Quality Selection in Two-Sided Markets:
A Constrained Price Discrimination Approach*

PRELIMINARY AND INCOMPLETE

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Abstract

Online platforms collect rich information about participants, and then share this information back with participants to improve market outcomes. In this paper we study the following information disclosure problem of a two-sided market: how much of its available information about sellers’ quality should the platform share with buyers to maximize its revenue?

One key innovation in our analysis is to reduce the study of optimal information disclosure policies to a constrained price discrimination problem. The information shared by the platform induces a “menu” of equilibrium prices and sellers’ expected qualities. Optimization over feasible menus yields a price discrimination problem. The problem is constrained because feasible menus are only those that can arise in the equilibrium of the two sided-market for some information disclosure policy.

We analyze this constrained price discrimination problem, and apply our insights to two distinct two-sided market models: one in which the platform chooses prices and sellers choose quantities (similar to ride-sharing), and one in which sellers

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choose prices (similar to e-commerce). We provide conditions under which a simple information structure of banning a certain portion of sellers from the platform, and not sharing any information about the remaining participating sellers maximizes the platform’s revenue.

1 Introduction

Online platforms have an increasingly rich plethora of information available about market participants; these include rating systems, public and private written feedback, purchase behavior, and many other sources. Using these sources, platforms have become increasingly sophisticated in classifying the quality of the sellers that participate in their platform (for example, see Tadelis (2016) and Garg and Johari (2019)). This information can be used to both increase platform revenue, and to enhance the welfare of the platform’s participants. For example, platforms such as ridesharing and cleaning services platforms remove low quality sellers from their platform. Platforms can also boost the visibility of high quality sellers with certain badges, as is done by online marketplaces such as Amazon Marketplace and eBay. We refer broadly to such market design choices by platforms as quality selection.

In this paper, we study quality selection in two-sided markets. In particular, we investigate the optimal amount of information about the sellers’ quality that a two-sided market platform should share with buyers in order to maximize its own revenue. Our results characterize conditions under which simple information structures, such as just banning a portion of low quality suppliers or giving badges to high quality suppliers, emerge as optimal designs. Methodologically, our main innovation is to reduce the platform’s revenue maximization problem to a constrained, tractable price discrimination problem; this connection is likely of independent interest in the study of two-sided market design.

We consider two-sided market models with heterogeneous buyers and heterogeneous sellers. Sellers are heterogeneous in their quality levels and buyers are heterogeneous in their trade-offs between quality and price. The platform decides on an information structure, that is, how much of the information it has about the sellers’ quality to share with the buyers. The platform’s goal is to choose an information structure that maximizes the platform’s revenue. The information structure can consist of banning a certain portion of the sellers, but also richer structures that share more granular information with buyers about the quality of participating sellers.

As noted above, one of our paper’s key observations is that the platform’s revenue
maximization problem reduces to a constrained price discrimination problem. We show that every information structure induces a certain subset of price-expected quality pairs that we call a menu, from which the buyers can choose. Platforms can use the information they collect about the sellers’ quality to induce a menu in many different ways. For example, giving badges to high quality sellers can influence the prices such sellers charge, the quantities they sell, and their market entry decisions (Hui et al., 2018). Similarly, banning some low quality sellers can also influence the prices, the quantities sold, and the participating sellers’ quality. Importantly, the prices and expected qualities are equilibrium objects. Not only must the buyers’ incentive compatibility and individual rationality constraints be satisfied, as in a standard price discrimination problem (e.g., Mussa and Rosen (1978)), but the total supply must also equal the total demand. This restricts the choice of menus available to the platform; this is the sense in which the price discrimination problem that we study in this paper a constrained price discrimination problem.

Our main structural result shows that finding the optimal menu in the constrained price discrimination problem is equivalent to finding the optimal information structure. This equivalence proves to be beneficial for two reasons. First, characterizing the solution of a constrained price discrimination problem (see Section 3) is generally easier than solving for the optimal information structure in a two-sided market model. Second, the constrained price discrimination problem is general and can capture different market arrangements. Using this equivalence, we provide a broad set of conditions under which a simple information structure in which the platform bans a certain portion of low quality sellers and does not share any information about the participating sellers maximizes the platform’s revenue. This resembles a common practice in ride-sharing and cleaning services platforms (in these cases review scores of participating providers are typically so high that they do not reveal much information (Tadelis, 2016)). We provide a simple example in Section 2 that illustrates the key features of our analysis.

We then apply this equivalence to study two different two-sided market models. In the first model, the platform chooses prices and sellers choose quantities. In the second model, the sellers choose prices, and quantities are determined in equilibrium. The first model is motivated by platforms where the platform chooses the prices and the sellers choose the quantities (e.g., how many hours to work); such a setting is loosely motivated by labor platforms such as ridesharing and cleaning services. The second model is motivated by online marketplaces where the sellers choose the prices, and the quantities are determined in equilibrium (e.g., such as Amazon Marketplace).
In both models, the platform’s decisions (the platform decides on an information structure and prices in the first model, and on an information structure in the second model) generate a game between buyers and sellers. Given the platform’s decisions there are four equilibrium requirements. First, the sellers choose their actions (prices or quantities) to maximize their profits. Second, the buyers choose whether to buy the product and if so, from which (expected) quality to buy it to maximize their utility. Third, given the information structure that the platform chooses, the buyers form beliefs about the sellers’ quality that are consistent with Bayesian updating and with the sellers’ actions. Fourth, we require market clearing: the total supply equals the total demand.

We show that each equilibrium of the game induces a certain subset of price-quality pairs; each pair consists of a price, and the expected quality of sellers offering at that price. The platform’s goal is to choose a menu that maximizes the platform’s revenue. The set of possible menus that the platform can choose from depends on the equilibrium outcomes of the game. Hence, characterizing this set can be challenging. For our first model, we show that for every information structure there exists a strictly convex optimization problem whose unique solution yields the unique menu of induced price-quality pairs. For the second model, we explicitly provide the menu that each information structure induces. In each setting, we then leverage the analysis of the constrained price discrimination problem to solve for the platform’s optimal information disclosure, providing sharp characterizations of the optimal solution.

The rest of the paper is organized as follows. Section 1.1 discusses related literature. In Section 2 we describe a simple example that captures the main features of our analysis. In Section 3 we study the general constrained price discrimination problem. In Section 4 we present the two-sided market models and provide our main results. In Section 5 we provide a summary, followed by an Appendix containing proofs.

1.1 Related Literature

Our paper is related to several strands of literature. We discuss each of them separately below.

Nonlinear pricing. Nonlinear pricing schemes are widely studied in the economics and management science literature (see Wilson (1993) for a textbook treatment). The price discrimination problem that we consider in this paper is closest to the classical second-degree price discrimination problem (Mussa and Rosen, 1978) and (Maskin and Riley, 1984).
The problem that the platform solves in our setting differs from the previous literature on price discrimination in two major aspects. First, the costs for the platform are zero. This is because in the two-sided market models that we study, the costs of producing a higher quality product are incurred by the sellers and not by the platform. Hence, the platform’s revenue maximization problem reduces to a constrained price discrimination problem with no costs. Second, the platform cannot simply choose any subset of price-quality pairs (menus) that satisfies the incentive compatibility and individual rationality constraints. The set of menus from which the platform can choose is determined by the additional equilibrium requirements described in the introduction.

These differences significantly change the analysis and the platform’s optimal menu. First, a key part of our analysis is to incorporate equilibrium constraints into the price discrimination problem, introducing significant additional complexity. In addition, under the regularity assumption that the virtual valuation function is increasing, Mussa and Rosen (1978) show that the optimal menu assigns different qualities of the product to different types. In contrast, the results in our paper are drastically different: under certain regularity assumptions, the optimal menu assigns the same quality of the product to different types.\(^1\)

**Information design.** There is a vast recent literature on how different information disclosure policies influence the decisions of strategic agents and equilibrium outcomes in different settings. Applications include Bayesian persuasion (Aumann and Maschler (1966) and Kamenica and Gentzkow (2011)), dynamic contests (Bimpikis et al., 2019a), matching markets (Ostrovsky and Schwarz, 2010), queuing theory (Lingenbrink and Iyer, 2019), games with common interests (Lehrer et al., 2010), exploration in recommendation systems (Papanastasiou et al. (2017) and Immorlica et al. (2019)), social networks (Candogan, 2019), the retail industry (Lingenbrink and Iyer (2018) and Drakopoulos et al. (2019)), and many more.

In this paper we focus on the amount of information about the sellers’ quality that a two-sided market platform should share with buyers. Our information disclosure policy problem is different from the previous literature because the platform faces equilibrium constraints when informing buyers about the sellers’ quality; these constraints emerge

\(^1\)Another difference from most of the previous literature is that in our model each menu is finite (i.e., there is a finite number of price-quality pairs), and thus the standard techniques used to analyze the price discrimination problems in the previous literature cannot be used. Bergemann et al. (2011) study a price discrimination problem with a finite menu in order to study a setting with limited information. However, because the platform’s costs are 0 in our setting, we cannot use the Lloyd-Max optimality condition that Bergemann et al. (2011) employ.
because actual two-sided market outcomes are determined endogenously by buyers’ and sellers’ behavior, subsequent to the information disclosure choices of the platform. There are at least three salient characteristics of our setting. First, the platform does not have full information about the sellers’ quality. Second, buyers’ beliefs about the sellers’ quality can depend on the sellers’ actions (in addition to the standard dependence of the buyers’ beliefs on the platform’s information disclosure policy). For example, if the sellers choose quantities (e.g., how many hours to work) these quantities influence the expected qualities.\(^2\) Third, the prices and the sellers’ expected qualities must form an equilibrium in the two-sided market (i.e., the total supply equals the total demand). Overall, these constraints significantly limit the platform’s feasible information structures, and therefore, the revelation principle cannot be applied as is typically done in the Bayesian persuasion literature.

**Two-sided market platforms.** Recent papers study how platforms can use information and other related market design levers to improve market outcomes. In the context of matching markets, Arnosti et al. (2018) and Kanoria and Saban (2019) suggest different restrictions on the agents’ actions in order to mitigate inefficiencies that arise in those markets. Vellodi (2018) studies the role of design of rating systems in shaping industry dynamics. In Romanyuk and Smolin (2019) the platform designs what buyer information the sellers should observe before the platform decides to form a match. The paper most closely related to ours is the contemporaneous work by Bimpikis et al. (2019b) that studies the interaction between information disclosure and the quantity and quality of the sellers participating in the platform. As in our paper, in the papers noted above the full disclosure policy is not necessarily optimal, and hiding information can increase the social welfare and/or the platform’s revenue.

### 2 A Simple Motivating Model

In this section we provide a simple model that illustrates many important features of our paper. While this model ignores important features of our more general model, it will be helpful to highlight important aspects of our analysis and main results.

Consider a platform where heterogeneous sellers and heterogeneous buyers interact. There are two types of sellers: high quality sellers \(q_H\) and low quality sellers \(q_L\) with \(q_H > q_L\). The platform knows the sellers’ quality and considers two options. Option \(B\)

\(^2\)Because the buyers’ beliefs are consistent with the sellers’ actions, our model also relates to the adverse selection literature (see Akerlof (1970)).
is to ban the low quality sellers and keep only the high quality sellers on the platform. Option $K$ is to keep both low quality and high quality sellers on the platform and share the information about the sellers’ quality with the buyers.

The total supply of products by sellers whose quality level is $i = H, L$ is given by the function $S_i(p_i^j)$. When the platform chooses option $j = B, K$, $p_i^j$ is the price of the product sold by sellers whose quality level is $i = H, L$. We assume that the total supply is increasing in the price. The total supply can also depend on the mass of sellers whose quality level is $i = H, L$ and on the sellers’ costs. In our two-sided market models the supply function will be micro-founded, but we abstract away from these details for now.

On their part, buyers are heterogeneous in how much they value quality relative to price. A buyer with type $m$ that decides to purchase from a seller whose quality level is $i = H, L$ has a utility $mq_i - p_i^j$. We normalize the utility associated to not buying to zero. The distribution of the buyers’ types is described by a probability distribution function $F$. We assume that $F$ admits a density function $f$. The buyers choose to buy or not to buy the product from sellers whose quality level is $i = H, L$ in order maximize their own utility. The buyers’ decisions generate demand for quality $i = H, L$ sellers $D^K_i(p^K_L, p^K_H)$ when the platform chooses option $K$, and demand for quality $H$ sellers $D^K_H(p^K_H)$ when the platform chooses the option $B$.

The platform’s goal is to choose an option that maximizes the total transaction value given that prices form an equilibrium, in the sense that supply equals demand. Note that if the platform charges commissions from each side of the market, maximizing the total transaction value is equivalent to maximizing the platform’s revenue. For this reason, we will refer to the platform’s objective as revenue or total transaction value interchangeably. If the platform chooses option $B$, then the total transaction value is $p^K_B D^K_B(p^K_H)$ and the equilibrium requirement is $S^K_H(p^K_H) = D^K_B(p^K_H)$. If the platform chooses option $K$, then the total transaction value is

$$p^K_H D^K_H(p^K_L, p^K_H) + p^K_L D^K_L(p^K_L, p^K_H)$$

and the equilibrium requirements are

$$S^K_H(p^K_H) = D^K_H(p^K_L, p^K_H)$$

and

$$S^K_L(p^K_L) = D^K_L(p^K_L, p^K_H).$$

(1)

Assume that the prices that satisfy the equilibrium requirements are unique. That is, $(p^K_L, p^K_H)$ are the unique prices that solve the equations in (1) and $p^K_H$ is the unique price
that solves $D_H^B(p_H^B) = S_H(p_H^B)$. In this case, the platform’s revenue maximization problem reduces to a constrained price discrimination problem. Choosing option $B$ is equivalent to showing the buyers the price-quality pair $(q_H, p_H^B)$, while choosing option $K$ is equivalent to showing the buyers the price-quality pairs $(q_H, p_H^K)$ and $(q_L, p_L^K)$. Hence, each option is equivalent to a subset of price-quality pairs that we call a menu and the platform’s goal is to choose the menu with the higher revenue. In other words, the platform’s revenue maximization problem is to choose a subset of price-quality pairs $C \in \mathcal{C}$ to maximize the total transaction value

$$\sum_{(p_i, q_i) \in C} p_i D_i(C)$$

where $D_i(C)$ is the mass of buyers that choose the price-quality pair $(p_i, q_i)$ under the menu $C$ and $\mathcal{C}$ is the set of possible menus.

In our simple model, $\mathcal{C}$ contains only two menus. In Section 3 when we introduce our general model we study a price discrimination problem with a rich set of possible menus $\mathcal{C}$ defined by a general constraint set. Furthermore, in the model we consider in this section, the sellers’ qualities are fixed and the prices are constrained by the equilibrium requirements. In the general two-sided market models (see Sections 4.2 and 4.3), the expected qualities are also determined in equilibrium.

While the price discrimination problem in this example is simple, we later show that we can solve a general constrained price discrimination problem with similar arguments (see Section 3). We analyze the price discrimination problem in two stages. In the first stage, we compare the revenue from option $K$ (showing the price-quality pairs $(q_H, p_H^K)$ and $(q_L, p_L^K)$) to the revenue from the infeasible option $I$: showing the price-quality pair $(q_H, p_H^K)$. Option $I$ is infeasible because while the pair $(q_H, p_H^K)$ and $(q_L, p_L^K)$ clears the market, only showing $(q_H, p_H^K)$ will generally not do so.

Note that the equilibrium requirements imply that the price of the product sold by high quality sellers is higher than the price of the product sold by low quality sellers, i.e., $p_H^K > p_L^K$. If the platform were to choose option $I$ then fewer buyers would participate in the platform compared to option $K$, but the participating buyers would pay the higher price $p_H^K$. Option $I$ would be better than option $K$ if and only if the revenue gains from the participating buyers that pay a higher price when choosing $I$ instead of $K$ outweigh the revenue losses from the mass of buyers that do not participate in the platform when choosing $I$ instead of $K$. This depends on the *elasticity of the density function* $\partial \ln f(m) / \partial \ln m$. Intuitively, when the density function’s elasticity is not too “high” the mass of buyers that the platform loses is not too “high”. We show in Theorem 1 a general
version of the following: when the density function’s elasticity is bounded below by −2, option I yields more revenue than option K (see a detailed analysis of the elasticity condition in Section 3).

In the second stage of the analysis, we compare the revenue from option B to the revenue from (infeasible) option I. The equilibrium requirements imply that \( p_B^H \geq p_K^H \). To see this, note that \( D_B^H(p_K^H) \geq D_K^H(p_L^K, p_H^K) = S_H(p_K^H) \), i.e., the demand for high quality sellers in option B is greater than the demand for high quality sellers in option K when the price is \( p_H^K \). This follows because for some buyers, buying from the high quality sellers yields a positive utility that is smaller than the utility from buying from the low quality sellers. Hence, in option B, these buyers buy from the high quality sellers, while in option K they buy from the low quality sellers. Thus, the demand for high quality sellers under the price \( p_K^H \) exceeds the supply. Because the supply is increasing and the demand is decreasing in the price, we must have \( p_B^H \geq p_K^H \).

If the platform shows the buyers the menu \( (q_H, p) \) only the buyers whose valuations satisfy \( mq_H - p \geq 0 \) buy the product from the high quality sellers. Thus, \( pD_B^H(p) = p(1 - F(p/q_H)) \). When the density function’s elasticity is bounded below by −2, the function \( F(m)m \) is convex (see Section 3), and hence, the revenue function \( R_H(p) := p(1 - F(p/q_H)) \) is concave in the price \( p \). Thus, as shown in Figure 1 below, option B yields more revenue than option I if the equilibrium price \( p_B^H \) is lower than the unconstrained price that maximizes the platform’s revenue \( p_M^H \) ignoring equilibrium conditions, that is:

\[
p_M^H = \arg\max_{p \geq 0} p \left( 1 - F\left( \frac{p}{q_H} \right) \right).
\]

![Figure 1: The platform’s revenue as a function of the price.](image)

\(^3\)Note that if the supply is perfectly elastic (for example, in the Bertrand competition that we study in Section 4.3.2 the supply is perfectly elastic), then we have \( p_B^H = p_K^H \). This follows because higher demand does not increase the price when the supply is perfectly elastic. In this case, the second stage of the analysis is not necessary.
Intuitively, the equilibrium price $p_{BH}^H$ is lower than the price that maximizes the platform’s revenue $p_{MH}^H$ if the total supply of high quality sellers is large enough. In particular, if the total supply of high quality sellers exceeds the total demand under the price $p_{MH}^H$, then the equilibrium price $p_{BH}^H$ must be lower than $p_{MH}^H$ to achieve market clearing. To see this more clearly, consider $F$ to be a uniform distribution on $[0,1]$ and the high quality sellers’ supply function to be given by $S_H(p) = p\phi_H/c_H$, where $c_H > 0$ is the cost of producing a unit of the product and $\phi_H > 0$ is the mass of high quality sellers. Note that the equilibrium price $p_{BH}^H$ is the unique price that solves the equation $p\phi_H/c_H = 1 - p/q_H$. Hence, $p_{BH}^H = q_H c_H / (\phi_H q_H + c_H)$. A straightforward calculation shows that $p_{MH}^H = q_H/2$. Thus, $p_{MH}^H \geq p_{BH}^H$ if and only if $\phi_H q_H \geq c_H$. So, if the sellers’ production cost $c_H$ is low or the mass of high quality sellers is high enough, then the total supply of high quality sellers is large enough and $p_{MH}^H \geq p_{BH}^H$.

In our simple model, the condition $p_{MH}^H \geq p_{BH}^H$ can be replaced by the weaker condition $R_H(p_{BH}^H) \geq R_H(p_{MH}^H)$. We use a similar version of this weaker condition to prove that a menu that consists of only one price-quality pair is optimal in the general constrained price-discrimination problem (see Definition 1 in Section 3). In the general two-sided market models that we study in Section 4, the qualities are also determined in equilibrium and the set of possible menus that the platform can choose from can be very large. Extending the weaker condition from the price discrimination problem to this general two-sided market setting would be impractical because it would require checking a version of the weaker condition for every menu. Nevertheless, in Theorem 2 we show that it is enough to check the condition that the total supply of sellers exceeds the total demand under the price that maximizes the platform’s revenue for only one specific set of sellers. Then, we can use a similar argument to the arguments in the second stage of the analysis above to show that a menu that consists of only one price-quality pair is optimal.

We conclude that when the elasticity of the density function is not too low, and the supply of high quality sellers is large enough, then option $B$ yields more revenue than option $K$. That is, banning low quality sellers and keeping only the high quality sellers yields more revenue than keeping both low quality and high quality sellers on the platform. In the next sections we study this result in the context of more general two-sided market models and information structures.

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Generally, the influence of the sellers’ quality $q_H$ on the condition $p_{MH}^H \geq p_{BH}^H$ is more complicated. An increase in the sellers’ quality increases both the equilibrium price and the price that maximizes the platform’s revenue.
3 A Constrained Price Discrimination Problem

In the simple model of the previous section, we observed that the platform’s problem of choosing how much information to share with the buyers about the sellers’ quality reduces to a price discrimination problem with constraints on the menu that can be chosen by the platform. In this section, we study a general constrained price discrimination problem; the simple model in the previous section is a special case. In the price discrimination problem we consider, a platform can charge different prices for different qualities of a product. The platform chooses a subset of price-quality pairs, i.e., a menu, from a feasible space of possible menus (referred to as the constraint set). The constraint set restricts the possible choices of menus available to the platform.

In the two-sided market models that we study in Section 4, the constraint set is determined by the endogenously-determined equilibrium in these markets: i.e., the price-quality pairs in the menu must form an equilibrium, in the sense that the prices and qualities agree with the buyers’ and sellers’ optimal actions, and supply equals demand. In this section, we consider a general constraint set. The platform’s problem is to choose a subset of price-quality pairs that belongs to the constraint set in order to maximize the total transaction value, while knowing only the distribution of valuations of possible buyers. As previewed in the simple model of the previous section, in Section 4 we will show that the information disclosure problem of the platform in our two more general two-sided market models reduces to the general constrained price discrimination problem.

3.1 Preliminaries

In this subsection we collect together basic concepts needed for our subsequent development.

**Menus.** A menu $C$ is a finite set of price-quality pairs.

**Constraint set.** We denote by $C$ the set of all possible menus from which the platform can choose. $C$ is called a constraint set.

**Buyers.** We assume a continuum of buyers. Given a menu, the buyers choose whether to buy a unit of the product and if so, at which price-quality pair to buy it. Each buyer has a type that determines how much they value quality relative to price. The utility of a type $m$ buyer over price-quality combinations is $mq - p$. The type distribution is given by a continuous cumulative distribution function $F$. We assume that $F$ is supported on an interval $[a, b] \subseteq \mathbb{R}_+ := [0, \infty)$. 

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Platform optimization problem and optimal menus. Given the constraint set $\mathcal{C}$, the platform chooses a menu $C = \{(p_1, q_1), \ldots, (p_k, q_k)\} \in \mathcal{C}$ to maximize the total transaction value, subject to the standard incentive compatibility and individual rationality constraints.

In other words, the platform chooses a menu $C \in \mathcal{C}$ to maximize:

$$\pi(C) := \sum_{(p, q) \in C} p D_i(C),$$

where $D_i(C)$ is the total mass of buyers that choose the price-quality pair $(p_i, q_i)$ when the platform chooses the menu $C \in \mathcal{C}$. That is,\(^5\)

$$D_i(C) := \int_a^b 1\{m:mq_i−p_i≥0\}(m)1\{m:mmq_i−p_i=\max_{(p, q) \in C} m q_i−p_i\}F(dm),$$

where $1_A$ is the indicator function of the set $A$. A menu $C' \in \mathcal{C}$ is called optimal if it maximizes the total transaction value, i.e., $C' = \arg\max_{C \in \mathcal{C}} \pi(C)$.

Price-maximal menus. Let $\mathcal{C}_p = \{C \in \mathcal{C} : D_i(C) > 0 \text{ for all } (p, q) \in C\}$ be the set that contains all the menus $C$ such that the mass of buyers that choose the price-quality pair $(p_i, q_i)$ is positive for every $(p_i, q_i) \in C$. A menu $C \in \mathcal{C}_p$ is called price-maximal if for all $(p, q) \in [0, \infty) \times [0, \infty)$ such that $C \cup \{p, q\} \in \mathcal{C}_p$, we have $p \leq p'$ for some $(p', q') \in C$. Intuitively, a menu $C$ is price-maximal if it is not feasible to add a price-quality pair to $C$ with positive demand and a higher price than all the other prices in the menu $C$.

The next Lemma shows that the optimal menu (if it exists) is price maximal. The proof of this Lemma follows from Step 4 in the proof of Theorem 1. The constraint set $\mathcal{C}$ that we study in the context of our two-sided market models will typically admit price-maximal menus.

**Lemma 1** Let $C \in \mathcal{C}$ be a menu. If $C$ is not price-maximal then $C$ is not optimal.

$k$-separating menus. A menu $C = \{(p_1, q_1), \ldots, (p_k, q_k)\} \in \mathcal{C}$ is said to be $k$-separating for a positive integer $k$ if $C$ contains exactly $k$ different price-quality pairs. That is, a $k$-separating menu $C$ satisfies $|C| = k$ where $|C|$ is the number of price-quality pairs on the menu $C$. We let $C_k \subseteq \mathcal{C}$ be the set of all $k$-separating menus. For the rest

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\(^5\)If there is a subset of price-quality pairs $C'$ such that for some type $m$ buyer we have $mq_i−p_i ≥ 0$ and $mq_i−p_i = \max_{(p, q) \in C'} m q_i−p_i$ for all $(p_i, q_i) \in C'$ then we assume that the buyer chooses the price-quality pair with the highest index, i.e., $\max_{i \in C'} i$. This assumption does not change our analysis because $F$ does not have atoms.
of the section, we assume without loss of generality that \( p_1 \leq p_2 \leq \ldots \leq p_k \) for every \( k \)-separating menu \( C = \{(p_1, q_1), \ldots, (p_k, q_k)\} \).

### 3.2 Optimality of 1-Separating Menus

The main result of this section (Theorem 1) shows that under certain conditions, a 1-separating menu is optimal. Translating to the two-sided market model, this means that the platform potentially bans a portion of the sellers and provides no further quality information to buyers about the remaining sellers.

Our theorem shows that this result obtains under two key conditions on the model, each of which is related to conditions discussed in Section 2. The first is convexity of \( F(m)m \). The second involves ensuring that the platform has a sufficiently “rich” set of 1-separating menus available. We now discuss each condition in turn.

**Convexity of \( F(m)m \).** First, we require \( F(m)m \) to be convex in \( m \). If we suppose that \( F \) has a strictly positive and continuously differentiable density \( f \), then an elementary calculation shows that \( F(m)m \) is convex if and only if:

\[
\frac{\partial f(m)}{\partial m} \frac{m}{f(m)} = \frac{f'(m)m}{f(m)} \geq -2.
\]

In other words, the *elasticity* of the density function must be bounded below by \(-2\) (as noted in Section 2). As noted in our discussion there, this condition essentially ensures that when the platform chooses the optimal 1-separating menu, the revenue losses from buyers who choose not participate in the platform are outweighed by the revenue gains from sales made under the 1-separating menu.

A number of distributions satisfy this condition, e.g., power law distributions \( F(m) = d + cm^k \) for some constants \( k > 0, c, d \); beta distributions \( f(m) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} m^{\alpha-1} (1 - m)^{\beta-1} \) with \( \beta \leq 1 \), where \( \Gamma \) is the gamma function); and Pareto distributions \( F(m) = 1 - \left(\frac{c}{m}\right)^{\alpha} \) on \([c, \infty)\), where \( c \geq 1 \) is a constant). It is also worth noting that the condition that \( F(m)m \) is convex is distinct from monotonicity of the so-called *virtual value function* \( r(m) := m - (1 - F(m))/f(m) \), a condition that plays a key role in the price discrimination literature.\(^\text{6}\)

To see the dependence on the density function’s elasticity, consider a simple price

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\(^\text{6}\)See Mussa and Rosen (1978) and Maskin and Riley (1984), and more generally the mechanism design literature (e.g., Myerson (1981)), for use of the monotonicity of the virtual valuation function. Convexity of \( F(m)m \) can be shown to be equivalent to monotonicity of the product of the virtual valuation with the density, \( r(m)f(m) \).
discrimination setting inspired by the example of Section 2. In particular, suppose that the platform has only two price-quality pairs available: \((p_L, q_L) = (1, 1.5)\) and \((p_H, q_H) = (2, 4)\), and the platform can either choose the 1-separating menu \(\{(p_H, q_H)\}\) consisting of high quality only, or the full (2-separating) menu \(\{(p_L, q_L), (p_H, q_H)\}\) consisting of both qualities. In Figure 2 we demonstrate the consequences of different elasticities of \(f\). In the figures in the left column, the platform chooses the full menu, the black color represents the buyers that choose not to participate in the platform, the green color represents the buyers that choose \(L\), and the red color represents the buyers that choose \(H\). In the figures in the right column, the platform chooses the 1-separating high quality menu, the black color represents the buyers that choose to not participate in the platform, and the orange color represents the buyers that choose to buy the product.

The 1-separating high quality menu yields more revenue than the full menu if and only if the area between the points \(B\) and \(C\) times \(p_H\) is greater than or equal to the area between the points \(A\) and \(C\) times \(p_L\), that is, the revenue losses from losing the participation in the platform of buyers whose valuations are between 1.5 and 2 are smaller than the revenue gains from charging the participating buyers whose valuations are between 2 and 2.5 the higher price. Intuitively, when the elasticity is lower, this difference is higher. In other words, when the elasticity is lower, the full menu is more attractive because the platform loses too much revenue when choosing the 1-separating high quality menu instead.

1-richness. Second, we require a condition that ensures that the 1-separating menus in the constraint set are sufficiently “rich”. This condition, which we call 1-richness, implies that for every menu \(C = \{(p_1, q_1), \ldots, (p_k, q_k)\}\) that is price-maximal and hence might be optimal (see Lemma 1), we can find a 1-separating menu that yields a higher transaction value than the 1-separating menu \(\{(p_k, q_k)\}\).

We have the following definition.

**Definition 1** We say that a constraint set \(C\) is 1-rich if for every price-maximal menu,\(^7\) \(C = \{(p_1, q_1), \ldots, (p_k, q_k)\}\) there exists a 1-separating menu \(C' \in C\) such that \(\pi(\{p_k, q_k\}) \leq \pi(C')\).

We now provide two examples of constraint sets that are 1-rich.

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\(^7\)Recall that we assume without loss of generality that \(p_1 \leq p_2 \leq \ldots \leq p_k\) for every menu \(C = \{(p_1, q_1), \ldots, (p_k, q_k)\}\).
Example 1  

(i) In this example, the platform can choose any subset of price-quality pairs from a pre-fixed set of price-quality pairs. Suppose that there is a given set $\mathcal{P}$ of $R$ price-quality pairs, $\mathcal{P} = \{(p_1, q_1), \ldots, (p_R, q_R)\}$. Then the constraint set is $\mathcal{C}_\mathcal{P} = 2^\mathcal{P}$ where $2^\mathcal{X}$ is the set of all subsets of a set $\mathcal{X}$.

(ii) In this example, the platform can choose any finite string $(p_1, q_1, \ldots, p_k, q_k)$ in $\mathbb{R}^k$ for $k \leq N$ where $N \geq 1$, $p_i \in [0, \overline{p}]$ and $q_i \in [0, \overline{q}]$ for all $1 \leq i \leq k$. That is, the constraint set is given by

$$\mathcal{C}_N = \{ C : C \text{ is a } k\text{-separating menu for } k \leq N \text{ such that } (p, q) \in [0, \overline{p}] \times [0, \overline{q}] \text{ for all } (p, q) \in C \}. $$

Both sets of menus introduced in Example 1 are 1-rich because for every menu $C = \{(p_1, q_1), \ldots, (p_k, q_k)\} \in \mathcal{C}$, the 1-separating menu $C' = \{p_k, q_k\}$ belongs to $\mathcal{C}$ and satisfies $\pi(\{p_k, q_k\}) = \pi(C')$. We note that the constraint set in Example 1 part (ii) is standard and was previously considered in the price discrimination literature (see for example Bergemann et al. (2011)).

The 1-richness condition is an important contribution of our work. Although it is a fairly simple condition to state, it proves to be a valuable “interface” to study two-sided markets. In particular, our subsequent analysis in Section 4 establishes interpretable

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The 1-richness condition is an important contribution of our work. Although it is a fairly simple condition to state, it proves to be a valuable “interface” to study two-sided markets. In particular, our subsequent analysis in Section 4 establishes interpretable
conditions on features of the market under which the constraint set is 1-rich. For example, in the two-sided market model in Section 4.3, the constraint set that the platform faces is the same as the constraint set in Example 1 part (i). We also note that although the constraint sets in Example 1 are 1-rich independently of the distribution function $F$, in general the 1-rich property may depend on the shape of $F$. For example, this is the case for the two-sided market model studied in Section 4.2. As a special case of this market model, in the simple example of Section 2, the condition that $p_M^H \geq p_B^H$ can be shown to imply 1-richness; this is a condition that depends on the distribution function $F$.

**Main result.** We can now state our main result using the previous two conditions. The following theorem that states our constrained price discrimination problem admits an optimal solution that is 1-separating. All the proofs in the paper are deferred to the Appendix.

**Theorem 1** Suppose that $F(m)$ is a convex function on $[a, b]$ and that $C$ is 1-rich. Assume that the set of all 1-separating menus $C_1 \in C$ is a compact subset of $\mathbb{R}^2$. Then there is an optimal 1-separating menu.

We note that Theorem 1 also holds in the case that the support of $F$ is unbounded. In particular, Theorem 1 holds when the support of $F$ is given by $[0, \infty)$.

We conclude with two additional results that expand on the main theorem above. First, the following corollary shows that for some menus $C \in C$, it is enough to show that the function $F(m)$ is convex on a subset of $[a, b]$ in order to prove that there exists a 1-separating menu that yields more revenue than the menu $C$. Thus, the menu that maximizes the total transaction value can still be 1-separating for a distribution function that is convex on a subset of the distribution’s support. For a $k$-separating menu $C = \{(p_1, q_1), \ldots, (p_k, q_k)\} \in C_p$, let $m_i(C) = (p_i - p_{i-1}) / (q_i - q_{i-1})$ for $i = 1, \ldots, k$ where $p_0 = q_0 = 0$. Corollary 1 follows immediately from the proof of Theorem 1.

**Corollary 1** Let $C = \{(p_1, q_1), \ldots, (p_k, q_k)\} \in C_p$ be a $k$-separating menu where $p_i < p_j$ if $i < j$. Suppose that $F(m)$ is convex on $[m_1(C), m_k(C)]$ and that $C$ is 1-rich. Then there exists a 1-separating menu $C^*$ that yields more revenue than $C$, i.e., $\pi(C) \leq \pi(C^*)$.

In addition, if $F(m)$ is convex on $[m_1(C), m_k(C)]$ for every price-maximal menu and $C_1$ is compact, then there is a 1-separating menu that maximizes the total transaction value.

---

8Note that $C \in C_p$ implies $m_i(C) < m_j(C)$ for $i < j$ and that $[m_1(C), m_k(C)] \subseteq [a, b]$ (see the proof of Theorem 1).
We can also show that when the function $F(m)m$ is not convex, we can find a 1-rich constraint set $C$ such that a 1-separating menu does not maximize the total transaction value.

**Proposition 1** Suppose that $F(m)m$ is not convex on $(a,b)$. Then there exists a constraint set $C$ that is 1-rich and a menu $C \in C$ that maximizes the total transaction value and yields strictly more revenue than any 1-separating menu in $C$.

The preceding two results naturally suggest it would be interesting to explore whether positive results can be proven when $F(m)m$ is convex on a subset of its support, and concave on the rest of the support. We conclude by briefly suggesting one conjecture in this direction; proving this result is ongoing work. We conjecture that when the distribution function is convex-concave, the optimal menu pools the low-type buyers (in the region where $F(m)m$ is convex) and separates the high-type buyers (in the region where $F(m)m$ is concave).

## 4 Two-Sided Market Models

In this section we consider two-sided market models with heterogeneous buyers and heterogeneous sellers, in which a platform has partial information about the sellers’ quality. The platform’s information is summarized by a finite partition of the set of possible sellers’ quality levels. The platform decides on an information structure to share with the buyers that is coarser than the partition that describes the platform’s initial information. The platform’s goal is to choose an information structure that maximizes the platform’s revenue. As we discussed in the introduction, we consider two different settings. In the first setting, the platform chooses prices and the sellers choose quantities (see Section 4.2). In the second setting, the sellers choose prices and quantities are determined in equilibrium (see Section 4.3).

### 4.1 Information Structures

In this section we describe the information the platform has about the sellers’ quality levels and the set of information structures from which the platform can choose.

**Seller quality.** Let $X$ be the set of possible sellers’ quality levels. We assume that $X$ is the interval $[0, \bar{x}]$ in $\mathbb{R}$. We denote by $\mathcal{B}(X)$ the Borel sigma-algebra on $X$ and by

\[\text{all our results hold for the case that } X \text{ is any compact set in } \mathbb{R}_+.\]
The space of all Borel probability measures on $X$. The distribution of the sellers’ quality levels is described by a probability measure $\phi \in \mathcal{P}(X)$.

**Platform’s information.** The platform’s information is summarized by a finite (measurable) partition $I_o = \{A_1, \ldots, A_l\}$ of $X$. We assume that $\phi(A_i) > 0$ for all $A_i \in I_o$. The platform has no information about the sellers’ quality levels if $|I_o| = 1$ where $|I_o|$ is the number of elements in the partition $I_o$.

**Information structures.** Given the platform’s information $I_o$, the platform chooses an information structure to share with buyers. We now define an information structure.

**Definition 2** An information structure $I$ is a family of disjoint sets such that every set in $I$ is a union of sets in $I_o$, i.e., $B \in I$ implies $\bigcup_i A_i = B$ for some sets $A_i \in I_o$.

While the class of information structures we study is relatively simple, it provides enough richness for our analysis. An interesting direction for future work is to expand our analysis to other information structures.

We now provide examples of information structures.\(^{10}\)

**Example 2** Suppose that $X = [0, 1]$, $I_o = \{A_1, A_2, A_3, A_4\}$, $A_j = [0.25(j - 1), 0.25j)$, $j = 1, \ldots, 4$. In this case, two examples of information structures are $I_1 = \{A_3, A_4\}$ and $I_2 = \{A_3 \cup A_4\}$. In the information structure $I_1$, the sellers whose quality levels belong to the sets $A_1$ and $A_2$ are “banned” from the platform, and the sellers whose quality levels belong to the sets $A_3$ and $A_4$ can participate in the platform. The platform shares the information it has about the sellers whose quality levels belong to the sets $A_3$ and $A_4$, i.e., the buyers know that the quality level of a seller in the set $A_4$ is between $0.75$ and $1$, and the quality level of a seller in the set $A_3$ is between $0.5$ and $0.75$. In the information structure $I_2$, the sellers whose quality levels belong to the sets $A_1$ and $A_2$ are banned from the platform and the platform does not share the information it has about the other sellers.

Given an information structure $I$, we also define the measure space $\Omega_I = (X, \sigma(I))$ where $\sigma(I)$ is the sigma-algebra generated by $I$. Recall that a function $p : (X, \sigma(I)) \to \mathbb{R}$ is $\sigma(I)$ measurable if and only if $p$ is constant on each element of $I$, i.e., $x_1, x_2 \in B$ and $B \in I$ imply that $p(x_1) = p(x_2) := p(B)$.

Given the platform’s initial information on the sellers’ quality levels $I_o$, we denote by $\mathbb{I}(I_o)$ the set of all possible information structures.

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\(^{10}\)Note that equilibrium conditions will be required to fully specify buyers’ beliefs on seller quality within each element of the information structure.
\textbf{\textit{k-separating information structures.}} We say that an information structure \( I \) is \textit{k-separating} if \( I \) contains exactly \( k \) elements, i.e., \(|I| = k\). For example, the information structure \( I_1 \) described in Example 2 is 2-separating and the information structure \( I_2 \) is 1-separating.

\section*{4.2 Two-Sided Market Model 1: Sellers Choose Quantities}

In this section we consider a model in which the platform chooses the prices, and the sellers choose the quantities.

The platform chooses an information structure \( I \in \mathbb{I}(I_o) \) and a \( \sigma(I) \) measurable pricing function \( p \). The measurability of the pricing function means that if the platform does not reveal any information about the quality of two sellers, i.e., the two sellers belong to the same set \( B \) in the information structure \( I \), then these sellers are given the same price under the platform’s choice of pricing function. The measurability condition is natural because the buyers do not have any information on the sellers’ quality except the information provided by the platform, so any rational buyer will not buy from a seller \( x \) whose price is higher than a seller \( y \) when \( x \) and \( y \) have the same expected quality.

With slight abuse of notation, for an information structure \( I = \{B_1, \ldots, B_n\} \), we denote a \( \sigma(I) \) measurable pricing function by \( p = (p(B_1), \ldots, p(B_n)) \) where \( p(B_i) \) is the price that every seller \( x \) in \( B_i \) charges. A pricing function \( p = (p(B_1), \ldots, p(B_n)) \) is said to be \textit{positive} if \( p(B_i) > 0 \) for all \( B_i \in I \).

An information structure \( I = \{B_1, \ldots, B_n\} \) and a pricing function \( p \) generate a game between the sellers and the buyers. The platform’s decisions and the structure of the game are common knowledge at the start of the game. In the game, the sellers choose quantities\(^\text{11}\), and the buyers choose whether to buy a product and if so, from which set of sellers \( B_i \in I \) to buy it. The platform’s decisions

Each equilibrium of the game induces a certain revenue for the platform. The platform’s goal is to choose an information structure and prices that maximize the platform’s equilibrium revenue. We now describe the buyers’ and sellers’ decisions in detail.

\subsection*{4.2.1 Buyers}

In this section we describe the buyers’ utility and decisions.

Buyers are heterogeneous in how much they value the quality of the product relative to its price; in particular, every buyer has a \textit{type} in \([a, b] \subseteq \mathbb{R}_+ := [0, \infty)\), with buyers’ types

\footnote{Here quantities can correspond, for example, to how many hours the sellers choose to work.}
distributed according to the probability distribution function \( F \) on \([a, b]\), with continuous probability density function \( f \). The buyers do not know the sellers’ quality levels, but they know the information structure \( I = \{ B_1, \ldots, B_n \} \) and the \( \sigma(I) \)-measurable pricing function \( p \) that the platform has chosen.

The buyers choose whether to buy a product and if so, from which set of sellers \( B_i \in I \) to buy it. A type \( m \in [a, b] \) buyer’s utility from buying a product from a type \( x \in B_i \) seller is given by

\[
Z(m, B_i, p(B_i)) = m\mathbb{E}_{\lambda_{B_i}}(X) - p(B_i).
\]

The probability measure \( \lambda_{B_i} \) describes the buyers’ beliefs about the quality levels of sellers in the set \( B_i \), and \( \mathbb{E}_{\lambda_{B_i}}(X) \) is the seller’s expected quality given the buyers’ beliefs \( \lambda_{B_i} \).

In equilibrium, the buyers’ beliefs are consistent with the sellers’ quantity decisions and with Bayesian updating.

A type \( m \) buyer buys a product from a type \( x \in B_i \) seller if \( Z(m, B_i, p(B_i)) \geq 0 \) and \( Z(m, B_i, p(B_i)) = \max_{B \in I} Z(m, B, p(B)) \), and does not buy it otherwise.

The total demand in the market for products sold by type \( x \in B_i \) sellers given the information structure \( I \) and the pricing function \( p \), \( D_I(B_i, p) \) is given by

\[
D_I(B_i, p) = \int_a^b 1\{Z(m, B_i, p(B_i)) \geq 0\} 1\{Z(m, B_i, p(B_i)) = \max_{B \in I} Z(m, B, p(B))\} F(dm).
\]

### 4.2.2 Sellers

In this section we describe the sellers’ decisions. Given the information structure \( I \) and the pricing function \( p \), a type \( x \in B_i \subseteq X \) seller’s utility is given by

\[
U(x, h, p(B_i)) = hp(B_i) - \frac{k(x)h^{\alpha+1}}{\alpha + 1}.
\]

The sellers choose a quantity \( h \in \mathbb{R}_+ \) in order to maximize their utility. For a type \( x \) seller, the cost of producing \( h \) units is given by \( k(x)h^{\alpha+1}/(\alpha + 1) \). The sellers’ cost function depends on their types and on the quantities that they sell. We assume that \( k \) is measurable and is bounded below by a positive number. We also assume that the cost

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\(^{12}\)All of our results hold if a type \( m \in [a, b] \) buyer’s utility is given by \( Z(m, B_i, p(B_i)) = mv(\lambda_{B_i}) - p(B_i) \) for some function \( v : \mathcal{P}(X) \to \mathbb{R}_+ \) that is increasing with respect to stochastic dominance. For example, the function \( v \) can capture buyers’ risk aversion.

\(^{13}\)If there are multiple sets \( \{ B_i \}_{B_i \in \mathcal{P}} \) such that for some type \( m \) buyer we have \( Z(m, B_i, p(B_i)) \geq 0 \) and \( Z(m, B_i, p(B_i)) = \max_{B \in I} Z(m, B, p(B)) \), then we break ties by assuming that the buyer chooses to buy from the set of sellers with the highest index, i.e., \( \max_{i \in \{i : B_i \in \mathcal{P}\}} i \).
of producing $h$ units is strictly convex in the quantity, i.e., $\alpha > 0$.

Let $g(x, p(B_i)) = \arg\max_{h \in \mathbb{R}_+} U(x, h, p(B_i))$ be the quantity that a type $x \in B_i$ seller chooses when the pricing function is $p = (p(B_1), \ldots, p(B_n))$. Note that $g$ is single-valued because $U$ is strictly convex in $h$. Let $S_i(B_i, p(B_i)) = \int_{B_i} g(x, p(B_i)) \phi(dx)$ be the total supply in the market of sellers with types $x \in B_i$.

### 4.2.3 Equilibrium

In this section we define the equilibrium concept that we use. Given the information structure and the pricing function that the platform chooses, there are four equilibrium requirements. First, the sellers choose quantities in order to maximize their utility. Second, the buyers choose whether to buy a product and if so, from which set of sellers to buy it in order to maximize their own utility. Third, the buyers’ beliefs about the sellers’ quality are consistent with Bayesian updating and with the sellers’ actions. Fourth, demand equals supply for each set $B_i$ that belongs to the information structure. We now define an equilibrium formally.

**Definition 3** Given an information structure $I = \{B_1, \ldots, B_n\}$ and a positive pricing function $p = (p(B_1), \ldots, p(B_n))$, an equilibrium is given by the buyers’ demand $\{D_i(B_i, p)\}_{i=1}^n$, sellers’ supply $\{S_i(B_i, p(B_i))\}_{i=1}^n$, and buyers’ beliefs $\{\lambda_{B_i}\}_{i=1}^n$ that satisfy the following conditions:

(i) **Sellers’ optimality:** The sellers’ decisions are optimal. That is,

$$g(x, p(B_i)) = \arg\max_{h \in \mathbb{R}_+} U(x, h, p(B_i))$$

is the optimal quantity for each seller $x \in B_i \in I$.

(ii) ** Buyers’ optimality:** The buyers’ decisions are optimal. That is, for each buyer $m \in [a, b]$ that buys from type $x \in B_i$ sellers, we have $Z(m, B_i, p(B_i)) \geq 0$ and $Z(m, B_i, p(B_i)) = \max_{B \in I} Z(m, B, p(B))$.

(iii) **Rational expectations:** $\lambda_{B_i}(A)$ is the probability that a buyer is matched to sellers whose quality levels belong to the set $A$ given the sellers’ optimal decisions, i.e.,

$$\lambda_{B_i}(A) = \frac{\int_A g(x, p(B_i)) \phi(dx)}{\int_{B_i} g(x, p(B_i)) \phi(dx)}$$

for all $B_i \in I$ and for all measurable sets $A \subseteq B_i$.\(^{14}\)

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\(^{14}\)We assume uniform matching within each set $B_i$. Further, if $\int_{B_i} g(x, p(B_i)) \phi(dx) = 0$ then we
(iv) Market clearing: For all \( B_i \in I \) the total supply equals the total demand, i.e.,

\[
S_I(B_i, p(B_i)) = D_I(B_i, p).
\]

\( D_I(B_i, p) \) and \( S_I(B_i, p(B_i)) \) are defined in Sections 4.2.1 and 4.2.2 respectively.

The equilibrium requirements limit the platform’s ability to design the market. The buyers’ beliefs about the expected sellers’ quality depends on the sellers’ quantity decisions, which the platform cannot control. Thus, the platform’s ability to influence the buyers’ beliefs by choosing an information structure is constrained. Furthermore, the prices and the expected sellers’ qualities must form an equilibrium (i.e., supply equals demand) in each set of the information structure. This equilibrium requirement is in addition to the more standard requirement in the market design literature that the buyers’ and sellers’ decisions are optimal. Hence, the platform cannot implement every pair of an information structure and pricing function. This motivates the following definition.

**Definition 4** An information structure and pricing function pair \((I, p)\) is called implementable if there exists an equilibrium \((D, S, \lambda)\) under \((I, p)\) where \( D = \{D_I(B_i, p)\}_{B_i \in I}, \)

\( S = \{S(B_i, p(B_i))\}_{B_i \in I}, \) and \( \lambda = \{\lambda_{B_i}\}_{B_i \in I}. \) We say that \((D, S, \lambda)\) implements \((I, p)\) if \((D, S, \lambda)\) is an equilibrium under \((I, p)\).

We denote by \( W^Q \) the set of all implementable pairs of an information structure and pricing function \((I, p)\). The platform’s goal is to choose an information structure \( I = \{B_1, \ldots, B_n\} \) and a pricing function \( p \) that maximize the total transaction value \( \pi^Q \) given by

\[
\pi^Q(I, p) := \sum_{B_i \in I} p(B_i) \min\{D_I(B_i, p), S_I(B_i, p(B_i))\}
\]

under the constraint that \((I, p)\) is implementable. That is, the platform’s revenue maximization problem is given by \( \max_{(I, p) \in W^Q} \pi^Q(I, p). \)

### 4.2.4 Equivalence with Constrained Price Discrimination

The main motivation for studying the constrained price discrimination problem that we analyzed in Section 3 is that the platform’s revenue maximization problem described

\[\text{define } \lambda_{B_i} \text{ to be the Dirac measure on the point } 0 = \min X.\]

\[\text{We can easily incorporate into the model commissions } \gamma_1, \gamma_2 \text{ on each side of the market. In this case the platform’s revenue is given by } \sum_{B_i \in I} p(B_i) \min\{D_I(B_i, p), S_I(B_i, p(B_i))\}(\gamma_1 + \gamma_2). \text{ Hence, for fixed commissions, the platform’s revenue maximization problem is equivalent to maximizing the total transaction value.}\]
above reduces to this constrained price discrimination problem. To see this, let \((I, \mathbf{p})\) be an information structure-pricing function pair where \(I = \{B_1, B_2, \ldots, B_n\}\) and \(\mathbf{p} = (p(B_1), \ldots, p(B_n))\). Let \(D = \{D_I(B_i, \mathbf{p})\}_{B_i \in I}, S = \{S(B_i, p(B_i))\}_{B_i \in I}, \) and \(\lambda = \{\lambda_{B_i}\}_{B_i \in I}\) be an equilibrium under \((I, \mathbf{p})\). Then \((I, \mathbf{p})\) induces a subset of price-expected quality pairs \(C\). The menu \(C\) is given by

\[
C = \{(p(B_1), \mathbb{E}_{\lambda_{B_1}}(X)), \ldots, (p(B_n), \mathbb{E}_{\lambda_{B_n}}(X))\}
\]

where \(\mathbb{E}_{\lambda_{B_i}}(X)\) is the equilibrium expected quality of the sellers that belong to the set \(B_i\).

Denoting, \(q_i := \mathbb{E}_{\lambda_{B_i}}(X)\), the menu \(C\) yields the total transaction value

\[
\pi(C) := \sum_{(p_i, q_i) \in C} p_i D_i(C)
= \sum_{B_i \in I} p(B_i) D_I(B_i, \mathbf{p})
= \sum_{B_i \in I} p(B_i) \min\{D_I(B_i, \mathbf{p}), S_I(B_i, p(B_i))\}
= \pi^Q(I, \mathbf{p}).
\]

The first equality follows from the definition of \(\pi\) (see Section 3). The third equality follows from the fact that \((I, \mathbf{p})\) is implementable. We conclude that the implementable information structure-pricing function pair \((I, \mathbf{p})\) yields the same revenue as the menu \(C\) that it induces.

We denote by \(C^Q\) the set of all menus \(C\) that are induced by some implementable \((I, \mathbf{p}) \in \mathcal{W}^Q\). With this notation, the platform’s revenue maximization problem is equivalent to the constrained price discrimination problem of choosing a menu \(C \in C^Q\) to maximize \(\sum p_i D_i(C)\) that we studied in Section 3. That is, we have \(\max_{(I, \mathbf{p}) \in \mathcal{W}^Q} \pi^Q(I, \mathbf{p}) = \max_{C \in C^Q} \pi(C)\).

An information structure is optimal if it induces a menu that maximizes the platform’s revenue. The next subsection studies optimal information structures in this model.

### 4.2.5 Results

In this section we present our main results regarding the two-sided market model where the sellers choose quantities and the platform choose prices.

Note that if \((I, \mathbf{p})\) induces the menu \(C\) and \(I\) is a \(k\)-separating information structure, then \(C\) is a \(k\)-separating menu. From the fact that the platform’s revenue maximization problem reduces to the constrained price discrimination problem, Theorem 1 implies that if \(C^Q\) is 1-rich and \(F(m)m\) is convex, then the optimal information structure is
1-separating, i.e., the optimal information structure consists of one element. In this subsection, we leverage this fact to show that a 1-separating information structure is optimal under certain conditions on the model’s primitives that ensure that $C^Q$ is 1-rich.

Let $\varphi^Q : \mathbb{I}(I_o) \rightarrow C^Q$ be the set-valued mapping from the set $\mathbb{I}(I_o)$ of all possible information structures to the set of menus $C^Q$ such that $C \in \varphi^Q(I)$ if and only if $C$ is a menu that is induced by some implementable $(I, p)$. That is, $\varphi^Q(I)$ contains all the menus that can be induced when the platform uses the information structure $I$. We note that the mapping $\varphi^Q$ is generally complicated and there is no simple characterization of this mapping.

However, we make substantial progress via the following proposition. In particular, it can be shown that associated to every information structure $I$ is a strictly convex program over the space of pricing functions $p$, such that $(I, p)$ is implementable if and only if the solution to the program is $p$. Since every strictly convex program has at most one solution, this result also implies that the cardinality of $\varphi^Q(I)$ is at most one; in other words, there is no more than one menu $C$ such that $C \in \varphi^Q(I)$.

**Proposition 2** For every information structure $I \in \mathbb{I}(I_o)$, there exists a strictly convex program over pricing functions such that $(I, p)$ is implementable if and only if the solution to the program is $p$. Therefore, there is at most one menu $C$ such that $C \in \varphi^Q(I)$.

To construct the claimed convex program in the preceding proposition, for every information structure $I = \{B_1, \ldots, B_n\}$ we define an associated excess supply function. We show that the excess supply function satisfies the law of supply, i.e., the excess supply function is strictly monotone\(^{16}\) on a convex and open set $P \subseteq \mathbb{R}^n$ such that if $p$ is an equilibrium price vector then $p \in P$. The excess supply function is the gradient of some function $\psi$. Thus, minimizing $\psi$ over $P$ is a strictly convex program that has a solution (minimizer) if and only if the solution is a zero of the excess supply function, i.e., an equilibrium price vector.

In the remainder of this subsection, we establish conditions for 1-richness of the space of menus induced under $\varphi^Q$; these conditions are analogous to those discussed for the simple model in Section 2. We start by defining an analog of the monopoly optimal price.\(^{16}\)

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\(^{16}\)A function $\zeta : P \rightarrow \mathbb{R}^n$ is strictly monotone on $P$ if for all $p = (p_1, \ldots, p_n)$ and $p' = (p'_1, \ldots, p'_n)$ that belong to $P$ and satisfy $p \neq p'$, we have

$$\langle \zeta(p) - \zeta(p'), p - p' \rangle > 0$$

where $\langle x, y \rangle := \sum_{i=1}^n x_i y_i$ denotes the standard inner product between two vectors $x$ and $y$ in $\mathbb{R}^n$. 

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Let 
\[ p^M(B) = \arg\max_{p \geq 0} p \left( 1 - F\left( \frac{p}{E_{\lambda B}(X)} \right) \right) \]
be the price that maximizes the platform’s revenue under the 1-separating information structure \{B\} ignoring equilibrium conditions. \( E_{\lambda B}(X) \) is the sellers’ expected quality under the 1-separating information structure \{B\}. In the Appendix we prove Lemma 3 that states that given an information structure, the sellers’ expected qualities do not depend on the prices as long as the prices are positive. This follows from the sellers’ cost function which implies that the sellers’ optimal quantity decisions are homogeneous in the prices. We assume for the rest of the section that \( E_{\lambda A_1}(X) < \ldots < E_{\lambda A_i}(X) \).

A menu \( \{(p^*(B), E^*_{\lambda B}(X))\} \) is called maximal if for every information structure \( I = \{B\} \) and every menu \( \{(p(B), E_{\lambda B}(X))\} \in \varphi^Q(I) \) we have \( E^*_{\lambda B}(X) \geq E_{\lambda B}(X) \) or \( p^*(B) \geq p(B) \) where at least one inequality is strict. We say that a 1-separating information structure \( I \) is maximal if \( C \in \varphi^Q(I) \) is a maximal menu. Let \( \mathcal{M} \) be the set of maximal information structures.

We denote by \( \{B^H\} \in \mathcal{C}^Q \setminus \{A_1\} \) the information structure with the highest equilibrium price among all the 1-separating information structures except \{A_1\}. That is, \( \{(p(B^H), E_{\lambda B^H}(X))\} \in \varphi^Q(\{B^H\}) \) and \( \{(p(B), E_{\lambda B}(X))\} \in \varphi^Q(\{B\}) \) imply \( p(B^H) \geq p(B) \) for every 1-separating information structure \{B\} such that \{B\} \( \neq \{A_1\} \) and \{B\} \( \in \mathcal{C}^Q \).

Theorem 2 shows that if
\[ S_{\{B^H\}}(B^H, p^M(B^H)) \geq D_{\{B^H\}}(B^H, p^M(B^H)) \tag{3} \]
and \( F(m)m \) is convex, then the optimal information structure is 1-separating and belongs to \( \mathcal{M} \). Inequality (3) says that under the information structure \{B^H\} and the price \( p^M(B^H) \), the supply exceeds the demand. This implies that under the information structure \{B^H\}, the equilibrium price is lower than the price that maximizes the platform’s revenue.\(^\text{17}\)

Checking if inequality (3) holds is straightforward given the model’s primitives. In order to prove that the optimal information structure is 1-separating we show that inequality (3) implies that \( \mathcal{C}^Q \) is 1-rich and then apply Theorem 1. Note that inequality (3) only imposes that the total supply of sellers exceeds the total demand under the price that

\(^{17}\)We note that if we introduce transfers or subsidies for each side of the market (i.e., the platform can pay sellers to increase supply) then the platform can always charge buyers and pay sellers in a way that inequality (3) holds and the subsidies do not influence the platform’s revenue.
maximizes the platform’s revenue for the single information structure \(B^H\) (see discussion in Section 2). We provide more details in Theorem 2.

The following Lemma shows that the set \(B^H\) belongs to the partition that describes the platform’s initial information \(I_o\). Hence, finding \(B^H\) only requires computing the equilibrium price for \(l - 1\) sets (recall that \(B^H \neq A_1\) from the definition of \(B^H\)).

**Lemma 2** We have \(B^H \in I_o \setminus A_1 = \{A_2, \ldots, A_l\}\).

The following example illustrates that inequality (3) holds if the sellers’ costs in \(B^H\) are low enough and/or the size of the supplier set \(B^H\) is large enough (similarly to Section 2).

**Example 3** Suppose that \(F(m)\) is the uniform distribution on \([0, 1]\), i.e., \(F(m) = m\) on \([0, 1]\). Assume also that \(\alpha = 1\). A direct calculation shows that \(p^M(B) = E_{\lambda_B}(X)/2\). Hence, inequality (3) holds if and only if

\[
1 - \frac{p^M(B^H)}{E_{\lambda_B}(X)} \leq p^M(B^H) \int_{B^H} k(x)^{-1} \phi(dx) \iff 1 \leq \int_{B^H} x k(x)^{-1} \phi(dx) \tag{4}
\]

where we use the fact that \(E_{\lambda_B}(X) \int_{B^H} k(x)^{-1} \phi(dx) = \int_{B^H} x k(x)^{-1} \phi(dx)\) (see Lemma 3 in the Appendix). Thus, the size of the set \(B^H\), the sellers’ qualities in \(B^H\), and the sellers’ costs in \(B^H\) determine whether inequality (3) holds. In order to determine the information structure \(\{B^H\}\) with the highest equilibrium price we can solve for the equilibrium price:

\[
1 - \frac{p^{eq}(B^H)}{E_{\lambda_B}(X)} = p^{eq}(B) \int_B k(x)^{-1} \phi(dx) \iff p^{eq}(B) = \frac{\int_B x k(x)^{-1} \phi(dx)}{\int_B k(x)^{-1} \phi(dx)(1 + \int_B x k(x)^{-1} \phi(dx))} \tag{5}
\]

and choose the set \(B \in \{A_2, \ldots, A_l\}\) with the highest equilibrium price.

Assume further, as in Section 2, that for all \(A_i \in I_o\), \(q_i\) and \(c_i\) are the quality and cost of every seller in \(A_i\), respectively. That is, the platform knows the sellers’ costs and the sellers’ quality levels. Denoting \(\phi_j := \phi(A_j)\), inequality (4) is equivalent to \(q_j \phi_j \geq c_j\) where \(A_j = B^H\). Note that this is exactly the same condition as the condition in the second stage of the analysis in Section 2. To find \(j\), note that the equilibrium price is given by \(q_i c_i/(c_i + \phi_i q_i)\) (see inequality (5)). Hence we can find \(j\) by solving \(j = \arg\max_{2 \leq i \leq n} q_i c_i/(c_i + \phi_i q_i)\).

**Theorem 2** Assume that \(F(m)\) is strictly convex on \([a, b]\). Assume that inequality (3) holds.
Then there exists a 1-separating information structure $I^*$ such that

$$(I^*, p^*) = \arg\max_{(I, p) \in \mathcal{W}^q} \pi^Q(I, p).$$

That is, there exists a 1-separating information structure $I^*$ that maximizes the platform’s revenue.

Furthermore, the optimal 1-separating information structure $I^*$ belongs to $\mathcal{M}$, so it is maximal.

When the support of $F$ is unbounded it can be the case that inequality (3) trivially holds because the supply under the price that maximizes the platform’s revenue tends to infinity. For example, suppose that $F$ has the Pareto distribution, i.e., $F(m) = 1 - 1/m^\beta$ on $[1, \infty)$. Then $F(m)m$ is convex for $\beta < 1$. In this case, the support of $F$ is unbounded\(^{18}\) so $p^M$ is not necessarily well defined. Indeed, for every $q > 0$ we have

$$\lim_{p \to \infty} p \left(1 - F \left(\frac{p}{q}\right)\right) = \lim_{p \to \infty} p \left(\frac{q^\beta}{p^\beta}\right) = \infty.$$ 

Thus, the price that maximizes the platform’s revenue tends to infinity which means that the supply under this price tends to infinity and inequality (3) trivially holds.

Under the conditions of Theorem 2, the optimal 1-separating information structure depends on the distribution function $F$ and on other parameters of the two-sided market model. Theorem 2 shows that the optimal 1-separating information structure is maximal. In the proof of Lemma 2 in the Appendix we show that every 1-separating information structure that is not an element of $I_o$ is not maximal. Hence, we have the following Corollary.

**Corollary 2** Suppose that the assumptions of Theorem 2 hold. The optimal 1-separating information structure is an element of $I_o = \{A_1, \ldots, A_l\}$.

Intuitively, inequality (3) implies that the platform cannot implement the price that maximizes its revenue because the market is demand-constrained. In this case, given a fixed sellers’ expected quality, the platform prefers a high equilibrium price (see Figure 1 in Section 2). In other words, the optimal 1-separating information structure is maximal. Choosing a 1-separating information structure $\{B\}$ that consists of a union of sets of sellers is not maximal. Removing the set of sellers with the lowest expected quality in

\(^{18}\) All our results hold also in the case that the support of $F$ is unbounded.
\{B\} yields a new 1-separating information structure \{B^*\}. For a fixed price, the total supply under \{B^*\} is lower than under \{B\}. Thus, the equilibrium price and the expected sellers’ quality under \{B^*\} is higher than under \{B\}. Thus, \{B\} is not maximal.

The information structure that bans all the sellers except the highest quality sellers is maximal and hence potentially optimal. When the equilibrium price under this information structure is higher than the equilibrium price under any other 1-separating information structure, then the set of maximal information structures \mathcal{M} consists of one element, and thus, the information structure that bans all the sellers except the highest quality sellers is optimal. The following Corollary shows that this is the case when the size of the set \(A_t\) is low and/or the costs of the sellers in \(A_t\) are high compared to other sets in \(I_o\). This is intuitive because in this case the supply of sellers in the set \(A_t\) is low, and hence, the equilibrium price under the information structure \{\(A_t\}\} is high compared to the other information structures in \(I_o\).

**Corollary 3** Assume that \(F(m)m\) is convex on \([a, b]\) and that inequality (3) holds. If

\[
\int_{A_t} k(x)^{-1/\alpha} \phi(dx) = \min_{A_j \in I_o} \int_{A_j} k(x)^{-1/\alpha} \phi(dx)
\]

then the optimal 1-separating information structure is \(A_t\). That is, banning all sellers except the highest quality sellers is optimal for the platform.

Combining conditions (3) and (6), we conclude from Theorem 2 that when the total supply of high quality sellers is not too large and not too small, and the elasticity of the density function is not too low, banning all sellers but the highest quality sellers is optimal for the platform.

### 4.3 Two-Sided Market Model 2: Sellers Choose Prices

In this section we consider a model in which the sellers choose the prices and the quantities are determined in equilibrium.

The platform chooses an information structure \(I \in \mathbb{I}_o\) (see Section 4.1). An information structure generates a game between buyers and sellers. In this game, sellers make entry decisions first. After the entry decisions, in each set of sellers that belongs to the information structure, the participating sellers engage in Bertrand competition. Buyers form beliefs about the sellers’ quality and choose whether to buy a product and if so, from which set of sellers to buy it.
Each equilibrium of the game induces a certain revenue for the platform. The platform’s goal is to choose the information structure that maximizes the platform’s equilibrium revenue. We now describe the sellers’ and buyers’ decisions in detail.

4.3.1 Buyers

In this section we describe the buyers’ decisions. The buyers make their decisions after the sellers’ entry and pricing decisions have been made. We denote by $H(B_i) \subseteq B_i$ the set of quality $x \in B_i$ sellers that participate in the platform and by $p_x$ the price that a quality $x \in \bigcup_{B_i \in I} H(B_i)$ seller charges.

As in Section 4.2.1, the buyers’ heterogeneity is described by a type space $[a, b] \subset \mathbb{R}_+$, and buyers’ types are distributed according to a probability distribution function $F$ on $[a, b]$. The buyers do not know the sellers’ quality levels, but they know the information structure $I = \{B_1, \ldots, B_n\}$ that the platform has chosen. Because the buyers do not have any information about the sellers’ quality aside from the information structure $I$, and there are no search costs or frictions, the buyers that decide to buy a product from quality $x \in B_i$ sellers buy it from the seller (or one of the sellers) with the lowest price in $B_i$.

The preceding requirement implies that sellers cannot use prices in order to signal quality. That is, two sellers with quality levels $x_1, x_2$ such that $x_1 \in B_i, x_2 \in B_i$ for some set $B_i$ in the information structure $I$ cannot disclose information about their quality level to the buyers. Because the main focus of this section is examining the platform’s quality selection decisions, we abstract away from information that sellers can disclose to buyers. In particular, our model abstracts away from the possibility that the sellers signal their quality through higher prices. This may be an interesting avenue for future research.

Given the information structure $I = \{B_1, \ldots, B_n\}$ and the sets of sellers that participate in the platform $\{H(B_i)\}_{B_i \in I}$, $H(B_i) \subseteq B_i$, the buyers form beliefs $\lambda_{B_i} \in \mathcal{P}(X)$ about the quality level of type $x \in B_i$ sellers.\footnote{With slight abuse of notation we use a similar notation to the notation of Section 4.2.1.} In equilibrium, the buyers’ beliefs are consistent with the sellers’ entry decisions and with Bayesian updating, that is, $\lambda_{B_i}$ describes the conditional distribution of $\phi$ given $H(B_i)$, i.e., $\lambda_{B_i}(A) = \phi(A|H(B_i))$ where $\phi(A|H(B_i)) := \frac{\phi(A \cap H(B_i))}{\phi(H(B_i))}$ for every (measurable) set $A$ and all $B_i \in I$ such that $\phi(H(B_i)) > 0$.

We denote by $p(B_i) = \inf_{x \in H(B_i)} p_x$ the lowest price among the sellers in the set $B_i$. A type $m \in [a, b]$ buyer’s utility from buying a product from quality $x \in B_i$ sellers is given
In particular, when 

max_{B \in I} Z(m, B, p(B)),

and does not buy a product otherwise.

The total demand in the market for products that are sold by type \( x \in B_i \) sellers \( D_I(B_i, p(B_1), \ldots, p(B_n)) \) who charge the lowest price in \( B_i \) is given by

\[
D_I(B_i, p(B_1), \ldots, p(B_n)) = \int_a^b 1_{\{Z(m, B_i, p(B_i)) \geq 0\}} 1_{\{Z(m, B_i, p(B_i)) = \max_{B \in I} Z(m, B, p(B))\}} F(dm).
\]

(7)

The demand is zero for sellers that do not charge the lowest price in \( B_i \).

### 4.3.2 Sellers

In this section we describe the sellers’ decisions. Sellers first choose whether to participate in the platform or not. In each set \( B_i \in I \) that belongs to the information structure participating sellers price their products simultaneously and engage in price competition with other sellers whose quality levels belong to the set \( B_i \in I \). Because a buyer that decides to buy a product from a quality \( x \in B_i \) seller buys it from the seller (or one of the sellers) who charges the lowest price in the set \( B_i \), the price competition between the sellers resembles Bertrand competition.

A quality \( x \in B_i \subseteq X \) seller that participates in the platform sells a quantity of \( h_I(B_i, H(B_i), p_x, p(B_1), \ldots, p(B_n)) \) units if the set of participating sellers is \( H(B_i) \), the price that \( x \) charges is \( p_x \in \mathbb{R}_+ \), and \( p(B_i) = \inf_{x \in H(B_i) \setminus \{x\}} p_x \) is the lowest price among the other sellers in the set \( H(B_i) \). We denote by \( M_I(B_i, p(B_1), \ldots, p(B_n)) \) the total mass of sellers whose quality levels belong to \( B_i \) and who charge the price \( p(B_i) \). The quantity allocation function \( h_I \) is determined in equilibrium and is given by

\[
h_I(B_i, H(B_i), p_x, \mathbf{p}) = \begin{cases} 
\infty & \text{if } p_x < p(B_i), \quad D_I(B_i, \mathbf{p}) > 0 \\
\frac{D_I(B_i, \mathbf{p})}{M_I(B_i, \mathbf{p})} & \text{if } p_x = p(B_i), \quad D_I(B_i, \mathbf{p}) > 0 \\
0 & \text{if } p_x > p(B_i), \text{ or } D_I(B_i, \mathbf{p}) = 0
\end{cases}
\]

(8)

where \( \mathbf{p} := (p(B_1), \ldots, p(B_n)) \) and we define \( D_I(B_i, \mathbf{p}) / M_I(B_i, \mathbf{p}) = \infty \) if \( M_I(B_i, \mathbf{p}) = 0 \) and \( D_I(B_i, \mathbf{p}) > 0 \). This quantity allocation resembles the quantity allocation in the standard Bertrand competition model with a continuum of sellers. In particular, when

\[
Z(m, B_i, p(B_i)) = m \mathbb{E}_{\lambda_{B_i}}(X) - p(B_i).
\]
multiple sellers’ charge the same price, the buyers’ demand splits evenly between the sellers.

A quality \( x \in B_i \subseteq X \) seller’s utility from participating in the platform is given by

\[
U(x, H(B_i), p_x, p(B_1), \ldots, p(B_n)) = h_I(B_i, H(B_i), p_x, p(B_1), \ldots, p(B_n))(p_x - c(x)).
\]

We assume that the cost function \( c \) is positive and constant on each element of the partition \( I_o \), i.e., \( x_1, x_2 \in A_i \) and \( A_i \in I_o \) imply \( c(x_1) = c(x_2) = c(A_i) \). The assumption that the cost function \( c \) is constant on each element of the partition \( I_o \) means that the cost function is measurable with respect to the platform’s information, i.e., the platform knows the sellers’ costs but not the sellers’ quality levels. This assumption is not essential to our results. We also assume that the cost function is increasing, i.e., \( c(A_i) < c(A_j) \) for \( i < j \). This assumption means that producing higher quality products costs more. A quality \( x \in X \) seller’s utility from not participating in the platform is normalized to 0.

### 4.3.3 Equilibrium

In this section we define the equilibrium concept that we use for the game described above. For simplicity, we focus on a symmetric equilibrium in the sense that for all \( B_i \in I \), all the sellers that participate in the platform charge the same price. With slight abuse of notation, we denote this price by \( p(B_i) \), i.e., \( p_x = p(B_i) \) for all \( x \in H(B_i), B_i \in I \).

**Definition 5** Given an information structure \( I = \{B_1, \ldots, B_n\} \), an equilibrium consists of a vector of positive prices \( p = (p(B_1), \ldots, p(B_n)) \in \mathbb{R}^{|I|} \), positive masses of sellers that participate in the platform \( \{M_i(B_i, p)\}_{B_i \in I} \), positive masses of demand \( \{D_i(B_i, p)\}_{B_i \in I} \), and buyers’ beliefs \( \lambda = (\lambda_{B_i})_{B_i \in I} \) such that

(i) **Sellers’ optimality:** The sellers’ decision are optimal. That is,

\[
p(B_i) = \arg\max_{p_x \in \mathbb{R}_+} U(x, H(B_i), p_x, p(B_i))
\]

is the price that seller \( x \in H(B_i) \) charges. Seller \( x \in B_i \) enters the market, i.e., \( x \in H(B_i) \), if and only if \( U(x, H(B_i), p(B_i), p) \geq 0 \).

(ii) **Buyers’ optimality:** The buyers’ decisions are optimal. That is, for each buyer \( m \in [a, b] \) that buys from type \( x \in B_i \) sellers, we have \( Z(m, B_i, p(B_i)) \geq 0 \) and \( Z(m, B_i, p(B_i)) = \max_{B_i \in I} Z(m, B, p(B)) \).

(iii) **Rational expectations:** \( \lambda_{B_i}(A) \) is the probability that a buyer is matched to sellers
whose quality levels belong to the set $A$ given the sellers’ entry decisions, i.e.,

$$\lambda_{B_i}(A) = \phi(A|H(B_i)) = \frac{\phi(A \cap H(B_i))}{\phi(H(B_i))}$$

for every (measurable) set $A$ and for all $B_i \in I$ such that $\phi(H(B_i)) > 0$.\textsuperscript{20}

(iv) Market clearing: For all $B_i \in I$ we have

$$M_I(B_i, p) h_I(B_i, H(B_i), p(B_i), p) = D_I(B_i, p)$$

where $M_I(B_i, p) = \phi(H(B_i))$ is the mass of sellers in $B_i$ that participate in the platform; $D_I(B_i, p)$ and $h_I(B_i, H(B_i), p(B_i), p)$ are defined in Sections 4.3.1 and 4.3.2, respectively.

We say that an information structure $I$ is implementable if there exists an equilibrium $(p, D, M, \lambda)$ under $I$ where $D = \{D_I(B_i, p)\}_{B_i \in I}$, $M = \{M_I(B_i, p)\}_{B_i \in I}$, and $\lambda = \{\lambda_{B_i}\}_{B_i \in I}$. We denote by $W^P$ the set of all implementable information structures.

The platform’s goal is to choose an implementable information structure to maximize the total transaction value $\pi^P$ given by

$$\pi^P(I) := \sum_{B_i \in I} p(B_i) \min\{D_I(B_i, p), M_I(B_i, p) h_I(B_i, H(B_i), p(B_i), p)\}.$$ 

### 4.3.4 Equivalence with Constrained Price Discrimination

As in Section 4.2.3, the platform’s revenue maximization problem described above reduces to the constrained price discrimination problem that we analyzed in Section 3. To see this, note that every implementable information structure $I = \{B_1, B_2, \ldots, B_n\}$ and associated equilibrium prices $p$ induces a menu $C$ that is given by $C = \{(p(B_1), \mathbb{E}_{\lambda_{B_1}}(X)), \ldots, (p(B_n), \mathbb{E}_{\lambda_{B_n}}(X))\}$ where $\mathbb{E}_{\lambda_{B_i}}(X)$ is the equilibrium expected quality of the sellers that belong to the set $B_i$ and $p = (p(B_1), \ldots, p(B_n))$ is the vector of equilibrium prices. The implementable information structure $I$ yields the same revenue as the menu $C$ that it induces (see Section 4.2.3). We denote by $C^P$ the set of all menus $C$ that are induced by some implementable information structure $I \in W^P$. With this notation, the platform’s revenue maximization problem is equivalent to the constrained price discrimination problem of choosing a menu $C \in C^P$ to maximize $\sum p_i D_i(C)$ that we studied in Section 3.

\textsuperscript{20}If $\phi(H(B_i)) = 0$ then we define $\lambda_{B_i}$ to be the Dirac measure on the point 0.
4.3.5 Results

In this section we present our main results regarding the two-sided market model in which the sellers choose the prices.

Let $\varphi^P : \mathbb{I}(I_o) \Rightarrow \mathcal{C}^P$ be the set-valued mapping from the set $\mathbb{I}(I_o)$ of all possible information structures to the set of menus $\mathcal{C}^P$ such that $C \in \varphi^P(I)$ if and only if $C$ is a menu that is induced by some implementable information structure $I$. As opposed to the two-sided market model that we study in Section 4.2, the mapping $\varphi^P$ can be explicitly characterized in the current setting. This is because Bertrand competition pins down the equilibrium prices.

For an information structure $I = \{B_1, \ldots, B_n\}$ let $L(I) = \{G_1, \ldots, G_n\}$ be an information structure such that $G_j \in I_o$ for all $G_j \in L(I)$ and $G_j$ is the set with the lowest index among the blocks of $B_j$, i.e., among the sets $\{A_k\}$ such that $B_j = \cup_k A_k$. The following proposition shows that if $C \in \varphi^P(I)$ then $C = \{(c(G_1), \mathbb{E}_{\lambda_{G_1}}(X)), \ldots, (c(G_n), \mathbb{E}_{\lambda_{G_n}}(X))\}$.

**Theorem 3** (i) Let $I$ be any information structure. Suppose that $C \in \varphi^P(I)$. Then

$$C = \{(c(G_1), \mathbb{E}_{\lambda_{G_1}}(X)), \ldots, (c(G_n), \mathbb{E}_{\lambda_{G_n}}(X))\}$$

where $L(I) = \{G_1, \ldots, G_n\}$ and $\lambda_{G_i}(A) = \phi(A \cap G_i)/\phi(G_i)$ for every measurable set $A$.

(ii) $\mathcal{C}^P$ is 1-rich.

The following Corollary follows immediately from Theorem 1 and Theorem 3.

**Corollary 4** Assume that $F(m)m$ is convex on $[a,b]$. Then there exists a 1-separating information structure that maximizes the platform’s revenue.

As in the previous section, we obtain a sharp characterization of the optimal information structure. Note that as previewed in Section 2, in this model the 1-rich condition is immediately satisfied because supply is perfectly elastic due to Bertrand competition.

5 Conclusions

In this paper we study optimal information disclosure for online platforms. A key part of our analysis is showing that the platform’s problem reduces to a constrained price discrimination problem, where the constraints are given by equilibrium requirements. We use this equivalence to provide sharp characterizations of optimal information disclosures.
policies for a couple of two-sided market models. In current and future work we plan to make tighter connections between our results and analysis and real-world platforms. We also plan to further leverage our price discrimination formulation to derive optimal information disclosure policies under different assumptions than the ones we have focused on in this manuscript.

A Appendix

A.1 Proofs of Section 3

Proof of Theorem 1. Let \( C = \{ (p_i, q_i)_{i=1}^n \} \in \mathcal{C} \) be a menu such that \( p_k \leq p_j \) for all \( k < j \) and \( n > 1 \). We can assume\(^{21}\) that the demand for each price-quality pair in \( C \) has a positive mass. That is

\[
D_i(C) = \int_a^b 1_{\{m_{q_i} - p_i \geq 0\}} 1_{\{m_{q_i} - p_i = \max_{i=1,\ldots,n} m_{q_i} - p_i\}} F(dm) > 0
\]

for all \( 1 \leq i \leq n \). Note that \( D_i(C) > 0 \) for all \( 1 \leq i \leq n \) implies that \( q_k < q_j \) for all \( k < j \).

**Step 1.** The revenue from the menu \( C \) is given by

\[
\pi(C) = \sum_{i=1}^n p_i \left( F(m_{i+1}) - F(m_i) \right)
\]

where \( m_{n+1} = b \) and the numbers \( \{m_i\}_{i=1}^n \) satisfy \( m_i \in [a, b] \) for all \( 1 \leq i \leq n \) and

\[
m_i q_i - p_i = m_i q_{i-1} - p_{i-1}
\]

where \( q_0 = p_0 = 0 \).

**Proof of Step 1.** The proof of Step 1 is standard (see Maskin and Riley (1984)). We provide it here for completeness.

Because \( q_n > q_j \) for all \( 1 \leq j \leq n - 1 \), if for some \( 1 \leq j \leq n - 1 \) and \( m \in [a, b] \) we have

\[
m(q_n - q_j) \geq p_n - p_j
\]

\(^{21}\)If for some \((p_k, q_k)\) in \( C \) we have \( D_k(C) = 0 \), then the menu \( C \setminus \{ (p_k, q_k) \} \) has the same revenue as the menu \( C \). Thus, we can consider the menu \( C \setminus \{ (p_k, q_k) \} \) instead of the menu \( C \).
then
\[ m'(q_n - q_j) \geq p_n - p_j \]
for all \( m' \in [m, b] \). Thus, if for some \( m \in [a, b] \) we have
\[ mq_n - p_n \geq \max\{ \max_{1 \leq j \leq n-1} mq_j - p_j, 0 \} \]
then inequality (9) holds for all \( m' \in [m, b] \). In other words, if a type \( m \) chooses the price-quality pair \((p_n, q_n)\), then every type \( m' \) with \( m \leq m' \leq b \) chooses the price-quality pair \((p_n, q_n)\).

Let
\[ W_n := \{ m \in [a, b] : mq_n - p_n \geq \max\{ \max_{1 \leq j \leq n-1} mq_j - p_j, 0 \} \} \]
be the set of types that choose the price-quality pair \((p_n, q_n)\). Define \( m_n = \min W_n \). If \( D_n(C) > 0 \) implies that the set \( W_n \) is not empty. From the fact that \( m \in W_n \) implies \( m' \in W_n \) for all \( m \leq m' \leq b \), \( W_n \) equals the interval \([m_n, b]\). Thus,
\[ D_n(C) = \int_a^b 1_{W_n}(m) F(dm) = F(b) - F(m_n) = F(m_{n+1}) - F(m_n) \]
where \( m_{n+1} := b \) so \( F(m_{n+1}) = 1 \).

Define \( m_i = \min W_i \) where we define the sets
\[ W_i := \{ m \in [a, m_{i+1}] : mq_i - p_i \geq \max\{ \sup_{1 \leq j \leq i-1} mq_j - p_j, 0 \} \} \]
for all \( 1 \leq i \leq n-1 \). If \( D_i(C) > 0 \) implies that \( W_i \) is not empty. Thus, \( m_i \) is well defined. From the same argument as the argument above, if a type \( m \in W_i \) chooses the price-quality pair \((p_i, q_i)\), then every type \( m' \) with \( m \leq m' \leq m_{i+1} \) chooses the price-quality pair \((p_i, q_i)\). Thus, \( W_i \) equals the interval \([m_i, m_{i+1}]\) and
\[ D_i(C) = \int_a^b 1_{W_i}(m) F(dm) = F(m_{i+1}) - F(m_i) > 0 \]
for all \( 1 \leq i \leq n \).

Note that \( W_1 = \{ m \in [a, m_2] : mq_1 - p_1 \geq 0 \} \). The continuity of the function \( mq_1 - p_1 \) implies that \( m_1 = \min W_1 \) satisfies \( m_1 q_1 - p_1 = 0 \). Using continuity again and the definition of \( m_2 \) we conclude that \( m_2 q_2 - p_2 = m_2 q_1 - p_1 \). Similarly, \( m_i q_i - p_i = m_i q_{i-1} - p_{i-1} \) for all \( 2 \leq i \leq n \).
Thus, the revenue from the menu $C$ is given by

$$\pi(C) = \sum_{i=1}^{n} p_i D_i(C) = \sum_{i=1}^{n} p_i (F(m_{i+1}) - F(m_i))$$

where $m_{n+1} = b$ and the numbers $\{m_i\}_{i=1}^{n}$ satisfy $m_i \in [a, b]$ for all $1 \leq i \leq n$ and $m_i q_i - p_i = m_i q_{i-1} - p_{i-1}$, $q_0 = p_0 = 0$.

**Step 2.** The function $f(x, y) = x F\left(\frac{z}{y}\right)$ is convex on $E = \{(x, y) : x/y \in [a, b], y > 0\}$.

**Proof of Step 2.** Recall that the perspective function $\overline{f}(x, y) = yg\left(\frac{z}{y}\right)$ is convex on $E$ whenever $g$ is convex on $[a, b]$. Suppose that $g(x) = F(x) x$. Then $g$ is convex on $[a, b]$ from the theorem’s assumption. Thus,

$$\overline{f}(x, y) = yg\left(\frac{x}{y}\right) = yF\left(\frac{x}{y}\right) \cdot \frac{x}{y} = x F\left(\frac{x}{y}\right) = f(x, y)$$

is convex on $E$.

**Step 3.** Let $0 = d_0 < d_1 < \ldots < d_k$ and $0 = z_0 < \ldots < z_k$. Assume that $(z_i - z_{i-1})/(d_i - d_{i-1}) \in [a, b]$ for all $1 \leq i \leq k$. Then

$$z_k F\left(\frac{z_k}{d_k}\right) \leq \sum_{i=1}^{k} (z_i - z_{i-1}) F\left(\frac{z_i - z_{i-1}}{d_i - d_{i-1}}\right). \quad (10)$$

**Proof of Step 3.** From Step 2 the function $f(x, y) = x F\left(\frac{z}{y}\right)$ is convex on $E$. From Jensen’s inequality we have

$$k^{-1} \sum_{i=1}^{k} x_i F\left(\frac{k^{-1} \sum_{i=1}^{k} x_i}{y_i}\right) = f\left(k^{-1} \sum_{i=1}^{k} (x_i, y_i)\right) \leq k^{-1} \sum_{i=1}^{k} f(x_i, y_i) = k^{-1} \sum_{i=1}^{k} x_i F\left(\frac{x_i}{y_i}\right)$$

for all $(x_1, \ldots, x_k)$ and $(y_1, \ldots, y_k)$ such that $(x_i, y_i) \in E$ for all $i = 1, \ldots, k$. Thus,

$$\sum_{i=1}^{k} x_i F\left(\frac{\sum_{i=1}^{k} x_i}{\sum_{i=1}^{k} y_i}\right) \leq \sum_{i=1}^{k} x_i F\left(\frac{x_i}{y_i}\right).$$

Let $z_i - z_{i-1} = x_i \geq 0$ and $d_i - d_{i-1} = y_i > 0$. Note that $\sum_{i=1}^{k} x_i = z_k$ and $\sum_{i=1}^{k} y_i = d_k$ to conclude that inequality (10) holds.

**Step 4** The menu that maximizes the total transaction value is price-maximal.

**Proof of Step 4.** Assume that $C$ is not price-maximal. Then there exists a price-quality pair $\{p_{n+1}, q_{n+1}\}$ such that $p_{n+1} > p_n$ and $C \cup \{p_{n+1}, q_{n+1}\}$ belongs to $C_p$, i.e.,
$D_i(C) > 0$ for all $1 \leq i \leq n + 1$. From Step 1, we have $m_iq_i - p_i = m_iq_{i-1} - p_{i-1}$ for all $i$ (recall that $q_0 = p_0 = 0$). This implies that 

$$m_i = \frac{p_i - p_{i-1}}{q_i - q_{i-1}}.$$

for all $i$. We have

$$\pi(C \cup \{p_{n+1}, q_{n+1}\}) - \pi(C) = \sum_{i=1}^{n} p_i (F(m_{i+1}) - F(m_i)) + p_{n+1}(1 - F(m_{n+1}))$$

$$- \sum_{i=1}^{n-1} p_i (F(m_{i+1}) - F(m_i)) - p_n(1 - F(m_n))$$

$$= p_n \left( F\left(\frac{p_{n+1} - p_n}{q_{n+1} - q_n}\right) - F\left(\frac{p_n - p_{n-1}}{q_n - q_{n-1}}\right) \right) + p_{n+1} \left( 1 - F\left(\frac{p_{n+1} - p_n}{q_{n+1} - q_n}\right) \right)$$

$$- p_n \left( 1 - F\left(\frac{p_n - p_{n-1}}{q_n - q_{n-1}}\right) \right) > 0.$$

Thus, $C$ is not optimal. The inequality follows from the facts that $p_{n+1} > p_n$ and $D_{n+1} = 1 - F((p_{n+1} - p_n)/(q_{n+1} - q_n)) > 0$. We conclude that the menu that maximizes the total transaction value (if it exists) is price-maximal.

**Step 5.** Let $C^* = \{(p_n, q_n)\}$. We have

$$\pi(C) \leq \pi(C^*).$$

**Proof of Step 5.** From Step 1 we have

$$\pi(C) = \sum_{i=1}^{n} p_i (F(m_{i+1}) - F(m_i))$$

$$= \sum_{i=1}^{n-1} p_i \left( F\left(\frac{p_{i+1} - p_i}{q_{i+1} - q_i}\right) - F\left(\frac{p_i - p_{i-1}}{q_i - q_{i-1}}\right) \right) + p_n \left( 1 - F\left(\frac{p_n - p_{n-1}}{q_n - q_{n-1}}\right) \right)$$

$$= p_n - \sum_{i=1}^{n} (p_i - p_{i-1})F\left(\frac{p_i - p_{i-1}}{q_i - q_{i-1}}\right).$$

The first equality follows from Step 1. In the second equality we use the fact that $F(m_{n+1}) = F(b) = 1$. 

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Let $C^* = \{(p_n, q_n)\}$. Using Step 1 again we have

$$\pi(C^*) = p_n \left(1 - F\left(\frac{p_n}{q_n}\right)\right)$$

Thus, we have $\pi(C) \leq \pi(C^*)$ if and only if

$$p_n F\left(\frac{p_n}{q_n}\right) \leq \sum_{i=1}^{n} (p_i - p_{i-1}) F\left(\frac{p_i - p_{i-1}}{q_i - q_{i-1}}\right).$$

(11)

From Step 1, $m_i = (p_i - p_{i-1}) / (q_i - q_{i-1}) \in [a, b]$ for all $1 \leq i \leq n$. Thus, from Step 3, inequality (11) holds. We conclude that $\pi(C) \leq \pi(C^*)$.

We proved that for any menu $C = \{(p_i, q_i)_{i=1}^{n}\} \in C$ such that $D_i(C) > 0$ for all $1 \leq i \leq n$ we have $\pi(C) \leq \pi(C^*)$ where $C^* = (p_n, q_n)$. From Step 4, we can assume that $C$ is a price-maximal menu. Because $C$ is 1-rich, there exists a 1-separating menu $C' \in C$ such that $\pi(C^*) \leq \pi(C')$. We conclude that for any menu $C \in C$ there exists a 1-separating menu $C'$ such that $\pi(C) \leq \pi(C')$. Thus,

$$\sup_{C \in C} \pi(C) \leq \max_{C \in C_1} \pi(C)$$

which proves the theorem. The maximum on the right side of the last inequality is attained because the distribution function $F$ is continuous and $C_1$ is a compact set. □

**Proof of Proposition 1.** Suppose that $g(z) = F(z)z$ is not convex on $(a, b)$. Then there exist non-negative numbers $z_1 \in (a, b)$, $z_2 \in (a, b)$ and $0 < \lambda < 1$ such that

$$g(\lambda z_1 + (1 - \lambda) z_2) > \lambda g(z_1) + (1 - \lambda) g(z_2).$$

Let $k_1, k_2, d_1, d_2$, and $0 < \theta < 1$ be such that $k_1 \geq 0$, $k_2 \geq 0$, $d_1 > 0$, $d_2 > 0$, $d_1 z_1 = k_1$, $d_2 z_2 = k_2$, and $\theta d_1 = \lambda (\theta d_1 + (1 - \theta) d_2)$.

Note that $1 - \lambda = (1 - \theta) d_2 / (\theta d_1 + (1 - \theta) d_2)$.

Denote $d_\theta := \theta d_1 + (1 - \theta) d_2$ and $k_\theta := \theta k_1 + (1 - \theta) k_2$. Note that

$$\lambda z_1 + (1 - \lambda) z_2 = \frac{\theta d_1 k_1}{d_\theta d_1} + \frac{(1 - \theta) d_2 k_2}{d_\theta d_2} = \frac{k_\theta}{d_\theta}.$$

We have

$$\theta d_1 g\left(\frac{k_1}{d_1}\right) + (1 - \theta) d_2 g\left(\frac{k_2}{d_2}\right) = d_\theta \left(\frac{\theta d_1}{d_\theta} g\left(\frac{k_1}{d_1}\right) + \frac{(1 - \theta) d_2}{d_\theta} g\left(\frac{k_2}{d_2}\right)\right) < d_\theta g\left(\frac{k_\theta}{d_\theta}\right).$$

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We conclude that the function \( f(x, y) := yg\left(\frac{x}{y}\right) = xF\left(\frac{x}{y}\right) \) is not convex on \( E^* = \{(x, y) : x/y \in (a, b), y > 0\} \).

Since \( f \) is continuous and not convex it is not midpoint convex.\(^{22}\)

Thus, there exists \((x_1, y_1) \in E^*\) and \((x_2, y_2) \in E^*\) such that

\[
f\left(\frac{(x_1, y_1)}{2} + \frac{(x_2, y_2)}{2}\right) > \frac{f(x_1, y_1)}{2} + \frac{f(x_2, y_2)}{2}.
\]

If \(x_1 = x_2 = 0\) then the left-hand-side and the right-hand-side of the last inequality equal 0 which is a contradiction, so we have \(x_1 + x_2 > 0\).

Assume in contradiction that \(\frac{x_2}{y_2} = \frac{x_1}{y_1}\). We have

\[
f\left(\frac{(x_1, y_1)}{2} + \frac{(x_2, y_2)}{2}\right) > \frac{f(x_1, y_1)}{2} + \frac{f(x_2, y_2)}{2}
\]

\[
\Leftrightarrow (x_1 + x_2)F\left(\frac{x_1 + x_2}{y_1 + y_2}\right) > x_1 F\left(\frac{x_1}{y_1}\right) + x_2 F\left(\frac{x_2}{y_2}\right)
\]

\[
\Leftrightarrow F\left(\frac{x_1 + x_2}{y_1 + y_2}\right) > F\left(\frac{x_1}{y_1}\right)
\]

\[
\Rightarrow \frac{x_1 + x_2}{y_1 + y_2} > \frac{x_1}{y_1} \Leftrightarrow \frac{x_2}{y_2} > \frac{x_1}{y_1},
\]

which is a contradiction. Thus, \(\frac{x_2}{y_2} \neq \frac{x_1}{y_1}\).

Assume without loss of generality that \(\frac{x_2}{y_2} > \frac{x_1}{y_1}\). Then \(x_2 > 0\).

Let \(p_2 > p_1\) and \(q_2 > q_1\) be such that \(p_2 - p_1 = x_2 > 0\), \(p_1 = x_1\), \(q_2 - q_1 = y_2\) and \(y_1 = q_1\). Define the menus \(C = \{(p_1, q_1), (p_2, q_2)\}\), \(C^* = \{(p_1, q_1)\}\), and \(C^{**} = \{(p_2, q_2)\}\).

Let \(C = \{C, C^*, C^{**}\}\). Then \(C\) is 1-rich. We now show that \(D_1(C) > 0\), \(D_2(C) > 0\) and that \(C\) yields more revenue than the 1-separating menus \(C^*\) and \(C^{**}\).

Note that \(\frac{x_2}{y_2} > \frac{x_1}{y_1}\) implies

\[
m_2 = \frac{p_2 - p_1}{q_2 - q_1} > \frac{p_1}{q_1} = m_1
\]

where \(m_1\) and \(m_2\) are defined in Step 1 in the proof of Theorem 1.

Since \(F\) is supported on \([a, b]\), \(F\) is strictly increasing on \([a, b]\). Note that \(m_1\) and \(m_2\) belong to \((a, b)\) so \(m_2 > m_1\) implies that \(F(m_2) > F(m_1)\). We have \(D_1(C) = F(m_2) - F(m_1) > 0\). In addition, because \(m_2 = x_2/y_2\) and \((x_2, y_2) \in E^*\) we have \(m_2 < b\),

\(^{22}\)Recall that the function \(f : E^* \rightarrow \mathbb{R}\) is midpoint convex if for all \(e_1, e_2 \in E^*\) we have \(f((e_1 + e_2)/2) \leq (f(e_1) + f(e_2))/2\). A continuous midpoint convex function is convex. We conclude that \(f\) is not midpoint convex.
so $D_2(C) = 1 - F(m_2) > 0$.

Inequality (12) implies that

$$p_2 F\left(\frac{p_2}{q_2}\right) > (p_2 - p_1) F\left(\frac{p_2 - p_1}{q_2 - q_1}\right) + p_1 F\left(\frac{p_1}{q_1}\right).$$

Because $D_1(C) > 0$ and $D_2(C) > 0$, from Step 5 in the proof of Theorem 1, the last inequality implies $\pi(C) > \pi(C^*)$ where $C^* = \{(p_1, q_1)\}$.

The menu $C^* = \{(p_1, q_1)\}$ does not maximize the total transaction value because

$$\pi(C^{**}) = p_1 \left(1 - F\left(\frac{p_1}{q_1}\right)\right) < p_2 \left(1 - F\left(\frac{p_2 - p_1}{q_2 - q_1}\right)\right) + p_1 \left(F\left(\frac{p_2 - p_1}{q_2 - q_1}\right) - F\left(\frac{p_1}{q_1}\right)\right) = \pi(C)$$

where the equalities follow from Step 1 in the proof of Theorem 1.

We conclude that the 2-separating menu $C$ yields more revenue than the 1-separating menus $C^*$ and $C^{**}$.

A.2 Proofs of Section 4

We first prove the following Lemma:

**Lemma 3** Fix an information structure $I = \{B_1, B_2, \ldots, B_n\}$ in $\mathbb{I}(I_o)$. Then, for every positive pricing function $p$ we have

$$\mathbb{E}_{\lambda_{B_i}}(X) = \int_{B_i} x(k(x))^{-1/\alpha} \phi(dx) \int_{B_i} x(k(x))^{-1/\alpha} \phi(dx).$$

The probability measure $\lambda_{B_i}$ is given in Equation (2) in Section 4. That means the expected sellers’ qualities do not depend on the prices.

**Proof of Lemma 3.** Fix an information structure $I = \{B_1, B_2, \ldots, B_n\}$ in $\mathbb{I}(I_o)$.

Given a positive pricing function $p$, the optimal quantity of a seller $x$ in $B_i$, $g(x, p(B_i)) = \arg\max_{h \in \mathbb{R}^+} U(x, h, p(B_i))$ is given by

$$g(x, p(B_i)) = \left(\frac{p(B_i)}{k(x)}\right)^{1/\alpha}. \quad (13)$$

Hence, we have

$$\mathbb{E}_{\lambda_{B_i}}(X) = \int_{B_i} x \lambda_{B_i}(dx) = \frac{\int_{B_i} x g(x, p(B_i)) \phi(dx)}{\int_{B_i} g(x, p(B_i)) \phi(dx)} = \frac{\int_{B_i} x(k(x))^{-1/\alpha} \phi(dx)}{\int_{B_i} (k(x))^{-1/\alpha} \phi(dx)}.$$
Thus, the expected sellers' quality $\mathbb{E}_{\lambda_{B_1}}(X)$ does not depend on the prices when the pricing function is positive. ■

**Proof of Proposition 2.** For the rest of the proof except for Step 3, we fix an information structure $I = \{B_1, B_2, \ldots, B_n\}$ in $\mathbb{I}(I_0)$ and assume that $\mathbb{E}_{\lambda_{B_1}}(X) < \ldots < \mathbb{E}_{\lambda_{B_n}}(X)$ where the expected sellers' quality $\mathbb{E}_{\lambda_{B_i}}(X)$ is given in Lemma 3.

Let $P$ be the set of all pricing functions such that the demand for each set $B_i \in I$, $D_I(B_i, p)$ is greater than 0, each price is greater than 0, and the prices are ordered according to an ascending order. That is,

$$P = \{p \in \mathbb{R}_+^n : D_I(B_i, p) > 0 \text{ for all } i = 1, \ldots, n, 0 < p(B_1) < \ldots < p(B_n)\}.$$

To simplify notation, for the rest of the proof we denote $p_i = p(B_i)$, $p_i' = p'(B_i)$, $s_i(p_i) = S_I(B_i, p(B_i))$, $\mathbb{E}_{\lambda_{B_i}}(X) = q_i$, and $d_i(p) = D_I(B_i, p)$. Note that $p \in P$ implies $0 < q_1 < \ldots < q_n$ (recall that Lemma 3 implies that the expected sellers' quality $q_i$ does not depend on the prices).

Define the function $\psi : P \rightarrow \mathbb{R}$ by

$$\psi(p) = \sum_{i=1}^{n} \frac{p_i^{\frac{1}{a}} \int_{B_i} k(x)^{-1/a} \phi(dx)}{(1 + 1/\alpha)} - p_n + \sum_{i=0}^{n-1} F_2\left(\frac{p_i + 1 - p_i}{q_i + 1 - q_i}\right)(q_i + 1 - q_i) \quad (14)$$

where $F_2(x) = \int_{a}^{x} F(m)dm$ is the antiderivative of $F$ and $q_0 = p_0 = 0$. Note that $p \in P$ implies that for every $1 \leq i \leq n - 1$ we have $a \leq (p_i + 1 - p_i)/(q_i + 1 - q_i) \leq b$ (see Step 1 in the proof of Theorem 1). Because the function $F$ is continuous, the fundamental theorem of calculus implies that the function $F_2$ is differentiable and $F_2' = F$. Thus, $\psi$ is continuously differentiable.

Let $\nabla \psi$ be the gradient of $\psi$ and let $\nabla_i \psi$ be the $i$th element of the gradient. A direct calculation shows that for $1 \leq i \leq n - 1$ we have

$$\nabla_i \psi(p) = p_i^{\frac{1}{a}} \int_{B_i} k(x)^{-1/a} \phi(dx) - F_2'\left(\frac{p_i + 1 - p_i}{q_i + 1 - q_i}\right) + F_2'\left(\frac{p_i - p_i-1}{q_i - q_i-1}\right)$$

$$= p_i^{\frac{1}{a}} \int_{B_i} k(x)^{-1/a} \phi(dx) - F\left(\frac{p_i + 1 - p_i}{q_i + 1 - q_i}\right) + F\left(\frac{p_i - p_i-1}{q_i - q_i-1}\right)$$

$$= s_i(p_i) - d_i(p).$$

The last equality follows from Step 1 and Step 5 in the proof of Theorem 1, the fact that
\( p \in P \), and Equation (13) (see the proof of Lemma 3). Similarly,

\[
\nabla_n \psi(p) = p_n^{1/\alpha} \int_{B_n} k(x)^{-1/\alpha} \phi(dx) - 1 + F\left( \frac{p_n - p_{n-1}}{q_n - q_{n-1}} \right) = s_n(p_i) - d_n(p).
\]

Thus, the excess supply function is given by \( \nabla \psi(p) = (\nabla_1 \psi(p), \ldots, \nabla_n \psi(p)) \) where \( \nabla_i \psi(p) = s_i(p_i) - d_i(p) \) for all \( i \) from 1 to \( n \). Note that \( \nabla \psi(p) = 0 \) implies that \((I, p)\) is implementable.

Our goal is to prove that \((I, p)\) is implementable if and only if \( p \) is the unique minimizer of \( \psi \). To show that \( \psi \) has at most one minimizer we prove that \( \psi \) is strictly convex on the convex set \( P \). We proceed with the following steps:

**Step 1.** The set \( P \) is bounded, convex and open in \( \mathbb{R}^n \).

**Proof of Step 1.** We first show that \( P \) is bounded. Let \( \bar{p} = q_n b \) and let \( p = (p_1, \ldots, p_n) \) be a vector such that \( p_i > \bar{p} \) for some \( 1 \leq i \leq n \). Then

\[
mq_i - p_i \leq bq_n - p_i < bq_n - \bar{p}.
\]

Hence \( d_i(p) = 0 \). That is, \( p \) does not belong to \( P \). We conclude that \((\bar{p}, \ldots, \bar{p})\) is an upper bound of \( P \) under the standard product order on \( \mathbb{R}^n \). Clearly, \( P \) is bounded from below. Hence, \( P \) is bounded.

We now show that \( P \) is a convex set in \( \mathbb{R}^n \). Let \( p, p' \in P \) and \( 0 < \lambda < 1 \).

We need to show that \( \lambda p + (1 - \lambda) p' \in P \). First note that

\[
0 < \lambda p_1 + (1 - \lambda) p'_1 < \ldots < \lambda p_n + (1 - \lambda) p'_n
\]

so we only need to show that \( d_i(\lambda p + (1 - \lambda) p') > 0 \) for all \( i = 1, \ldots, n \). Let \( 1 \leq i \leq n - 1 \). Because \( d_i(p) > 0 \) and \( d_i(p') > 0 \) we have \( F\left( \frac{p_{i+1} - p_i}{q_{i+1} - q_i} \right) - F\left( \frac{p_i - p_{i-1}}{q_i - q_{i-1}} \right) > 0 \) and \( F\left( \frac{p_{i+1} - p_i}{q_{i+1} - q_i} \right) - F\left( \frac{p'_i - p'_{i-1}}{q_i - q_{i-1}} \right) > 0 \). Strict monotonicity of \( F \) on its support implies \( \frac{p_{i+1} - p_i}{q_{i+1} - q_i} > \frac{p'_i - p'_{i-1}}{q_i - q_{i-1}} \) and \( \frac{p'_{i+1} - p'_i}{q_{i+1} - q_i} > \frac{p'_{i} - p'_{i-1}}{q_i - q_{i-1}} \). Hence,

\[
\frac{\lambda p_{i+1} + (1 - \lambda) p'_{i+1} - (\lambda p_i + (1 - \lambda) p'_i)}{q_{i+1} - q_i} > \frac{\lambda p_i + (1 - \lambda) p'_i - (\lambda p_{i-1} + (1 - \lambda) p'_{i-1})}{q_i - q_{i-1}}.
\]

Using again the strict monotonicity of \( F \) we conclude that

\[
F\left( \frac{\lambda p_{i+1} + (1 - \lambda) p'_{i+1} - (\lambda p_i + (1 - \lambda) p'_i)}{q_{i+1} - q_i} \right) - F\left( \frac{\lambda p_i + (1 - \lambda) p'_i - (\lambda p_{i-1} + (1 - \lambda) p'_{i-1})}{q_i - q_{i-1}} \right) > 0.
\]
That is, \( d_i(\lambda p + (1 - \lambda)p') > 0 \). Similarly we can show that \( d_n(\lambda p + (1 - \lambda)p') > 0 \). Thus, \( P \) is a convex set.

Because \( d_i(p) \) is continuous for all \( 1 \leq i \leq n \), it is immediate that the set \( P \) is an open set in \( \mathbb{R}^n \).

**Step 2.** The function \( \psi \) is strictly convex on \( P \).

**Proof of Step 2.** We claim that \( \nabla \psi \) is strictly monotone on \( P \), i.e., for all \( p = (p_1, \ldots, p_n) \) and \( p' = (p'_1, \ldots, p'_n) \) that belong to \( P \) and satisfy \( p \neq p' \), we have

\[
\langle \nabla \psi(p) - \nabla \psi(p'), p - p' \rangle > 0
\]

where \( \langle x, y \rangle := \sum_{i=1}^n x_i y_i \) denotes the standard inner product between two vectors \( x \) and \( y \) in \( \mathbb{R}^n \). Because \( P \) is a convex set it is well known that \( \nabla \psi \) is strictly monotone on \( P \) if and only if \( \psi \) is strictly convex on \( P \).

Let \( p, p' \in P \) and assume that \( p \neq p' \).

Because \( g \) is strictly increasing in \( p_i \), \( k \) is a positive function, and \( \phi(B_i) > 0 \), the supply function \( s_i(p_i) = p_i^{1/\alpha} \int_{B_i} k(x)^{-1/\alpha} \phi(dx) \) is strictly increasing in the price \( p_i \). Thus, \( s_i(p_i) > s_i(p'_i) \) if and only if \( p_i > p'_i \). Combining the last inequality with the fact that \( p \neq p' \) implies

\[
\sum_{i=1}^n (p_i - p'_i)(s_i(p_i) - s_i(p'_i)) > 0.
\]

Let \( p_0 = p'_0 = 0 \). We have

\[
\sum_{i=1}^n (p_i - p'_i)(d_i(p) - d_i(p')) = \sum_{i=1}^{n-1} (p_i - p'_i) \left( F \left( \frac{p_{i+1} - p_i}{q_{i+1} - q_i} \right) - F \left( \frac{p_i - p_{i-1}}{q_i - q_{i-1}} \right) \right) \\
- \sum_{i=1}^{n-1} (p_i - p'_i) \left( F \left( \frac{p'_{i+1} - p'_i}{q_{i+1} - q_i} \right) - F \left( \frac{p'_i - p'_{i-1}}{q_i - q_{i-1}} \right) \right) \\
+ (p_n - p'_n) \left( F \left( \frac{p'_n - p'_{n-1}}{q_n - q_{n-1}} \right) - F \left( \frac{p_n - p_{n-1}}{q_n - q_{n-1}} \right) \right) \\
= \sum_{i=1}^n (p_i - p_{i-1} - (p'_i - p'_{i-1})) \left( F \left( \frac{p'_i - p'_{i-1}}{q_i - q_{i-1}} \right) - F \left( \frac{p_i - p_{i-1}}{q_i - q_{i-1}} \right) \right) \\
\leq 0.
\]
The last inequality follows from the monotonicity of $F$. Thus,

$$
\langle \nabla \psi(p) - \nabla \psi(p'), p - p' \rangle = \sum_{i=1}^{n} (s_i(p_i) - d_i(p) - (s_i(p_i') - d_i(p'))(p_i - p_i') 
$$

$$
= \sum_{i=1}^{n} (p_i - p_i')(s_i(p_i) - s_i(p_i')) - \sum_{i=1}^{n} (p_i - p_i')(d_i(p) - d_i(p')) 
$$

$$
> 0.
$$

We conclude that $\nabla \psi$ is strictly monotone. Hence, $\psi$ is strictly convex.

**Step 3.** $(I, p)$ is implementable if and only if $p$ is the unique minimizer of $\psi$.

**Proof of Step 3.** Suppose that $(I, p)$ is implementable where $I = \{B_1, B_2, \ldots, B_n\}$ and $p = (p(B_1), \ldots, p(B_n))$. Let $D = \{D_I(B_i, p)\}_{B_i \in I}$, $S = \{S(B_i, p(B_i))\}_{B_i \in I}$, and $\lambda = \{\lambda_{B_i}\}_{B_i \in I}$ be an equilibrium under $(I, p)$.

Because $(I, p)$ is implementable we have $p(B_i) > 0$ for all $B_i \in I$ and

$$
D_I(B_i, p) = S_I(B_i, p(B_i)) = \int_{B_i} g(x, p(B_i)) \phi(dx) > 0
$$

where the last inequality follows because $g$ is positive (see the proof of Lemma 3) and $\phi(B_i) > 0$. We can assume without loss of generality that $E_{\lambda_{B_1}}(X) < \ldots < E_{\lambda_{B_n}}(X)$. To see this, note that if $E_{\lambda_{B_i}}(X) = E_{\lambda_{B_j}}(X)$ for some $i < j$ then $\min\{D_I(B_i, p), D_I(B_j, p)\} = 0$ which contradicts the implementability of $(I, p)$. Thus, relabeling if needed, we can assume $E_{\lambda_{B_i}}(X) < E_{\lambda_{B_j}}(X)$ for all $i < j$. This implies that $p(B_i) < p(B_j)$ for all $i < j$.

Thus, $p$ belongs to $P$. Hence, $\nabla \psi(p) = 0$ for some $p \in P$. Because $\psi$ is strictly convex on the convex set $P$, there is at most one $p \in P$ such that $\nabla \psi(p) = 0$. We conclude that for every information structure $I \in \mathbb{I}(I_o)$ there exists at most one pricing function $p$ such that $(I, p)$ is implementable.

Furthermore, because the set $P$ is an open set, we have $\nabla \psi(p) = 0$ if and only if $p$ is the unique minimizer of the strictly convex function $\psi$. We conclude that $(I, p)$ is implementable if and only if $p$ is the unique minimizer of $\psi$. ■

**Proof of Lemma 2.** Let $I = \{B\}$ be a 1-separating information structure and assume that $B \neq A_i$ for all $A_i \in I_o$. Thus, $B$ is a union of at least two elements of $I_o$. Let $k$ be highest index among these elements. Hence, $E_{\lambda_{A_j}}(X) < E_{\lambda_{A_k}}(X)$ for all $A_j \subseteq B$. We have
\[\mathbb{E}_{\lambda_B}(X) = \frac{\int_B x(k(x))^{-1/\alpha} \phi(dx)}{\int_B (k(x))^{-1/\alpha} \phi(dx)} = \frac{\sum_{A_i:B \subseteq B} \int_{A_i} x(k(x))^{-1/\alpha} \phi(dx)}{\sum_{A_i:A_i \subseteq B} \int_{A_i} (k(x))^{-1/\alpha} \phi(dx)} \leq \frac{\int_{A_k} x(k(x))^{-1/\alpha} \phi(dx)}{\int_{A_k} (k(x))^{-1/\alpha} \phi(dx)} = \mathbb{E}_{\lambda_{A_k}}(X).\]

The first and last equalities follow from Lemma 3. The inequality follows from the elementary inequality \(\sum_{i=1}^{n} x_i / \sum_{i=1}^{n} y_i \leq \max_{1 \leq i \leq n} x_i / y_i\) for positive numbers \(x_1, \ldots, x_n\) and \(y_1, \ldots, y_n\).

Assume that \((I, p(B))\) is implementable and that it induces the menu \(((p(B), \mathbb{E}_{\lambda_B}(X)))\). Then \(\mathbb{E}_{\lambda_B}(X) \leq \mathbb{E}_{\lambda_{I}}(X)\).

We claim that \(p(B) < p(A_k)\) where \(p(A_k)\) is the (unique) equilibrium price under the information structure \(A_k\). To see this, note that

\[S_{A_k}(B, p(B)) = \int_{A_k} \left(\frac{p(B)}{k(x)}\right)^{1/\alpha} \phi(dx) \leq \int_{B} \left(\frac{p(B)}{k(x)}\right)^{1/\alpha} \phi(dx) = S_I(B, p(B)) = D_I(B, p(B)) = 1 - F\left(\frac{p(B)}{\mathbb{E}_{\lambda_B}(X)}\right) \leq 1 - F\left(\frac{p(B)}{\mathbb{E}_{\lambda_{A_k}}(X)}\right) = D_{A_k}(B, p(B)).\]

The first inequality follows from the facts that \(B \supseteq A_k\) and \(\phi(B \setminus A_k) > 0\). The second inequality follows from the fact that \(F\) is increasing. Hence, the demand exceeds the supply under the price \(p(B)\). From Step 2 in the proof of Theorem 2 we have \(p(B) < p(A_k)\). Thus, \(B \neq B^H\) which proves the Lemma.

**Proof of Theorem 2.** Let 1. For any two 1-separating information structures \(\{B_1\}\) and \(\{B_2\}\) we have \(p^M(B_2) \geq p^M(B_1)\) whenever \(\mathbb{E}_{\lambda_{B_2}}(X) \geq \mathbb{E}_{\lambda_{B_1}}(X)\).
**Proof of Step 1.** Because $F(m)m$ is strictly convex, $p^M(B)$ is single-valued for every 1-separating information structure $B$. In addition, we clearly have $a \leq p^M(B)/E_{\lambda_B}(X) \leq b$.

Assume in contradiction that $p^M(B_2) < p^M(B_1)$ and $E_{\lambda_{B_2}}(X) \geq E_{\lambda_{B_1}}(X)$. Then $p^M(B_2)/E_{\lambda_{B_2}}(X) < p^M(B_1)/E_{\lambda_{B_1}}(X)$. The first order conditions and the fact that the strict convexity of $F(m)m$ on $[a,b]$ implies that the function $F(m) + mf(m)$ is strictly increasing on $[a,b]$ imply

$$0 = 1 - \left( F\left( \frac{p^M(B_2)}{E_{\lambda_{B_2}}(X)} \right) + \frac{p^M(B_2)}{E_{\lambda_{B_2}}(X)} f\left( \frac{p^M(B_2)}{E_{\lambda_{B_2}}(X)} \right) \right)$$

which is a contradiction.

**Step 2.** Let $\{B\}$ be a 1-separating information structure and let $\{(p(B),E_{\lambda_B}(X))\} \in \varphi^Q(\{B\})$. Then for every $p > 0$ we have $S_{\{B\}}(B,p) \geq D_{\{B\}}(B,p)$ if and only if $p \geq p(B)$.

**Proof of Step 2.** Assume in contradiction that $p(B) > p > 0$ and $S_{\{B\}}(B,p) \geq D_{\{B\}}(B,p)$. Recall that the sellers’ expected quality $E_{\lambda_B}(X)$ does not depend on the price (see Lemma 3). We have

$$1 - F\left( \frac{p}{E_{\lambda_B}(X)} \right) = D_{\{B\}}(B,p) \leq S_{\{B\}}(B,p)$$

$$= \int_B g(x,p)\phi(dx)$$

$$< \int_B g(x,p(B))\phi(dx)$$

$$= 1 - F\left( \frac{p(B)}{E_{\lambda_B}(X)} \right)$$

which is a contradiction to the fact that $F$ is increasing. The strict inequality follows because $g$ is strictly increasing in the price and $\phi(B) > 0$ (see the proof of Lemma 3). The last equality follows from the fact that $\{(p(B),E_{\lambda_B}(X))\} \in \varphi^Q(\{B\})$. This proves that $S_{\{B\}}(B,p) \geq D_{\{B\}}(B,p)$ implies $p \geq p(B)$. The other direction is proven in a similar manner.

**Step 3.** We have

$$S_{\{B\}}(B,p^M(B)) \geq D_{\{B\}}(B,p^M(B)).$$

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for every 1-separating information structure \( \{B\} \) that belongs to \( \mathcal{M} \setminus \{A_1\} \).

**Proof of Step 3.** Let \( \{B\} \in \mathcal{M} \setminus \{A_1\} \) and \( \{(p(B), \mathbb{E}_{\lambda_B}(X))\} \in \varphi^Q(\{B\}) \). From the definition of the information structure \( \{B^H\} \) we have \( p(B^H) \geq p(B) \). Because the information structure \( \{B\} \) is maximal, the last inequality implies \( \mathbb{E}_{\lambda_B}(X) \geq \mathbb{E}_{\lambda_{B^H}}(X) \).

Thus, Step 1 implies \( p^M(B) \geq p^M(B^H) \).

From the Theorem’s assumption and Step 2 we also have \( p^M(B^H) \geq p(B^H) \). We conclude that

\[
p(B) \leq p(B^H) \leq p^M(B^H) \leq p^M(B).
\]

Using Step 2 again we have \( S_{\{B\}}(B, p^M(B)) \geq D_{\{B\}}(B, p^M(B)) \).

**Step 4.** \( C^Q \) is 1-rich.

**Proof of Step 4.** Let \( (I, \mathbf{p}) \) be implementable where \( I = \{B_1, B_2, \ldots, B_n\} \). Let \( C = \{(p(B)), \mathbb{E}_{\lambda_{B_1}}(X)), \ldots, (p(B_n)), \mathbb{E}_{\lambda_{B_n}}(X))\} \) be the menu that is induced by \( (I, \mathbf{p}) \).

Suppose that \( (D, S, \lambda) \) implements \( (I, \mathbf{p}) \). We can assume that \( D(B_i, \mathbf{p}) > 0 \) for all \( B_i \in I \) and \( 0 < p(B_1) < \ldots < p(B_n) \) (see the proof of Proposition 2). Note that \( D(B_i, p) > 0 \) for \( B_i \in I \) implies \( 0 < \mathbb{E}_{\lambda_{B_1}}(X) < \ldots < \mathbb{E}_{\lambda_{B_n}}(X) \).

Consider the 1-separating information structure \( I' = \{B_n\} \).

We claim that there exists a \( p^q(B_n) \geq p(B_n) \) such that \( (I', p^q(B_n)) \) is implementable and \( (I', p^q(B_n)) \) induces the menu \( \{(p^q(B_n), \mathbb{E}_{\lambda_{B_n}}(X))\} \).

From Step 1 in the proof of Theorem 1, we have \( D_{I'}(B_n, p(B_n)) = 1 - F \left( \frac{p(B_n)}{\mathbb{E}_{\lambda_{B_n}}(X)} \right) \).

Note that there exists a \( \bar{p} > p(B_n) \) such that \( D_{I'}(B_n, \bar{p}) = 0 \) (for example we can choose \( \bar{p} = \mathbb{E}_{\lambda_{B_n}}(X)b \)).

Define the excess demand function \( \tau : [p(B_n), \bar{p}] \to \mathbb{R} \) by \( \tau(\cdot) = D_{I'}(B_n, \cdot) - S_{I'}(B_n, \cdot) \). From the definition of \( \bar{p} \) we have \( \tau(\bar{p}) < 0 \).

Note that

\[
\tau(p(B_n)) = D_{I'}(B_n, p(B_n)) - S_{I'}(B_n, p(B_n)) \\
= D_{I'}(B_n, p(B_n)) - S_I(B_n, p(B_n)) \\
\geq D_I(B_n, \mathbf{p}) - S_I(B_n, p(B_n)) = 0
\]

The first equality follows from the definition of \( \tau \). The second equality follows from the fact that \( S_I(B_n, p(B_n)) = S_{I'}(B_n, p(B_n)) = \int_{B_n} g(x, p(B_n))\phi(dx) \), i.e., seller \( x \)'s optimal quantity decision does not change when the information structure changes. The inequality follows from the definition of the demand function. The last equality follows from the fact that \( (I, \mathbf{p}) \) is implementable.
Because the distribution function $F$ and the optimal quantity function $g$ are continuous, the excess demand function $\tau$ is continuous on $[p(B_n), \bar{p}]$. Thus, from the intermediate value theorem, there exists a $p^q(B_n)$ in $[p(B_n), \bar{p}]$ such that $\tau(p^q(B_n)) = 0$. We conclude that $(I', p^q(B_n))$ is implementable and that $p^q(B_n) \geq p(B_n)$.

We now show that there exists a 1-separating menu $C \in \mathcal{C}^Q$ that yields more revenue than then menu $\{(p(B_n), E_{\lambda n}(X))\}$. This implies that $\mathcal{C}^Q$ is 1-rich (see Definition 1). We consider two cases.

**Case 1.** $B_n \in \mathcal{M}$. Note that $B_n \neq A_1$ because $0 < E_{\lambda n}(X) < \ldots < E_{\lambda B_n}(X)$.

From Step 2 and Step 3 we have $p^q(B_n) \leq p^M(B_n)$. We conclude that $p(B_n) \leq p^q(B_n) \leq p^M(B_n)$. The convexity of $F(m)m$ implies that $p \left(1 - F \left(\frac{p}{E_{\lambda B_n}(X)}\right)\right)$ is concave in $p$. Thus,

$$p(B_n)D(B_n, p(B_n)) = p(B_n) \left(1 - F \left(\frac{p(B_n)}{E_{\lambda B_n}(X)}\right)\right) \leq p^q(B_n) \left(1 - F \left(\frac{p^q}{E_{\lambda B_n}(X)}\right)\right) = p^q(B_n)D(B_n, p^q(B_n)).$$

We conclude that the menu $\{(p^q(B_n), E_{\lambda B_n}(X))\} \in \mathcal{C}^Q$ yields more revenue than the menu $\{(p(B_n), E_{\lambda B_n}(X))\}$.

**Case 2.** $B_n \neq \mathcal{M}$. Because $\mathcal{M}$ is not empty, there exists some $\{B\} \in \mathcal{M}$ such that $\{(p(B), E_{\lambda B}(X))\} \in \varphi^Q(\{B\})$, $p(B) \geq p(B_n)$, and $E_{\lambda B}(X) \geq E_{\lambda B_n}(X)$. From Step 2 and Step 3 we have $p(B) \leq p^M(B)$.

Hence, we have $p(B_n) \leq p(B) \leq p^M(B)$ which implies

$$p(B_n) \left(1 - F \left(\frac{p(B_n)}{E_{\lambda B_n}(X)}\right)\right) \leq p(B) \left(1 - F \left(\frac{p(B)}{E_{\lambda B}(X)}\right)\right) \leq p(B) \left(1 - F \left(\frac{p(B)}{E_{\lambda B}(X)}\right)\right).$$

That is, the menu $\{(p(B), E_{\lambda B}(X))\} \in \mathcal{C}^Q$ yields more revenue than the menu $\{(p(B_n), E_{\lambda B_n}(X))\}$.

We conclude that $\mathcal{C}^Q$ is 1-rich. Theorem 1 implies that the optimal menu is 1-separating, i.e., the optimal information structure is 1-separating.

From case 2 above, for every 1-separating menu $C$ that does not belong to $\mathcal{M}$ there exists a 1-separating menu in $\mathcal{M}$ that yields more revenue than $C$. We conclude that the optimal 1-separating information structure belongs to $\mathcal{M}$. ■

**Proof of Corollary 3.** Let $I = \{B\}$ be a 1-separating information structure and
assume that \( B \neq A_l \). Assume that \((I, p(B))\) is implementable and that it induces the menu \( \{(p(B), E_{\lambda_B}(X))\} \). From a similar argument to the arguments in the proof of Lemma 2 we have \( E_{\lambda_B}(X) \leq E_{\lambda_{A_l}}(X) \). Let \( p(A_l) \) be the (unique) equilibrium price under the information structure \( \{A_l\} \). We have

\[
S_{A_l}(B, p(B)) = \int_{A_l} \left( \frac{p(B)}{k(x)} \right)^{1/\alpha} \phi(dx)
\]

\[
\leq \int_B \left( \frac{p(B)}{k(x)} \right)^{1/\alpha} \phi(dx)
\]

\[
\leq D_{A_l}(B, p(B)).
\]

The first inequality follows from inequality (6) and the fact that \( B \supseteq A_k \) for some set \( A_k \in I_o \). The second inequality follows from the same arguments as the arguments in the proof of Lemma 2. From Step 2 in the proof of Theorem 2 we have \( p(B) \leq p(A_l) \).

Thus, the set of maximal information structures \( \mathcal{M} \) consists of one element \( \{A_l\} \). From Theorem 2 the optimal information structure is 1-separating. Theorem 2 also implies that the optimal information structure belongs to \( \mathcal{M} \). Thus, \( A_l \) is the optimal information structure. ■

**Proof of Theorem 3.** Let \( I = \{B_1, \ldots, B_n\} \) be an information structure and suppose that \( C \in \varphi^P(I) \).

(i) Let \( p = (p(B_1), \ldots, p(B_n)) \) be the equilibrium price vector associated with \( C \). We claim that \( p(B_i) = c(G_i) \) where \( L(I) = \{G_1, \ldots, G_n\} \).

If \( p(B_i) < c(G_i) \) then for every seller \( x \in B_i \) we have \( U(x, H(B_i), p(B_i), p) < 0 \) so the mass of sellers that participate in the platform equals to 0 which contradicts the implementability of \( I \). If \( p(B_i) > c(G_i) \) then the sellers’ pricing decisions are not optimal. Sellers in \( G_i \subseteq B_i \) can decrease their price and increase their utility. Thus, \( I \) is not implementable. We conclude that \( p(B_i) = c(G_i) \) for all \( B_i \in I \).

For all \( B_i \in I \), only the sellers \( G_i \subseteq B_i \) participate in the platform under the equilibrium price vector \( p = (c(G_1), \ldots, c(G_n)) \). Thus, the proof of part (i) follows.

(ii) To prove that \( C^P \) is 1-rich it is enough to prove that \( (c(G_n), E_{\lambda_{G_n}}(X)) \in \varphi^P(\{B_n\}) \). First note that \( D_{\{B_n\}}(B_n, c(G_n)) \geq D_I(B_n, (c(G_1), \ldots, c(G_n))) > 0 \) (see the proof of Theorem 2). Furthermore, it is optimal for all the sellers in \( G_n \subseteq B_n \) to participate in the platform and for all the sellers in \( B_n \setminus G_n \) to not participate under the price \( c(G_n) \).

So \( E_{\lambda_{G_n}}(X) \) is the sellers’ expected quality given the sellers’ optimal entry decisions and the price \( c(G_n) \). Also, it is easy to see that the price \( c(G_n) \) maximizes the participating
sellers’ utility. From the quantity allocation function $h_I$ it follows immediately that the market clearing condition is satisfied. We conclude that $(c(G_n), \mathbb{E}_{\lambda G_n}(X)) \in \varphi^P(\{B_n\})$.

References


