Intermediary Funding Cost and Short-Term Risk Premia

Wenhao Li and Jonathan Wallen*

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Abstract

We build a theory of short term risk premia dynamics based on funding costs. The theory builds on a framework of intermediary asset pricing in a market microstructure setting. Financial intermediaries facilitate trading by market making. To fund these trading activities, intermediaries earn a risk premium. This risk premium increases in intermediary leverage and asset idiosyncratic risks. We test our theory across multiple asset classes, including equities, bonds, and currencies. Conditional on a large price shock, high intermediary leverage and asset idiosyncratic risk raise short term risk premia by about 100 to 170 basis points. We also find evidence of risk sharing and capacity constraints among intermediaries. Intermediary leverage and asset idiosyncratic volatility are important factors in explaining the time series of risk premia in equities, bonds, and currencies.

JEL classification: E44, G12, G21, G23

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*Wenhao Li: Graduate School of Business, Stanford University. Jonathan Wallen: Graduate School of Business, Stanford University. We are grateful to Arvind Krishnamurthy, Benjamin Hébert, Darrell Duffie, Jonathan Berk, Jiacui Li, and Hanno Lustig for helpful discussions.
1. Introduction

A canonical assumption of the asset pricing literature is that a representative household is marginal in pricing assets. Financial intermediaries between the household and asset simply serve as a frictionless pass-through or veil. The recent intermediary asset pricing literature challenges this canonical assumption by studying explicitly the role of financial intermediaries. For example, Brunnermeier and Pedersen (2009) explains how the connection between intermediary funding liquidity and asset liquidity may lead to negative liquidity spirals and a flight to quality. He and Krishnamurthy (2013) explains crisis dynamics of risk premia through a model of constrained intermediaries as marginal in pricing assets. Brunnermeier and Sannikov (2014) points out the highly nonlinear amplification effects of the financial friction in macroeconomics. In currency markets, Gabaix and Maggiori (2014) shows how intermediaries help explain exchange rate dynamics. All these papers posit that a friction separates the balance sheets of households and financial intermediaries. The limited risk-bearing capacity of intermediaries results in asset prices following supply and demand forces, as shown in Garleanu et al. (2009). In brief, a growing theory literature identifies intermediary capital as special.

In this paper, we apply the intermediary asset pricing framework to a market microstructure setting to explain short term risk premia dynamics. We develop a theoretical model of order imbalances and market clearing intermediaries with heterogenous funding costs. Clear testable predictions arise from our model. (1) Short-term risk premia increase in both intermediary leverage and asset idiosyncratic risk. (2) Return volatility increases with intermediary leverage. (3) Increases to intermediary leverage reduces trading. (4) Intermediaries with higher leverage increasingly participate when order imbalances are large. With data from multiple asset classes, including bonds, equities, and currencies, we provided evidence supporting these hypotheses.

The core economic mechanisms of our model are as follows. Intermediaries absorb order imbalances from their clients. To fund these short-term positions, intermediaries borrow from the repo markets, which requires pledging collateral and posting margin. The cost of this borrowing varies by intermediary leverage. Riskier, more leveraged intermediaries have higher funding costs. Additionally, the riskiness of the asset is positively related to the funding requirement. Consequently, intermediaries price a risk premium in taking on client liquidity demands. This short-term risk premium is positively related to intermediary leverage and asset idiosyncratic risks, defined as the residual variance of returns to avoid confounding with leverage. Furthermore, asset return volatility endogenously increases with intermediary leverage, as liquidity shocks have larger impact with higher funding costs caused by higher leverage. Consequently, risk premia dynamics generate predictable asset price reversal patterns.

The novelty of our model is its characterization of intermediary leverage as a state variable for short term risk premia. The existence of price reversals is well established in both the
theoretical and empirical literature. Duffie (2010) posits a theory of slow moving capital, where intermediaries absorb short-term order imbalances. Since these intermediaries are capital constrained and risk-averse, they require a risk premium to do so. Empirically, this short run price reversal fact was documented as early as Cox and Peterson (1994). Similarly, the pricing of idiosyncratic volatility is well established.\(^1\) A debate in the literature is on whether idiosyncratic risk bears a positive or negative risk premium. Tinic and West (1986), Lehmann (1990), and Malkiel and Xu (2002) all find that portfolios of stocks with higher idiosyncratic volatility have higher average returns. However, Ang et al. (2009) finds that stocks with recent past high idiosyncratic volatility have low future average returns across 23 developed markets.

We posit that idiosyncratic risk is priced negatively (positively) whether intermediaries are long (short) the asset. This interpretation potentially reconciles mixed empirical evidence within the literature.

Testing our model predictions across three different asset classes, we find strong evidence of intermediary asset pricing. Conditional on a large negative price shock, average short-term risk premia are about 50 to 80 basis points across equity, bond, and currency markets. These risk premia are positively related to aggregate market leverage of primary dealers and asset idiosyncratic risks, measured by the residual variance of asset returns. A one standard deviation increase in both the aggregate leverage and asset idiosyncratic risk increase such risk premia between 30 to 250 basis points on average across equities, bonds and currencies. Consistent with intermediaries being broadly marginal in asset pricing as in He et al. (2016), these risk premia dynamics span many asset classes. Moreover, asset return volatility increases with intermediary leverage, both in and out of crisis. On average one percentage increase in aggregate leverage increases about 2 percentage of return volatility.

To highlight the intermediary funding cost, we identify the effect of intermediary leverage shocks on market making activity. We construct a unique dataset on US bond market intermediation. This market is ideal because of the large role that financial specialists play in clearing the illiquid market. Furthermore, the intermediaries tend to specialize in trading particular bonds. Following an increase to leverage or relative leverage, defined as individual leverage over aggregate leverage, the intermediary has smaller trade size, lower total trading volume, and less number of trades. Moreover, when other intermediaries have increased leverage, the current intermediary increase its trading, which shows evidence of intermediary competing by funding costs. The economic effect of capital shocks is significant. When leverage increase by 1, the number of trades decline by about 2 percent. When relative leverage increase by 1 percent, the number of trades decline by about 20 percent.

\(^1\)Notable papers on this topic include Tinic and West (1986), Lehmann (1990), Barberis and Huang (2001), Malkiel and Xu (2002), Ang et al. (2006), Stambaugh et al. (2015), Han and Lesmond (2011), and Chen and Petkova (2012).
Related to this research, Adrian et al. (2014) (AEM) and He et al. (2016) (HKM) document the importance of intermediary capital as a risk factor in pricing the cross-section of asset returns. The existence of this common risk factor is evidence in favor of an intermediary asset pricing framework. However, this macro-evidence is one of many potential explanations for why asset prices may fall during volatile periods. An increase in risk aversion in response to market shocks, a la Malmendier and Nagel (2011), may be an underlying confounding factor. Risk aversion both decreases asset prices and constrains the capital extended to intermediaries. We circumvent many concerns related to competing slow moving explanations by focusing on short term risk premia. We document a day-to-day effect of intermediary leverage on short term risk premia through funding costs. Intermediary asset pricing explains not only the risk premia associated with financial crises, but also that of daily market making.

Of note is that market liquidity may be another channel through which order flow generates risk premia. Empirically, Stambaugh (2003) identify a liquidity risk factor and argue that this state variable is important for levered financial specialists. This finding presents an endogeneity problem – Does financial specialist leverage determine liquidity or vice-versa? Kondor and Vayanos (2016) tackle this issue through a dynamic model of liquidity provision, where liquidity is increasing with intermediary capital. We note the deep connection between the intangible concept that is liquidity and intermediary capital. The subsequent focus on the funding cost of intermediaries follows from its measurability and the centrality of dealers in financial markets.

The remainder of this paper is structured as follows. Section 2 describes the model, provides solutions and testable predictions. Section 3 describes our dataset and empirically tests the theory. Section 4 concludes.

2. The Model

We present a model to explain the dynamics of risk premia through intermediary funding costs and asset volatility. The model robustly predicts that intermediary leverage and idiosyncratic volatility increase short term risk premia. Moreover, the model uniquely predicts that the distribution of intermediary leverage matters for pricing idiosyncratic risks. The model is designed to explicitly explain the effect of intermediary leverage on the pricing of assets. This measurable economic mechanism avoids relying on difficult to quantify state variables such as risk aversion or liquidity. The appendix explores extensions to the model including risk aversion dynamics and a wealth framework similar to He and Krishnamurthy (2013).
2.1. Setup

We consider a continuous-time infinite-horizon economy with $t \in [0, \infty)$. We fix a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. All stochastic processes considered in this paper are adapted with respect to $\mathcal{F}_t$. The economy has a unit mass of risk-neutral deep pocket households and a unit mass of risk-neutral intermediaries, denoted by $[0, 1]$. A finite number of assets are traded by both households and intermediaries. The intermediaries have a special search technology and optimally provide liquidity to households. Households experience liquidity shocks arriving at a Poisson process. Since the probability of two shocks arriving at the same time is zero and each shock almost surely hits a new household, we can separately price each asset. In the following discussions, we consider one asset with fundamental value process $X_t$, with properties defined later.

When a household is hit by order $Q$ at time $t$, its utility is

$$u(Q) = \mathbb{E}_t[Q(P_\tau + \pi - X_\tau)]$$

where $\tau$ is the time until trading with an intermediary, $P_\tau$ is the price of transaction at time $\tau$, and $\pi$ reflects the total possible gain from trading. Households search for intermediaries with intensity $\lambda_h$ and Nash bargain over $P_\tau$. Households have the outside option to continue searching, while intermediaries have no trade as an outside option.

We assume intermediary $i \in [0, 1]$ has a multiplier $\xi_i \in \mathbb{R}_{++}$ such that its leverage is $\xi_i L_t$ at time $t$ and the distribution of $\xi_i$ has a cumulative-density-function $F(\cdot)$ and density $f(\cdot)$ with support $\mathbb{R}_{++}$. This specification is to guarantee stationarity. \footnote{Note that for simplicity of exposition, we assume that leverage is positive instead of no less than 1. It should be interpreted as debt over equity in the model.} The default probability to a funding position at the repayment date $\tau$ is $\nu(L_\tau) \in [0, 1)$, such that $\nu(\cdot)$ is continuous, strictly increasing and $\nu(0) = 0$. The process $L_t$ is an Ito process\footnote{This assumption can be weaked into the following one: The process $L_t$ is a martingale and is a Markov process, and satisfies the monotone property: Starting from $L_t = \ell$, the future leverage $L_s$ is increasing with $\ell$ a.s. We need these properties for comparative statics.} with

$$L_t = L_0 \exp(-\frac{1}{2} \int_0^t \sigma^2_s ds - \int_0^t \sigma_s dB_s)$$

where $\sigma_t > 0$ is an adapted and bounded process.

To clearly identify the systematic and idiosyncratic risk components, we decompose the fundamental process as follows

$$X_t = \alpha + \beta \cdot L_t + \tilde{\sigma} \xi_t$$

where we assume that $\xi_t$ is i.i.d. with zero mean and unit variance, and is independent from

\footnote{2Note that for simplicity of exposition, we assume that leverage is positive instead of no less than 1. It should be interpreted as debt over equity in the model.}
the leverage process.\(^4\) By this assumption, we get \(X_t\) a martingale with respect to \(\mathcal{F}_t\). For ease of interpretation, we restrict the idiosyncratic volatility \(\tilde{\sigma} \geq 0\).\(^5\) As a result, an increase in \(\tilde{\sigma}\) is a mean preserving spread.

Intermediaries fund their positions from a group of risk neutral, deep pocket households. We assume a perfectly competitive funding market such that intermediaries can immediately find their funding at a competitive price. For a long position, the purchased asset is posted as collateral with margin. The intermediary allocates a nonnegative margin for the trading, which is costly as there is an opportunity cost for equity capital denoted by \(c\). For a short position, the cash from sale is posted as collateral along with margin.

After a household finds an intermediary, they enter Nash bargaining over the trading gain. Information is symmetric between households and intermediaries. The household has bargaining power \(\kappa\), and the outside option to randomly search another intermediary. The benefit of continued search is the possibility of finding an intermediary with lower leverage that provides better prices. The cost is a loss of utility from time discounting. The intermediary has an outside option of not trading. After trading, the intermediary search for other households to offload the inventory with intensity \(\lambda_d\) at the fundamental value\(^6\). We neglect the inter-dealer market without loss of generality, because the search between households and intermediaries already captures the key economic mechanism of funding costs impacting risk premia.

### 2.2. Leverage, Volatility and Margin

The margin is set by a competitive funding market so that in expectation lenders break-even. We assume the interest rate \(r\) is fixed. Because discount rate is zero in the model, this interest rate reflects the additional interest charged above and beyond the discount rate. Thus the funding cost variation is reflected in the margin requirement\(^7\).

Now we consider the funding of a long position of intermediary \(i\), whose leverage is \(\xi_i L_t\) at time \(t\). The whole process is illustrated in Figure 1. The search intensity is \(\lambda_d \in (0, \infty)\). During the period, the aggregate leverage \(L_t\) may change over time. Although not needed in the model, we may interpret the leverage as slow moving while asset fundamental is fast moving, which motivates our assumption of an exogenous leverage process, as leverage is relatively fixed at the trading horizon. At time \(\tau\), with probability \(1 - \nu(L_t \xi_i)\), the intermediary will not default.

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\(^4\)This assumption makes \(X_t\) a non-continuous process, but guarantees that only \(L_t\) is the state variable.

\(^5\)Alternatively, we may define the residual as \(-\epsilon_t\) and then get a nonnegative volatility. We assume the residual volatility to be constant for simplicity of comparative statics, but all the main results hold if we instead use a Markov process \(\sigma_t\), with the monotone property that the future path increases monotonically with current level.

\(^6\)We avoid another layer of bargaining for simplicity, and main results are robust if we assume the offloading price is determined by a bargaining.

\(^7\)We can also assume a fixed margin but a changing interest rate to reflect funding costs. Our main results are robust to this alternative assumption.
and repay the full amount $X_t e^{r(\tau-t)}$. However, with probability $\nu(L_t \xi)$, the intermediary will default, in which case the lender can sell the collateral at fundamental price $X_\tau$ (or just keep holding, which by risk-neutrality assumption also has a value of $X_\tau$) and also keep the margin $m_+ X_t$, but get at most the required amount $X_t e^{r(\tau-t)}$. We assume that the margin has to be nonnegative. Thus the margin is taken as the maximum of 0 and the one that solves the following break-even conditions

$$E_t[(1 - \nu(\xi L_\tau))X_t e^{r(\tau-t)} + \nu(\xi L_\tau) \min\{m_+ X_t + X_\tau, X_t e^{r(\tau-t)}\}] = X_t$$

$$E_t[(1 - \nu(\xi L_\tau))(X_\tau + X_t e^{r(\tau-t)}) + \nu(\xi L_\tau) \min\{m_- X_t + X_\tau, X_\tau + X_t e^{r(\tau-t)}\}] = E_t[X_\tau]$$

where the benchmark of borrowing/lending is expressed in terms of the fundamental value process $X_t$.\(^8\) When there is no default, the intermediary makes full-payment. In default, the lender keeps the margin and collateral. In case of a long position, the collateral is the security and in case of a short position is cash.

From the asset return decomposition (1), we can rewrite the margin equations into

$$E_t[(1 - \nu(\xi L_\tau))e^{r(\tau-t)} + \nu(\xi L_\tau) \min\{m_+ + \frac{1}{X_t}(\alpha + \beta L_\tau + \bar{\sigma} \varepsilon_\tau), e^{r(\tau-t)}\}] = 1$$

$$E_t[(1 - \nu(\xi L_\tau))\left(\frac{X_\tau}{X_t} + e^{r(\tau-t)}\right) + \nu(\xi L_\tau) \min\{m_- + 1, \frac{\alpha + \beta L_\tau + \bar{\sigma} \varepsilon_\tau}{X_t} + e^{r(\tau-t)}\}] = E_t\left(\frac{X_\tau}{X_t}\right)$$

where I use $\xi$ to replace $\xi_i$ for notational simplicity. Denote the solution of (2) as $\hat{m}_+(X_t, L_t, \xi; \bar{\sigma})$ and the solution of (3) as $\hat{m}_-(X_t, L_t, \xi; \bar{\sigma})$, which can take values in $\mathbb{R}$. Then the margin should

\(^8\)For the risk neutral households that have not experienced a liquidity shock, they have negligible order size and thus $X_t = P_t$. 

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Fig. 1. Illustration of the Funding of a Long Position
Denote the leverage multiplier of the intermediary picked up by the investor as $\xi$. We have the following proposition.

**Proposition 1.** The margin requirements $m_+(X_t, L_t, \xi; \tilde{\sigma})$ and $m_-(X_t, L_t, \xi; \tilde{\sigma})$ are increasing in $\tilde{\sigma}_t$, $L_t$ and $\xi$. Moreover, as the asset becomes fully hedgable (idiosyncratic risk approaches 0), the margin requirement is zero

$$\lim_{\xi \to 0} m_+(X_t, L_t, \xi; \tilde{\sigma}) = \lim_{\xi \to 0} m_-(X_t, L_t, \xi; \tilde{\sigma}) = 0$$

**Proof.** To study the monotonicity of $m_+$, we only have to study the monotonicity of $\hat{m}_+$ because $m_+$ preserves the monotonicity.

First, taking the expectation of (2) inside, we have

$$\mathbb{E}_t[(1 - \nu(\xi L_\tau))e^{-r(\tau-t)}] + \mathbb{E}_t[\nu(\xi L_\tau)\mathbb{E}[\min\{m_+ + \frac{1}{X_t}(\alpha + \beta L_\tau + \tilde{\sigma}\varepsilon_\tau), e^{-r(\tau-t)}\} | \mathcal{F}_t, L_\tau]] = 1$$

Because everything inside the inner conditional expectation is measurable with respect to the sigma algebra of $L_\tau$ and $\mathcal{F}_\tau$, and $\varepsilon_\tau$ has conditional mean 0, an increase in $\tilde{\sigma}$ decreases the inner expectation, by the concavity of min function. To compensate this decrease, $m_+$ have to increase.

Second, by definition of the leverage process, an increase in $L_t$ increases $L_\tau$ a.s., thus putting more weights on the default part of the payout. Because

$$\min\{m_t + \frac{1}{X_t}X_\tau, e^{r(\tau-t)}\} \leq e^{r(\tau-t)}$$

The left side of (2) decreases. To compensate this decrease, $m_t$ should increases. Similar arguments apply to $\xi$.

Finally, the limit of default probability is

$$\lim_{\xi \to 0} \nu(\xi L_\tau) = 0$$

by dominated convergence theorem. Thus when $\xi \to 0$, $\hat{m}_+$ has to go to $-\infty$, and thus $m_+$ goes to 0. The proof is similar for $m_-$. \qed

Proposition 1 makes explicit the connection among leverage, volatility, and margin requirements in a general setting. We do not impose specific functional form of leverage or the asset
value process. The margin requirement impacts the pricing of the asset through short term risk premia.

2.3. Household’s Search and Bargaining

Households affected by a liquidity shock search for intermediaries to negotiate a trading price. Households have the outside option to search for another intermediary. By the same random matching technology as in Duffie et al. (2005), the search intensity is \( \lambda_h \cdot 1 = \lambda_h \). Denote the value of the outside option as \( V_o(X_t, L_t) \) when the intermediary buys the asset, and \( V_o(X_t, L_t) \) when the intermediary sells the asset. Both \( V_+ \) and \( V_- \) are exogenous when the household bargains with the intermediary because the household does not have commitment power. In what follows, we will only present results for the case of households demanding liquidity from intermediaries position.

In the Nash bargaining, the excess utility ratio should be \( \kappa/(1 - \kappa) \),

\[
\frac{P_+ + \pi - X_t - V_o(X_t, L_t)}{X_t - E_t[\int_0^{\tau_d} c \cdot m_+(X_t, L_t, \xi; \tilde{\sigma}) P_t dt] - P_t} = \frac{\kappa}{1 - \kappa}
\]

(4)

where \( \tau_d \sim \text{Poisson}(\lambda_d) \) is the time for the intermediary to find another household for offloading inventory. Evaluating the integral and solving the price, we can express the price as a function of aggregate leverage, individual intermediary’s leverage, and current fundamental value:

\[
P_+(X_t, L_t, \xi) = \frac{X_t - (1 - \kappa)(\pi - V_o(X_t, L_t))}{1 + \kappa c \cdot m_+(X_t, L_t, \xi; \tilde{\sigma}) \lambda d + r}
\]

(5)

Equation (5) has rich interpretations. First, the price increases in the fundamental value \( X_t \), as expected, but the rate of increase is influenced by the margin requirement. Second, as shown later, \( \pi \geq V_o(X_t, L_t) \), and (5) shows that when the investor has better bargaining power, the selling price is higher, i.e. there is less price impact. Third, when the opportunity cost of equity capital is higher, the selling price is lower as the intermediary requires more compensation. Fourth, a reduction of search friction in households finding other intermediaries increases the selling price. Finally, the price of selling to an intermediary with higher leverage (\( \xi \) is higher) is lower.

Next, we proceed to solve the outside option \( V_o(X_t, L_t) \) for households. Note that the household has an optimal stopping problem. When a household finds an intermediary at \( L_t = L \), \( X_t = x \) and multiplier \( \xi \), the stopping region is

\[
S = \{ P_+(X, L, \xi) + \pi - X \geq V_o(X, X) \}
\]
With (5), the region is defined by

$$S = \left\{ \frac{-X_c \cdot m_+(X, L, \xi; \tilde{\sigma}) \frac{1}{\lambda d + r}}{1 + c \cdot m_+(X, L, \xi; \tilde{\sigma}) \frac{1}{\lambda d + r}} + \pi \geq V^o_+(X, L) \right\}$$

And the outside option is determined recursively by

$$V^o_+(X, L) = E[e^{-r \tau_0} \max\{P(X_{\tau_0}, L_{\tau_0}, \tilde{\xi}) + \pi - X_{\tau_0}, V^o_+(X_{\tau_0}, L_{\tau_0})\}|X_0 = X, L_0 = L]$$

Thus the problem is significantly influenced by the pricing with outside option, which has to be determined endogenously.

When the asset price is $X$, the aggregate leverage is $L$, and the specific leverage is $\xi$, the value function of selling for households is

$$V_+(X, L, \xi) = \max\{\pi + \frac{(1 - \kappa)V^o_+(X, L) - (1 - \kappa)\pi - \kappa c \cdot m_+(X, L, \xi; \tilde{\sigma}) \frac{1}{\lambda d + r}X}{1 + \kappa c \cdot m_+(X, L, \xi; \tilde{\sigma}) \frac{1}{\lambda d + r}}, V^o_+(X, L)\}$$

The problem is too hard to solve with general bargaining power $\kappa$. Without loss of intuition, we assume $\kappa = 1$, i.e. the intermediaries are so competitive that they make zero profit from intermediation. Although stark, this still captures the effects of funding cost on the pricing of idiosyncratic risks. Then prices are

$$P_+(X_t, L_t, \xi) = \frac{X_t}{1 + c \cdot m_+(X_t, L_t, \xi; \tilde{\sigma}) \frac{1}{\lambda d + r}}$$

$$P_-(X_t, L_t, \xi) = \frac{X_t}{1 - c \cdot m_-(X_t, L_t, \xi; \tilde{\sigma}) \frac{1}{\lambda d + r}}$$

which only depend on current asset price, aggregate leverage, and the leverage of specific intermediary. The household subject to liquidity shock will solve an optimal stopping problem with stopping region of both low $L_t$ and $\xi$. The problem can be formulated as

$$V_+(x, L, \xi) = \max\{P_+(x, L, \xi) + \pi - x, E[e^{-r \tau_0}V_+(X_{\tau_0}, L_{\tau_0}, \tilde{\xi})|X_0 = x, L_0 = L]\}$$

$$V_-(x, L, \xi) = \max\{x + \pi - P_-(x, L, \xi), E[e^{-r \tau_0}V_-(X_{\tau_0}, L_{\tau_0}, \tilde{\xi})|X_0 = x, L_0 = L]\}$$

where $\tau_0$ is the time to find another intermediary starting from time 0. With (6) and (7), we can get

$$V_+(x, L, \xi) = \max\{\pi - \frac{c \cdot m_+(x, L, \xi; \tilde{\sigma}) \frac{1}{\lambda d + r}x, E[e^{-r \tau_0}V_+(X_{\tau_0}, L_{\tau_0}, \tilde{\xi})|X_0 = x, L_0 = L]\}$$
\begin{align*}
V_-(x, L, \xi) = \max\{\pi - \frac{c \cdot m_-(X_t, L_t, \xi; \tilde{\sigma}) \frac{1}{\lambda_d + \tau}}{1 - c \cdot m_-(X_t, L_t, \xi; \tilde{\sigma}) \frac{1}{\lambda_d + \tau}} x, E[e^{-r\tau_0}V_- (X_{\tau_0}, L_{\tau_0}, \tilde{\xi}) | X_0 = x, L_0 = L]\}
\end{align*}

Then we have the following proposition.

**Proposition 2.** There exists thresholds \( \bar{\ell}_+(x, L) \in \mathbb{R}_+ \) \((\bar{\ell}_-(x, L) \in \mathbb{R}_+) \) \(^9\) when a household with selling (buying) shock meets an intermediary given the state \((x, L)\), such that only intermediaries with leverage below \( \bar{\ell}_+(x, L) \) \((\bar{\ell}_-(x, L))\) trade the asset. An increase in intermediary search efficiency \( \lambda_d \) reduces the threshold, while increase in trading gain \( \pi \) increases the threshold.

**Proof.** First, for the case of household selling assets, the stopping region is

\begin{align*}
\{\xi : \pi - \frac{c \cdot m_+(X, L, \xi; \tilde{\sigma}) \frac{1}{\lambda_d + \tau}}{1 + c \cdot m_+(X, L, \xi; \tilde{\sigma}) \frac{1}{\lambda_d + \tau}} x \geq E[e^{-r\tau_0}V_+ (X_{\tau_0}, L_{\tau_0}, \tilde{\xi}) | X_0 = x, L_0 = L]\}
\end{align*}

Because the left side decreases with \( \xi \in \mathbb{R}_+ \), there must be a threshold \( \bar{\ell}_+ \in \mathbb{R}_+ \) such that the household stops if and only if \( \bar{\ell}_+ \leq \bar{\ell}_+(x, L) \). The same arguments get through for households buying assets.

Second, with higher \( \lambda_h \), the left-hand side is lower, and thus the threshold \( \mathbb{R}_+ \) decreases. The same for increase in \( \pi \).

\( \square \)

This is a unique feature of our model. The supply of market making is increasing in intermediary leverage. The marginal intermediaries has higher leverage when liquidity demand is high. The distribution of intermediary leverage matters with respect to the dynamics of short term risk premia. This finding challenges assumption of theoretical models that treat intermediaries as a unit mass of specialists that perfectly risk share.

### 2.4. Equilibrium Risk Premia and Return Volatility

When there is no order imbalance, the funding cost is not incurred and the asset price follows \( X_t \). However, when there is order imbalance, the asset is evaluated either at \( P_+(X_t, L_t, \xi) \) for sell order imbalances or at \( P_-(X_t, L_t, \xi) \) for buy order imbalances. Denote the set of sell-order arrival times as \( \mathcal{T}_+(\omega), \omega \in \Omega \), and the set of buy-order arrival times as \( \mathcal{T}_-(\omega), \omega \in \Omega \). Then the price can be expressed as

\begin{align*}
P_t &= 1_{t \notin \mathcal{T}_+ \cup \mathcal{T}_-} X_t + 1_{t \in \mathcal{T}_+} 1_{t \leq \ell_+(X_t, L_t)} \frac{X_t}{1 + c \cdot m_+(X_t, L_t, \xi; \tilde{\sigma}) \frac{1}{\lambda_d + \tau}} \nonumber \\
&\quad + 1_{t \in \mathcal{T}_-} 1_{t \geq \ell_-(X_t, L_t)} \frac{X_t}{1 - c \cdot m_-(X_t, L_t, \xi; \tilde{\sigma}) \frac{1}{\lambda_d + \tau}} \quad (8)
\end{align*}

\( \quad ^9\text{Here } \mathbb{R}_+ = \mathbb{R}_+ \cup \infty. \)
We define the idiosyncratic risk premia as the amount of reversal in return when there is a demand shock, i.e.

\[ R^+_t = 1_{\xi_t \leq \ell_-(X_t, L_t)} \frac{X_t}{P_+(X_t, L_t, \xi_t)} - 1 = 1_{\xi_t \leq \ell_-(X_t, L_t)} c \cdot m_+(X_t, L_t, \xi_t; \tilde{\sigma}) \frac{1}{\lambda_d + r} \]  

(9)

\[ R^-_t = 1_{\xi_t \leq \ell_-(X_t, L_t)} \frac{X_t}{P_-(X_t, L_t, \xi_t)} - 1 = 1_{\xi_t \leq \ell_-(X_t, L_t)} c \cdot m_-(X_t, L_t, \xi_t; \tilde{\sigma}) \frac{1}{\lambda_d + r} \]  

(10)

**Proposition 3** (Leverage, Idiosyncratic Volatility and Risk Premia). The idiosyncratic risk premia, measured by the influence of demand shocks, increases with (a) cross-section and time-series leverage and (b) idiosyncratic volatility. Moreover, only intermediaries with individual leverage lower than a threshold price the asset.

**Proof.** From (9), (10), and proposition 1, it is easy to see that the risk premia increases in both cross-section leverage \( \xi \) and time-series leverage \( L \). Moreover, lower quantile of the leverage distribution influences the price of the asset.

Thus our model produces the sharp prediction that the lower quantiles of leverage distribution matters more for pricing assets. Finally, we can measure the amount of return volatility, defined by the quadratic variation in returns to fundamental \( R_t = X_t/P_t \).

**Proposition 4** (Endogenous Return Volatility). The idiosyncratic return volatility measured by quadratic variation of returns increases with aggregate leverage.

**Proof.** We notice that liquidity shocks increase influence of demand shocks, as in (8), thus increasing the quadratic variation of returns.

### 2.5. Extension: Funding Capacity Constraint

In the model, the size of liquidity shocks has no effect on risk premia. This unintuitive result comes from the assumption of perfectly elastic funding costs with respect to size. In effect, each intermediary has unlimited funding capacity. To study how aggregate liquidity shock size affects trading and risk premia, we assume that each intermediary has a funding capacity \( q > 0 \), which is smaller than the liquidity shock size \( Q \). As a result, to fully realize trading gains, the investor has to trade with \( n(Q) = \lceil Q/q \rceil > 1 \) intermediaries. This capacity constraint effectively increases investor more urgency, which raises the trading thresholds \( \ell_+ \) and \( \ell_- \). In what follows, we will vary liquidity shock size \( Q \) but fix capacity \( q \) to study the impact of an increase in liquidity shock size.

**Proposition 5** (Funding Capacity Constraint and Supply Shock). Assume that each intermediary has a funding capacity \( q \in (0, Q) \), where \( Q \) is the size of the liquidity shock. Then an
increase in $Q$ increases $\bar{\ell}$. Specifically, when the increase in $Q$ causes an increase in $n(Q)$, the increase in $\bar{\ell}$ is strict.

2.6. Alternative Models

In the model, we focus on the funding channel to explain risk premia dynamics. Since the dynamics happen over a few days, new debt, not equity, funding clear the market. There is another possible channel: a wealth effect of intermediaries, similar to He and Krishnamurthy (2013). To compare the similarity and differences between these two explanations, we build another model with risk averse intermediaries, shown in Appendix B. We summarize our findings in that alternative model as follows.

- Idiosyncratic risk premia increases with the interaction of idiosyncratic risks and intermediary leverage. This is intuitive because leverage reflects intermediary’s capacity to bear risks. Thus with a higher leverage, the per unit increase in volatility requires higher compensation.
- Return volatility increases endogenously with intermediary leverage.

As a result, we find a common prediction that idiosyncratic risk volatility increases with intermediary leverage. However, we also find a difference that can distinguish the wealth effects and margin effects. The wealth effects predict a strong interaction between leverage and volatility. However, the margin channel predicts that both leverage and volatility separately increase risk premia.

3. Empirical Results

Our theory applies to multiple asset classes and in all periods where intermediaries hold risky assets, not only in crises. To empirically test the model, we use three asset classes, equities, bonds and currencies. Furthermore, we check the robustness of our results by excluding financial crises.

3.1. Data

For the three markets, equities, bonds, and currencies, we construct standardized datasets for comparability. The sample spans 1990 to 2015 for equities and currencies, which covers two global financial crises (Asia crisis of 1997 and the Great Recession of 2008). Including these two major crises enables us to test the model predictions in normal periods and financial crises. Bond data is constrained by the inception of TRACE in July, 2002\textsuperscript{10}. Building on the

\textsuperscript{10}The national association of securities dealers first launched TRACE on July 1st, 2002.
methodology of HKM, we create a daily variant of their quarterly intermediary capital measure as follows.

\[ L_t = \frac{\sum_{i=1}^{n} \text{Market Equity}_{i,t} + \text{Book Debt}_{i,t}}{\sum_{i=1}^{n} \text{Market Equity}_{i,t}} \]  

(11)

Thus the leverage we are measuring is the reciprocal of the capital ratio in HKM. Our constructed daily series \( \{L_t\} \) when subsampled to the quarterly frequency is over 96% correlated with the quarterly HKM series. In measuring intermediary leverage, we also keep firm specific ratios. In doing so, we are able to explore the effects of idiosyncratic capital shocks on intermediation activity.

First, we source return series for equities from CRSP, US bonds from TRACE, and currencies from Bloomberg. We focus on the S&P500 equities to set a high bar for finding intermediary capital effects on risk premia. Drechsler and Drechsler (2014) document a shorting premium in smaller, more illiquid stocks and a strong association with known asset pricing anomalies. By only including S&P500 equities, we avoid such concerns. Similarly, for US corporate bonds, we choose the top 1,000 traded bonds\(^{11}\). Furthermore, we follow the methodology of Dick-Nielsen (2009) in eliminating erroneously recorded trades in TRACE. For currencies, we include 12 major currencies, covering Australia, Canada, Denmark, Eurozone, Japan, Korea, Norway, New Zealand, Sweden, Switzerland, and United Kingdom. The US dollar serves as the base currency. Table 1 presents summary statistics of the data.

From these return series, we compute a measure of idiosyncratic risk following Foster and Nelson (1996). We project returns on a parsimonious set of liquid, traded factors. The intuition is that intermediaries may be partially hedging the exposure to common factors. Since we are interested in the idiosyncratic risk known to market participants contemporaneously, we choose a one-sided rolling regression window. In effect, we are fitting a conditional linear factor model

\[ R_{i,t} = \alpha_{i,t} + \beta_{i,t} F_t + \varepsilon_{i,t} \]  

(12)

over three month rolling windows. We define the idiosyncratic risk of asset \( i \) for time \( t \) to be

\[ I_{i,t} = \text{Var}(\varepsilon_{i,t}) \]  

(13)

where the variance is compute over three months of residuals. This corresponds to the idiosyncratic variance \( \tilde{\sigma}_t^2 \) in the model. We use both the traded S&P500 index (futures) and the 10-year Treasury rate as the risk factors for bonds and equities. Our theory requires that

\(^{11}\)Another reason for only using the top 1,000 traded bonds is that corporate bond trading is very sporadic, which makes it hard to get consecutive daily prices from the trading data. Using the most frequently traded bonds alleviate this problem.
when we extract idiosyncratic risks and when we do regressions on idiosyncratic risk premia, we should use the same set of liquid traded assets. We follow the theory by also controlling these two factors in regressions on idiosyncratic risk premia. To measure idiosyncratic risk in the currency market, we instead use option implied volatility against the USD, which is a more robust measure of idiosyncratic risks for currencies.

To identify periods where intermediaries are absorbing large order imbalances in the short run, we use tails of daily asset returns, which is a noise proxy. The intuition is that following large trading volume associated with substantial price movements, intermediaries are more likely to be active in clearing the market. Using proprietary data from the Taiwan Stock Exchange, Andrade et al. (2008) confirm this intuition. Consequently, we focus on periods following large price movements. To clear the market during price shocks, intermediaries hold inventory. For compensation, intermediaries charge a risk premium. The risk premium magnifies the price shock and reverts as intermediaries offload their inventory. We measure the risk premia as the subsequent five-day return after the shock.

$$RP_{i,t} = \frac{\Delta P_{i,t \rightarrow t+5}}{P_t}$$

where $RP_{i,t}$ is the value of short-term risk premia for asset $i$ in time $t$, $P_{i,t}$ is the price (or exchange rate against USD) of asset, and $\Delta P_{i,t \rightarrow \tau} = P_{i,\tau} - P_{i,t}$ is the price difference. Mapping to the model, $RP_{i,t}$ theoretically increases with both leverage and idiosyncratic risk, and it is a proxy of the liquidity component in price movement, i.e. the magnitude of reversal in returns, in the short-run. We define short-run as one trading week (5 days). This is consistent with Collin-Dufresne and Daniel (2014) who find a half-life of 2.5 days for the temporary component of price shocks. This 5 day estimate of risk premia biases down the results. If intermediaries offload inventories more slowly than 5 days, then we underestimate risk premia.

Figure 2 illustrates a series of examples showing full, partial, and no return reversals and the corresponding risk premia. To show the relationship between the key variables defined above, Figure 3 jointly plots them across equity, bond, and currency markets. We find a strong co-movements of intermediary leverage, aggregate idiosyncratic risks, and the idiosyncratic risks. These plots provide a visual check of our theory, and it turns out that our predictions are quite visible from data.

Insofar as we are interested in identifying an association between aggregate intermediary capital and idiosyncratic risk premia, the above data is sufficient. However, another novel contribution of this paper is to study the heterogeneity of intermediary capital and identify causal relationships. To do so, we utilize the granular identification available through Fixed Income Securities Database (FISD). Unlike the anonymously reported transactions in TRACE,
FISD reports dealer identity.\textsuperscript{12} The cost to this granular identification is the sample. FISD reports only dealer trades with insurance companies and health maintenance organizations. However, we do have a longer time series, Jan 1994 to Dec 2014. To the extent that this subset of trades are representative of trading activity in the US bond market, our findings are generalizable. With the known dealer identity, we construct a novel dataset matching the intermediation activity of specific dealers with their capital ratios. Using this dataset, we define three different measures of trading activity of intermediaries, including capital intermediated, number of trades, number of new bonds intermediated. Due to semi-infrequent bond trading, we aggregate trading at the monthly frequency. Consequently, we may measure the effect of idiosyncratic and aggregate capital shocks on dealer-specific activity in the US bond market.

To assess the risk sharing properties of intermediaries, we compare aggregated vs. segmented intermediary capital. Using the dealer identity information in FISD, we construct the following measure of individual intermediary capital specialized in trading bond $i$.

$$
L^{id}_{i,t} = \frac{\sum_{j \in I(i,t)} \left( \text{Market Equity}_{j,t} + \text{Book Debt}_{j,t} \right)}{\sum_{j \in I(i,t)} \text{Market Equity}_{j,t}}
$$

(15)

In (15), $I(i,t)$ is the set of intermediaries that trade bond $i$ at time $t$. Note that the median number of dealers for a bond over its lifetime is two. This measure of bond specific intermediary capital may yield insight into the extent to which intermediary capital is slow moving.

### 3.2. Short-Term Risk Premia

The intermediary asset pricing literature derives insights into the non-linear behavior of risk premia during crises. However, an inherent difficulty in empirically verifying such predictions is the scarcity of financial crises. In an exceptionally comprehensive dataset on financial crises, Krishnamurthy and Muir (2016) find a negative relationship between credit spreads and the severity of financial crisis across 19 countries and 44 crises. These 44 financial crises span from 1869 to 2014 and a variety of financial intermediaries and market conditions. Whether evidence of intermediary asset pricing may convincingly be found using case studies of financial crises is beyond the scope of this paper.

Instead of studying financial crises and long term risk premia effects, we find identification in idiosyncratic order imbalances and idiosyncratic risk premia. Drawing from the price reversals literature, we know that predictable reversals do occur (Cox and Peterson, 1994). Empirically, we identify event days across equity, bond, and currency markets where intermediaries probably

\textsuperscript{12}The identity of dealer in this database is name, which is subject to input errors and hard to uniquely identify. We manually map dealer names to their cusips and ticker, which we then use to merge with CRSP/Compustat Merged Database.
absorbed large, undiversifiable positions. Due to potential shorting constraints, we focus on tail negative returns that are outside two standard deviation of past return volatility, which by definition doesn’t use future information. The intuition is that intermediaries primarily offload inventory during positive price shocks. However, intermediaries clear the market during negative price shocks by taking on inventory.

According to Proposition 3, the idiosyncratic risk premia with positive intermediary holding should increase in both intermediary leverage and idiosyncratic risks. Thus we hypothesize that conditional on a price shock, short term risk premia are positively related to both intermediary leverage and asset idiosyncratic risk. We empirically test this proposition:

\[ RP_{i,t} = \alpha_i + T_t + \beta_1 \cdot I_{i,t-1} + \beta_2 \cdot L_{t-1} + \gamma_1 \cdot r_t + \gamma_2 \cdot \xi_t + \varepsilon_{i,t} \]  

(S1)

Of note is that this specification uses only real-time publically available information, which is a strict subset of information available to financial specialists. In effect, our estimates of risk premia are lower bounds. Market makers may more optimally manage order flows and inventories than this simple trading strategy.

We include asset fixed effects (\(\alpha_i\)) to capture unobservable features of the asset that may affect risk premia. Similarly, we include year dummies (\(T_t\)) to absorb any common factors that may change over our sample. For instance, the rise of high frequency traders has increased the speed at which price discovery occurs within financial markets. Brogaard et al. (2014) find that high frequency traders trade in the direction of permanent price changes, especially following macroeconomic news announcements. More rapid price discovery may increase the size of price shocks. Additionally, we control for the risk free rate (10 year US Treasury yield, denoted by \(r_t\)) and the S&P500 return (\(\xi_t\)). These two controls are liquid traded assets, which may have been used to partially hedge the exposure in the risky asset. Aggregate intermediary leverage (\(I_{i,t-1}\)) is averaged over the past month for smoothness. Standard errors are clustered by asset and year. We lag the explanatory variables by one day to only use real time information.

We find evidence of intermediaries pricing risk premia based on asset idiosyncratic risk and funding costs. Table 2 displays the primary regression results.

Specifically, for bonds, as shown in columns (1) and (2) of both Table 2, the risk premia increase with asset idiosyncratic risk and intermediary leverage. Funding costs, not asset volatility, explains risk premia over the full sample (1). However, excluding the financial crisis of 2008 (2), we also see an effect of asset volatility on risk premia. This finding is consistent with the intermediary dependence and illiquidity of the corporate bond market. Funding costs matter more among our sample of large, liquid bonds, which are overwhelmingly investment grade.

For equities, as shown in columns (3) and (4) of Table 2, short term risk premia increase
significantly with asset volatility and intermediary leverage. The effects tend to be larger, but not significantly different, when we include the financial crisis of 2008. Both the magnitude of risk premia and price shocks are larger in the equity market, relative to the bond market. This difference in part explains the difference in magnitude between the funding cost coefficients.

For currencies, as shown in columns (5) and (6) of both Table 2, we do not find evidence of an effect of funding costs on short term risk premia. Potential explanations to this may be the greater complexities of cross-border capital flows and their relation to exchange rate. For instance, Bruno and Shin (2014) document the currency appreciation channel of cross border capital flows. Furthermore, central banks play an active role in managing exchange rates. Central banks are not directly subject to any such capital constraints or even costs of capital. We continue to explore the potentially different risk premia dynamics in currency markets with respect to intermediary funding costs. These caveats aside, we do find evidence of an effect of idiosyncratic asset volatility on short term risk premia.

Finally, results are economically significant. To demonstrate this, for each year we group price pressure events by whether the leverage or idiosyncratic risk exceed its 70% quantile. Results are in Table 3. Within a given year, intermediaries earn 37.5 more bps in bond markets and 104 bps more for equities when funding costs are high. Similarly, intermediaries earn 100, 67, and 96 more bps for bonds, equities, and currencies with high idiosyncratic risk. Short term risk premia across all three markets vary in response to asset volatility and funding costs.

### 3.3. Return Volatility

According to Proposition 4, return volatility increases with intermediary leverage. The intuition follows from increased volatility in risk premia due to higher funding costs. To formally test this model implication, we design the following empirical specification.

\[
I_{i,t} = \alpha_i + T_t + \beta L_{t-1} + \gamma_1 \cdot r_t + \gamma_2 \cdot \xi_t + \varepsilon_{i,t}
\]  

(S2)

In (S2), we include asset fixed effects \((\alpha_i)\) to capture unobservable features of the asset that may affect risk premia. Similarly, we include year dummies \((T_t)\) to absorb any variation in common factors, which explains risk premia. Proposition 7 predicts that \(\beta\) is significantly positive.

Empirically, we find evidence that intermediary leverage in the previous month increases current idiosyncratic volatility. Table 4 finds this effect across all three asset classes. Furthermore, the finding is robust to the exclusion of financial crises. The findings are economically significant: a one percentage increase in intermediary leverage increases return variance by 0.77 to 1.79 percent for bond and equity markets and 0.13 to 0.56 percent for currency markets.
3.4. Funding and Market Making Activity

The model predicts that greater funding costs decrease market making activity, as shown in Proposition 2. To test this prediction, we design the following empirical specifications:

\[ \text{Intermediation}_{j,t} = \alpha + \beta_1 \cdot L^{id}_{j,t-1} + \beta_2 \cdot A^{id}_{j,t-1} + \varepsilon_{j,t} \]  

(S3)

where intermediation may be measured by trade size, trading volume or number of trades, \( L^{id}_{j,t} \) is the individual leverage, \( A^{id}_{j,t-1} \) is the individual intermediary’s balance sheet size, which controls the heterogeneity of scale among intermediaries. To test the robustness of leverage on trading activity, we also define a measure of relative leverage

\[ \tilde{L}^{id}_{j,t} = \frac{L^{id}_{j,t}}{L_t} \]  

(16)

which reflects the relative distribution of intermediary leverage. We hypothesize that intermediaries with relatively higher leverage are less active in market making. This follows from the search mechanism in the model, where clients find intermediaries with lower funding costs to obtain better prices. With this new variable, we define an alternative test

\[ \text{Intermediation}_{j,t} = \alpha + \beta_1 \cdot \tilde{L}^{id}_{j,t-1} + \beta_2 \cdot A^{id}_{j,t-1} + \varepsilon_{j,t} \]  

(S4)

Across multiple definitions of market making, we find evidence that intermediaries with higher funding costs are less active. Table 5 displays the primary results. We find very large negative effects of individual dealer leverage on its trade size, monthly volume, and monthly number of trades. These findings are consistent with our model implications. As a robustness check, we consider relative leverage as in specification (S4). Table 6 shows robust findings for trade size and monthly number of trades.

3.5. Market Level Risk Sharing

We study the extent of intermediary risk sharing. To do so, we compare segmented vs. aggregate intermediary capital. Segmented capital includes only the capital of intermediaries who have recently traded the asset. Aggregate intermediary capital is the sum of all intermediary capital, irrespective of activity in the particular asset. This separation follows from the intuition that intermediaries specialize in trading particular assets. For instance, the median bond within our sample has two active intermediaries over its lifetime.

We assess the efficiency of intermediary risk sharing by measuring the extent to which segmented or aggregate intermediary capital explains activity in a particular asset. Efficient risk sharing is consistent with aggregate intermediary capital explaining the market making activity
for bond $i$. We empirically test this hypothesis with the following regression specification.

$$\text{Intermediation}_{i,t} = \alpha_i + \gamma_1 \cdot L_{i,t-1}^d + \gamma_2 L_{t-1} + \varepsilon_{i,t} \quad (S5)$$

where $L_{t}$ denotes aggregate leverage, while $L_{i,t-1}^S$ is the segmented leverage of dealers trading bond $i$ in period $t$. We include bond fixed effects to capture unobservable characteristics of the asset that impact its trading activity. To avoid any endogenous, mechanical relationship between trading activity and intermediary leverage, we lag leverage by one month. Standard errors are clustered by asset.

For the market as a whole, we find strong evidence of order sharing. We compare aggregate with segmented intermediary capital, which is only the capital of intermediaries that traded bond $i$ in period $t$. Table 8 presents results indicative of risk sharing among intermediaries. In response to a one standard deviation decrease in aggregate capital in previous month, intermediary market making decreases 15.75 percent. However, for segmented intermediary capital, the effects are statistically and economically insignificant. Consequently, aggregate capital impacts the market making activity of intermediaries for particular assets. In effect, when one intermediary pulls back, another fills the gap.

### 3.6. Liquidity Shock and Participation in Intermediation

With intermediary funding capacity constraint, a liquidity demand shock raises the threshold $\bar{\ell}_+$ by which households search for another intermediary, and thus the high-leverage intermediaries will get more trading volume, as shown in Proposition 5. We empirically test this prediction following

$$\text{Trading Volume}_{i,t} = \alpha_j + \gamma_1 \cdot \Delta \text{Total Volume}_t + \gamma_2 L_{i,t-1}^d \cdot \gamma_3 L_{t-1} + \varepsilon_{i,t} \quad (S6)$$

where $\text{Trading Volume}_{i,t}$ is the log trading volume of intermediary $i$ for month $t$; $\Delta \text{Total Volume}_t$ is the change in log aggregate trading volume for month $t$; $L_{i,t-1}^d$ is the leverage of individual intermediary $i$ for month $t - 1$; $L_{t-1}$ is the aggregate leverage of intermediaries for month $t - 1$.

Consistent with the economic intuition of a supply curve to market making, we find that high leverage intermediaries absorb increases to aggregate trading volume. In Table 7, column (1) confirms that an increase in trading volume mechanically increases aggregate intermediary market making. Columns (2) and (3) split the sample into above and below average leverage intermediaries. The high leverage intermediaries increase their market making activity by 1.5 percent per 1 percent increase in aggregate trading volume. However, low leverage intermediaries do not significantly respond to changes to aggregate trading volume. These findings are consistent with predictions in Proposition 5, and confirms the economic mechanism that larger
liquidity shocks are effectively more urgent, and thus willing to trade with intermediaries of higher leverage.

3.7. Differentiating Wealth Effects and Margin Effects

As discussed in section 2.6, a key difference between the funding based mechanism and the wealth mechanism is that the wealth mechanism uniquely predicts that only the interaction of leverage and idiosyncratic risks matter for short-term risk premia. As a result, a natural test of the two mechanisms would be including leverage, idiosyncratic risks, and their interaction in the same regression. We design the following regression specification:

$$ RP_{i,t} = \alpha_i + T_t + \beta_0 \cdot I_{i,t-1} \cdot L_{t-1} + \beta_1 \cdot I_{i,t-1} + \beta_2 \cdot L_{t-1} + \gamma_1 \cdot r_t + \gamma_2 \cdot \xi_t + \varepsilon_{i,t} $$  \hspace{1cm} (S7)

where $RP_{i,t}$ is defined in (14). To get robust and stable results, we use indicators of high leverage and high idiosyncratic risks, defined as exceeding the 70% quantile within the same year. We find that the interaction term is unstable and insignificant, as shown in Table 9, while the individual effects of leverage and idiosyncratic risks are almost not influenced. These results reject the hypothesis that there is a significant wealth effect in the pricing of short-term risk premia.

4. Conclusion

We extend the theory of intermediary asset pricing to a market microstructure setting to explain short run risk premia. Our model characterizes the process by which intermediary market making generates risk premia. The primary, novel implication of this model is that intermediary leverage is a state variable for the magnitude of short run risk premia. The model explicitly links intermediary leverage, idiosyncratic risk and risk premia. The model predicts that risk premia increase with both intermediary leverage (funding costs) and idiosyncratic volatility of asset returns.

Empirically, we test our model across multiple asset classes, including bonds, equities, and currencies. Across markets, intermediaries earn on average risk premia in the range of 50 to 80 basis points by clearing order imbalances in periods of large negative tail asset returns. These risk premia increase by about 37 bps for bonds, 103 bps for equities and insignificantly for currencies when intermediary leverage is high. Similarly for high idiosyncratic risk assets, risk premia increase by 100 bps for bonds, 67 bps for equities, and 96 bps for currencies. Furthermore, these risk premia dynamics are present in large and liquid assets (S&P500 equities, 600 most traded bonds, and 12 major developed market currencies), which we expect to be
efficiently priced. Consistent with our model, these findings are robust to excluding recessions. Intermediary leverage prices short term risk premia in all periods. Currently, we are working on expanding the set of asset classes to include derivative markets, sovereign debt, and commodity markets. These findings complement the empirical findings of AEM and HKM by pointing out that intermediary capital is an important risk factor.

To more convincingly establish causality, we identify the effect of idiosyncratic capital shocks to intermediaries on market making activity in the US bond market. Following an idiosyncratic negative shock to capital in the previous month, intermediaries pull back from market making activities. The financial specialist intermediates less capital and participates in fewer trades. However, for the market as a whole, we find evidence of risk sharing. Despite heavy dealer specialization in US bond markets, aggregate capital determines market making activity. When an intermediary experiences a capital shock, others increase their market making activities. Furthermore, we find evidence of intermediary capacity constraints. In periods of larger liquidity demand, intermediaries with higher leverage participate more. Consequently, the distribution of intermediary leverage is an important factor in explaining risk premia in equities, bonds and currencies.
References


Appendix A.   Empirical Evidence

Table 1: Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>Bond (TRACE)</th>
<th>Equity (CRSP)</th>
<th>Currency (Bloomberg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Assets</td>
<td>528</td>
<td>1025</td>
<td>33</td>
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<tr>
<td>Observations</td>
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<td>206238</td>
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<td>Pressure Events</td>
<td>133871</td>
<td>172906</td>
<td>6215</td>
</tr>
<tr>
<td>Average Day 0 Return</td>
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<td>-5.28%</td>
<td>-1.08%</td>
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<tr>
<td>Average 5-Day Return</td>
<td>0.56%</td>
<td>0.67%</td>
<td>0.83%</td>
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<tr>
<td>Reversal Response</td>
<td>49%</td>
<td>8%</td>
<td>26%</td>
</tr>
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<td>Average Idiosyncratic Vol</td>
<td>2.15%</td>
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<td>Average Systematic Vol</td>
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<td>Idiosyncratic Vol/Systematic Vol</td>
<td>5.6</td>
<td>6.2</td>
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</table>
Fig. 2. Examples of Price Reversals and Idiosyncratic Risk Premia
Fig. 3. Idiosyncratic Risk Premia, Intermediary Leverage, and Idiosyncratic Risks
### Table 2: Short-Term Risk Premia on Idiosyncratic Risk and Leverage

**Dependent variable: Short-Term Risk Premia**

<table>
<thead>
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<th></th>
<th>Bond (1)</th>
<th>Equity (2)</th>
<th>Equity (3)</th>
<th>Equity (4)</th>
<th>Currency (5)</th>
<th>Currency (6)</th>
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<tr>
<td>I</td>
<td>1.355***</td>
<td>0.216</td>
<td>2.039***</td>
<td>2.171***</td>
<td>6.330***</td>
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<td></td>
<td>(0.329)</td>
<td>(0.162)</td>
<td>(0.247)</td>
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<td>29.356**</td>
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<tr>
<td>R²</td>
<td>0.039</td>
<td>0.039</td>
<td>0.027</td>
<td>0.029</td>
<td>0.064</td>
<td>0.069</td>
</tr>
</tbody>
</table>

* p<0.1; ** p<0.05; *** p<0.01

Note: This table shows the regression results of short-term risk premia, defined in equation (14) as the reversal return after liquidity shocks, regressed on asset idiosyncratic risk and aggregate intermediary leverage. Controls include asset fixed effects, year fixed effects, risk free rate (10 year US Treasury yield) and the S&P500 return. We lag the variables by one day to avoid capturing any potential endogenous effects. Asset-Year clustered standard errors are shown in the parentheses.
Table 3: Short-Term Risk Premia Regressed on Idiosyncratic Risk and Leverage Indicators

<table>
<thead>
<tr>
<th></th>
<th>Bond</th>
<th>Equity</th>
<th>Currency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>HighI</td>
<td>99.742***</td>
<td>53.140***</td>
<td>66.913***</td>
</tr>
<tr>
<td></td>
<td>(31.363)</td>
<td>(9.150)</td>
<td>(16.324)</td>
</tr>
<tr>
<td>HighLvg</td>
<td>37.513***</td>
<td>23.598**</td>
<td>103.647***</td>
</tr>
<tr>
<td></td>
<td>(14.363)</td>
<td>(9.673)</td>
<td>(27.897)</td>
</tr>
<tr>
<td>Crisis excluded</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Asset Fixed effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Year Fixed effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Clustered errors</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Degree of freedom</td>
<td>132943</td>
<td>108523</td>
<td>171854</td>
</tr>
<tr>
<td>Observations</td>
<td>133,488</td>
<td>109,067</td>
<td>172,906</td>
</tr>
<tr>
<td>R²</td>
<td>0.033</td>
<td>0.059</td>
<td>0.031</td>
</tr>
</tbody>
</table>

Note: This table shows the regression results of short-term risk premia, defined in equation (14) as the reversal return after liquidity shocks, regressed on indicators of asset idiosyncratic risk and aggregate intermediary leverage. HighI is defined as 1 if the asset idiosyncratic risk is higher than 70% quantile of its distribution in the same year, and HighLvg is defined similarly. Controls include asset fixed effects, year fixed effects, risk free rate (10 year US Treasury yield) and the S&P500 return. We lag the variables by one day to avoid capturing any potential endogenous effects. Asset-Year clustered standard errors are shown in the parentheses.
<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(Aggregate Leverage)</td>
<td>2.068***</td>
<td>1.329***</td>
<td>1.142***</td>
<td>0.832***</td>
<td>0.801**</td>
<td>0.203</td>
</tr>
<tr>
<td></td>
<td>(0.299)</td>
<td>(0.432)</td>
<td>(0.230)</td>
<td>(0.230)</td>
<td>(0.336)</td>
<td>(0.278)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crisis Excluded</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Asset Fixed effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Year Fixed effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Clustered errors</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Degree of freedom</td>
<td>1078665</td>
<td>906785</td>
<td>4018657</td>
<td>3650530</td>
<td>204032</td>
<td>181068</td>
</tr>
<tr>
<td>Observations</td>
<td>1,079,208</td>
<td>907,327</td>
<td>4,019,710</td>
<td>3,651,580</td>
<td>204,093</td>
<td>181,127</td>
</tr>
<tr>
<td>R²</td>
<td>0.691</td>
<td>0.659</td>
<td>0.537</td>
<td>0.534</td>
<td>0.628</td>
<td>0.616</td>
</tr>
</tbody>
</table>

*p<0.1; **p<0.05; ***p<0.01

Note: This table shows the regression results of individual asset return variance on aggregate intermediary leverage. We lag the aggregate leverage by one day to avoid capturing any potential endogenous effects. Controls include asset fixed effects, year fixed effects, risk free rate (10 year US Treasury yield) and the S&P500 return. Asset-Year clustered standard errors are shown in the parentheses. Crisis periods are from the NBER recessions.
### Table 5: Leverage and Trading Activity

<table>
<thead>
<tr>
<th></th>
<th>Trade Size</th>
<th>Total Volume</th>
<th>log(No. of Trades)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>Individual Leverage</td>
<td>−51.165*</td>
<td>−61.382***</td>
<td>−63.351</td>
</tr>
<tr>
<td></td>
<td>(27.194)</td>
<td>(23.016)</td>
<td>(45.005)</td>
</tr>
<tr>
<td>Total Asset</td>
<td>1.709***</td>
<td>1.760***</td>
<td>3.423***</td>
</tr>
<tr>
<td></td>
<td>(0.100)</td>
<td>(0.141)</td>
<td>(0.316)</td>
</tr>
</tbody>
</table>

| Crisis Excluded? | No         | Yes          | No                  | Yes                | No                | Yes               |
| Year Fixed effects | Yes        | Yes          | Yes                 | Yes               | Yes               | Yes               |
| Clustered errors | Yes        | Yes          | Yes                 | Yes               | Yes               | Yes               |
| Degree of freedom | 1474444    | 1263370      | 6587                | 5521               | 6587              | 5521              |
| Observations     | 1,474,468  | 1,263,391    | 6,611               | 5,542              | 6,611             | 5,542             |
| R²               | 0.011      | 0.012        | 0.538               | 0.536              | 0.423             | 0.416             |

*p<0.1; **p<0.05; ***p<0.01

Note: This table shows the regression results of intermediation activity individual leverage and total assets. Regressions (1) and (2) are on trade-level data, while (3) to (6) are on dealer-level aggregated data. We lag the relative leverage by one day to avoid capturing any potential endogenous effects. Controls include year fixed effects. Year clustered standard errors are shown in the parentheses. Crisis periods are from the NBER recessions.
Table 6: Leverage Distribution and Trading Activity

<table>
<thead>
<tr>
<th></th>
<th>Trade Size</th>
<th>Total Volume</th>
<th>log(No. of Trades)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>Individual Leverage/</td>
<td>−608.75***</td>
<td>−667.63***</td>
<td>−437.38</td>
</tr>
<tr>
<td>Aggregate Leverage</td>
<td>(185.96)</td>
<td>(176.20)</td>
<td>(436.96)</td>
</tr>
<tr>
<td>TotalAsset</td>
<td>1.72***</td>
<td>1.77***</td>
<td>3.42***</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.14)</td>
<td>(0.32)</td>
</tr>
</tbody>
</table>

Crisis Excluded? | No | Yes | No | Yes | No | Yes
Year Fixed effects | Yes | Yes | Yes | Yes | Yes | Yes
Clustered errors | Yes | Yes | Yes | Yes | Yes | Yes
Degree of freedom | 1474444 | 1263370 | 6587 | 5521 | 6587 | 5521
Observations | 1,474,468 | 1,263,391 | 6,611 | 5,542 | 6,611 | 5,542
R² | 0.004 | 0.004 | 0.54 | 0.54 | 0.42 | 0.42

*p<0.1; **p<0.05; ***p<0.01

Note: This table shows the regression results of intermediation activity on relative leverage as defined in (16) and total assets. Regressions (1) and (2) are on trade-level data, while (3) to (6) are on dealer-level aggregated data. We lag the relative leverage by one day to avoid capturing any potential endogenous effects. Controls include year fixed effects. Year clustered standard errors are shown in the parentheses. Crisis periods are from the NBER recessions.
Table 7: Supply Shock and Change in Intermediation

<table>
<thead>
<tr>
<th></th>
<th>Dependent Variable: Total Trading Volume</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>All</td>
<td>High Leverage</td>
<td>Low Leverage</td>
</tr>
<tr>
<td>Change in Aggregate Volume (log)</td>
<td></td>
<td>0.5947</td>
<td>1.5075</td>
<td>0.1988</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.07)**</td>
<td>(2.64)***</td>
<td>(1.10)</td>
</tr>
<tr>
<td>Individual Leverage</td>
<td></td>
<td>-0.009</td>
<td>-0.0036</td>
<td>-0.3566</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3.85)***</td>
<td>(1.75)*</td>
<td>(7.44)***</td>
</tr>
<tr>
<td>Aggregate Leverage</td>
<td></td>
<td>0.0159</td>
<td>-0.014</td>
<td>0.0284</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4.18)***</td>
<td>(4.05)***</td>
<td>(5.46)***</td>
</tr>
<tr>
<td>Bank FE</td>
<td></td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>$R^2$</td>
<td></td>
<td>0.72</td>
<td>0.75</td>
<td>0.61</td>
</tr>
<tr>
<td>$N$</td>
<td></td>
<td>5986</td>
<td>1265</td>
<td>4721</td>
</tr>
</tbody>
</table>

*p<0.1; **p<0.05; ***p<0.01

Note: This table shows the regression results of trading volume among different leverage groups on the change in aggregate volume, individual leverage, and aggregate leverage. It illustrates whether intermediaries with higher leverage increasingly participate when order imbalances are larger. We lag the leverage variables by one day to avoid capturing any potential endogenous effects. Controls include bank fixed effects.
<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Capital</th>
<th>Trades</th>
<th>New Bonds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>Individual Leverage</td>
<td>−0.009**</td>
<td>−0.011***</td>
<td>−6.459***</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.004)</td>
<td>(0.507)</td>
</tr>
<tr>
<td>Aggregate Leverage</td>
<td>0.016***</td>
<td>0.024***</td>
<td>3.234***</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.005)</td>
<td>(0.525)</td>
</tr>
<tr>
<td>Crisis Excluded?</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Intermediary Fixed effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Degree of freedom</td>
<td>5948</td>
<td>5281</td>
<td>5948</td>
</tr>
<tr>
<td>Observations</td>
<td>5,986</td>
<td>5,319</td>
<td>5,986</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.722</td>
<td>0.716</td>
<td>0.692</td>
</tr>
</tbody>
</table>

*p<0.1; **p<0.05; ***p<0.01

Note: This table shows the regression results of intermediary market making activity in the US bond market on individual and aggregate intermediary leverage as in specification (S5). Because aggregate leverage is weighted across all intermediaries, the coefficients on aggregate leverage is similar to the average leverage of the other intermediaries. Controls include intermediary fixed effects. We lag the variables by one day to avoid capturing any potential endogenous effects. The sample period is January 1994 to December 2014. Recessions periods are from the NBER recessions.
Table 9: A Test on Wealth Mechanism with Risk Premia Regressed on Indicators

<table>
<thead>
<tr>
<th></th>
<th>Bond (1)</th>
<th>Equity (2)</th>
<th>Equity (3)</th>
<th>Currency (4)</th>
<th>Currency (5)</th>
<th>Currency (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HighI*HighLvg</td>
<td>88.926</td>
<td>−12.536</td>
<td>71.785</td>
<td>32.965</td>
<td>−34.095</td>
<td>−40.666</td>
</tr>
<tr>
<td></td>
<td>(69.723)</td>
<td>(16.334)</td>
<td>(44.783)</td>
<td>(28.133)</td>
<td>(48.772)</td>
<td>(57.273)</td>
</tr>
<tr>
<td>HighI</td>
<td>67.037***</td>
<td>57.051***</td>
<td>41.699**</td>
<td>36.429***</td>
<td>106.637**</td>
<td>115.782**</td>
</tr>
<tr>
<td></td>
<td>(10.262)</td>
<td>(12.911)</td>
<td>(18.084)</td>
<td>(12.099)</td>
<td>(49.895)</td>
<td>(51.111)</td>
</tr>
<tr>
<td>HighLvg</td>
<td>8.363</td>
<td>27.558***</td>
<td>79.781***</td>
<td>79.142***</td>
<td>19.525</td>
<td>12.442</td>
</tr>
</tbody>
</table>

Crisis excluded | No | Yes | No | Yes | No | Yes
Asset Fixed effects | Yes | Yes | Yes | Yes | Yes | Yes
Year Fixed effects | Yes | Yes | Yes | Yes | Yes | Yes
Clustered errors | Yes | Yes | Yes | Yes | Yes | Yes
Degree of freedom | 132943 | 108523 | 171854 | 147558 | 6153 | 5395
Observations | 133,488 | 109,067 | 172,906 | 148,608 | 6,215 | 5,455
R² | 0.040 | 0.063 | 0.032 | 0.036 | 0.066 | 0.072

Note: This table shows the regression results of short-term risk premia, defined in equation (14) as the reversal return after liquidity shocks, regressed on indicators of asset idiosyncratic risk, aggregate intermediary leverage, and their interaction. HighI is defined as 1 if the asset idiosyncratic risk is higher than 70% quantile of its distribution in the same year, and HighLvg is defined similarly. Controls include asset fixed effects, year fixed effects, risk free rate (10 year US Treasury yield) and the S&P500 return. We lag the variables by one day to avoid capturing any potential endogenous effects. Asset-Year clustered standard errors are shown in the parentheses.
Appendix B. A Model of Wealth Effects

We consider a continuous-time infinite-horizon economy with $t \in [0, \infty)$ and build our model based on He and Krishnamurthy (2012, 2013). As shown in Figure 4, we consider the intermediated investors and intermediaries as endogenous in our model, but a larger fraction of the economy as exogenous. The focus of our model is to study how intermediaries influence the pricing of idiosyncratic risks.

There is a systematic asset with an exogenous price process\(^{13}\) $S_t$, and return

$$\xi_t := \frac{dS_t}{S_t} = \mu_{\xi,t}dt + \sigma_{\xi,t}dZ_t$$

\(^{13}\)All random variables and stochastic processes are defined on a probability space $(\Omega, F, P)$, with a filtration $\{F_t : t \geq 0\}$ that is right continuous.

$Z_t$ is a standard Brownian motion. We can interpret this asset as the S&P500 index. Alternatively, we should think of it as the equivalent asset of a fully diversified portfolio held by the intermediary. Because intermediaries are only a small fraction of all the economy, the systematic asset are not changing the price of this systematic asset much, and thus we model the price as exogenous. This means that the supply is endogenous, i.e. in equilibrium intermediaries will choose how much they would hold this asset, depending on the state of the world.

There is an idiosyncratic asset with exogenous dividend process

$$\frac{dD_t}{D_t} = \mu_D dt + \sigma_D dZ_t + \tilde{\sigma}_D dB_t$$

where $\mu_D > 0$, $\sigma_D > 0$, and $\tilde{\sigma}_D > 0$ are constants, $D_0 > 0$ is given, and $B_t$ is another Brownian motion that is uncorrelated with $Z_t$, i.e. $Cov_t(Z_t, B_t) = 0$. We interpret the randomness $dB_t$ as an idiosyncratic risk, not hedgeable by the hedging asset. The dividend process is exogenous, but the price of the idiosyncratic asset is endogenously determined. Denote the default free borrowing rate as $r_t$ (continuously compounded). Denote the price and total return on the risky asset as $P_t$ and $R_t$, respectively. By definition

$$dR_t = \frac{dP_t + D_t dt}{P_t}$$

Because $Z_t$ and $B_t$ are the only stochastic parameters in the economy, we conjecture an equilibrium form of

$$dR_t = \mu_t dt + \sigma_t dZ_t + \tilde{\sigma}_t dB_t$$
Fig. 4. An Illustration of the Model Setup

**B.1. Investors**

As illustrated in Figure 4, there is a continuum of identical investors\textsuperscript{14} and intermediaries. We model the investors as overlapping generations. We index time as $t, t + \delta, t + 2\delta \cdots$, and consider the continuous time limit when $\delta$ becomes of order $dt$. A unit mass of generation $t$ investors are born with the same wealth $W^h_t$, and live in period $t$ and $t + \delta$, each with the objective

$$\max \rho^h \delta \left( \frac{c^h_t}{1 - \gamma^h} \right)^{1 - \gamma^h} + (1 - \rho^h \delta) E_t \left[ \frac{(W^h_{t+\delta})^{1 - \gamma^h}}{1 - \gamma^h} \right]$$

where $c^h_t$ is the consumption of the investor at period $t$, $W^h_{t+\delta}$ is a bequest for generation $t + \delta$ investors, and $\gamma^h$ is the risk aversion of investors. The parameter $\rho^h$ is an impatience parameter. Higher $\rho^h$ corresponds to greater weight to today’s consumption and less weight on tomorrow’s bequest for the next generation.

Furthermore, we assume that generation $t$ investors receive labor income at date $t$ of $lS_t$, where $l > 0$ is a constant. Thus labor income is proportional to the systematic of the economy. This specification avoids the problem where investors vanish from the economy as in Dumas (1989).

**B.2. Intermediaries**

There is a unit mass of generation $t$ specialists born with the same wealth $W_t$, and they live in period $t$ and $t + \delta$. At every $t$, investors invest in intermediaries that are run by specialists. The intermediation relation is short-term, i.e. only lasts from $t$ to $t + \delta$. We assume that $W_t$, the wealth of the specialist, is all invested in the equity of intermediary. The investor has wealth $W^h_t$, and invests $i_t$ in the intermediary as equity. As in He and Krishnamurthy (2013) an agency friction seperates the balance sheet of investors from that of intermediaries. A “skin

\textsuperscript{14}Additionally, we have noise traders who are represented as exogenous demand shocks
in the game” constraint limits the ability of intermediaries to raise capital:

\[ i_t \leq mW_t \] (22)

The returns are shared proportional to contributed equity capital, i.e. the specialist earns \( W_t/(W_t + i_t) \) and the investor earns \( i_t/(W_t + i_t) \). Both agents may at the endogenously determined risk free rate, \( r_t \). Total investment is the sum of wealth \( W_t + W^h_t \). However, only the intermediary may invest in the risk free asset. Consequently, the leverage of the intermediary is

\[ L_t = \frac{W_t + W^h_t}{W_t + i_t} \] (23)

We model standard CRRA utility for the specialists who operate the intermediary:

\[ \max \rho \delta \left( \frac{c_t}{1 - \gamma} \right) + (1 - \rho \delta) E_t \left[ \left( \frac{W_{t+\delta}}{1 - \gamma} \right) \right] \] (24)

where \( c_t \) is the consumption of the specialist at period \( t \), \( W_{t+\delta} \) is a bequest for generation \( t + \delta \) specialists, and \( \gamma \) is the relative risk aversion of the specialist.

Denote the intermediary’s fraction of investment in the idiosyncratic asset as \( x_t \) and fraction of investment in the hedging asset as \( y_t \). Similarly, these are also the fractions of investment by the specialist. The specialist wealth evolves as follows

\[ \frac{W_{t+\delta} - W_t}{\delta} = \frac{W_t x_t (dR_t - r_t dt)}{\text{return from idiosyncratic asset}} + \frac{W_t y_t (d\xi_t - r_t dt)}{\text{return from hedging asset}} + \frac{W_t r_t dt - c_t dt}{\text{riskfree return, consumption}} \]

with the limit \( \delta \to 0 \) as

\[ dW_t = W_t x_t (dR_t - r_t dt) + W_t y_t (d\xi_t - r_t dt) + W_t r_t dt - c_t dt \] (25)

The investor wealth evolves endogenously according to

\[ \frac{W^h_{t+\delta} - W^h_t}{\delta} = \frac{i_t x_t (dR_t - r_t dt)}{\text{return from idiosyncratic asset}} + \frac{i_t y_t (d\xi_t - r_t dt)}{\text{return from hedging asset}} + \frac{W^h_t r_t dt - c^h_t dt}{\text{riskfree return, consumption}} + \frac{Y_t dt}{\text{labor income}} \]

with the limit \( \delta \to 0 \) as

\[ dW^h_t = i_t x_t (dR_t - r_t dt) + i_t y_t (d\xi_t - r_t dt) + W^h_t r_t dt - c^h_t dt + Y_t dt \] (26)
B.3. Individual Optimization Problems

For above setup, we solve the optimization problems of the investors and specialists. We provide solutions when \( \delta \to 0 \).

First, by Ito’s lemma, the investor’s optimization problem can be rewritten as

\[
\max_{c_t^h, i_t} \rho^h \left( \frac{c_t^h}{1-\gamma^h} \right) dt + (1 - \rho^h dt) E_t \left[ \left( \frac{W_t^h}{1-\gamma^h} \right) - \gamma^h W_t^h \right] dW_t^h - \frac{1}{2} W_t^h \left( \frac{\gamma^h - 1}{\gamma^h} \right) (dW_t^h)^2 \\
\text{s.t.} \quad 0 \leq i_t \leq mW_t
\]

With equation (26), the optimization is

\[
\max_{c_t^h, i_t \leq mW_t} \left\{ \frac{\rho^h}{1-\gamma^h} \left( \frac{c_t^h}{1-\gamma^h} \right) + \left( W_t^h \right)^{\gamma^h+1} \left( i_t x_t (\mu_t - r_t) + i_t y_t (x_t \sigma_t) + W_t^h c_t^h + c_t^h \right) \right\} - \frac{1}{2} \left( \frac{1}{W_t^h} \right)^\gamma h \left( \left( i_t x_t \right)^2 (\sigma_t^2 + \tilde{\sigma}_t^2) + \left( i_t y_t \right)^2 \sigma_t^2 + 2i_t x_t y_t \sigma_t \tilde{\sigma}_t \right) \right\}
\]

(27)

Since the objective function is jointly strictly concave in \( \{c_t, i_t\} \), first order conditions are both necessary and sufficient conditions for optimality. The first order for consumption is

\[
c_t^h = \rho^{1/\gamma^h} W_t^h
\]

(28)

Absent from capital constraint, the first order condition for \( i_t \) is

\[
W_t^h (x_t (\mu_t - r_t) + y_t (x_t \sigma_t - r_t)) - \gamma^h i_t \left( \left( \frac{\sigma_t^2 + \tilde{\sigma}_t^2}{\gamma^h} \right) x_t^2 + \frac{\sigma_t^2 y_t^2}{\gamma^h} + 2x_t y_t \sigma_t \tilde{\sigma}_t \right) = 0
\]
i.e.

\[
i_t = \frac{1}{\gamma^h \left( \frac{\sigma_t^2 + \tilde{\sigma}_t^2}{\gamma^h} \right) x_t^2 + \frac{\sigma_t^2 y_t^2}{\gamma^h} + 2x_t y_t \sigma_t \tilde{\sigma}_t} \frac{W_t^h}{W_t^h}
\]

However, as long as

\[
\frac{1}{\gamma^h \left( \frac{\sigma_t^2 + \tilde{\sigma}_t^2}{\gamma^h} \right) x_t^2 + \frac{\sigma_t^2 y_t^2}{\gamma^h} + 2x_t y_t \sigma_t \tilde{\sigma}_t} W_t^h \geq mW_t
\]

(29)

the capital constraint should be binding. This region is of interest because when constrained, financial intermediary wealth influences risk premiums.

Second, by Ito’s lemma the specialist’s optimization problem can be rewritten as

\[
\max_{c_t^h, x_t, y_t} \rho \left( \frac{c_t^h}{1-\gamma} \right) dt + (1 - \rho dt) E_t \left[ \left( \frac{W_t^h}{1-\gamma} \right) - W_t^{-\gamma} dW_t^h - \frac{1}{2} W_t^{-\gamma-1} (dW_t^h)^2 \right]
\]

(28)
With equation (25), the optimization is

$$
\max_{c_t, x_t, y_t} \left\{ \rho^{\frac{1}{1-\gamma}} + W_t^{-\gamma+1}(x_t(\mu_t - r_t) + y_t(\mu_{\xi,t} - r_t) + r_t - \frac{c_t}{W_t}) - \frac{1}{2}W_t^{-\gamma+1}\gamma((x_t)^2(\sigma_t^2 + \bar{\sigma}_t^2) + (y_t)^2\sigma_{\xi,t}^2 + 2x_t y_t\sigma_{\xi,t}\sigma_t) \right\}
$$

(30)

Since the objective function is jointly strictly concave in \(\{c_t, x_t, y_t\}\), first order conditions are both necessary and sufficient conditions for optimality. The first order condition for consumption \(c_t\) yields

$$
c_t = \rho^{1/\gamma}W_t
$$

(31)

First order conditions for \(x_t\) and \(y_t\) result in a system of linear equations

$$
\begin{align*}
\mu_t - r_t - \gamma(\sigma_t^2 + \bar{\sigma}_t^2)x_t - \gamma y_t\sigma_{\xi,t}\sigma_t &= 0 \\
\mu_{\xi,t} - r_t - \gamma\sigma_{\xi,t}^2 y_t - \gamma\sigma_{\xi,t}\sigma_t x_t &= 0
\end{align*}
$$

(32)

with solution

$$
x_t = \frac{1}{\gamma\bar{\sigma}_t^2}(\mu_t - r_t - \frac{\sigma_t}{\sigma_{\xi,t}}(\mu_{\xi,t} - r_t))
$$

(33)

$$
y_t = \frac{1}{\gamma\sigma_{\xi,t}^2\sigma_t^2}((\mu_{\xi,t} - r_t)(\sigma_t^2 + \bar{\sigma}_t^2) - (\mu_t - r_t)\sigma_t\sigma_{\xi,t})
$$

(34)

In equilibrium, \(x_t\) and \(y_t\) will be determined by market clearing. Idiosyncratic risk is priced if and only if the intermediary sector holds a non-zero position of the idiosyncratic asset, i.e. \(x_t \neq 0\). When \(x_t = 0\), only the systematic component is priced

$$
\mu_t - r_t = \frac{Cov_t(dR_t, d\xi_t)}{Var_t(d\xi_t)}(\mu_{\xi,t} - r_t)
$$

(35)

The proof follows from rearranging equation (33). Although simple, it has profound implications. First, if intermediary the asset holding \(x_t\) fluctuates around 0, we will observe positive and negative idiosyncratic risks. However, on average about zero risk premia. This is part of the reason why previous empirical research finds conflicting evidence on how idiosyncratic risk is priced. Consequently, to measure the pricing of idiosyncratic risk, we should estimate the transient component to returns, e.g. liquidity shocks. This idea is applied in our empirical section when we construct our measure of the idiosyncratic risk premia.

B.4. Market Equilibrium

We define a market equilibrium that allows us to explicitly characterize how intermediary leverage endogenously effects idiosyncratic risks and how these two factors jointly price risk premia. The definition of an equilibrium is as follows.
Definition 1. An equilibrium for the economy is a set of idiosyncratic asset return $R_t$ (together with $\mu_t, \sigma_t, \tilde{\sigma}_t$), price $P_t$, risk free borrowing rate $\{r_t\}$, and decisions $\{i_t, x_t, y_t, c_t, c_t^h\}$ such that

1. Given the price, return, and risk free rate, decisions solve the optimization problems of investors (equation (27)) and specialists (equation (30)).
2. The idiosyncratic asset market clears

$$x_t(W_t + i_t) - Q_tP_t = 0$$

(36)

In equation (36), $-Q_t$ is the exogenous demand process of the idiosyncratic asset from noise traders. $Q_t > 0$ requires intermediaries to be long the asset.
3. The debt market clears

$$\begin{align*}
(W_t^h - i_t) &= (x_t + y_t - 1)(W_t + i_t) \\
\text{lending of investors} & \quad \text{borrowing of intermediaries}
\end{align*}$$

(37)

4. The consumption good market clears,

$$c_t + c_t^h = lS_t + Q_tD_t$$

(38)

For technical reasons, we assume that $Q_t$ should not be too negative so that total consumption $Q_tD_t + lS_t > 0$. Furthermore, $Q_t$ is deterministic\(^{15}\) and differentiable in $t$. Finally, to make the model solvable, we assume that a fraction $\delta \in (0, 1)$ of the total asset is devoted to the trading of idiosyncratic asset when there are nonzero liquidity shocks, which results in the following identity.

$$x_t(W_t + i_t) = \delta(W_t^h + W_t)1_{Q_t \neq 0}$$

(39)

B.5. Constrained and Unconstrained Region

The model solution is separated into two parts: the capital constraint binds or not. We will provide explicit conditions for a binding capital constraint in equilibrium. With equation (32), (29) is equivalent to

$$\frac{\gamma}{\gamma^h} W_t^h \geq mW_t$$

Thus we get the following lemma.

\(^{15}\)An alternative way to model demand process is to use compound Poisson process. This will introduce a jump term in asset prices and add additional complexities, but the intuition remain the same.
**Lemma 1.** In equilibrium, the capital constraint is binding if and only if

\[ \frac{1}{\gamma h} W_t^h \geq \frac{1}{\gamma} m W_t \]  

(40)

Lemma 1 explicitly links the wealth ratio and risk aversion ratio of specialists and investors to determine whether the capital constraint binds. Suppose \( \gamma^h < \gamma \) and a non-binding capital constraint, the total investment from the investors is

\[ i_t = \frac{\gamma}{\gamma^h} W_t^h > W_t^h \]

which means that investors borrow money from intermediaries. This result is quite unrealistic. Thus we restrict ourselves to \( \gamma^h \geq \gamma \), i.e. investors are more risk averse than specialists.

In the binding region, we have

\[ i_t = m W_t \]  

(41)

and

\[ L_t = \frac{W_t + W_t^h}{(m + 1) W_t} \]  

(42)

In the non-binding region, we have

\[ i_t = \frac{\gamma}{\gamma^h} W_t^h \]  

(43)

and

\[ L_t = \frac{W_t + W_t^h}{W_t + \frac{\gamma}{\gamma^h} W_t^h} \]  

(44)

Define the critical value of leverage as

\[ L^* = \frac{m \gamma^h / \gamma + 1}{m + 1} \]

When (40) holds, intermediary leverage \( L_t \in [L^*, \infty) \). When (40) doesn’t hold, intermediary leverage \( L_t \in [1, L^*) \). Thus intermediary leverage can fully characterize whether the capital constraint is binding.

Using the consumption good clearing in (38), we have

\[ \hat{\rho}^{1/\gamma} W_t + (\hat{\rho}^h)^{1/\gamma} W_t^h = l S_t + Q_t D_t \]

To simplify expressions, we denote \( \hat{\rho} = \rho^{1/\gamma} \) and \( \hat{\rho}^h = (\rho^h)^{1/\gamma^h} \). Then we can express \( W_t^h \) as

\[ W_t^h = \frac{l S_t + Q_t D_t}{\hat{\rho}^h} - \frac{\hat{\rho}}{\hat{\rho}^h} W_t \]  

(45)
With (42), (44) and (45), we can express $W_t$ as a function of $L_t$:

$$W_t = \begin{cases} 
\frac{lS_t + Q_tD_t}{(m + 1)(\hat{\rho} L_t - (\hat{\rho} - \hat{\rho})} & \text{if } L_t \in [L^*, \infty) \\
(\gamma^h - \gamma L_t)(lS_t + Q_tD_t) & \text{if } L_t \in [1, L^*) \\
(\gamma^h \hat{\rho} - \gamma \hat{\rho})L_t - \gamma^h(\hat{\rho} - \hat{\rho}) & \text{if } L_t \in [L^*, \infty) 
\end{cases}$$

(46)

Specialist’s wealth, $W_t$, decreases with leverage, $L_t$, when $L_t \geq L^*$. In the other case, when $L_t < L^*$, specialist wealth $W_t$ also decreases with leverage, $L_t$, given more assumptions on risk aversion and impatience parameters. Summarizing these two cases, we find under mild assumptions, $W_t$ strictly decreases in $L_t$. This one-to-one inverse mapping from $L_t$ to $W_t$ allows us to use intermediary leverage rather than wealth as a state variable.

B.6. Equilibrium Risk Premia

From the definition of leverage (23), equilibrium condition (37), and assumption (39), we get

$$x_t = \delta L_t 1_{Q_t \neq 0}, \quad y_t = (1 - \delta 1_{Q_t \neq 0}) L_t$$

(47)

From (33), we get

$$\mu_t - r_t = \frac{\sigma}{\sigma_{\xi_t}} (\mu_{\xi_t} - r_t) + \gamma \delta L_t 1_{Q_t \neq 0} \sigma_t^2$$

(48)

Misleadingly, $\gamma L_t \sigma_t^2 > 0$ for all $Q_t \neq 0$. This seems to indicate that the idiosyncratic risk premia is always positive for nonzero liquidity shocks. However, we need to properly scale by the direction of order imbalance. From (36) and (47), we get

$$\delta 1_{Q_t \neq 0}(W_t + W_t^{}) = P_t Q_t$$

(49)

Thus, we have $P_t > 0$ when $Q_t > 0$, but $P_t < 0$ when $Q_t < 0$. The effective return to the asset is

$$\text{sign}(Q_t) dR_t = \frac{E_t[dP_t] + D_t dt}{|P_t|} = \text{sign}(Q_t) (r_t + \frac{\text{Cov}_t(dR_t, d\xi_t)}{\text{Var}_t(d\xi_t)} (\mu_{\xi_t} - r_t) + \gamma \delta L_t \sigma_t^2)$$

A sufficient conditions could be $\gamma^h \geq \gamma$ and specialist wealth $\hat{\rho}^h > \hat{\rho}$. Then we get $\gamma^h \hat{\rho}^h - \gamma \hat{\rho} > 0$ and

$$\langle \gamma^h \hat{\rho}^h - \gamma \hat{\rho} \rangle L_t - \gamma^h(\hat{\rho}^h - \hat{\rho}) \geq \gamma^h \hat{\rho}^h - \gamma \hat{\rho} - \gamma^h(\hat{\rho}^h - \hat{\rho}) = (\gamma^h - \gamma) \hat{\rho} > 0$$

In the numerator,

$$\gamma^h - \gamma L_t \geq \gamma^h - \gamma L^* = \frac{\gamma^h - \gamma}{m + 1} \geq 0$$

Thus the numerator decreases in $L_t$ while the denominator increases in $L_t$ and both are positive, which means that specialist’s wealth $W_t$ decreases in $L_t$. 

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16 A sufficient conditions could be $\gamma^h \geq \gamma$ and specialist wealth $\hat{\rho}^h > \hat{\rho}$. Then we get $\gamma^h \hat{\rho}^h - \gamma \hat{\rho} > 0$ and
where the function

\[
\text{sign}(Q_t) = \begin{cases} 
1 & \text{if } Q_t > 0 \\
0 & \text{if } Q_t = 0 \\
-1 & \text{if } Q_t < 0 
\end{cases}
\]

When \( Q_t > 0 \), the effective return is positively related to \( \gamma L_t \tilde{\sigma}_t^2 \). To the contrary, when \( Q_t < 0 \), the effective return is negatively related to \( \gamma L_t \tilde{\sigma}_t^2 \). This is intuitive: when \( Q_t > 0 \) (\( Q_t < 0 \)), the intermediary is long (short) the asset and earns a positive (negative) risk premium. Note that \( P_t < 0 \) in the model is merely a modeling convenience, which should be interpreted as low asset price.

With \( \text{Cov}_t(dR_t, d\xi_t) = \sigma_t \sigma_\xi dt \), and \( \text{Var}_t(d\xi_t) = \sigma^2_{\xi,t} dt \), we get the following proposition.

**Proposition 6** (Equilibrium Risk Premium for the Idiosyncratic Asset). The excess return of the idiosyncratic asset has the following form

\[
\frac{\text{sign}(Q_t)(\mu_t - r_t)}{\text{effective excess return}} = \frac{\text{Cov}_t(\text{sign}(Q_t)dR_t, d\xi_t)}{\text{Var}_t(d\xi_t)(\mu_{\xi,t} - r_t)} + \frac{\text{sign}(Q_t)\gamma \delta L_t \tilde{\sigma}_t^2}{\text{idiosyncratic risk premia}}
\]

The premium on investor demand shocks is thus

\[
\Delta \mu_t = \text{sign}(Q_t)\gamma \delta L_t \tilde{\sigma}_t^2
\]

which we define as the “short-term risk premia”.

Note that the interaction of leverage and idiosyncratic risk determines the short-term risk premia in Proposition 6. With respect to the familiar CAPM, the hedging asset represents a liquid, tradeable “market portfolio”. Alternatively, a consumption based asset pricing framework yields\(^{17}\) \( (\mu_t - r_t)dt = \text{Cov}(c_t^{-\gamma} dc_t, dR_t) \). This follows from specialists trading the asset and pricing it with their own pricing kernel. However, this expression does not yield testable implications due to a lack of data on specialist consumption.

Proposition 6 is quite general. It also holds for assets without dividend (e.g. currency). This is because of a reliance on only the two return processes (17) and (20) and the intermediary’s optimization problem. Furthermore, it applies in both the constrained and unconstrained regions. Consequently, idiosyncratic risk is priced in all periods, not uniquely in crises.

Proposition 6 yields testable empirical implications. For negative \((-Q_t < 0)\) demand shocks, the excess return of idiosyncratic assets is positively related to the interaction of leverage and idiosyncratic risks. For positive \((-Q_t > 0)\) demand shocks, the relationship is negative. When there is no demand shock \((-Q_t = 0)\), the risk premium is simply zero.

\(^{17}\)See the Appendix for a rigorous proof.
B.7. Equilibrium Asset Price

To close the model, we need to solve for equilibrium asset price and volatilities. First, we use equation (49) to get

$$P_t = \delta \frac{1}{Q_t} (W_t + W_t^h)$$

(52)

when $Q_t \neq 0$. This is the key equation that we will use to solve equilibrium returns. When $Q_t \neq 0$, the intermediary sector is not devoting capital to the trading of the idiosyncratic asset, and thus price is undetermined in the model. To make following discussions meaningful, without explicit statement, we assume $Q_t \neq 0$.

With (45) and (52), we can solve the equilibrium price of the risky asset as

$$P_t = \frac{\delta}{\hat{\rho}^h} \left( lS_t + \left( \hat{\rho}^h - \hat{\rho} \right) W_t^h \right)$$

(53)

Similar to He and Krishnamurthy (2012), we assume that $\hat{\rho} > \hat{\rho}^h$ and $\gamma^h \geq \gamma$ in all the following discussions. This can be achieved by having investor risk aversion $\gamma^h$ and impatience $\rho^h$ relatively large. Under such conditions, asset prices are positively related to intermediary wealth, and leverage is strictly negatively related to intermediary wealth, as discussed in section B.5.

Equation (53) shows that when $Q_t > 0$, i.e. intermediaries are long the asset, the price of the idiosyncratic asset increases with intermediary capital, but decreases with the size of demand shocks. This is because intermediaries have limited wealth, and they require a lower price to hold more assets.

B.8. Equilibrium Asset Return Volatilities

To solve equilibrium asset volatilities, we use equations (19) and (20) to express

$$dP_t = P_t \mu_t dt + P_t \sigma_t dZ_t + P_t \tilde{\sigma}_t dB_t - D_t dt$$

(54)

With solutions (47) and the assumption $Q_t \neq 0$, the equilibrium wealth differential of the specialist in (25) becomes

$$dW_t = W_t \delta L_t ((\mu_t - r_t) dt + \sigma_t dZ_t + \tilde{\sigma}_t dB_t) + W_t (1 - \delta) L_t ((\mu_{\xi,t} - r_t) dt + \sigma_{\xi,t} dZ_t) + W_t r_t dt - \rho W_t dt$$

$$= W_t ((\delta \mu_t + (1 - \delta) \mu_{\xi,t}) L_t + (1 - L_t) r_t - \rho) dt + W_t L_t (\delta \sigma_t + (1 - \delta) \sigma_{\xi,t}) dZ_t + W_t L_t \delta \tilde{\sigma}_t dB_t$$

Taking differentials on both sides of (53) with Itô’s formula, match the terms on $dB_t$, and express $W_t$ in terms of $L_t$, we get the following proposition.
Proposition 7 (Idiosyncratic Volatility). In equilibrium, the idiosyncratic volatility $\tilde{\sigma}_t$ is proportional to the volatility of dividend growth,

$$\tilde{\sigma}_t = F(L_t) \cdot \tilde{\sigma}_D$$

(55)

where $F(L_t)$ is an increasing function of $L_t$ in the constrained region $L_t \in [L^*, \infty)$. When $\delta$ is sufficiently small, it is also an increasing function of $L_t$ in the unconstrained region $L_t \in [1, L^*)$.

Because typically one asset only takes up a small fraction of intermediaries’ balance sheet, the assumption of $\delta$ being small is reasonable. Proposition 7 has clear testable implications. When intermediaries have high leverage, idiosyncratic volatility increases. In a richer model to describe crisis dynamics, we may add an additional channel where asset price volatility decreases intermediary capital. In such an extension, high intermediary leverage generates asset price volatility, which increases intermediary leverage. Then there will be an endogenous leverage spiral similar to the illiquidity spiral of Brunnermeier and Pedersen (2009). However, this is beyond the scope of the paper. We focus on showing the implications of intermediary asset pricing on idiosyncratic risk premia in all states of the world, not only crises.