Initiation of Merger and Acquisition Negotiation with Two-Sided Private Information*

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Abstract

In a dynamic model of merger negotiation with two-sided private information and two-sided endogenous initiation, the paper investigates (1) what determines the timing of M&A initiation, (2) who initiates the M&A negotiation, and (3) why bid premia are higher for bidder-initiated deals than target-initiated deals. The key driving force for the results is that the timing of initiation can reveal information about the target’s private signal of its stand-alone value, and the bidder’s private signal about its valuation for the target’s firm. The model predictions are consistent with the empirical literature that emphasizes the role of private information in deal-initiation.

Empirical evidence shows that who initiates an M&A deal varies from case to case, and this matters for how much bid premium the targets receive. Masulis and Simsir (2015) document that 36.7% of deals are target-initiated, while 63.3% are bidder-initiated. Also, targets receive lower bid premium for target-initiated deals (48.8%) than in bidder-initiated deals (62.8%). Moreover, the difference remains after controlling for observable target and bidder financial conditions, implying

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that private information might be at play. Aktas, de Bodt and Roll (2010) also show that target firms receive 32.1% points lower bid premium in target-initiated deals than in bidder-initiated deals, with a sample restricted to one-to-one negotiations. Despite the evidence above, there is little in the theoretical literature that discusses initiation of M&A deals. To fill in the gap in the literature, we build a framework with two-sided private information to explain endogenous initiations of M&A negotiations. We study the following research questions.

When a target and a bidder are considering a one-to-one negotiation,

1. What determines the timing of the initiation of the M&A negotiation?
2. Who becomes the initiator?
3. Why target firms receive lower bid premium in target-initiated deals than in bidder-initiated deals?

We focus on one-to-one negotiations instead of auctions. One-to-one negotiations are very common in practice. According to Boone and Mulherin (2007) and Aktas, de Bodt and Roll (2010), about 50% of M&A deals are one-to-one negotiation, while the other 50% are public or private auction. Moreover, the variation in the choice of selling mechanisms cannot explain the difference in bid premium between target-initiated and bidder-initiated deals. First, in the sub-sample of one-to-one negotiation, the difference in bid premium due to different initiators not only exists, but is also higher than the difference in the entire sample (Aktas, de Bodt and Roll (2010), Masulis and Simsir (2015)). Second, in the entire sample, bid premium does not vary significantly with the choice of auctions vs negotiations. Therefore, although target-initiated deals are more likely to be auctions and bidder-initiated deals are more likely to be negotiations (Aktas, de Bodt and Roll (2010)), who initiates remains to be the key determinant for the difference in bid premium between target-initiated deals and bidder-initiated deals. Finally, the intuition of this paper should be able to extend to auctions where both the target and the bidders have private information. This is because while the paper focuses on the seller-buyer competition, in an auction with many buyers and one seller, both the seller-buyer competition and the buyer-buyer competition exist.

The key driving force of the timing of initiation is the target’s private information about its
stand-alone value, and the bidder’s private information about its valuation on the target firm. In particular, if a firm delays initiation, it is able to obtain a higher bid premium by either (i) initiating later and pretending to be a stronger initiator, or (ii) waiting for the opponent to initiate. For the latter case in particular, the firm is able to hide its private signal and blend in the crowd of stronger types, therefore maintaining its information advantage as a non-initiator and obtaining higher payoff. Indeed, it is exactly such information advantage of the non-initiator that explains the empirically documented difference in bid premium between target-initiated deals and bidder-initiated deals due to non-observables (Masulis and Simsir (2015)). In addition, the latter “waiting for the opponent to initiate” motive gives a flavor of “war of attrition” to the game, making the initiation decisions strategic substitutes between the two firms. That is, if one firm initiates earlier, the opponent would like to postpone initiation longer.

This paper also shows that although higher bid premium followed by later initiation is desirable, weaker types such as targets with lower stand-alone value and bidders with higher valuation for the target firms rationally choose to initiate the deal early. This is because the marginal benefit for delaying initiation is lower for those weaker types. As a result, our model predicts that target-initiated deals happen when both the target’s stand-alone value and the bidder’s value for the target’s firm are relatively low but is unobserved by the public; a deal is bidder-initiated when both the private signals of the target’s stand-alone value and the bidder’s value for the target’s firm are relatively high.

Finally, the predictions of our model are consistent with the empirical evidence of the difference in bid premium between target-initiated deals vs bidder initiated deals. In addition, our results are also consistent with the fact that such difference increases with the uncertainty about the target firm.
Related Literature

Although there is a rich literature on M&A, both empirical and theoretical research on the initiation of M&A deal are very limited. One exception in empirical research is Masulis and Simsir (2015), which reveals that target financial or economic weakness, target financial constraints and economy wide shocks are important motives for target-initiated deals. They also find that average bid premiums, target cumulative abnormal returns (CAR) measured around the merger announcement dates and the deal value to EBITDA multiples of target-initiated deals is significantly lower than in bidder-initiated deals. However, this gap cannot be explained by weaker financial conditions of targets immediately prior to merger announcements. After adjusting for self-selection, they find evidence that the private information held by target firms is the main driver of the lower premiums observed in target-initiated deals. Another empirical paper is Aktas, de Bodt and Roll (2010), which also show that target firms receive 32.1% points lower bid premium in target-initiated deals than in bidder-initiated deals within a sample restricted to one-to-one negotiations.

There are a few exceptions in theoretical research. Gorbenko and Malenko (2014) study the initiation of M&A by bidders who are privately informed about their valuation of the target firm. Gorbenko and Malenko (2015) allow for both the bidders and the target to initiate an auction, but the seller does not have any private information. Cong (2015) also considers target and bidder initiation of an auction on real options, but target private information is still absent. Most importantly, none of the three papers explain the difference of bid premium between target-initiated deals and bidder-initiated deals, which is the focus of our paper.

In addition, the paper is related to the literature of dynamic signaling (Admati and Perry (1987), Daley and Green (2012)) and war of attrition (Bulow and Klemperer (1999), Abreu and Gul (2000)). Actually, it is a non-trivial combination of the two literature. First, simple dynamic signaling models with one-sided initiation do not explain the difference between bid premium in target-initiated deals and bidder-initiated deals, not does it have the feature of initiation as strategic substitutes. Second, unlike standard models on war of attrition in which the payoff after one agent gives up do not depend on the time at which the agent gives up, in our model the
payoffs after initiation are endogenously determined by the initiation timing precisely because of
the feature of dynamic signaling.

Finally, the paper is related to the bargaining literature. Cramton (1992) introduced a bar-
gaining model with two-sided private information. However, he made an unnatural assumption
that beliefs stay fixed even if receiving an offer that is inconsistent with the previously formed
belief. In addition, he assumed a particular type of bargaining protocol, while our model allows
for a general set of bargaining protocols.

The rest of the paper is organized as follows. Section 1 provides the assumptions and the
equilibrium concept; Section 2 solves the model; Section 3 show the robustness of our results
under more general bargaining protocols; Section 4 illustrates that our model generates realistic
empirical predictions; Section 5 concludes.

1 The Model

1.1 Model Setup

The story begins at $t = 0$. There are one bidder and one target, both are risk neutral with discount
rate $r$.

The target’s stand-alone value if no initiation or if the negotiation breaks down is $c \in [l, h]$, with $l \geq 0$. Denote the p.d.f. and c.d.f. of $c$ as $f_c(\cdot)$ and $F_c(\cdot)$. Assume $f_c(\cdot)$ is continuously
differentiable with $f_c(c) > 0$ for all $c \in [l, h]$, and is line symmetric around the center of the
support, $\frac{l+h}{2}$. The target is privately informed about its stand-alone value.

The bidder’s value on the target’s firm is $v \in [A + l, A + h]$, while the bidder’s stand-alone
value is normalized to zero. Denote the p.d.f. and c.d.f. of $v$ as $f_v(\cdot)$ and $F_v(\cdot)$. Assume $f_v(\cdot)$ is
continuously differentiable with $f_v(v) > 0$ for all $v \in [A + l, A + h]$, and is line symmetric around
the center of the support, $A + \frac{l+h}{2}$. The bidder is privately informed about its value of the target’s
firm.

Further, we assume that $c$ and $v$ are independently distributed. One can imagine a big firm such
as Google is trying to buy a small start-up firm. It’s likely that the start up after being incorporated into Google is completely different from what it is as a stand-alone firm due to synergies. Hence we restrict our attention to the case where the synergies are independent of target’s stand-alone value.

Also, we consider the case with common knowledge of gains from trade, or \( A + l > h \). That is, the lowest bidder value \( A + l \) is strictly higher than the highest target stand-alone value \( h \). This corresponds to the “gap case” in the bargaining literature.

Starting at \( t = 0 \), both firms can initiate at any time \( t \in [0, +\infty) \), and time is continuous. Bargaining between the two parties starts once any firm initiates. Following Nash (1953), we assume the bargaining protocol after initiation to be a *Nash Demand Game* with pure cash offers. In this one-shot bargaining game, the target submits its price \( p_T \), and the bidder submits its price \( p_B \) simultaneously. If the price asked by the target is smaller or equal to the price bidder offers \( (p_T \leq p_B) \), the bidder pays its offered price and the target gets its asked price; otherwise, the target gets its outside option \( c \), and the bidder gets its outside option zero. Therefore the payoffs to the target and the bidder if deal succeeds are \( p_T \) and \( v - p_B \) respectively; if the bargaining fails, their payoffs are \( c \) and 0.

Although we focus on this particular type of bargaining protocol for tractability, we will show in an extension that the same results still hold for more general bargaining protocols as long as they satisfy certain properties.

To wrap up the model setup, we define formally the belief system and the strategy profile of the game.

**Definition 1.1.** A belief system of the game is \( \mu_T = \{ \mu_{T,t} \}_{t=0}^{+\infty}, \mu_B = \{ \mu_{B,t} \}_{t=0}^{+\infty} \), where \( \mu_{T,t}(\cdot) : H_t \to \Delta ([l, h]) \) is the target’s time-\( t \) belief on the bidder’s type based on the history up to time \( t \), \( \mu_{B,t}(\cdot) : H_t \to \Delta ([A + l, A + h]) \) is the bidder’s time-\( t \) belief on the target’s type based on the history up to time \( t \).

**Definition 1.2.** A pure strategy profile of the game is \( \{ \tau_T (\cdot), \tau_B (\cdot), p_T (\cdot), p_B (\cdot) \} \), where \( \tau_T (\cdot) : [l, h] \to [0, +\infty) \) is the timing of initiation for the target as a function of its type, \( \tau_B (\cdot) : [l, h] \to [0, +\infty) \) is the timing of initiation for the bidder as a function of its type.
\[ t = 0 \]
\[ \tau_T(c), \tau_B(v) \]
\[ t = \tau_I, I \in \{T, B\} \]
\[ p_T(c, h_{\tau_I}), p_B(v, h_{\tau_I}) \]

Net Payoffs:
\[ p_T - c, v - p_B \]

Figure 1.1: Time line

\([A + l, A + h] \rightarrow [0, +\infty)\) is the timing of initiation for the bidder as a function of its type. Let the actual initiation time be \( \tau_I \), where \( I \in \{T, B\} \), then define \( p_T(\cdot) : [l, h] \times \mathcal{H}_{\tau_I} \rightarrow [0, +\infty) \) is the target’s asked price in the Nash Demand Game after initiation, based on its type and the history observed until the actual time of initiation, \( p_B(\cdot) : [A + l, A + h] \times \mathcal{H}_{\tau_I} \rightarrow [0, +\infty) \) is the bidder’s offered price in the Nash Demand Game after initiation, based on its type and the history observed until the actual time of initiation.

Notice that the initiation strategies \( \tau_T(\cdot) \) and \( \tau_B(\cdot) \) are only functions of the firm’s own types. That is, the firms’ decisions on the initiation time is time-consistent. This is because although a firm can deduce information about the opponent’s type from the fact that the opponent has not initiated yet by time \( t \), this has already been taken account at \( t = 0 \).

Figure 1.1 summarizes the game with a time line.

### 1.2 Equilibrium Concept and Refinement

The equilibrium concept we apply is Perfect Bayesian Nash Equilibrium, and we look for equilibria where the initiation strategies \( \tau_T(\cdot) \) and \( \tau_B(\cdot) \) are separating\(^1\) and continuously differentiable. Furthermore, the initiation strategies are such that \( \tau_B^{-1}(v) = A + l + h - \tau_T^{-1}(v) \) for all \( v \in [A + l, A + h] \). This restriction is actually a symmetric requirement on the initiation strategies for an equivalent game that would be introduced later.

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\(^1\)This is the reason why we focus on pure strategies.
**Definition 1.3.** A Perfect Bayesian Nash Equilibrium in the game is a strategy profile \( \{\tau_T, \tau_B, p_T, p_B\} \) and belief system \( (\mu_T, \mu_B) \), where

(i) In the continuation game characterized by \( \{c, v, \mathcal{H}_r, \mu_T(\cdot), \mu_B(\cdot)\} \), \( p_T(\cdot) \) and \( p_B(\cdot) \) are mutually best responses;

(ii) Given \( \{p_T(\cdot), p_B(\cdot)\} \), \( \tau_T(\cdot) \) and \( \tau_B(\cdot) \) are mutually best responses;

(iii) Bayes’s Rule is used to generate belief system \( (\mu_T(\cdot), \mu_B(\cdot)) \) whenever possible.

In the continuation game in the form of Nash Demand Game, there are multiple equilibria. To pin down a unique equilibrium, we follow Nash (1953) and add a vanishing perturbation. In particular, recall that in a standard Nash Demand Game, deal succeeds if and only if \( p_T \leq p_B \). In a Nash Demand Game with a vanishing perturbation, the deal succeeds if and only if \( p_T \leq p_B + \varepsilon \) with \( \mathbb{E}(\varepsilon) = 0 \). Let \( \operatorname{Var}(\varepsilon) \to 0 \), we pin down the unique equilibrium. In addition, in the initiation game we impose the “D-1” criterion by Banks and Sobel (1987) to rule out pooling equilibrium.

## 2 Solving the Model

### 2.1 Transforming the Problem into a Symmetric Game

From Definition 1.2, we see that the two participants in the game are asymmetric. In order to achieve tractability, we transform the original asymmetric problem into an equivalent symmetric problem by changing of variables. In particular, define

\[
\begin{align*}
c_1 &= c, \\
c_2 &= a - v,
\end{align*}
\]

where \( a = A + h + l \).

That is, we keep the target’s type \( c \) intact, while we redefine the bidder’s type to be \( c_2 = a - v \). Since \( v \in [A + l, A + h] \), \( c_2 \) lies in \( [l, h] \), which is the same support of the target’s type. In addition, because both \( c \) and \( v \)'s p.d.f. are line symmetric with respect to the center of their supports, \( c_2 \)
Under (2.1) and (2.2), the net payoff to the target, the net payoff to the bidder, and the condition under which the deal succeeds in a Nash Demand Game is exactly the same for the original problem and the transformed problem.

and $c$ (hence also $c_1$) follows the same distribution. Therefore, if we denote $f(\cdot) = f_c(\cdot)$, then $c_i$ are i.i.d. on $[l,h]$, with p.d.f. $f(\cdot)$. That is, the distributions of the redefined types of the two firms are completely symmetric.

Next, we convert the original Nash Demand Game in the bargaining stage into a new symmetric one. Since only the difference in payoffs for between a successful deal and a failed deal matters, we only consider this net payoff. In the original problem, the net payoff for the target for a successful deal is $p_T - c$, while the net payoff for the bidder is $v - p_B - c$. Plugging in (2.1), the net payoffs for the target and the bidder become $p_T - c_1$ and $v - p_B - c_2$. Now define

$$ p_1 = p_T, $$
$$ p_2 = a - p_B. $$

Then the net payoffs to the target and the bidder become $p_1 - c_1$ and $p_2 - c_2$. Furthermore, the deal succeeds in the original Nash Demand Game if and only if $p_T \leq p_B$. With (2.2), this condition is equivalent to $p_1 \leq a - p_2$.

Therefore, the original Nash Demand Game is transformed into a new symmetric Nash Demand Game by changing of variables in (2.1) and (2.2). In this new game, firm $i$ has private information about its outside option $c_i$, which lies in $[l,h]$ and has p.d.f. $f(\cdot) = f_c(\cdot)$. Both firms submit an amount of claim $p_i$ out of $a = A + h + l$, which is the total amount of money available. If $p_1 + p_2 \leq a$, both get their requests; otherwise, each obtains their outside option $c_i$. Also, the net payoffs to firm $i$ is $p_i - c_i$, for $i = 1, 2$. Table 2.1 summarizes the two games in the bargaining stage.

<table>
<thead>
<tr>
<th>Original</th>
<th>Net Payoff to Target</th>
<th>Net Payoff to Bidder</th>
<th>Deal succeeds iff</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p_T - c$</td>
<td>$v - p_B$</td>
<td>$p_T \leq p_B$</td>
</tr>
</tbody>
</table>

| Transformed| $p_1 - c_1$         | $p_2 - c_2$         | $p_1 + p_2 \leq a$ |

Table 2.1: **Equivalence of the Original and the Transformed Problem in the Bargaining Stage**

Under (2.1) and (2.2), the net payoff to the target, the net payoff to the bidder, and the condition under which the deal succeeds in a Nash Demand Game is exactly the same for the original problem and the transformed problem.
Finally, since the continuation game after changing of variables is symmetric, the transformed initiation game is also symmetric. Therefore, we first solve the new symmetric initiation game, and then convert it back and find the corresponding equilibrium in the original game.

2.2 Solving the Transformed Symmetric Problem

We look for symmetric separating equilibrium with continuously differentiable initiation strategies for the transformed problem. Denote $\tau_i(c_i), i = 1, 2$ as the time of initiation for firm $i$ of type $c_i$, then we focus on equilibrium with $\tau_1(\cdot) = \tau_2(\cdot) = \tau(\cdot) \in C^1$, with $\tau(\cdot)$ either strictly positive or strictly negative on $[l, h]$.

Suppose such $\tau(\cdot)$ exists. We then solve the game backwards by starting with the continuation game after initiation.

2.2.1 The Continuation Game After Initiation

Suppose firm $i$ has initiated the bargaining at $t$. Since the initiation strategies are separating strategies, then the initiator firm $-i$ thinks it knows firm $i$’s type $c_i$ for sure, while firm $i$’s posterior of firm $-i$’s type given firm $-i$ has not initiated yet is a truncation of the prior. Denote the type firm $-i$ believes the initiator firm $i$ to be as $x_i$, and support of firm $i$’s posterior of firm $-i$’s type as $[x_{-i}, y_{-i}]$.

Formally, we use subscript $I$ and $N$ for the “Initiator” and “Non-Initiator”. The continuation game is characterized by (i) the initiator and the non-initiator’s true outside options $(c_I, c_N)$; (ii) the non-initiator’s belief on initiator’s outside option $\hat{c}_I = x_I$, (iii) the initiator ’s belief on the non-initiator’s outside option $\hat{c}_N \in [x_N, y_N]$. Note that we need to consider both on-equilibrium-path continuation game where beliefs are consistent with the true types, and off-equilibrium-path continuation game where beliefs are not.

We start with the on-equilibrium-path case with $c_I = x_I$, and $c_N \in [x_N, y_N]$. In this case, the Nash Demand Game with the refinement of a vanishing perturbation gives a equilibrium summarized in the lemma below.
Lemma 2.1. The Nash Demand Game with the refinement of a vanishing perturbation has a unique Bayesian Nash Equilibrium in the continuation game after initiation. The equilibrium submitted prices \( \{ p_I = p_I (x_I, x_N, y_N), p_N = p_N (x_I, x_N, y_N) \} \) solves the following problem

\[
\max_{\{p_I, p_N\}} \frac{1}{2} \mathbb{E} \left[ \log (p_I - c_I) | c_I = x_I \right] + \frac{1}{2} \mathbb{E} \left[ \log (p_N - c_N) | c_N \in [x_N, y_N] \right]
\]

s.t. \( p_I + p_N = a, \)
\( p_I \geq x_I, \)
\( p_N \geq y_N. \)  \hspace{1cm} (2.3)

Proof. We extend the discrete-type results in Harsanyi and Selten (1972) for Nash Demand Game with incomplete information to obtain the objective function and the feasibility constraint (the first line of constraint). The second the and third lines of constraints are the individual rationality conditions for firm \( I \) and firm \( N. \)  \( \square \)

The solution to Problem (2.3) in the lemma above has the following properties.

Lemma 2.2. The equilibrium submitted prices \( \{ p_I (x_I, x_N, y_N), p_N (x_I, x_N, y_N) \} \) have the following properties.

1. Pooling Claims: given beliefs, \( p_i \) does not depend on firm \( i \)'s true type \( c_i, \) \( i = I, N. \)

2. Efficiency: \( p_I + p_N = a. \)

3. Strict IR: \( p_I > x_I, \) and \( p_N > c_N, \) \( \forall c_N \in [x_N, y_N]. \)

4. Monotonicity I: \( p_I (x_I, x_N, y_N) \) is strictly increasing in \( x_I \) and strictly decreasing in \( x_N, y_N; \)
\( p_N (x_I, x_N, y_N) \) is strictly increasing in \( x_N, y_N \) and strictly decreasing in \( x_I. \)

5. Monotonicity II: \( p_I (x_I, x_I, y_N) \) is \( C^1 \) in \( x_I, \) with \( \frac{dp_I(x_I, x_N, y_N)}{dx_I} > 0. \)

6. Symmetry: \( p_I (x_I, x_I, x_I) = \frac{a}{2}. \)

Proof. See Appendix.  \( \square \)

Next, we consider the off-equilibrium-path case where the true types are inconsistent with the beliefs. That is, \( c_I \neq x_I, \) or \( c_N \notin [x_N, y_N]. \) The optimal strategy in this case by the two firms are
Lemma 2.3. (Consistent Action or Opt Out) Consider $i = I$ or $N$. When firm $i$’s true type $c_i$ is inconsistent with firm $-i$’s belief of it, firm $i$’s optimal claim is

$$
\begin{cases}
    p_i(x_I, x_N, y_N) & \text{if } p_i(x_I, x_N, y_N) > c_i; \\
    0 & \text{otherwise}.
\end{cases}
$$

Proof. See Appendix.

To illustrate the lemma above, consider a firm whose true type is inconsistent with the opponent’s belief. If acting as if its type is consistent with the opponent’s belief gives it strictly positive net payoff, it follows the equilibrium action indicated by the opponent’s belief; otherwise, it opts out by claiming zero and obtains its outside option $c_i$.

2.2.2 The Initiation Game

Given the equilibrium in the continuation game, we now solve for the initiation stage. Since we look for symmetric separating equilibrium, the initiation strategy $\tau(\cdot)$ is strictly monotonic. Therefore the inverse function of $\tau(\cdot)$ exists. We denote $x(t) = \tau^{-1}(t)$ as the type that initiates at time $t$ in equilibrium. Since belief is consistent, this is also the type that the non-initiator believes the initiator to be if observing initiation time $t$.

We will first assume that $x'(t) > 0$ (higher types initiates later), and we will verify that this is indeed an Perfect Bayesian Equilibrium. Then we will show that symmetric separating equilibrium with $x'(t) < 0$ does not exist.

Suppose $x'(t) > 0$. After initiation, the initiator has completely revealed its type to its opponent by the initiation timing, while the non-initiator’s type is revealed to be higher than the initiator’s type. That is, the non-initiator believes the initiator’s type to be $x(t)$, while initiator believes the non-initiator’s type to be in the range of $[x(t), h]$.

Define

$$p(x) = p_I(x, x, h)$$
where $p_I(\cdot)$ is defined in Lemma 2.1 with $x_I = x_N = x$ and $y_N = h$. Then $p(x(t))$ is the initiator’s payoff if the initiation time is $t$. According to the property of Efficiency in Lemma 2.2, the non-initiator’s payoff is $a - p(x(t))$. Then we have the following two properties of $p(\cdot)$.

**Proposition 2.1.** *(Stronger initiator gets higher claim)* $p'(x) > 0 \ \forall x \in (l, h)$

*Proof.* It’s a straightforward application of Monotonicity II in Lemma 2.2. \qed

**Proposition 2.2.** *(Initiator claim < non-initiator claim)* $p(x) < a - p(x), \ \forall x \in (l, h)$.

*Proof.* Monotonicity I and Corollary 5.2 give this result. \qed

Proposition 2.1 and 2.2 illustrate the two important forces in the trade-offs on initiation timing. Proposition 2.1 claims that if the initiator is believed to be of higher outside option (a stronger type), then it’s able to get a higher price as an initiator. Since $x'(t) > 0$ (higher type initiates later), this force generates an incentive for weaker types to postpone initiation so as to pretend a stronger initiator.

Proposition 2.2, on the other hand, states that the price the initiator obtains is smaller than that of the non-initiator. This is because the initiator reveals its type to be $x(t)$ for sure, while the non-initiator is able to blend in the crowd of $[x(t), h]$. This loss of information advantage due to initiation induces weaker types to postpone initiation so that they could pool with a population of higher types.

Figure 2.1 gives an illustration of the trade-off firms face with respect to initiation timing. If firm $i$ initiates at time $t$, the price obtained is $p(x(t))$. If waiting till $t + \Delta$, there are both benefit and cost. There are three beneficial forces. First, one could postpone losing outside option $c_i$ by $\Delta$, and this benefit is higher for firms with higher $c_i$. The fact that the benefit of waiting is higher for higher types generates a Single Crossing Property, which allows higher types to separate from lower types by waiting for longer.

Second, if the opponent does not initiate before $t + \Delta$ so firm $i$ initiates at that time, firm $i$ obtains price $p(x(t + \Delta))$. This price is higher than the price it gets as a type-$x(t)$ initiator because (i) $p'(x) > 0$ (Proposition 2.1), and (ii) $x'(t) > 0$. 


Third, if the opponent firm initiates right before $t + \Delta$, firm $i$ avoids initiating at $t + \Delta$. That is, instead of signals its type to be $x(t + \Delta)$ for sure and obtains $p(x(t + \Delta))$, it manages to blend in the stronger crowd of $[x(t + \Delta), h]$ and achieves a price $a - p(x(t + \Delta)) > p(x(t + \Delta)) > p(x(t))$. Note that this price is not only higher than $p(x(t))$, but also higher than the price given to a stronger initiator ($p(x(t + \Delta))$). The difference between $a - p(x(t + \Delta))$ and $p(x(t + \Delta))$ stems from the non-initiator’s information advantage.

Finally, there is one force against waiting: any gain would be discounted.

The following proposition demonstrates the trade-offs on initiation timing with the HJB equation and pins down the solution for the initiation timing for the transformed problem.

**Proposition 2.3.** There is a symmetric separating equilibrium with $C^1$ strategy in which $\tau'(\cdot) > 0$. $\tau(\cdot)$’s inverse function $x(\cdot)$ follows the ODE:

\[
\begin{align*}
\text{initiates now} & \quad \frac{r [p(x(t)) - x(t)]}{1 - F(x(t))} [a - 2p(x(t))] + \frac{x'(t) p'(x(t))}{1 - F(x(t))} \\
\text{the opponent initiates} & \quad \text{itself initiates later}
\end{align*}
\]

\[x(0) = l.\]

In addition, neither symmetric separating equilibrium with $\tau'(\cdot) < 0$, nor pooling equilibrium exist.

*Proof.* See Appendix.

Discussion of Proposition 2.3

**Strategic Substitutes.** Equation (2.4) demonstrates the decision of initiation firm $i$ has to make. Given firm $i$’s belief on the opponent’s strategy $\tau(\cdot) = x^{-1}(\cdot)$, and given the opponent’s belief $x(\cdot)$
on firm \(i\)'s type as a function of the initiation time, firm \(i\) chooses the optimal initiation timing. If not initiating at \(t\), but waiting for another \(dt\) instead, two cases would happen. With probability 
\[x'(t) \frac{f(x(t))}{1-F(x(t))} dt,\]
the opponent's type locates in \([x(t), x(t+dt)]\) and initiates. Then firm \(i\) was able to blend in the crowd of \([x(t+dt), h]\) and obtains a net benefit of 
\[a - p(x(t+dt)) - p(x) \approx a - 2p(x(t)) > 0.\]
With probability 
\[1 - x'(t) \frac{f(x(t))}{1-F(x(t))} dt \approx 1,\]
the opponent's type locates in \([x(t+dt), h]\) instead. Then firm \(i\) initiates at \(t+dt\), and obtains a net payoff of 
\[p(x(t+dt)) - p(x(t)) \approx p'(x(t)) dt.\]

Note that when the opponent initiates faster \((x'(t)\) higher, or \(\tau'(c)\) lower), the probability of the opponent initiating \((x'(t) \frac{f(x(t))}{1-F(x(t))} dt)\) is higher, hence there is more benefit for firm \(i\) to wait. Therefore, if one firm initiates faster, the other would postpone initiation. That is, initiation is a **strategic substitute** between the two firms. This is because the concern of losing information advantage due to initiation gives this model a flavor of **war of attrition**, and in models of war of attrition, initiation is commonly strategic substitute.

**The non-existence of equilibrium with \(\tau' < 0\).** We’ve shown that the only symmetric separating equilibrium with \(\tau(\cdot) \in C^1\) must have \(\tau' > 0\). This is a result of the first benefit of waiting illustrated in Figure 2.1. Since the benefit of delaying the loss of the outside option \(c_i\) is higher for higher types, higher types initiates later than lower types in equilibrium.

### 2.3 Solving the Original Initiation Game

Denote \(\tau^*(\cdot)\) as the solution to \(\tau\) in Proposition 2.3, then with Equation (2.1) and (2.2), we convert the equilibrium of the transformed symmetric problem into the original problem with a target and a bidder. Recall that the equilibrium time of initiation for a target with type \(c\) and a bidder with type \(v\) are denoted as \(\tau_T(c)\) and \(\tau_B(v)\) respectively.

**Proposition 2.4.** The equilibrium time of initiation \(\tau_T(c)\) and \(\tau_B(v)\) are such that

\[
\tau_T(c) = \tau^*(c)
\]

\[
\tau_B^{-1}(v) = a - \tau^*-1(v).
\]
where $\tau^* (\cdot)$ is the equilibrium timing for the transformed problem.

**Implications from Proposition 2.4**

First, $\tau^* > 0$ implies that $\tau_T^* > 0$ and $\tau_B^* < 0$. That is, seller with higher stand-alone value initiates later, while buyer with lower value for the seller’s firm initiates later.

Second, we are able to identify when a deal would be target-initiated. A deal is target-initiated when $c < a - v$. That is, when both the target’s stand-alone value and the bidder’s value for the target’s firm are relatively low. A deal is bidder-initiated when $c > a - v$, i.e., when both the target’s stand-alone value and the bidder’s value for the target’s firm are relatively high.

Third, we identify when there would be long delay and short delay. That is, there would be long delay when $c$ is higher and $v$ is lower, while short delay happens when $c$ is lower and $v$ is higher.

Figure 2.2 illustrates the results above.
3 Extension to General Bargaining Protocols

Although in the main part of the paper we focus on Nash Demand Game, all our results hold for more general bargaining protocols as long as certain properties of the protocols are satisfied.

Suppose in the continuation game after initiation in the transformed game, firm $i$ thinks firm $-i$’s type $c_{-i} \in [x_{-i}, y_{-i}]$, with $x_{-i} \leq y_{-i}$, for $i = 1, 2$. Define $P_i(x_1, y_1, x_2, y_2, c_1, c_2)$ as firm $i$’s equilibrium claim out of total pie $a$ if firm $i$ believes that firm $-i$’s type lies in $[x_{-i}, y_{-i}]$ for $i = 1, 2$, and the true types of the two firms are actually $c_1$ and $c_2$. Denote $\tilde{P}_i(x_1, y_1, x_2, y_2, c_1, c_2)$ as firm $i$’s equilibrium claim out of total pie $a$ if firm $i$ believes that firm $-i$’s type lies in $[x_{-i}, y_{-i}]$ for $i = 1, 2$, and if the beliefs are indeed consistent with the true types ($c_i \in [x_i, y_i]$ for $i = 1, 2$).

**Proposition 3.1.** The results in Proposition 2.3 as long as the following are true:

1. *(Pooling Claims)* $\tilde{P}_i(x_1, y_1, x_2, y_2, c_1, c_2) = p_i(x_1, y_1, x_2, y_2)$, for $i = 1, 2$. That is, $\tilde{P}_i$ does not depend on $c_1$ or $c_2$.

2. *(Per mutability)* $p_i(x_1, y_1, x_2, y_2) = p_2(x_2, y_2, x_1, y_1)$.

3. *(Efficiency)* $p_i(x_1, y_1, x_2, y_2) + p_2(x_1, y_1, x_2, y_2) = a$.

4. *(Strict Individual Rationality)* $p_i(x_1, y_1, x_2, y_2) > c_i, \forall c_i \in [x_i, y_i]$, for $i = 1, 2$.

5. *(Monotonicity I)* $p_i(x_1, y_1, x_2, y_2)$ is strictly increasing in $x_1, y_1$, and strictly decreasing in $x_2, y_2$, for $i = 1, 2$.

6. *(Monotonicity II)* Let $p(x) = p_i(x, x, x, h)$, then $p(\cdot)$ is $C^1$ and $p'(x) > 0$.

7. *(Consistent action Or Opt out)*

$$P_i(x_1, y_1, x_2, y_2, c_1, c_2) = \begin{cases} 
  p_i(x_1, y_1, x_2, y_2) & \text{if } p_i(x_1, y_1, x_2, y_2) > c_i \\
  0 & \text{if otherwise.}
\end{cases}$$

The first six assumptions are about $\tilde{P}_i(x_1, y_1, x_2, y_2, v_1, v_2)$, which is $i$’s equilibrium claim if the beliefs are consistent. The Pooling Claims assumption presumes that when the beliefs are consistent, all types of $c_i \in [x_i, y_i]$ pool on the same claim $p_i(x_1, y_1, x_2, y_2)$. This assumption simplifies our analysis dramatically. The Per mutability assumption says that $p_i(x_1, y_1, x_2, y_2)$ does not depend on the labeling of the two firms. This assumption creates symmetry between
the two players, allowing for a tractable symmetric equilibrium; the Efficiency assumption implies that there is no money left on the table; the Strict Individual Rationality assumption reflects that under the common claim $p_i(x_1, y_1, x_2, y_2)$, all types of $c_i \in [x_i, y_i]$ receive strictly positive net profit from trade; the Monotonicity I assumption states that firm $i$’s payoff is strictly increasing in its own outside option, and strictly decreasing in its opponent’s outside option; the Monotonicity II assumption requires the distribution of $c_i$ to be even enough. The last assumption considers that case if the true types could be inconsistent with the beliefs. It claims that as long as trade happens, the equilibrium claim by firm $i$ has to be $p_i(x_1, y_1, x_2, y_2)$, which is the claim as if both types are consistent with the beliefs. The Strict Individual Rationality and the Consistent action or Opt out assumptions makes it possible for a pure strategy separating equilibrium to exist.

4 Empirical Applications

The empirical predictions of our model are consistent with the empirical literature about initiation timing.

**Fact.** Masulis and Simsir (2015): Target firms receive lower bid premium in target-initiated deals than in bidder-initiated deals. The difference remains after controlling observable target financial conditions.

**Corollary 4.1.** This is true in our model: $E\left[\frac{p(c)}{E(c)} - 1\mid c < a - v\right] < E\left[\frac{a - p(a - v)}{E(c)} - 1\mid a - v < c\right]$.

**Remark.** The left hand side is the average bid premium for all target-initiated deals ($c < a - v$), while the right hand side is the average bid premium for all bidder-initiated deals. In Masulis and Simsir (2015), bid premium is defined as the deal value divided by the target market value 2 months ago minus 1. To fit in our model, we approximate the target’s market value 2 months ago with the unconditional mean $E(c)$.

The proof is very simple but worth noting: $E\left[\frac{p(c)}{E(c)} - 1\mid c < a - v\right] = E\left[\frac{p(a - v)}{E(c)} - 1\mid a - v < c\right] < E\left[\frac{a - p(a - v)}{E(c)} - 1\mid a - v < c\right]$. The first equality means that the price in expectation paid to the initiator is the same no matter who initiates. The second inequality is due to the fact that the
initiator obtains less than the non-initiator due to the information disadvantage \( p(\cdot) < a - p(\cdot) \). Note that the magnitude of the difference in bid premium is determined by the information advantage of an non-initiator.

**Fact.** Masulis and Simsir (2015): The difference of bid premium between bidder-initiated and target-initiated deals increases with the uncertainty of the target firms.

**Example.** Suppose \( c \) and \( v \) follows uniform distribution, and \( U [\hat{l}, \hat{h}] \) is a mean-preserving spread of \( U [l, h] \). Then \( E \left[ \frac{a-p(a-v)}{E(c)} - 1 | a > v \right] - E \left[ \frac{p(c)}{E(c)} - 1 | a < v \right] \) is higher for the mean-preserving spread \( U [\hat{l}, \hat{h}] \) than for \( U [l, h] \).

The intuition is as follows. According to the proof of 4.1, the difference in bid premium is determined by \( E \left[ \frac{a-p(a-v)}{E(c)} - 1 | a - v < c \right] - E \left[ \frac{p(a-v)}{E(c)} - 1 | a - v < c \right] = E \left[ \frac{a-2p(a-v)}{E(c)} - 1 | a - v < c \right] = E \left[ \frac{a-2p(x)}{E(c)} - 1 \right] \), where \( x = \min (a - v, c) \). If the support of \( c \) and \( a - v \) (i.i.d.) are more dispersed, the distribution of \( x = \min (a - v, c) \) shifts to the left. Since \( p' > 0 \), \( E \left[ \frac{a-2p(x)}{E(c)} - 1 \right] \) is higher, hence the difference is higher too.

## 5 Conclusion and Future Extensions

This paper builds a framework with two-sided private information to explain endogenous initiations of M&A negotiations. We study the following research questions. What determines the timing of the initiation of the M&A negotiation? Who becomes the initiator? Why target firms receive lower bid premium in target-initiated deals than in bidder-initiated deals?

The key driving force of the timing of initiation is the target’s private information about its stand-alone value, and the bidder’s private information about its valuation on the target firm. Although later initiation in induces higher gains, weaker types like targets with lower stand-alone value and bidders with higher valuation for the target firms rationally choose to initiate the deal early. The predictions of our model are consistent with the empirical facts that motivate our research.
As for future extensions, we will try to obtain a tractable equilibrium when the settings cannot be transformed into a symmetric one. In this way, we could consider the following extensions: different discount rates; different bargaining power in the negotiation stage; stock offers; interdependent value and adverse selection.

Appendix

Proof for Lemma 2.2

Proof. Since the objective function is concave, the solution to the problem is unique. The solution is unique, hence the claims are pooled across types. The Permutability holds due to the equal weights on the two terms. The Efficiency assumption holds trivially. Since \( p_i - c_i \) is inside \( \log \), \( p_i > c_i \) has to be true for all \( c_i \) and for \( i = 1, 2 \), so Strict Individual Rationality holds. The Monotonicity I and II can be shown by taking the first order condition of the objective function and applying the Implicit Function Theorem. The last property is trivial. \( \Box \)

Proof to Lemma 2.3

Proof. Consider firm \( i \)'s choice. Suppose the outcome in the initiation stage implies to firm \(-i\) that \( c_i \in [x_i, y_i] \). Then if \( p_i (x_1, y_1, x_2, y_2) - c_i > 0 \), firm \( i \) submits claim \( p_i (x_1, y_1, x_2, y_2) \), which is consistent with such beliefs. To see this, suppose firm \(-i\) submits \( p_{-i} (x_1, y_1, x_2, y_2) \). If firm \( i \) asks for more than \( p_i (x_1, y_1, x_2, y_2) \), it would be higher than \( a - p_{-i} (x_1, y_1, x_2, y_2) \), resulting in failure of trade. Hence deviation upwards reduces firm \( i \)'s payoff from \( p_i (x_1, y_1, x_2, y_2) - c_i > 0 \) to zero, therefore is not profitable. Moreover, asking for less than \( p_i (x_1, y_1, x_2, y_2) \) is trivially not profitable. If \( p_i (x_1, y_1, x_2, y_2) - c_i \leq 0 \), then trade cannot happen and firm \( i \) get its outside option. \( \Box \)

Corollary 5.1. \( p (x) > x \) and \( a - p (x) > h \).

Proof. It directly comes from the Strict IR and Efficiency in Lemma 2.2. \( \Box \)

Per mutuality and Efficiency implies:
Corollary 5.2. $p(h) = \frac{a}{2}$.

Proof. It's a result of applying Efficiency and Symmetry in Lemma 2.2.

Proof for Proposition 2.3

Proof. Pooling equilibrium can be ruled out with the D-1 criterion by Banks and Sobel (1987). Suppose there exists a pooling equilibrium with all types pooling at time $t_p \geq 0$. Suppose we observe agent $i$ initiating at time $t > t_p$ instead. Since higher types have higher marginal benefit of waiting, the highest type must be the $c_i$ with the largest set of opponents’ actions that make agent $i$ better-off than in the equilibrium. That is,

$$\{a_{-i} \in A_{-i} | u_i(c_i, t, a_{-i}) > u_i(c_i, t_p, a_{-i})\} \subset \{a_{-i} \in A_{-i} | u_i(h, t, a_{-i}) > u_i(h, t_p, a_{-i})\}, \forall c_i \in [l, h]$$

Therefore, any agent deviating to $t > t_p$ is regarded as type $h$ according to the D-1 criterion. Then type $h$ would like to deviate to initiating at $t_p + \Delta$, which only increases the cost of discounting marginally, but improves on the payoff in the bargaining game substantially due to jump in belief. Contradiction.

We now compute a symmetric separating equilibrium with continuously differentiable strategy, where $\tau'(c) > 0$ or equivalently $x'(t) > 0$. Then we will show that $\tau'(c) < 0$ cannot be true.

Given the opponent $-i$'s strategy $x^{-1}(c)$ and its belief $x(t)$ on firm $i$, the problem of firm $i$ with true outside option $v$ is

$$\max_{\hat{t}} U(c, \hat{t})$$

where

$$U(c, \hat{t}) = \int_0^\hat{t} e^{-rt} \max[a - p(x(t)) - v, 0] x'(t) f(x(t)) dt + e^{-r\hat{t}} \max[p(x(\hat{t})) - v, 0] \left[1 - F(x(\hat{t}))\right]$$

(5.1)

Note that the “max” term comes from Lemma 2.3. The equilibrium condition requires that at the optimum, $x(\hat{t}) = v$. 

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Instead of solving the problem directly, we solve another problem, and then we show that its solution also solves the original problem. Define

\[ \bar{U}(v, \hat{t}) = \int_0^t e^{-rt} (a - p(x(t)) - v) \cdot x'(t) f(x(t)) dt + e^{-rt} \left( p(x(\hat{t})) - v \right) \cdot \left[ 1 - F(x(\hat{t})) \right]. \]

F.O.C to \( \hat{t} \) and replace \( v \) with \( x(\hat{t}) \), we have the solution \( \bar{x}(\cdot) \):

\[ \bar{x}'(\hat{t}) = \frac{r \left[ p(\bar{x}(\hat{t})) - \bar{x}(\hat{t}) \right]}{[a - 2p(\bar{x}(\hat{t}))] \cdot \frac{f(\bar{x}(\hat{t}))}{1 - F(\bar{x}(\hat{t}))} + p'(\bar{x}(\hat{t}))}, \quad (5.2) \]

Note that if we replace \( \bar{x}(\cdot) \) with \( \hat{t}(\cdot) \) and replace \( \bar{x}(\hat{t}) \) with \( v \), this gives the same which is the same ODE as 2.4. By Assumption 5.1, 2.1 and 2.2, we know that \( p(c) - c > 0 \), \( p'(c) > 0 \), and \( a - 2p(c) > 0 \). Therefore, \( \hat{t}'(c) > 0 \). The right hand side of 2.4 is bounded for \( c \in [l, h] \), therefore it’s integrable. Hence

\[ \hat{t}(c) = \bar{t}(l) + \int_l^c \frac{[a - 2p(s)] \cdot \frac{f(s)}{1 - F(s)} + p'(s)}{r \left[ p(s) - s \right]} ds. \]

Therefore \( \hat{t}(l) \) uniquely pins down the solution to the ODE.

Next, we show that under \( \bar{x}(\cdot) \), this solution from F.O.C. is indeed the global maximizer for \( \bar{U}(c, \hat{t}) |_{x(\cdot) = \bar{x}(\cdot)} \). Let \( t^* \) be the maximizer for \( \bar{U}(c, \hat{t}) \). Then it is sufficient to show that \( \frac{\partial \bar{U}(c, \hat{t})}{\partial \hat{t}} |_{x(\cdot) = \bar{x}(\cdot)} > 0 \) for \( \hat{t} < t^* \), and \( \frac{\partial \bar{U}(c, \hat{t})}{\partial \hat{t}} |_{x(\cdot) = \bar{x}(\cdot)} < 0 \) for \( \hat{t} > t^* \). Since \( \bar{x}'(\cdot) > 0 \), it is sufficient to show that \( \bar{U}(c, \hat{t}) \) is supermodular. That is, \( \frac{\partial^2 \bar{U}}{\partial c \partial \hat{t}} |_{x(\cdot) = \bar{x}(\cdot)} \geq 0 \). Since \( \frac{\partial^2 \bar{U}}{\partial c \partial \hat{t}} |_{x(\cdot) = \bar{x}(\cdot)} = e^{-rt} r \left[ 1 - F(\bar{x}(\hat{t})) \right] > 0 \), the solution from F.O.C is indeed the global maximizer.

Then we show if \( x(\cdot) = \bar{x}(\cdot) \), \( t^* \) also solves problem 5.1. By Corollary 2.2, \( a - p(x(t)) - c > p(x(t)) - c \). By Assumption 2.1, \( c \leq h \) and \( x(t) \leq h \), we have \( p(x(t)) - c \geq p(h) - c \geq p(h) - h \). Corollary 5.1 implies that \( p(h) - h > 0 \), hence \( a - p(x(t)) - c > 0 \). Therefore \( \max \left[ a - p(x(t)) - c, 0 \right] = a - p(x(t)) - c \).

By the construction above, \( \bar{x}(t^*) = c \). Therefore \( p(\bar{x}(t^*)) - c = p(c) - c \). Together with
Corollary 5.1, \( \max [p(\bar{x}(t^*)) - c, 0] = \max [p(c) - c, 0] = p(c) - c = p(\bar{x}(t^*)) - c. \)

Hence when \( t \) is in the neighborhood of \( t^* \), \( \frac{\partial U(c, t)}{\partial t} \big|_{x(\cdot) = \bar{x}(\cdot)} = \frac{\partial \tilde{U}(c, \hat{t})}{\partial \hat{t}} \big|_{x(\cdot) = \tilde{x}(\cdot)}. \) Hence \( \frac{\partial U(c, \hat{t})}{\partial \hat{t}} \big|_{x(\cdot) = \tilde{x}(\cdot), \hat{t} = t^*} = 0. \) Then we check if \( \hat{t} = t^* \) is the global maximizer for \( U(c, \hat{t}) \big|_{x(\cdot) = \tilde{x}(\cdot)}. \) Since \( p(\bar{x}(t^*)) - c = p(c) - c > 0, \) and \( \ddot{x}' > 0, \) \( p' > 0, \) either \( p(\bar{x}(t)) - c > 0 \) for all \( t \geq 0, \) or there exists \( 0 \leq \hat{t} < t^* \), such that \( p(x(\hat{t})) - c = 0. \) If the latter case is true,

\[
U(v, \hat{t}) \big|_{x(\cdot) = \tilde{x}(\cdot)} = \begin{cases} \tilde{U}(c, \hat{t}) \big|_{x(\cdot) = \tilde{x}(\cdot)}, & \text{if } \hat{t} \geq t^*; \\ H(c, \hat{t}) \big|_{x(\cdot) = \tilde{x}(\cdot)}, & \text{if } 0 \leq \hat{t} < t^*. \end{cases}
\]

where \( H(c, \hat{t}) \big|_{x(\cdot) = \tilde{x}(\cdot)} = \int_0^t e^{-rt} (a - p(\bar{x}(t)) - c) \cdot \ddot{x}'(t) f(x(t)) dt. \)

By the analysis for \( \tilde{U}(c, \hat{t}) \big|_{x(\cdot) = \tilde{x}(\cdot)}, \) \( \hat{t} = t^* \) is indeed the global maximizer for \( U(c, \hat{t}) \big|_{x(\cdot) = \tilde{x}(\cdot)} \) for \( \hat{t} \in [\hat{t}, \tilde{\tau}(h)]. \) Since \( \frac{\partial H(c, \hat{t})}{\partial \hat{t}} \big|_{x(\cdot) = \tilde{x}(\cdot)} = e^{-r\hat{t}} (a - p(\bar{x}(t)) - c) f(\bar{x}(\hat{t})) \ddot{x}'(\hat{t}) > 0, \) \( \hat{t} = t^* \) is the global maximizer for \( U(c, \hat{t}) \big|_{x(\cdot) = \tilde{x}(\cdot)}. \)

Moreover, we show that \( \tilde{\tau}(l) = 0. \) Suppose \( \tilde{\tau}(l) > 0. \) That is, firm \( -i \)'s lowest type \( l \) does not initiate immediately. Then firm \( i \) of type \( l \) would deviate from \( \tilde{\tau}(l) \) by initiating immediately at \( t = 0. \) This is because the lowest return it would get by deviating in this way is \( e^{-r \cdot 0} \max [p(l) - l, 0] \cdot (1 - F(l)) > e^{-r \cdot l} \max [p(l) - l, 0] \cdot (1 - F(l)). \) That is, if it deviates and if it is regarded as the lowest type, it still gets strictly higher payoff than initiating at \( \tilde{\tau}(l). \) Therefore in equilibrium, \( \tilde{\tau}(l) = 0. \)

Finally, we show that \( \tilde{\tau}'(v) < 0 \) cannot be a symmetric separating equilibrium with continuously differentiable strategy. Suppose otherwise. Then the firm \( i \)'s problem becomes:

\[
\max_{\hat{t}} \hat{U}(c, \hat{t})
\]

where \( \hat{U}(c, \hat{t}) = \int_0^{\hat{t}} e^{-rt} \max [a - \dot{p}(x(t)) - c, 0] \cdot (-x'(t)) f(x(t)) dt + e^{-r \hat{t}} \max [\dot{p}(x(\hat{t})) - c, 0] \cdot F(x(\hat{t})) \) (5.3)

where \( \dot{p}(x) = p_I(x, x, l). \)
In a separating equilibrium, at the optimal $\hat{t} = t^*$ and under the equilibrium $x^*(\cdot)$, we must have $x(t^*) = c$. In addition, since in equilibrium, both firm’s true types are consistent with their perceived types in the continuation stage, we can apply Strict IR in Lemma 2.2 and conclude that they obtain strictly positive terminal payoff. Therefore, at $t^*$ and under the equilibrium $x^*(\cdot)$,

$$
\hat{U}(c, t^*) = \int_0^{t^*} e^{-rt} \left[ a - \hat{p}(x^*(t)) - c \right] \cdot \left( -x^*(t) \right) f(x^*(t)) \, dt + e^{-rt} \left[ \hat{p}(x^*(t^*)) - c \right] \cdot F(x^*(t^*)).
$$

F.O.C. to $t^*$ and let $x^*(t^*) = c$, we have

$$
x^*(t) = -\frac{r (\hat{p}(x^*(t)) - x^*(t))}{[a - 2\hat{p}(x^*(t))] \frac{f(x^*(t))}{F(x^*(t))} - \hat{p}'(x^*(t))}.
$$

Note that this ODE comes from F.O.C., hence is necessary condition. Since $\frac{\partial^2 U(c, t^*)}{\partial v \partial t^*}|_{x(\cdot) = x^*(\cdot)} = e^{-rt^*} r F(x^*(t^*)) > 0$, $\hat{U}(c, \hat{t}) |_{x(\cdot) = x^*(\cdot)}$ is submodular in the neighborhood of $t^*$. Therefore, even if $x^*(t) < 0$, $\hat{U}(c, \hat{t}) |_{x(\cdot) = x^*(\cdot)}$ achieves local minimum at $\hat{t} = t^*$. Therefore such separating equilibrium does not exist. 

$\Box$
References


