Performance Evaluation in the Presence of Latent Factors

(Draft available upon request; comments welcome)

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Motivation

Suppose the data-generating process of mutual fund returns is:

$$r_{it}^e = \alpha_i + \beta_i^T V_t + \gamma_i^T Z_t + \epsilon_{it}, \; i = 1, \ldots, N, \; t = 1, \ldots, T, \quad (1)$$

where

- $\alpha_i$: factor-adjusted expected return ("real" alpha);
- $V_t$ and $Z_t$: two sets of systematic factors;
- $\epsilon_{it}$: idiosyncratic noise satisfies that $E(\epsilon_{it}\epsilon_{js}) = 0$ if $i \neq j$ or $t \neq s$.

Suppose one estimates $\{\alpha_i\}_{i=1}^N$ in (1) with the following model by OLS:

$$r_{it}^e = \tilde{\alpha}_i + \tilde{\beta}_i^T V_t + \eta_{it}. \quad (2)$$

Is this a problem?
Assume $Z_t = \Phi + \Gamma V_t + W_t$, where $W \perp \perp V$ and $E(W) = 0$.

Then,

$$\tilde{\alpha}_i = \alpha_i + \gamma_i^T \Phi$$

and

$$\tilde{\beta}_i^T = \beta_i^T + \gamma_t^T \Gamma.$$

The OLS estimator of $(\tilde{\alpha}_i, \tilde{\beta}_i^T)$ in model (2) is biased for $(\alpha_i, \beta_i^T)$, unless $(1, V)$ and $Z$ are independent. That is, $\Phi = 0$ and $\Gamma = 0$. 
It is not unusual that econometricians estimate the “real” alpha by

\[ r_{it}^e \sim \tilde{\alpha}_i + \tilde{\beta}_i^m \text{MKT}_t + \tilde{\beta}_i^s \text{SMB}_t + \tilde{\beta}_i^h \text{HML}_t + \tilde{\beta}_i^u \text{MOM}_t + \eta_{it}. \]

where

- MKT, SMB, HML, and MOM are the Fama-French-Carhart factors.

Many mutual fund investors even only account for the market factor.

- Berk-van Binsbergen (2016) and Barber-Huang-Odean (2016).

There could be other factors that affect cross-sectional fund returns.

- Indeed, many other factors (beyond the FFC ones) are found to determine cross-sectional stock returns.

One wants to account for those factors to estimate “real” alpha, whether these factors are due to risk or mispricing.

- Pástor-Stambaugh (2002) and Barber-Huang-Odean (2016).
The challenges

▶ However, which factors we should include in the regression model?

1. There are hundreds of factors documented, although the list itself might not be exclusive.
2. These documented factors might be measured with errors.
3. Mutual fund strategies could change over time so that the set of factors that determine mutual fund returns might also change.

▶ We offer an econometric approach that solves these problems.
The confounding-factor approach

We model the returns of fund $i$ by

$$r_{it}^e = \alpha_i + \beta_i^T V_t + \gamma_i^T Z_t + \epsilon_{it}, \ i = 1, \ldots, N, \ t = 1, \ldots, T. \quad (5)$$

Here,

- $V_t = (V_t^1, \ldots, V_t^d)$ contains $d$ specified “robust” factors ($d$ can be 0);
  - For example, we use the FFC four factors.
- $Z_t = (Z_t^1, \ldots, Z_t^w)$ contains $w$ confounding factors;
  - These factors are not specified in advance and are instead estimated from mutual fund returns.
- $\epsilon_{it}$ is heteroskedastic noise satisfying that
  - $\epsilon_{it} \sim N(0, \sigma_i^2)$ and $E(\epsilon_{it}\epsilon_{js}) = 0$ if $i \neq j$ or $t \neq s$. 
The model:

\[ r_{it}^e = \alpha_i + \beta_i^T V_t + \gamma_i^T Z_t + \epsilon_{it}. \]  

(5)

- **The idea:** without forcing any particular ex-ante specification, we can isolate fund returns that are due to exposures to factors beyond the “robust” ones.

- We provide a procedure to estimate model (5) consistently and efficiently.
In preparation, we apply, fund-by-fund, the standard time-series regression of $r_{it}^e$ on $1$ and $V_t$.

Let $\{\hat{\tau}_i\}_{i=1}^N$ and $\{\hat{\zeta}_{it}, i = 1, \ldots, N, t = 1, \ldots, T\}$ denote the intercepts and the residuals, respectively.

Our estimation procedure consists of two steps:

1. Factor analysis on $\{\hat{\zeta}_{it}\}$ to recover the space spanned by $Z_t$.
2. Cross-sectional robust regression to get consistent estimate of $\{\alpha_i\}_{i=1}^N$. 
Step 1: Factor analysis

- Apply the quasi-maximum likelihood method of Bai-Li (2012) to estimate $\gamma \equiv (\gamma_1, \ldots, \gamma_N)^T$ and $\Sigma \equiv \text{diag} \left( \{\sigma_i^2\}_{i=1}^N \right)$:

$$\left(\hat{\gamma}, \hat{\Sigma}\right) = \arg \min_{\tilde{\gamma}, \tilde{\Sigma}} \left\{ -\log |\tilde{\gamma}(\tilde{\gamma})^T + \tilde{\Sigma}| - \text{tr} \left\{ S \left[ \tilde{\gamma}(\tilde{\gamma})^T + \tilde{\Sigma} \right]^{-1} \right\} \right\},$$

- where $S = 1/T \sum_t \left( \hat{\zeta}_t - \bar{\hat{\zeta}}_t \right) \left( \hat{\zeta}_t - \bar{\hat{\zeta}}_t \right)^T$ is the variance-covariance matrix of $\hat{\zeta}_t \equiv \left( \hat{\zeta}_{1t}, \ldots, \hat{\zeta}_{Nt} \right)^T$.

- Under certain technical conditions of Bai-Li (2012), $\hat{\gamma}$ and $\hat{\Sigma}$ are consistent estimates of $\gamma$ and $\Sigma$, respectively.

- This likelihood approach is similar to Principal Components Analysis, but is consistent for a fixed $N$ and more efficient.
Step 2: Cross-sectional robust regression

- Apply the cross-sectional robust regression (RR) to estimate $\alpha_i$ as follows:

$$\hat{\alpha}_i = \hat{\tau}_i - \hat{\lambda} \hat{\gamma}_i,$$

where

$$\hat{\lambda} = \arg \min_{\lambda} \sum_{i=1}^{N} \rho \left( \frac{\hat{\tau}_i - \lambda \hat{\gamma}_i}{\hat{\sigma}_i} \right)$$

and $\rho$ is a robust loss function satisfying Assumption 1 (next page).
Step 2: Cross-sectional robust regression

**Assumption 1:** (i) $\rho(0) = 0$, (ii) $\rho(x)$ is non-increasing when $x \leq 0$ and is non-decreasing when $x > 0$, (iii) the first derivative of $\rho$ exists and is bounded, and (iv) $\rho'' > 0$ in a neighborhood of 0.

▶ **Example:** the Tukey’s bisquare function:

$$
\rho(x) = \begin{cases} 
  x \left(1 - \left(\frac{x}{c}\right)^2\right)^2 & \text{if } |x| \leq c \\
  0 & \text{if } |x| > c
\end{cases}
$$

▶ The quadratic loss function of OLS doesn’t satisfy this assumption as its first derivative is not bounded.
Robust loss function

![Loss function comparison](image)

- Least squares
- Absolute value
- Huber
- Tukey
Intuition of robust regression

- Image some funds (in red) have large “real” alpha (vertical axis).
- OLS (purple line) is less likely to identify those high-alpha funds than RR (blue line), because the quadratic loss function puts too much weight on them.

Figure: Robust regression v.s. OLS
Statistical theory

Theorem 1 (Wang-Zhao-Hastie-Owen): Given Assumption 1 and $\|\alpha\|_1/N \to 0$ as $N \to \infty$, and assume the conditions for accurate factor analysis in Bai and Li (2012). Then $\hat{\alpha}_j \overset{p}{\to} \alpha_j$ for any fixed $j$, if $N, T \to \infty$ and $(\log N)^2/T \to 0$.

Proposition 1: $\hat{\alpha}$ is as efficient as the OLS estimator if $Z$ were observed.
Mutual fund dataset

- CRSP mutual fund dataset from 1985-2015.
- Standard filters to obtain U.S. actively managed equity mutual funds.
- Use more than 5000 distinct funds in total.
- Compare the confounding-factor model (5) with other linear factor models in identifying common-factor-related returns of mutual funds.
Test 1: Analysis of residuals

- If fund exposures to common factors are stripped out, residuals of the returns of two different funds should have zero correlation.

- At the beginning of each year from 1990-2015, calculate return residuals under a given factor model based on monthly returns in the past five years.

  - The other factor models include (i) the FFC four-factor model, (ii) the Fama-French (FF) five-factor model, and (iii) several multi-factor models that augment the FF five-factor model with the betting-against-beta factor (Frazzini-Pedersen (2013)), the liquidity factor (Pástor-Stambaugh (2003)), or the three industry factors (Pástor-Stambaugh (2002)).

- Compute pairwise correlations of residuals under a given factor model.
Test 1: Results

(a) average pairwise correlations

(b) interquarter pairwise correlations
Test 2: Simulation exercise

- At the beginning of each year from 1990-2015, estimate fund loadings on the FFC four factors based on past five-year data.

- Simulate three latent factors that are normally distributed with annualized average returns of 5% and annualized Sharpe ratios of 50%, 40%, and 30%, respectively.

- Simulate the same number of funds as in the data such that
  1. loadings on the FFC factors are drawn from the empirical distributions;
  2. loading on the three latent factors is the “Q” matrix in the QR decomposition of a normally distributed matrix;
  3. 67% funds have zero “real” alpha;
  4. 33% funds have “real” alpha that is normally distributed with mean zero and standard deviation of 5% per year.

- Compare the estimated alpha to the “real” alpha.
## Test 2: Results

<table>
<thead>
<tr>
<th>Year</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>0.2</td>
</tr>
<tr>
<td>1995</td>
<td>0.4</td>
</tr>
<tr>
<td>2000</td>
<td>0.6</td>
</tr>
<tr>
<td>2005</td>
<td>0.8</td>
</tr>
<tr>
<td>2010</td>
<td>1.0</td>
</tr>
<tr>
<td>2015</td>
<td></td>
</tr>
</tbody>
</table>

**Figure:** Cross-sectional correlations between “real” alpha and estimated alpha.
Many mutual fund investors don’t account for common factors (beyond the market factor) when evaluating mutual fund managers.

- Berk-van Binsbergen (2016) and Barber-Huang-Odean (2016).
- This investor behavior leads to a significant mismatch between mutual fund scale and skill (Song (2017)).

We should expect a negative relationship between a fund’s future return performance and its prior factor-related returns, defined as

$$\Delta_{c_i} \equiv \alpha_i - \alpha_{i}^{\text{capm}}.$$  

Here, $\alpha_i$ is in (5) and $\alpha_{i}^{\text{capm}}$ is the CAPM alpha of fund $i$. 
Past factor-related returns and future returns

For each year from 1990-2015, sort all funds into deciles based on $\Delta_{ci}^{cf}$ in (6), as estimated by monthly returns over the past five years.

- Decile 10 consists of those funds with the highest past-five-year $\Delta_{ci}^{cf}$.

Calculate AUM-weighted returns for each decile over the next year.

Compare the time series of AUM-weighted returns over the entire sample period (1990-2015) for all decile portfolios.
Past factor-related returns and future returns

(a) Net future $\alpha^{\text{capm}}$

(b) Net future $\alpha^{\text{ffc}}$
We propose a linear model of mutual fund returns with latent factors.

- Account for the effects of common factors on cross-sectional fund returns, without forcing any ex-ante specification of the factors.

We develop a two-step approach to recover the latent factors and estimate the parameters consistently and efficiently.

- Identify factor-related returns better than many other linear factor models.

Be wise when investing in mutual funds!


