A Theory of Must-Have

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Abstract

“Must-have” items —items that retailers need to compete effectively in the marketplace— have played a central role in many recent antitrust cases, notably U.S. v AT&T et al. (2018). We advance a theory formalizing the notion of “must-have” and study its antitrust implications in two different settings: abuse of dominance and vertical mergers. We show that a manufacturer of a must-have item can monopolize an adjacent, competitive market under circumstances in which a manufacturer of a standard monopoly item cannot. We also show that a vertical merger involving a manufacturer of a must-have item is more likely to be anticompetitive than one involving a manufacturer of a standard monopoly product.

Keywords: Monopolization, Vertical Mergers, Tying, Wholesale Markets.

JEL classifications: D43, K21, L12, L42

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1 Introduction

At trial, much time was spent debating the “must-have” status of Turner’s programming content. According to the Government, distributors literally “must-have” Turner’s content in order “to compete effectively” (...) Defendants countered that the term “must-have” is simply a marketing phrase used to mean “popular” and, similarly, that Turner content is not actually necessary to allow distributors to operate their business successfully.

On November 20, 2017, the United States Department of Justice (DOJ), in an unprecedented move, sued to block AT&T/DirecTV’s $85.4 billion bid for Time Warner. According to the U.S. Government, the merger would substantially lessen competition in the multichannel television market by enabling the combined company to use Time Warner’s must-have channels (e.g., TNT, HBO, CNN) to hinder AT&T/DirecTV’s rivals. As no settlement among the parties was reached, U.S. v. AT&T, et al. (2018) became the first fully litigated vertical merger case in the U.S. in 40 years, and one of the most closely watched antitrust cases in decades.¹

Although by far the most prominent example, the above is by no means the only antitrust case in which must-have items, items that retailers need to “compete effectively” in the marketplace, have emerged. Nor are authorities’ concerns confined exclusively to the effect of such items on vertical mergers. For example, in 2013 Cablevision accused Viacom of abuse of market power² when the latter tied its popular, allegedly must-have networks (e.g., Nickelodeon, MTV) to its less valuable channels. The accusation claimed that Viacom’s strategy was designed to prevent Cablevision from distributing competing networks that consumers likely would prefer.³

Given their central role in many of the most recent, important antitrust cases in decades, the absence of any formal economic model explaining what makes a particular product a must-have, and the associated antitrust implications of such items, is highly surprising. We aim to fill this gap by providing the first formal theory of must-have items, and using this new theory to answer the following questions:

¹On June 12, 2018, in a highly controversial decision (Salop, Wright and Rybnicek, 2018), Judge Richard Leon approved the merger without conditions, marking an historic defeat for the DOJ. The approval decision was upheld unanimously on February 28, 2019, by a panel of three Judges of the U.S. Court of Appeals for the DC Circuit. The same day, the DOJ communicated that no further actions would be undertaken on the case.
²Cablevision Systems Corp. v. Viacom International Inc. (Civil Action No. 13 CIV 1278 (LTS) (JLC), S.D.N.Y. 2013)
³Similar issues were also raised and investigated by the EU Commission in 2005 before approving the conglomerate merger between Procter & Gamble and Gillette. See Procter & Gamble / Gillette (EU Case No COMP/M.3732)
(i) Are there such things as “must-have” items?
(ii) If so...
   a. Under what circumstances do they emerge?
   b. What are the antitrust implications?

We begin our analysis by providing a precise definition of the concept of must-have (alternative interpretations are discussed below). We posit that an item is considered a “must-have” if its removal from a distributor’s lineup causes a large number of final consumers to switch to a rival distributor for items other than the removed item itself. Consequently, a must-have item is more than a “monopoly” item, a unique item that a retailer cannot easily substitute for; it is also an item without which a retailer cannot compete effectively for the remaining products in her lineup.4

To investigate whether such items exist, we propose a simple model of retail competition with three main features: (a) multiple products, (b) heterogeneous consumers, and (c) one-stop shopping. More precisely, we consider two Bertrand retailers, \( R_1 \) and \( R_2 \), competing to serve a group of final consumers with heterogeneous valuations for two items, \( A \) and \( B \). Some consumers are interested only in product \( A \), others only in \( B \), and the remainder are interested in both \( A \) and \( B \). Furthermore, a fraction of consumers are forced to one-stop shop. Product \( A \) comes in a single variety with no close substitute, while \( B \) comes in different varieties, all very close substitutes for one another. Both retailers have constant returns to scale and compete by simultaneously setting prices for each product and for the bundle of the two if they carry both.

Equipped with this simple yet realistic model, we show that must-have items do exist. In particular, product \( A \) satisfies our definition of must-have if and only if there exists an important number of one-stop shoppers who are simultaneously interested in purchasing both \( A \) and \( B \). If, however, this condition is not met, then \( A \) loses its must-have status and becomes nothing more than a standard monopoly product.5

To understand this result, start from a situation in which both retailers carry both products, and consider the removal of \( A \) from one of the retailer’s lineup, say from retailer \( R_2 \). The key is that \( A \)’s removal makes \( R_2 \)’s offering vertically inferior to \( R_1 \)’s for those consumers who are interested in both products and are also forced to one-stop shop. The latter induces a fraction of these consumers to switch their purchases of \( A \) and \( B \) from \( R_2 \) to \( R_1 \). Thus, \( A \)’s removal from \( R_2 \)’s lineup ends up affecting \( R_2 \)’s sales for product \( B \), as well.

From a retailer’s perspective, it is evident that one-stop-shopping economies create complementarity between \( A \) and \( B \): not carrying \( A \) makes it more difficult to sell \( B \). However, this

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4Throughout this paper, retailers are referred to with female pronouns.
5In our baseline model, exogenous one-stop shopping is a necessary condition for the emergence of must-have products. We show, however, that in more complicated models, one-stop-shop economies can also arise endogenously, but only for certain correlations in consumers’ valuations.
“must-have complementarity” is unique; it is an endogenous equilibrium object that emerges thanks to the presence of retail competition, so its magnitude depends on both retailers’ wholesale prices. Thus, as we will see, it is not comparable to, and has different implications than, exogenous technological complementaries that emerge, for example, from valuation or cost externalities. Interestingly, as we elaborate in Section 2.3, current market conditions in the cable television industry appear to support the emergence of must-have channels.

Having established the existence of must-have items, we move on to understand their implications for antitrust policy. We extend our baseline retail model to incorporate upstream manufacturers and study two applications increasingly relevant for antitrust: abuse of dominance and vertical mergers. While notoriously different, the two applications share a common result: must-have items prove to be more harmful than standard monopoly products.

**Abuse of Dominance.** Our first application analyzes must-have items in the context of upstream monopolization and abuse of dominance. For that, we consider an upstream manufacturer of $A$ and $B$ facing a more efficient fringe of $B$ suppliers. We abstract from scale economies and first-mover advantages and allow manufacturers to make take-it-or-leave-it offers, which may include two-part tariffs and tying provisions. In this simple setting, we show that the manufacturer of $A$ and $B$ can exploit $A$’s must-have status to monopolize the adjacent, otherwise competitive market $B$ in circumstances in which a manufacturer of a standard monopoly product cannot. The monopolization of market $B$ is accomplished by tying the purchase of $A$ and $B$, and exploiting the fact that downstream retailers need product $A$ to compete effectively for $B$.

Interestingly, this leverage mechanism does not rely on any market feature previously thought to be essential for monopolization, such as first-mover advantages, ex-ante commitments, scale economies, or contractual frictions. Furthermore, unlike alternative theories, monopolization can arise even without any foreclosure of $B$’s rival suppliers.

This new theory of monopolization seems to fit particularly well several recent antitrust cases, including *Cablevision v. Viacom* and *Cascade Health Solutions v. PeaceHealth.*\(^6\) Moreover, it provides support to the Federal Communications Commission’s (FCC) long-standing concern regarding the bundling of (loosely defined) must-have broadcast programming in re-transmission consent negotiations.\(^7\)

**Vertical Mergers.** In the second application, which is motivated by recent mergers in the multichannel television market, we examine the effects of must-have items on vertical mergers. In particular, we analyze the implications of a vertical merger between an upstream manufacturer of a must-have product $A$ and a retailer that distributes both $A$ and $B$, when the latter

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\(^6\) *Cascade Health Solutions v. PeaceHealth* (515 F.3d 883, 895, 9th Cir. 2008)

\(^7\) We return to this example in Section 3.6.
A retailer faces competition downstream.

For this purpose, we develop a tractable yet sufficiently rich model that allows us to incorporate must-have items into the Nash bargaining theory of harm used, for example, to oppose the Comcast/NBC-Universal and AT&T/Time Warner mergers (Murphy, 2010; Shapiro, 2018) —a theory based on bilateral and simultaneous (linear-price) negotiations between upstream suppliers and downstream distributors.  

It has been argued informally that the mere presence of must-have items increases the anti-competitive potential of vertical mergers by providing the merging entity with more bargaining leverage to negotiate higher prices for A from rival distributors. This argument contains a fatal flaw, however: must-have items are not merger-specific, so the leverage conveyed by A’s must-have status is also at the manufacturer’s disposal before the merger. The latter does not imply that must-have items are therefore innocuous, but it does suggest that in order to be a reason of increased concern, must-have items must interact with the merger in some nontrivial way.

We show that such interaction does indeed exist. More precisely, vertical mergers increase items’ must-have potential, and therefore provide the merging entity with more bargaining leverage than what the upstream manufacturer had before the merger. Intuitively, because the magnitude of the “must-have complementarity” is endogenous and depends on retailers’ wholesale prices, in the presence of must-have items a merger between M and R1 decreases R2’s outside option in her negotiation with M: by decreasing the wholesale price at which R1 can obtain product A, the merger allows R1 to be more aggressive downstream and steal a higher fraction of one-stop shoppers from R2 if the latter does not carry A. The reduced outside option then allows the merging entity to negotiate higher prices for A with R2, making the merger more likely to be anticompetitive. We refer to this new effect as the must-have merger effect, which can be shown, at least in our model, to be substantial, enough to turn an otherwise neutral or pro-competitive merger into an uncompetitive one.

Note that in our theory, not carrying A does not force a retailer to exit the market, even though its business is adversely impacted. We cannot think of any item in any industry whose removal from a distributor’s lineup would leave the retailer in such a life-threatening situation to be forced to exit the market. To some, however, this is precisely what makes an item a must-have: an “essential input” without which a competitor cannot operate downstream (Fumagalli, 2008).

Interestingly, even though must-have items are recurrently mentioned in the U.S. Government’s case against AT&T/Time Warner, they are never formally included in the merger evaluation, possibly due to lack of solid theoretical foundations for the concept. We hope our theory helps in that regard.

Horn and Wolinsky (1988) were the first to introduce the Nash-in-Nash bargaining protocol into merger analysis (see also Collard-Wexler, Gowrisankaran and Lee, 2019). Since then, it has become a workhorse of empirical work in wholesale contexts where the terms of trade are determined by bilateral negotiations between upstream suppliers and downstream distributors (e.g., Crawford and Yurukoglu, 2012; Lee, 2013; Gowrisankaran, Nevo and Town, 2015; Ho and Lee, 2017; Crawford et al., 2018).
Motta and Calcagno, 2018, pp. 471-472)

This “essential input” interpretation of must-have has two significant drawbacks compared to our notion. First, it is highly restrictive, resulting in too high a standard of proof for authorities and policymakers. More importantly, however, is the fact that, as we will see throughout this paper, the possibility of exiting the market is not necessary for anticompetitive harm to emerge. Consequently, the “essential input” interpretation of must-have only serves to divert attention from a real anticompetitive threat: the fact that a must-have item, viewed through the lens of our less stringent definition, can be highly detrimental to social welfare and consumer surplus.

Related literature.— We are the first to formalize the concept of must-have in a theory where such qualification arises endogenously as an equilibrium outcome. Despite its novelty, our theory is intimately connected to several strands of the literature.

First, our theory closely relates to the literature on retail bundling (e.g., McAfee, McMillan and Whinston, 1989; Armstrong and Vickers, 2010; Chen and Riordan, 2013) and one-stop shopping (e.g., Bliss, 1988; Chen and Rey, 2012; Johnson, 2017). None of these papers, however, combines the same ingredients as our baseline model.

Second, our application on monopolization and abuse of dominance tightly connects to the vast literature on vertical restraints and foreclosure. We are certainly not the first to show how a multi-product manufacturer may be able to extend monopoly power from one market to an adjacent, competitive one (e.g., Whinston, 1990; Nalebuff, 2004; Peitz, 2008; Greenlee, Reitman and Sibley, 2008; Calzolari and Denicolo, 2015). However, by construction, no previous paper has studied how must-have items could lead to monopolization. Therefore, our main result is not that monopolization is possible, but rather that a manufacturer of a must-have item can monopolize under circumstances in which a manufacturer of a standard monopoly product cannot.

Third and finally, our second application relates to the literature on the evaluation of vertical mergers. We contribute to this literature on two fronts. First, we develop a tractable, yet sufficiently rich model that allows us to incorporate must-have items into the Nash bargaining theory of harm used, for example, to oppose the Comcast/NBC-Universal and AT&T/Time Warner mergers (Murphy, 2010; Shapiro, 2018). Second, for the more empirically oriented

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10 In fact, as we will see in the conclusion, this particular issue constituted one of the most significant setbacks of the U.S. Government’s case against AT&T and Time Warner.

11 Choné and Linnemer (2016) build upon Aghion and Bolton (1987) to study the anticompetitive implications of nonlinear pricing when a fraction of a buyer’s demand is uncontestable. They refer to such items as “must-stock items.” Their concept of must-stock is therefore not endogenous and more closely related to the idea of a monopoly product, a product without close substitutes, than to our equilibrium notion.

12 For a complete overview, see Whinston (2006) and Fumagalli, Motta and Calcagno (2018).

13 See Riordan (2008) for an excellent survey on the theory and some antitrust cases. For empirical papers on the topic see Chipty (2001), Suzuki (2009), and Crawford et al. (2018).
reader, we show how the omission of must-have items in vertical-merger evaluation can lead to an underestimation of anticompetitive harm.

The rest of this paper is organized as follows: Section 2 presents a model of must-have, formalizing our definition. Section 3 contains the application on the abuse of dominance. Section 4 covers the application on vertical mergers. Section 5 concludes. The Appendix contains proofs to selected propositions and lemmas. Remaining proofs and additional results can be found in the online Appendix.

2 A Model of Must-Have

2.1 The Set-up

Consider two products, $A$ and $B$, and two Bertrand retailers, $R_1$ and $R_2$, simultaneously setting prices.$^{14,15}$ There is a single variety of $A$, but $N \geq 2$ perfectly homogeneous varieties of $B$, denoted by $B_1, \ldots, B_N$. Think of $A$ as a product for which no close substitutes exist, and $B$ as a set of generic varieties that are widely available, all of which are approximately the same to consumers. Hence, $A$’s removal from a retailer’s lineup has a meaningful effect, while the same is not true for the removal of any of the individual varieties of $B$.

We allow for the costs of $A$ and $B$ to vary across retailers. $R_i$ faces a wholesale price of $w_{Ai}$ for $A$, and $w_{Bi}$ for any of the varieties of $B$.$^{16}$ Wholesale prices are common knowledge. Given that all varieties of $B$ are perfect substitutes, and further, that they all cost the same to $R_i$, from hereon we will refer to all such varieties as simply product $B$.

Final consumers differ in their valuations of the two goods. A unit mass of consumers value good $A$ in $v_A$, and an equal mass value good $B$ in $v_B$. While we assume $v_A$ to be uniformly distributed in the unit interval, we assume $v_B$ to be the same across consumers and equal to $b \leq 1/2$. To be consistent with the idea that $B$ is widely available, we assume $\max\{w_{B1}, w_{B2}\} \leq b$ so that both retailers find it profitable to sell standalone units of $B$.

Notice that from a retailer’s perspective, consumer demand for multiple products can arise because of two (not mutually exclusive) reasons: (i) a single group of consumers is interested in both $A$ and $B$, or (ii) two different groups of consumers are interested in only one of them.

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$^{14}$Recall from footnote 4 that throughout this paper, retailers are referred to with female pronouns.

$^{15}$Our insights readily extend to settings with imperfect competition, such as the Hotelling linear-city model. Notice, however, that our definition of must-have is linked inextricably to the idea of retail competition; hence, some level of retail competition will is needed.

$^{16}$Although exogenously given, for now, these wholesale prices will become endogenous in Sections 3 and 4 once we explicitly incorporate upstream manufacturers to study the effects of must-have items in vertical negotiations.

$^{17}$We adopt these valuations for tractability. In particular, they allow us to write retailers’ equilibrium profits in closed form, greatly simplifying analysis. As shown in the online Appendix, however, we arrive at the same qualitative results if $b > 1/2$, or alternatively, if we let $v_B$ to be uniformly and independently distributed over the unit interval.
To capture these possibilities, we allow for different levels of overlap between the mass of consumers interested in product $A$ and the mass interested in $B$. In particular, we assume that with probability $\mu \in [0,1]$, independent of $v_A$, a consumer who values $A$ also values $B$.\footnote{With this formulation, we avoid any market-size effect. To see this, suppose that goods are available at zero cost to a group of consumers, 100 of whom have preferences for $B$ and another 100 of whom have preferences for $A$. The total number of consumers in the group can vary from 200 to 100 as $\mu$ varies from 0 to 1, but the total number of units sold is invariant to $\mu$.}

Finally, as prevalent in many retail markets, final consumers usually incur in shopping costs when visiting multiple retailers, costs that at times are significant enough that some decide to one-stop-shop (Bliss, 1988; Chen and Rey, 2012; Johnson, 2017). Shopping costs may reflect the opportunity cost of time spent in traffic and parking, selecting products, and so forth, or the increasing burden of dealing with multiple retailers, such as to pay multiple bills or contact different companies for customer service and technical support, among others. We let $z \in [0,1]$, independent of $\mu$ and $v_A$, be the probability that any given consumer is a one-stop shopper, i.e., faces a prohibitively high shopping cost, while $1 - z$ is the probability that such a consumer can visit multiple retailers. While this does not affect consumers who value only one good, it does make a difference to those who value both. A fraction $1 - z$ of these consumers are now free to shop at different retailers, while the remaining fraction $z$ visit a single store.\footnote{Alternatively, we could assume that consumers face a shopping cost $c \in [0, +\infty)$, drawn from some cumulative distribution $G(c)$. Our results do not change as long as $G(c)$ is such that a fraction of consumers one-stop shop in equilibrium.}

The game unfolds as follows. First, $R1$ and $R2$ simultaneously set prices $(p_{A1i}, p_{B1i}, p_{AB1i})$, where $p_{Ai}$ and $p_{Bi}$ denote $Ri$’s stand-alone prices for products $A$ and $B$, respectively, and $p_{ABi} \leq p_{Ai} + p_{Bi}$ is the price for the joint purchase of both goods. Then, after observing all retail prices, final consumers decide where to shop and what to buy.\footnote{When necessary, we use the following standard tie-breaking rules: (i) consumers indifferent between the bundle or a single good always purchase the bundle; (ii) consumers indifferent to purchasing $k \in \{A, B, AB\}$ from $i$ or $j$ visit the retailer with the lowest wholesale price for $k$. If wholesale prices are the same, they visit $Ri$ with probability $1/2$. It is possible to relax all these assumptions at the cost of some additional notation.}

Having described of the game, we can now offer a formal definition of must-have, capturing in a precise statement the notion introduced in the previous section.

**Definition (Must-have Item).** Consider an equilibrium in which the marginal profit contribution of $A$ is non-negative for both retailers.\footnote{A non-negative marginal contribution is a necessary condition for a retailer to carry product $A$ willingly. This requirement is needed here because retailers’ costs are assumed to be exogenous, but can be dispensed once upstream manufacturers are introduced.} Product $A$ is a must-have for $Ri$ if its removal from her lineup (i.e., if $w_{Ai} \to +\infty$) causes some consumers in the new equilibrium to switch their purchases of $B$ to a rival retailer.

Note that our definition of must-have involves an equilibrium notion: to assess whether $A$ is must-have for $Ri$ or not, we compare the equilibrium outcome before and after $A$ is removed from $Ri$’s lineup.
2.2 Retail Demand

We begin by characterizing retailers’ demand. Suppose retailers offer prices \((p_{Ai}, p_{Bi}, p_{ABi})\)\(_{i=1,2}\), where \(p_{ABi} \leq p_{Ai} + p_{Bi}\) without loss of generality. Define \(p_k = \min\{p_{k1}, p_{k2}\}\), where \(k = A, B, AB\), and \(R_k\) as the retailer offering the lowest price for choice \(k\).\(^{22}\) Omitting for a moment the possibility of price ties, we have that those consumers who value only \(A\) will buy a total of \((1 − \mu)(1 − p_A)\) units from retailer \(R_A\), provided that \(p_A < 1\), while those who value only \(B\) will buy a total of \((1 − \mu)\) units from \(R_B\), provided that \(p_B < 1\). These decisions are summarized for \(p_{A1} < p_{A2}\) and \(p_{B1} > p_{B2}\) in the first two rows of Table 1.

The decision of consumers who value both goods is more involved. For a fraction \(z\) of these consumers, one-stop shopping forces them to decide whether to buy both products from \(R_{AB}\), just \(B\) from \(R_B\), or just \(A\) from \(R_A\). Due to our valuation structure, however, these consumers will always purchase \(B\), implying that their actual choice is between buying just \(B\) or buying both \(A\) and \(B\). Consequently, since the consumer indifferent to these two options has valuation \(p_{AB} − p_B\) for good \(A\), consumers with \(v_A \geq p_{AB} − p_B\) will buy a total of \(\mu(z(1 − p_{AB} + p_B))\) units of each good from \(R_{AB}\), provided that \(p_{AB} − p_B < 1\), while the remaining consumers will buy a total of \(\mu(z(p_{AB} − p_B))\) units of \(B\) from \(R_B\). These decisions are summarized in rows 3 and 4 of Table 1 under the additional assumption that \(p_{AB1} < p_{AB2}\).

Table 1: Example of Retailers’ Demand from Different Consumer Groups\(^{(a)}\)

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\begin{array}{cccc}
\text{Consumers that} & \text{Retailer 1} & \text{Retailer 2} \\
& \text{Units of } A & \text{Units of } B & \text{Units of } A & \text{Units of } B \\
1. \text{Value only } A & (1 − \mu)(1 − p_{A1}) & 0 & 0 & 0 \\
2. \text{Value only } B & \mu(z(1 − p_{AB1} + p_{B2})) & \mu(z(1 − p_{AB1} + p_{B2})) & 0 & 0 \\
3. \text{Value both - } v_A \text{ high}\(^{(b)}\) & \mu(1 − z)(1 − p_{A1}) & 0 & 0 & \mu(1 − z)(1 − p_{A1}) \\
\text{One-stop shoppers} & 0 & 0 & 0 & \mu(z(p_{AB1} − p_{B2})) \\
4. \text{Value both - } v_A \text{ low}\(^{(c)}\) & \mu(1 − z)(1 − p_{A1}) & 0 & 0 & \mu(1 − z)(1 − p_{A1}) \\
\text{Two-stop shoppers} & 0 & 0 & 0 & \mu(1 − z)p_{A1} \\
5. \text{Value both - } v_A \text{ high}\(^{(c)}\) & \mu(1 − z)(1 − p_{A1}) & 0 & 0 & \mu(1 − z)(1 − p_{A1}) \\
\text{Two-stop shoppers} & 0 & 0 & 0 & \mu(1 − z)p_{A1} \\
6. \text{Value both - } v_A \text{ low}\(^{(c)}\) & \mu(1 − z)(1 − p_{A1}) & 0 & 0 & \mu(1 − z)(1 − p_{A1}) \\
\end{array}
\]

\(^{(a)}\) The table is constructed assuming \(p_{A1} < p_{A2}, p_{B1} > p_{B2}\), and \(p_{A1} + p_{B2} < p_{AB1} < p_{AB2}\).

\(^{(b)}\) \(v_A = p_{AB1} − p_{B2}\) is the valuation of the indifferent consumer between groups 3 and 4.

\(^{(c)}\) \(v_A = \min\{p_{A1} + p_{B2}, p_{AB1}\} − p_{B2} = p_{A1}\) is the valuation of the indifferent consumer between groups 5 and 6.

For the remaining \(1 − z\) fraction of consumers who value both goods, the decision is slightly different. They must decide whether to buy both \(A\) and \(B\), either from \(R_A\) and \(R_B\) or from \(R_{AB}\), whatever is cheaper; just \(B\) from \(R_B\); or just \(A\) from \(R_A\). Again, these consumers will always purchase \(B\), implying that their actual choice is between buying just \(B\) for \(p_B\) or buying

\[^{22}\]Appendix A provides a formal, detailed derivation of these demands.
both $A$ and $B$ for $\tilde{p}_{AB} = \min\{p_A + p_B, p_{AB}\}$. Since the indifferent consumer between these two options has valuation $\tilde{p}_{AB} - p_B$ for good $A$, consumers with $v_A \geq \tilde{p}_{AB} - p_B$ will buy a total of $\mu(1 - z)(\tilde{p}_{AB} + p_B)$ units of each good, while the remaining consumers will buy a total of $\mu(1 - z)(\tilde{p}_{AB} - p_B)$ units of $B$. These decisions are summarized in rows 5 and 6 of Table 1 under the additional consideration that $\tilde{p}_{AB} = p_{A1} + p_{B2} < p_{AB1}$.

2.3 Must-Have Items

To illustrate what makes product $A$ a must-have, consider a situation in which both retailers are equally efficient in the provision of $A$, i.e., $w_{A1} = w_{A2} = w_A < 1$, but $R2$ is more efficient in the provision of $B$, i.e., $w_{B2} < w_{B1} \leq b$. In this case, the retail equilibrium is as follows:

**Lemma 1.** If $w_{A1} = w_{A2} = w_A < 1$ and $w_{B2} < w_{B1} \leq b$, then there is no bundling in equilibrium and stand-alone equilibrium prices are $p_{A1}^* = p_{A2}^* = w_A$ and $p_{B1}^* = p_{B2}^* = w_{B1}$. Retailers profits are then given by:

$$\pi_{R1}^* = 0 \quad \pi_{R2}^* = w_{B1} - w_{B2}$$

**Proof.** See Appendix D. ■

That is, the retail equilibrium follows a standard Bertrand logic. Because of one-stop shopping, retailers compete on three fronts simultaneously: for the two stand-alone goods and the bundle. In each front, they engage in Bertrand-like competition, so retail bundling does not arise. Being more efficient, $R2$ is the sole seller of $B$, selling a total of 1 unit. She sells $\mu$ units to those consumers who value both goods (consumers with $v_A \geq w_A$ buying both $A$ and $B$, and those with $v_A < w_A$ buying only $B$) and $1 - \mu$ units to those caring only about $B$.

Suppose now that good $A$ is “removed” from $R2$’s lineup (i.e., $w_{A2} \rightarrow +\infty$), while $R1$ continues to carry it. As the following lemma states, in this new situation, retailers continue competing in Bertrand fashion for good $B$, but $R1$ now charges monopoly prices for good $A$:

**Lemma 2.** If $R1$ does not carry $A$ and $w_{B2} \leq w_{B1} \leq b$, then equilibrium prices are $p_{A1}^* = (1 + w_{A1})/2$, $p_{B1}^* = p_{B2}^* + \epsilon = w_{B1}$, and $p_{AB1}^* = p_{A1}^* + p_{B1}^*$. Retailers profits are then given by

$$\pi_{R1}^* = (1 - w_A)^2/4 \quad \pi_{R2}^* = (w_{B1} - w_{B2})\left[1 - \frac{\mu z(1 - w_A)}{2}\right]$$

**Proof.** See Appendix D. ■

Comparing (1) and (2), it is evident that good $A$’s removal has badly hit $R2$’s profits. She can see them reduced by as much as 50% as $\mu \rightarrow 1$, $z \rightarrow 1$, and $w_A \rightarrow 0$. The reason is because in this new situation, $R2$ is no longer the sole seller of $B$, despite being more efficient. All consumers interested in purchasing both goods who value $A$ in $p_{A1}^*$ or more, and are forced
to one-stop shop, go to visit R1, reducing R2’s business for B in exactly $\mu z(1 - w_A)/2$ units. Product A therefore classifies as a must-have for R2, as its removal from her lineup caused consumers to switch some of their purchases of B to R1.23

What is remarkable about this example is that R2 was making no profit on A originally, as $p_{A1}^* = p_{A2}^* = w_A$ before A’s removal, yet carrying A was still critical for R2 not to lose sales on the profitable product B to its rival R1. What explains this striking result? The key is that A’s removal from R2’s lineup created vertical differentiation among retailers. In particular, it made R2’s offering vertically inferior to R1’s for those consumers interested in both products who are forced to one-stop shop. This immediately suggests that the must-have status of good A appears to be intimately linked to the presence of one-stop shoppers interested in both products (i.e., $\mu z > 0$).24

From a retailer’s perspective, it is evident that one-stop-shopping economies create complementarity between A and B: not carrying A makes it more difficult to sell B. This “must-have” complementarity, however, is very different from complementarities that arise from exogenous technological reasons, for instance, from a valuation externality in which $v_B$ increases when B is consumed in conjunction with A. Indeed, as we will see in the applications that follow, two characteristics make the “must-have complementarity” unique. The first is that it is intimately related to retail competition (so it is completely absent under monopoly retailing), and hence, its magnitude is endogenous to retailers’ wholesale prices. The second is that a manufacturer of A will not be able to appropriate this complementarity using transfers (e.g., two-part tariffs), something he can do when the complementarity stems from exogenous technological reasons.

Building on the previous example, we can run the following thought experiment. For a given set of wholesale prices ($w_{A1}, w_{A2}, w_{B1}, w_{B2}$), compute first the corresponding retail equilibrium prices (the full characterization of the retail market equilibrium can be found in Appendix B). Then remove A from Ri’s lineup (i.e., let $w_{Ai} \rightarrow +\infty$), and recompute the equilibrium prices. Finally, compare the two equilibria and evaluate the impact of this removal on Ri’s sales of B units. If we execute this procedure for all possible combinations of wholesale prices, we obtain the following result that formalizes our previous intuition:

**Proposition 1.** There exists a set of wholesale prices ($w_{A1}, w_{A2}, w_{B1}, w_{B2}$) such that A qualifies as must-have for Ri if and only if $\mu z > 0$.

---

23 It is also easy to see in the example that A’s marginal profit contribution is non-negative for both retailers.

24 Two additional ingredients operate in the background of this vertical differentiation argument: the presence of retail competition and heterogeneity in consumers’ valuation of product A. First, retail competition allows consumers to switch retailers in response to A’s removal. Although we have assumed Bertrand competition so far, such an extreme environment is not required for our results. Second, the fact that high valuation consumers are obtaining a strictly positive surplus in consumer markets allows product portfolio decisions to create vertical differentiation among retailers. Note, however, that introducing heterogeneity in consumers’ valuation for product B will not turn any individual variety of B into a must-have, as many other varieties of B are readily available to retailers.
Proof. See Appendix D.

Thus, if $\mu z > 0$, we say that product $A$ has must-have potential, as we can potentially find a set of wholesale prices that makes $A$ a must-have. Proposition 1 is the building block of Sections 3 and 4, where we introduce upstream manufacturers, allowing wholesale prices to be endogenously determined, and study whether an upstream firm can exploit this potential to its advantage.

Of the market conditions identified in Proposition 1, the one that deserves further discussion is one-stop shopping ($z > 0$). In our setting, $\mu z$ consumers are forced to one-stop shop because of high shopping costs. One may wonder whether the same phenomenon could arise endogenously, in the absence of shopping costs ($z = 0$) but due to retailers’ pricing (i.e., due to retail bundling). Although not for the model we have considered so far, it can be shown (see the online Appendix) that when valuations for $A$ and $B$ are negatively correlated, good $A$ can emerge as must-have despite $z = 0$. In a nutshell, under negatively correlated valuations, whenever $R1$ carries $A$ and $B$ while $R2$ only carries $B$, $R1$ has an incentive to offer $A$ alone and also in a bundle so as to better discriminate consumers that value $A$ highly and $B$ not as much. Thus, in contrast with Lemma 2 above, under a negative correlation of valuations, bundling in a strict sense does emerge in equilibrium in the critical configuration in which one retailer carries $A$ and $B$, and the other carries only $B$. The strict emergence of bundling then generates a similar (endogenous) one-stop-shopping effect: all consumers interested in buying both products go to $R1$, with a consequent reduction in the number of consumers visiting $R2$ looking for $B$ units.

Unlike the case attributable to high shopping costs, this alternative way to generate one-stop shopping, while plausible, is not as prevalent. It requires valuations to be highly negatively correlated. As we also show in the online Appendix, it does not work, for instance, if valuations are uniformly distributed over the unit square. This is because, if valuations are not highly negatively correlated, it is not in $R1$’s best interest to bundle $A$ and $B$ when facing a more efficient rival $R2$ that carries only $B$. Note that this results marks a sharp contrast to the case in which both retailers carry both products, under which bundling emerges for a much wider range of valuation distributions, including uniform distributions over the unit square (Armstrong and Vickers, 2010). The implication is obvious: observing bundling in environments in which all retailers carry multiple products (i.e., under a symmetric portfolio configuration) does not imply that we will continue to observe bundling in the counterfactual scenario in which one retailer drops one of the products. Thus, observing bundling in environments in which all retailers carry multiple products does not say much as to whether a particular item qualifies as must-have or not.

Having formalized our definition of must-have and established the conditions for their emergence (consumer interest in multiple products and one-stop-shopping economies), we now briefly discuss how these insights can be applied to a real-world setting. Consider the debate about
the existence of must-have channels in the market for subscription-based television services. In
the early 1990s, competition between service providers was weak. Most households in the U.S.
accessed television services through monopoly cable providers operating in exclusive franchise
areas. Since then, thanks to the emergence of satellite television providers like DirecTV and
the entry of telecom companies like AT&T and Verizon, competition for distribution has in-
tensified. According to Chipty (2016), 99% of households today can choose from at least three
distributors and 35% can choose from at least four.\(^{25}\) It is also known that households tend to
value, to some extent, channel variety, and that shopping costs are not trivial, as most, if not
all, households purchase video programming services from a single distributor (Crawford and
Yurukoglu, 2012; Crawford et al., 2018).\(^ {26}\) Hence, according to Proposition 1, current market
conditions in the cable television industry appear consistent with market conditions supporting
the emergence of must-have channels.

Having characterized the existence of must-have items, we now move to analyze their an-
titrust implications. In the next two sections, we incorporate upstream manufacturers and
study two applications increasingly relevant for antitrust: abuse of dominance and vertical
mergers.

3 Application I: Abuse of Dominance

In our first application, we show how the manufacturer of a must-have item can leverage
the must-have condition to monopolize an adjacent market under circumstances in which a
monopoly manufacturer cannot.

3.1 The Setup

The setting we examine is identical to that of section 3, except we now model manufacturers
explicitly.

Manufacturers.\(^ {27}\) We assume that \(A\) is manufactured by firm \(M\) at no cost. Good \(B\), in
turn, is produced by a fringe of competitive suppliers at unit cost \(c < b\), and also by firm \(M\)
but at a higher cost, \(c + \Delta < b\).\(^ {28}\)

Contracts. The fringe will always optimally offer the linear contract \(w_{Bi} = c\), for \(i = 1, 2\). In

\(^{25}\)See also Shapiro (2018), pp. 28-29.

\(^{26}\)Note, however, that one-stop-shopping effects might be weakening due to the emer-
gence of over-the-top (OTT) services such as Hulu, HBO Now, and YouTube TV, among

\(^{27}\)Throughout this paper, male pronouns refer to manufacturers.

\(^{28}\)In the extensions, we replace the fringe by a strategic rival, also more efficient than \(M\) in producing \(B\).
M’s case, we restrict attention to two-part-tariff schemes, though we allow M to condition the terms of trade over both product lines jointly. When that is the case, we say that M is offering a portfolio contract.

**Timing.** On date 1, manufacturers simultaneously approach retailers R1 and R2 with take-it-or-leave-it offers that are observed by all participants. On date 2, retailers simultaneously decide which offers to accept, and sign contracts accordingly, establishing the retail costs governing the terms of trade between parties. Any fixed fee T is paid when the contract is signed, while wholesale unit prices w are paid as units are ordered. On date 3, and after observing contracting decisions, retailers compete for final consumers by simultaneously setting retail prices \( p_{Ai}, p_{Bi}, p_{ABi} \), the outcome being summarized by Lemmas B.1-B.4 in Appendix B.

![Timeline for Application I](image)

**Figure 1: Timeline for Application I**

### 3.2 Benchmark Case: No Must-Have

Consider first the benchmark situation in which A is not a must-have. Our first result states that in this case, M cannot obtain more than 1/4 which is equal to A’s monopoly profits.

**Proposition 2.** Suppose \( \mu_z = 0 \). The equilibrium is essentially unique. In all equilibria, both retailers procure A from M and B from the fringe, \( p_{Ai}^* = 1/2, p_{Bi}^* = c, p_{ABi}^* \geq p_{Ai}^* + p_{Bi}^* \), \( \pi_M^* = 1/4 \), and \( \pi_{Ri}^* = 0 \), for \( i = 1, 2 \).

**Proof.** See Appendix E. ■

An informal proof for this striking result is as follows. Note that the only way for M to obtain strictly more than A’s monopoly profit is to be able to extract part of the profits generated by B. However, since retailers are Bertrand competitors, the industry can only make profits in B if one retailer, say Ri, refrains from buying from the fringe and accepts a \( w_{Bi}^M > c \) from M. Profits generated from product B would then be \( w_{Bi}^M - c \), as retail competition for B is softened. M’s problem, however, is that those profits are now in \( Rj \)’s hand, and he can do nothing to extract them. The reason is that \( Rj \) can always reject any advances from M and buy B from the fringe to secure exactly \( w_{Bi}^M - c \). Anticipating this, M refrains from selling B.

---

29 In the extensions, we also consider the case in which M is restricted to offering linear prices.

30 Whenever a retailer is indifferent between accepting or rejecting a manufacturer’s offer, we assume the retailer accepts the offer. Furthermore, if a retailer is indifferent between two or more offers, she accepts the one with smaller fixed fees (which can be justified by adding an arbitrarily small amount of uncertainty in the number of units the retailer expects to sell during the duration of the contract); otherwise, she flips a coin.
and appropriates the monopoly rents generated by $A$. Retailers then procure $A$ from $M$ and $B$ from the fringe, and the statement of Proposition 2 follows.

It may seem surprising at first that $M$’s profit in Proposition 2 does not differ from what he would obtain if $A$ and $B$ were two completely unrelated markets; that is, if $M$ did not supply $B$ ($\Delta \to \infty$), and consumers who care for $A$ did not care for $B$ ($\mu = 0$). The reason, however, is that the present setting exhibits none of the conditions that the literature has established as necessary for $M$ to be able to exploit these market linkages and successfully leverage his monopoly power on $A$ over to the competitive market $B$.

Whinston (1990) and Nalebuff (2004), for example, demonstrate that by committing ex-ante (i.e., before a rival supplier of $B$ shows up) to sell $A$ and $B$ only as a bundle, $M$ can discourage efficient entry into market $B$, allowing $M$ to monopolize market $B$. However, both theories rely on the assumptions that $M$ moves first and can commit, and that rival suppliers must pay a fixed cost upon entry. Greenlee, Reitman and Sibley (2008) and Calzolari and Denicolo (2015) dispense with these assumptions, but assume instead that $M$ cannot entirely appropriate $A$’s monopoly profit ($1/4$ in our case) in the first place. In Greenlee, Reitman and Sibley (2008), this is obtained by restricting $M$ to offering linear contracts; Calzolari and Denicolo (2015) assume that buyers have private information about their demands. By conditioning the sale of $A$ to $B$, whether with bundling discounts or exclusive deals, $M$ can relax these price/information restrictions at the expense of (more efficient) suppliers, which are foreclosed.

None of these assumptions are present in our setting, ultimately explaining why $M$ cannot monopolize market $B$ when $A$ is a standard monopoly product ($\mu z = 0$). We now turn to the case in which $A$ qualifies as must-have.

### 3.3 Monopolization with Must-Have

Suppose $\mu z > 0$ and consider what would happen if $M$ were to approach retailers with the following pair of offers: the portfolio exclusivity contract

$$(w_{A1}^{Me} = 0, w_{B1}^{Me} = b, T_{AB1}^{Me} = 1/4 - \epsilon)$$

to $R1$, with $\epsilon \to 0$, and nothing to $R2$. Under this contract $R1$ is entitled, after paying a fixed fee of $1/4 - \epsilon$, to purchase $A$ and $B$ at unit costs of 0 and $b$, respectively, so long as she agrees to be $M$’s exclusive distributor in both categories.

It is clear that $R2$ will necessarily sign with the fringe, as she has no other offer available. But what will $R1$ do? If she rejects $M$’s offer and signs with the fringe, she anticipates zero profits as the continuation play involves both retailers carrying only $B$ at the same cost $c$. If she instead accepts $M$’s offer, the continuation play involves $(w_{A1}, w_{B1}) = (0,b)$ and $(w_{A2}, w_{B2}) = (+\infty,c)$, so by Lemma 2 of Section 2 (or equivalently, Lemma B.4 of Appendix B), we know that $R1$
anticipates a profit of $1/4 - T_{AB1}^M = \epsilon > 0$. Thus, following the proposed offers, we know that $R2$ will sign with the fringe and $R1$ will sign with $M$.

Prices in the retail market will then be $p_{A1}^* = 1/2$, $p_{B1}^* = p_{B2}^* + \epsilon = b$, and $p_{AB1}^* \geq p_{A1}^* + p_{B1}^*$. $R2$ will sell $B$ to: (i) all consumers that value only $B$, (ii) all two-stop shoppers interested in both goods, and (iii) only half of the one-stop shoppers interested in $A$ and $B$ (i.e., those with $v_A < p_{A1}^* = 1/2$). $R1$, in turn, will sell units of $A$ to all those consumers with $v_A \geq p_{A1}^* = 1/2$, and will sell $B$ to all those one-stop shoppers who visited her store to purchase $A$. This scenario is summarized in Table 2.

### Table 2: Equilibrium Units Sold to the Different Groups of Consumers

<table>
<thead>
<tr>
<th>Consumers that</th>
<th>Retailer 1</th>
<th>Retailer 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Units of A</td>
<td>Units of B</td>
</tr>
<tr>
<td>1. Value only $A$</td>
<td>$(1 - \mu)/2$</td>
<td>$0$</td>
</tr>
<tr>
<td>2. Value only $B$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>3. Value both - $v_A$ high(b)</td>
<td>$\mu z/2$</td>
<td>$\mu z/2$</td>
</tr>
<tr>
<td>One-stop shoppers</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Value both - $v_A$ low(c)</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>One-stop shoppers</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Value both - $v_A$ high(c)</td>
<td>$\mu(1 - z)/2$</td>
<td>$0$</td>
</tr>
<tr>
<td>Two-stop shoppers</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Value both - $v_A$ low(c)</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>Two-stop shoppers</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Equilibrium prices $p_{A1} = 1/2$, $p_{A2} = +\infty$, $p_{B1}^* = b$, $p_{B2}^* = b - \epsilon$, $p_{AB1}^* = 1/2 + b$, $p_{AB2}^* = +\infty$.

(b) $v_A = p_{AB1}^* - p_{B2}^* = 1/2 + \epsilon$ is the valuation of the indifferent consumer between groups 3 and 4.

(c) $v_A = \min\{p_{A1}^* + p_{B2}^*, p_{AB1}^*\} - p_{B2}^* = 1/2$ is the valuation of the indifferent consumer between groups 5 and 6.

$R1$ will then get $1/4$ and $\epsilon$ before and after fixed fees, respectively, and $R2$ will get $\pi^*_R = (b - c)(1 - \mu z/2)$, while $M$ will obtain

$$\pi^*_M = T_{AB1}^M + \mu z(b - c - \Delta)/2 = 1/4 - \epsilon + \mu z(b - c - \Delta)/2 > 1/4$$

What is striking about this example is that $M$ is obtaining strictly more than $A$’s monopoly profit by selling units of $B$ that he should not be selling, as he is not only more inefficient in the production of $B$ than the fringe, but he is also charging more for these units. With the help of his must-have item, $M$ has been able to leverage market power from market $A$ to $B$, increasing the prices paid by final consumers in the latter, and capturing $\mu z(b - c - \Delta)/2$ of those rents. On the way to monopolizing $B$, $M$ has hurt not only consumers but also rival suppliers of $B$, reducing their sales from 1 to $1 - \mu z/2$ units. Thus, fringe suppliers have been partially foreclosed from the market.

The mechanism behind this result works as follows. First, when $M$ abstains from making an offer to $R2$ and ties the availability of $A$ to the purchase of $B$ in $R1$’s contract, he is essentially
changing the nature of R1’s procurement decision. R1 is no longer searching for the best deal for B (which is being offered by the fringe), but for the best deal for B conditional on also securing A. This allows M to charge a very high price for B while still obtaining R1’s acceptance. Second, because R1 has now a high unit cost for B (equal to the monopoly price b), competition in that market is softened dramatically. Finally, M is able to appropriate a fraction of the profits created from such softening, thanks to the must-have nature of good A: R1 will still be selling units of B to one-stop shoppers despite having a higher unit cost for B than R2.

Note that if A loses its must-have status, say, because \( \mu = 0 \) or because shopping is costless for all consumers (\( z = 0 \)), M can still soften market B with the same portfolio offers above. However, his profits would be strictly less than 1/4, as he will not be able to divert sales of B from R2 to R1, and still needs to leave R1 with \( \epsilon > 0 \) to secure her acceptance (see Proposition 2). Hence, \( \mu z > 0 \) is needed, though \( \mu z \to 0 \) is sufficient for monopolization to arise. Loosely speaking, however, we would expect this kind of monopolization to occur only when \( \mu z \) is sufficiently large (and \( c + \Delta \ll b \)), in order to justify the potential risks such a strategy could entail; for instance, the increased antitrust scrutiny it could generate.

If, however, A retains its must-have status but portfolio offers are banned, we are back to the no-monopolization outcome of Proposition 2, as the next proposition establishes.

**Proposition 3.** If portfolio offers are banned, so M is forced to sell A and B on a stand-alone basis, then the essentially unique equilibrium outcome is the one described in Proposition 2: \( \pi^*_M = 1/4, p^*_A = 1/2, p^*_B = c, \pi^*_M = 1/4, \) and \( \pi^*_R = 0, \) for \( i = 1, 2. \)

**Proof.** See Appendix E. □

Intuitively, when portfolio offers are banned, M’s terms for A are fixed when retailers are considering which contract for B to take. Hence, retailers will always search for the best deal for B in isolation, implying that both retailers will necessarily end up procuring B from the fringe. But if so, product B will be sold downstream at the competitive price \( c \), and M has again no choice but to content himself with being a monopoly in A. This same no-monopolization outcome would also prevail if consumers were to bypass the retailers by buying directly from the fringe, as doing so would also ensure a stand-alone price of c for B in the retail market.

We have illustrated how M can profitably extend monopoly power from one market to another. The leverage mechanism behind this strategy —using portfolio offers to soften competition in the adjacent market and then exploiting A’s must-have status to appropriate part of that benefit— is novel, among many other reasons, because it does not require of scale economies or any first-mover advantage. However, the contract analyzed so far might be suboptimal. Characterizing M’s optimal/equilibrium course of action comes next. This exercise is not only interesting in its own right; it also helps better appreciate the mechanism’s novelty.
3.4 Optimal Monopolization Offers

Three observations greatly simplify the characterization of M’s optimal offers: (i) the fringe always offers B at cost; (ii) M makes take-it-or-leave-it-offers; and (iii) there are unrestricted transfers between M and the two retailers.

Conditions (i)-(iii) imply that M’s problem can be rewritten as choosing wholesale prices \((w^M_{A1}, w^M_{B1})_{i=1,2}\) and B production allocations (in-house or from the fringe) so as to maximize total industry profit subject to retailers’ obtaining their outside options (i.e., the payoff a retailer can secure from unilaterally rejecting M’s instructions while simultaneously sourcing B from the fringe at c).\(^{31}\) Fixed fees are then adjusted to guarantee that all parties receive their promised payoffs.

In a nutshell, wholesale prices that result in retail prices closer to monopoly levels, or more efficient production allocations (such as allowing retailers to procure units of B from the fringe), increase industry profits but force M to leave a greater share in the hands of retailers. The next proposition summarizes the resolution to this tradeoff:

**Proposition 4.** Let

\[
\bar{\Delta} = \frac{(b-c)}{2} - \frac{z(b-c)^2}{8}
\]

M’s optimal offers are then characterized as follows:

1. *(Monopolization with Partial Foreclosure)* If \(\Delta \leq \bar{\Delta}\), then M offers the portfolio exclusivity contract

\[
(w^M_{A1} = 0, w^M_{B1} = b, T^M_{AB1} = 1/4 - \epsilon)
\]

to R1 and nothing to R2. Under these offers, the fringe is partially foreclosed. On the equilibrium path, R1 accepts M’s portfolio contract and sells stand-alone units of A and bundles, while R2 accepts the fringe’s offer and sells stand-alone units of B. Retail prices are at their monopoly levels \(p^*_A = 1/2, p^*_B = b, p^*_{AB} = 1/2 + b\), and M’s profits are \(\pi^*_M = 1/4 + \mu z(b - c - \Delta)/2\).

2. *(Monopolization with No Foreclosure)* If \(\Delta > \bar{\Delta}\), then M offers

\[
(w^M_{A1} = 1/2 - z(b-c)/4, w^M_{B21} = b, T^M_{AB1} = (1 - \mu)z(b-c)/8 - \epsilon) \quad \text{and} \quad (w^M_{B1} = +\infty, T^M_{B1} = +\infty)
\]

where \(\epsilon \to 0\). Under these offers, the fringe is not foreclosed at all. On the equilibrium path, R1 accepts M’s portfolio contract but sells only stand-alone units of A. R2, in turn,

\(^{31}\)Alternatively, the problem can be recast more elegantly as the following transfer pricing problem: M chooses transfer prices \((w^M_{A1}, w^M_{B1}, w^f_{Bi})_{i=1,2}\), where \(w^f_{Bi} \in \{c, +\infty\}\), subject to the constraint that Ri’s profits must be greater than or equal to the profits she would obtain when \((w^M_{A1} = +\infty, w^M_{B1} = +\infty, w^f_{Bi} = c)\), and Rj plays according to whatever M determines.
accepts M’s offer for A and the fringe’s offer for B. She then sells bundles and stand-alone units of B. Retail prices are \( p_{A1}^* = p_{A2}^* = 1/2 \), \( p_{B1}^* = p_{B2}^* = b \), and \( p_{AB1}^* = p_{AB2}^* = 1/2 + b - z(b - c)/4 \), and M’s profits are \( 1/4 + \mu z(b - c)/4 + \mu z^2(b - c)^2/16 \).

**Proof.** See the online Appendix. ■

Note that under no circumstances is it optimal for M to practice full foreclosure. As we show in the online Appendix, M’s maximum profit, if he does decide to do so is \( \pi_M^* = 1/4 + \mu z(b - c)/2 - \Delta \), which is strictly lower than the profit he can obtain from the (partial foreclosure) offers of the previous section. The reason is that extending the latter offers to full foreclosure carries no benefit. It does not help reduce retailers’ outside options, and is costlier for M due to the additional B units he must now produce at cost \( c + \Delta \).

M’s optimal solution, then, is to have the fringe produce at least some units. If M is slightly less efficient than the fringe (i.e., \( \Delta < \bar{\Delta} \)), M opts for partial foreclosure, exactly as described in the previous section. To see why, note first that M can always set one retailer’s outside options, say R1’s, to approximately zero and at no cost, by either (i) offering R2 product B at the same wholesale price at which R1 can procure it from the fringe, or (ii) letting R2 procure units of B from the fringe directly. Next suppose \( \Delta = 0 \). We then have that the best way for M to reduce R2’s outside option, while maximizing surplus, is to let R2 buy from the fringe and to provide R1 with goods A and B at marginal costs \( w_{A1} = 0 \) and \( w_{B1} = b \), respectively (along with the fixed fee \( T_{AB1} \) that leaves her with \( \epsilon \to 0 \)). This leads to the fringe being partially foreclosed, as M sells strictly positive units of B through the bundles offered by R1.

However, as \( \Delta \) increases (i.e., \( \Delta > \bar{\Delta} \)), M might rather have the fringe supplying all B units and R2 being the retailer selling all these units, whether stand-alone or in a bundle. Although this arrangement allows M to save on production costs, it increases R2’s outside option because M can no longer set \( w_{A1} = 0 \): doing so would destroy too much surplus from the intense competition for bundles that this would entail (bundles would be priced at \( b \)). M solves this tension by increasing \( w_{A1} \) to the point at which the marginal loss from surplus destruction (from permitting bundles to be sold below monopoly prices) is equal to the marginal gain from outside-option reduction.

In either case, M moves away from the industry profit-maximizing outcome in order to contain retailers’ outside options, whether by producing himself (when \( \Delta \) is small) or by inducing retailers to charge less than monopoly prices (when \( \Delta \) is larger). Portfolio offers and A’s must-have status, however, continue to be essential for the scheme characterized in Proposition 4 to work.

We are clearly not the first to show how a multi-product manufacturer (M) may be able to

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32 Full foreclosure can appear as an equilibrium outcome, however, if the fringe is replaced by a strategic single product rival S who faces increasing returns to scale. See the discussion in the next subsection.
extend monopoly power from one market \((A)\) to an adjacent market \((B)\).\(^{33}\) Earlier theories (e.g., Whinston, 1990; Nalebuff, 2004; Greenlee, Reitman and Sibley, 2008; Calzolari and Denicolo, 2015) exhibit important differences from ours, however. Notably, our theory builds on the role of retailers as intermediaries between manufacturers and final consumers, which these other theories fail to explicitly account for. The leverage mechanism in our theory works by affecting retailers’ decision about which products to carry and how these decisions affect retailers’ outside options.

In the presence of must-have items, portfolio offers allow \(M\) to exploit a particular type of complementarity that endogenously arises between products \(A\) and \(B\): the fact that not carrying \(A\) negatively affects sales of \(B\) by making a retailer appears vertically inferior to a competing retailer carrying both products. This allows the manufacturer of a must-have item \(A\) to increase prices in adjacent competitive markets while simultaneously reducing retailers’ outside options.\(^{34}\) This explains, for instance, why monopolization of \(B\) is possible even without any foreclosure whatsoever (when \(\Delta > \bar{\Delta}\) in Proposition 4).\(^{35}\)

It is important to highlight that this “must-have complementarity” is rather special. Had the complementarity arisen for exogenous technological reasons, for example, from a valuation externality (i.e., when consuming \(B\) is more valuable if consumed in conjunction with \(A\)), then monopolization would not have followed. The key is that the “must-have complementarity” cannot be fully extracted by \(M\), giving him incentives to use it as leverage in his negotiation to retailers.

Indeed, note that all consumers that buy product \(A\) get some surplus, some more than others. Absent first-degree price discrimination, this surplus can never be extracted from consumers. When \(\mu z > 0\), moreover, this surplus creates the aforementioned “must-have complementarity.” There is no contract, however, that allows \(M\) to entirely appropriate this complementarity, as the surplus is in the hands of final consumers, not in the hands of retailers, implying that retailers’ participation constraints bind before \(M\) is able to extract \(A\) marginal contribution fully. Being unable to extract this surplus, \(M\) then uses portfolio offers to exploit it as leverage to monopolize market \(B\) with the adverse consequences this entails.

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\(^{33}\)This argument has also been used in single-product environments under the assumption that retail buyers have “non-contestable” (i.e., units that are going to be purchased by the retailer in any event), and “contestable” portions of demand (i.e., units for which the retailer is willing to find substitutes). The adjacent market is then viewed as the “contestable” portion.

\(^{34}\)Thus our model is also very different to the exclusionary theories in the single-product realm (e.g., Rasmusen, Ramseyer and Wiley, 1991; Segal and Whinston, 2000; Simpson and Wickelgren, 2007; Asker and Bar-Isaac, 2014). Our mechanism, by working through a multi-product interaction (i.e., a must-have condition), does not rely on any of the elements that, one way or another, these other mechanisms do require (Whinston, 2006; Fumagalli, Motta and Calcagno, 2018): first-mover advantage accompanied with ex-ante commitment (Ide, Montero and Figueroa, 2016), scale economies, and lump-sum payments from manufacturers to retailers.

\(^{35}\)This is in sharp contrast with most monopolization mechanisms, including Asker and Bar-Isaac (2014), which also rely on competing retailers. Our monopolization mechanism also departs from theirs in that ours is invariant to the number of retailers (see section 3.5), while theirs fails to work with an increasing number of retailers.
3.5 Extensions

Before we move onto the model’s discussion and main implications we briefly discuss four extensions showing the robustness of our results. A formal treatment, when indicated, can be found in the online Appendix.

**Imperfect Competition.** As we move away from the Bertrand limit, upstream competition will not fully permeate downstream and retailers will enjoy some oligopoly rents on good $B$. But as we show in the online Appendix, none of our results change, as these extra rents remain small compared to the extra rents $M$ can obtain from monopolizing market $B$ with portfolio offers.

**Linear Prices.** If $M$ is restricted to linear contracts, he can still use $A$ as leverage to monopolize market $B$, though under a more restricted set of circumstances. In particular, the monopolization strategy may no longer be profitable when the must-have effect (i.e., $\mu z$) is too small.

**Single-Product Strategic Rival.** The leverage mechanism just outlined will also continue to hold if the fringe is replaced by a strategic single product rival $S$. In the online Appendix, for instance, we characterize a partial foreclosure equilibrium similar in spirit to the partial foreclosure outcome of section 3.2. Introducing $S$ allows us also to study the effect of scale economies. In particular, we consider the presence of a single-product rival with marginal cost $\kappa$ and fixed cost $K$, such that $\kappa + K = c$ (to keep overall efficiency fixed). We then show that if $K \in [K, c]$, where $K > 0$, then there can be a full foreclosure equilibrium in which $M$ improves upon the payoffs obtained in Proposition 4. Thus, while scale economies are not needed for the mechanism to work, they may help $M$ to arrive at better outcomes.

**More than two retailers.** Adding more Bertrand retailers does not meaningfully change our results, either. For instance, in the case of $\Delta \leq \bar{\Delta}$, $M$ would approach each additional retailer $i \geq 3$ with the portfolio exclusivity offer ($w_{Ai}^{Me} = 1/2, w_{Bi}^{Me} = b, T_{ABi}^{Me} = -\epsilon$), leaving him with the same payoff as before.\(^{36}\)

3.6 Discussion and Implications

We have shown how the manufacturer of an item with must-have potential can exploit that potential to monopolize an otherwise competitive market in circumstances that a manufacturer of a standard monopoly product cannot. This new theory of harm relies neither on first-mover advantages nor ex-ante commitments, scale economies, and/or contractual frictions, as previous theories do. Furthermore, this new theory exhibits some surprising features, such as the possibility of monopolization without any foreclosure of rival manufacturers whatsoever.

\(^{36}\)Note that slotting allowances (i.e., negative fixed fees) can be dispensed with as soon as we depart from Bertrand competition.
Our theory not only contributes to the literature a novel leverage mechanism but, more importantly, it carries important antitrust implications. Take, for instance, the existing obligation for broadcast television stations and multichannel video programming distributors (MVPDs) to negotiate retransmission consent in “good faith.” Under the U.S. Communications Act of 1934, MVPDs are forbidden from retransmitting the signal of a broadcast station without the latter’s consent. The Act, however, also requires broadcast stations and distributors to negotiate for retransmission consent in “good faith.” Just recently, in December 2014, the U.S. Congress directed the FCC to review the circumstances under which there may be a violation of such obligation,\(^\text{37}\) which led the FCC to release a Notice of Proposed Rulemaking seeking comments on how to address potential updates.\(^\text{38}\)

Central to the FCC’s inquiry was the question as to whether the bundling/tying of broadcast signals with other networks should be considered to be in compliance with this “good faith” standard. In particular, while the FCC stated that bundling is presumptively consistent with this standard, it also stipulated that practices intended to gain or sustain market power represent a violation of such obligation. The FCC’s main concern was that broadcasters may require MVPDs to purchase less popular programming in order to have access to (loosely defined) “must-have” broadcast programming, forcing MVPDs to pay for programming they do not want, and passing the additional costs to final consumers. Note that \textit{Cablevision v. Viacom} revolves around the exact same issue, the only difference being that bundling involved cable networks, some more popular than others.\(^\text{39}\) The FCC then invited opinions as to whether market circumstances have changed, so that bundling of broadcast with non-broadcast programming might be problematic.

Our theory shed some light on these issues. As we have shown in this section, owners of channels satisfying our must-have definition can effectively use them, in combination with tying/portfolio offers, to monopolize otherwise competitive markets. Coupled with the fact that current market conditions in this industry seem suitable for the emergence of must-have channels (see discussion in section 2.3), we conclude that the current bundling/tying practices in retransmission consent negotiations could potentially violate the good faith standard.

The theory developed in this section also offers a new perspective on the evaluation of conglomerate mergers such as Procter & Gamble/Gillette and Pernod Ricard/Diageo/Seagram Spirits.\(^\text{40}\) Typically, these mergers receive less antitrust scrutiny than horizontal ones simply because they involve, for the most part, unrelated products that do not compete with each

\(^{37}\text{See Pub. L. No. 113-200, § 103(c), 128 Stat. 2059 (2014).}\)

\(^{38}\text{“Implementation of Section 103 of the STELA Reauthorization Act of 2014/Totality of the Circumstances Test”, 80 FR 59706 (10/02/2015).}\)

\(^{39}\text{Similar issues have also arisen in bundling/tying cases outside the multichannel video programming industry, such as \textit{3M v. LePage’s} (324 F.3d 141, 3d Cir. 2003) and in \textit{Cascade Health Solutions v. PeaceHealth}.}\)

\(^{40}\text{EU Case No COMP/M.2268.}\)
other. But consider a situation in which a supplier of good A is proposing to merge with one of the few suppliers of an unrelated good B. If, according to our definition, A qualifies as a must-have, this conglomerate merger might be deeply problematic, perhaps more so than if two suppliers of B merge. Our theory also proposes a potential remedy for such situations: by prohibiting portfolio offers involving different product categories, the antitrust authority can limit the potential damage that such deals can entail.

4 Application II: Vertical Mergers

Compared to the previous section, the focus of this application is somewhat different: instead of providing an entirely new theory of harm, here we incorporate must-have items into the workhorse model used in the evaluation of recent vertical mergers to study how their presence might alter, if at all, antitrust predictions. Consequently, we begin by adapting our model in the simplest possible way to accommodate the Nash bargaining theory of harm used to oppose the Comcast/NBC-Universal and AT&T/Time Warner mergers (Murphy, 2010; Shapiro, 2018).

4.1 The Setup

As before, there are two products, A and B, two competing retailers, R1 and R2, and two manufacturers, M and a fringe. However, since the Nash bargaining theory of harm does not rely on M being a multi-product firm, here we assume that M produces only A. Our aim is to study how a vertical merger between M and R1 affects the negotiated terms between M and R2.

Manufacturers. A is supplied by a monopoly manufacturer M at no cost, while a fringe of competitive producers supplies B at cost $c < b$.

Consumer valuations and retail markets. We depart from our original formulation in that consumers are now split evenly in two independent retail markets, $m = 1, 2$, each of size 1/2. Consumers’ valuations, moreover, are assumed to be market- and retailer-specific. More precisely, and similar to Bernheim and Madsen (2017), we assume that retailer Ri has a “home” advantage in market $m = i$, in that a consumer in that market values product $l \in \{A, B\}$ in $v_l + \gamma$ when purchased from Ri, and in $v_l$ when purchased from $R_j \neq Ri$, where $\gamma > 0$.

The parameter $\gamma$ captures the degree of retailer differentiation. It is needed to ensure that after M merges with R1, there are gains from trade to be made between M and R2, even though M has direct access to all final consumers through R1.\footnote{If $\gamma = 0$, as in our original formulation, M will refuse to deal with R2 after merging with R1, as this would only introduce more competition in the market.} Throughout this section, we assume that $\gamma$ is sufficiently high so that post-merger M and R2 have an incentive to reach an
agreement, but not so high that pre-merger retailers become local monopolies in their respective “home” markets. The range of values for $\gamma$ that comply with both conditions will be specified more precisely as the analysis unfolds.

Finally, as in Sections 2 and 3, $\mu \in [0,1]$ is the probability that a consumer who values $A$ also values $B$, and $z \in [0,1]$ is the fraction of one-stop shoppers.

**Contracts.** Following the Nash bargaining theory of harm, we assume that supplier-retailer relationships are governed by linear contracts when parties are independent entities. While this is irrelevant for the fringe as it continues to (optimally) offer $w_{B1} = w_{B2} = c$, it is not for $M$ (note that superscripts are no longer needed, as $M$ does not supply $B$). We denote by $w_{Ai}$ $M$’s negotiated terms with $R_i$ for product $A$.

Note that the fringe’s contract will always be accepted in equilibrium by both retailers, as no fixed fees are involved, and $M$ is unable to make portfolio offers. Hence, from here on, we can omit the fringe’s existence and assume that $w_{B1} = w_{B2} = c$.

**The Vertical Merger.** We assume that once $M$ vertically integrates with $R_1$ (i) he always delivers $A$ at a wholesale price of zero, his actual cost of production, and (ii) $M$ and $R_1$ completely internalize each other’s profits when making decisions in the game.

**Timing and Bargaining Protocol.**

*Pre-Merger.* On date 1, $M$ simultaneously negotiates with $R_1$ and $R_2$ over wholesale prices $w_{A1}$ and $w_{A2}$, respectively. We employ Nash-in-Nash bargaining as our solution concept (Horn and Wolinsky, 1988). Bargaining weights are assumed to be the same in both bilateral negotiations, and equal to $\beta$, and $1 - \beta$ for $M$ and $R_i$, respectively. On date 2, and after observing the terms of trade governing all wholesale transactions, retailers compete for final consumers by simultaneously setting retail prices $(p_{Ai}^{(m)}, p_{Bi}^{(m)}, p_{ABi}^{(m)})_{i=1,2}$ in each retail location $m = 1, 2$.

![Timeline for Application II](image-url)

Figure 2: Timeline for Application II

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42Thus, there is no scope for “vertical separation” a l`a Bonanno and Vickers (1988)
Post-Merger. The post-merger timing is identical to the pre-merger timing, except that on date 1, \(M\) negotiates only with \(R2\) using a Nash bargaining protocol anticipating that \(w_{A1} = 0\) on date 2.

### 4.2 Downstream Equilibrium at each Retail Location

The fact that consumer valuations are now market- and retailer-specific modifies the retail pricing equilibrium of the last stage. The resulting equilibrium, however, follows closely the spirit of the retail equilibrium of the baseline model.

Here we present two of the lemmas characterizing the retail pricing equilibrium at location \(m = i\), which we will use in this section. The full characterization of the retail pricing equilibrium can be found in Appendix C.

**Lemma 3.** Consider location \(m = i\). If \(w_{A1} \leq w_{A2} + \gamma\) there is no retail bundling in equilibrium.\(^{43}\) Retail prices are then given by \(p_{Ai}^{(i)*} = \min\{w_{A2} + \gamma, (1 + \gamma + w_{A1})/2\}\), \(p_{A2}^{(i)*} = w_{A2}\), \(p_{B1}^{(i)*} = c + \gamma\), \(p_{B2}^{(i)*} = c\). Retailers’ equilibrium profits at this retail location are then

\[
\pi_{Ri}^{(i)*} = \begin{cases} 
(1 + \gamma - w_{A1})^2/4 + \gamma & \text{if } (1 + \gamma + w_{A1})/2 \leq w_{A2} + \gamma \\
(w_{A2} - w_{A1} + \gamma)(1 - w_{A2}) + \gamma & \text{otherwise}
\end{cases}
\]

and \(\pi_{Rj}^{(i)*} = 0\).

**Lemma 4.** Consider location \(m = i\). If \(R_i\) does not carry \(A\) and \(w_{A2} \leq 1\), then there is no retail bundling in equilibrium. Retail prices are then given by \(p_{Ai}^{(i)*} \to \infty\), \(p_{A2}^{(i)*} = (1 + w_{A2})/2\), \(p_{B1}^{(i)*} = c + \gamma - \epsilon\) (with \(\epsilon \to 0\)), and \(p_{B2}^{(i)*} = c\). Retailers’ equilibrium profits at this retail location are then \(\pi_{Ri}^{(i)*} = \gamma[1 - \mu_z(1 - w_{A2})/2]\) and \(\pi_{Rj}^{(i)*} = (1 - w_{A2})^2/4\).

### 4.3 Benchmark Case: No Must-Have

As a benchmark, we begin by analyzing the case in which \(A\) does not qualify as a must-have (\(\mu_z = 0\)). Our goal is to illustrate how the Nash bargaining theory of harm works, setting the stage to incorporate must-have items in the next subsection.

So, assume \(\mu_z = 0\), and consider the pre-merger situation in which manufacturers and retailers are independent entities. A Nash-in-Nash equilibrium is a pair of wholesale prices \((w_{A1}^*, w_{A2}^*)\) such that \(w_{A1}^*\) is the (generalized) Nash solution to the bargaining problem between \(M\) and \(R_i\), given that both parties correctly anticipate the price \(w_{A2}^*\) that will be agreed between

\(^{43}\)Recall that \(w_{B1} = w_{B2} = c\).
M and Rj. That is, given $w_{A_j}^*$, $w_{A_i}^*$ solves:

$$
\max_{w_{A_i}} (\pi_M(w_{A_i}, w_{A_j}^*) - \pi_M(\infty, w_{A_j}^*))^{\beta} (\pi_{Ri}(w_{A_i}, w_{A_j}^*) - \pi_{Ri}(\infty, w_{A_j}^*))^{1-\beta}
$$

subject to

$$
\begin{align*}
\pi_M(w_{A_i}, w_{A_j}^*) &\geq \pi_M(\infty, w_{A_j}^*) \\
\pi_{Ri}(w_{A_i}, w_{A_j}^*) &\geq \pi_{Ri}(\infty, w_{A_j}^*)
\end{align*}
$$

(3)

where $\pi_M(w_{A_i}, w_{A_j}^*)$ and $\pi_{Ri}(w_{A_i}, w_{A_j}^*)$ are the parties’ payoffs when they reach a deal at $w_{A_i}$, and $\pi_M(\infty, w_{A_j}^*)$ and $\pi_{Ri}(\infty, w_{A_j}^*)$ are their (disagreement) payoffs (i.e., outside options) in case they fail to reach an agreement.

**Proposition 5.** Suppose that $\mu z = 0$, then the unique symmetric pre-merger equilibrium, $w_{A_1}^* = w_{A_2}^* = w_A^*$, involves Bertrand pricing if and only if $\gamma < (1 + \beta)/(1 + 3\beta) \equiv \bar{\gamma}$. The equilibrium wholesale price for $A$, $w_A^*$, is strictly between 0 and 1/2, and is given by the (negative square root) solution to the quadratic equation:

$$
\beta \gamma (1 - 2w_A^*) = (1 - \beta)w_A^* (1 - w_A^* - \gamma/2)
$$

(4)

Prices at each retail location are then given by $p_{Ai}^{(i)*} = w_A^* + \gamma$, $p_{Aj}^{(i)*} = w_A^*$, $p_{Bi}^{(i)*} = c + \gamma$ and $p_{Bj}^{(i)*} = c$ for $i = 1, 2$.

**Proof.** See the online Appendix. ■

An informal proof of Proposition 5 is as follows. If the retail market equilibrium involves Bertrand pricing, we know from Lemma 3 that in a neighborhood around $(w_{A1}, w_{A2}) = (w_A^*, w_A^*)$, M’s and Rj’s profits and disagreement payoffs are given by the following (recall that each retail market has size 1/2):

$$
\begin{align*}
\pi_M &\equiv \pi_M(w_{A_i}, w_{A_j}) = w_{A_i}(1 - w_{A_j})/2 + w_{A_j}(1 - w_{A_i})/2 \\
\pi_{Ri} &\equiv \pi_{Ri}(w_{A_i}, w_{A_j}) = (w_{A_j} - w_{A_i} + \gamma)(1 - w_{A_j})/2 + \gamma/2 \\
\bar{\pi}_M &\equiv \pi_M(\infty, w_{A_j}) = w_{A_j}(1 - w_{A_j})/4 + w_{A_j}(1 + \gamma - w_{A_j})/4 \\
\bar{\pi}_{Ri} &\equiv \pi_{Ri}(\infty, w_{A_j}) = \gamma/2
\end{align*}
$$

(5) – (8)

In case of agreement, Rj obtains (quality-adjusted) Bertrand profits on both products in market $m = i$ and nothing in market $m \neq i$; in case of disagreement, she retains only the Bertrand profits on product B in market $m = i$.

Because $M$ and $Ri$ then set $w_{A_i}$ so as to solve (3), the equilibrium is obtained by simultaneously solving the pair of first-order conditions

$$
\frac{\beta}{\pi_M - \bar{\pi}_M} \frac{\partial \pi_M}{\partial w_{A_i}} + \frac{1 - \beta}{\pi_{Ri} - \bar{\pi}_{Ri}} \frac{\partial \pi_{Ri}}{\partial w_{A_i}} = 0
$$

(9)
for $i = 1, 2$. Substituting (5)-(8) into (9), applying symmetry, and solving yields (4). Bertrand pricing at the retail market then requires $w^*_A < 1 - \gamma$, which holds if and only if $\gamma < \bar{\gamma}$.\textsuperscript{44}

Suppose now that $M$ proposes to merge with $R1$. How should an antitrust authority react to such a proposal? As with the mergers of Comcast and NBC-Universal or AT&T and Time Warner, the answer is not straightforward because opposing forces are at work. One force is the so-called elimination of double marginalization (EDM) effect: the fact that $R1$ would have access to $M$’s products at a lower price, presumably at cost. The EDM effect results in fiercer competition in the downstream market, benefiting final consumers. Acting in the opposite direction, however, is the so-called increased bargaining leverage (IBL) effect. The IBL effect states that after the merger, $M$ would have incentives to (bilaterally) negotiate higher wholesale prices for his products with rival distributors; in this case, with $R2$. And, all else being equal, increasing the cost to rival distributors would necessarily result in higher retail prices, to the detriment of final consumers.

**Proposition 6.** Suppose $\mu z = 0$ and that $M$ and $R1$ merge. In the post-merger equilibrium the integrated firm $M$-$R1$ deals with $R2$ if and only if $\gamma \geq (5 - \sqrt{7})/9 \equiv \bar{\gamma}$. When that is the case, the IBL effect pushes $w_{A2}$ upward while the EDM effect pushes both $w_{A1}$ and $w_{A2}$ downward.

**Proof.** See Appendix F. ■

Figure 3 illustrates the workings of the two different effects. Whether the EDM effect dominates the IBL effect is ultimately an empirical matter. The figure depicts a situation of a post-merger equilibrium’s value of $w_{A2}$ being above its pre-merger level $w^*_A$, but this need not always be the case.

The first important aspect of Proposition 6 is that $\gamma$ needs to be bounded away from zero; otherwise, $M$ would stop trading with $R2$ after the merger and offer $A$ exclusively through its downstream affiliate $R1$ in both retail locations. We will work under the assumption that $\gamma \geq \bar{\gamma}$,\textsuperscript{45} as the Nash-bargaining theory of harm is not a refusal to deal argument.\textsuperscript{46}

To convey some intuition of the IBL and EDM effects, recall that after the merger: (i) $M$ delivers product $A$ to $R1$ at $w_{A1} = 0$; (ii) $M$ and $R1$ completely internalize each other’s profits in their decisions at any stage of the game; and (iii) the negotiation between $M$ and $R2$ still follows a Nash-bargaining protocol, so in equilibrium, (9) continues to hold for $R2$. However, because of (ii), $\pi_M$ and $\bar{\pi}_{M2}$ are now equal to:\textsuperscript{47}

\textsuperscript{44}As explained in the proof of the proposition, in the region $1/(1 + 2\beta) < \gamma < (1 + \beta)/(1 + 3\beta)$ there is also a candidate for a symmetric equilibrium with monopoly pricing for good $A$ in both locations (i.e., $(1 + w_{A} + \gamma)/2 < w_{A} + \gamma$). While robust to local deviations, this candidate does not survive global deviations.

\textsuperscript{45}Note that $\bar{\gamma} > \gamma$ for all $\beta \in [0, 1]$.

\textsuperscript{46}“... the Government does not allege that a post-merger Turner would be incentivized to start actually engaging in long-term blackouts with distributors. That is so, as Professor Shapiro concedes, because withholding Turner content would not be profitable to the merged entity...”. Judge R. Leon - USA v. AT&T, et al. (2018).
\[ \pi_M = \frac{(w_{A2} + \gamma)(1 - w_{A2})}{2} + \frac{\gamma}{2} + w_{A2}(1 - w_{A1})/2 \]  \hspace{1cm} (10) \\
\[ \bar{\pi}_{M2} = \frac{(1 + \gamma - w_{A1})(1 + \gamma + w_{A1})}{8} + \frac{\gamma}{2} + \frac{1 - w_{A1}(1 + w_{A1})}{8} \]  \hspace{1cm} (11) 

Consider first the IBL effect, which is driven by the following changes. First, \( M \)'s post-merger profits become more sensitive to changes in \( w_{A2} \) (i.e., \( \partial \pi_M / \partial w_{A2} \) is larger than its pre-merger level, regardless of \( w_{A1} \)), both because \( M \) completely internalizes \( R_1 \)'s profits and because \( R_2 \)'s ability to compete in retail market 1 is impaired as \( w_{A2} \) increases. And second, while both (10) and (11) are larger than their pre-merger levels, as \( M \) internalizes \( R_1 \)'s profit both on and off path, the difference \( \pi_M - \bar{\pi}_{M2} \) is significantly smaller post-merger. The reason is simple: \( \bar{\pi}_{M2} \) jumps by an amount equal to \( R_1 \)'s monopoly profits on good \( A \) in both retail markets, while \( \pi_M \) jumps only an amount equal to \( R_1 \)'s equilibrium profit on good \( A \) in market 1. A larger \( \partial \pi_M / \partial w_{A2} \), together with a smaller \( \pi_M - \bar{\pi}_{M2} \), improves \( M \)'s bargaining position against rival distributors; hence, for any given \( w_{A1} \), \( w_{A2} \) must necessarily go up in order for (9) 

**Figure 3: Effects of a Vertical Merger (No Must-Have)**

The solid and dashed lines depict \( w_{A2}(w_{A1}) \) (the Nash solution in the \( M-R_2 \) negotiation for a given \( w_{A1} \)), before and after the merger, respectively. The pre-merger situation is given by \((w_{A1}^*, w_{A2}^*)\), the intersection of \( w_{A2}(w_{A1}) \) (Pre-Merger) with the 45° line. The IBL effect then corresponds with an upward shift of \( w_{A2}(w_{A1}) \), so for any given \( w_{A1} \), \( M \) can extract a higher \( w_{A2} \). The direct EDM effect is simply the drop of \( w_{A1} \) from \( w_{A1}^* \) to 0; the indirect EDM effect is the drop in \( w_{A2} \) occasioned by the decrease in \( w_{A1} \).

\(^{47}\)This assumes that Bertrand pricing prevails post-merger in both locations (alternative pricing regimes are considered in the proof of the proposition).
to continue to hold.

Next, consider the post-merger reduction of the wholesale price \( w_{A1} \) to its marginal cost of zero. As illustrated in Figure 3, this has the effect of reducing retail prices through two different channels. The first channel is the EDM direct effect, the fact that \( R1 \) has a lower wholesale cost for \( A \). In our model (see Lemma 3), this implies that the retail equilibrium price of \( A \) in market 2 goes down from its pre-merger level \( w_{A}^{*} + \gamma \) to \( \gamma \). The second channel, the EDM indirect effect, is largely a consequence of the first. Anticipating the drop in market 2’s retail prices, and therefore the erosion of \( R2 \)’s profits, \( R2 \) can now demand a \( w_{A2} \) from \( M \) that is lower than the pre-merger level. A lower \( w_{A2} \), in turn, extends the fall in retail prices to market 1, as well. This overall drop in retail prices is the result of the EDM effect.

4.4 Vertical Mergers and Must-Have Items

Assume now that \( A \) classifies as must-have \(( \mu_{z} > 0 \)). Relative to the benchmark case with \( \mu_{z} = 0 \), the only change is that retailers’ outside options are no longer (8), but instead:

\[
\bar{\pi}_{Ri} = \frac{\gamma}{2} \left( 1 - \frac{\mu_{z}(1 - w_{Aj})}{2} \right)
\]

as seen from Lemma 4. Consequently, now in case of disagreement, \( R_i \) obtains only a fraction of the Bertrand profits on product \( B \) in market \( m = i \). This is due to good \( A \)’s being a must-have item: if \( R_i \) fails to reach an agreement with \( M \) and does not carry \( A \), then some consumers in location \( m = i \) will switch their purchases of \( B \) to \( R_j \).

Consider first the pre-merger situation. Given the reduction in retailers’ outside options, it is not surprising that \( A \)’s must-have status should allow \( M \) to negotiate better (pre-merger) terms with both retailers.

**Proposition 7.** Suppose \( \mu_{z} > 0 \), then the unique symmetric pre-merger equilibrium involves Bertrand pricing if and only if \( \gamma < \left( 1 + \beta + \mu_{z} \right)/(1 + 3\beta + 2\beta\mu_{z}) \equiv \bar{\gamma}(z) \). The equilibrium wholesale price for \( A \), \( w_{A}^{**} \) is given by the (negative square root) solution to the quadratic equation:

\[
\beta\gamma(1 + \mu_{z}/2)(1 - 2w_{A}^{**}) = (1 - \beta)w_{A}^{**}(1 - w_{A}^{**} - \gamma/2)
\]

where \( w_{A}^{**} \in (0, 1/2) \) and is increasing in \( A \)’s must-have potential \(( \mu_{z} \)).

**Proof.** As the proof follows closely that of Proposition 5, hence, it is omitted. Showing that \( w_{A}^{**} \) is increasing in \( \mu_{z} \) is immediately apparent from looking at (13) and using \( w_{A}^{**} \in (0, 1/2) \).

Consider now a scenario in which \( M \) and \( R1 \) merge. While it is true that \( A \)’s must-have status provides \( M \) with more bargaining leverage by reducing \( R2 \)’s outside option, this argu-
ment, in and of itself, is not sufficient to claim that the vertical merger is more anticompetitive. The reason is that this additional leverage is also at M’s disposal before the merger takes place, as Proposition 7 shows. This is not to imply, however, that must-have items are therefore innocuous in vertical merger evaluations. As we show next, must-have items produce a subtler anticompetitive effect, the must-have merger (MHM) effect, which is generated by the interaction of A’s must-have status with the vertical merger itself.

To understand this new effect, note from (12) that A’s must-have status not only reduces R2’s outside option, but also makes it depend on $w_{A1}$. This occurs because the number of consumers that R2 loses to R1 when A is removed from her lineup is not exogenous but depends on the equilibrium prices that retailers charge in the retail market. Thus, the lower $w_{A1}$, the lower the price R1 charges in the retail market for A, and hence, the higher the fraction of one-stop shoppers that switch to R1, having been attracted by this lower price (and the fact that R2 is not carrying A). In other words, because the “must-have complementarity” emerges endogenously due to the combination of one-stop shopping and retail competition, its magnitude is not fixed but depends crucially on the terms signed by rival retailers.

Hence, in the presence of must-have items, the EDM direct effect (i.e., the decrease of $w_{A1}$ from its pre-merger level to zero) not only induces the EDM indirect effect, but also generates an increase in A’s must-have potential, providing M-R1 with more bargaining leverage relative to what M had before the merger. This extra boost in bargaining leverage is the MHM effect, as summarized in the following proposition.

**Proposition 8.** Suppose $\mu z > 0$ and that M and R1 merge. In the post-merger equilibrium the integrated firm M-R1 deals with R2 if and only if $\gamma \geq (5 + 2z - \sqrt{7 + 8z + 2z^2})/(3 + z)^2 \equiv \gamma(z)$. When that is the case, there is an effect in addition to the IBL and EDM effects documented in Proposition 6: the MHM (must-have merger) effect. This latter effect allows M to negotiate higher post-merger terms with R2 that are increasing in A’s must-have potential ($\mu z$).

**Proof.** See Appendix F. ■

The existence of the MHM effect has two critical implications for merger evaluation. First, vertical mergers involving manufacturers of must-have items should raise more antitrust concerns than mergers involving manufacturers of standard monopoly products. Whether the MHM effect can be significant enough to, for example, turn a pro-competitive transaction into an anticompetitive one is ultimately an empirical matter, beyond the scope of our analytical application. The second implication is the need to recognize that the number of consumers lost in the event of a breakdown in negotiations—a critical input for estimating the outside option of rival distributors—is not invariant to the merger. A naive estimation using pre-merger data without making any adjustments to take this fact into account will always underestimate the anticompetitive potential of the merger at hand.
The latter point can be illustrated with the aid of Figure 4, which depicts the post-merger negotiation between \( M \) and \( R2 \) under two different sets of assumptions. The solid curve represents the terms that \( M \) and \( R2 \) negotiate as a function of \( w_{A1} \) under the correct assumption that \( R2 \)'s outside option varies with \( w_{A1} \). The dashed curve represents the terms under the naive assumption that \( R2 \)'s outside option remains constant at its pre-merger level, i.e., \( \bar{\pi}_{R2} = \gamma/2 - \mu z \gamma (1 - w_{A1}^*)/4 \). As should be fairly intuitive by now, the solid curve is a clockwise rotation of the dashed line around \( w_{A1}^* \). Thus, a “naive” merger evaluation would predict that \( w_{A2} \) would increase from \( w_{A2}^* \) to \( w_{A2}' \), when in reality it will increase to \( w_{A2}'' \). The extent of this underestimation, the difference between \( w_{A2}' \) and \( w_{A2}'' \), is precisely the MHM effect. 48

The possibility of underestimating the anticompetitive harm of a vertical merger involving must-have items is more than a theoretical curiosity. In fact, it could have already happened. As argued in Section 2, current market conditions in the multichannel television industry seem suitable for the emergence of must-have channels. Hence, it is possible that HBO and a subset of Time Warner’s Turner Networks (e.g., CNN, TNT, TBS) could have qualified as must-have according to our definition. However, from reading publicly available documents regarding the

48Although calculating the extent of under-estimation would certainly require a richer model than ours, our model can still illustrate that the MHM effect can, in principle, be substantial. Letting \( z = \mu = 1, \beta = \gamma = 1/2, \) and \( w_{A1} \) to drop to marginal cost after the merger, we obtain that the combined IBL and EDM effects take \( w_{A2} \) from its pre-merger level of 0.407 to 0.449, a 10.4% increase. Adding the MHM effect, \( w_{A2} \) jumps to 0.5, a 22.9% increase from its pre-merger level.
government’s evaluation of the AT&T/Time Warner merger (e.g., Shapiro, 2018, pp. 50-55), it
seems that the fraction of consumers lost to Time Warner’s content removal was assumed to
be the same pre- and post-merger.\textsuperscript{49,50}

5 Final Remarks

The concept of must-have—an item that a retailer must carry in order to “compete effectively”—
has taken center stage in highly contentious antitrust cases. However, we have seen no effort
made to formalize its meaning (e.g., distinguishing a must-have from a standard monopoly
product), let alone its allegedly anticompetitive effects. In this paper, we have advanced a
theory with a precise definition of what “compete effectively” means and its implications for
competition policy.

While must-have and monopoly products share the (exogenous) property of having no close
substitutes, two crucial differences distinguish the two. The first is that must-have is an end-
dogenous qualification. The second difference concerns their differing antitrust implications. A
manufacturer of a must-have item can be more detrimental to welfare and consumer surplus
than the manufacturer of a standard monopoly product. We have established, for example, that
a must-have manufacturer can monopolize an adjacent, competitive market in circumstances
in which a monopoly manufacturer cannot. We have also shown that in the case of vertical
mergers, a must-have manufacturer can negotiate much higher (post-merger) prices from rival
distributors than could a monopoly manufacturer.

We cannot conclude without referring to alternative interpretations of must-have, most
notably the one embraced by Judge Leon and others: must-have as an “essential input,” that
is, an input without which a competitor cannot operate downstream (Fumagalli, Motta and
Calcagno, 2018, pp. 471-472). Such a product would exist, for example, if not carrying it
prevented a retailer from reaching a minimum scale of operation, forcing her to exit the market.
Note that under this alternative definition, the distinction we have developed between a must-
have and a standard monopoly product becomes irrelevant. All that matters is that the retailer
not carrying such a product suffers a revenue loss sufficiently large to end operations. Whether
retailers compete, and multi-product consumers one-stop shop is, therefore, irrelevant.

This definition, however, seems very restrictive, resulting in too high a standard of proof for
authorities and policymakers, as it would be quite challenging to prove that a retailer would
ever be in such a life-threatening situation. This particular issue constituted, in fact, one of the
most significant setbacks of the U.S. Government’s case against AT&T and Time Warner:

\textsuperscript{49}Murphy (2010) appears to have followed a similar approach in evaluating the Comcast/NBC-Universal merger.
\textsuperscript{50}This omission is not entirely surprising since we are the first to document the existence of the MHM effect.
Based on the evidence, I agree with defendants that Turner’s content is not literally “must-have” in the sense that distributors cannot effectively compete without it. The evidence showed that distributors have successfully operated, and continue to operate, without the Turner networks or similar programming.

Judge Richard Leon - 17-2511 USA v. AT&T, et al. (RJL) (2018)

Our theory, in contrast, shows that must-have items —properly defined— require a less stringent, more easily falsifiable set of market conditions, and can lead to very anticompetitive outcomes.
Appendix A  Retail Demands

Without loss of generality, let $p_{ABi} \leq p_{Ai} + p_{Bi}$, for $i = 1, 2$. We use the convention that if $R_i$ does not carry product $l = A, B$ (i.e., $w_{li} \to +\infty$), then $p_{li} \to +\infty$ and $p_{ABi} \to +\infty$.

As mentioned in the main text, we focus on the case in which both retailers carry product $B$ (i.e., $\max\{w_{B1}, w_{B2}\} \leq b$), and at least one of them carries $A$ (i.e., $\min\{w_{A1}, w_{A2}\} < 1$). This is consistent with our interpretation of $A$ as a “unique” product for which there are no close substitutes, and $B$ as a set of generic varieties, all of which are available to retailers.

Retail demands come from three groups of consumers. Demands from the first group, those who value only one good, are given by:

$$q_{Ai} = \max\{1 - p_{Ai}, 0\} \mathbb{T}(p_{Ai}, p_{Aj}, w_{Ai}, w_{Aj})$$

$$q_{Bi} = \mathbb{I}\{p_{Bi} \leq b\} \mathbb{T}(p_{Bi}, p_{Bj}, w_{Bi}, w_{Bj})$$

where

$$\mathbb{T}(p_{ki}, p_{kj}, w_{ki}, w_{kj}) = \begin{cases} 
1 & \text{if } [p_{ki} < p_{kj}] \cup [p_{ki} = p_{kj}] \cap [w_{ki} < w_{kj}] \\
1/2 & \text{if } [p_{ki} = p_{kj}] \cap [w_{ki} = w_{kj}] \\
0 & \text{otherwise}
\end{cases}$$

for $k = A, B, AB$.

Obtaining demands from the second group, those who value both goods but must one-stop shop, is more involved since their option set expands to $k = A, B, AB$. Let $u_1^*(v_A)$ denote the maximum utility that a consumer from this group with valuation $v_A$ can obtain when shopping at $i = 1, 2$:

$$u_1^*(v_A) = \max\{v_A + b - p_{ABi}, v_A - p_{Ai}, b - p_{Bi}\}$$

This maximum utility comes from purchasing either $B$, or $A$ and $B$, but never just $A$. To see the latter, suppose the contrary: (i) $v_A - p_{Ai} > v_A + b - p_{ABi}$ and (ii) $v_A - p_{Ai} > b - p_{Bi}$. From (i) we have that $p_{Ai} + b < p_{ABi}$, and using $p_{Bi} \leq b$, from the fact that $\max\{w_{B1}, w_{B2}\} \leq b$, we arrive at $p_{Ai} + p_{Bi} < p_{ABi}$, a contradiction. Given this, a consumer in this second group would either purchase $B$ for $p_{AB}$, or $A$ and $B$ for $p_{AB}$, with the indifferent consumer being that with valuation $v_A = p_{AB} - p_B$. From here, we can readily obtain the retail demands:

$$q_{Bi} = \max\{p_{AB} - p_{Bi}, 0\} \mathbb{T}(p_{Bi}, p_{ Bj}, w_{Bi}, w_{Bj})$$

$$q_{ABi} = \max\{1 - p_{ABi} + p_B, 0\} \mathbb{T}(p_{ABi}, p_{ABj}, w_{ABi}, w_{ABj})$$

where $w_{ABi} \equiv w_{Ai} + w_{Bi}$.

Obtaining demands from the third group, those who value both goods and can shop costlessly, requires consideration of an additional possibility relative to the previous case: the fact that some consumers may buy $A$ and $B$ from different retailers whenever $p_A + p_B < p_{AB}$. Following the same reasoning as before, consumers in this group will also buy either $B$, or $A$ and $B$. Thus, depending on whether the indifferent consumer between these two options is $v_A = p_{AB} - p_B$, when $p_{AB} \leq p_A + p_B$, or
Given the demands specified above, Ri’s profits can be written as:

\[
\pi_i = (p_{Ai} - w_{Ai})[(1 - \mu)\hat{q}_{Ai} + \mu(1 - z)\hat{q}_{Ai}]
+ (p_{Bi} - w_{Bi})[(1 - \mu)\hat{q}_{Bi} + \mu z \hat{q}_{Bi} + \mu(1 - z)\hat{q}_{Bi}]
\]

\[
(p_{ABI} - w_{AI} - w_{B1})[\mu z \hat{q}_{ABI} + \mu(1 - z)\hat{q}_{ABI}]
\]

**Appendix B  Pricing Equilibria: Baseline Model**

In this appendix, we present the lemmas that characterize the retail pricing equilibrium of our baseline model. Notice that Lemmas 1 and 2 in the main text are particular cases of Lemmas B.1 and B.4, respectively.

**Lemma B.1.** If \(w_{A2} \leq w_{A1}\) and \(w_{B2} \leq w_{B1}\), then there is an essentially unique\(^{51}\) pure-strategy equilibrium. There is no retail bundling and equilibrium standalone prices are \(p^*_{A1} = w_{A1}\), \(p^*_{A2} = \min\{w_{A1}, (1 + w_{A2})/2\}\), and \(p^*_{B1} = p^*_{B2} = w_{B1}\). Retailers’ payoffs are then \(\pi^*_{R1} = 0\) and

\[
\pi^*_{R2} = \begin{cases} 
(1 - w_{A2})^2/4 + w_{B1} - w_{B2} & \text{if } (1 + w_{A2})/2 \leq w_{A1} \\
(w_{A1} - w_{A2})(1 - w_{A1}) + w_{B1} - w_{B2} & \text{otherwise}
\end{cases}
\]

**Lemma B.2.** If \(w_{A2} > w_{A1}\), \(w_{B2} \leq w_{B1}\) and \(w_{A2} + w_{B2} \leq w_{A1} + w_{B1}\), then there exists a pure-strategy equilibrium characterized by prices \(p^*_{A1} = \min\{w_{A2}, (1 + w_{A1})/2\}\), \(p^*_{A2} = w_{A2}\), \(p^*_{B1} = p^*_{B2} = w_{B1}\), \(p^*_{AB1} = p^*_{AB2} = w_{A1} + w_{B1}\), and payoffs:

\[
\pi^*_{R1} = \begin{cases} 
(1 - \mu)(1 - w_{A1})^2/4 & \text{if } (1 + w_{A1})/2 \leq w_{A2} \\
(1 - \mu)(w_{A2} - w_{A1})(1 - w_{A2}) & \text{otherwise}
\end{cases}
\]

and \(\pi^*_{R2} = (w_{B1} - w_{B2}) - \mu(w_{A2} - w_{A1})(1 - w_{A1})\). Furthermore, if \(z > 0\) this equilibrium is essentially unique.\(^{52}\)

**Lemma B.3.** If \(w_{A1} + w_{B1} - w_{B2} \leq w_{A2} \leq (1 + w_{A1})/2 + w_{B1} - w_{B2}\), and \(w_{B2} \leq w_{B1}\), then there is an essentially unique pure-strategy equilibrium. Equilibrium prices are given by \(p^*_{A1} = \min\{w_{A2}, (1 + w_{A1})/2\}\), \(p^*_{A2} = w_{A2}\), \(p^*_{B1} = p^*_{B2} + \epsilon = w_{B1}\) (with \(\epsilon \rightarrow 0\)), and \(p^*_{AB1} = p^*_{AB2} = w_{A2} + w_{B2}\). Retailers’

\(^{51}\)When necessary, we apply the usual refinement in asymmetric Bertrand environments, ruling out equilibria in which a retailer prices a product below cost but expects to sell no units of it. We then say that an equilibrium is essentially unique if all equilibria surviving the refinement above give to retailers and to each group of final consumers the same profits and consumer surplus, respectively.

\(^{52}\)If \(z = 0\) and \(\mu > 0\) then there are multiple equilibria; \(z > 0\) with \(z \rightarrow 0\), however, suffices to guarantee uniqueness.
equilibrium profits are then

\[ \pi_{R1}^* = \begin{cases} 
(1 - \mu)(1 - w_{A1})^2/4 + \Gamma & \text{if } (1 + w_{A1})/2 \leq w_{A2} \\
(1 - \mu)(w_{A2} - w_{A1})(1 - w_{A2}) + \Gamma & \text{otherwise}
\end{cases} \]

and \( \pi_{R2}^* = (w_{B1} - w_{B2})(1 - \mu(1 - w_{A2} - w_{B2} + w_{B1})) \), where \( \Gamma \equiv \mu(w_{A2} + w_{B2} - w_{A1} - w_{B1})(1 - w_{A2} - w_{B2} + w_{B1}) \)

**Lemma B.4.** If \( (1 + w_{A1})/2 + w_{B1} - w_{B2} \leq w_{A2} \), and \( w_{B2} \leq w_{B1} \), then there is essentially unique pure-strategy equilibrium characterized by prices \( p_{A1}^* = (1 + w_{A1})/2, p_{A2}^* = w_{A2}, p_{B1}^* = p_{B2}^* + \epsilon = w_{B1} \) (with \( \epsilon \to 0 \)), \( p_{AB1}^* = p_{A1}^* + p_{B1}^* \), and \( p_{AB2}^* = p_{A2}^* + p_{B2}^* \), and payoffs \( \pi_{R1}^* = (1 - w_{A1})^2/4 \) and \( \pi_{R2}^* = (w_{B1} - w_{B2})(1 - \mu z(1 - w_{A1})/2) \).

### Appendix C Pricing Equilibria: “Home Advantage” Model

In this appendix, we provide all the lemmas that characterize the retail equilibrium in the model examined in Section 4. The equilibrium follows closely the spirit of Lemmas B.1-B.4 characterizing the retail equilibrium in the baseline model. Finally, notice that Lemma C.1 is identical to Lemma 3 (we are stating it here as well for completeness), while Lemma 4 is a particular case of Lemma C.4. For the proofs, see the online Appendix.

**Lemma C.1.** Consider location \( m = i \). If \( w_{Ai} \leq w_{Aj} + \gamma \), there is no retail bundling in equilibrium. Retail prices are then given by \( p_{Ai}^{(i)} = \min\{w_{Aj} + \gamma, (1 + \gamma + w_{Ai})/2\}, p_{Aj}^{(i)} = w_{Aj}, p_{Bi}^{(i)} = c + \gamma, p_{Bj}^{(i)} = c \). Retailers’ equilibrium profits at this retail location are then:

\[ \pi_{R1}^{(i)*} = \begin{cases} 
(1 + \gamma - w_{Ai})^2/4 + \gamma & \text{if } (1 + \gamma + w_{Ai})/2 \leq w_{Aj} + \gamma \\
w_{Aj} - w_{Ai} + \gamma(1 - w_{Aj}) + \gamma & \text{otherwise}
\end{cases} \]

and \( \pi_{R2}^{(i)*} = 0 \).

**Lemma C.2.** Consider location \( m = i \). If \( w_{Aj} + \gamma < w_{Ai} \leq w_{Aj} + 2\gamma \), retail equilibrium prices are given by \( p_{Ai}^* = w_{Ai}, p_{Aj}^* = \min\{w_{Ai} - \gamma, (1 + w_{Aj})/2\}, p_{Bi}^* = c + \gamma, p_{Bj}^* = c, p_{ABi}^* = w_{Aj} + c + 2\gamma \), and \( p_{ABj}^* = w_{Aj} + c \). Retailers’ equilibrium profits at this retail location are then \( \pi_{R1}^* = \gamma - \mu(w_{Ai} - w_{Aj} - \gamma)(1 - w_{Aj}) \) and

\[ \pi_{R2}^* = \begin{cases} 
(1 - \mu)(1 - w_{A1})^2/4 & \text{if } (1 + w_{A1})/2 \leq w_{A2} \\
(1 - \mu)(w_{Ai} - w_{Aj} - \gamma)(1 - w_{Ai} + \gamma) & \text{otherwise}
\end{cases} \]

**Lemma C.3.** Consider location \( m = i \). If \( w_{Aj} + 2\gamma < w_{Ai} \leq (1 + w_{Aj})/2 + 2\gamma \), equilibrium retail prices are given by \( p_{Ai}^* = w_{Ai}, p_{Aj}^* = \min\{w_{Ai} - \gamma, (1 + w_{Aj})/2\}, p_{Bi}^* = c + \gamma, p_{Bj}^* = c, p_{ABi}^* = w_{Ai} + c \), and \( p_{ABj}^* = w_{Ai} + c - 2\gamma \). Retailers’ equilibrium profits at this retail location are then \( \pi_{R1}^* = \gamma[1 - \mu(1 - w_{Ai} + 2\gamma)] \) and

\[ \pi_{R2}^* = \begin{cases} 
(1 - \mu)(1 - w_{A1})^2/4 + \tilde{\Gamma} & \text{if } (1 + w_{A1})/2 \leq w_{A2} - \gamma \\
(1 - \mu)(w_{Ai} - w_{Aj} - \gamma)(1 - w_{Ai} + \gamma) + \tilde{\Gamma} & \text{otherwise}
\end{cases} \]

where \( \tilde{\Gamma} \equiv \mu(w_{Ai} - w_{Aj} - 2\gamma)(1 - w_{Ai} + 2\gamma) \).
Lemma C.4. Consider location \( m = i \). If \((1 + w_{A_B})/2 + 2\gamma < w_{A_i}\), there is no retail bundling in equilibrium. Retail prices are then given by \( p_{A_i}^{(i)*} = w_{A_i}, p_{A_B}^{(i)*} = (1 + w_{A_B})/2, p_{B_i}^{(i)*} = c + \gamma - \epsilon \) (with \( \epsilon \to 0 \)), and \( p_{B_j}^{(i)*} = c \). Retailers’ equilibrium profits at this location are then \( \pi_{R_i}^{(i)*} = \gamma(1 - \mu z(1 - w_{A_j})/2) \) and \( \pi_{R_j}^{(i)*} = (1 - w_{A_j})^2/4 \).

Appendix D  Proofs for Section 2

Proof of Lemmas 1 and 2

Preliminaries. We begin by introducing an upper bound on equilibrium profits that will be instrumental for the proofs that follow. Let \((p_{A_i}^*, p_{B_i}^*, p_{A_B}^*)_{i=1,2}\) be a pure-strategy equilibrium of the game, and \((\pi_{R1}^*, \pi_{R2}^*)\) the corresponding equilibrium profits.

Now, fix \( R_j \)'s equilibrium prices and consider an auxiliary setting where \( R_i \) (and only \( R_i \)) can discriminate between the different groups of consumers; that is, she is able to charge (i) prices \((s_{A_i}, s_{B_i})\) to those consumers interested in only one product, (ii) prices \((x_{A_i}, x_{B_i}, x_{A_B})\) to those interested in both products who face no shopping costs, and (iii) prices \((y_{B_i}, y_{A_B})\) to those interested in both products but are forced to one-stop shop.\(^{53}\)

Define then:

\[
\hat{\theta}_{R_i}^* = \max_{(s_{A_i}, s_{B_i})} \left\{ (s_{A_i} - w_{A_i})(1 - s_{A_i}) \mathbb{I}_{\{s_{A_i} \leq p_{A_i}^*\}} + (s_{B_i} - w_{B_i}) \mathbb{I}_{\{s_{B_i} \leq p_{B_i}^*\}} \right\}
\]

\[
\hat{\theta}_{R_i}^* = \max_{(y_{A_B})} \left\{ (y_{A_B} - w_{A_i} - w_{B_i})(1 - y_{A_B}) + \min(y_{A_B}, p_{B_i}^*) \right\} \mathbb{I}_{\{y_{A_B} \leq p_{A_B}^*\}} + (y_{B_i} - w_{B_i}) \min\{y_{A_B}, p_{A_B}^*\} - y_{B_i} \mathbb{I}_{\{y_{B_i} \leq p_{B_i}^*\}} \right\}
\]

\[
\hat{\theta}_{R_i}^* = \max_{(x_{A_i}, x_{B_i}, x_{A_B})} \left\{ (x_{A_i} - w_{A_i})(1 - x_{A_i}) \mathbb{I}_{\{x_{A_i} \leq p_{A_i}^*\}} \mathbb{I}_{\{x_{A_i} + \min(x_{B_i}, p_{B_i}^*) < \min(x_{A_B}, p_{A_B}^*)\}} + (x_{B_i} - w_{B_i})(1 - \mathbb{I}_{\{\min(x_{A_B}, p_{A_B}^*) \leq \min(x_{A_i}, p_{A_i}^*) + x_{B_i}\}} \min(x_{A_B}, p_{A_B}^*) + x_{B_i}) \mathbb{I}_{\{x_{B_i} \leq p_{B_i}^*\}} + (x_{A_B} - w_{A_i} - w_{B_i})(1 - x_{A_B} + \min(x_{B_i}, p_{B_i}^*) \mathbb{I}_{\{x_{A_B} \leq \min(x_{A_i}, p_{A_i}^*) + \min(x_{B_i}, p_{B_i}^*)\}} \mathbb{I}_{\{x_{A_B} \leq \min(x_{A_i}, p_{A_i}^*) + \min(x_{B_i}, p_{B_i}^*)\}} \right\}
\]

where, for simplicity, we omit the tie-breaking function \( T(\cdot) \) used in Appendix A. Clearly, \( \pi_{R_i}^* \leq (1 - \mu)\hat{\theta}_{R_i}^* + \mu(1 - z)\hat{\theta}_{R_i}^* + \mu z \hat{\theta}_{R_i}^* \equiv \hat{\pi}_{R_i}^* \) for any \((p_{A_i}^*, p_{B_i}^*, p_{A_B}^*)\), since we have expanded \( R_i \)'s action space to a larger set of prices. Therefore, if \( R_i \)'s equilibrium profits reach \( \hat{\pi}_{R_i}^* \), it must be that \( R_i \) has no incentives to deviate.

Proof of Lemma 1. Here we will only show that the outcome stated in the lemma is indeed an equilibrium. The proof for uniqueness follows that in the proof in Lemma B.1. Given that \( R_2 \) is charging less than or equal to \( R_1 \)'s unit costs for \( k = A, B, AB \), it is easy to see that \( R_1 \) has no strictly profitable deviation.

To see that \( R_2 \) has no incentive to deviate either, note that \( R_2 \)'s best response to \((p_{A_i}^*, p_{B_i}^*, p_{A_B}^*)\) in

\(^{53}\)Note that it suffices to consider only \((y_{B_i}, y_{A_B})\) for one-stop shoppers instead of \((y_{A_i}, y_{B_i}, y_{A_B})\), given that their choice is between buying \( B \) or \( AB \), never \( A \) alone. That is not true, however, for two-stop shoppers, as they might purchase \( A \) from one retailer and \( B \) from the other (hence, even if the consumer ends up purchasing \( AB \) or only \( B \), \( R_i \) might still be selling standalone units of \( A \).
the auxiliary setting, which is given by \( s_{A2}^* = x_{A2}^* = w_A, s_{B2}^* = x_{B2}^* = y_{B2}^* = w_B, \) and \( x_{AB2}^* = y_{AB2}^* = w_A + w_B, \) leads to an auxiliary payoff of \( \theta_{R2}^* = \theta_{R2} = \tilde{\theta}_{R2}^* = \tilde{\pi}_{R2}^*, \) which is equal to \( \pi_{R2}^*. \)

**Proof of Lemma 2.** Here we will only show that the outcome stated is indeed an equilibrium. The proof for uniqueness follows that in the proof in Lemma B.4. To check that \( R1 \) does not have incentives to deviate, we show that \( \pi_{R1}^* = \tilde{\pi}_{R1}. \) Using \( p_{B2}^* = w_B - \epsilon, \) we obtain \( s_{A1}^* = x_{A1}^* = (1 + w_A)/2, \)

\[
\begin{align*}
\hat{s}_{B1}^* &= \hat{x}_{B1}^* = y_{B1}^* \in [w_B, \infty), \\
x_{AB1}^* &\in [x_{A1}^* + x_{B1}^*, \infty), \\
y_{AB1} &= (1 + w_A)/2 + w_B.
\end{align*}
\]

Thus \( \hat{\theta}_{R1}^* = \hat{\theta}_{R1} = \hat{\theta}_{R1} = (1 - w_A)^2/4, \) and therefore \( \tilde{\pi}_{R1}^* = \pi_{R1}^*. \)

Consider now \( R2 \)'s optimal response to \( R1 \)'s equilibrium prices

\[
\max_{p_{B2} - w_{B2}} \left[(1 - \mu) + \mu (1 - z) 1\{p_{B2} < p_{A1}^* - p_{A3}^* \} + (p_{AB2}^* - p_{B2}) (\mu z + \mu (1 - z) 1\{p_{B2} > p_{A3}^* - p_{A1}^* \}) \right] 1\{p_{B2} < p_{B2}^*\}
\]

from which we obtain \( p_{B2}^* = w_B - \epsilon, \) with \( \epsilon \to 0 \) (this latter is needed to attract the two-stop shoppers interested in both products, and where we are using the fact that \( w_B - w_{B2} \leq b \leq 1/2, \) yielding a profit of \( (w_B - w_{B2}) [1 - \mu z (1 - w_A)^2/2]. \) Hence, given \( R1 \)'s equilibrium prices, \( R2 \) is playing optimally.

**Proof of Proposition 1**

First we need to show that if \( \mu z > 0, \) then we can always find \( \mathbf{w} = (w_{A1}, w_{A2}, w_{B1}, w_{B2}) \) such that \( A \) qualifies as must-have for at least one retailer. This is simple. As described in the text, when \( w_{A1} = w_{A2} \) and \( w_{B1} < w_{B2}, \) \( A \) qualifies as must-have for \( R_i. \)

Second, we need to show that if a \( \mathbf{w} \) exists such that \( A \) qualifies as must-have for at least one retailer, then \( \mu z > 0. \) We prove the latter by proving the contrapositive: if \( \mu z = 0, \) then such set \( \mathbf{w} \) does not exist.

Without loss of generality assume \( w_{B2} \leq w_{B1}. \) Consider first the case in which \( w_{B1} = w_{B2}. \) Regardless of whether \( w_{A1} > w_{A2} \) or \( w_{A1} \leq w_{A2}, \) retailers will price \( A \) and \( B \) according to Lemma B.1. So, if \( A \) is removed from either retailer’s lineup, Lemma B.4 shows no profit loss from selling \( B \) for either retailer; hence, \( A \) cannot qualify as must-have for either of them.

Consider now the case in which \( w_{B2} < w_{B1}. \) Since \( \mu z = 0, \) we know from Lemma B.4 that if \( A \) is removed from \( R2 \)'s lineup, \( R2 \) would continue selling all units of \( B; \) hence, \( A \) cannot classify as must-have for \( R2. \) Showing the same for \( R1 \) is more involved. Note that for \( A \) to qualify as must-have for \( R1, R1 \) must be selling strictly positive units of \( B \) before \( A \)'s removal. We will show, however, in that case, \( A \)'s marginal profit contribution for \( R2 \) must be negative, going against our definition of must-have, which requires marginal contributions to be non-negative for both retailers.

Indeed, when \( w_{B2} < w_{B1} \) then \( R1 \) sells strictly positive units of \( B \) only when \( w_{A1} + w_{B1} - w_{B2} \leq w_{A2} < (1 + w_A)/2 + w_{B1} - w_{B2}; \) that is, when the retail pricing equilibrium is characterized by Lemma B.3. But in such a Lemma \( R2 \) would be strictly better off not having to carry \( A. \) Doing so increases the equilibrium price of the bundle, \( p_{AB1}^* \), from \( w_{A2} + w_{B2} \) to \( (1 + w_A)/2 + w_{B1} \) (Lemma B.4), and with that, \( R2 \)'s profits, as they are strictly increasing in \( p_{AB1}^* \) since she is, in any case, selling \( B \) units only in stand-alone fashion. Hence, under this configuration of wholesale prices, \( A \)'s marginal contribution to \( R2 \)'s profits is strictly negative. Consequently, when \( \mu z = 0, \) there is no combination of wholesale prices \( \mathbf{w} = (w_{A1}, w_{A2}, w_{B1}, w_{B2}) \) such that \( A \) classifies as must-have.
Appendix E  Selected Proofs for Section 3

Proof of Proposition 2

Claim E.1. In any equilibrium \( \pi_M^* = 1/4 \) and both retailers procure \( B \) from the fringe.

Proof First, we show that \( \pi_M^* \geq 1/4 \). Indeed, if \( M \) approaches both retailers with the linear-price contract \((w_A^M = 1/2, T_A^M = 0)_{i=1,2}\) and no offer for \( B \), both retailers will accept \( M \)’s offers for \( A \), and source \( B \) from the fringe. Their wholesale prices will then be \( w_{A1} = w_{A2} = 1/2 \) and \( w_{B1} = w_{B2} = c \), so, by Lemma 1 of Section 2 (or equivalently, Lemma B.1 of Appendix B), retail equilibrium prices would be \( p_{A1}^* = 1/2, p_{B1}^* = c \). It then immediately follows that \( M \) obtains a profit of 1/4.

To show that \( \pi_M^* \leq 1/4 \), let \( w_{B1} \) and \( w_{B2} \) be retailers’ equilibrium marginal costs for \( B \) after the contracting stage has ended, and without loss of generality, set \( w_{B1} \geq w_{B2} \). Note that a retailer can always reject \( M \)’s offers for \( A \) and simply purchase \( B \) from the fringe. Hence, from Lemma B.4 we have:

\[
\pi_{Ri}^* \geq \max\{0, w_{Bj} - c\} \quad \text{for} \ i = 1, 2 \text{ and } j \neq i
\]

Furthermore, inspecting Lemmas B.1-B.4, we conclude that \( p_{B1}^* = p_{B2}^* = \max\{w_{B1}, w_{B2}\} = w_{B1} \). Hence, in any equilibrium, \( \pi_M^* + \pi_{R1}^* + \pi_{R2}^* \) must be less than or equal to \( \Pi^* \), the maximum industry profits subject to \( p_{B1}^* = \min\{p_{B1}^*, p_{B2}^*\} = w_{B1} \). Based on the retail demands presented in section 2.2, that \( A \)’s cost of production is zero and the most efficient source of \( B \) is the fringe, industry profits can be written as:

\[
\Pi = (1 - \mu)p_A^*(1 - p_A^*) + (1 - \mu)(p_B - c) \\
+ \mu z(p_{AB} - c)(1 - p_{AB} + p_B) + \mu z(p_B - c)(p_{AB} - p_B) \\
+ \mu(1 - z)(\tilde{p}_{AB} - c)(1 - \tilde{p}_{AB} + \tilde{p}_B) + \mu(1 - z)(p_B - c)(\tilde{p}_{AB} - \tilde{p}_B)
\]

(14)

where, as in Section 2.2, \( p_k = \min\{p_{k1}, p_{k2}\} \), where \( k = A, B \), \( AB \) and \( \tilde{p}_{AB} = \min\{p_A + p_B^*, p_{AB}\} \). The first line shows the contribution from the two groups of consumers who value only one good, while the next two lines show the contributions from the two groups that value both goods: the one-stop shoppers (line 2) and the multi-stop shoppers (line 3).

Maximizing (14) subject to the constraint \( p_B \leq p_B^* = w_{B1} \), yields \( p_A^* = 1/2, p_B^* = w_{B1} \), and \( p_{AB} = \tilde{p}_{AB} = 1/2 + w_{B1} \), resulting in \( \Pi^* = 1/4 + w_{B1} - c \). Finally, because \( \pi_{Ri}^* \geq \max\{0, w_{Bj} - c\} \), we then have that \( \pi_M^* \leq \Pi^* - \pi_{R1}^* - \pi_{R2}^* \leq 1/4 + w_{B1} - c - (w_{B1} - c) = 1/4 \). Hence, \( \pi_M^* \leq 1/4 \), and therefore \( \pi_M^* = 1/4 \). But the latter immediately implies that \( w_{B1} = w_{B2} = c \); that is, both retailers must be procuring \( B \) from the fringe. Otherwise, there exists a retailer \( i \) such that \( \pi_{Ri}^* > 0 \), which would imply that \( \pi_M^* < 1/4 \), a contradiction. ■

Claim E.2. In any equilibrium (i) \( A \) will be sold downstream at a price of \( 1/2 \); (ii) there cannot be any retail bundling; and (iii) retailers get zero profits.

Proof Since \( w_{B1} = w_{B2} = c \), the retail pricing equilibrium is necessarily given by Lemma B.1, which immediately implies there will not be any retail bundling. Moreover, for \( \pi_M^* = 1/4 \), we must have that \( A \) is being sold downstream at a price of \( 1/2 \), and that retailers are getting zero profits. ■

The latter implies that the equilibrium, if it exists, will be essentially unique (i.e., profits and surplus accrued by manufacturers, retailers, and consumers are the same across all equilibria). Hence, to prove
this proposition, we have only to prove that an equilibrium of the game exists. That is easy: if $M$ offers both retailers the non-discriminatory contract
\[
(w^M_{A_i} = 1/2, T^M_{A_i} = 0)
\]
for product $A$ and no contract for $B$, then both retailers will accept $M$’s contract for $A$, and also the fringe’s offer for $B$. This leads to all necessary conditions for an equilibrium of the game. It is then not difficult to prove that players have no incentives to deviate, so the above constitutes an equilibrium of the game. ■

**Proof of Proposition 3**

Suppose then that, in equilibrium, $R_i$ accepted contracts for $B$ (from either from $M$, the fringe, or both) such that her cost of procuring $B$ is $w_{Bi}$. Because portfolio offers are banned, from the retailers’ perspective $M$’s offers for $A$ are fixed when the retailers are considering which contract for $B$ to take. Hence, retailers will always accept the contract for $B$ that offers the most favorable terms. This immediately implies that $w_{Bi} \leq c$ for $i = 1, 2$, as a retailer has always the option to procure $B$ from the fringe at $c$. But if so, then $\max\{w_{B1}, w_{B2}\} \leq c$, and therefore $\min\{p^*_B1, p^*_B2\} \leq \max\{w_{B1}, w_{B2}\} \leq c$, irrespective of $M$’s offers for good $A$. Finally, since the fringe is more efficient than $M$ in producing $B$, it is still true that $\Pi^1$, the maximum industry profits subject to $\min\{p^*_B1, p^*_B2\} \leq c$, is equal to $1/4$. Hence, when portfolio offers are banned, we have that in any equilibrium, $\pi^*_M = 1/4$ and both retailers procure $B$ from the fringe. From hereon, the proof follows the same steps as the proof of Proposition 2. ■

**Appendix F Selected Proofs for Section 4**

**Proof of Proposition 6**

The proof is organized as follows: in Part 1, we show that $M$-R1 deals with R2 if and only if $\gamma \geq (5 - \sqrt{7})/9 \equiv \gamma_1$. In Part 2, we show that the IBL makes $w_{A2}$ increase above its pre-merger level. And in Part 3, we cover the EDM effects.

**Part 1.** Take some $\gamma > 0$ and define $W(\gamma)$ as the set of all $w_{A2} \geq 0$ for which an agreement between $M$-R1 and R2 is possible, that is, the set of all $w_{A2} \geq 0$ that simultaneously satisfy: (i) $\pi_M(w_{A1} = 0, w_{A2}; \gamma) > \pi_M(w_{A1} = 0, w_{A2} \to +\infty; \gamma)$ and (ii) $\pi_{R2}(w_{A1} = 0, w_{A2}; \gamma) > \pi_{R2}(w_{A1} = 0, w_{A2} \to +\infty; \gamma)$.

We want to find
\[
\gamma = \inf\{\gamma > 0 : W(\gamma) \neq \emptyset\}
\]
In doing so, let’s conjecture that $\gamma < 1$, so we can concentrate on $\gamma \in (0, 1)$. From Lemmas 3, 4, C.2 and C.4, we know that:
\[
\pi_{R2}(0, w_{A2}; \gamma) = \begin{cases} 
\gamma/2 + (\gamma - w_{A2})/2 & \text{if } w_{A2} \leq \gamma \\
< \gamma/2 & \text{if } \gamma < w_{A2} < 1/2 + 2\gamma \\
\gamma/2 & \text{if } 1/2 + 2\gamma \leq w_{A2}
\end{cases}
\]
Since $\pi_{R2}(0, \gamma; \gamma) = \pi_{R2}(0, +\infty; \gamma)$, we have that $W(\gamma) \subseteq [0, \gamma)$, so from now on we can restrict attention
to \( w_{A2} \in [0, \gamma) \) (i.e., Lemma 3). We then have

\[
\pi_M(0, w_{A2}; \gamma) = \begin{cases} 
(w_{A2} + \gamma)(2 - w_{A2})/2 & \text{if } w_{A2} < (1 - \gamma)/2 \\
(1 + \gamma)^2/8 + (w_{A2} + \gamma)/2 & \text{if } (1 - \gamma)/2 \leq w_{A2} < \gamma
\end{cases}
\]

where the region \((1 - \gamma)/2, \gamma)\) is non-empty if and only if \( \gamma > 1/3 \).

From the fact that when \( w_{A2} = \gamma \), \( R2 \)'s payoff is exactly the same as if she did not trade with \( M \), we can state that:

**Claim F.1.** \( W(\gamma) \neq \emptyset \) if and only if \( \pi_M(0, \gamma; \gamma) > \pi_M(0, +\infty; \gamma) \)

**Proof.** Start by noting the following property \((P)\) for all \( w_{A2} \in [0, \gamma)\): (i) \( \pi_M(0, w_{A2}; \gamma) \) is strictly increasing and continuous in \( w_{A2} \) and (ii) \( \pi_{R2}(0, w_{A2}; \gamma) \) is strictly decreasing and continuous in \( w_{A2} \).

\((\implies)\) If \( W(\gamma) \neq \emptyset \), then there exist some \( w'_{A2} \geq 0 \) such that \( \pi_M(0, w'_{A2}; \gamma) > \pi_M(0, +\infty; \gamma) \) and \( \pi_{R2}(0, w'_{A2}; \gamma) > \pi_{R2}(0, +\infty; \gamma) \). But the latter implies \( w'_{A2} < \gamma \), which, in turn, and together with \((P)\)(i), implies that \( \pi_M(0, \gamma; \gamma) > \pi_M(0, w'_{A2}; \gamma) > \pi_M(0, +\infty; \gamma) \).

\((\impliedby)\) If \( \pi_M(0, \gamma; \gamma) > \pi_M(0, +\infty; \gamma) \), then, from \((P)\)(i) and \((P)\)(ii) we can always find some \( \epsilon > 0 \) but sufficiently small, such that \( \pi_M(0, \gamma - \epsilon; \gamma) > \pi_M(0, +\infty; \gamma) \) and \( \pi_{R2}(0, \gamma - \epsilon; \gamma) > \pi_{R2}(0, +\infty; \gamma) \).

Consequently, Claim F.1 implies that \((16)\) is given by the minimum \( \gamma \) such that:

\[
\pi_M(0, \gamma; \gamma) > \pi_M(0, +\infty; \gamma) \tag{17}
\]

where \( \pi_M(0, +\infty; \gamma) = (1 + \gamma)^2/8 + \gamma/2 + 1/8 \). To find this minimum, we split our search into two regions. The first region is \( \gamma \geq 1/3 \). In this region, \((17)\) holds if and only if \( \gamma > 1/4 \). But since \( \gamma \geq 1/3, \gamma_1 = 1/3 \). The second region is \( \gamma < 1/3 \). Since in this region \( \gamma < (1 - \gamma)/2 \), we have that \((17)\) holds if and only if \( \gamma(2 - \gamma) > \pi_M(0, +\infty; \gamma) \); that is, if and only if \( \gamma > \gamma_2 = (5 - \sqrt{7})/9 \). Thus, \( \gamma = \min\{\gamma_1, \gamma_2\} = (5 - \sqrt{7})/9 \). We conclude this part by verifying that our conjecture that \( \gamma < 1 \) was indeed true.

**Part 2.** Suppose that after the merger \( w_{A1} \) remains at its pre-merger level \( w_A^* \) and that \( M \) internalizes a fraction \( \kappa \in (0, 1] \) of \( R1 \)'s profits (eventually we will let \( \kappa \) go to unity). We know from \((3)\) that the post-merger negotiation between \( M \) and \( R2 \) is governed by the first-order condition:

\[
f(w_{A1}, w_{A2}; \kappa) = \frac{\beta}{\Delta \pi_M} \frac{\partial \pi_M}{\partial w_{A2}} + \frac{1 - \beta}{\Delta \pi_{R2}} \frac{\partial \pi_{R2}}{\partial w_{A2}} \tag{18}
\]

where \( \Delta \pi_j = \pi_j - \pi_{\bar{j}} \) for \( j \in \{M, R2\} \). Our goal here is to demonstrate that \( w_{A2}^{BL} > w_A^* \), where \( f(w_A^*, w_{A2}^{BL}; 1) = 0 \).

Since before the merger, retailers competed in a Bertrand manner for product \( A \), there will be a value of \( \kappa \), say \( \bar{\kappa} \in (0, 1] \), below which the post-merger competition for product \( A \) in both markets will still follow a Bertrand-pricing logic, which requires that \((1 + w_{Ai} + \gamma)/2 > w_{Aj} + \gamma \) in location \( i = 1, 2 \)
and \( j \neq i \). In this case, payoffs (5) and (7) change to:

\[
\pi_M = w_A^1(1 - w_{A2})/2 + w_{A2}(1 - w_A^*)/2 + \kappa[(w_{A2} - w_A^*)/(1 - w_{A2})/2 + \gamma/2
\]

\[
\bar{\pi}_M = w_A^1(1 - w_A^*)/4 + w_A^*(1 + \gamma - w_A^*)/4 + \kappa[(1 + \gamma - w_A^*)^2/8 + (1 - w_A^*)^2/8 + \gamma/2]
\]

Letting \( \kappa < \bar{\kappa} \) and differentiating (18) with respect to \( w_{A2} \) yields:

\[
\frac{\partial f(\cdot)}{\partial w_{A2}} = -\frac{\beta \kappa}{\Delta \pi_M} - \frac{\beta}{(\Delta \pi_M)^2} \left( \frac{\partial \pi_M}{\partial w_{A2}} \right)^2 - \frac{1 - \beta}{(\Delta \pi_M)^2} \left( \frac{\partial \pi_{R2}}{\partial w_{A2}} \right)^2 < 0
\]

A decreasing \( f(\cdot) \) implies that \( f(\cdot) \) crosses zero only once and from above.

Similarly, differentiating (18) with respect to \( \kappa \) yields

\[
\frac{\partial f(\cdot)}{\partial \kappa} = \frac{\beta}{\Delta \pi_M} \frac{\partial^2 \pi_M}{\partial w_{A2} \partial \kappa} - \frac{\beta}{(\Delta \pi_M)^2} \frac{\partial \pi_M}{\partial w_{A2}} \frac{\partial \Delta \pi_M}{\partial \kappa}
\]

It is easy to see that

\[
\frac{\partial^2 \pi_M}{\partial w_{A2} \partial \kappa} = \frac{1}{2} - w_{A2} + \frac{w_A^*}{2} - \frac{\gamma}{2} > 0
\]

from the Bertrand-pricing assumption that \((1 + w_A^* + \gamma)/2 > w_{A2} + \gamma\), and that

\[
\frac{\partial \Delta \pi_M}{\partial \kappa} = \frac{(w_{A2} - w_A^* + \gamma)(1 - w_{A2})}{2} - \frac{(1 + \gamma - w_A^*)^2}{8} - \frac{(1 - w_A^*)^2}{8} < 0
\]

from the fact that \( \pi_{R1}(w_{A1}, w_{A2}) \leq \pi_{R1}(w_{A1}, +\infty) \). Hence, \( f(\cdot) \) is increasing in \( \kappa \).

But if \( f(\cdot) \) is increasing in \( \kappa \) and crosses zero only once and from above, then the equilibrium value of \( w_{A2} \), the one that solves \( f(w_{A2}, w_A^*; \kappa) = 0 \) and that we denote by \( w_{A2}^*(w_A^*, \kappa) \), is increasing in \( \kappa \). Therefore, if Bertrand pricing for \( A \) prevails all the way to \( \kappa = 1 \), then we necessarily have that \( w_{A2}^*(w_A^*, \kappa) > w_A^* \), as claimed.

It remains to see what happens if Bertrand pricing for \( A \) in location 1 prevails only up to some \( \kappa < 1 \) (Bertrand pricing continues prevailing in location 2 since \( w_{A2}^*(w_A^*, \kappa) \geq w_A^* \)). If so, we must have:

\[
w_{A2}^{IBL} = w_{A2}^*(\kappa = 1) > \min\{w_A^* + \gamma, (1 + w_A^* - \gamma)/2\}
\]

But \( \min\{w_A^* + \gamma, (1 + w_A^* - \gamma)/2\} > w_A^* \) (recall that under Bertrand pricing, \( w_A^* < 1 - \gamma \)), so again we arrive at \( w_{A2}^{IBL} = w_{A2}^*(\kappa = 1) > w_A^* \).

\textit{Part 3.} The fall of \( w_A^* \) below its pre-merger level is a direct consequence of the merger. Hence we focus on showing that \( \bar{w}_{A2}^{EDM} < w_{A2}^{IBL} \), where \( \bar{w}_{A2}^{EDM} \) is given by \( f(0, w_{A2}^{EDM}; 1) = 0 \) and \( w_{A2}^{IBL} \) is given by \( f(w_A^*, w_{A2}^{IBL}; 1) = 0 \). In order to show this result, we need to consider several cases.

The first case is when \( w_{A2}^{IBL} \) and \( w_A^* \) are such that there is monopoly-pricing for \( A \) in location 1 (i.e., \((1 + w_A^* + \gamma)/2 \leq w_{A2}^{IBL} + \gamma\)) but Bertrand pricing for \( A \) in location 2 (i.e., \( w_A^* + \gamma \leq (1 + w_{A2}^{IBL} + \gamma)/2 \)). Substituting the corresponding payoffs into (18) and solving \( f(w_A^*, w_{A2}^{IBL}; 1) = 0 \), we quickly arrive at

\[
w_{A2}^{IBL} = \beta(w_A^* + \gamma) + (1 - \beta)(1 + w_A^*)/4
\]

\[\text{Note that the additional condition for Bertrand pricing stated in Lemma 6, i.e., } w_{Ai} - \gamma \leq w_{Aj} \text{ for } i = 1, 2 \text{ and } j \neq i, \text{ also holds true since } W(\gamma) \neq 0.\]
Thus, if the solution to \( f(w_{A1}, w_{A2}; 1) = 0 \) remains in the monopoly-pricing region as we reduce \( w_{A1} \) below its pre-merger level all the way to zero, it can be readily seen from (19) that \( w_{A2}^{EDM} = \beta \gamma + (1 - \beta)/4 < w_{A2}^{IBL} \), as claimed. If, however, the solution to \( f(w_{A1}, w_{A2}; 1) = 0 \) falls into the Bertrand-pricing region as we reduce \( w_{A1} \) below its pre-merger level, then it must be true that \( w_{A2}^{EDM} < (1 - \gamma)/2 \). But because \( w_{A2}^{IBL} \) was such that \((1 - \gamma)/2 < (1 + w_{A} - \gamma)/2 \leq w_{A2}^{IBL} \), this corroborates that \( w_{A2}^{EDM} < w_{A2}^{IBL} \).

The second case is when \( w_{A2}^{IBL} \) and \( w_{A}^{*} \) are such that there is Bertrand-pricing for \( A \) in location 1 (i.e., \((1 + w_{A} + \gamma)/2 \leq w_{A2}^{IBL} + \gamma \)) and Bertrand-pricing for \( A \) in location 2. Now, we know that in the Bertrand-pricing region \( w_{A2}^{IBL} \) is the (negative square root) solution to the quadratic equation

\[
\beta(w_{A}^{*} - w + \gamma)(2 - \gamma - 2w - w_{A}^{*}) = (1 - \beta)((w + \gamma)(1 - w) + w(1 - w_{A}^{*}) - (2 + 2\gamma + \gamma^2 - 2(w_{A}^{*})^2)/4)
\]

(20)

where, by Proposition 5, \( w_{A}^{*} \) corresponds to the solution to (4) and \( \gamma < \bar{\gamma} \equiv (1 + \beta)/(1 + 3\beta) \).

As in the previous case, there are two possibilities to address. If \( w_{A2}^{EDM} \) falls in the Bertrand-pricing region, then \( w_{A2}^{EDM} \) can be obtained directly from (20) after making \( w_{A}^{*} = 0 \). In this case, it is not difficult to show that \( w_{A2}^{EDM} < w_{A2}^{IBL} \) for all \( \gamma \in [\gamma, \bar{\gamma}] \) and \( \beta \in (0, 1] \) (when \( \beta = 0 \), \( w_{A2}^{EDM} = w_{A2}^{IBL} \)).

If, on the other hand, \( w_{A2}^{EDM} \) falls in the monopoly-pricing region, then from (19), we have that \( w_{A2}^{EDM} = \beta \gamma + (1 - \beta)/4 \), which restricts \( \gamma \) to be greater than \((1 + \beta)/2(1 + 2\beta) \) (otherwise \( w_{A2}^{EDM} \leq (1 - \gamma)/2 \), a contradiction). It is again not difficult to show that \( w_{A2}^{EDM} < w_{A2}^{IBL} \) for all \( \gamma \in [(1 + \beta)/2(1 + 2\beta), \bar{\gamma}] \) and \( \beta \in [0, 1] \).

**Proof of Proposition 8**

The first three parts of the proof (involving, respectively, \( \gamma(z) \), the IBL effect, and the EDM effect) follow closely those of the proof of Proposition 6, so they are largely omitted here, except to point out that \( \bar{\gamma}(z) = 1/(3 + z) \geq 1/4 \) for all \( z \in [0, 1] \) and \( \gamma(z) = (5 + 2z - \sqrt{7 + 8z + 2z^2})/(3 + z)^2 < \bar{\gamma}(z) \) for all \( z \in [0, 1] \), so \( \gamma(z) = \bar{\gamma}(z) \). Thus, it remains to demonstrate the MHM effect, which is done by formally documenting the clockwise rotation of Figure 4.

Recall that \( w_{A2}^{(\gamma)}(w_{A1}) \) is the solution to \( f(w_{A1}, w_{A2}; 1) = 0 \), where \( f(\cdot) \) is given by (18), when \( R_{2}' \)’s outside option is invariant to \( w_{A1} \), i.e., \( \pi_{(R_{2})}^{(-)} = \gamma/2 - \mu z \gamma(1 - w_{A}^{*})/4 \), and that \( w_{A2}^{(\gamma)}(w_{A1}) \) is the solution to \( f(w_{A1}, w_{A2}; 1) = 0 \) when \( R_{2}' \)’s outside option varies with \( w_{A1} \), i.e., \( \pi_{(R_{2})}^{(+)} = \gamma/2 - \mu z \gamma(1 - w_{A1})/4 \).

Thus, at \( w_{A1} = w_{A}^{*} \), \( \pi_{(R_{2})}^{(-)} = \pi_{(R_{2})}^{(+)} \), \( \pi_{(R_{2})}^{(-)} = \pi_{(R_{2})}^{(+) - \pi_{(R_{2})}^{(-)}} \), so \( w_{A2}^{(-)}(w_{A}^{*}) = w_{A2}^{(+)}(w_{A}^{*}) \). When \( w_{A1} < w_{A}^{*} \), \( \pi_{(R_{2})}^{(+)} > \pi_{(R_{2})}^{(-)} \), and hence, \( \Delta \pi_{R_{2}}^{(+)} \equiv \pi_{R_{2}}^{(+)} - \pi_{R_{2}}^{(-)} \geq \Delta \pi_{R_{2}}^{(-)} \) (and more so the larger \( \mu z \) is). And since \( \partial \pi_{R_{2}}^{(+)}/\partial w_{A2} > 0 \) and \( \partial \pi_{R_{2}}^{(-)}/\partial w_{A2} < 0 \), a jump in \( \Delta \pi_{R_{2}} \) in (18) necessarily implies that \( w_{A2} \) must go up for \( f(w_{A1}, w_{A2}; 1) = 0 \) to continue holding. Finally, when \( w_{A1} > w_{A}^{*} \), \( \pi_{R_{2}}^{(+)} > \pi_{R_{2}}^{(-)} \) and hence, \( \Delta \pi_{R_{2}}^{(+)} < \Delta \pi_{R_{2}}^{(-)} \). This time, a drop in \( \Delta \pi_{R_{2}} \) necessarily implies that \( w_{A2} \) must go down for \( f(w_{A1}, w_{A2}; 1) = 0 \) to continue holding.
References


