responds to the changing marginal utilities of all categories theoretically reconciles, at least in the context of online media consumption, the diversity—relevance dilemma.

We evaluate the proposed approach on a clickstream dataset from a large international news website and find that the Multi-Category Utility based approach makes more accurate content category recommendations than Collaborative Filter alternatives. Moreover, by combining the proposed approach, which finds the most relevant category at each stage of a session, with Collaborative Filters, which recommend the most relevant items within that category, we are able to recommend specific articles 6%–14% more accurately than using only the Collaborative Filters. At the same time, the proposed approach matches the diversity of observed consumption more
closely than Collaborative Filters do—exhibiting a lower concentration or diversification bias. Finally, using a policy simulation we estimate that under the proposed recommendation strategy readers would be expected to consume 57% more articles on the website than they do under the currently implemented strategy. This increased engagement has direct revenue implications for the publishers.

Most personalized recommender systems learn consumers’ preference for different types of items from their choices; few learn consumers’ preference for diversity. We augment the literature in recommendation diversification by offering a way to learn consumers’ preference for diversity from the items they sequentially select within different consumption occasions. As a result, the level of diversity is personalized to each consumer and each consumption occasion. The list diversification strategies might still be relevant for recommendations at each page-view, where managing ambiguous consumer preferences and providing diverse choices to enhance the consumer’s experience are important goals. The proposed multi-category utility based approach could be overlaid on these strategies to broaden the scope of diversification to the entire consumption occasion, thus leading to more relevant and appropriately diverse recommendations.

The utility theoretic approach taken in this paper allows one to borrow and incorporate various theories of consumption—satiation and variety seeking, inertia, cost of search vs. following recommendations, consumer learning, etc.—that have been developed in the Consumer Behavior literature. Yet such approaches have thus far been relatively rare in the personalized recommender systems literature. We believe that modeling these factors in a utility theoretic framework, as illustrated in the present study, offers a systematic way to accurately interpret consumer choices and design better personalized recommender systems.

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Appendix I. Data Description

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*Table 1 The list of categories.*

Appendix II. Model Estimation

Likelihood Function

The likelihood function for individual $u$ can be written as:

$$ L(Data^u | a_u, s_u, C_r^u, C_{u,cons}, C_{u,switch}, \alpha^0_{ru}) = \prod_{t=1}^{N_u} P(Decision^u_t | a_u, s_u, C_r^u, C_{u,cons}, C_{u,switch}, \alpha^0_{ru}) $$

User $u$ makes the set of $N_u$ decisions within the data collection period. The probability of observing $Data^u$ is the product of the probability of observing all $N_u$ decisions for this user. Each decision $Decision^u_t$ results in one of three types of observations: selection of a recommended article,
selection of an article in the self-guided mode, exit from the site. Probability of each outcome can be computed in the following manner.

1. The probability of user \( u \) selecting an article at the \( n \)th rank in the recommendation list is

\[
P\left( \text{Channel} = R, \text{Rank} = n \mid a_u, s_u, C_u^R, C_u^{\text{rank}}, C_u^{\text{switch}}, \alpha_{ru}^0 \right)
\]

\[
= P\left( \text{Rank} = n \mid a_u, s_u, C_u^{\text{rank}}, \text{Channel} = R \right)
\times P\left( \text{Channel} = R \mid a_u, s_u, C_u^R, C_u^{\text{rank}}, C_u^{\text{switch}}, \alpha_{ru}^0 \right)
\]

2. There are two paths for user \( u \) to select an article from category \( k \) via self-guided channel when she is within a session: she may directly opt for self-guided channel after reading the previous article, or she may consider recommendations first after reading the article but deeming no article in the list to bring positive net utility opt for the self-guide channel to find an article by herself. Therefore, the probability of observing a user to select an article in category \( k \) in the self-guided mode is sum of the probability of two mutually exclusive outcomes:

\[
P\left( \text{Channel} = S, \text{Category} = k \mid a_u, s_u, C_u^R, C_u^{\text{switch}}, \alpha_{ru}^0 \right)
\]

\[
= P\left( \text{Category}_1 = k \mid \text{Channel}_1 = S, a_u, s_u, C_u^{\text{switch}} \right)
\times P\left( \text{Channel}_1 = S \mid a_u, s_u, C_u^R, C_u^{\text{switch}}, \alpha_{ru}^0 \right)
\]

\[
+ P\left( \text{Category}_2 = k \mid \text{Channel}_2 = S, \text{Rank}_1 = \text{None}, \text{Channel}_1 = R, a_u, s_u, C_u^{\text{switch}} \right)
\times P\left( \text{Channel}_2 = S \mid \text{Rank}_1 = \text{None}, \text{Channel}_1 = R, a_u, s_u, C_u^{\text{switch}}, \alpha_{ru}^0 \right)
\]

\[
\times P\left( \text{Channel}_1 = R \mid a_u, s_u, C_u^R, C_u^{\text{switch}}, \alpha_{ru}^0 \right)
\]

\[
\times P\left( \text{Rank}_1 = \text{None} \mid \text{Channel}_1 = R, a_u, s_u, C_u^{\text{switch}} \right)
\times P\left( \text{Channel}_1 = R \mid a_u, s_u, C_u^R, C_u^{\text{switch}}, \alpha_{ru}^0 \right)
\]

a. The first article of the session is always obtained through the self-guided channel. So, the probability of the selected category is \( P\left( \text{Category}_1 = k \mid \text{Channel}_1 = S, a_u, s_u \right) \).

3. There are two paths for user \( u \) to exit the website: she may choose to exit directly after reading any article or after considering recommendations first but not clicking on any link deeming that no articles in the list will bring the positive net utility. Therefore, the probability of an exit observation is the sum of the probability for these two mutually exclusive outcomes:

\[
P\left( \text{Channel} = E \mid a_u, s_u, C_u^R, C_u^{\text{switch}}, \alpha_{ru}^0 \right)
\]

\[
= P\left( \text{Channel}_1 = E \mid a_u, s_u, C_u^R, C_u^{\text{switch}}, \alpha_{ru}^0 \right)
\]

\[
+ P\left( \text{Channel}_2 = E \mid \text{Rank} = \text{None}, \text{Channel}_1 = R, a_u, s_u, C_u^{\text{switch}}, \alpha_{ru}^0 \right)
\]

\[
\times P\left( \text{Rank}_1 = \text{None} \mid \text{Channel}_1 = R, a_u, s_u, C_u^{\text{switch}} \right)
\]

\[
\times P\left( \text{Channel}_1 = R \mid a_u, s_u, C_u^R, C_u^{\text{switch}}, \alpha_{ru}^0 \right)
\]
The expressions for computing the probabilities of choosing channel/category/position within the recommendation list are specified in Section 3.

**MCMC Algorithm**

The six user specific parameters of interest—a_u, s_u, C^r_u, C^r_u, cons, c^switch_u, and α^0_r_u—are treated as random variables. The full hierarchical Bayesian model is presented below:

1. \( a_u \sim \log N(\theta_a, \Sigma_a) \)
2. \( \theta_a \sim N(\theta_{\theta_a}, \Sigma_{\theta_a}) \)
3. \( \Sigma_a \sim IW(S_a^{-1}, v_a) \)
4. \( s_u \sim \log N(\theta_s, \sigma_s) \)
5. \( C^r_u \sim \log N(\theta_r, \sigma_r) \)
6. \( \theta_r \sim N(\theta_{\theta_r}, \sigma_{\theta_r}) \)
7. \( \sigma_r \sim IW(S_r^{-1}, v_r) \)
8. \( C^r_u, cons \sim \log N(\theta_{cons}, \sigma_{cons}) \)
9. \( \theta_{cons} \sim N(\theta_{\theta_{cons}}, \sigma_{\theta_{cons}}) \)
10. \( \sigma_{cons} \sim IW(S_{cons}^{-1}, v_{cons}) \)
11. \( c^switch_u \sim \log N(\theta_{switch}, \sigma_{switch}) \)
12. \( \theta_{switch} \sim N(\theta_{\theta_{switch}}, \sigma_{\theta_{switch}}) \)
13. \( \sigma_{switch} \sim IW(S_{switch}^{-1}, v_{switch}) \)
14. \( \alpha^0_r_u \sim \log N(\theta_{ru}, \sigma_{ru}) \)

The lower case regular weight symbols are used for scalar variables, lower case bold symbols are used for vector variables, and upper case Greek alphabets are used for matrix parameters.

\( \log N() \) is the log-Normal distribution taking multivariate form when its mean parameter is a vector. \( N() \) is the Normal distribution taking the multivariate form when the mean parameter is a vector. \( IW() \) is the inverse Wishart distribution.
The parameters of the distribution of \( a_u, c_u^r, c_u^{cons}, \) and \( c_u^{switch} \) are estimated. The parameters of the distributions of \( s_u \) and \( a_u^0 \) are constants and specified in advance; thus these two distributions provide regularization for the two estimates.

No closed form expressions exist for maximum a-posteriori estimates of the user-specific parameters. Therefore, following the popular practice for estimation of hierarchical Bayesian models we use a Markov Chain Monte Carlo (MCMC) algorithm. We sample each of the six blocks of the parameters sequentially in a Gibbs way, i.e., each block of variables is drawn from a distribution conditional on the latest values of other five blocks of parameters and the data. However, as it is intractable to directly draw from this conditional distribution we sample each parameter using Metropolis-Hasting algorithm. This procedure is also referred to as “Metropolis within Gibbs” (Rossi et al. 2005). The details of the algorithm as implemented is given below.

In each iteration of the algorithm following six steps are repeated to draw one sample of all the parameters.

Step 1: Sample \( a_u \)

A sample of the parameter, \( a_u^* \), is drawn from a log-Normal proposal distribution centered on the current value of the parameter, i.e., \( a_u^* \sim \text{LogN}(\log(a_u), \sigma_a^2) \). The sampled \( a_u^* \) is accepted with the probability:

\[
\min \left\{ \frac{L(Data^u, s_u, C_u^r, C_u^{cons}, C_u^{switch}, a_u^0) \log N(a_u^*|\theta_a, \Sigma_a)}{L(Data^u, s_u, C_u^r, C_u^{cons}, C_u^{switch}, a_u^0) \log N(a_u|\theta_a, \Sigma_a)} \prod_k a_{ku}^*, 1 \right\}
\]

where the last terms in the numerator and denominator come from the Jacobian of the log-Normal proposal distribution—necessary due to the log transformation of the variable in the proposal distribution (Xu et al. 2014). All the other parameter values are taken from their latest sample. The parameters \( \theta_a, \Sigma_a, S_a, v_a \) of the distribution of \( \theta_a \) and \( \Sigma_a \) are initiated with \( \bar{\theta}_a = 0, \Sigma_\theta_a = 10^5 I, S_a = I, v_a = 1 \) at the start of the estimation. From the sampled \( a_u \) for all users in this iteration, we estimate the parameters of the posterior distribution \( \theta_{\theta_a} \) (Normal mean), \( \Sigma_{\theta_a} \) (Normal variance), \( S_a \) (inverse Wishart scale matrix) and \( v_a \) (inverse Wishart degree of freedom) using the following update formulae for conjugate priors:

\[
\theta_{\theta_a} = \Sigma_{\theta_a} \left( \sum_{u=1}^{U} \log(a_u) \right) ' (\Sigma_a^{-1} + \bar{\theta}_a \Sigma_{\theta_a}^{-1})'
\]

\[
\Sigma_{\theta_a} = (U \Sigma_a^{-1} + \Sigma_{\theta_a}^{-1})^{-1}
\]
\[ S_a = \sum_{u=1}^{U} (\log(a_u) - \theta_a)(\log(a_u) - \theta_a)' + S_a \]

\[ \nu_a = U + \sigma_a \]

Based on the updated values of \( \theta_a, \Sigma_a, S_a, \nu_a \), we draw a new sample of \( \theta_a \sim MVN(\theta_a, \Sigma_a) \) and \( \Sigma_a \sim IW(S_a^{-1}, \nu_a) \).

**Step 2: Sample \( s_u \)**

We set the parameters of the prior over \( s_u \) to \( \theta_s = 5 \) and \( \sigma_s = 1 \). The next draw of \( s_u \) comes from a log-Normal proposal distribution centered on the current value of \( s_u \), i.e., \( s_u^* \sim \log N(\log(s_u), \sigma_s^2) \). The accepting probability for \( s_u^* \) is given as:

\[
\min \left\{ \frac{L(Data^u|a_u, s_u^*, \theta^{r}_u, \Sigma^{r}_u, \sigma^{r}_u, a^{0}_r)}{L(Data^u|a_u, s_u, \theta^{r}_u, \Sigma^{r}_u, \sigma^{r}_u, a^{0}_r)} : \log N(s_u^*|\theta_s, \sigma_s) \right\}
\]

\( s_u \) is modeled as a random variable following a log Normal distribution with known parameters, thus there is no need to update \( \theta_s \) and \( \sigma_s \).

**Step 3: Sample \( C^{r}_u \)**

The next draw of \( C^{r}_u \) comes from a log-Normal proposal distribution centered on the current value of the parameter, i.e., \( C^{r}_u \sim \log N(\log(C^{r}_u), \sigma^{r}_u^2) \). The accepting probability is:

\[
\min \left\{ \frac{L(Data^u|a_u, s_u, \theta^{r}_u, \Sigma^{r}_u, \sigma^{r}_u, a^{0}_r)}{L(Data^u|a_u, s_u, \theta^{r}_u, \Sigma^{r}_u, \sigma^{r}_u, a^{0}_r)} : \log N(C^{r}_u|\theta_r, \sigma_r) \right\}
\]

The parameters \( \theta_{\theta_r}, \sigma_{\theta_r}, s_r, \nu_r \) of the distribution of \( \theta_r \) and \( \sigma_r \) are initiated as \( \bar{\theta}_{\theta_r} = 0, \bar{\sigma}_{\theta_r} = 10^5, \bar{s}_r = 1, \bar{\nu}_r = 1 \) at the start of the estimation. After sampling \( C^{r}_u \) for all users, we estimate the posterior parameters \( \theta_{\theta_r} \) (Normal mean), \( \sigma_{\theta_r} \) (Normal variance), \( s_r \) (Inverse Wishart scale matrix) and \( \nu_r \) (Inverse Wishart degree of freedom) using the update formula for conjugate priors:

\[
\theta_{\theta_r} = \Sigma_{\theta_r}^{-1} \left( \sum_{u=1}^{U} \log(C^{r}_u) \right) \sigma_{\theta_r}^{-1} + \bar{\theta}_{\theta_r} \sigma_{\theta_r}^{-1}
\]

\[
\sigma_{\theta_r} = \left( U \sigma_r^{-1} + \bar{\sigma}_{\theta_r}^{-1} \right)^{-1}
\]

\[
\nu_r = \sum_{u=1}^{U} (\log(C^{r}_u) - \theta_r)(\log(C^{r}_u) - \theta_r)' + \bar{s}_r
\]
\[ v_r = U + \bar{v}_r \]

Based on the updated \( \theta_{\theta^r}, \sigma_{\theta^r}, s_r, v_r \), we draw a new sample of \( \theta_r \sim N(\theta_{\theta^r}, \sigma_{\theta^r}) \) and \( \Sigma_r \sim IW(s_r^{-1}, v_r) \).

**Step 4: Sample \( C_{u,cons}^r \)**

The next draw of \( C_{u,cons}^r \) comes from a log-Normal proposal distribution centered on the current value of the parameter, i.e., \( C_{u,cons}^r * \sim LogN(\log(C_{u,cons}^r), \bar{\sigma}_{cons}^2) \). This sample is accepted with the probability

\[
\min \left\{ \frac{L(Data^u | a_u, s_u, C_u^r, C_{u,cons}^r, C_u^{switch}, a_{ru}^{0}) \log N(C_{u,cons}^r * | \theta_{cons}, \sigma_{cons}) C_{u,cons}^{r *}}{L(Data^u | a_u, s_u, C_u^r, C_{u,cons}^r, C_u^{switch}, a_{ru}^{0}) \log N(C_{u,cons}^r | \theta_{cons}, \sigma_{cons}) C_{u,cons}^{r}} \right\}
\]

The parameters \( \theta_{\theta_{cons}}, \sigma_{\theta_{cons}}, s_{cons}, v_{cons} \) of the distribution of \( \theta_{cons} \) and \( \sigma_{cons} \) are initiated as \( \bar{\theta}_{cons} = 0, \bar{\sigma}_{cons} = 10^5, s_{cons} = 1, \bar{\sigma}_{cons} = 1 \) at the start of the estimation. After sampling \( C_{u,cons}^r \) for all users, we estimate the posterior parameters: \( \theta_{cons} \) (Normal mean), \( \sigma_{cons} \) (Normal variance), \( s_{cons} \) (Inverse Wishart scale matrix) and \( v_{cons} \) (Inverse Wishart degree of freedom) using the update formula for conjugate priors:

\[
\theta_{\theta_{cons}} = \sigma_{\theta_{cons}} \left( \left( \sum_{u=1}^{U} \log(C_{u,cons}^r) \right) s_{cons}^{-1} + \bar{\theta}_{\theta_{cons}} \bar{\sigma}_{\theta_{cons}}^{-1} \right) \left( \sum_{u=1}^{U} \log(C_{u,cons}^r) \right) - \theta_{cons} \right) + s_{cons}
\]

\[
\sigma_{\theta_{cons}} = \left( U s_{cons}^{-1} + \bar{\sigma}_{\theta_{cons}}^{-1} \right)^{-1}
\]

\[
s_{cons} = \sum_{u=1}^{U} (\log(C_{u,cons}^r) - \theta_{cons}) (\log(C_{u,cons}^r) - \theta_{cons})' + s_{cons}
\]

\[
v_{cons} = U + \bar{v}_{cons}
\]

Based on the updated \( \theta_{\theta_{cons}}, \sigma_{\theta_{cons}}, s_{cons}, v_{cons} \), we draw a new sample of \( \theta_{cons} \sim N(\theta_{\theta_{cons}}, \sigma_{\theta_{cons}}) \) and \( \Sigma_{cons} \sim IW(s_{cons}^{-1}, v_{cons}) \).

**Step 5: Sample \( C_{u}^{switch} \)**

The next draw of \( C_{u}^{switch} \) comes from a log-Normal proposal distribution centered on the current value of the parameter, i.e., \( C_{u}^{switch *} \sim LogN(\log(C_{u}^{switch}), \bar{\sigma}_{swi}^2) \). The acceptance probability is:

\[
\min \left\{ \frac{L(Data^u | a_u, s_u, C_u^r, C_{u,cons}^r, C_u^{switch *}, a_{ru}^{0}) \log N(C_{u,cons}^r * | \theta_{swi}, \sigma_{swi}) C_{u,cons}^{r *}}{L(Data^u | a_u, s_u, C_u^r, C_{u,cons}^r, C_u^{switch}, a_{ru}^{0}) \log N(C_{u,cons}^r | \theta_{swi}, \sigma_{swi}) C_{u,cons}^{r}} \right\}
\]
The parameters $\theta_{\text{swi}}, \sigma_{\text{swi}}, s_{\text{swi}}, v_{\text{swi}}$ of the distribution of $\theta_{\text{swi}}$ and $\sigma_{\text{swi}}$ are initiated as: $\theta_{\text{swi}} = 0, \sigma_{\text{swi}} = 10^5, s_{\text{swi}} = 1, v_{\text{swi}} = 1$ at the start of the estimation. After sampling $C_u^{\text{switch}}$ for all users, we estimate the posterior parameters $\theta_{\text{swi}}$ (normal mean), $\sigma_{\text{swi}}$ (normal variance), $s_{\text{swi}}$ (inverse Wishart scale matrix) and $v_{\text{swi}}$ (inverse Wishart degree of freedom) using the update formula for conjugate priors:

$$\theta_{\text{swi}} = \Sigma_{\text{swi}} \left( \left( \sum_{u=1}^{U} \log(C_u^{\text{switch}}) \right) \sigma_{\text{swi}}^{-1} + \Theta_{\text{swi}} \sigma_{\text{swi}}^{-1} \right)$$

$$\sigma_{\text{swi}} = \left( U \sigma_{\text{swi}}^{-1} + \Theta_{\text{swi}}^{-1} \right)^{-1}$$

$$s_{\text{swi}} = \sum_{u=1}^{U} (\log(C_u^{\text{switch}}) - \theta_{\text{swi}})(\log(C_u^{\text{switch}}) - \theta_{\text{swi}})' + s_{\text{swi}}$$

$$v_{\text{swi}} = U + s_{\text{swi}}$$

Based on the updated $\theta_{\text{swi}}, \sigma_{\text{swi}}, s_{\text{swi}}, v_{\text{swi}}$ we draw a new sample of $\theta_{\text{swi}} \sim N(\theta_{\text{swi}}, \sigma_{\text{swi}})$ and $\sigma_{\text{swi}} \sim IW(s_{\text{swi}}^{-1}, v_{\text{swi}})$.

Step 6: Sample $\alpha_{ru}^{0}$.  

The next draw of $\alpha_{ru}^{0}$ comes from a log-Normal proposal distribution centered on the latest value of the parameter, i.e., $\alpha_{ru}^{0} \sim LogN(\log(\alpha_{ru}^{0}), \sigma_{ru}^{2})$. This draw is accepted with the following probability

$$\min \left\{ \frac{L(\text{Data}|a_{u}, s_{u}, C_{u}, C_{\text{cons}}^{\text{u}}, C_{\text{switch}}^{\text{u}}, \alpha_{ru}^{0}, \gamma)}{L(\text{Data}|a_{u}, s_{u}, C_{u}, C_{\text{cons}}^{\text{u}}, C_{\text{switch}}^{\text{u}}, \alpha_{ru}^{0*}) \log N(\alpha_{ru}^{0*}|\theta_{ru}, \sigma_{ru}) \alpha_{ru}^{0*}}, 1 \right\}$$

$\alpha_{ru}^{0}$ is modeled to be random variable following log-Normal distribution with fixed parameters ($\theta_{ru} = 5$ and $\sigma_{ru} = 1$). Thus, there is no need to update $\theta_{ru}$ and $\sigma_{ru}$.

We adjust the variance or step size of each proposal distributions adaptively over iterations such that the average acceptance rate of the corresponding parameter is between 0.1 and 0.4 (Haario et al. 2006). The outlined steps are run for 50,000 iterations. The first 30,000 samples were discarded to ensure that the samples are not dependent on initialization. Mean and the variance of the parameters were computed from the last 20,000 samples.