Dealer Funding Costs: Implications for the Term Structure of Dividend Risk Premia

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Abstract

I show how debt-overhang funding costs to derivatives dealers’ shareholders for carrying and hedging inventory affect mid-market derivatives prices. An implication is that some supposed “no-arbitrage” pricing relationships, such as options put-call parity, frequently break down. I also explore the implications for measuring the term structure of S&P 500 dividend risk premia.

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I. Introduction

Derivatives dealers, acting as intermediaries, provide immediacy to ultimate investors by temporarily absorbing their net trade demands. In doing so, dealers have net funding requirements for carrying and hedging their inventories. A standard textbook presentation of “no-arbitrage” pricing assumes that dealers finance any net cash funding requirements for their market-making positions and related hedges at risk-free market interest rates. In reality, however, dealers often fund their cash requirements at other rates that are influenced by the credit risk of the dealer or by the type of collateral supplied by the dealer (Andersen, Duffie, and Song (2016)).

Here, I depart from the usual funding-rate assumptions of standard “no-arbitrage” pricing models. I build a structural model of a derivatives dealer’s balance sheet and account for more realistic funding costs. In this model, dealers provide immediacy to investors while hedging themselves through long and short positions in the underlying asset. Dealers are often cash-constrained, so have net cash funding requirements for their hedging positions. I allow a dealer to consider various alternative financing strategies. Assuming that dealers maximize their shareholders’ total equity market value, I show that repo financing is generally preferred by dealers over general unsecured debt issuance or secondary equity offerings.

As explained by Andersen, Duffie, and Song (2016), debt-overhang funding costs to dealers’ shareholders are an important determinant of dealers’ trading and pricing decisions. Here, I show that dealers may frequently prefer to finance derivatives hedging positions in repo markets. This means that the net cost to dealers for providing executable quotes depends on the spread between repo rates for the underlying asset and risk-free market interest rates. It follows that this spread has an impact on equilibrium mid-market derivatives prices.

An implication is that some supposed “no-arbitrage” pricing relationships, such as options put-call parity, can frequently break down to an economically important degree, without adjusting for dealers’ funding costs. Reliance on these “no-arbitrage” pricing relationships for other asset-pricing results may thus lead to unexpected results. Because of transactions costs, shorting access, capital-raising frictions, and other frictions, even deep-pocket sophisticated investors have difficulty exploiting the associated low-risk “arbitrage” opportunities. Hedge funds, for example, normally rely for funding on dealers’ prime brokerage services.

For example, van Binsbergen, Brandt, and Koijen (2012) (BBK) estimate S&P 500 dividend risk premia by relying on put-call parity to infer the prices of maturity-specific dividends paid by S&P 500 equities, known as “dividend strips.” That is, in order to derive a synthetic
option-implied dividend strip price, BBK apply the put-call parity formula

\[ \hat{P}_{t,T} = S_t + p_{t,T} - c_{t,T} - Ke^{-(T-t)\hat{y}_{t,T}}, \]  

(1)

where \( \hat{P}_{t,T} \) is the suggested synthetic option-implied price of dividends paid between times \( t \) and \( T \), \( S_t \) is the stock price, and \( p_{t,T} \) and \( c_{t,T} \) are the mid-market prices of European puts and calls respectively with exercise date \( T \) and strike price \( K \). For this purpose, BBK use the LIBOR-swap-implied zero-coupon rate \( \hat{y}_{t,T} \) as a proxy for the risk-free interest rate \( y_{t,T} \). The LIBOR-swap-implied zero curve is normally calculated from LIBOR rates, Eurodollar futures, and LIBOR-swap rates. In the maturity spectrum of between one and two years that BBK study, the LIBOR-swap-implied zero-coupon rate is generally much lower than dealers’ unsecured term-borrowing rate, and it is not available for financing to dealers.¹

Based on my supporting theory for dealer financing costs, the mid-market prices of call and put options with strike price \( K \) and maturity \( T \) satisfy

\[ c_{t,T} - p_{t,T} + Ke^{-(T-t)\hat{y}_{t,T}} = S_t e^{(T-t)\rho_{t,T}} - \mathcal{P}_{t,T}, \]  

(2)

where \( \rho_{t,T} \) is the spread between the preferred financing rate of a dealer for hedging the option positions and the risk-free rate \( y_{t,T} \), and \( \mathcal{P}_{t,T} \) is the market value of dividends paid between times \( t \) and \( T \). The first term on the right-hand side of (2) reflects the funding costs to a dealer’s shareholders for hedging. I show that a dealer’s preferred financing rate is often the associated repo rate. The second term reflects the dividend income to the dealer due to the equity hedging position. As a result, the dividend strip price should be derived from a financing-cost-adjusted (FCA) put-call parity formula, given by

\[ \mathcal{P}_{t,T} = S_t e^{(T-t)\rho_{t,T}} + p_{t,T} - c_{t,T} - Ke^{-(T-t)\hat{y}_{t,T}}. \]  

(3)

I will show that the discrepancy between the parity-implied dividend strip price \( \hat{P}_{t,T} \) in (1) and \( \mathcal{P}_{t,T} \) in (3) is caused mainly by ignoring the dealer funding costs for hedging dealing inventory.

My analysis focuses on long-dated S&P 500 options contracts with a time to exercise of

¹ The LIBOR-swap-implied zero-coupon rates are different to LIBOR rates in general. LIBOR rates are the average short-term unsecured financing rates of a panel of large active banks, and the maximum maturity of LIBOR rates is one year. Intuitively, the LIBOR-swap-implied zero-coupon rate is the rate for a hypothetical borrower whose credit quality is reset at the end of every floating rate coupon date (often every three months) to the average current quality of a panel of large active banks. As a result, it is generally much lower than the unsecured term-financing rate because a dealer’s actual term-financing rate reflects the market expected credit deterioration of the dealer over a number of successive coupon periods. See Section III.C for a detailed discussion.
between one and two years, consistent with the maturity spectrum of dividend risk premia estimated by BBK. Dealers are heavily involved in intermediating these long-dated options. Using the S&P 500 repo rates provided by a large dealer bank, I show that the costs to dealers of financing option hedging positions indeed have an economically important impact on parity-implied dividend strips prices, consistent with the predictions of my model. I also show that dealers’ funding costs explain a good portion of the time-series variation in parity-implied dividend strip prices.

The following example from the equity index market is illustrative of the importance of adjusting for dealer funding costs, in order to obtain reasonable synthetic pricing for equity dividend strips from longer-dated options. On August 20, 2013, European calls on the Eurostoxx 50 index (SX5E) with a strike price of €2800 and time to exercise of 1.33 years and 2.33 years, traded at prices of €208.5 and €263.9, respectively. European puts with the same strike price and time to exercise traded at prices of €314.0 and €433.8, respectively. The SX5E spot price was €2788.0. Applying the methodology used by BBK (1) to the Eurostoxx 50 example thus leads to an imputed market value for the SX5E dividends paid between December 19, 2014 to December 18, 2015, in Euros, of

\[
\hat{P}_{1.33,2.33} = \hat{P}_{2.33} - \hat{P}_{1.33} \approx 85.1,
\]

using the 1.33-year and 2.33-year EURIBOR-swap-implied zero-coupon rates of 0.38% and 0.54%, respectively (3). The 2015 SX5E dividend futures whose payoff is the SX5E dividends paid during the same period, traded at €103.4. This implies an estimation bias for the annualized risk premium for the 2015 SX5E dividend of

\[
\frac{1}{2.33} \log \left( \frac{103.4 \times e^{-2.33 \times 0.32\%}}{85.1} \right) \approx 8\%,
\]

where I use the associated overnight index swap (OIS) zero rate of 0.32% as a proxy for the risk-free rate (4)

I will show that this large estimation error for the parity-implied 2015 SX5E dividend

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2Eurostoxx 50 index options are traded on the Eurex exchange. SX5E index and the SX5E options prices are obtained from Bloomberg. The EURIBOR-swap-implied zero-coupon rates are also obtained from Bloomberg.

3Based on formula (1), \( \hat{P}_{2.33} = 2788.0 + 433.8 - 263.9 - 2800 \times e^{-2.33 \times 0.54\%} \approx 193 \) and \( \hat{P}_{1.33} = 2788.0 + 314.0 - 208.5 - 2800 \times e^{-1.33 \times 0.38\%} \approx 108 \).

4Eurostoxx 50 dividend futures are also traded on the Eurex exchange. Specifically, the payoff of the 2015 SX5E dividend futures is equal to the declared ordinary gross dividends of SX5E that go ex-dividend between December 19, 2014 and December 18, 2015. The SX5E dividend futures price is obtained from Bloomberg.

5The OIS zero rate is obtained from Bloomberg. See Section III for a discussion of OIS rates.
price is mainly caused by ignoring optimal or actual dealer financing costs for hedging their options positions. Applying the FCA put-call parity formula \[3\] leads to an implied dividend price in Euros, of

\[ P_{1.33,2.33} = P_{2.33} - P_{1.33} \approx 103.0. \]

Here, I have substituted into the FCA parity formula the actual 1.33-year and 2.33-year financing rates of 0.84% and 1.09% reported by Crédit Suisse (2013), respectively. As a proxy for the risk-free rates, I use the associated overnight index swap (OIS) zero rates of 0.18% and 0.32%, respectively. The resulting implied dividend futures price, \[P_{1.33,2.33} \times e^{2.33\times0.32\%} \approx 103.4,\] coincides with the observed SX5E dividend futures price.

The observation that the cost to dealer shareholders of financing hedge positions at the equity repo rate can cause a break-down in put-call parity is implicit in option-pricing methods already used by some market participants, as reported by Piterbarg (2010) and Lou (2014). These authors do not, however, offer a supporting model. Building on the marginal-valuation shareholder-preference theory developed by Andersen, Duffie, and Song (2016), my model shows that using repurchase agreements to finance a dealer’s hedging-related cash requirements is normally a preferred funding strategy for the dealer’s shareholders. I also show how the underlying repo rate affects equilibrium mid-market derivatives prices. I then use the model to explore the implications for measuring the term structure of S&P 500 dividend risk premia.

This paper is related to research that tests standard “no-arbitrage” pricing relationships. Examples include Brennan and Schwartz (1990), and Roll, Schwartz, and Subrahmanyam (2007), among many others. This literature typically relies on standard option put-call parity or futures cost-of-carry formula (without adjustment for dealer financing costs at rates other than the risk free rate), and often documents a break-down of “no-arbitrage” pricing relationships. Potential explanations for this breakdown offered by this literature include poor market liquidity of the underlying stocks, short-sell constraints, and other frictions. My paper provides another theoretical explanation for this breakdown. My explanation is likely to be more relevant for less liquidly traded positions that rely on greater access to dealer’s balance sheets, such as longer-dated equity options. Garleanu, Pedersen, and Poteshman (2009) address the implications of dealer immediacy for option pricing through the effect of inventory risk bearing, but do not account for the effect of dealers’ preferred cash financing strategies.

This paper also fits into the literature that examines the term structure of equity dividend

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\[P_{2.33} = 2788.0 \times e^{2.33\times(1.09\%-0.32\%)} + 433.8 - 263.9 - 2800 \times e^{-2.33\times0.32\%} \approx 228.0 \text{ and } P_{1.33} = 2788.0 \times e^{1.33\times(0.84\%-0.18\%)} + 314.0 - 208.5 - 2800 \times e^{-2.33\times0.18\%} \approx 125.0.\]

Section III provides a detailed discussion of OIS rates.
risk premia. van Binsbergen, Brandt, and Koijen (2012) use longer-dated option prices and the put-call parity to infer the prices of equity dividend strips. They document, based on the synthetic prices of dividend strips, that the returns on S&P 500 short-term dividend strategies are higher than the average return of the index itself. I show that the put-call parity should be adjusted significantly for longer-dated options in order to obtain reasonable synthetic pricing for equity dividend strips. Other explanations have been suggested for the findings in BBK. For example, Schulz (2015) argues that taxes could possibly explain the average high return on parity-implied dividend strips in BBK. Boguth, Carlson, Fisher, and Simutin (2013) argue that microstructure noise can be exacerbated when computing returns of parity-implied dividend strips. van Binsbergen and Koijen (2015) provide an excellent survey of related recent research.

van Binsbergen, Hueskes, Koijen, and Vrugt (2013), van Binsbergen and Koijen (2015), Cejnek and Randl (2015), and Cejnek and Randl (2016) study dividend risk premia by relying directly on dividend swap (futures) pricing data. Dividend-swap-implied dividend strip prices do not rely significantly on access to dealer balance sheets, and are therefore unlikely to be affected by the forces considered in this paper. In any case, those papers do not appear to provide support for the earlier suggestion of BBK of an average downward sloping term structure of S&P 500 dividend risk premia, consistent with the dealer-funding price distortions that I address in this paper. However, van Binsbergen and Koijen (2015) document that short-term dividend strips have outperformed the corresponding index in Europe which would be consistent with a downward-sloping term structure of dividend risk premia in Europe.

The rest of this paper is organized as follows. In Section II, I present the supporting theory, using a structural model of derivatives dealers’ balance sheets. I show that the rates at which dealers prefer to finance derivatives hedging positions have an impact on equilibrium mid-market derivatives prices. As an example of my theory, I show how parity-implied dividend strip prices should be adjusted for dealer funding costs for carrying and hedging inventory in Section III. I also test the model’s predictions in Section IV. Section V concludes. Supporting calculations and proofs are found in appendices.

van Binsbergen and Koijen (2015) rely on a sample of dividend swap prices from 2002 to 2014. They estimate that the monthly holding-period returns of 1-year and 1-to-2 year S&P 500 dividend strips are lower than that of the S&P 500 by 2.76% and 0.24% per year, respectively, over 2002-2014 (Table 2 of van Binsbergen and Koijen (2015)). Cejnek and Randl (2015) use a sample of S&P 500 dividend swap prices from 2006 to 2013 and Cejnek and Randl (2016) use a sample from 2005 to 2015. They also report that the returns of 1-year to 5-year S&P 500 dividend strategies underperform those of the benchmark index. (See Figure 1 and Table 2 of Cejnek and Randl (2015), and Table 1 of Cejnek and Randl (2016).) van Binsbergen and Koijen (2015) also document that the point estimate of short-end equity risk premia is higher than that of the index premium in Japan and in the UK, although the results are not statistically significant.
II. Model of Dealer Quotes

The model of dealer quotes developed in this section is an application of the marginal valuation shareholder-preference theory developed by Andersen, Duffie, and Song (2016).

A. Model Setup

I consider a model with periods 0 and 1. The risk-free gross rate of return is $Y$. That is, one can invest 1 at time zero and receive riskless payoff of $Y$ at time 1. A risky security, known as the “underlying,” pays $D_1$ at time 1 and then has an ex-dividend liquidation market value of $S_1$. An exogenous stochastic discount factor $M_1 > 0$ is used to discount future cash flows. That is, any asset with a cum-dividend value of $C_1$ at time 1 has a market value at time zero of $E(M_1 C_1)$. The underlying therefore has a market value of

$$S_0 = E(M_1 D_1) + E(M_1 S_1).$$

The market value of the dividend $D_1$ is $P = E(M_1 D_1)$.

There is also a forward contract on the underlying, by which an initially determined forward price $F$ is exchanged at time 1 for $S_1$. There are two kinds of agents, “end users” and “dealers.” End users have an exogenously given aggregate inelastic demand for the derivative at time 0. Dealers, acting as intermediaries, take the other side of end-user demand.

Dealers are competitive. For simplicity, I assume that dealers have identical legacy assets and liabilities at time 1 of $A$ and $L$, before considering new derivatives positions. The random variables $A$ and $L$ have finite expectations and a continuous joint probability density. A dealer defaults on the event $\mathcal{D} = \{A < L\}$, which is assumed to have a strictly positive probability. In that case, the dealer’s shareholders get zero and the dealer’s creditors recover a fraction $\kappa \leq 1$ of the dealer’s asset. Therefore, the dealer’s equity shareholders have a claim to $(A - L)^+$ before considering new trades.

Dealers hedge any new forward positions with end users through long and short positions in the underlying asset. I don’t endogenize this hedging motive. In practice, dealers do hedge their derivatives inventories (Piterbarg (2010) and Hull and White (2015)). I assume that dealers do not have ready cash on their balance sheets to fund hedge positions. Dealers obtain any necessary cash from external capital markets, choosing from among the financing options: (i) issue unsecured debt, (ii) issue equity, and (iii) place the underlying asset out on

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10I fix a probability space with a probability measure. All expectations are defined with respect to this probability measure.

11The following results also apply if the liability $L$ is deterministic.
repo. These external capital markets for financing are assumed to be competitive and based on symmetric information.

For simplicity, I focus on a forward contract, rather than the long-call short-put synthetic forward position considered by BBK. The equity hedge of a forward is essentially the same as the equity hedge of the long-call short-put position, on a delta basis. The funding costs to a dealer’s shareholders for hedging the forward therefore imply an adjustment to the dealer’s forward price quotes that are essentially the same as the total quote funding cost adjustment for the long-call short-put position.

**B. Individual Dealer’s Problem**

Suppose an end user asks a dealer for quotes on a forward position of size $q > 0$ on the underlying. The case of negative $q$, by which the end user sells a forward position, is treated in Appendix A. For simplicity, I assume that the end user is default free. In order to hedge the forward position, the dealer buys $q$ units of the underlying. My main objective is to compute the dealer’s reservation forward offer price, that offer price leaving the dealer’s shareholders indifferent to the entering forward position, after considering the effects of financing the forward hedge. Under the assumption that dealers maximize their equity market capitalization, dealers prefer to enter the new position if and only if the offer price is higher than the reserve offer price.

To this end, I follow Andersen, Duffie, and Song (2016) by characterizing the first-order impact of the new derivatives positions on the dealer’s equity market capitalization. That is, I calculate the first derivative of the value of the claim for the dealer’s shareholders, per unit of the claim. This first-order approach is reasonable unless the size of the trade is large relative to the dealer’s balance sheet, which would rarely be the cases for major dealers.

I will now show that the dealer strictly prefers to finance the hedge in the equity repo market, provided that the repo rate is not excessive.

**Case 1: Financing with Unsecured Debt.** I first consider the dealer’s potential choice to finance hedging positions by issuing unsecured debt. Let $s(q)$ denote the market credit spread on the newly-issued debt that is necessary to finance the underlying hedge for a forward position of $q$ units. The credit spread $s(q)$ is determined by the new forward position and the dealer’s legacy balance sheet. The detailed calculation of $s(q)$ is provided in Appendix A.

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12 The delta of a derivatives position is the partial derivative of the market value of the position with respect to the underlying price.

13 By including end-user default doesn’t change the results of the model. See Andersen, Duffie, and Song (2016) for a more general case, in which the end-user has strictly positive default probability.
After entering the new forward, hedging, and financing positions, the dealer’s shareholders have a claim to \((A + q(S_1 + D_1) - q(S_1 - F) - qS_0(Y + s(q)) - L)^+\) at time 1. The marginal impact of the net cash flows on the dealer’s equity market capitalization is

\[
G = \frac{\partial E[M_1(A + q(S_1 + D_1) - q(S_1 - F) - qS_0(Y + s(q)) - L)^+]}{\partial q} \bigg|_{q=0},
\]

assuming that the derivative is well defined. Appendix A includes a proof of the following result:

**Lemma 1:** If the dealer finances its hedging position by issuing unsecured debt, the marginal value \(G\) of the trade to shareholders is well defined and given by

\[
G = E[M_1 1_{D^c}(F + D_1 - S_0(Y + s))],
\]

where \(D^c = \{A \geq L\}\) is the event that the dealer does not default, and \(s\) is the dealer’s original unsecured credit spread. The reservation offer price \(\hat{F}\), that at which the marginal value \(G\) to shareholders is zero, is

\[
\hat{F} = S_0(Y + s) - E[1_{D^c}M_1D_1]/E[1_{D^c}M_1].
\]

That is, with debt financing, shareholders strictly prefer to enter at least some positive amount of the new position if and only if the offer price is higher than the reservation offer price \(\hat{F}\).

**Case 2: Financing Through Repo.** Suppose that the dealer instead finances the hedge in the repo market. In the opening leg of the repo, dealers supply the underlying and receive cash from a repo counterparty. In the closing leg of the repo, for every unit of cash received at time zero, the dealer must return \(\Psi_0\) in cash at time 1. That is, the one-period repo rate is \(\Psi_0 - 1\). At time 1, the repo counterparty will return the underlying asset to the dealer. For simplicity, I abstract from issues of over-collateralization (hair-cut). I also assume the repo counterparty is default free.\(^{14}\)

With repo financing of the hedge, the total equity claim is

\[
(A + qD_1 + qF - qS_0\Psi_0 - L)^+.
\]

Appendix A proves the following result.

**Lemma 2:** If the dealer finances the hedge in the repo market, the marginal value to share-

\(^{14}\)By including over-collateralization and repo counterparty default doesn’t change the model predictions.
holders of the net trade is well defined and given by

$$\tilde{G} = E[M_11_Dc(F + D_1 - S_0\Psi_0)].$$  \hfill (5)

The associated reservation forward offer price is

$$\tilde{F} = S_0\Psi_0 - \frac{E(M_11_DcD_1)}{E(M_11_Dc)}.$$

When I later apply this result to the measurement of dividend risk premia, the underlying is the S&P 500 index, and one period has a duration of between one and two years. In this case, the repo rate $\Psi_0 - 1$ is generally higher than the associated risk-free market rate $Y$, for at least one of the following reasons:

- An equity index is risker than general repo collateral, such as treasuries or U.S. agency debt instruments. For example, [Hu, Pan, and Wang (2015)] document that the average overnight equity tri-party repo rate was higher than the average treasury tri-party repo rate by 44 basis points between 2005-2008. This spread increased to about 80 basis points during the financial crisis;

- At this relatively long maturity, of at least one year, counterparty risk is not trivial. In practice, dealers often roll over short-term repo positions, rather than use long-term repo positions.\textsuperscript{15}

Appendix [A] considers the case in which the hedge is financed with a secondary equity offering, and shows that equity financing is the least favorable funding strategy for the dealer’s shareholders. Because of this, and because equity financing is rarely used in practice for transaction-level dealer financing, I will not consider it further.

\textbf{C. Equilibrium Derivatives Prices}

From now on, I assume that $\Psi_0 < Y + s$. That is, the repo rate is assumed to be lower than the dealer’s unsecured financing rate. This would typically follow from the fact that if a dealer defaults, a repo counterparty can rely on the collateral first, and then enter a claim for any shortfall, pari passu with unsecured creditors. The alternative case is rare in practice, for the applications that I will consider. In late 2015, however, the U.S. treasury general collateral (GC) repo rates has been higher than than LIBOR at maturities of one to three months. This is considered an extreme anomaly and has never occurred before (Skarecky [2015]).

\textsuperscript{15}For additional discussion of how dealers model and calibrate repo rates, see Combescot [2013].
PROPOSITION 1: Given any forward offer price $F$, the marginal value $\tilde{G}$ of the trade to shareholders under repo financing is strictly higher than the marginal value $G$ under unsecured debt financing.

That is, provided the repo rate $\Psi_0 - 1$ is not excessive, the dealer’s shareholders strictly prefer to fund derivatives hedging positions in the repo market.

So far, I have focused on the situation in which an end user wants to buy a forward position from the dealer. For the opposite case in which end users request bid quotes, dealers would typically establish the associated hedges through reverse repo in the repo market in practice. Appendix A calculates the marginal impact of the new positions on the dealer’s equity market value in this case.

I assume that dealers maximize their shareholders values. In a competitive bidding upon the request from end users, dealers would choose to finance the hedging positions in the repo market. The next result states that the the equilibrium forward bid and offer prices are identical.

PROPOSITION 2: Dealers strictly prefer to finance forward hedges in the repo market (over the alternatives of equity financing and unsecured debt financing). Any dealer’s reservation forward bid and offer prices are identical and given by

$$F = S_0 \Psi_0 - \frac{E(M_1 D c_1 D_1)}{E(M_1 D c_0)}.$$  \hfill (6)

In practice, dealers’ bid-offer quotes also include profit margins and frictional costs for overhead and inventory risk bearing, so that bid-offer spreads are usually positive. I omit these frictions for simplicity. If these frictional costs are similar for long and short positions, the mid-market price (the average of bid and offer prices) is well approximated by (6).

Andersen, Duffie, and Song (2016) show that debt-overhang funding costs to dealers’ shareholders for hedging are an important determinant of equilibrium derivatives prices. By showing that dealers have a preference to fund forward hedging positions in the repo market, I show that the repo rate for an asset underlying a forward contract (such as an equity index forward) can have an important impact on equilibrium mid-market forward prices.

III. Implied Dividend Strip Prices

This section applies the prior results to options put-call parity and the calculation of synthetic dividend strip prices.
A. Extension to Multi-Period Case

From now on, I assume for simplicity that the dealer’s survival event is independent of the stochastic discount factor $M_1$ and the dividend $D_1$. The mid-market forward price is then

$$F_m = S_0 \Psi_0 - \frac{E(M_1 D_1)}{E(M_1)} = S_0 \Psi_0 - Y \mathcal{P}.$$  

The first term on the right hand side reflects the funding costs to the dealer’s shareholders for hedging. The second term reflects the dividend income to the dealer that is associated with the underlying hedging position. Thus, the implied market price of the dividend $D_1$ is

$$\mathcal{P} = E(M_1 D_1) = S_0 \frac{\Psi_0}{Y} - F_m.$$

(7)

I will apply the model to a setting in which the underlying is the S&P 500 index and one period has a duration equal to the time to maturity of a dividend strip. I have so far assumed that the dividend of the underlying is paid at the end of the period. In reality, the dividends associated with the S&P 500 index are paid frequently over time. For this purpose, I will consider the stream of stochastic dividends of the underlying paid between time $t$ and $T$, and let $\mathcal{P}_{t,T}$ denote the market value of the claim to this dividend stream. I also follow an industry convention of using continuously compounding rates (measured on an annualized basis).

The funding-cost-adjusted pricing formula (7) can be readily extended to this case of interim dividends\textsuperscript{16} with the result that

$$\mathcal{P}_{t,T} \equiv \sum_{i=1}^{T-t} E_t(M_{t,t+i} D_{t+i}) = S_t e^{(T-t) \rho_{t,T}} - F_m e^{-(T-t) y_{t,T}},$$

(8)

where $E_t$ denotes conditional expectation at time $t$, $M_{t,t+i}$ is the stochastic discount factor at time $t$ for cash flows at time $t+i$, $D_{t+i}$ is the dividend paid at time $t+i$, and $\rho_{t,T} \equiv \psi_{t,T} - y_{t,T}$ is the continuously compounding spread between the S&P 500 repo rate $\psi_{t,T}$ and the risk-free market interest rate $y_{t,T}$ between times $t$ and $T$. In practice, the overnight index swap (OIS) zero-curve is a normal benchmark for the risk-free curve\textsuperscript{17}

\textsuperscript{16} The supporting calculations for the case with interim dividends are identical, and thus are omitted for brevity. See Andersen, Duffie, and Song (2016) for a multi-period structural model of dealer balance sheet.

\textsuperscript{17} The OIS rate is the fixed rate on an overnight index swap, which pays a predetermined fixed rate in exchange for receiving the compounded daily federal funds rate over the term of the contract. Hull and White (2013) provide a discussion of OIS rates.
B. Synthetic Pricing for Equity Dividend Strips

Based on the standard options put-call parity, the implied synthetic price \( \hat{P}_{t,T} \) of the S&P 500 dividend strip, as calculated by van Binsbergen, Brandt, and Koijen (2012), is

\[
\hat{P}_{t,T} = S_t - F_m e^{-(T-t)\hat{y}_{t,T}}. \tag{9}
\]

As mentioned earlier, BBK use the LIBOR-swap-implied zero-coupon rate \( \hat{y}_{t,T} \) as a proxy for the risk-free rate \( y_{t,T} \). Using (9), BBK then use mid-market call and put prices with the same strike price to infer the forward price \( F_m \).

Comparing (8) and (9), the dividend market value \( P_{t,T} \) implied by the dealer-preferred financing method and the value \( \hat{P}_{t,T} \) estimated by BBK differ by

\[
P_{t,T} - \hat{P}_{t,T} = S_t(e^{(T-t)\psi_{t,T}} - 1) - F_m(e^{-(T-t)y_{t,T}} - e^{-(T-t)\hat{y}_{t,T}}). \tag{10}
\]

Up to a first-order approximation,

\[
P_{t,T} - \hat{P}_{t,T} \approx (S_t - F_m)(\hat{y}_{t,T} - y_{t,T})(T-t) + S_t(\psi_{t,T} - \hat{y}_{t,T})(T-t), \tag{11}
\]

recalling that \( \psi_{t,T} \) is the continuously-compounding S&P 500 repo rate. The two terms on the right-hand side of (11) correspond to two potential roles of interest rates in synthetic pricing for equity dividend risk premia: (i) using the LIBOR-swap-implied zero curve in place of the risk-free curve, and (ii) ignoring optimal or actual dealer financing costs for carrying and hedging dealing inventory.

I briefly discuss the LIBOR-swap-implied zero curve that BBK rely on in their estimate of S&P 500 dividend strip prices. I also compare the LIBOR-swap-implied zero-coupon rate with LIBOR rate.

C. LIBOR-swap-implied Zero-coupon Rate

Although closely related, the LIBOR-swap-implied zero-coupon rates are different to LIBOR rates in general. LIBOR rates are the average short-term unsecured financing rates of a panel of large active banks, and the maximum maturity of LIBOR rates is one year. In other words, there is no LIBOR rate available in the maturity spectrum of between one and two years that BBK study.

The LIBOR-swap-implied zero curve is normally derived from LIBOR rates, Eurodollar futures, and LIBOR-swap rates, and the maturity can be as long as more than ten years. Intuitively, the LIBOR-swap-implied zero-coupon rate is the rate for a hypothetical borrower
whose credit quality is reset at the end of every floating rate coupon date (often every three months) to the average current quality of a panel of large active banks. As a result, it is much lower than the unsecured term-financing rate because a dealer’s actual term-financing rate reflects the market expected credit deterioration of the dealer over a number of successive coupon periods. See Duffie and Singleton (1997), Collin-Dufresne and Solnik (2001), and Feldhütter and Lando (2008) for more details.

In summary, the LIBOR-swap-implied zero-coupon rate that BBK use to estimate one-year to two-year dividend risk premia is not available for financing to dealers. In fact, the LIBOR-swap-implied rate is a reasonable proxy for risk-free rate before the financial crisis. For example, the average spread between the two-year LIBOR-swap-implied zero rate and the two-year OIS zero rate was 12 basis points between 2001 to 2007.

D. Potential Role of Funding Costs

I have displayed in (11) the two potential roles of interest rates in BBK’s estimates of dividend strip prices. Before the financial crisis, the first term in (11), \((S_t - F_m)(\hat{y}_{t,T} - y_{t,T})(T - t)\), was not significant for the following reasons: (i) The difference between the S&P 500 index value and the implied forward price of the S&P 500 index, \(S_t - F_m\), is usually at most a few percent of the index value \(S_t\), unless interest rates are extremely high and the maturities are extremely long. This was not a concern for the case addressed by BBK. (ii) Before the financial crisis, the LIBOR-swap-implied zero rate was a reasonable proxy for risk-free rate. BBK study the S&P 500 dividend risk premia mainly during the pre-crisis period. Excluding the period between 2008-2009 doesn’t seem to affect their results. As a result, the first term, \((S_t - F_m)(\hat{y}_{t,T} - y_{t,T})(T - t)\), was small relative to the second, \(S_t(\psi_{t,T} - \hat{y}_{t,T})(T - t)\), during the BBK sample period.

The main concern in the parity-implied dividend strip prices is ignoring the preferred dealer financing source for hedging. For my purpose of analyzing the estimates of dividend risk premia by BBK, I therefore rewrite (11) as

\[
\hat{P}_{t,T} \approx P_{t,T} - S_t \hat{\rho}_{t,T}(T - t),
\]

(12)

where \(\hat{\rho}_{t,T} \equiv (\psi_{t,T} - \hat{y}_{t,T})\). I will show this product is large enough to have an economically

---

18 The OIS zero rates and the LIBOR-swap-implied zero rates are obtained from Bloomberg. I also check the LIBOR-swap-implied zero rates with OptionMetrics. I obtain similar results for the two data sources.

19 To see this, I rewrite \(\text{(8)}\) as \(S_t - F_m \approx \hat{P}_{t,T} + F_m(T - t)y_{t,T} - S_t(T - t)\hat{\rho}_{t,T}\). The net values of the short-term dividend strips are a small fraction of the index value.

20 See Table 3 of BBK (2012). Since the financial crisis, however, even the LIBOR-swap-implied zero curve is no longer a reasonable proxy for the risk-free term structure.
important impact on synthetic pricing for dividend strips.

IV. Empirical Results

This section tests the model predictions in Section III using S&P 500 repo rates provided by a dealer bank.

A. The S&P 500 Repo Rate

A large U.S. dealer bank\textsuperscript{21} generously provided the term structure of the S&P 500 repo rates on the last day of each month from January 2013 to January 2016. Although these S&P 500 repo rate observations are from the post-crisis period, they are also informative of the potential magnitudes of spreads between S&P 500 repo rates and the LIBOR-swap-implied zero rates during the pre-crisis period.

Figure 1 displays 1-year spreads and 2-year spreads between the S&P 500 repo rates and the corresponding LIBOR-swap-implied zero rates from January 2013 to January 2016. The reported annualized 1-year and 2-year S&P 500 repo rates are on average 28 basis points and 32 basis points higher than, respectively, the associated LIBOR-swap-implied zero rates.

The following example is illustrative of the importance of dealer funding costs in estimating the S&P 500 dividend risk premia.

Example: I consider the annualized risk premium of buying a two-year S&P 500 synthetic dividend strip and holding it to maturity. Based on (12), the estimated effect of dealers’ funding costs for the two-year dividend risk premium is

\[
\frac{1}{2} \log \frac{P_{t,t+2}}{P_{t,t+2}} \approx \frac{\hat{\rho}_{t,t+2} + \hat{P}_{t,t+2}/S_t}{P_{t,t+2}/S_t} \approx 8.39%.
\]

For the purpose of this calculation, I take $\hat{\rho}_{t,t+2}$ to be 32 basis points (bps), which is the average reported spread between the two-year S&P 500 repo rate and the two-year LIBOR-swap-implied zero rate. I take $\hat{P}_{t,t+2}/S_t$ to be 380.1 bps, the sample average based on monthly data from January 2013 to January 2016, as estimated in Section IV.B.

Thus, in the setting of this example, if one were to use the standard put-call parity for estimating dividend strip prices, one would over-estimate the annualized holding-period return of two-year dividend strip by about 8%.

\textsuperscript{21}A major dealer provided the S&P 500 repo rates and the LIBOR-swap-implied zero curves. To judge the accuracy of the swap-implied zero curves, I compare them with the LIBOR-swap-implied zero-curves supplied by OptionMetrics before August 31, 2015. I obtain similar results for the two data sources.
Figure 1: Spread (in basis points) between the S&P 500 repo rate and the LIBOR-swap-implied zero-coupon rate from January 2013 to January 2016. Data source: a major dealer bank.

B. Funding Costs on Parity-Implied Dividend Prices

To test the theory developed in Section III, I follow the method of BBK, computing the parity-implied dividend strip prices on the last day of each month from January 2013 to January 2016 by relying on the put-call parity formula (1),

\[ \hat{P}_{t,T} = S_t + p_{t,T} - c_{t,T} - Ke^{-(T-t)\hat{y}_{t,T}}. \]

For this purpose, I obtain the closing best-bid and best-offer quotes of all S&P 500 option contracts supplied by OptionMetrics and Bloomberg. I also obtain the closing prices of S&P 500 index and the LIBOR-swap-implied zero curves from OptionMetrics and Bloomberg.\footnote{OptionMetrics supplies data before August 31, 2015. I obtain the option data and LIBOR-swap-implied zero curves between September 2015 to January 2016 from Bloomberg. I obtain similar results by excluding the samples from September 2015 to January 2016.} I collect quotes on call option contracts and put option contracts with the same strike price and time to maturity. For each of these put and call matches, I use put-call parity to calculate the implied dividend strip prices. As in BBK, I use mid-market options quotes taking the median across all prices for a given maturity. (Taking the mean rather than the median would not discernably change my results.) To obtain dividend prices at constant maturities, I follow BBK by interpolating over the available maturities.

<table>
<thead>
<tr>
<th>maturity</th>
<th>mean</th>
<th>standard deviation</th>
<th>mean</th>
<th>standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-year</td>
<td>34.62</td>
<td>3.28</td>
<td>1.86%</td>
<td>0.13%</td>
</tr>
<tr>
<td>1.5-year</td>
<td>52.61</td>
<td>4.15</td>
<td>2.83%</td>
<td>0.17%</td>
</tr>
<tr>
<td>2-year</td>
<td>70.53</td>
<td>5.20</td>
<td>3.80%</td>
<td>0.24%</td>
</tr>
</tbody>
</table>

Table I presents summary statistics for 1-year, 1.5-year, and 2-year parity-implied synthetic dividend prices from January 2013 to January 2016. The average implied market values of these synthetic dividend strips, in dollars, are 34.62, 52.61 and 70.53, respectively. Over the sample period, the parity-implied dividend strip prices are on average 1.86%, 2.83%, and 3.80% of the S&P 500 index value, respectively. These estimates are similar to those of BBK over their sample period from 1996 to 2009.

I have shown (see (12)) that the parity-implied dividend strip price $\hat{P}_{t,T}$ is negatively correlated with the funding cost to dealers of hedging inventory, given by $S_t\hat{\rho}_{t,T}(T - t)$. To test this, I estimate the following model:

$$\hat{P}_{t,t+h} = \frac{\hat{P}_{t,t+h}}{S_t} = \alpha + \beta(\hat{\rho}_{t,t+h}h) + \gamma^TX_t + \epsilon_{t+h},$$  \hspace{1cm} (13)

where $\hat{P}_{t,t+h}$ is the parity-implied dividend price with maturity $h$, $\hat{\rho}_{t,t+h}$ is the spread at maturity $h$ between the S&P 500 repo rate and LIBOR-swap-implied zero rate, and $X_t$ contains a series of control variables, including, for example, the CBOE VIX index, the TED spread, corporate bond spread. I assume that the residuals $\epsilon_{t+h}$ satisfy the standard conditions for ordinary-least-squares estimation.

The interested parameter is $\beta$ in (13). There are no reasons to believe that the funding costs to dealers’ shareholders of hedging S&P 500 index options should have a significant impact on the true market values of short-term dividend strips. As a result, if the synthetic prices calculated from the standard put-call parity are reasonable estimates of the dividend strip prices, we should expect to find

$$\beta = 0.$$  \hspace{1cm} (14)

I refer equation (14) as the null hypothesis.

Table II reports the regression results. As one can see, the estimate of $\beta$ is negative.
Table II  Regression results for (13). The period is from January 2013 through January 2016, and the frequency is monthly. TED spread is the difference between 3-month LIBOR rate and 3-month T-bill interest rate, VIX is the CBOE volatility index, 3M LIBOR is the 3-month LIBOR rate based on U.S. Dollar, HML and SMB are Fama-French HML and SMB factors, Baa-Fed Funds is Moody’s seasoned Baa corporate bond yield minus federal funds rate, Term premium is the term premium on 10-year zero coupon U.S. treasury bond. Standard errors are reported in parentheses.

<table>
<thead>
<tr>
<th>maturity $h$</th>
<th>1.5-year</th>
<th>1.75-year</th>
<th>2-year</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$-0.73$</td>
<td>$-1.02$</td>
<td>$-0.89$</td>
</tr>
<tr>
<td>(0.31)</td>
<td>(0.37)</td>
<td>(0.33)</td>
<td>(0.36)</td>
</tr>
<tr>
<td>TED spread</td>
<td>$5.49 \times 10^{-3}$</td>
<td>$9.17 \times 10^{-3}$</td>
<td>$8.04 \times 10^{-3}$</td>
</tr>
<tr>
<td>(0.017)</td>
<td>(0.019)</td>
<td></td>
<td>(0.022)</td>
</tr>
<tr>
<td>VIX</td>
<td>$4.49 \times 10^{-5}$</td>
<td>$3.89 \times 10^{-5}$</td>
<td>$2.62 \times 10^{-5}$</td>
</tr>
<tr>
<td>(9.53 $\times 10^{-5}$)</td>
<td>(1.08 $\times 10^{-4}$)</td>
<td>(1.25 $\times 10^{-4}$)</td>
<td></td>
</tr>
<tr>
<td>3M LIBOR</td>
<td>$-0.025$</td>
<td>$-0.030$</td>
<td>$-0.029$</td>
</tr>
<tr>
<td>(0.021)</td>
<td>(0.024)</td>
<td></td>
<td>(0.027)</td>
</tr>
<tr>
<td>HML</td>
<td>$-1.89 \times 10^{-4}$</td>
<td>$-2.01 \times 10^{-4}$</td>
<td>$-1.92 \times 10^{-4}$</td>
</tr>
<tr>
<td>(1.19 $\times 10^{-4}$)</td>
<td>(1.35 $\times 10^{-4}$)</td>
<td>(1.57 $\times 10^{-4}$)</td>
<td></td>
</tr>
<tr>
<td>SMB</td>
<td>$5.69 \times 10^{-5}$</td>
<td>$1.37 \times 10^{-4}$</td>
<td>$2.14 \times 10^{-4}$</td>
</tr>
<tr>
<td>(1.47 $\times 10^{-4}$)</td>
<td>(1.67 $\times 10^{-4}$)</td>
<td>(1.94 $\times 10^{-4}$)</td>
<td></td>
</tr>
<tr>
<td>Baa-Fed Funds</td>
<td>$4.20 \times 10^{-3}$</td>
<td>$5.12 \times 10^{-3}$</td>
<td>$5.65 \times 10^{-3}$</td>
</tr>
<tr>
<td>(1.39 $\times 10^{-3}$)</td>
<td>(1.56 $\times 10^{-3}$)</td>
<td>(1.78 $\times 10^{-3}$)</td>
<td></td>
</tr>
<tr>
<td>Term premium</td>
<td>$-5.56 \times 10^{-3}$</td>
<td>$-6.26 \times 10^{-3}$</td>
<td>$-6.74 \times 10^{-3}$</td>
</tr>
<tr>
<td>(1.72 $\times 10^{-3}$)</td>
<td>(1.93 $\times 10^{-3}$)</td>
<td>(2.24 $\times 10^{-3}$)</td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.11</td>
<td>0.49</td>
<td>0.17</td>
</tr>
</tbody>
</table>
and statistically significant for all the three maturities of 1.5-year, 1.75-year, and 2-year. In other words, we could readily reject the null hypothesis that $\beta = 0$. Moreover, the magnitude of $\beta$ is economically important. For example, the estimated parameter $\beta$ in the multi-variate regression is $-1.17$ for the 2-year maturity, with a $t$-statistic of $-3.26$. This would correspond, for a spread $\hat{\rho}_{t,t+2}$ of 32 bps, (the average spread from data reported in Section [IV.A]) to an upward bias in the estimated parity-implied strip price of around 17%. That is, the cost to dealers of financing S&P 500 index hedges has a significant impact on parity-implied dividend strip prices, in accordance with the predictions of my model of dealer financing.

BBK also highlight the “excess” volatility of their estimated parity-implied dividend strip prices. The regression (13) shows that a moderately large portion of the variation in the parity-implied dividend prices can be attributed to funding costs for S&P 500 index hedges. For example, the reported $R^2$ implies that funding costs along could explain about 20 percent of the variation (sample variance) in 2-year parity-implied dividend prices over the sample period.

I also estimate the following regression on innovations of parity-implied dividend prices and repo spreads:

$$\Delta \hat{P}_{t,t+h} / S_t = \tilde{\alpha} + \tilde{\beta}(\Delta \hat{\rho}_{t,t+h}) + \tilde{\gamma}^T \Delta X_t + \epsilon_{t+h},$$

(15)

where $\Delta \hat{P}_{t,t+h} / S_t = \hat{P}_{t+\delta,t+\delta+h} / S_{t+\delta} - \hat{P}_{t,t+h} / S_t$, $\Delta \hat{\rho}_{t,t+h} = \hat{\rho}_{t+\delta,t+\delta+h} - \hat{\rho}_{t,t+h}$, and $\Delta X_t = X_{t+\delta} - X_t$, with $\delta = 1/12$, corresponding to monthly frequency. Table II reports the results. Consistent with the model predictions, the estimate of $\tilde{\beta}$ is negative and statistically significant at all maturities, and none of the other control variables is statistically significant. Further, the variation in the innovations of parity-implied dividend prices is largely explained by the changes in dealer funding costs.

To summarize, these simple regressions show that put-call-parity-implied dividend prices are negatively correlated with the costs to dealers for financing of hedges, consistent with the predictions of my model in Section III. Although I rely on S&P 500 repo rates observations and option prices from the post-crisis period, these data are likely to be informative for the pre-crisis period.

C. Bid-Offer Spreads of Long-Dated Options

I have shown that some supposed “no-arbitrage” pricing relationships frequently break down due to dealers’ funding costs of carrying and hedging derivatives inventory. A natural
Table III  Regression results for (15). The period is from January 2013 through January 2016, and the frequency is monthly. TED spread is the difference between 3-month LIBOR rate and 3-month T-bill interest rate, VIX is the CBOE volatility index, 3M LIBOR is the 3-month LIBOR rate based on U.S. Dollar, HML and SMB are Fama-French HML and SMB factors, Baa-Fed Funds is Moody’s seasoned Baa corporate bond minus federal funds rate, Term premium is the term premium on 10-year zero coupon U.S. treasury bond. Standard errors are reported in parentheses.

<table>
<thead>
<tr>
<th>maturity h</th>
<th>1.5-year</th>
<th>1.75-year</th>
<th>2-year</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-1.26</td>
<td>-1.20</td>
<td>-1.33</td>
</tr>
<tr>
<td></td>
<td>(0.52)</td>
<td>(0.50)</td>
<td>(0.47)</td>
</tr>
<tr>
<td>TED spread</td>
<td>0.023</td>
<td>0.026</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.020)</td>
<td></td>
</tr>
<tr>
<td>VIX</td>
<td>$8.86 \times 10^{-5}$</td>
<td>$1.16 \times 10^{-4}$</td>
<td>$1.47 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>$(9.00 \times 10^{-5})$</td>
<td>$(9.63 \times 10^{-5})$</td>
<td>$(1.06 \times 10^{-4})$</td>
</tr>
<tr>
<td>3M LIBOR</td>
<td>-0.014</td>
<td>5.88 $\times 10^{-3}$</td>
<td>-0.021</td>
</tr>
<tr>
<td></td>
<td>(0.036)</td>
<td>(0.039)</td>
<td></td>
</tr>
<tr>
<td>HML</td>
<td>$1.24 \times 10^{-4}$</td>
<td>$-1.22 \times 10^{-4}$</td>
<td>$-1.01 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>$(8.52 \times 10^{-5})$</td>
<td>$(9.13 \times 10^{-5})$</td>
<td>$(1.12 \times 10^{-4})$</td>
</tr>
<tr>
<td>SMB</td>
<td>$-2.15 \times 10^{-4}$</td>
<td>$-1.77 \times 10^{-4}$</td>
<td>$-1.60 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>$(1.48 \times 10^{-4})$</td>
<td>$(1.59 \times 10^{-4})$</td>
<td>$(1.76 \times 10^{-4})$</td>
</tr>
<tr>
<td>Baa-Fed Funds</td>
<td>$-1.04 \times 10^{-3}$</td>
<td>$-1.02 \times 10^{-3}$</td>
<td>$-1.41 \times 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>$(2.76 \times 10^{-3})$</td>
<td>$(2.96 \times 10^{-3})$</td>
<td>$(3.27 \times 10^{-3})$</td>
</tr>
<tr>
<td>Term premium</td>
<td>$-1.22 \times 10^{-3}$</td>
<td>$-1.13 \times 10^{-3}$</td>
<td>$-7.12 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>$(2.52 \times 10^{-3})$</td>
<td>$(2.70 \times 10^{-3})$</td>
<td>$(3.00 \times 10^{-3})$</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.16</td>
<td>0.44</td>
<td>0.22</td>
</tr>
</tbody>
</table>
question is whether some sophisticated investors, such as hedge funds or “deep-pocket”
investors, could take advantage of the implied seemingly low-risk “arbitrage” opportunity.

In practice, possibly due to bid-ask spreads, shorting access, and capital-raising frictions,
among other capital market frictions, it is generally difficult for investors to take advantage
of the implied opportunities. Hedge funds, for example, normally rely on funding on dealers’ prime brokerage services. Besides, investors incur options bid-ask spreads in order to
establish the synthetic dividend positions. The bid-ask spreads for long-dated options contracts are normally wide. For example, between 2002 to 2007, the average bid-offer spread for long-dated S&P 500 index options with maturities between one year and two years is $2.77. The average estimate by BBK of the 1.5-year synthetic dividend price is $32.65. This implies a substantial proportional transaction cost of about 8.5%.

V. Conclusion

When providing immediacy to ultimate investors, derivatives dealers usually have net
cash funding requirements for hedging their dealing inventories. I show that repo financing of
the hedging positions is generally preferred by a dealer’s shareholders over general unsecured
debt issuance or secondary equity offerings. Under the assumption that dealers maximize
their shareholders’ total equity market value, I show that equilibrium mid-market derivatives
prices depend on the spread between repo rates for the underlying asset and market risk-free
rates.

An implication is that some supposed “no-arbitrage” pricing relationships, which rely
on the notion that any net funding needs are financed at market risk-free rates, frequently
break down. In particular, I explore the implications for measuring the term structure of
S&P 500 dividend risk premia.

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Boguth, Oliver, Murray Carlson, Adlai Fisher, and Mikhail Simutin, 2013, Leverage and

23BBK use mid-market options prices.

24I collect the daily best-offer and best-bid prices of all the S&P 500 options contracts with a time-to-
exercise of between one and two years from OptionMetrics. I exclude contracts with zero open interest. I
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**Appendix A  Proofs and Other Calculations for Section II**

This appendix supplies proofs of Lemmas 1 and 2 and Proposition 1.

**Proof of Lemma 1** Because I have assumed a competitive capital market with complete information, creditors offering the new debt break even. That is, the market credit spread \(s(q)\) on the new debt, which is issued to finance the hedging position, solves

\[
Y = E \left[ M_1 \left( 1_{\mathcal{D}(q)}(Y + s(q)) + \frac{1_{\mathcal{D}(q)} \kappa (A + q(S_1 + D_1) + q(S_1 - F)^-) + q(S_1 - F)^+ (Y + s(q))}{L + qS_0(Y + s(q)) + q(S_1 - F)^+ (Y + s(q))} \right) \right],
\]

where \(\mathcal{D}(q)\) is the dealer’s survival event \(\{A+q(S_1+D_1)-q(S_1-F)-qS_0(Y+s(q))-L>0\}\). By letting \(q\) go to zero, it is easy to see that \(\lim_{q \to 0} s(q)\) exists, and

\[
\lim_{q \to 0} s(q) = s = \frac{Y^2 E[M_1 1_{\mathcal{D}}(1-\kappa A/L)]}{1 - Y E[M_1 1_{\mathcal{D}}(1-\kappa A/L)]},
\]
where \( s \) is the dealer’s original unsecured credit spread.

If the dealer finances the hedging position by issuing new debt, then the marginal value of the portfolios to its shareholders is

\[
G = \left. \frac{\partial E[M_1(A + q(S_1 + D_1) - q(S_1 - F) - qS_0(Y + s(q)) - L^+)]}{\partial q} \right|_{q=0}.
\]

The objective is to show that the derivatives exists and is given by

\[
G = E[M_1 \mathbf{1}_{D^c}(F + D_1 - S_0(Y + s))].
\]

By definition,

\[
G = \lim_{q \to 0} E[M_1 \mathbf{1}_{D^c(q)}(A + q(D_1 + F) - L - qS_0(Y + s(q)))] - E[M_1 \mathbf{1}_{D^c}(A - L)]
\]

\[
= \lim_{q \to 0} \frac{E[M_1 \mathbf{1}_{D^c(q)}(q(D_1 + F) - qS_0(Y + s(q)))]}{q} + E[M_1 \mathbf{1}_{D^c(q)}(A - L)]
\]

It is easy to see that

\[
\lim_{q \to 0} \frac{E[M_1 \mathbf{1}_{D^c(q)}(q(D_1 + F) - qS_0(Y + s(q)))]}{q} = \lim_{q \to 0} E[M_1 \mathbf{1}_{D^c(q)}(D_1 + F - S_0(Y + s(q)))]
\]

\[
= E[M_1 \mathbf{1}_{D^c}(D_1 + F - S_0(Y + s))],
\]

where the last equality is due to the fact that \( A \) and \( L \) have finite expectations, allowing interchangeability of the limit and expectation.

Notice that

\[
\mathbf{1}_{D^c(q)} - 1_{D^c} = \mathbf{1}_{D^c(q) \cap D} - 1_{D(q) \cap D^c},
\]

and

\[
|A - L| \leq q|D_1 + F - (Y + s(q))S_0|
\]

in the events \( D^c(q) \cap D \) and \( D(q) \cap D^c \). Thus,

\[
\lim_{q \to 0} \frac{E[M_1|(1_{D^c(q)} - 1_{D^c})(A - L)|]}{q} \leq \lim_{q \to 0} \frac{E[M_1|1_{D^c(q) \cap D}(A - L)| + E[M_1|1_{D(q) \cap D^c}(A - L)|]}{q}
\]

\[
\leq \lim_{q \to 0} E[M_1|(1_{D^c(q) \cap D} + 1_{D(q) \cap D^c})(D_1 + F - S_0(Y + s(q)))]].
\]
By the Lebesgue Dominated Converge Theorem,
\[
\lim_{q \to 0} E[M_1|(1_{D^c(q) \cap D} + 1_{D(q) \cap D^c})(D_1 + F)|] = E \left[ M_1 \lim_{q \to 0} |(1_{D^c(q) \cap D} + 1_{D(q) \cap D^c})(D_1 + F)| \right] = 0,
\]
where the last equality is due to the fact that \(A\) and \(L\) have a continuous joint density. Because \(\lim_{q \to 0} s(q)\) exist, I also have
\[
\lim_{q \to 0} E \left[ (1_{D^c(q) \cap D} + 1_{D(q) \cap D^c})S_0(r + s(q)) \right] = 0.
\]
Thus,
\[
\lim_{q \to 0} E[M_1|(1_{D^c(q)} - 1_{D^c})(A - L)|] = 0,
\]
and I have shown that
\[
G = E[M_11_{D^c}(F + D_1 - S_0(Y + s))].
\]

**Proof of Lemma 2:** If the dealer finances the hedging position through the repo market, then the total equity claim is \(E[M_1(A + q(S_1 + D_1) - q(S_1 - F) - qS_0\Psi_0 - L)^+]\), where \(\Psi_0 - 1\) is the underlying repo rate.

Thus, the marginal value of the portfolios to its shareholders is
\[
\tilde{G} = \frac{\partial E[M_1(A + q(S_1 + D_1) - q(S_1 - F) - qS_0\Psi_0 - L)^+]}{\partial q} \bigg|_{q=0}.
\]

Similar calculations from the proof of Lemma 1 apply here, and one can easily show that \(\tilde{G}\) exists and is given by
\[
\tilde{G} = E[M_11_{D^c}(F + D_1 - S_0\Psi_0)].
\]

**The Case of Equity Financing:** I also consider the case that the dealer finances the hedging positions by issuing new equities. Because the investors in a competitive market for the newly issued equity break even on the purpose of shares, then the market value of the legacy shareholders’ equity is given by
\[
E[M_1(A + q(S_1 + D_1) - q(S_1 - F) - L)^+] - qS_0.
\]

One can show as in the proof of Lemma 1 that the marginal value of the portfolios to the dealer’s legacy shareholders exists and is given by
\[
\hat{G} = E[M_11_{D^c}(D_1 + F)] - S_0.
\]
Proof of Proposition 1: I have assumed that \( \Psi_0 < Y + s \). That is, the repo rate of the underlying is assumed to be lower than the dealer’s unsecured financing rate. Thus, it is straightforward to see that

\[ \tilde{G} > G. \]

On the other hand, if the repo rate is higher than the dealer’s unsecured financing rate, that is, \( \Psi_0 \geq Y + s \), then

\[ \tilde{G} \leq G. \]

Now I show that \( G > \hat{G} \), that is, the marginal value to shareholders under unsecured debt financing is strictly higher than the marginal value under equity financing. It suffices to show that

\[ E[M_1 1_{De}(Y + s)] < 1. \]

Recall that the dealer’s unsecured credit spread

\[ s = \frac{Y^2 E[M_1 1_D(1 - \kappa A/L)]}{1 - YE[M_1 1_D(1 - \kappa A/L)]}. \]

Thus, I only need to show

\[ Y(E[M_1 1_{De}] + E[M_1 1_D(1 - \kappa A/L)]) < 1. \]

This is an immediate result due to \( 1_D(1 - \kappa A/L) < 1_D \) and \( YE(M_1) = 1 \).

The Case of Bid Quotes: If an end user wants to sell a forward position, a dealer usually hedges its position through reverse repo in the repo market. That is, in the opening leg, the dealer receives the underlying and supplies cash to the repo counterparty, where the cash is from selling the underlying. In the closing leg, the dealer buys back underlying and return the underlying, together with dividend, in exchange for cash and interest rate, which is the repo rate.

In this case, the total equity claim to the dealer’s shareholder is

\[ (A + qS_0 \Psi_0 + q(S_1 - F) - q(S_1 + D_1) - L)^+. \]

Following similar calculations as in the proof of Lemma 1, the marginal value of the net trade to shareholders is \( E[M_1 1_{De}(S_0 \Psi_0 - F - D_1)] \).