BARGAINING MICRO-FOUNDATIONS OF DECENTRALIZED MARKETS, COMPETITION, AND REPUTATION

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Abstract. Existing literature shows that it is possible for rational players to establish one-sided and two-sided reputation in a bilateral bargaining environment by mimicking irrational, $r$-insistent players, and that this reputation build-up can drastically change the Rubinstein (1982) outcome, by causing delay in reaching agreement. Furthermore, the literature on outside options in bilateral bargaining suggests that unless outside options are very large, they do not affect these bargaining outcomes. We are interested in the impact on the bargaining outcome from endogenizing outside options—namely, bargaining in markets. In the presence of competition in decentralized search markets, can rational agents mimic irrationality to build reputation, when can irrational types on both sides of the market trade in equilibrium, and what are the consequences for delay and efficiency? We develop discrete, hybrid and continuous-time models of bargaining in markets with and without irrational types by combining reputational bargaining of Abreu and Gul (2000) with a continuous-time version of the Rubinstein and Wolinsky (1985) model of bargaining in markets. Applications include decentralized search markets for labor, exotic assets, over-the-counter securities, venture capital funding, and repurchase agreements.

1. Introduction & Literature Review

We model non-cooperative, reputational bargaining in markets to analyze under what market conditions irrational agents are able to trade in equilibrium, when rational agents can benefit from mimicking irrational agents, and what causes delay in reaching agreement. By considering bilateral bargaining in a market setting, we are trying to understand the underlying micro-structure of decentralized markets.

Rubinstein (1982) analyzes a discrete-time alternating offer bargaining game and formulates a unique subgame perfect Nash equilibrium, where bargaining is efficient (i.e., there is no delay in reaching agreement). The finite-horizon bargaining game is solved via backwards induction, considering the expected continuation pay-off from rejection; the infinite horizon game is solved by exploiting the stationary structure of the game and finding a unique subgame perfect Nash equilibrium which is stationary. The result underscores the relative impatience of agents (i.e., rate of discounting) as the primary determinant of how much bargaining power an agent has and what fraction of the surplus the agent receives. As both agents get infinitely patient or as time between offers approaches zero, both agents receive 1/2 the surplus. Namely, the more patient agent receives a larger equilibrium share. Furthermore, there is a benefit from being the first proposer; by the second round, the size of the pie has shrunken due to discounting, and thus, the proposer needs to compensate his opponent for less.

A slightly unsettling feature of this otherwise clean and computationally tractable model, when compared to real-life bargaining outcome, is the complete efficiency result. In reality, bargaining often involves delay to reach agreement: workers engage in strikes, and prolonged exchanges of offers and counter-offers are inevitably the defining features of the bargaining process. The focus of bargaining literature has shifted to answering the question: how do we explain delay in bargaining models?

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An important result in the context of our model, was the impact of outside options on the Rubinstein bargaining model. Binmore, Shaked, and Sutton (1985) show that outside options alter the bargaining outcome only if the outside option pay-off is higher than the equilibrium bargaining outcome without outside options (i.e., Rubinstein (1982) equilibrium pay-offs). Since subgame perfect Nash equilibrium refines Nash equilibrium in dynamic models by eliminating non-credible threats, Binmore, Shaked, and Sutton (1985) show that outside options can only be credibly used to leverage one’s bargaining power if the outside option is large enough. If the outside option doesn’t yield a significant pay-off, no threat of rejecting your opponent’s offer and taking the outside option is credible.

Myerson (1991) and Abreu and Gul (2000) introduce behavioral agents in the bargaining model trying to incorporate delay, where rational agents mimic irrational bargaining postures to establish reputation. These models are consistent in that as the probability of irrationality goes to zero, delay and inefficiency vanish.

Myerson (1991) introduces the notion of “$r$-insistent” agents who always propose allocations where they offer fraction $r$ for themselves and reject any proposals which give them less than fraction $r$ of the surplus. Myerson (1991) shows that even if the probability of one’s opponent being a $r$-insistent type is small, one-sided reputation can be built where rational agents mimic the irrational type and successfully get the irrational demand. Thus, one-sided reputation can be built; however, as the bargaining frictions disappear, delay in reaching agreement vanishes.

Abreu and Gul (2000) analyze a model where each agent can be a $r$-insistent type with some probability, and two-sided reputation building can be supported. There is delay in this equilibrium since rational agents mimic irrationality by mixing over whether or not to concede and reveal rationality at each time. The Coase Conjecture is a technical condition which confirms that reputation building is profitable for an agent by ensuring that it is a dominant strategy for a player who reveals rationality, to immediately accept his opponent’s irrational demand if he believes his opponent is irrational with any positive probability. Abreu and Gul (2000) show that the discrete time model converges in continuous time to a war of attrition, where due to the Coase Conjecture condition always being satisfied (since the outside option is 0), the action space for an agent is two dimensional (insist your irrationality or concede to your opponent’s irrational demand). Furthermore, in the limit, this result holds, regardless of the exact bargaining mechanism, as long as offers can be made sufficiently frequently. They also extend the model to include multiple irrational types.

Analogous to Binmore, Shaked, and Sutton (1985) accessing the impact of outside options on non-reputational Rubinstein bargaining, Compte and Jehiel (2002) analyze the impact of outside options on reputation building in Myerson (1991) and Abreu and Gul (2000). Similarly, they find that unless the outside option is extremely attractive (i.e., if the outside option yields a pay-off higher than conceding to irrational opponent’s demand), the presence of outside options does not alter the reputation building and delay persists.

Another strand of bargaining literature looks at endogenizing the outside option\textsuperscript{2} by analyzing bilateral bargaining in markets, where the outside option is the ability to find and bargain with a new partner in the (‘search’) market. Rubinstein and Wolinsky (1985) analyze a model which abstracts from the flows of agents in and out of the market. Once a pair reaches agreement, they permanently exit the market and are replaced by clones. In their discrete-time model, each agent meets a new partner with some probability in each period, after which he forsakes his old partner and starts bargaining with the new partner. If an agent’s partner finds a new partner but the agent doesn’t find a new partner, he remains partnerless for the period and rejoins the matching process next period. If neither agent in a bargaining relationship finds a new partner, then bargaining continues with each agent being the proposer with probability 1/2. In case a new partnership is formed, each agent is the proposer with probability 1/2. Furthermore, it is assumed that there are a continuum of agents on both sides (i.e., continuum of buyers and sellers) such that once a partnership breaks up, there

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is 0 probability that the agent gets paired with a partner with whom he was paired with previously and did not reach agreement. Rubinstein and Wolinsky (1985) derive a unique, semi-stationary subgame perfect Nash equilibrium with efficiency in reaching agreement. However, Rubinstein and Wolinsky (1985) does not converge to the Walrusian competitive market result as frictions go to zero. In response, Gale (1987, 1986a, 1986b) develop models of bargaining in markets, in non-stationary environments with divisible goods, which converge to the Walrusian competitive market result as frictions go to zero.

We now consider a model akin to Rubinstein and Wolinsky (1985), except we maintain the alternating offer structure from Rubinstein (1982) and introduce irrational types, to analyze under what market conditions reputation can be built up. In our set up, there is no exogenous outside option (i.e., opting out disagreement pay-off), however, there exists an ‘endogenous outside option’ (finding a new bargaining partner) which appears via an exogenous matching process. Similar to Rubinstein and Wolinsky (1985) we abstract from modelling the flows in and out of market; instead we concentrate on the matching process of new partners, where a fraction of the population is irrational. We are interested in asking under what conditions we get delay in reaching agreement and reputation build up where rational agents mimic irrationality.

A recent paper Atakan and Ekmekci (2013), models reputational bargaining in markets, where there are inflows and outflows of both rational and irrational agents and equilibria with steady state of each type are considered. Their model endogenizes the choice of opting out of a given partnership, after which a new partner is found after a certain delay (search friction). To ensure a steady state, they have to introduce a portion of unmatched agents leaving the market for exogenous reasons and they also introduce an exogenous outside option which players can take and exit the market without reaching agreement. We believe that some of these modelling choices force irrational types to somehow exit the market, for the sake of allowing a steady state to be established. We believe that introducing this whole slew of exit options for irrational types — able to exogenously leave the market or to take an outside option — marginalizes the more fundamental questions of whether irrational types can in fact trade in the market and if so, how does their participation affect and get affected by the market environment.

Thus, we abstract from these market modelling complications and consider reputational bargaining, generalizing Abreu and Gu (2000) to include exogenous arrival of new bargaining partners, in a market model, building on Rubinstein and Wolinsky (1985). In section 2, we motivate the importance of modelling decentralized markets in the context of reputational bargaining and competition, by presenting some applications. Next, in section 3, we develop the basic framework of the model. Then, in section 4, for consistency with previous Rubinstein bargaining literature, we derive an alternating-offer discrete-time version of Rubinstein and Wolinsky (1985) and analyze its limiting properties of the equilibrium. In section 5, we construct a hybrid model, following Abreu and Pearce (2007) but in a market setting, with discrete-time alternating offers but continuous-time offer acceptance, to approach closer to a war of attrition style bargaining model. Then, in section 6, we build a continuous-time reputational bargaining model and analyze under what market environments the Coase Conjecture condition is satisfied, and hence, a mimicking mixed strategy equilibrium (where rational agents mimic irrationality and play a war of attrition style optimal stopping game) exists. Finally, in section 7, we revisit our applications in light of the theoretical results we draw from the analytical conclusions and numerical simulations of our model. By interpreting our model in the context of these applications, we draw micro-economic intuitions for unemployment and labor market frictions, fire-sales and demand squeezes for exotic over-the-counter (OTC) assets, liquidity squeezes in the repo market, and venture capital market movements.

2. Applications & Interpretation

We consider alternating offer, bilateral bargaining based trade in a decentralized market, where pairs of buyers and sellers who have a mutual interest in trading, meet according to an exogenously parametrized stochastic process. When two agents meet, they initiate a bargaining process over the
terms of the transaction; namely, how to divide the surplus from trade. If two players reach an agreement over the surplus allocation, they trade and exit the market. Bargaining postures and acceptance/offer strategies will be determined by the market conditions: expectations over what the chances of meeting a new partner are, what the cost of delay from rejection is, and how the new partner might behave. Thus, the bargaining process is strategic and depends on what alternatives to trade the agent believes he faces in the market.

This decentralized market model captures trade interactions in a variety of different markets such as housing, labor, and over-the-counter markets for bonds, derivatives, structured products, commodities and currencies. In finance, search-and-bargaining models are applicable in markets for “mortgage-backed securities, corporate bonds, emerging-market debt, bank loans, derivatives, certain equity markets,” and instances where venture capitalists and managers search and bargain over the percent stake in the start-up sold in exchange for capital, investment banks and clients bargain over deal structure, or banks bargain over collateral and haircut terms of repo agreements. In a theoretical economics modelling context, we abstract from the exact details of the market, and can think of this model in two natural contexts: (1) sellers, each with 1 unit of an asset, and buyers, each with unit demand for the asset, negotiate over how to allocate the unit surplus from trade when negotiating the price of the transaction; or (2) a two-good exchange economy where type 1 traders have (1,0) endowment, type 2 traders have (0,1) endowment, and bargaining takes place between the traders over where to meet on the contract curve.

The focus of this decentralized market model is different from centralized market models, where a central market-maker intermediates all trades by publicly posting bid-ask prices. For example, a stock market exchange, such as NYSE or NASDAQ, amalgamates all price offers from buyers and sellers, publicly announces the bid-ask spread, and pairs buyers and sellers of stock who are willing to trade at the current market price. We analyze decentralized markets, or ‘search’ markets, where buyers and sellers have to search for trading partners, and privately negotiate and conduct transactions. For example, when a worker is seeking employment, he is one of many unemployed workers, underemployed workers, or workers dissatisfied with their current job, who seek new employment. Similarly, there are many firms which possibly want to fill job vacancies and seek to hire workers. Generally, in labor markets, there is no central organizing body which links qualified workers and firms with vacancies. There are search frictions in this market: workers have to endure the costs of waiting, finding vacancies, interviewing, and remaining unemployed until an employment contract is formed with a firm, and firms have to endure underproduction due to a shortage of labor and endure the costs of advertising and recruiting, to try to attract qualified workers.

Furthermore, trading in decentralized markets is not just a matter of finding a match or partner, but also involves haggling or bargaining over the terms of agreement. The terms of trade might be the wage contract with remuneration, overtime wages, perks, and retirement benefits in the context of labor markets, or the price of an asset or derivative security which both the buyer and seller agree upon in an OTC market or housing market. Since there is no centralized market price due to private bargaining, restrained information flow, and search frictions, we are interested in modelling bargaining strategies and postures used by agents when negotiating with a possible trading partner. We can interpret the behavioral, inflexible types as, for example: a worker who is persistent on a certain share because he or she wants to match the wage contract from previous employment, or a firm which may be committed to a certain share of the surplus due to search committee dynamics.

In the financial sector, OTC markets have mushroomed in recent years due to financial innovation (i.e., Mortgage-backed Securities (MBSs), structured finance, and exotic derivative securities); however, due to their inherent private nature, these transactions are less transparent and therefore subject to fewer regulations. In light of the 2007-8 financial crisis, when many OTC-traded asset markets such

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as MBSs, Credit Default Obligations, CDO-squareds, and Credit Default Swaps, faced turmoil, understanding the underlying pricing mechanisms and bargaining micro-foundations of decentralized markets is of particular importance today.4

3. The Model

There are two types of agents in the decentralized market: denoted \( i = 1, 2 \). When two agents of opposite types meet, they bargain over the allocation of a unit surplus associated with the trade. The market operates over discrete time, where the amount of time (number of periods) that an agent has been in the market is denoted by \( t \in \{0, 1, 2, 3, \ldots \} \), where \( t = 0 \) is the time that the agent enters the market. We place no limit on \( \max \{t\} \), thus an agent may remain indefinitely in the market.

We assume preferences such that agent type \( i \) has rate of time preference \( r_i \in [0, 1] \), such that if agent type \( i \) agrees on a fraction \( k \in [0, 1] \) of the surplus in period \( t \geq 0 \), his utility is \( ke^{-r_i \Delta t} \), where \( \Delta \) is the time between 2 consecutive periods. Thus, the utility from never leaving the market is 0. Consequently, agents only derive utility if they agree on a division of the surplus and trade occurs.

A fraction \( q_i \) of agents of type \( i \) are irrational, commitment types who always offer the partition \( \theta_i \in [0, 1] \) for themselves and \( 1 - \theta_i \) to their partner, and accept only those offers which give them share \( k \geq \theta_i \). The remaining fraction \( 1 - q_i \) of agents of type \( i \), are rational, fully flexible, utility maximizing agents. We assume that the irrational agents’ offers are incompatible: \( \theta_1 + \theta_2 > 1 \). We note that extending the analysis to include behavioral players with multiple irrational types, as analyzed in Abreu and Gul (2000), is possible but beyond the scope of this paper. After two agents reach an agreement, they leave the market. We assume a steady state where out-flow of departure of agents is matched by an equal inflow of agents of each type (i.e., when agents trade, they leave the market, and they are replaced by clones).

We assume that at the end of the period, matching takes place according to the following stochastic process: with probability \( \alpha \), a type 1 agent meets a new partner and with probability \( \beta \), a type 2 agent meets a new partner. Assume that an agent meets at most one new partner and assume that \( \alpha, \beta \in (0, 1] \) are fixed over time. This follows from our assumption that after a pair consummates a match and leaves the market, identical clones take their place in the market. Thus, the numbers of type 1 and 2 agents in the market (upon which the parameters \( \alpha \) and \( \beta \) are presumed to depend on) are constant.

Agents can bargain with at most one partner and bargaining does not have to be concluded within one period. After the matching process, the agent is in one of three states: has no partner, has the same partner with whom he was bargaining in the previous period, or has a new partner. If the agent has no partner, he waits for the next period’s matching stage. If the agent has the same partner as in the previous period, the agent who is not proposer in the previous period is the proposer in the current period (i.e., alternating offer akin to Rubinstein (1982)). If the agent has a new partner, bargaining takes place where one of the parties is selected at random with probability 1/2 to propose a partition. The party which does not propose, will then choose to accept (action denoted “A”) or reject (action denoted “R”) the proposal. If the proposal is accepted, the parties trade and leave the market. If a proposal is rejected, the same bargaining procedure will be repeated in the next period unless the matching process matches one or more of the agents with a new partner.

The order of the events for type 1 (analogous for type 2) within a period is as follows: first, the matching stage takes place. If the agent was not in the process of bargaining in the previous period, with probability \( \alpha \), he meets a new partner and proceeds to the bargaining stage, while with probability \( 1 - \alpha \), he remains without a partner. If the agent was in the process of bargaining in the previous period, however, did not reach agreement, then with probability \( \alpha \) he meets a new partner, abandons his previous partner, and proceeds to bargain with the new partner, with probability \( (1 - \alpha)(1 - \beta) \) he and his opponent both do not meet a new partner and proceed to continue bargaining with each

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other, and with probability \((1 - \alpha)\beta\) he does not find a new partner while his old partner does, so he remains without a partner until the next period.

We assume that agents have perfect recall, so that a possible history of an agent at a certain stage of his market life is a sequence of all the observations made by him up to that stage. As in Rubinstein and Wolinsky (1985): “During any period, \(T\), the agent receives the relevant information sequentially. That is, he gets one by one the answers to the following questions in the following order: (I) Does he have a partner at period \(T\)? If so, did he have this same partner at period \(T - 1\)? (II) Who was selected to make an offer at \(T\)? (III) What was the offer at period \(T\)? (IV) What was the response to that offer?” This information is accumulated until the end of period \(T - 1\) in a period \(T\) history. Using the notation from Rubinstein and Wolinsky (1985), let us denote by \(H^T_o\), the set of all possible histories which end with the information that agent has a partner at period \(T\) and \(i\) is selected to make the offer. Let us denote by \(H^T_r\), the set of all possible histories which end with information that the agent has a partner at period \(T\) who has made an offer which \(i\) responds to by accepting or declining.

Our assumption that the probabilities \(\alpha\) and \(\beta\) are constants is appropriate for large markets in which variations due to inflow and outflow of agents are small. In this case, the important information is the likelihood with which an agent will find a new partner and the “intensity of his fear that his partner will abandon him.” This model can be linked to having \(N_1\) type 1 agents and \(N_2\) type 2 agents as primitives. Assuming there are \(M\) new matches in each period and \(N_1\) and \(N_2\) are large so that the probability of being rematched with a previous partner is small, \(\alpha = M/N_1\), \(\beta = M/N_2\). Alternatively, we can replace this large population, zero probability assumption, with an assumption that the agent remembers the identity of his partner only when he is in the process of bargaining and doesn’t remember the identities of previous bargaining partners. Thus even if an agent meets the same partner with whom he has already bargained before and broken off, the agent treats him as a new partner. Both of these assumptions are equivalent.

\[\text{DATE } t\]
\[\text{Paired} \quad \text{Not Paired} \quad \text{New to Market}\]
\[\text{Accept} \quad \text{Reject} \]
\[\text{Exit Market}\]

\[\text{DATE } t+1\]
\[\text{NATURE}\]
\[\alpha \quad 1-\alpha\]
\[\text{New Partner} \quad \text{No New Partner}\]
\[q \quad 1-q \quad \beta \quad 1-\beta\]
\[\text{PRIOR}\]
\[\text{Irrational} \quad \text{Rational} \quad \text{Not Paired} \quad \text{Same Partner (alt. offer)} \quad \text{Update Posterior}\]

\textbf{Figure 1.} Discrete-time model: generic 2-period game tree over periods \(t\) and \(t + 1\).

\(^5\text{Osborne and Rubinstein (1990).}\)
4. Discrete-Time Model Without Irrational Types


Let us now take the model from section 3 and set \( q_1 = q_2 = 0 \) so that there are only rational types in the model. We derive the alternating offer analog of Rubinstein and Wolinsky (1985). We further simplify, to get more manageable explicit solutions, by assuming \( r_1 = r_2 \), such that \( e^{-r_1 \Delta} = e^{-r_2 \Delta} = \delta \) where \( \Delta \) is the time between periods. The argument developed below extends to generic non-symmetric discount rate, however, equilibrium expressions are cleaner with this assumption.

We define a strategy for agent \( i \), \( f \in S_i \), as a sequence of decisions, \( f = (f^t)_{t=0}^{\infty} \), such that \( f^t \) is a function over all histories in \( H^t_{\alpha}, H^t_{r_1} \). If history \( h^t \in H^t_{\alpha} \), then agent \( i \) chooses to make an offer: \( f^t(h^t) \in [0,1] \) and if history \( h^t \in H^t_{r_1} \), then agent \( i \) responds to the offer by accepting or rejecting: \( f^t(h^t) \in \{ A, R \} \). Due to the stationary nature of the game, we naturally choose to focus on semi-stationary strategies, where after every history where an agent has just met a new partner, the bargaining strategy employed is the same. Let us define \( U_i(f,g)(h^T) \) as agent \( i \)'s expected utility after history \( h^T \) from employing strategy \( f \in S_i \) while all possible partners employ the semi-stationary strategy \( g \in S_j \).

We define a market equilibrium as a pair of semi-stationary strategies \((f^*, g^*)\) such that all type 1 agents employ strategy \( f^* \), all type 2 agents employ strategy \( g^* \), and for any history \( h^T \), \( U_1(f^*,g^*)(h^T) \geq U_1(f,g^*)(h^T) \forall f \in S_1 \) and \( U_2(f^*,g^*)(h^T) \geq U_2(f^*,g^*)(h^T) \forall g \in S_2 \). Thus, our definition of market equilibrium entails a subgame perfect Nash equilibrium in semi-stationary strategies which are protected against any possible unilateral deviation (semi-stationary or not).

We look for a market equilibrium in which type 1 agents offer \((x, 1-x)\) and type 2 agents offer \((1-y, y)\) in every period in which 1, 2 are proposers. Players 1 and 2 get a new bargaining partner in each period with probability \( \alpha \) and \( \beta \) respectively. Let \( V_1 \), \( V_2 \) be the pay-off to player 1, 2 from having no opponent in a period.

Agent 1’s indifference condition between accepting and rejecting 2’s offer of 1 − y:

\[
1 - y = \delta (1 - \alpha)(1 - \beta)(x) + \delta \alpha (x/2 + (1 - y)/2) + \delta (1 - \alpha) \beta V_1
\]

(1)

Agent 2’s indifference condition between accepting and rejecting 1’s offer of 1 − x:

\[
1 - x = \delta (1 - \alpha)(1 - \beta)(y) + \delta \beta ((1 - x)/2 + (y)/2) + \delta (1 - \beta) \alpha V_2
\]

(2)

Agent 1’s pay-off from having no partner:

\[
V_1 = \delta \alpha ((1 - y)/2 + x/2) + \delta (1 - \alpha) V_1
\]

\[
\Rightarrow V_1 = \frac{\alpha \delta / 2 (1 - y + x)}{1 - \delta + \delta \alpha}
\]

(3)

Agent 2’s pay-off from having no partner:

\[
V_2 = \delta \beta ((1 - x)/2 + y/2) + \delta (1 - \beta) V_2
\]

\[
\Rightarrow V_2 = \frac{\beta \delta / 2 (1 - x + y)}{1 - \delta + \delta \beta}
\]

(4)

Solving this system \{ (1), (2), (3), (4) \} yields the equilibrium \((x^*, y^*)\):

\[
x^* = \frac{2 - 2 \delta + \alpha \delta + \alpha \delta^2 - \alpha^2 \delta^2 - \alpha \beta \delta^2 + \alpha^2 \beta \delta^2}{2 - \alpha \delta - \beta \delta + 2 \alpha \beta \delta - 2 \delta^2 + 3 \alpha \delta^2 - \alpha^2 \delta^2 + 3 \beta \delta^2 - 4 \alpha \beta \delta^2 + \alpha^2 \beta \delta^2 - \beta^2 \delta^2 + \alpha^2 \beta^2 \delta^2}
\]

(5)

\[
y^* = \frac{2 - 2 \delta + \beta \delta + \beta \delta^2 - \beta^2 \delta^2 - \alpha \beta \delta^2 + \alpha^2 \beta \delta^2}{2 - \alpha \delta - \beta \delta + 2 \alpha \beta \delta - 2 \delta^2 + 3 \alpha \delta^2 - \alpha^2 \delta^2 + 3 \beta \delta^2 - 4 \alpha \beta \delta^2 + \alpha^2 \beta \delta^2 - \beta^2 \delta^2 + \alpha^2 \beta^2 \delta^2}
\]

(6)

There exists a unique market equilibrium \((f^*, g^*)\), where equilibrium strategies are
4.2. Properties and Analysis of the Market Equilibrium.

As a consistency check, notice that as we take \( \alpha = \beta = 0 \) (i.e., no arrival of partners), we get the equilibrium offers \( x^* = y^* \frac{1 - \delta}{1 - \delta + \alpha} = \frac{1}{1 + \delta} \), which are the equilibrium offers in Rubinstein (1982).

4.2.1. First Proposer Advantage and Value of Participating in Market. We note that in the relevant parameter range \( (\alpha, \beta, \delta \in [0,1]) \), we have \( x^* \geq 1 - y^* \) and \( y^* \geq 1 - x^* \), which shows the advantage of being the first proposer: larger share from offering a proposal which is accepted in equilibrium rather than receiving and accepting an offer in equilibrium.

Furthermore, we note that agent 1’s value from participating in the market is the value we derived earlier as the pay-off from not being paired in the market: thus, type 1 agent’s value from participating in the market is \( V_1^* = \frac{\delta^\alpha}{1 - \delta + \alpha} \left( \frac{1 - y^*}{2} + \frac{x^*}{2} \right) \) and type 2 agent’s value from participating in the market is \( V_2^* = \frac{\delta \beta}{1 - \delta + \beta} \left( \frac{1 - x^*}{2} + \frac{y^*}{2} \right) \).

4.2.2. Discount Factor \( \delta \) Limits. As we let impatience be small, i.e., take the limit as \( \delta \to 1 \), we get \((\overline{x^*}, \overline{y^*})\) where

\[
\frac{x^*}{\alpha + \beta} = \overline{x^*} = \frac{\alpha}{\alpha + \beta} \tag{7}
\]

\[
\frac{y^*}{\beta/(\alpha + \beta)} = \overline{y^*} = \frac{\beta}{\alpha + \beta} \tag{8}
\]

The allocations of surplus are thus determined by the probability of finding new partners \( \alpha, \beta \), and \( \lim_{\delta \to 1} x^* = \lim_{\delta \to 1} 1 - y^* = \lim_{\delta \to 1} V_1 = \overline{x^*} \).

On the other hand, if we consider extremely large impatience levels, i.e., take the limit as \( \delta \to 0 \), we get that \( \lim_{\delta \to 0} x^* = 1, \lim_{\delta \to 0} 1 - y^* = 0, \) and \( \lim_{\delta \to 0} V_1 = 0 \), since the loss due to discounting from one period delay overpowers all other factors. Both types of agents have the same bargaining power and surplus allocation is based on whether or not you are the proposer, which we have assumed to be determined randomly when a bargaining partnership is formed.

4.2.3. Length of Bargaining Period \( \Delta \) Limits. To explicitly denote the equilibrium allocations’ dependence on the length of time between consecutive time periods \( \Delta \), we let \( \alpha(\Delta) = \alpha \Delta, \beta(\Delta) = \beta \Delta, \) and \( \delta(\Delta) = e^{-r \Delta}, \) and substitute \( \alpha(\Delta), \beta(\Delta), \) and \( \delta(\Delta) \) into our equilibrium expressions \((x^*, y^*, V_1^*)\).

We interpret \( \alpha, \beta \) as the arrival rates of new partners and \( r \) is the rate of time preference. Taking the limit of this equilibrium as \( \Delta \to 0 \), using L'Hôpital’s rule we get

\[
\lim_{\Delta \to 0} x^* = \lim_{\Delta \to 0} 1 - y^* = \frac{r + \alpha}{\alpha + \beta + 2r} \quad \lim_{\Delta \to 0} V_1^* = \frac{\alpha}{\alpha + \beta + 2r} \tag{9}
\]

4.2.4. Population Size Dependence. Although the probabilities of meeting new partners are exogenously specified, we had interpreted these matching/search probabilities as functions of the population sizes \( N_1, N_2 \) of agents of types 1, 2. Without explicitly modelling this, let us fix \( M \) to be the per period rate of meeting an agent in the market. Thus substituting \( \alpha = \frac{M}{N_1} \) and \( \beta = \frac{M}{N_2} \) into (7) we get

\[
\lim_{\delta \to 1} x^* = \lim_{\delta \to 1} 1 - y^* = \frac{N_2}{N_1 + N_2} \tag{10}
\]
Thus, by law of total expectation,
\[
\lim_{\Delta \to 0} x^* = \lim_{\Delta \to 0} 1 - y^* = \frac{rN_1N_2/M + N_2}{2rN_1N_2/M + N_1 + N_2}
\] (11)

Thus, we see that the side with the smaller population in the market gets more, but not all, of the surplus from bargaining. This result coincides with general intuition that the agent who can more easily find new matches must be compensated more.

5. Hybrid Model Without Irrational Types

Let us take the model from section 3 and set \( q_1 = q_2 = 0 \), so that there are only rational types in the model. Furthermore, we now look at the continuous time version of the model above, where agents of type 1,2 find new partners governed by a Poisson process, where the arrival of new opponent is exponentially distributed with parameters \( \lambda_1, \lambda_2 \), and corresponding cumulative distribution functions \( G_1, G_2 \). This is a natural continuous-time generalization of our discrete model consisting of Bernoulli trials of finding a new partner every period: which converges to Poisson process of arrivals as we take limits,\(^6\) while the distribution of inter-arrival times of a Poisson Process is exponentially distributed.

Thus, there is a direct correspondence between parameters \( \alpha, \beta \) and \( \lambda_1, \lambda_2 \). Furthermore, discounting is now continuous: agents 1, 2 have instantaneous time preference of \( r_1, r_2 \). We again simplify for notational convenience, \( r_1 = r_2 = r \). Finally, we consider a hybrid model (as in Abreu and Pearce (2007)) where agents engage in alternating offer bargaining game if they do not find new partners, where they make alternating offers with discrete time interval \( \Delta \) between offers. However, the offers can be accepted at any time \( t \) in the time interval \( \Delta \) over which the offer remains on the table.

This hybrid set up of discrete time alternating offering with continuous time offer acceptance is assumed as a way to maintain Rubinstein bargaining structure (which relies on alternating offers) but also use war of attrition style acceptance action spaces. This incorporates an optimal stopping element in bargaining, which will be useful when we consider reputation building with behavioral types. Rubinstein bargaining model doesn’t sit well in continuous time, since the alternating offer structure is lost in the limit; thus we consider this hybrid model to approximate and get around this concern. With this hybrid set up, we are able to use a war of attrition style analysis without running into well-known problems of defining strategies and outcomes in continuous time; thus bypassing concerns of “openness” faced when modelling with continuous-time extensive games.

5.1. The Market Equilibrium.

In equilibrium, we will get efficiency, since there is no uncertainty (no irrational types). Furthermore, since all parameters are the same across all agents of the same type, we can impose symmetric strategy subgame perfect Nash equilibrium, where all of agents of types 1,2 use the same strategy. Finally, due to the stationary structure of the environment, we can impose stationarity of strategies. Thus, we look for a market equilibrium similar to the one derived in the discrete time model.

Let us define \( x, y \) as the stationary offers which agent 1 offers 2 and 2 offers 1 on the equilibrium path. Then, these offers must be such that they make the opponent indifferent between accepting immediately when the offer that is made, and waiting for time \( \Delta \). We note that if player \( i \) is in the period lasting time \( \Delta \), where \( j \) has made him an offer; if \( i \) is going to accept \( j \)’s offer, it is a dominant action for him to accept this offer immediately after the offer is made.

Thus, these offers \( x, y \) should satisfy:\(^7\)

\(^6\)Recall that binomial distribution with parameters \( n \) and \( p_n \) converges to the Poisson distribution with parameter \( r \), when \( np_n \to r \), as \( n \to \infty \).

\(^7\)In deriving this equilibrium, we use the following facts: For \( X \sim \text{Expon}(\lambda_1), Y \sim \text{Expon}(\lambda_2), X \perp \perp Y \). \( \mathbb{E}(X) = 1/\lambda_1 \), \( \Pr(X < Y) = \frac{1}{\lambda_1 + \lambda_2} \).

Let us define \( \mathbb{E}(X|X \leq \tau) = A \). By memoryless property of exponential distribution, \( \mathbb{E}(X|X \geq \tau) = \tau + 1/\lambda_1 \).

Thus, by law of total expectation,
\[
\mathbb{E}(X) = 1/\lambda_1 = (1 - e^{-\lambda_1\tau})A + e^{-\lambda_1\tau}(\tau + 1/\lambda_1).
\]

Thus, \( A = \frac{1}{1-e^{-\lambda_1\tau}} \).
\[ 1 - y = xe^{-(\lambda_2 - \lambda_1)\Delta}e^{-r\Delta} \]
\[ + \frac{1 - y + x}{2}(1 - e^{-\lambda_1\Delta})e^{-r\left(\frac{1-e^{-(\lambda_1\Delta+1)}}{\lambda_1(1-e^{-\lambda_1\Delta})}\right)} \]
\[ + \frac{1 - y + x}{2}e^{-\lambda_1\Delta}e^{-r(\Delta+1/\lambda_1)}(1 - e^{-\lambda_1\Delta}) \]
\[ 1 - x = ye^{-(\lambda_1 - \lambda_2)\Delta}e^{-r\Delta} \]
\[ + \frac{1 - x + y}{2}(1 - e^{-\lambda_2\Delta})e^{-r\left(\frac{1-e^{-(\lambda_2\Delta+1)}}{\lambda_2(1-e^{-\lambda_2\Delta})}\right)} \]
\[ + \frac{1 - x + y}{2}e^{-\lambda_2\Delta}e^{-r(\Delta+1/\lambda_2)}(1 - e^{-\lambda_2\Delta}) \]

These are player \( i \)'s indifference conditions (for \( i = 1, 2 \) respectively) between accepting \( j \)'s offer immediately and waiting for \( \Delta \) time, for \( j \neq i \):

- The first term is the case where neither agent gets new partner in time \( \Delta \).
- The second term is the case where player \( i \) gets new partner within time interval \( \Delta \), in which case with probability \( 1/2 \) he is proposer and with probability \( 1/2 \) he is receiver due to random draw of 1st mover.
- The third term is the case where player \( j \) gets new partner within time interval \( \Delta \) and player \( i \) gets an arrival after time \( \Delta \). By the memorylessness property of the exponential distribution, player \( i \) gets a new partner in \( \Delta + 1/\lambda_i \) units of time in expectation.

Solving the system for \((x, y)\) gives us the stationary market equilibrium values \((\hat{x}, \hat{y})\) satisfying market equilibrium strategies \((\hat{f}, \hat{g})\) for agents of type 1 and 2, where:

\[
\hat{f}(h^t) = \begin{cases} 
1 - \hat{x} & \text{if } h^t \in H^t_{1,1}, \\
A & \text{if } h^t \in H^t_{1,2} \text{ and last offer } 1 - y \geq 1 - y^* \\
R & \text{otherwise}
\end{cases}
\]

\[
\hat{g}(h^t) = \begin{cases} 
1 - \hat{y} & \text{if } h^t \in H^t_{2,1}, \\
A & \text{if } h^t \in H^t_{2,2} \text{ and last offer } 1 - x \geq 1 - x^* \\
R & \text{otherwise}
\end{cases}
\]

### 5.2. Consistency in Limits.

As a consistency check, comparing this hybrid model to our discrete time analog, we note that

\[
\lim_{\Delta \to 0} \hat{x} = \lim_{\Delta \to 0} 1 - \hat{y} = \frac{\lambda_1 - \lambda_1 e^{\frac{\Delta}{\lambda_2}} + r}{\lambda_1 + \lambda_2 - \lambda_2 e^{\frac{\Delta}{\lambda_2}} + \lambda_1 e^{\frac{\Delta}{\lambda_2}} + 2r} \quad (14)
\]

and

\[
\lim_{r \to 0} \hat{x} = \lim_{r \to 0} 1 - \hat{y} = \frac{\lambda_1}{\lambda_1 + \lambda_2} \quad (15)
\]

We conclude that our models are consistent, as in the limit as \( r \to 0 \) (analog of \( \delta \to 0 \)), we see that the equilibrium allocations are determined by the relative size of the arrival rates of new partners (analog of the arrival probabilities of new partners \( \frac{\alpha}{\alpha + \beta} \)).

---

\(^5\)This yields an efficient, symmetric SPNE, since there is no uncertainty. The explicit solutions for \( \hat{x}, \hat{y} \) can be explicitly derived but are cumbersome formulas which do not provide much intuition. Thus, instead of including these formulas, we derive and discuss limiting properties below and compare the results to our discrete model.
6. Continuous-Time Model With Irrational Types

6.1. The Coase Conjecture.

In the context of reputation-building bargaining, the Coase Conjecture is valid if: when your opponent is known to be rational and believes that you are a behavioral type with any positive probability, your opponent must give in right away to your possibly irrational demand. There are two important consequences of the Coase Conjecture result being satisfied. Firstly, it is necessary in an equilibrium where rational types mimic the irrational demands of behavioral agents. Secondly, it reduces the set of actions of each player at each time to a 2-action space: either continue mimicking irrational type’s demand or accept your opponent’s irrational demand. Thus, the 2-action space makes intermediate offers irrelevant and reduces the problem to an optimal stopping, war of attrition game.

We provide a condition, which we call the “Coase Conjecture condition,” such that if the condition is satisfied, the Coase Conjecture is valid. The Coase Conjecture condition compares the pay-off to rational player i from accepting the opponent’s irrational demand of \(1 - \theta_j\) to the most profitable alternative action. In Abreu and Gul (2000), the bargaining partnership is bilateral and there is no outside option (i.e., outside option pay-off is 0), thus, the Coase Conjecture condition trivially holds: \(1 - \theta_j \geq 0\), for all possible irrational types \(\theta_j \in (\frac{1}{2}, 1]\). Thus, they can always analyze reputation-building by reducing the game to a continuous-time war of attrition. On the other hand, all results in Compte and Jehiel (2002), where reputation cannot be built up, arise due to the underlying assumption (a violation of the Coase Conjecture condition) that \(1 - \theta_j \geq v_j\), where \(v_j\) is the player j’s outside option pay-off. Thus, rather than accepting the irrational opponent’s demand, the agent is always better off taking his outside option. Consequently, it is never optimal for a rational agent to try to build a reputation by mimicking irrational type’s demand.

In our paper, the Coase Conjecture condition

\[
1 - \theta_j \geq (e^{-r_j/\lambda_j}) \cdot V_j
\]

compares the pay-off of accepting the opponent’s irrational demand with the expected pay-off from waiting for a new partner. The expected time for the arrival of a new partner is \(1/\lambda_j\) and \(V_j\) is a rational player j’s expected pay-off from meeting a new bargaining partner, which in our model is endogenously determined in equilibrium (depending on prior probabilities of irrationality, irrational type’s demands, discount rates, and arrival rate of new partners for both agents).

For reputation-building in bargaining models to be reduced to simple, continuous-time war of attrition games—where rational players mimic the irrational demands of behavioral players and the bargaining game turns into an optimal stopping problem—as in Abreu and Gul (2000), it is necessary that the Coase Conjecture condition holds. We can think of more complex non-stationary war of attrition games which do not necessarily need the Coase Conjecture condition to hold. However, with a unique stationary irrational demand, \(\theta_i\), for each player \(i\), we obtain only a simple war of attrition bargaining game where, if the Coase Conjecture condition holds, there is possible reputation build up, where rational agent \(i\) either mimics being an irrational type by demanding \(\theta_i\) or concedes by accepting his opponent’s irrational demand, \(1 - \theta_j\).

6.2. Introducing Irrational Types.

We now allow for irrational types in the market: \(p, q \geq 0\). Let us first assume that the Coase Conjecture condition holds and thus solve the continuous-time war of attrition strategies: distribution functions \(F_1, F_2\) over conceding times. Upon solving for these potential equilibrium strategies \(F_1, F_2\)
(as in Abreu and Gul (2000)), we can then check under what market environment the endogenous outside option (value from meeting new opponent) satisfies the Coase Conjecture condition, thus, concluding, under what market conditions, equilibrium with reputational build-up and delay are possible.

Let us consider agent type $i$'s problem. Let us denote by $t \in [0, \infty]$, the time after starting bargaining with a new partner (i.e., $t=0$ is the time at which the agent is paired with a new partner). Let us denote by $V_i$, the expected pay-off to a rational agent of type $i$ from being matched with a new partner.

A rational agent $i$'s utility from conceding at time $t$ is

$$u_i(t) = \theta_i F_j(0) + \int_0^t \theta_i e^{-r_i x}(1 - G_i(x))(1 - G_j(x))dF_j(x)$$

$$+ (1 - \theta_j)e^{-r_j t}(1 - G_i(t))(1 - G_j(t))(1 - F_j(t))$$

$$+ \int_0^t V_i e^{-r_i x}(1 - G_j(x))(1 - F_j(x))dG_i(x)$$

$$+ \int_0^t V_i e^{-r_i (x+1/\lambda_i)}(1 - G_i(x))(1 - F_j(x))dG_j(x)$$

We are trying to determine the equilibrium functions $F_1, F_2$. This will allow us to compute $V_i$ and check under what conditions the Coase Conjecture is satisfied, the war of attrition reduction of the game is valid, reputation can be built up, and delay and inefficiency exist in equilibrium.

Maximizing the rational player $i$'s utility with respect to concession time $t$ (i.e., the agent must be indifferent between conceding at any time in the support of his mixed strategy distribution) gives

$$\frac{\partial}{\partial t} u_i(t) = \theta_i e^{-t(r_i + \lambda_i + \lambda_j)} f_j(t) - (1 - \theta_j)e^{-t(r_i + \lambda_i + \lambda_j)} f_j(t)$$

$$- (r_i + \lambda_i + \lambda_j)(1 - \theta_j)e^{-t(r_i + \lambda_i + \lambda_j)} (1 - F_j(t))$$

$$+ V_i e^{-t(r_i + \lambda_i + \lambda_j)} \lambda_i (1 - F_j(t))$$

$$+ V_i e^{-t(r_i + \lambda_i + \lambda_j) - r_i / \lambda_i} \lambda_j (1 - F_j(t))$$

$$= 0$$

This gives us the first-order linear differential equation:

$$f_j(t) = F_j(t) \left( \frac{(r_i + \lambda_i + \lambda_j)(1 - \theta_j) - V_i (\lambda_i + \lambda_j e^{-r_i / \lambda_i})}{(\theta_i + \theta_j - 1)} \right)^{\eta_j}$$

$$= \left( \frac{(r_i + \lambda_i + \lambda_j)(1 - \theta_j) - V_i (\lambda_i + \lambda_j e^{-r_i / \lambda_i})}{(\theta_i + \theta_j - 1)} \right)^{\eta_j}$$

Solving the differential equation by method of integrating factor shows that $F_j$ is one of the family of functions.

$$F_j(t) = 1 - c_j e^{-\eta_j t}$$

$F_i(t)$ is defined similarly by swapping subscripts. Now we need to solve for $c_1, c_2$ and $\eta_1, \eta_2$. Our first condition comes from standard war of attrition arguments, also used in Abreu and Gul (2000),

\[12\] Recall that in models with rational and irrational types, in equilibrium, we need to only consider rational players’ optimizing behavior, having specified irrational players’ actions. Thus, it suffices consider and maximize $u_i(t)$, a rational player $i$’s utility.
that in equilibrium, both players cannot concede with discrete positive probability at time \( t = 0 \), else it is strictly profitable for a player to deviate from this strategy, not concede at \( t = 0 \), and wait for \( \epsilon > 0 \) time before conceding. Thus, \( F_i(0) > 0 \rightarrow F_i(0) = 0 \), thus we obtain one boundary condition: 
\[(1 - c_1)(1 - c_2) = 0.\]

The other boundary condition arises from the claim proven in Abreu and Gul (2000); its analog can be proven in our model. The probability of irrationality of both players must reach 1 simultaneously at time we denote as \( \tau < \infty \). The intuition behind this claim is that if player \( i \) was rational and knew that player \( j \) is irrational, then \( i \) would concede immediately (assuming the Coase Conjecture condition is satisfied).

Thus by Bayes’ theorem:
\[
1 = \Pr(\text{player } i \text{ concedes by time } \tau \mid \text{player } i \text{ is rational}) \\
= \frac{\Pr(\text{player } i \text{ concedes by time } \tau \mid \text{player } i \text{ is rational})}{\Pr(\text{player } i \text{ is rational})} \\
= \frac{F_i(\tau)}{1 - z_i}.
\]

Thus,
\[
F_i(\tau) = 1 - e^{-\eta \tau} = 1 - z_i \tag{22}
\]

To work out which player(s) have \( c = 1 \), we solve \( F_i(T_i) = 1 - e^{-\eta T_i} = 1 - z_i \), which implies \( T_i = \frac{-\ln(z_i)}{\eta} \). If \( T_1 = T_2 \), where we call both players 1 and 2 being ‘equally strong,’ then \( c_1 = c_2 = 1 \). If \( T_i < T_j \), where we call player \( i \) ‘strong’ and player \( j \) ‘weak,’ then \( c_i = 1, c_j < 1 \). The terminology refers to how long it takes each player to build a reputation and convince his opponent of irrationality. The ‘weak’ player, \( j \), has to concede for a longer amount of time before convincing his opponent of his irrationality (with probability 1). The ‘weak’ player thus concedes with discrete probability, \( 1 - c_j > 0 \), at time 0 to ensure that both players finish conceding at the same time \( \tau = \min\{T_1, T_2\} \).

There are three conditions that need to be satisfied for such delay an mimicking equilibrium to exist: (1) \( \eta > 0 \), or equivalently, \( \tau \geq 0 \), (2) \( c_i = 1 \rightarrow c_j \leq 1 \), and (3) the Coase Conjecture condition.

6.2.1. **Symmetric Case.** Let us first consider the fully symmetric case with \( r_1 = r_2 = r \), \( \lambda_1 = \lambda_2 = \lambda \), \( z_1 = z_2 = z \), and \( \theta_1 = \theta_2 = \theta \in (\frac{1}{2}, 1) \]. We will thus look for symmetric mixed strategies, where the cumulative distribution of concession time follows \( F_1 = F_2 = F \). \( V_1 = V_2 = V \) is the expected pay-off for a rational agent from finding a new partner.

In the symmetric case, both players are ‘equally strong’, \( T_1 = T_2 = \tau \), therefore \( c = 1 \).

\[
F(t) = \begin{cases} 
1 - e^{-\eta t} & \text{if } t \leq \tau \\
1 - z & \text{if } t > \tau 
\end{cases} \tag{23}
\]

Recall, that the function \( F(t) \) is not a cumulative distribution function: it is the \textit{ex ante} probability that a player has conceded by time \( t \), prior to knowing whether he himself is rational or irrational.

If the player is irrational, he will never concede, so his cumulative distribution of concession times is trivially 0 for all \( t \). Let \( R(t) \) denote the cumulative distribution function of rational agent’s concession time. Then, by Bayes’ Rule we get normalization that:

\[
R(t) = \begin{cases} 
\frac{1 - e^{-\eta t}}{1 - z} & \text{if } t \leq \tau \\
1 & \text{if } t > \tau 
\end{cases} \tag{24}
\]

Next, we compute \( V \), a rational player’s expected pay-off from a new partner, using what we know about the equilibrium strategy. We know that the support over which a rational type concedes is \( t \in (0, \tau] \). Since a rational agent plays a mixed strategy in equilibrium, where he mixes over concession times in the support of \( F \), by definition, he must be indifferent between conceding at any time \( t \) in the support of \( F \). Consider a rational player giving up at time \( \epsilon > 0 \), where he gets pay-off
As we take \( \lim_{\epsilon \to 0} e^{-r\epsilon}(1 - \theta) = 1 - \theta \). Thus,

\[
V = 1 - \theta \tag{25}
\]

Thus, in the symmetric case, the Coase Conjecture condition is always satisfied, since \( 1 - \theta_j \geq e^{-r/\lambda}(1 - \theta_j) \). Consequently, the game can always be reduced from a structured alternating offer game to a continuous time war of attrition optimal stopping game, and a rational player will follow the equilibrium strategy determined by the cumulative distribution over concession times \( R(t) \), where

\[
R(t) = \begin{cases} 
1 - \frac{e^{-\eta t}}{1 - z}, & \text{if } t \leq \tau \\
1, & \text{if } t > \tau 
\end{cases} \tag{26}
\]

where \( \tau = \frac{-\ln(z)}{\eta} \geq 0 \) and \( \eta = \frac{r(1-\theta)+\lambda(1-\theta)(1-e^{-r/\lambda})}{(2\theta-1)} > 0 \).\(^{13}\)

It is worth noting that since \( e^{-r/\lambda} \leq 1 \), \( \eta \geq \eta_{ag} \) and \( \tau \leq \tau_{ag} \), where subscript “ag” refers to Abreu and Gul (2000). Thus, potential delay is reduced in a market setting with \( \lambda > 0 \).

From this simple, symmetric case, we can get intuition by evaluating how changing individual parameters impact delay (and hence efficiency). In the equilibrium, \( \tau \) measures the potential delay, as it is the last time by which a rational player must concede. Since \( \eta > 0 \), \( c_i = c_j = 1 \), and Coase Conjecture condition is always satisfied, such an equilibrium always exists. Over the next few pages, we provide a series of diagrams and explanations which illustrate the impact on \( \tau \) (potential delay), in the symmetric case of our model as compared to Abreu and Gul (2000), when:

- Changing the rate of discounting: \( r \)
- Changing the irrational type’s demand: \( \theta \)
- Changing the prior probability of irrationality: \( z \)
- Changing the rate of arrival of new partners: \( \lambda \)

\(^{13}\)We check that our distribution is consistent with Abreu and Gul (2000), since when \( \lambda = 0 \) (i.e., no arrivals of new partners) and \( \alpha_i = \alpha_j = \theta \) (symmetric case),

\[
F_{i,ag}(t) = F_{j,ag}(t) = F_{AG}(t) = \begin{cases} 
1 - e^{-r(1-\theta)t} = 1 - e^{-\eta_{ag} t}, & \text{if } t \leq \tau_{ag} \\
1 - z, & \text{if } t > \tau_{ag} 
\end{cases} \tag{27}
\]

as in Abreu and Gul (2000), since with \( \lambda = 0 \), \( \eta = \eta_{ag} \). Also, \( V(z = 0) = 1 - \theta = V_{ag} \) in the symmetric case. Note that “ag” refers to Abreu and Gul (2000).
• As impatience or rate of discounting, $r$, increases, potential delay, $\tau$, decreases. This is an intuitive result: the more impatient a player is, the higher his opportunity cost from delaying is, i.e., higher cost of mimicking irrationality and not having opponent concede. Thus, the more impatient players are, the less delay in reaching agreement. In the extreme case, as $r \to \infty$, $\tau \to 0$.

Figure 2. Numerical simulation of the effect on $\tau$ from changing $r$ in the symmetric case.
As the irrational type’s demand, $\theta$, increases, potential delay, $\tau$, increases. The more extreme irrational types make their demands, the less attractive the pay-off $1 - \theta$ is relative to $\theta$. Therefore, the gain from convincing an opponent of one’s irrationality is high, and thus rational players are willing to mimic irrational types for longer in the hopes that their opponent will concede. Thus, $\eta$ decreases (slower concession) in response to an increase in $\theta$, and thus, for fixed $z$, $\tau$ is higher. Consequently, there is more potential delay.

Figure 3. Numerical simulation of the effect on $\tau$ from changing $\theta$ in the symmetric case.
• As the proportion of irrational types in the population, \( z \), increases, potential delay, \( \tau \), decreases. Upon meeting a new partner, \( z \) is the prior belief of an agent that his opponent is irrational. If this prior is high, it naturally takes less time to convince one’s partner of being an irrational type. Thus, potential delay decreases.

**Figure 4.** Numerical simulation of the effect on \( \tau \) from changing \( z \) in the symmetric case.
As the arrival rate of new partners, $\lambda$, increases, potential delay, $\tau$, decreases asymptotically to a non-zero level, $\tau_{ag}/2$. Although in the symmetric case, $V = 1 - \theta$ from all new partners and there is no gain from a new partner from time $t = 0$ concession (i.e., $c_1 = c_2 = 1$), reputation needs to be built up starting from the prior every time a new partner is found, as previous partnership histories are private information. Thus, concessions must be made at a faster rate (increase in $\eta$) to compensate for increases in the arrival rate of new partners. As $\lambda \to \infty$, $\tau \to -\ln(z)(2\theta - 1)/2(1 - \theta)$, $\tau$’s limiting level corresponds to half the potential delay in Abreu and Gul (2000), which we denote by $\tau_{ag}/2$, because as $\lambda \to \infty$, an agent is without a partner half of the time.

![Figure 5. Numerical simulation of the effect on $\tau$ from changing $\lambda$ in the symmetric case.](image)

**6.2.2. Non-Symmetric Cases.** We use the equilibrium properties we know to derive conditions which must be satisfied for there to possibly be an equilibrium with reputational build-up, similar to what we have thus far analyzed. From the boundary conditions, we know that in equilibrium, $F_i(\tau) = 1 - c_i e^{-\eta_i \tau} = 1 - z_i$, thus

$$c_i = z_i e^{\eta_i \tau} = z_i e^{\eta_i \min\left\{\frac{-\ln(z_1)}{\eta_1}, \frac{-\ln(z_2)}{\eta_2}\right\}}$$ (28)

Furthermore, we can find the generic expression for $V_i$. A rational player $i$ concedes at time $\epsilon > 0$ as $\epsilon \to 0$ and thus is always guaranteed at least $1 - \theta_j$. However, if his opponent is the “weaker” player, he may also get $\theta_i - (1 - \theta_j)$ additional pay-off at time zero with probability $R_j(0)$, where $R_j(t) = \frac{F_j(t)}{1 - z}$, is a rational player $j$’s mixed strategy cumulative distribution function. Thus, the expected pay-off from a new partner is

$$V_i = z_j(1 - \theta_j) + (1 - z_j)((1 - \theta_j) + (\theta_i - (1 - \theta_j))R_2(0))$$

$$= (1 - \theta_j) + (\theta_i - (1 - \theta_j))F_j(0)$$

$$= (1 - \theta_j)c_j + \theta_i(1 - c_j)$$

Now, given the parameters of the model, we can check whether there are equilibria which satisfy the following: either $c_1 = 1$ and $c_2 \leq 1$, and/or $c_2 = 1$ and $c_1 \leq 1$. We can hypothesize each of these...
cases, \( c_i = 1 \) for \( i = 1, 2 \) and check whether there exists \( c_j \leq 1 \). Given our hypothesis that say \( c_i = 1 \),

\[
c_j = z_j e^{\eta_j \tau}
\]

\[
= z_j e^{\min\{-\ln(x_i), -\ln(x_j)\}}
\]

\[
= \frac{z_j}{z_i^{\eta_i/\eta_j}}
\]

Then, we calculate \( V_i \) and \( V_j \) to make sure that the Coase Conjecture condition is satisfied. Note that for the ‘weak’ player \( j \) (with \( c_j < 1 \)), \( V_j = 1 - \theta_i \) always satisfies the Coase Conjecture condition.\(^{14}\)

Thus, we want to check whether \( V_i \), the rational ‘strong’ player \( i \)'s expected pay-off from a new partner, satisfies the Coase Conjecture condition. The intuition behind this is that the ‘strong’ player (with \( c_i = 1 \)) gets a favourable ‘gift’ (receiving \( \theta_i > 1 - \theta_j \)) with discrete probability \( F_j(0) > 0 \) from a new ‘weak’ bargaining partner conceding at time 0. Even when the ‘strong’ player knows with probability 1 that his opponent is of the irrational type, if this ‘gift’ is very profitable and the arrival rate of new partners is high enough, then the ‘strong’ player may prefer to wait for an arrival instead of yielding to irrational opponent (contradicting the Coase Conjecture condition).

As rudimentary comparative statics, we look at how each of our model’s parameters — discount rates, proportion of irrational types, arrival rates of new partners, and irrational demands—affect surplus allocation by skewing the individual parameter while keeping everything else symmetric between the two sides.

\(^{14}\)In this case, \( 1 - \theta_i \geq e^{-r_j/\lambda_j} (1 - \theta_i) \), thus the Coase Conjecture condition holds trivially for the ‘weak’ player.
- Different Discount Rates:

The relative impatience of the two sides (incorporated by the discount rates) is one of the fundamental determinants of bargaining power in our model. Just as in Rubinstein (1982), the more impatient side has less bargaining power and gets a smaller expected pay-off in equilibrium, all other characteristics kept symmetric between the two sides. In our model, a 'weak' agent 2 ($r_2 \geq r_1$) concedes with discrete probability at time 0 ($c_2 \leq 1$) and thus gets pay-off $1 - \theta_1$, while a 'strong' agent 1 benefits from being conceded to with discrete probability at time 0 and gets expected pay-off $V_1$. We note that the potential delay ($\tau$) decreases as one side gets increasingly impatient, as deals are agreed upon faster. We see that if one side is too impatient, i.e., $r_2 > r^*_2$, the Coase Conjecture condition is violated, and such a mixed-strategy mimicking equilibrium which we focus on doesn’t exist.

![Figure 6](image_url)

**Figure 6.** Numerical simulation of the effect on $\tau$ and $e^{-r_1/\lambda_1}V_1$ from increasing $r_2$, thus making agent 2 more impatient, while leaving all other parameters symmetric amongst the two types of agents.
• Different Proportions of Irrational Types:

Other characteristics kept symmetric between the two sides, if agent 2 starts off believing that his bargaining partner, agent 1, is irrational with higher probability \( z_1 \geq z_2 \), a higher prior, it takes agent 1 a shorter time to build a reputation, thus \( \tau \) decreases and a ‘strong’ agent 1 gets a ‘gift’ from a ‘weak’ agent 2 conceding with discrete probability at time zero. If agent 2’s prior of agent 1’s irrationality is too high, i.e., \( z_1 > z^*_1 \), the Coase Conjecture condition is violated, and such a mixed-strategy mimicking equilibrium which we focus on doesn’t exist.

![Figure 7](image)

**Figure 7.** Numerical simulation of the effect on \( \tau \) and \( e^{-r_1/\lambda_1} V_1 \) from increasing \( z_1 \), agent 1’s prior probability of irrationality, while leaving all other parameters symmetric amongst the two types of agents.
• Different Arrival Rates of New Partners:

Other characteristics kept symmetric between the two sides, if agent 1 can find new partners more frequently than agent 2 ($\lambda_1 \geq \lambda_2$), his endogenously determined outside option is more valuable and can credibly be used as a threat by agent 1 to increase his bargaining power. As $\lambda_1$ increases, potential delay ($\tau$) decreases, ‘strong’ agent 1’s bargaining power increases, and ‘weak’ agent 2’s bargaining power decreases as he has to concede with larger discrete probability at time 0. If the arrival rates are too skewed in favor of agent 1, i.e., $\lambda_1 > \lambda_1^*$, then such a mixed strategy mimicking equilibrium which we focus on doesn’t exist, as agent 1’s bargaining power becomes too large and the Coase Conjecture condition is violated.

![Figure 8](image)

**Figure 8.** Numerical simulation of the effect on $\tau$ and $e^{-\tau_1/\lambda_1} V_1$ from increasing $\lambda_1$, arrival rate of new partners for agent 1, while leaving all other parameters symmetric amongst the two types of agents.
• Different Irrational Demands:

Other characteristics kept symmetric between the two sides, we note that \( \eta_i \) and \( \eta_j \) are not monotonic for all ranges of \( \theta_i \) and \( \theta_j \). As in Abreu and Gul (2000), the side with the lower irrational demand is ‘strong.’ However, the pay-off impact of differences in \( \theta_i \) and \( \theta_j \) are highly dependent on the other parameters \((z, \lambda, r)\) for \( i, j \). In our numerical example, for the defined parameter space, note that such a mixed strategy mimicking equilibrium only holds in the range \( \theta_1 \in [0.51, 0.7] \), i.e., \( \theta_1 < \theta_2 \), in figure 9. Given the parameter space, \( \theta^{**}_1 \approx 0.5617 \) yields a rational player 1 the highest expected pay-off from a new partner. Note that \( \tau \) monotonically increases in response to changes in \( \theta_1 \).

![Figure 9](image)

**Figure 9.** Numerical simulation of the effect on \( \tau \) and \( e^{-r_i/\lambda_i}V_i \) from increasing \( \theta_1 \), irrational demand for agent 1, while leaving all other parameters symmetric amongst the two types of agents.

6.3. **Equilibrium when the Coase Conjecture Condition is Violated.** If the Coase Conjecture condition is violated, \( 1 - \theta_j < e^{-r_i/\lambda_i}V_i \), in our model, it is violated only for the ‘strong’ player (as it trivially holds for the ‘weak’ player). Thus, a rational ‘strong’ player would never accept his ‘weak’ opponent’s irrational demand in equilibrium (since he prefers not to accept it even when he believes with probability 1 that his ‘weak’ opponent is irrational) since his endogenous outside option of waiting for a new bargaining partner is more profitable. Therefore, in equilibrium, a rational ‘strong’ player will mimic the irrational type and a rational ‘weak’ player will prefer to concede as early as possible, i.e., at time 0, by accepting his ‘strong’ opponent’s irrational demand (‘one-sided reputation’).

We note that this equilibrium is distinct from the equilibrium derived in Compte and Jehiel (2002), who consider a model where both players’ exogenous outside options violate the Coase Conjecture condition. In our model, if the Coase Conjecture condition is satisfied, both sides mimic irrationality. However, if the Coase Conjecture condition is violated, the ‘strong’ player mimics and the rational ‘weak’ player immediately concedes to the strong player.
7. Revisiting Applications

Our model elaborates on the relationship between supply and demand which is fundamental to the study of economics. The model explains how factors such as impatience, irrational demands, and competition affect each side’s bargaining power in a market. It is the relative bargaining power that determines the allocation of surplus from the transaction, whether it is the size of venture capitalist’s stake in a company, wage given to the worker by the employer, collateral and haircuts the borrower must give to the lender in a repo transaction, or the prices of assets in decentralized markets for commodities, real estate, derivatives, or exotic securities (such as MBSs, CDOs, CDSs). It is hard to empirically test the impact of bargaining delay and whether irrational mimicking-driven bargaining causes this delay. However, the impact of relative bargaining power on the allocation of surplus (i.e., wage negotiated by workers and price of asset agreed to by counter-parties) and how this allocation changes in response to a change in the market environment can be assessed using available data.

We should note that prices of assets are not the most ideal data with which to test the allocation of surplus since our model abstracts from addressing more complex issues such as fundamental values of assets. Our model best fits the framework of venture capitalist’s equity stake, since 100% of stake is easily normalized to 1 unit surplus, and bargaining over how much of the stake is allocated to the start-up manager compared to the venture capitalist (VC) is a clear division of the unit surplus. Nevertheless, data on the VC stake is not readily available, while data on wages, prices of assets, spreads on interest rates,... is available and has been analyzed in the empirical literature, so we tend to focus on the impact of relative bargaining power on prices when describing empirical applications of our model.

Finally, it is important to realize that our model is a highly simplified and stylized version of the applications we hope to contextualize. The main simplifications are that each individual buyer is identical and each individual seller is identical, and that each transaction generates a total surplus of 1 (i.e., each worker produces the same output and each asset traded is identical). Decentralized markets are generally decentralized, because these characteristics are heterogeneous: over-the-counter derivatives are highly idiosyncratic and each worker has different qualifications. Thus, these markets differ from centralized markets such as stock markets, where each share of stock is identical to every other share of stock for a particular firm. These idiosyncratic differences between each of the buyers and each of the sellers might also lead to different surpluses from trade for different pairs (i.e., a highly qualified worker with 20 years of work experience might be more productive and produce more output for a firm than a worker who has little prior experience). Nevertheless, our model can provide the framework to obtain key intuition for understanding important market events such as unemployment, fire-sales, demand crunches, and liquidity crises.

7.1. Labor Markets & Wage Negotiations. Search theory is widely used in modelling labor markets to describe the interactions between workers who seek employment and firms which have vacancies to fill. In most cases, labor markets are highly decentralized where search frictions and delay from labor strikes are widespread. It is important to understand that labor frictions arising from delay in bargaining, strikes, unemployed workers, and excess capacity all adversely affect production and output of the real economy. Although wage bargaining and labor negotiation data (such as offers, counter-offers, demands, and delay) is hard to find, we attempt to explain certain events in the labor market.\[^{15}\]

Let us call type 1 agents “firms” and type 2 agents “workers.” During recessions, aggregate demand dries up in the economy; thus, production falls, firms start downsizing, increase lay-offs, and decrease hiring ($\lambda_2$ decreases). On the flip side, unemployed workers searching jobs increases ($\lambda_1$ increases). We note that $\lambda_2$ changes can also encapsulate frictions arising from displaced workers and misaligned skill-set due to a structural shifts (all factors which have been raised in explaining the

recent increase in unemployment). The Beveridge Curve exhibits this very relationship of vacancy rates and unemployment rates being negatively correlated (see figure 10), suggesting that our interpretation of $\lambda_1$ and $\lambda_2$ moving in opposite directions during booms and busts holds in the data. Furthermore, impatience levels of unemployed workers having a hard time finding jobs increases ($r_2$ increases), while firms who cut production in response to an aggregate demand dry-up become less impatient ($r_1$ decreases). Not only are firms more likely to be irrational ($z_1$ increases) and ask for higher surplus, lower wages ($\theta_1$ increases); but workers also do not appreciate nominal wage cuts to begin with, thus, $\theta_2$ may remain unchanged. Thus, during business cycle downturns and recessions, labor loses bargaining power, wages decline, search frictions increase, and delay in reaching agreement (which can be loosely proxied by duration of unemployment) increases. In the data, we point out how unemployment and wages are negatively correlated (see figure 11), and how during recessions, duration of unemployment increases (see figure 12).

Our model can help contextualize an interesting debate which arose in the aftermath of the crisis in response to the American Recovery and Reinvestment Act of 2009 (colloquially known as the “stimulus”). Particularly, the subject of contention was the extension of the social safety net for unemployed workers promised by the stimulus: expanding the food stamp program, increasing unemployment insurance, increasing worker tax cuts, and extending other unemployment benefits comprised $296 billion of the $840 billion stimulus package. The debate was over how these welfare programs would affect the underlying incentives of unemployed workers (i.e., how $r$ and $\theta$ change as a response to the stimulus program).

Recent literature involving labor market adjustments due to long-term unemployment and displaced workers includes Arulampalam, Gregg, and Gregory (2001), Jacobson, LaLonde, and Sullivan (2003), and Couch and Placzek (2010).

For literature regarding the Beveridge Curve refer to Blanchard and Diamond (1990).

Figures reported from Executive Office of the President of the United States (2014).

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**Figure 10.** The graph shows the negative correlation between unemployment rate and vacancy rate (the Beveridge Curve) decomposed into the time-series of the two rates. This relationship seems to suggest that our interpretation of $\lambda_1$ and $\lambda_2$ moving in opposite directions during booms and busts holds in the data.

*Data source: Federal Reserve Economic Data, Federal Reserve Bank of St. Louis*
Figure 11. The graph shows the negative correlation between unemployment rate and wages: our model suggests that this result is partly due to workers losing bargaining power during economic slumps.

Data source: Federal Reserve Economic Data, Federal Reserve Bank of St. Louis

Economists such as Robert Barro,\textsuperscript{19} believed that these programs provided disincentives to unemployed workers to not actively search for jobs and instead free-ride on government support. This outlook suggests that as a result of the stimulus, workers are more patient despite being unemployed, due to increased unemployment benefits ($r_2$ decreases). Also, workers are less likely to participate vigorously in the job market due to the unemployment benefits providing disincentives ($\lambda_1$ decreases). Thus, delay increases: loosely proxied by an increase in duration of unemployment. Furthermore, $\theta_2$ also increases as the unemployment benefits have increased the ‘perks’ of staying unemployed, thus workers are less motivated to look for low-paying jobs, demand high wages, and resist nominal wage cuts relative to previous employment. On the other side of the debate, economists such as Paul Krugman,\textsuperscript{20} advocated that these programs work, do not disincentivize workers, but rather provide a safety net which takes care of basic needs and allow workers to more actively search for jobs ($\lambda_1$ increases), and impatience ($r_2$) and nominal rigidities ($\theta_2$) remain unchanged. These economists argue that the welfare program decreased delay (i.e., long term unemployment) and helped mitigate the impact of the Great Recession on the labor market and unemployment.\textsuperscript{21}

Accessing the validity of either side of this argument is beyond the scope of this paper, but we simply depict how different theories fit into our model and which parameter shifts explain each side of the argument: a mere depiction of an application of our model.

7.2. OTC Asset Markets: Fire-sales, Demand Squeezes, and Panics. Let us consider decentralized, over-the-counter asset markets, such as markets for commodities, derivative securities, collateralized debt obligations, mortgage-backed securities, credit default swaps, and exotic financial securities. These markets do not have central exchanges, such as the stock exchange, thus, there are

\textsuperscript{19}Barro (2011).
\textsuperscript{20}Krugman (2009).
\textsuperscript{21}We note that our model ignores important factors such as changes in aggregate demand, which Krugman would consider of first order of importance when evaluating the success of such policies.
search frictions: buyers and sellers in the market must individually search for counter-parties and engage in bilateral bargaining over terms of trade. Our model can help explain the phenomena of panics and resulting fire-sales from a microeconomic perspective. While papers such as Kiyotaki and Moore (1995) explain how systemic panics spread via credit channels and fire-sales, our model hopes to take one step back and explain how market environment might change during the lead up to the panic: what changes in characteristics of market participants cause skewed allocation of surplus from trade. We attempt to explain how and why assets are sold at fire-sale prices to begin with by looking at bilateral bargaining partnerships of individual agents in the market.\footnote{Other literature on search frictions and OTC markets includes Lagos, Rocheteau, and Weill (2011), Lagos and Rocheteau (2007, 2009), Bernardo and Welch (2004), Dufe, Grleanu, and Pedersen (2005), and Weill (2007).}

Let us call type 1 agents the “buyers” and type 2 agents the “sellers” of the asset. For example, consider the market for mortgage-backed securities. We abstract from the so-called “fundamental valuation” of the asset and consider the price of the asset to reflect market confidence and be determined by the relative bargaining power of buyers and sellers. Up until the start of the Great Recession in 2007-8, prices of MBSs were stable and the market was functioning well. The Great Recession reflected a sudden loss of confidence in these exotic financial products. Supply of these products had slowly increased in the market during normal times, thus $\lambda_1$ was large: population of sellers was large thus buyers could more frequently meet new counter-parties to bargain/trade with. However, after the loss of confidence in MBSs, the population of buyers in the market fell dramatically, thus $\lambda_2$ fell: sellers couldn’t find counter-parties. Furthermore, due to the run for liquidity (during periods of crisis, risk aversion increases and demand for liquidity rises), supply of MBSs on the market ballooned, thus $\lambda_1$ increased even further. Furthermore, lenders who lent to buyers on a short-term basis demanded cash and more liquid collateral, thus, these financial institutions wanted to sell and liquidate their MBS holdings, $r_2$ increased: sellers became increasingly impatient to sell their assets. Finally, buyers losing confidence in MBS would increase $z_1$ and $\theta_1$: buyers are more likely to act irrationally and ask

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure12.png}
\caption{The graph shows how during recessions (shaded regions on the graph), duration of unemployment—which we use as an imperfect proxy for delay in reaching agreement in our model—increases.\textit{Data source: Federal Reserve Economic Data, Federal Reserve Bank of St. Louis}}
\end{figure}
for even higher surplus (lower prices). We see that all of these changes caused bargaining power of buyers to increase while that of sellers to decrease. Thus, allocation of surplus shifts in favor of buyers during recessions, explaining the collapse of asset prices in the market.

7.3. Repo Market: Runs and Credit Squeezes. In an earlier section we depicted the application of our model to demand squeezes in asset markets. In a similar vein, we use our model to attempt to provide intuition for credit squeezes, such as the one witnessed by the financial sector during the Great Recession.

High leverage and increased reliance on short-term borrowing in the repo market were two phenomena which were characteristic of the recent financial boom and bust leading to the Great Recession. Interbank borrowing on the repo market became increasingly prevalent in the run-up to the crisis and when lenders lost confidence in the liquidity and solvency of their counter-parties, demands for collateral were tightened and haircuts were increased. Our model abstracts from addressing the underlying questions of insolvency, illiquidity, and how a crisis of confidence arises; however, we can provide intuition regarding the allocation of surplus from transactions and the relative bargaining power in instances of liquidity panics.

Let us call type 1 agents “borrowers” and type 2 agents "lenders" in the repo market. When banks and shadow banks who were borrowers faced a run on wholesale funding, as lenders lost confidence in the borrower’s ability to remain solvent and repay the debt obligations, the market environment changed. Many lenders left the market due to fear while banks reliant on repo borrowing (either insolvent or illiquid) increased in the market (recall Lehman Brothers and Bear Stearns), thus $\lambda_1$ decreased while $\lambda_2$ increased. Fearing contagion and liquidity crisis leading to a retail banking panic on deposits, these systematically important institutions faced incredibly short time deadlines in which they had to secure funding and prevent bankruptcy. Thus impatience on the part of borrowers $r_1$ became extremely high. Finally, due to the loss of confidence amongst lenders, $z_2$ and $\theta_2$ increased as lenders started to demand greater quantities of safer collateral such as cash and Treasury bills, rather than MBSs and CDOs which were facing runs and fire-sales as explained above. Thus, bargaining power was highly skewed towards the lenders in the repo market and financial institutions such as Lehman Brothers and Bear Stearns, which had grown highly dependent on short term repo borrowing, faced turmoil. Their assets lost value due to a widespread loss of confidence in asset-backed securities such as CDOs and MBSs, their borrowing costs increased as collateral demands tightened and haircuts increased, and they tried to liquidate exotic assets on the market at fire-sale prices to both decrease leverage, prevent run on the bank, and put up safe collateral.

In short, from our model, we get an intuitive micro-bargaining perspective on how credit constraints and fire-sales became an issue for highly leveraged financial institutions whose balance sheets were loaded with exotic, risky assets often times backed by sub-prime debt, during the Great Recession. We explain the bilateral bargaining underpinnings of collateral runs and flight to safety.

7.4. Venture Capital Markets. Our model fits the interaction between venture capitalists and start-up managers extremely well, because, essentially, there is a clear unit pie (100% equity ownership of the start-up) which the start-up and venture capitalist bargain over in exchange for capital investment in the company. This measure of splitting a unit pie of surplus via buying equity stake

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24Literature on credit squeezes and bank runs includes the seminal paper Diamond and Dybvig (1983), and empirical papers on the recent repo market run include Gorton and Metrick (2009), Gorton (2007), and Hordahl and King (2008).
25A loss of confidence in shadow banking is reflected in credit default swap spreads for AIG, Lehman, and Merrill Lynch. Figure 4 on page 19 of Brunnermeier (2008).
26Part IV, Chapter 18 of Financial Crisis Inquiry Report (2011) contains a detailed anecdotal account of the bargaining between Lehman Brothers and its counter-parties JP Morgan, Citigroup, and Bank of America in the repo market, who increased their collateral demands, asking for more collateral and collateral in form of $5 billion in cash. The account shows how J.P. Morgan and other counter-parties behaved like or mimicked irrational agents, by not willing to accept riskier collateral and smaller haircuts.
27Features fundamental to Kiyotaki and Moore (1995).
Figure 13. The graph, figure 2 on page 9 from Gorton and Metrick (2009), shows how during the Great Recession, haircuts on sub-prime structured products faced a run. Haircuts reaching 100% implies that sub-prime structured products were no longer accepted as collateral. This shows the need for fire-sales of sub-prime backed structured products (we modelled in section 7.2) and the loss of bargaining power for repo borrowers.

in the company is a better analog for our model than prices of assets or labor as analyzed above. However, this data is not readily available and the empirical literature on venture capital market doesn’t focus on equity stake allocation.

Furthermore, it is a market where supply and demand feed off each other: for example, venture capital firms mushroomed in the late 1990s along with technology start-ups during the Dot-com bubble. Thus, it seems that this market is rarely highly asymmetric in participation of start-ups and venture capitalists. We could draw a broad comparison to simultaneous supply and demand increases during the Dot-com bubble as both \( \lambda \)s increase. As a loose approximation, we do note that in the symmetric case of our model, potential delay decreases as \( \lambda \) increases, as seen in figure 5. This might explain the unbelievable rise of VC-backed technology firms and IPOs during the height of the Dot-com bubble.

8. Conclusion

We develop a search-and-bargain model to understand the role of reputation building in decentralized markets. We endogenize the outside option which is modelled by a stochastic process determining the arrivals of new bargaining partners, i.e., the search friction in the decentralized market. The market equilibrium (i.e., bargaining strategies and the resulting allocation of the unit surplus agreed to by the two sides) depends on the market environment, which we parameterize by the relative rates

\[ \text{Average Haircut on Structured Products} \]
\[ \text{Non-Subprime-Related Index} \]
\[ \text{Subprime-Related Index} \]

\[ 1/2/2007 \quad 1/2/2008 \quad 1/2/2009 \]

\[ 0\% \quad 20\% \quad 40\% \quad 60\% \quad 80\% \quad 100\% \quad 120\% \]

Literature describing the venture capital industry, deal sizes, industry and sector-wise analysis, and other empirical studies includes Gompers and Lerner (1999, 2004), and Hellman (1998). However, the data and empirical analyses in these papers cannot be used to test our model in the VC market. We would ideally want data on equity stake bargained for by the venture capitalist and start-up founder, and analyze how this allocation changed over the years in response to changes in number of VC firms and start-ups.

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\[ \text{29} \]
of impatience, arrival rates of new partners, prior probability of irrational types in the market, and irrational demand.

First, we develop a discrete-time model without behavioral types, akin to Rubinstein and Wolinsky (1985), but with alternating offers. We derive a stationary subgame perfect Nash equilibrium and find that the equilibrium is dependent on the relative discount rates, the relative population sizes of the two populations, and the length of time between offers. The side that is more patient and has a smaller population (higher likelihood of finding new bargaining partners each period) has greater bargaining power and thus is compensated with higher surplus in equilibrium.

Second, we build a hybrid model without behavioral types, similar to Abreu and Pearce (2007), where alternating offers are made in discrete time but can be accepted in continuous time. This continuous time generalization of our model, with Poisson arrivals of new bargaining partners, allows us to maintain the Rubinstein bargaining structure of alternating offers with war of attrition style, optimal stopping action space for accepting offers. We derive consistency in limits between the discrete and hybrid models: as agents become infinitely patient, surplus allocations are determined by the relative arrival rates of new partners. Since the exponentially distributed arrival rates of new partners are fundamentally rooted in population sizes, we once again find that allocation of surplus is determined by the element of competition arising from relative population size differences; the more scarcely populated side reaping higher surplus in equilibrium.

Third, we construct a continuous-time model with irrational types, generalizing Abreu and Gul (2000) to include the endogenized outside option of meeting new bargaining partners in the search market. We formulate the Coase Conjecture condition, which if satisfied, ensures that if a rational player, believing with any positive probability that his opponent is irrational, chooses to concede, he will concede immediately to the opponent’s irrational demand. We show that when the Coase Conjecture condition is satisfied, there is a mimicking equilibrium, where rational types mimic the irrational demands and engage in a war of attrition style, optimal stopping game, resulting in two-sided reputation building and potential delay in reaching agreement. In our model, the Coase Conjecture condition fails if one side’s outside option (expected pay-off from meeting a new partner) is too high, giving him very high bargaining power. In this case, there is one-sided reputation building and no delay in reaching agreement. Rational players on the ‘strong’ side mimic irrationality, rational players on the ‘weak’ side immediately concede to their opponents’ irrational demand, and irrational players on the ‘weak’ side are not able to trade in the market due to incompatible demands.

In analyzing the effect of the market environment on the value of the endogenized outside option, a player’s bargaining power, and the potential delay in reaching agreement, we first consider the symmetric case, where the Coase Conjecture condition is trivially satisfied and a mimicking equilibrium always exists. We find that potential delay in reaching agreement decreases with greater impatience, more frequent arrivals of new partners, higher prior probability of irrationality, and smaller irrational demands for both sides. As long as the arrival rate of new partners is positive, the potential delay in our model is always smaller than the potential delay in Abreu and Gul (2000). Second, we consider the asymmetric case, where we vary only one parameter between the two sides, leaving all other parameters symmetric. The side which is more patient, has a higher prior probability of irrationality, and a higher rate of arrival of new partners has greater bargaining power and higher equilibrium pay-off due to discrete probability concession at time zero by the ‘weak’ rational opponent. The player with a lower irrational demand has more bargaining power; however, the pay-off is not monotonic in changes in irrational demand. As these parameters become increasingly asymmetric, heavily favoring the ‘strong’ side of the market by increasing the endogenized outside pay-off, the Coase Conjecture condition is violated, and a one-sided reputation-building equilibrium exists, where only the ‘strong’ side can mimic irrationality in equilibrium.

These derived comparative statics of our model allow us to get intuition for how major market events such as unemployment increases during recessions, fire-sale of assets during busts, and repo market credit crunch during liquidity panics are driven by the micro-founded market conditions which skew the relative bargaining power between counter-parties in these decentralized, search markets.
References


